The search for a stochastic background of gravitational waves

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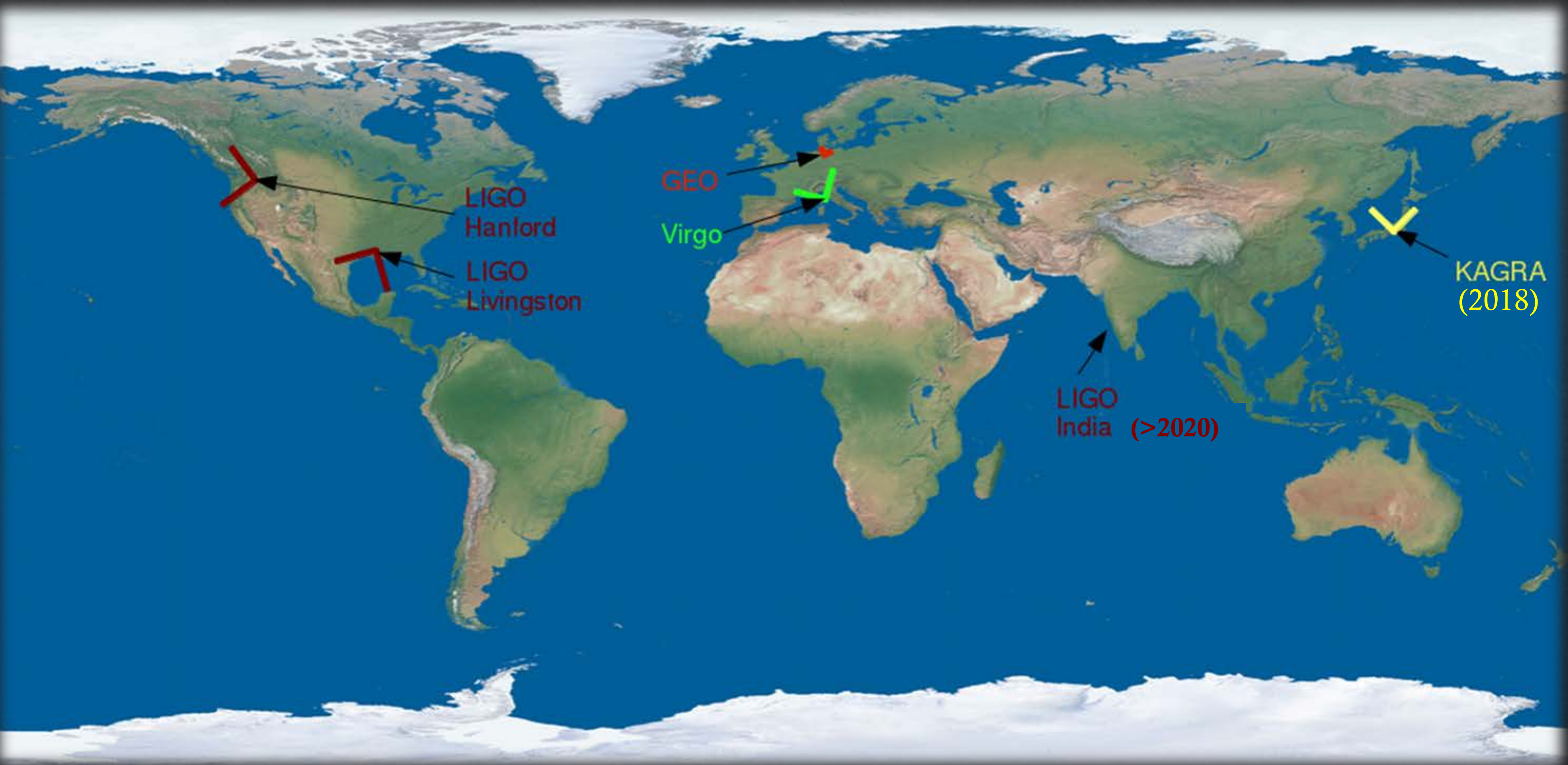
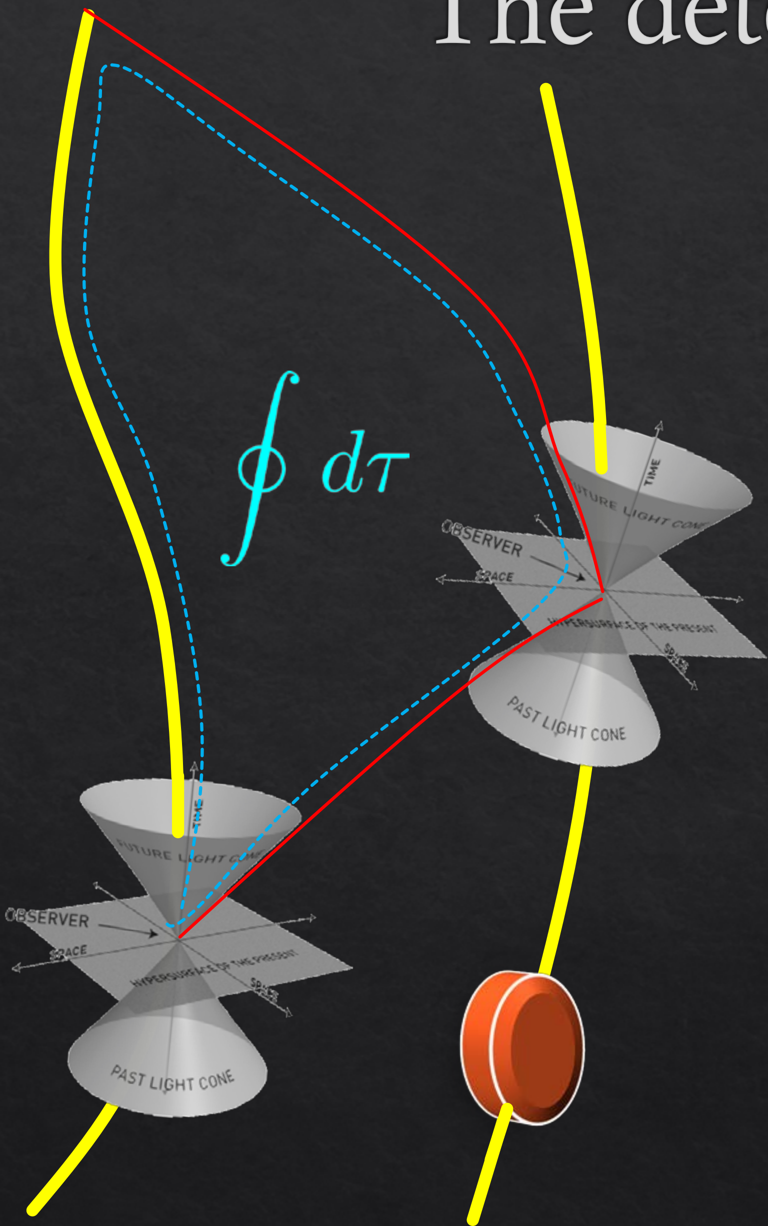
Workshop «String Theory and Inflation»
20 September 2016
Physics Department, University of Rome Tor Vergata

The Gravitational Wave Detectors Network



- Currently LIGO-H & LIGO-V operative (first scientific run ended, now stopped until the end of the year for upgrade)
- Virgo will join at the next scientific run.
- This will be important to improve sentitivity, coverage and estimation of parameters

The detector principle

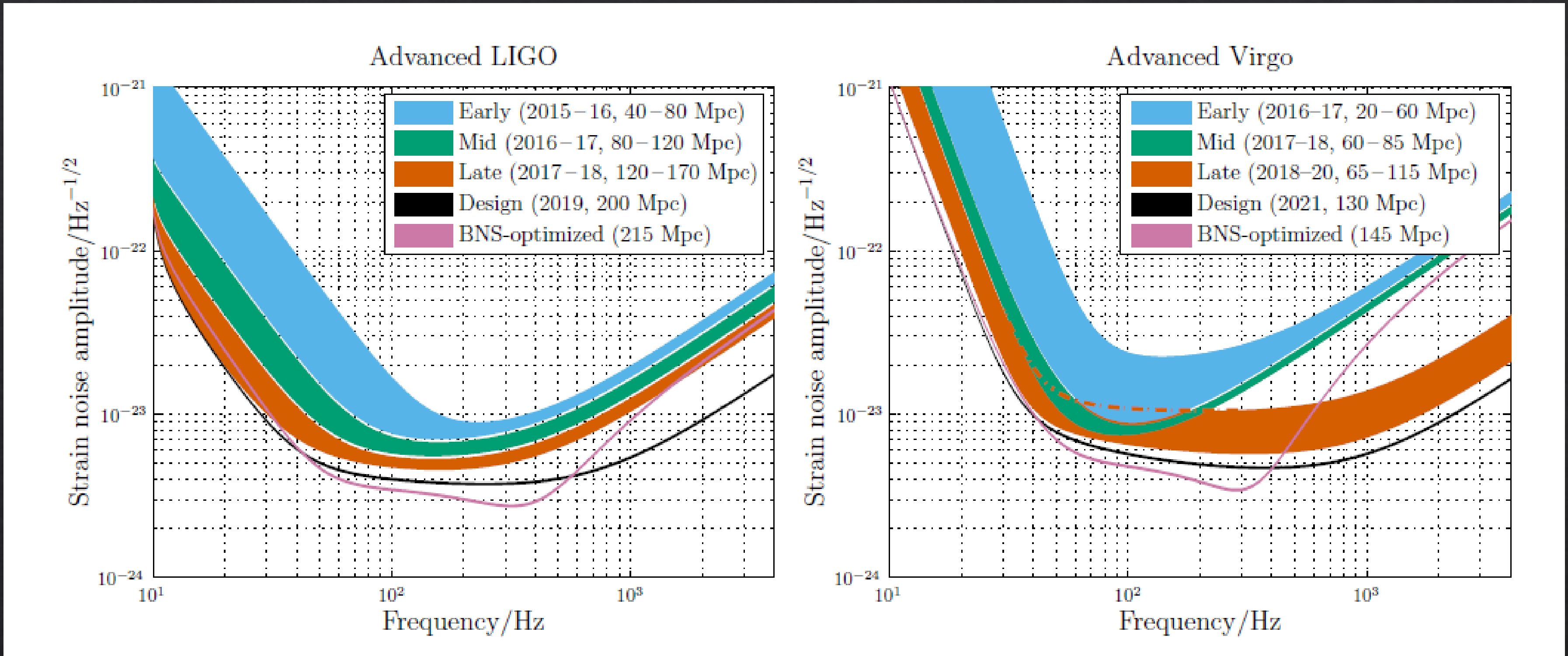


- Description can be coordinate dependent
- Physical observable is not
- Intuitive picture (when $\lambda_{GW} \gg \mathcal{F}L$): tidal force

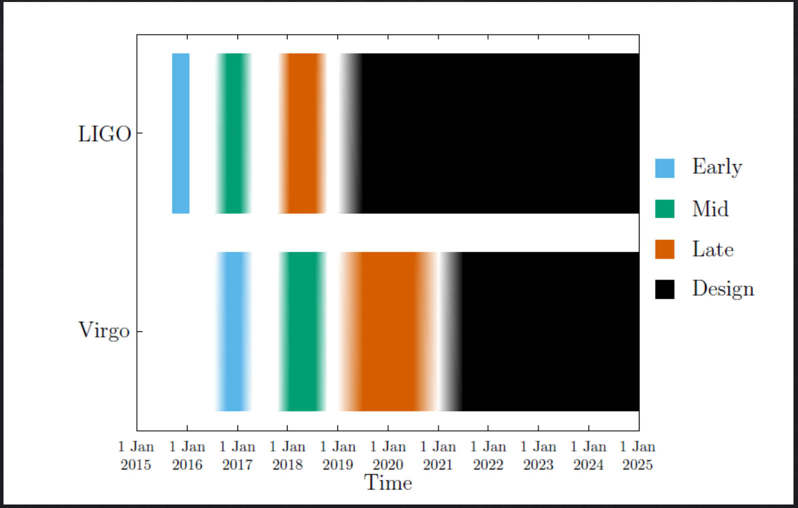
$$F_i = \frac{1}{2} m \frac{d^2 h_{ij}^{TT}}{dt^2} L_j$$

on the end mirrors.

The observing scenario



- Plan: a series of scientific runs with intermediate commissioning interruptions
- Sensitivity will increase in steps toward the design one
- Quite successful until now….



http://www.livingreviews.org/lrr-2016-1

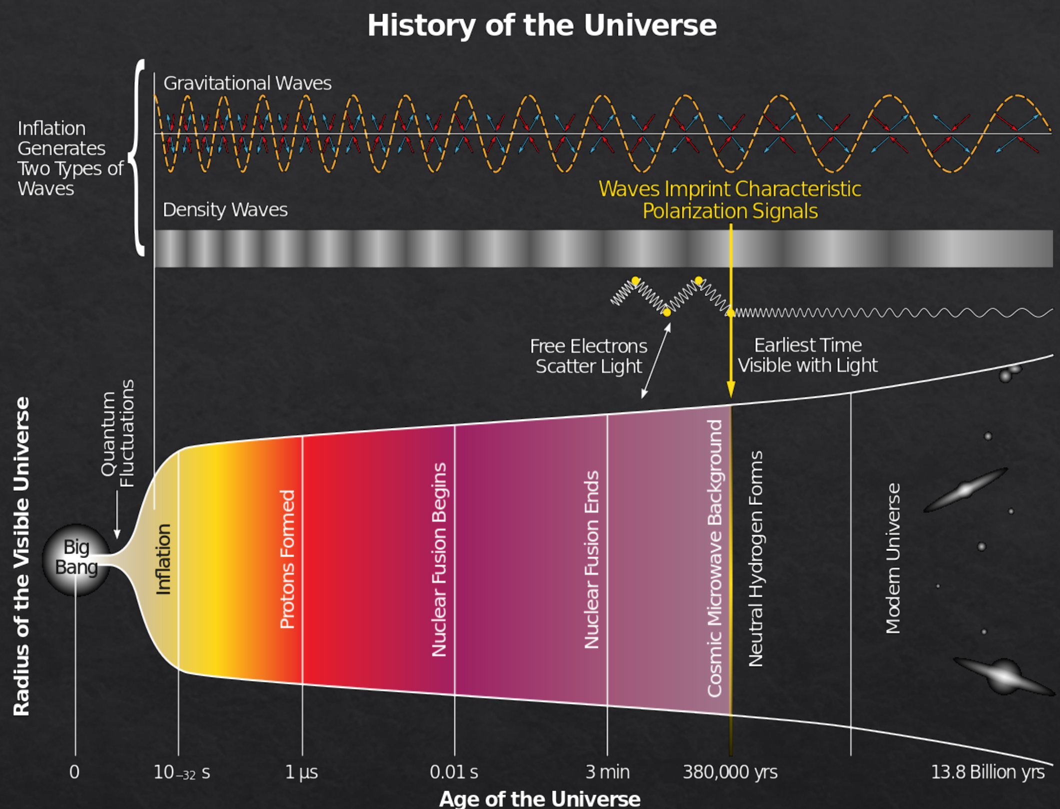
Stochastic background in a nutshell

A stochastic background can be seen as

- **a GW field which evolves from an initially random configuration**
- **the result of a superposition of many uncorrelated and unresolved sources**

Two different kinds:

- **Cosmological:**
  - signature of the early Universe: coupling of gravitational field is small!
  - *inflation, cosmic strings, phase transitions…*
- **Astrophysical:**
  - sources since the beginning of stellar activity
  - *compact binaries, supernovae, rotating NSs, core-collapse to NSs or BHs, supermassive BHs…*



Typical «first approximations» :

1) **Gaussian, because sum of many contributions**
2) **Stationary, because physical time scales much larger than observational ones**
3) **Isotropic (at least for cosmological backgrounds)**
4) **Unpolarized**

If these are true, SB is completely described by its power spectrum

Description (simplest model)

1. Correlation between GW modes

$$< h^{\star}_{A}(f,\hat{\Omega},\psi) h_B(f',\hat{\Omega}',\psi') >= \delta_{AB}\delta(f-f')\frac{\delta^2(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(\psi-\psi')}{2\pi}\frac{1}{2}S_h(f)$$

2. Connection with GW energy density

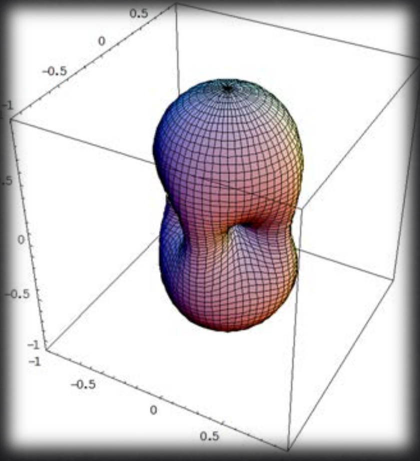
$$h_0^2\Omega_{gw}(f) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\log f} = \frac{4\pi^2 h_0^2}{3H_0^2} f^3 S_h(f)$$

3. Strain at the detector: sum over modes

$$h_{ij}(t,\vec{r}) = \sum_{P=+,\times}\int_{S^2} d\hat{\Omega}\,\varepsilon^P_{ij}(\hat{\Omega})\int_{-\infty}^{\infty} df\,\tilde{h}_P(f,\hat{\Omega})e^{i2\pi f(t-\hat{\Omega}\cdot\vec{r})}$$

4. Signal at the detector: projection on the detector's tensor

$$h(t,r) = D^{ij}h^{ij}(t,r)$$



How it is possible to (directly) detect it

- We have a vector-valued Gaussian stochastic process
- $\underline{x}_i = \left(x_i^{\text{Virgo}}, x_i^{\text{Hanford}}, \ldots \right) \qquad dP = \mathcal{N} \prod_f \exp\left(-\frac{1}{2} \tilde{\underline{x}}_f^{+} C^{-1}(f)\, \tilde{\underline{x}}_f \right) d\tilde{\underline{x}}_f$

$$\underline{x}_i = \underline{h}_i + \underline{n}_i \qquad \langle \underline{n}_i \otimes \underline{n}_j \rangle = I c_{ij}$$

- We must discriminate between two hypothesis:

$\mathcal{H}_0$

$$C = C_N = \begin{pmatrix} S_N^{\text{Virgo}} & 0 & \cdots & 0 \\ 0 & S_N^{\text{Hanford}} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & S_N^{\text{Kagra}} \end{pmatrix}$$

$\mathcal{H}_1$

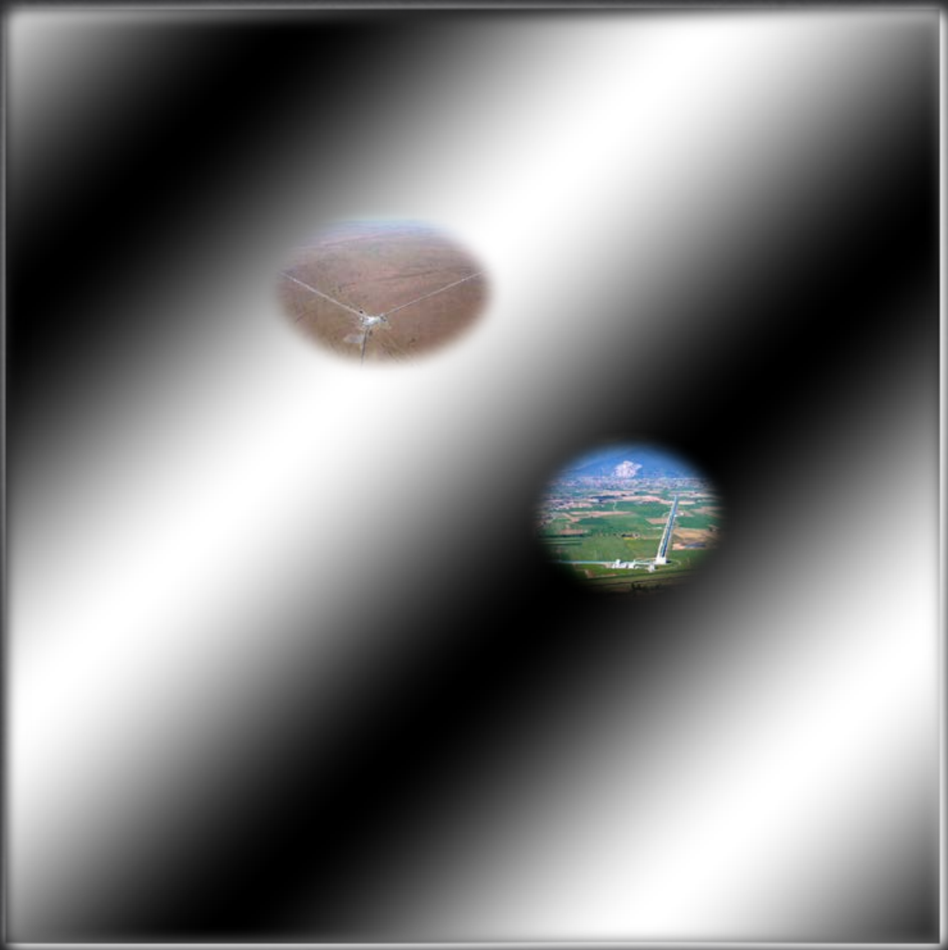
$$C = C_N + S_h \begin{pmatrix} \gamma_{V,V} & \gamma_{V,H} & \cdots & \gamma_{V,K} \\ \gamma_{V,H} & \gamma_{H,H} & \cdots & \gamma_{H,K} \\ \vdots & \vdots & \ddots & \cdots \\ \gamma_{V,K} & \gamma_{H,K} & \cdots & \gamma_{K,K} \end{pmatrix}$$

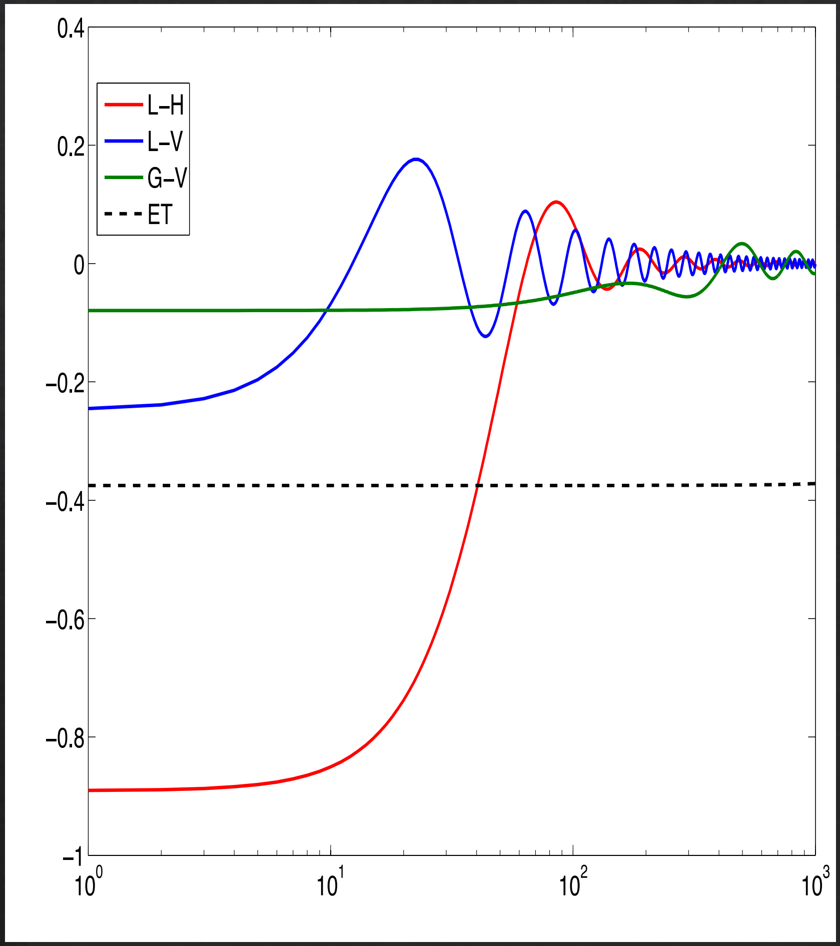
- Optimal statistic (two detectors):

$$Y = \lambda \int df \frac{\tilde{x}_1^{\star}(f) \tilde{x}_2(f) \gamma_{12}(f) S_h(f)}{S_{n,1}(f) S_{n,2}(f)}$$

Overlap reduction function (a.k.a. coherence)

$$\mathrm{SNR}_Y^2 := \frac{\mu_Y^2}{\sigma_Y^2} = 2T \int_0^\infty S_h^2(f) \frac{\gamma_{12}^2(f)}{S_{n,1}(f) S_{n,2}(f)} df$$





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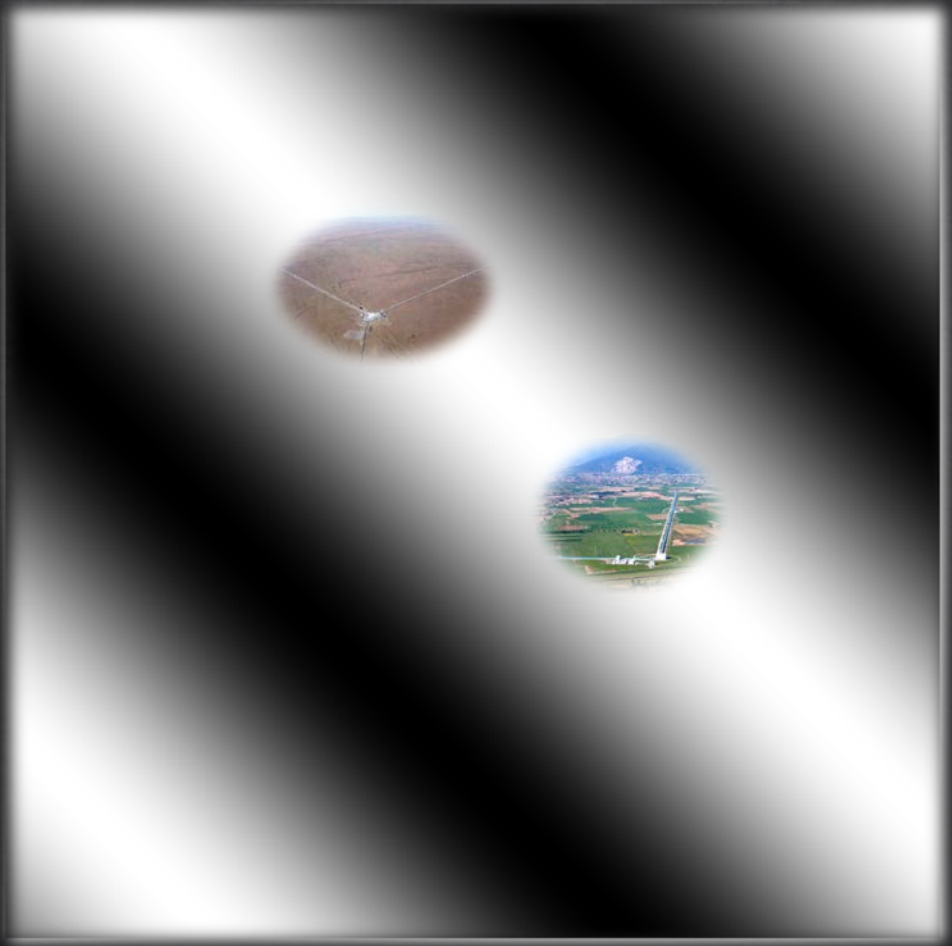
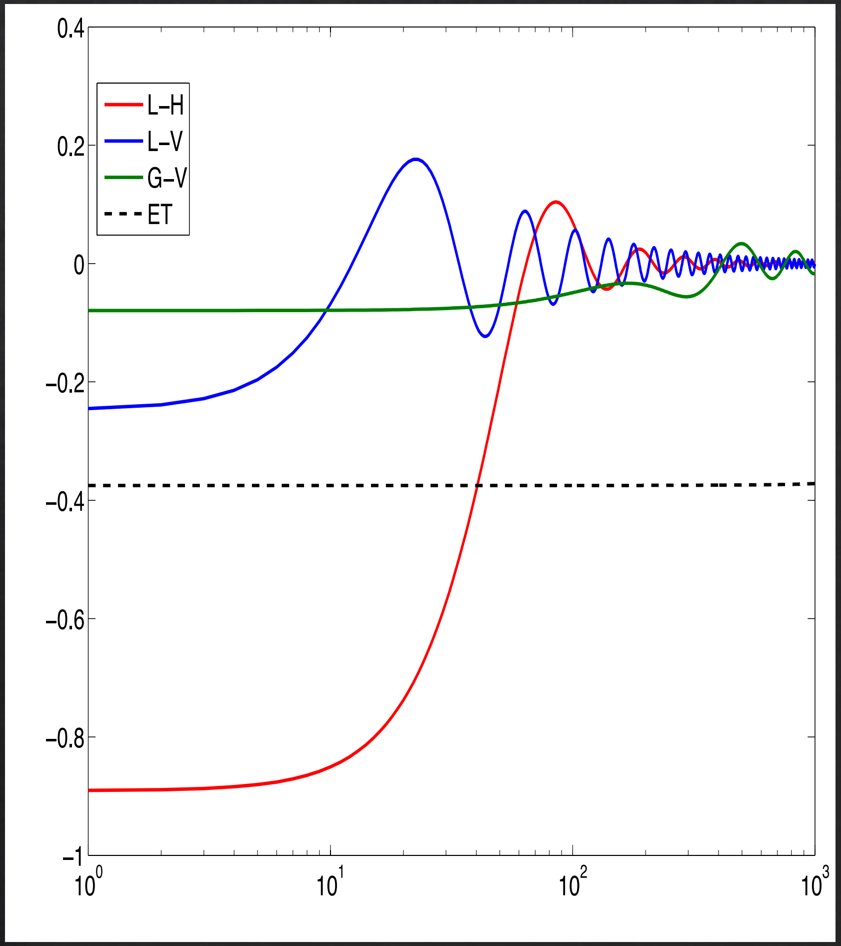
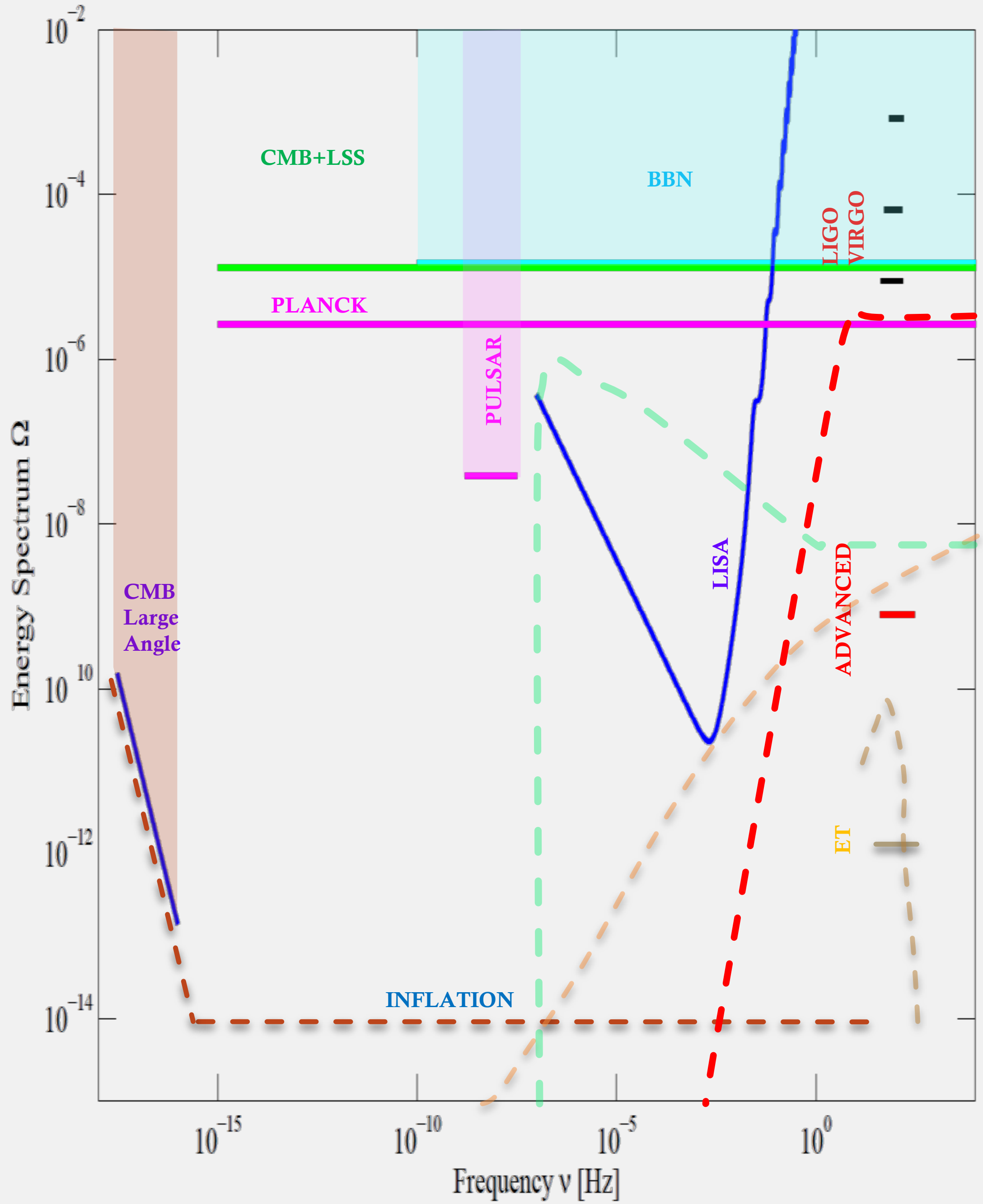




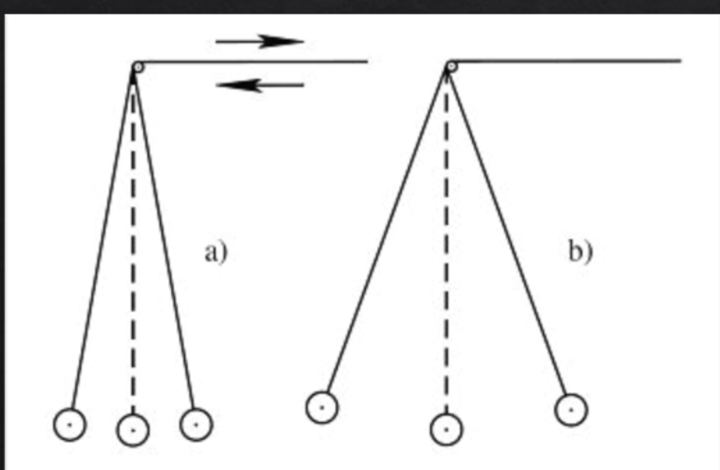
Upper limits & expected sensitivities

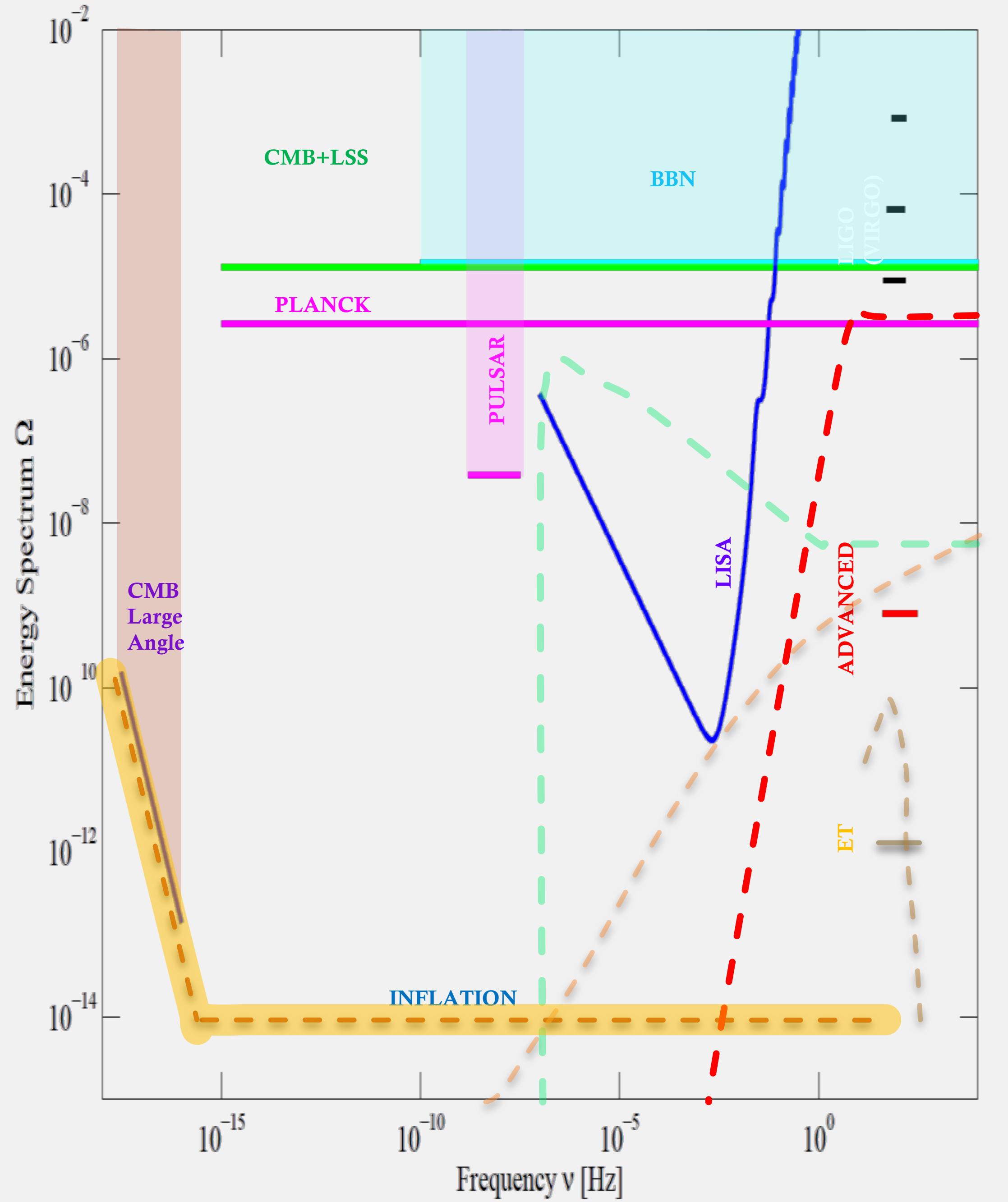
- **Big-Bang Nucleosynthesis model and observations** constrain the total GW energy at the time of BBN (integral bound)
- Similar bound from **CMB observations**
- Too much GW gives too much **large angle anisotropy** by the Sachs Wolfe effect
- Signal from **millisecond pulsars** works as a (big) interferometer:



Inflation

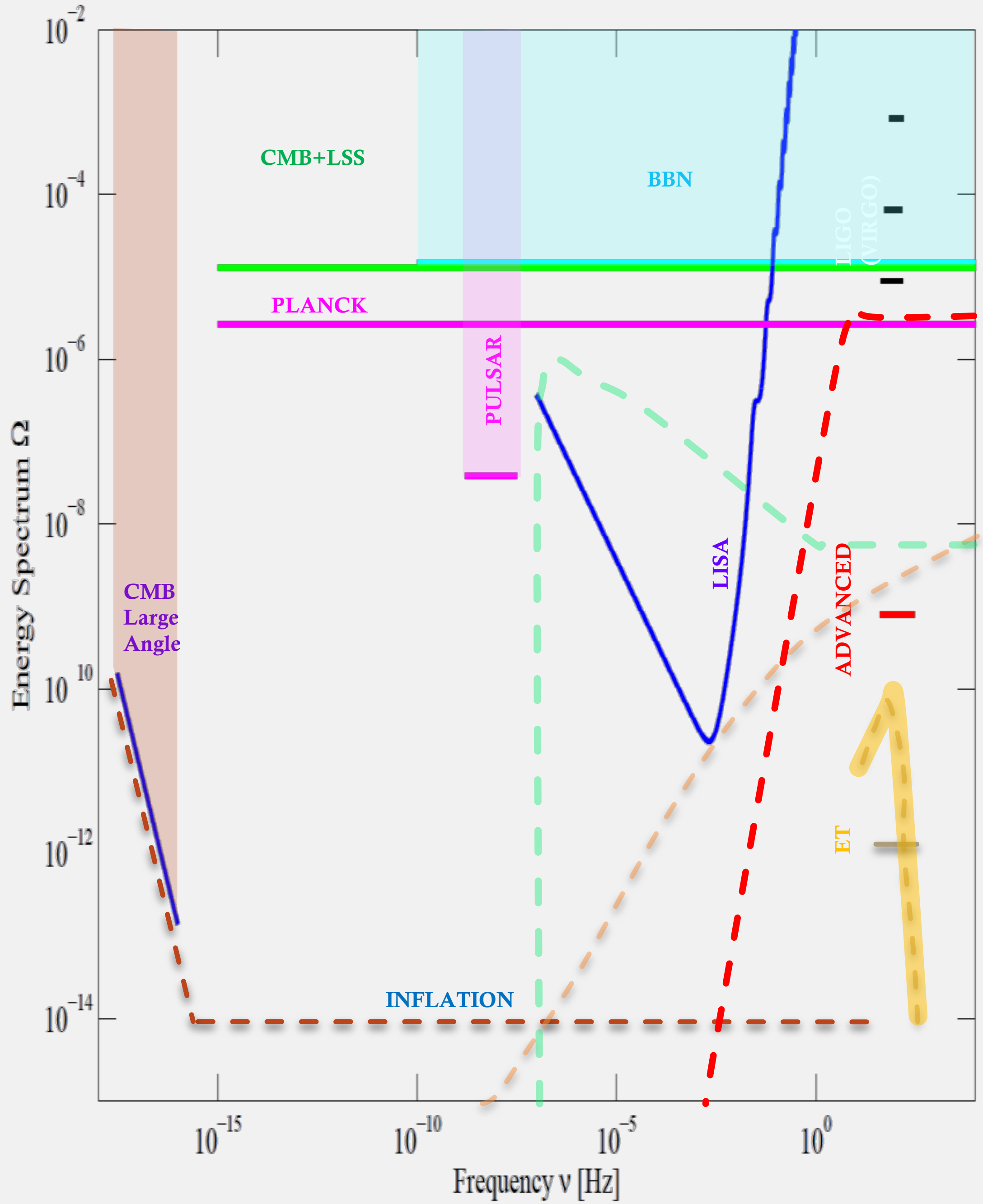
- Parametric amplification of vacuum quantum fluctuations
- Standard inflationary models are weakly dependent on frequency
- Tight bound: CMB large angle anisotropy
- Out of reach of advanced detectors by 5 orders of magnitude





Resonant preheating

- During a resonant preheating phase at the end of the inflation, inflaton energy can be transferred efficiently to other particles
- Can produce a significant GW background
- Spectrum peak depends on energy scale (here 10 GeV, higher frequencies for a larger scale)

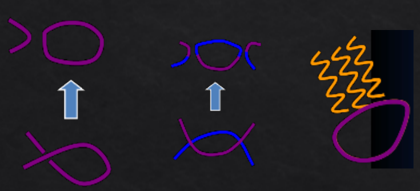


Easther & Lim, JCAP 0604, 010 (2006)
Easther et al., PLR 99, 221301 (2007)
Easther, Nucl. Phys. Proc. Suppl. 194, 33 (2009)

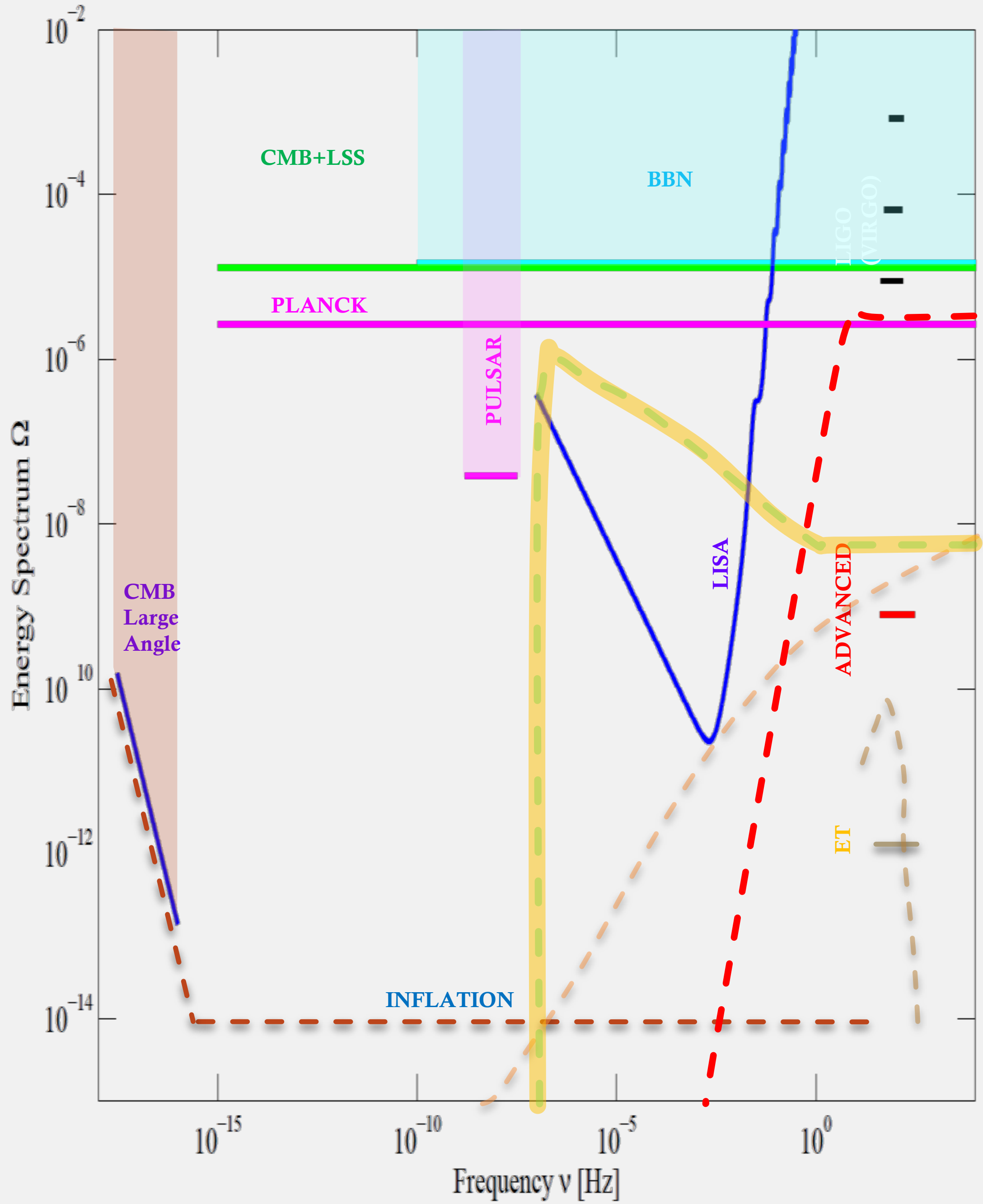
Cosmic (super)string models

- Dynamical network:
  - Strings entering in the horizon
  - Interconnection: loops generation
  - Radiation (GW and other fields): loop destruction

- Most efficient emission mechanisms: cusps and kinks
- Integrating over the whole universe leads to a GW background
- Large parameter space



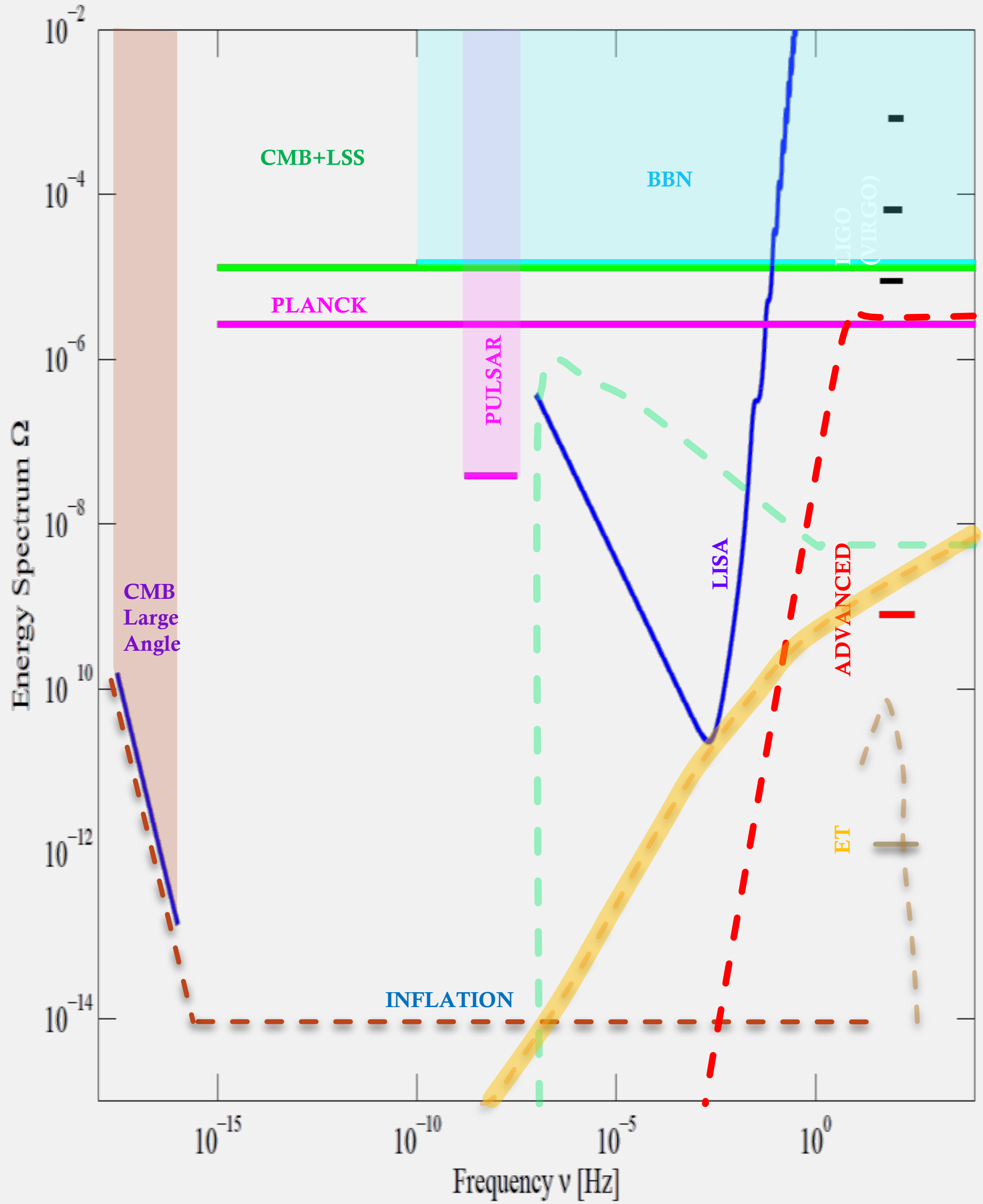
Damour & Vilenkin, PRL 85, 3761 (2000)
Siemens et al., PRL 98, 111101 (2007)
Olmez et al., PRD 81, 104028 (2010)



Axion-based inflation models

- Models include axion-gauge couplings
- Gauge backreaction on the inflaton extends inflation
- The late inflationary phase increases GW production at high frequencies

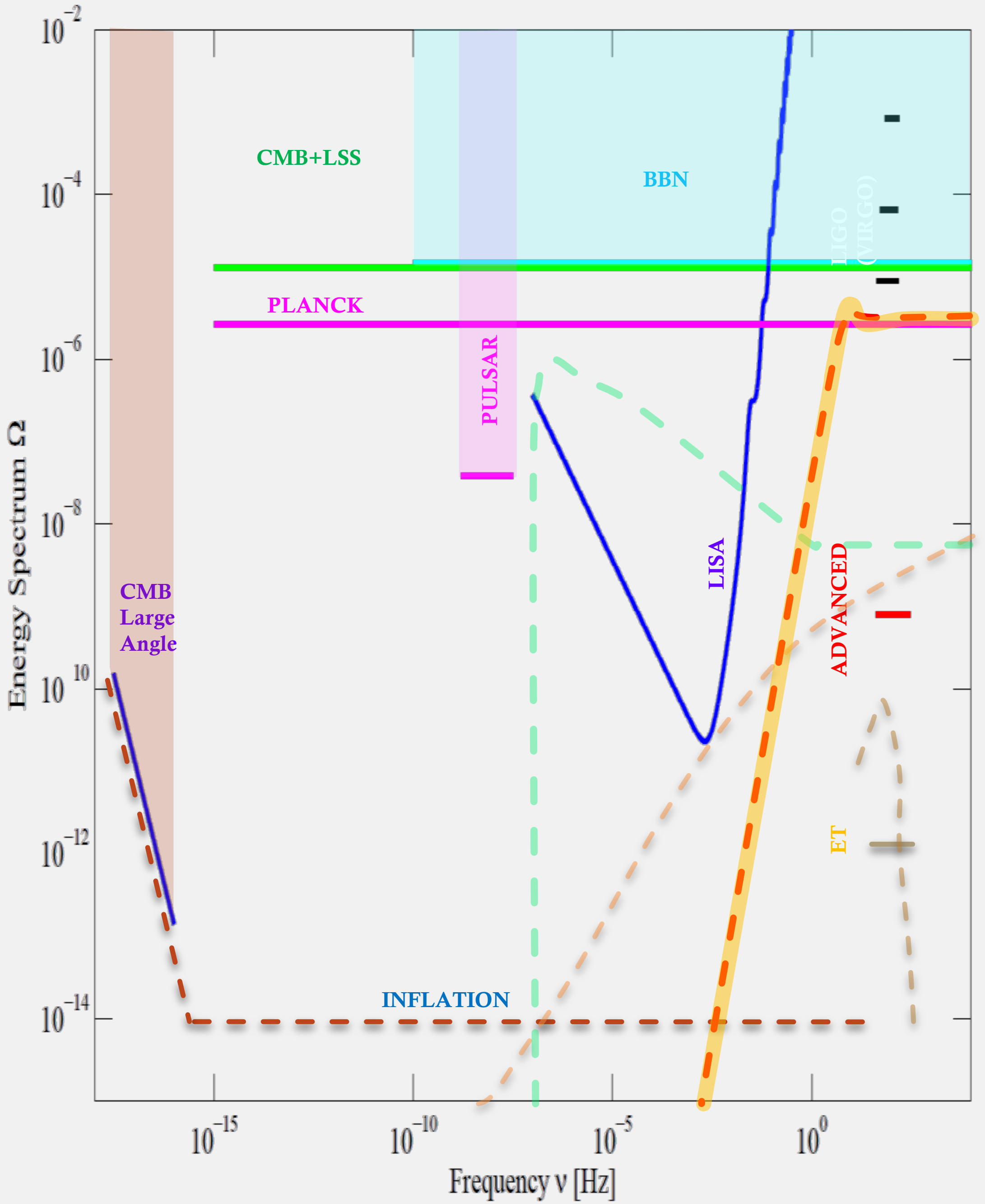
Barnaby, Pajer and Peloso - Phys. Rev. D 85, 023525



Pre-BB models

- Alternative cosmologies
- Evade the CMB large angle anisotropy bound
- Evade the BBN-CMB integral bound
- Can be significant at Virgo/LIGO frequencies

Gasperini & Veneziano, Phys. Rep. 373, 1 (2003)
Buonanno et al., PRD 55, 3330 (1997)



Isotropic upper limits

$$\Omega_{GW}(f) = \Omega_\alpha \left(\frac{f}{f_{ref}} \right)^\alpha$$

Best current upper limit

Frequency (Hz)	$f_{\rm ref}$ (Hz)	α	Ω_α	95% C.L. upper limit	Previous limits
41.5–169.25	...	0	$(-1.8 \pm 4.3) \times 10^{-6}$	5.6×10^{-6}	7.7×10^{-6}
170–600	...	0	$(9.6 \pm 4.3) \times 10^{-5}$	1.8×10^{-4}	...
600–1000	900	3	0.026 ± 0.052	0.14	0.35
1000–1726	1300	3	-0.077 ± 0.53	1.0	...

PRL 113, 231101 (2014)

Colocated interferometers

Band (Hz)	95% C.L. UL ($\times 10^{-3}$)
460–1000	0.77
460–537	1.11
537–628	2.12
628–733	1.18
733–856	2.53
856–1000	2.61

Phys. Rev. D 91, 022003 (20015)

O_1 analysis
(advanced detectors)
is in progress.

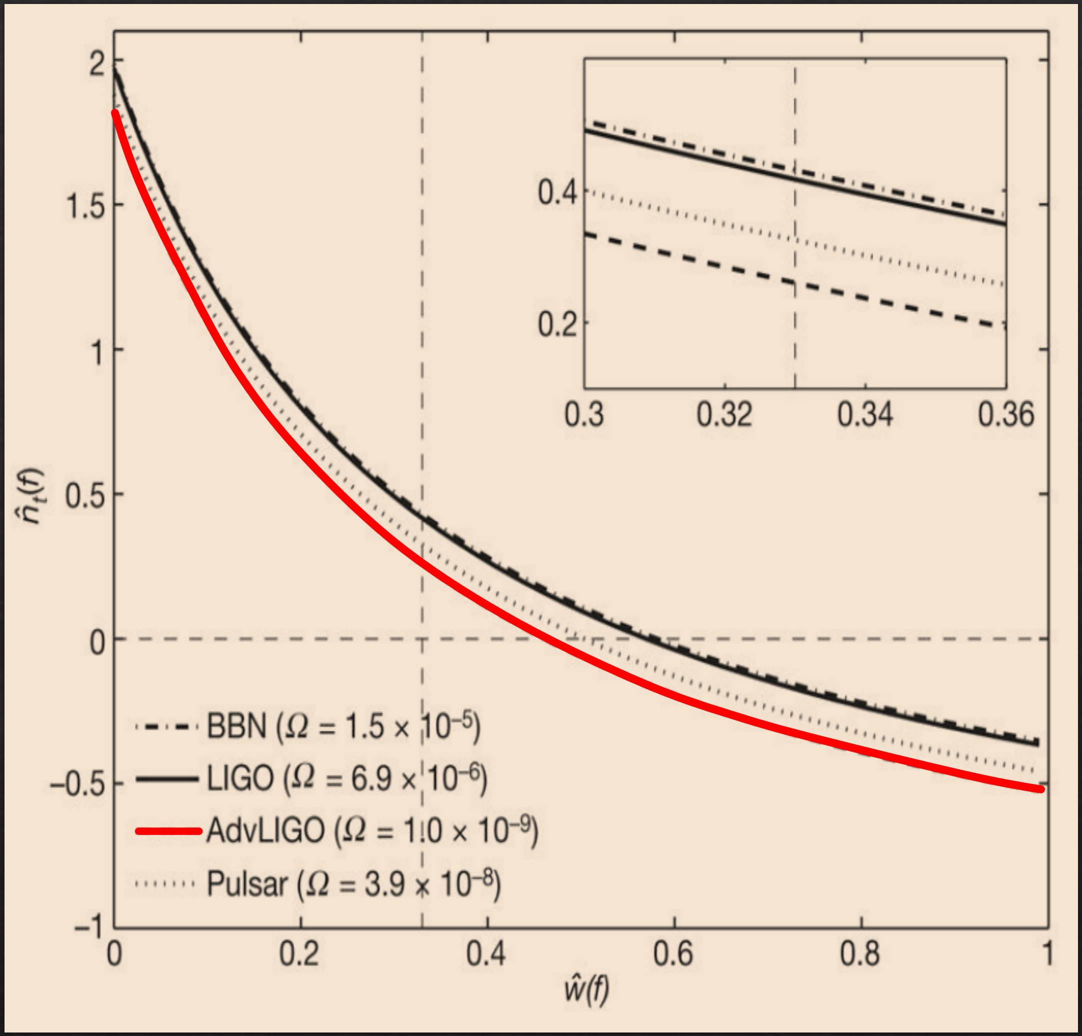
Constraint on early universe state equation

Spectrum parameterization:

$$\Omega_{GW}(f) = A f^{\hat{\alpha}(f)} f^{\hat{n}(f)} r$$

$$\hat{\alpha}(f) = 2\frac{3\hat{w}(f) - 1}{3\hat{w}(f) + 1}$$

- $\hat{n}_t(f)$ effective tensor tilt parameter
- r ratio of tensor and scalar perturbation amplitudes (here $r = 0.1$)
- $\widehat{w}(f)$ equation of state parameter

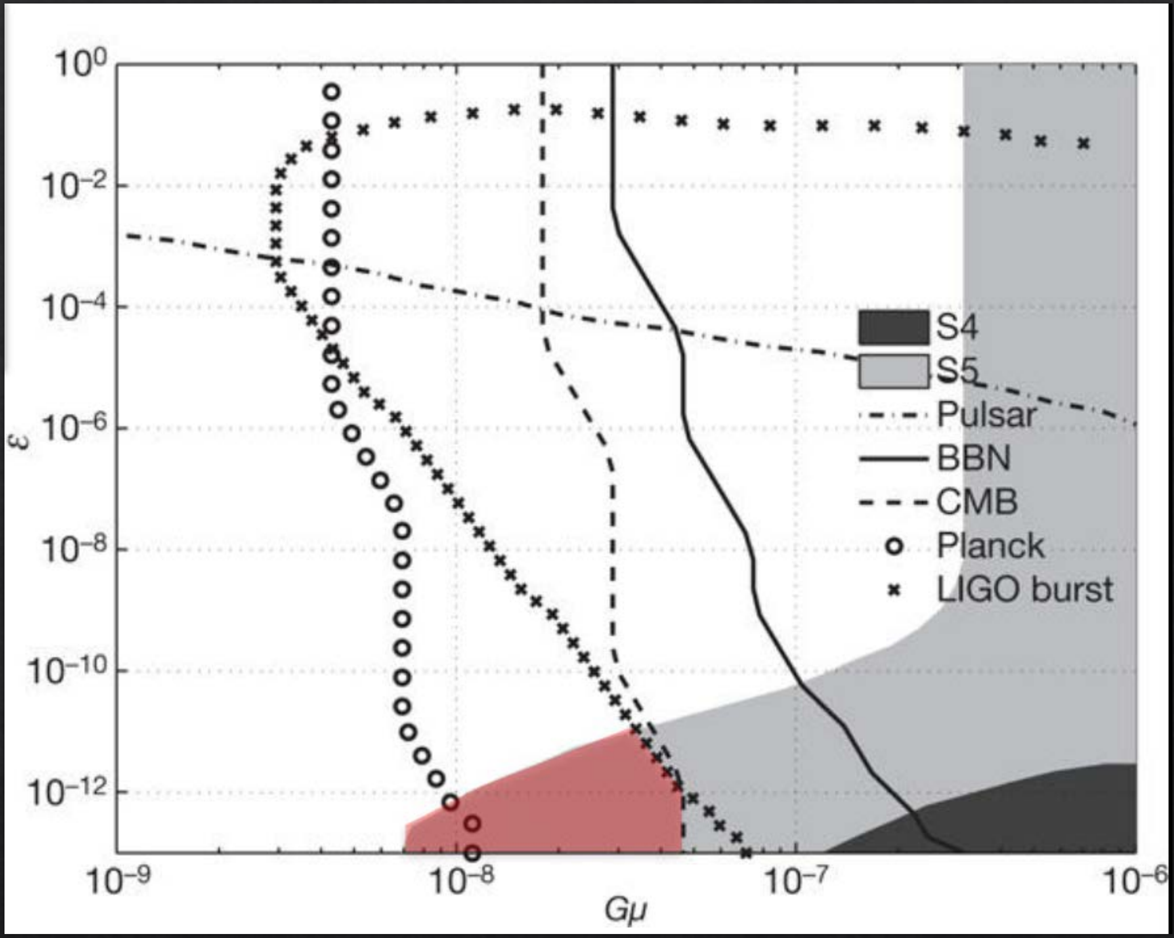


LSC/Virgo Collaboration, Nature 460, 990-994 (2009)
PRD 78, 043531 (2008)

Constraint on cosmic strings models

- Network of cosmic strings parameterized by:
- String tension μ
- Reconnection probability p
- Loop size (parameterized by ε)

- $G\mu<10^{-6}$ (CMB observations)
- ε unconstrained
- $10^{-4} < p < 1$ ($p = 10^{-3}$ here)



Region excluded: entire plane will be probed by advanced detectors.

LSC/Virgo Collaboration, Nature 460, 990-994 (2009)
PRD 80, 062002 (2009)
PRL 98, 111101 (2007)

Constraints on pre-Big-Bang models

Spectrum:

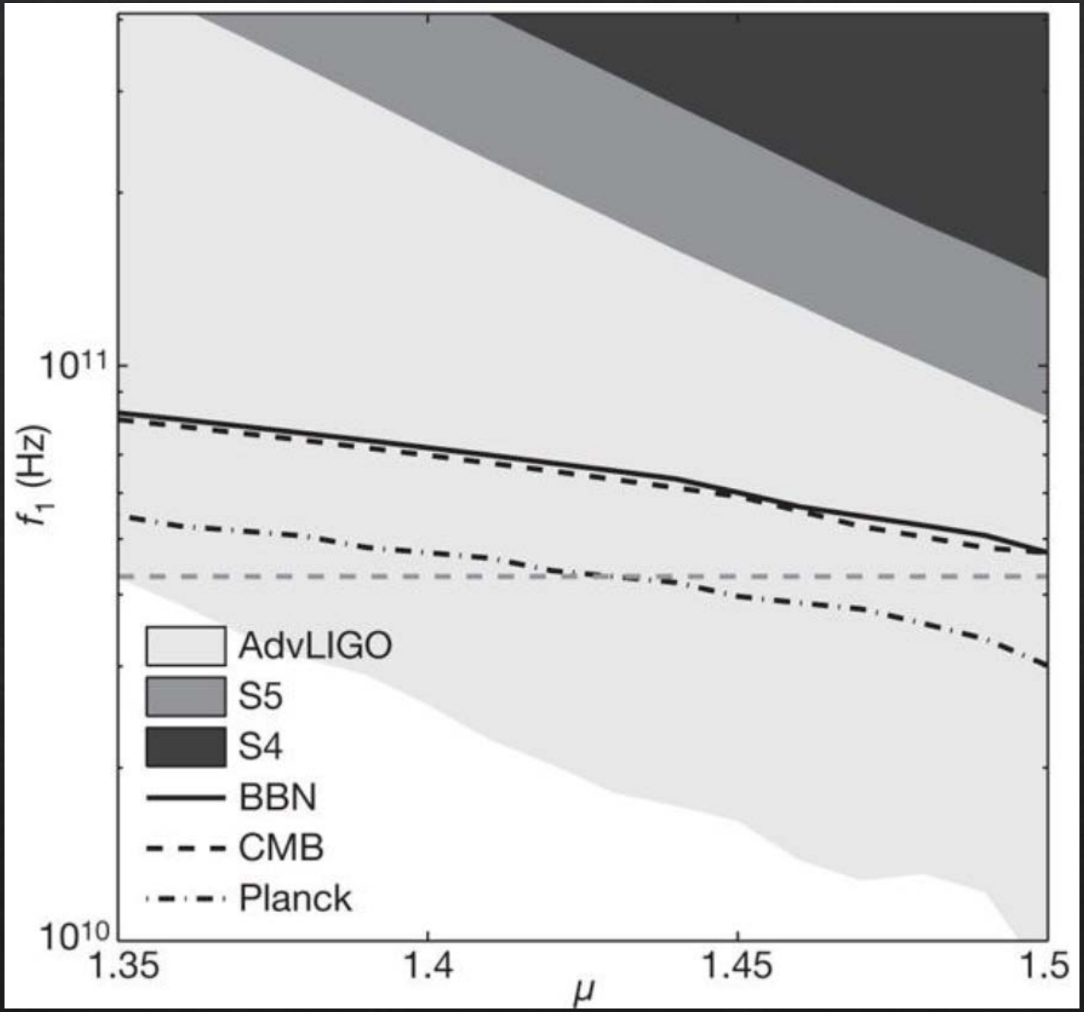
$$\Omega_{GW}(f) \propto f^3$$

below f_s , f_s = 30 Hz here

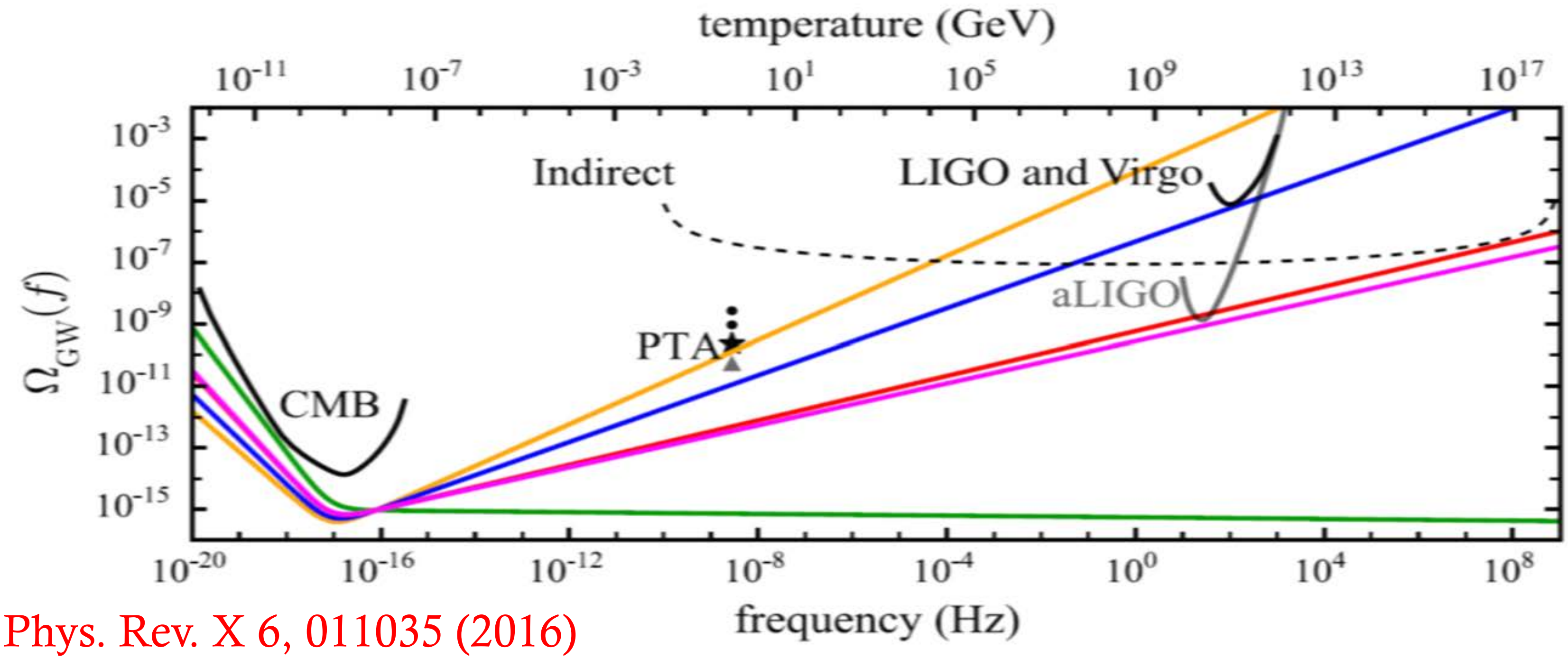
$$\Omega_{GW}(f) \propto f^{3-2\mu}$$

above f_s , f_s = 30 Hz here

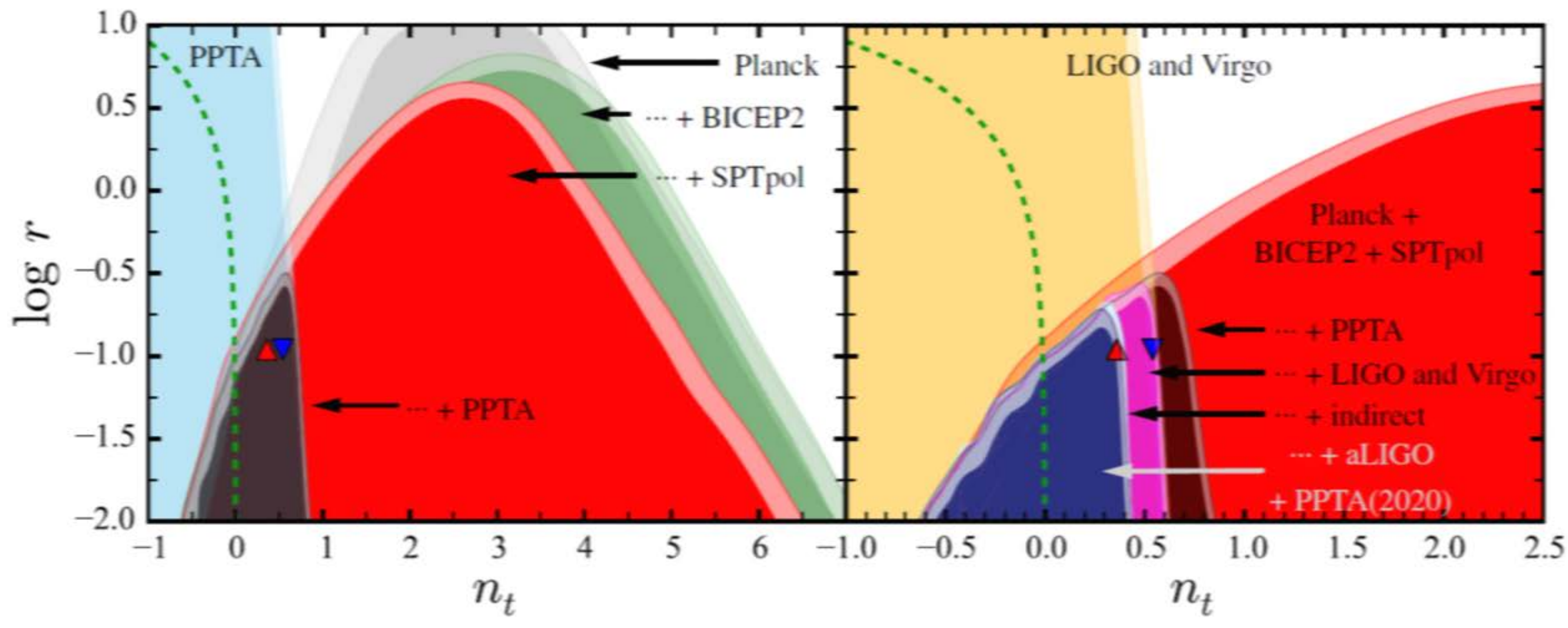
f_1 : cut off frequency
(a factor 10 from 4.3×10^{10})



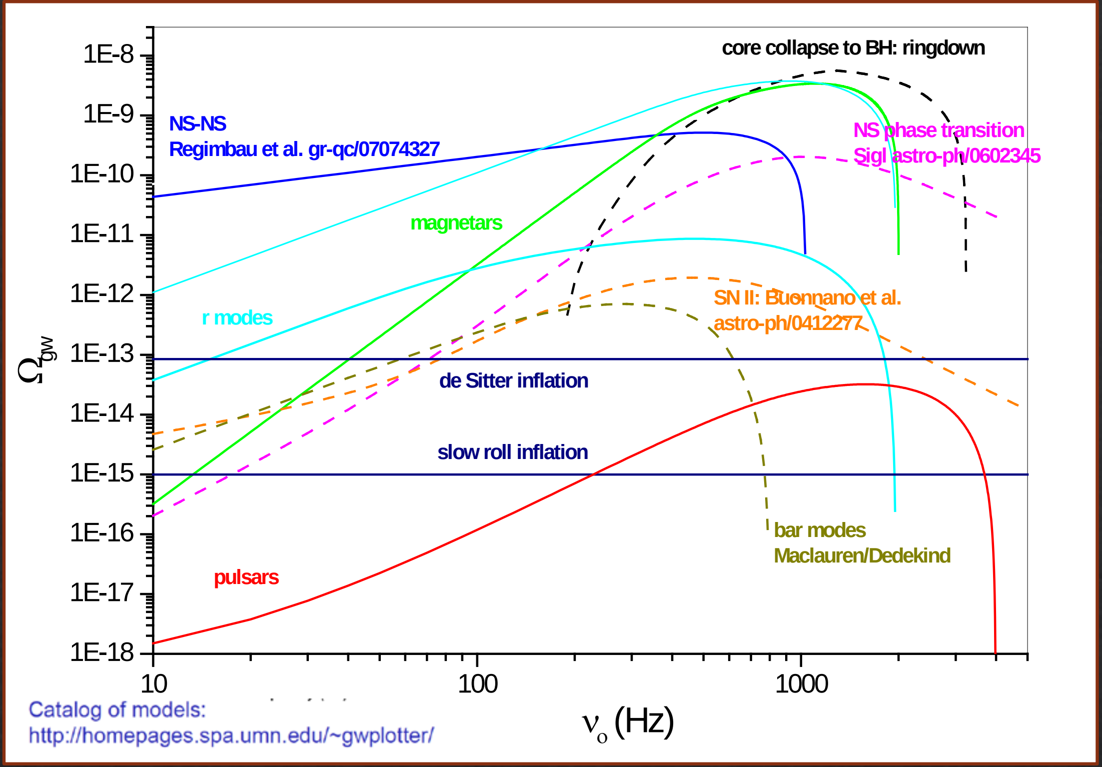
LSC/Virgo Collaboration, Nature 460, 990-994 (2009)
PRD 73, 063008 (2006)



Phys. Rev. X 6, 011035 (2016)



Astrophysical Stochastic Background



- **Core collapse supernovae**
  - **Neutron star formation:** *Blair & Ju 1996, Coward et al. 2001-02, Howell et al. 2004, Buonanno et al. 2005*
  - **Stellar Black Hole formation:** *Ferrari et al. 1999, de Araujo et al. 2000-04*
- **Neutron stars**
  - **tri-axial emission:** *Regimbau & de F. Pacheco 2001-06*
  - **bar or r-modes:** *Owen et al. 1998, Ferrari et al. 1999, Regimbau 2001*
  - **phase transitions:** *Sigl 2006*
- **Stellar Compact Binaries**
  - **near coalescence (NS, BH):** *Regimbau et al. 2006-07, Coward et al. 2005 (BNS), Howell et al. 2007 (BBH)*
  - **low frequency inspiral phase:** *Ferrari et al. 2002, Farmer & Phinney 2002, Cooray 2004 (WD-NS)*
- **Capture of compact objects by SMBHs** *:*

Barack & Cutler 2004

$$\Omega_{gw}(f) = \int P(\underline{\theta})\Omega_{gw}(f,\underline{\theta})d\underline{\theta}$$

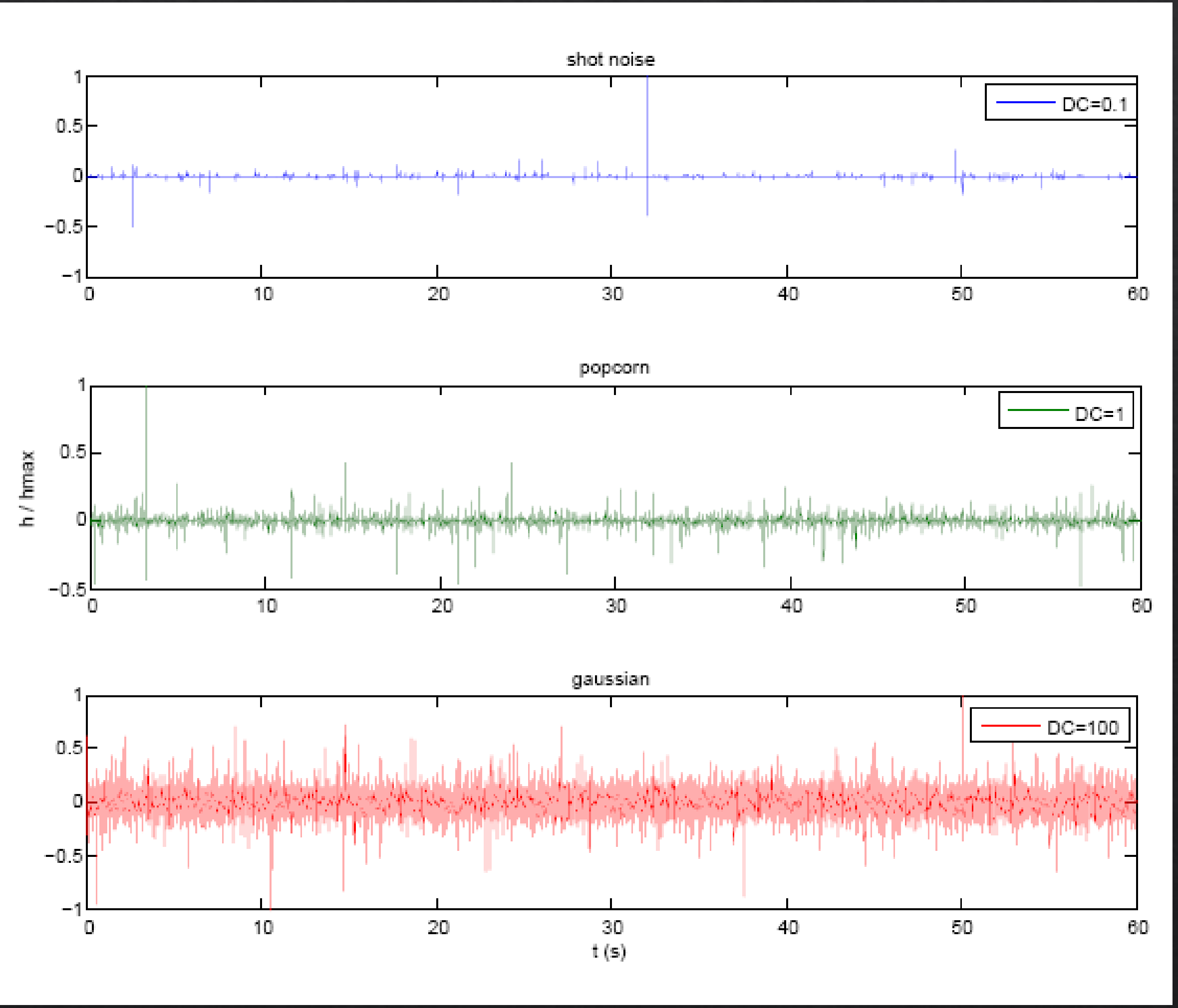
$$\Omega_{gw}(f,\underline{\theta}) = \frac{f}{\rho_c}\int \frac{dR}{dz}(z,\underline{\theta})\,\frac{1}{4\pi r^2(z)}\,\frac{dE_{gw}}{df}\,(\underline{\theta}, f(1+z))\,dz$$

Astrophysical Stochastic Background

$$D(z) = \int_0^z \tau \left(1 + z'\right) \frac{dR}{dz'} dz'$$

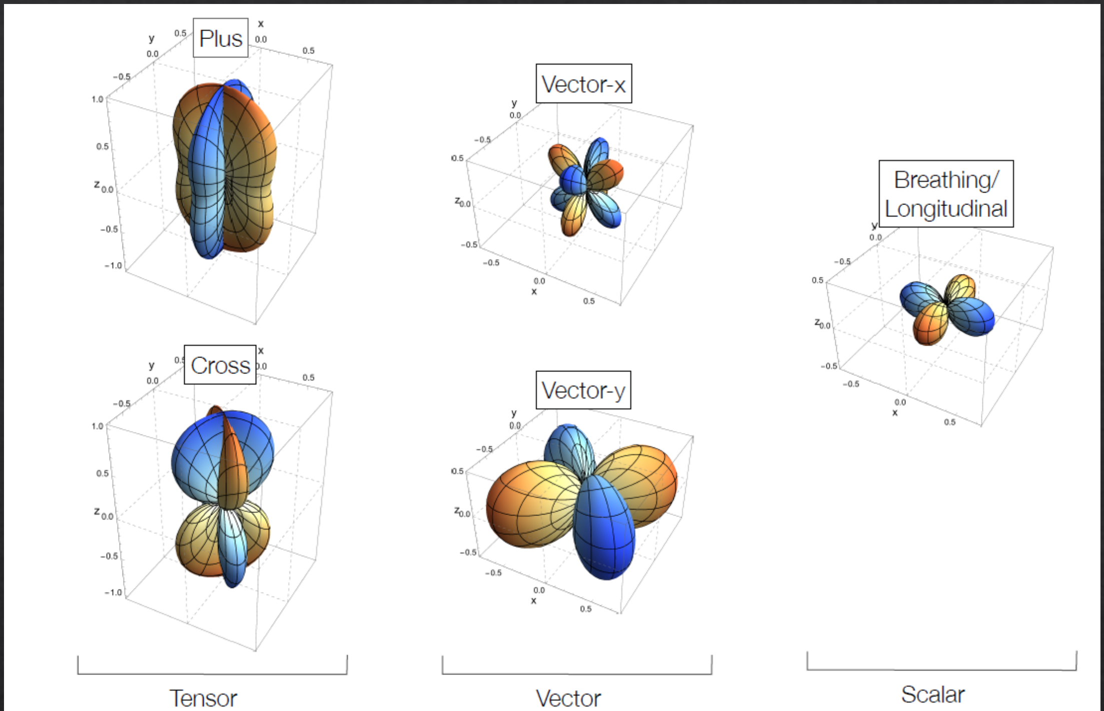
Duty cycle: ratio between the observed typical duration of the event and the average time between events.

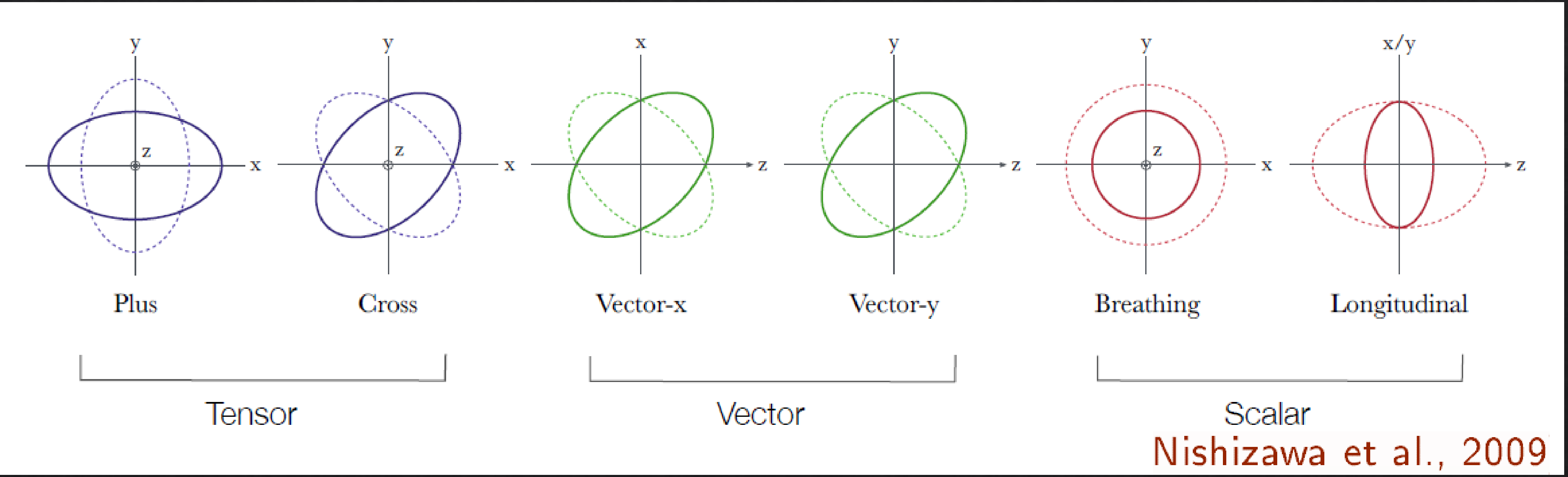
- $D \ll 1$: resolved sources
  - Burst data analysis, optimal filtering
- $D \simeq 1$: «popcorn noise»
  - Maximum likelihood statistic (Drasco et al. 2003)
  - Probability event horizon (Coward et al. 2005)
- $D \gg 1$: Gaussian stochastic background
  - Cross correlation statistic (isotropic/anisotropic)

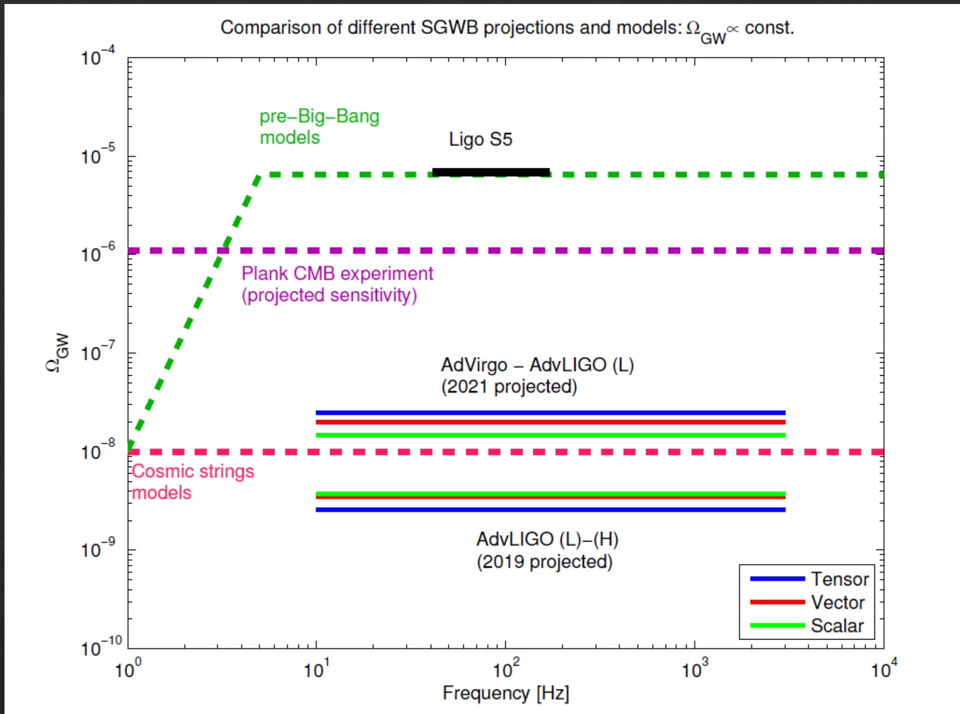


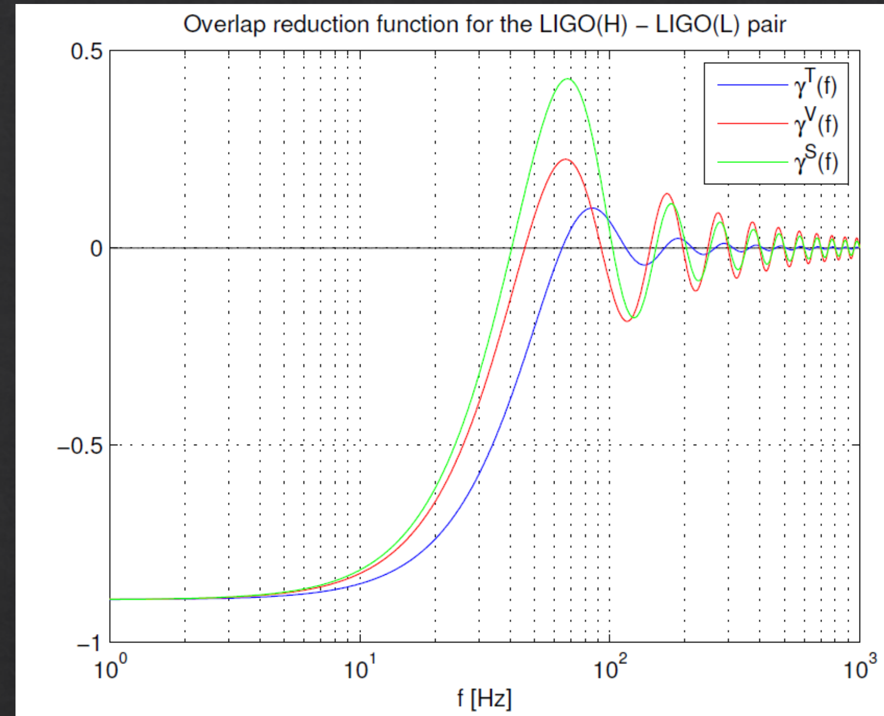
Non standard polarizations

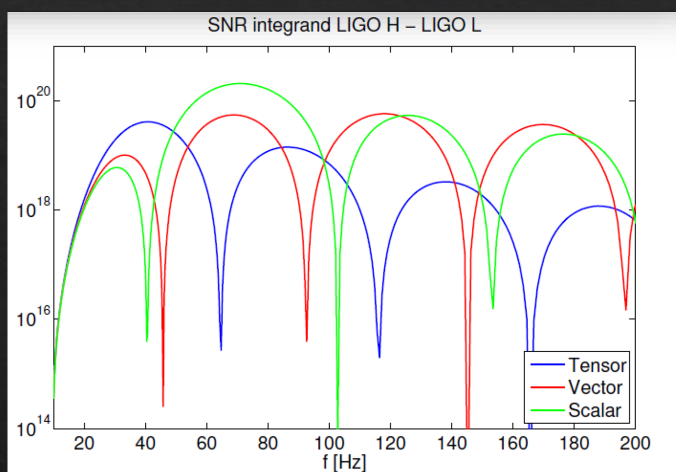
- Looking at alternative theories of gravity
- Indirect constraints on scalar modes
  - Binary pulsars
  - WMAP
- Need many detector pairs to disentangle different contributions

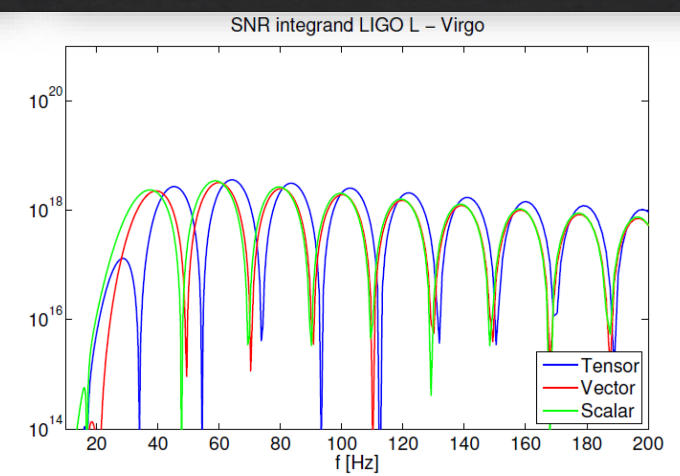






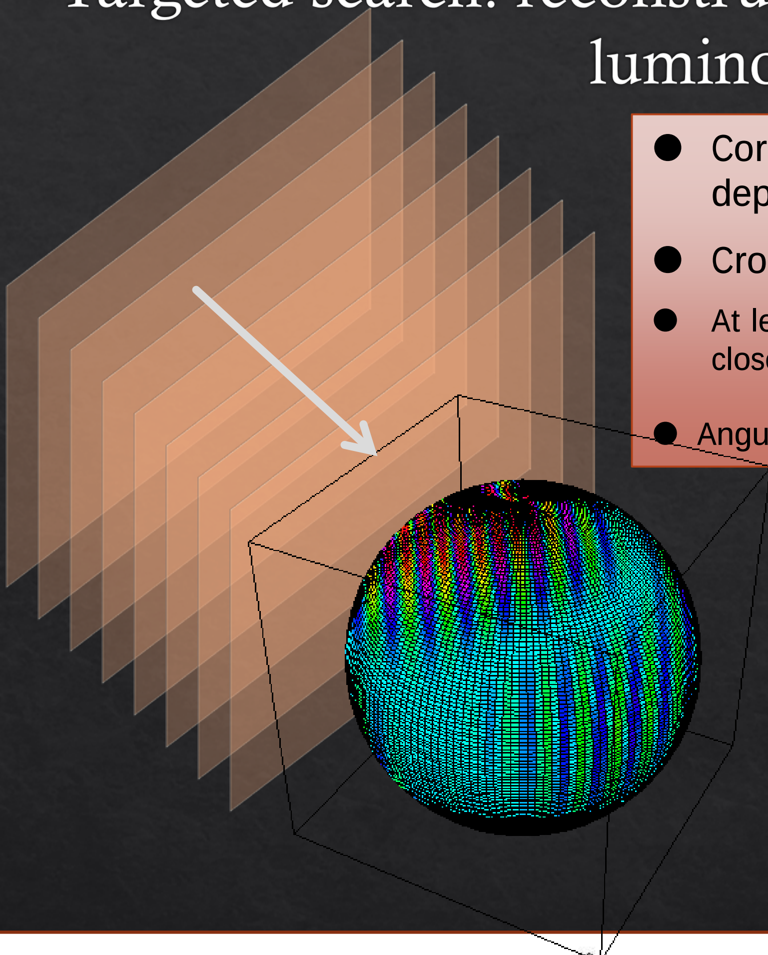




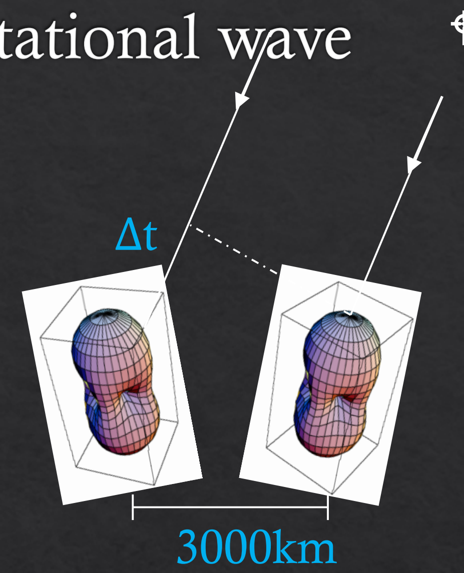


$$\Omega(f) = \Omega_T \left(\frac{f}{f_0}\right)^{\alpha_T} + \Omega_V \left(\frac{f}{f_0}\right)^{\alpha_V} + \Omega_S \left(\frac{f}{f_0}\right)^{\alpha_S}$$

Targeted search: reconstruct a map of the gravitational wave luminosity in the sky



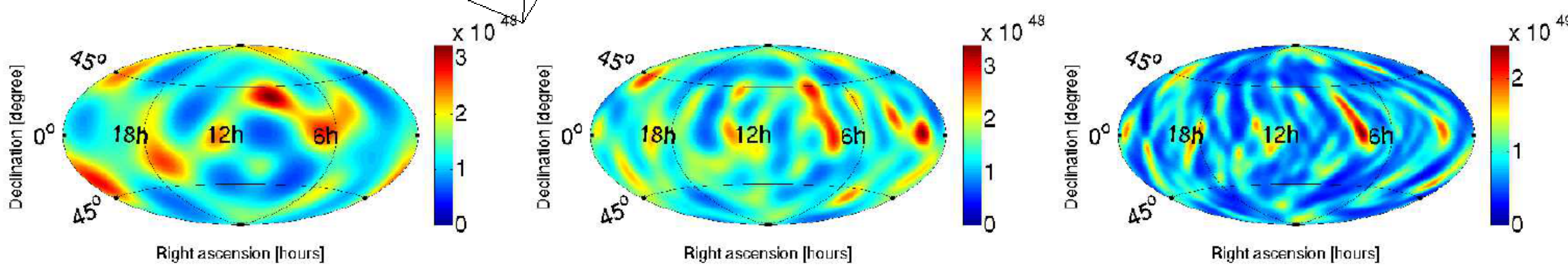
- Correct the direction-dependent modulation
- Cross-Correlate
- At least 3 detectors needed to close the inverse problem.
- Angular resolution limited by λ/D



$$\Omega_{gw}(f) = \frac{2\pi^2}{3H_0^2} f^2 S_h(f) \int_{S^2} d\hat{\Omega}\, \mathcal{P}(\hat{\Omega})$$

$$\mathcal{P}(\vec{\Omega}) \equiv \int \eta(\hat{\Omega}')\delta^2(\hat{\Omega},\hat{\Omega}')d\hat{\Omega}'$$ «radiometer» search

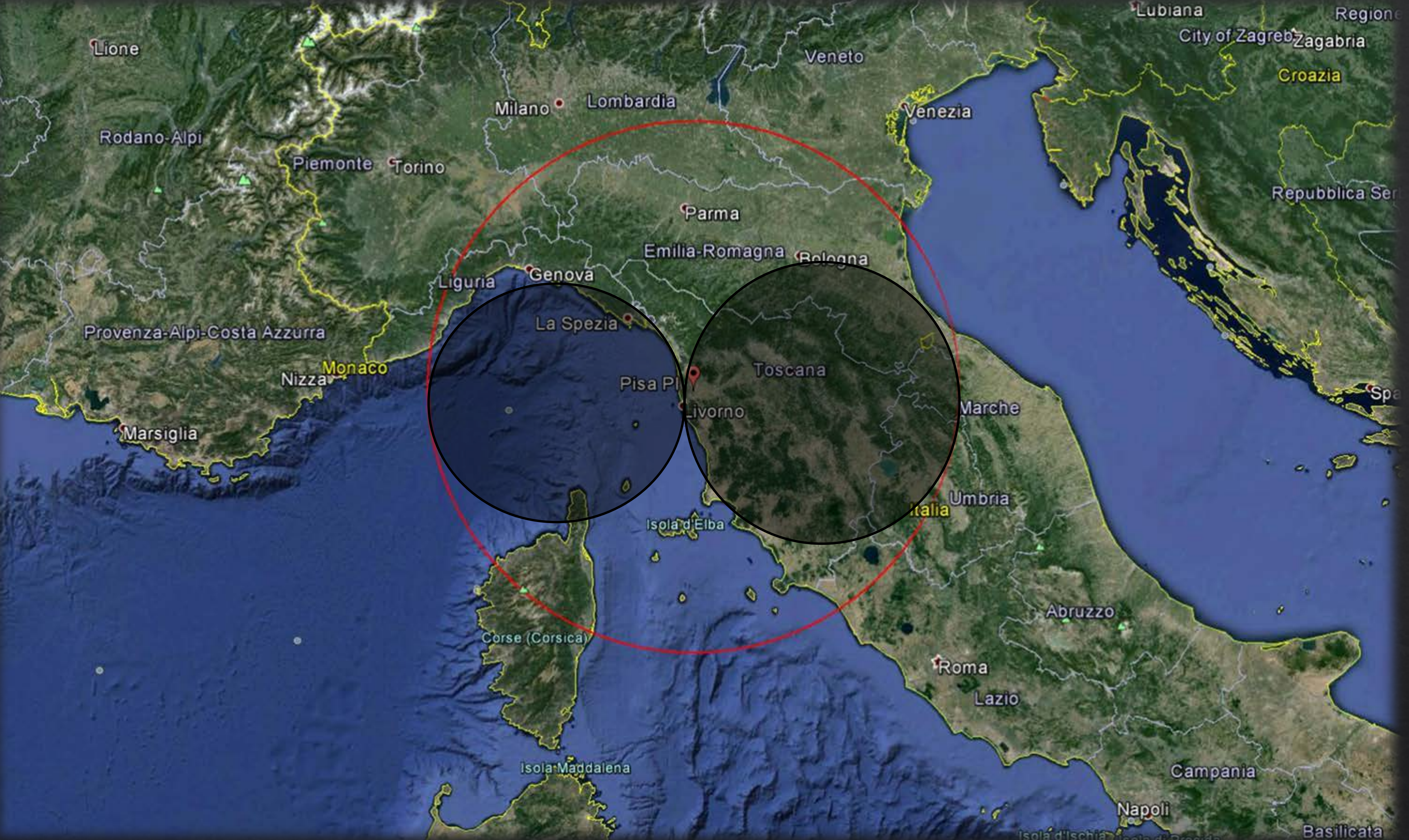
$$\mathcal{P}(\vec{\Omega}) \equiv \sum_{\ell,m} c_{\ell m} Y_{\ell m}(\hat{\Omega})$$ «SA» search



Virgo+WA 200 Hz

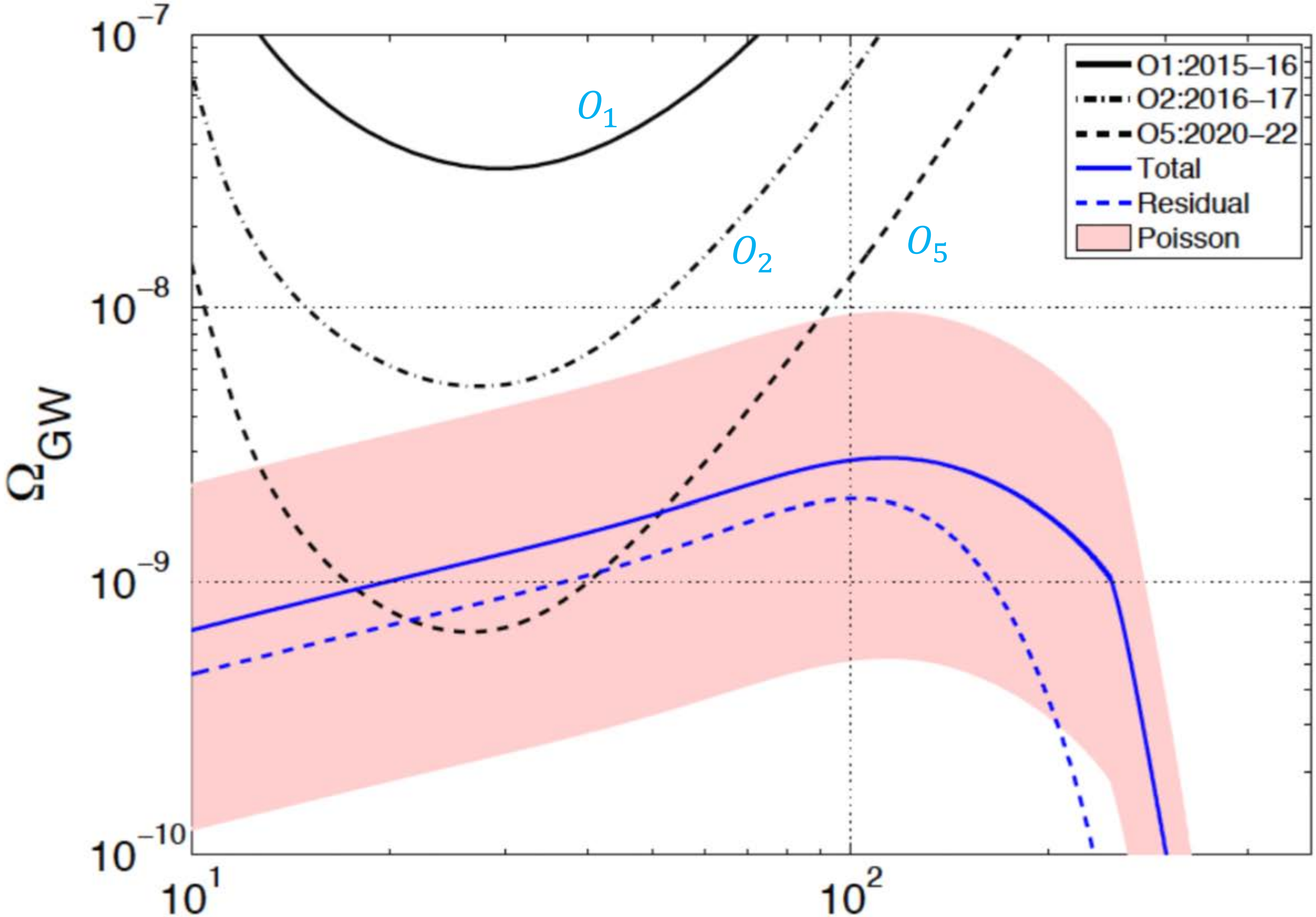
PRD76, 082003, 2007

Phys. Rev. Lett. 107, 271102



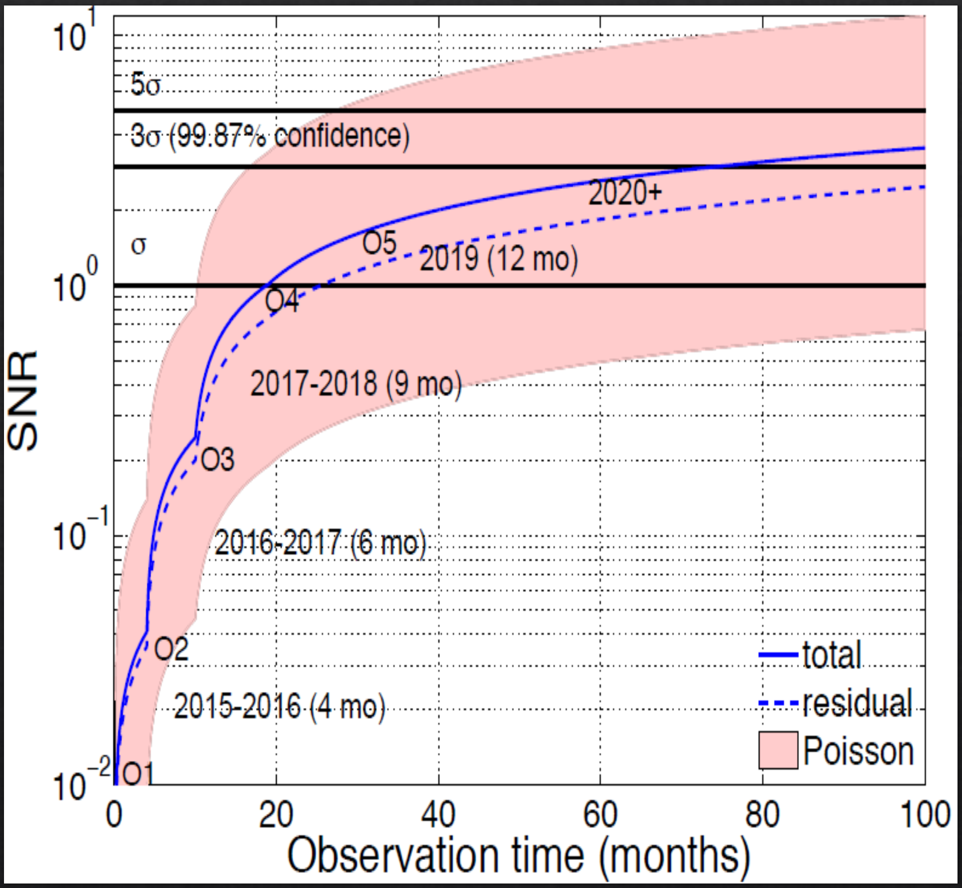
$\left(\frac{v}{c}\right)^2 \lesssim 0.36$ $\quad d \gtrsim 180\text{km}$ $\quad R_s \sim 90\text{km}$

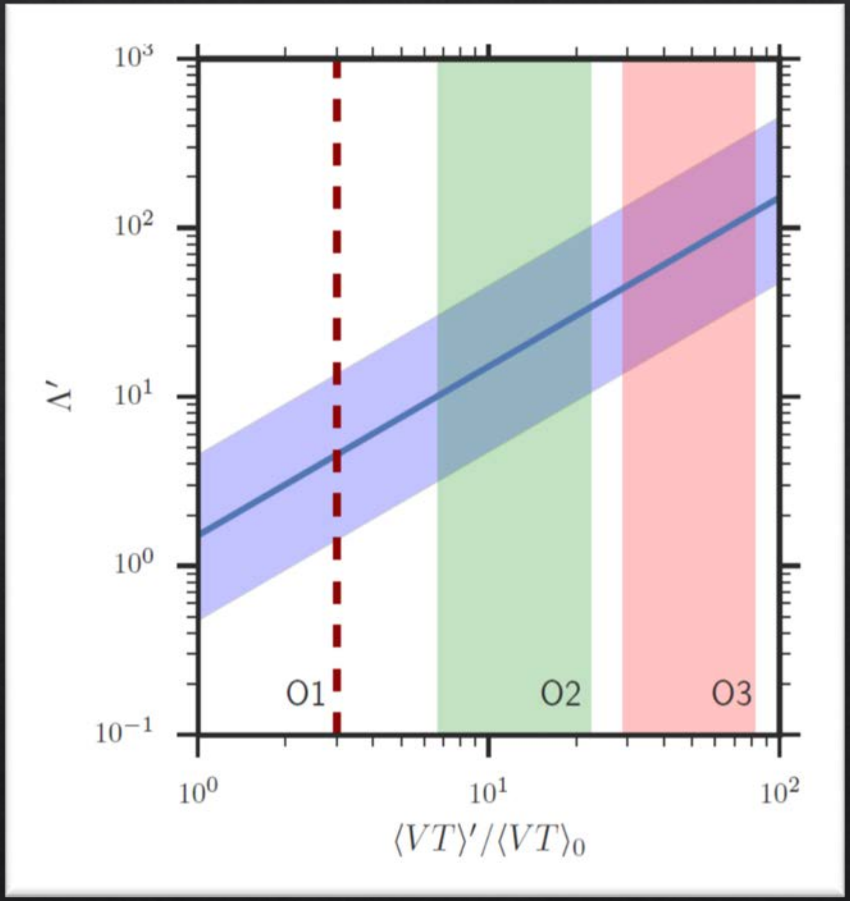
Implications of LIGO first detection



B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. 116, 131102

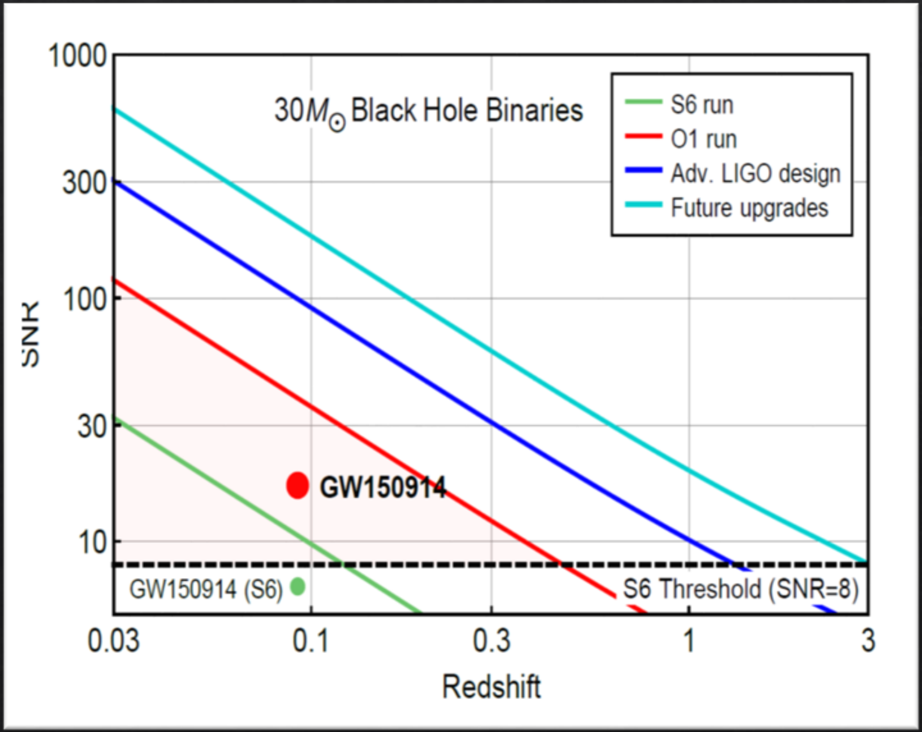
Perspectives





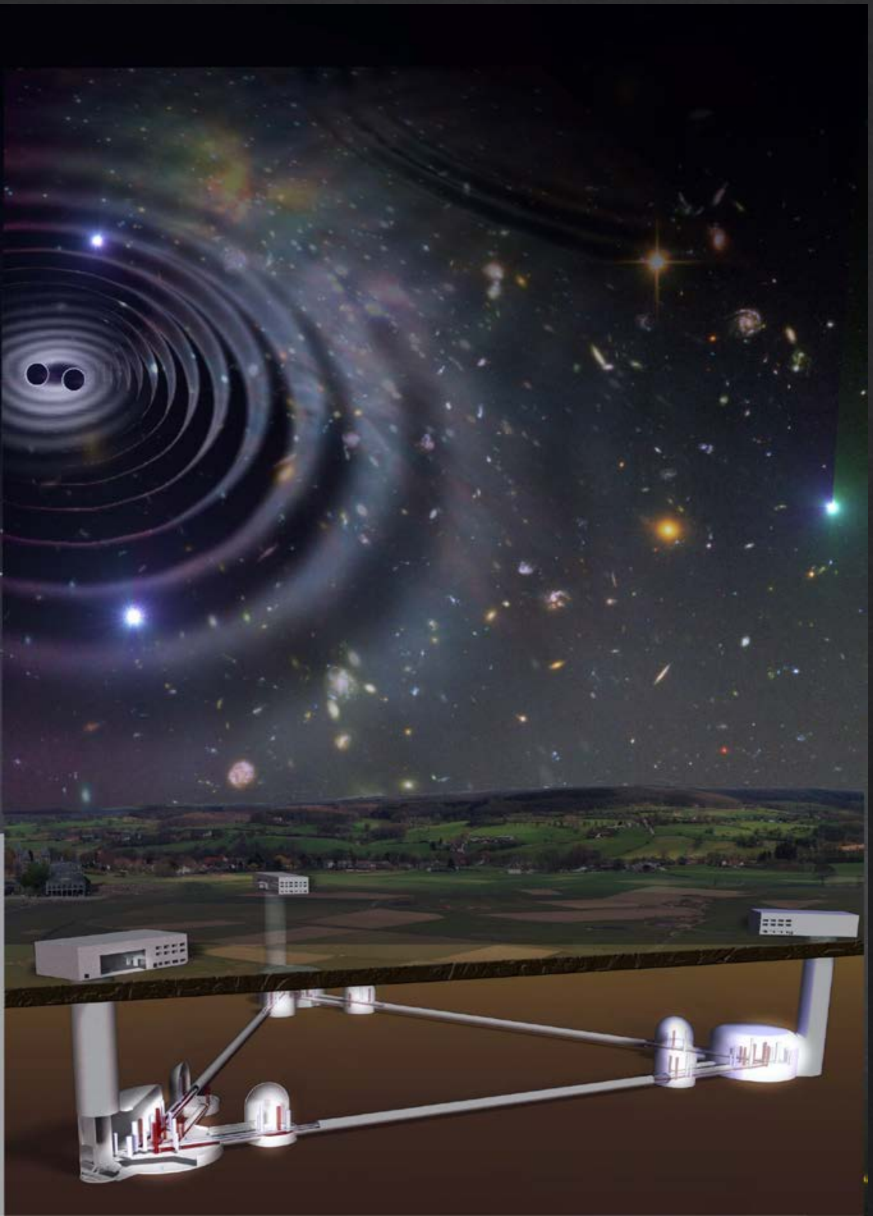
Expectations for future runs

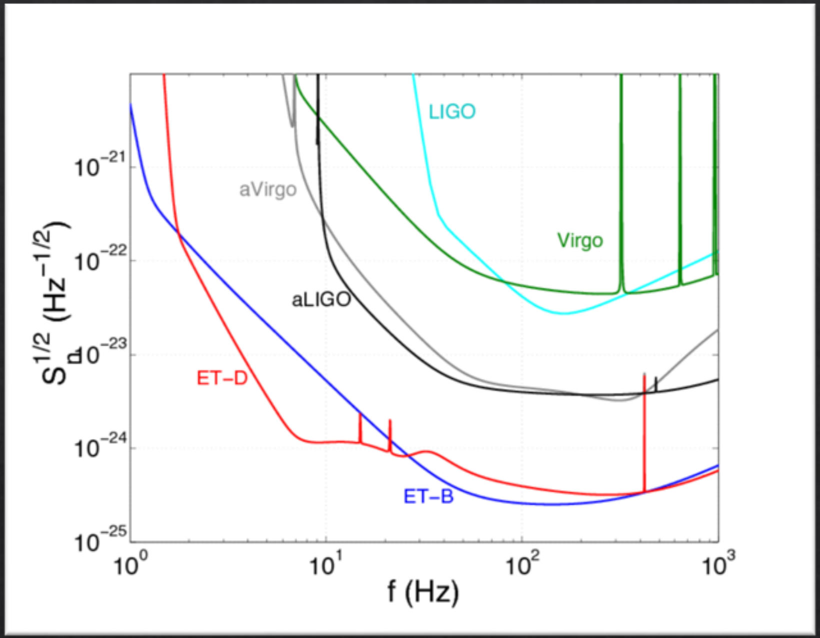
The median value and 90% credible interval for the expected number of highly significant events (FARs <1/century) as a function of surveyed time-volume in an observation

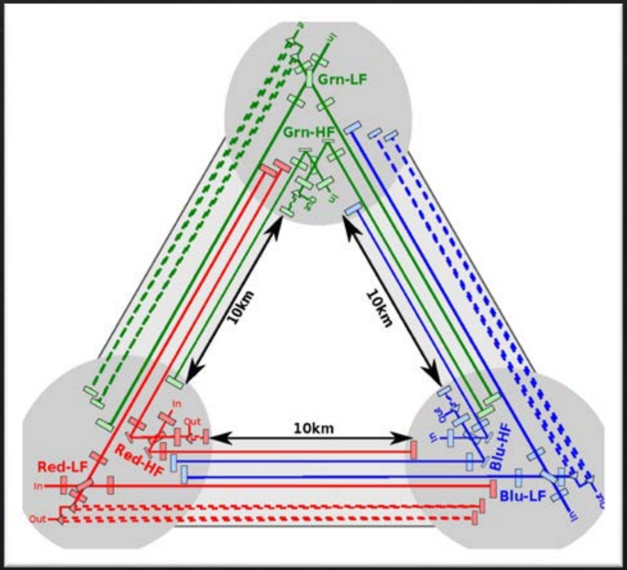


Signal-to-noise ratio vs z for 30 $M_\odot$ BBH

What next? The third generation detectors



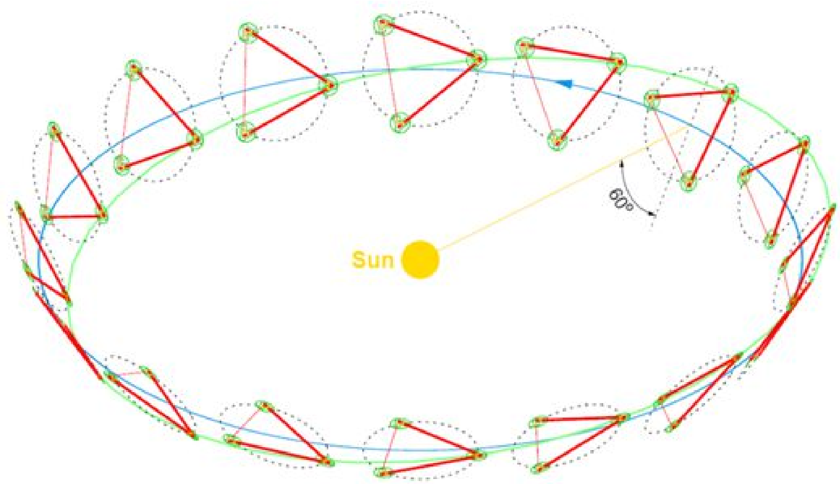




What next? Space detectors

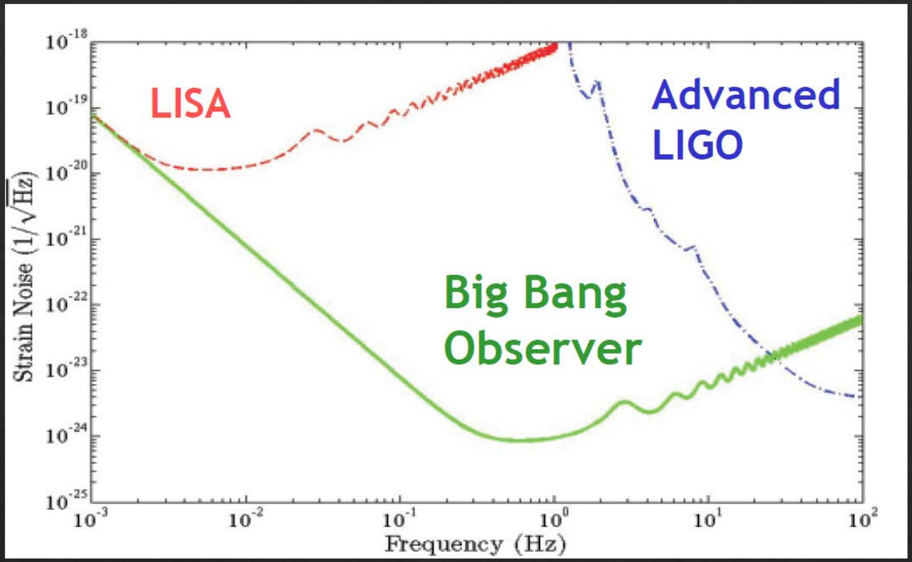
eLISA Space Based GW Detector

- Laser Interferometer in Space Antenna, LISA, provides unique capabilities
  - Immune to seismic noise
  - Long baseline provides 0.001 - 1Hz GW spectrum sensitivity needed for observing massive black hole mergers
- Multiple identical or similar detectors to improve detection confidence



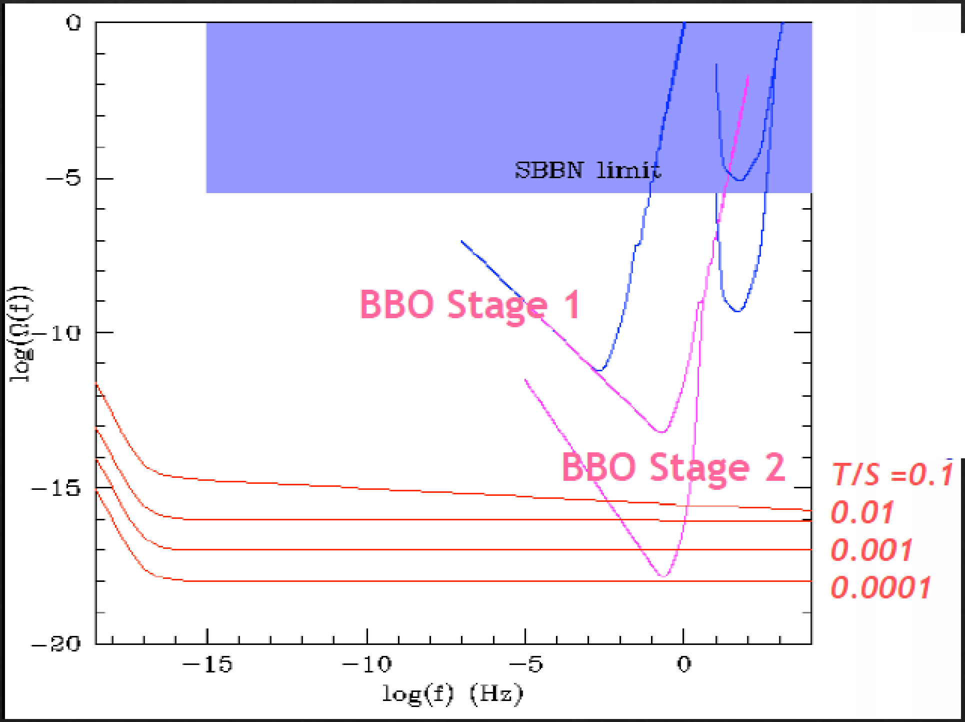
LISA: a mission to detect and observe gravitational waves, O Jennrich, in Gravitational Wave and Particle Astrophysics, Proc SPIE v5500

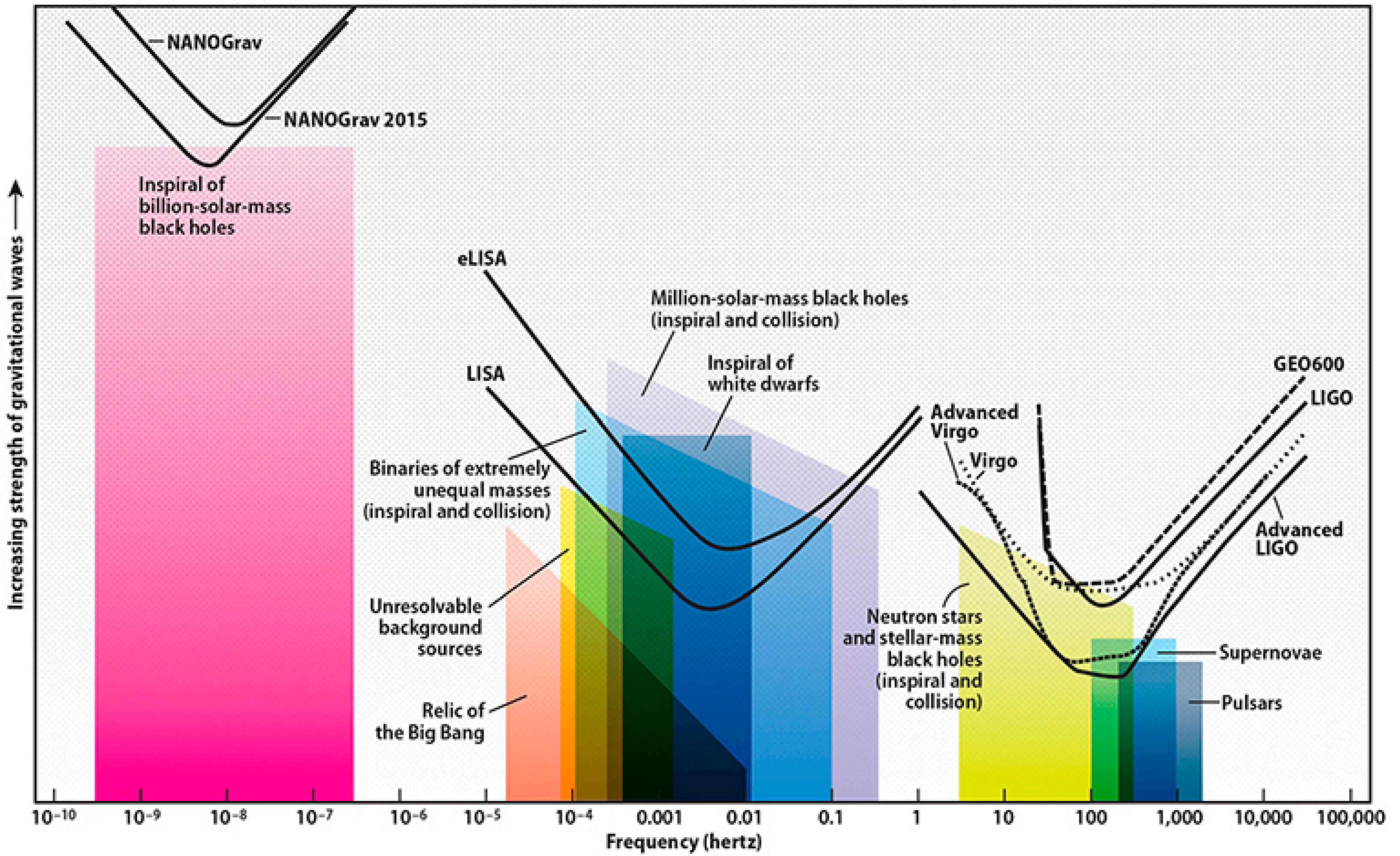
Big Bang Observer



- Fill the gap between (e)LISA & earthbound detectors
- Designed to detect SB produced by inflation
- Space based
- Shorter arm than (e)LISA

- Higher laser power
- Improved acceleration noise
- Two phases





Thank you for your attention....