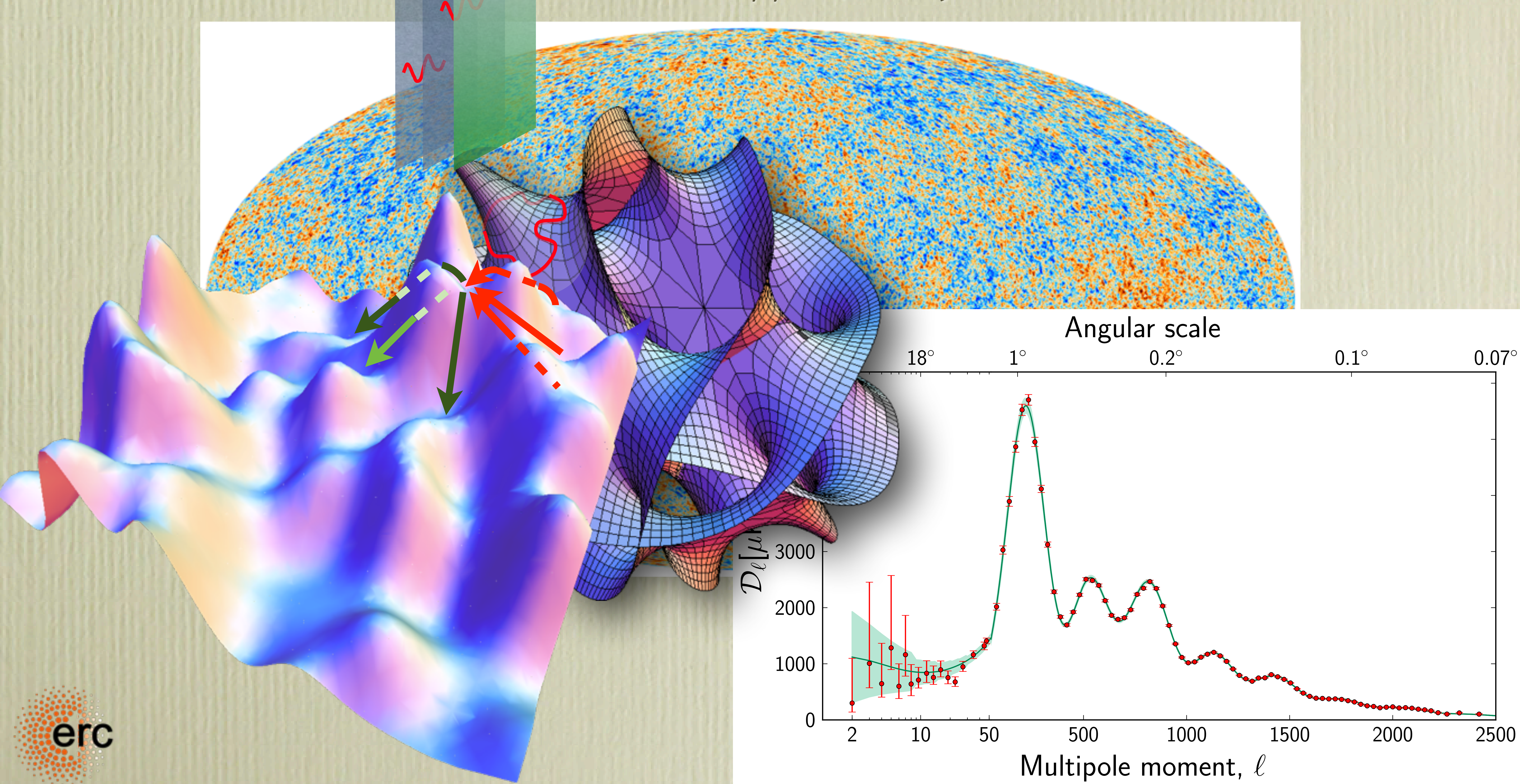
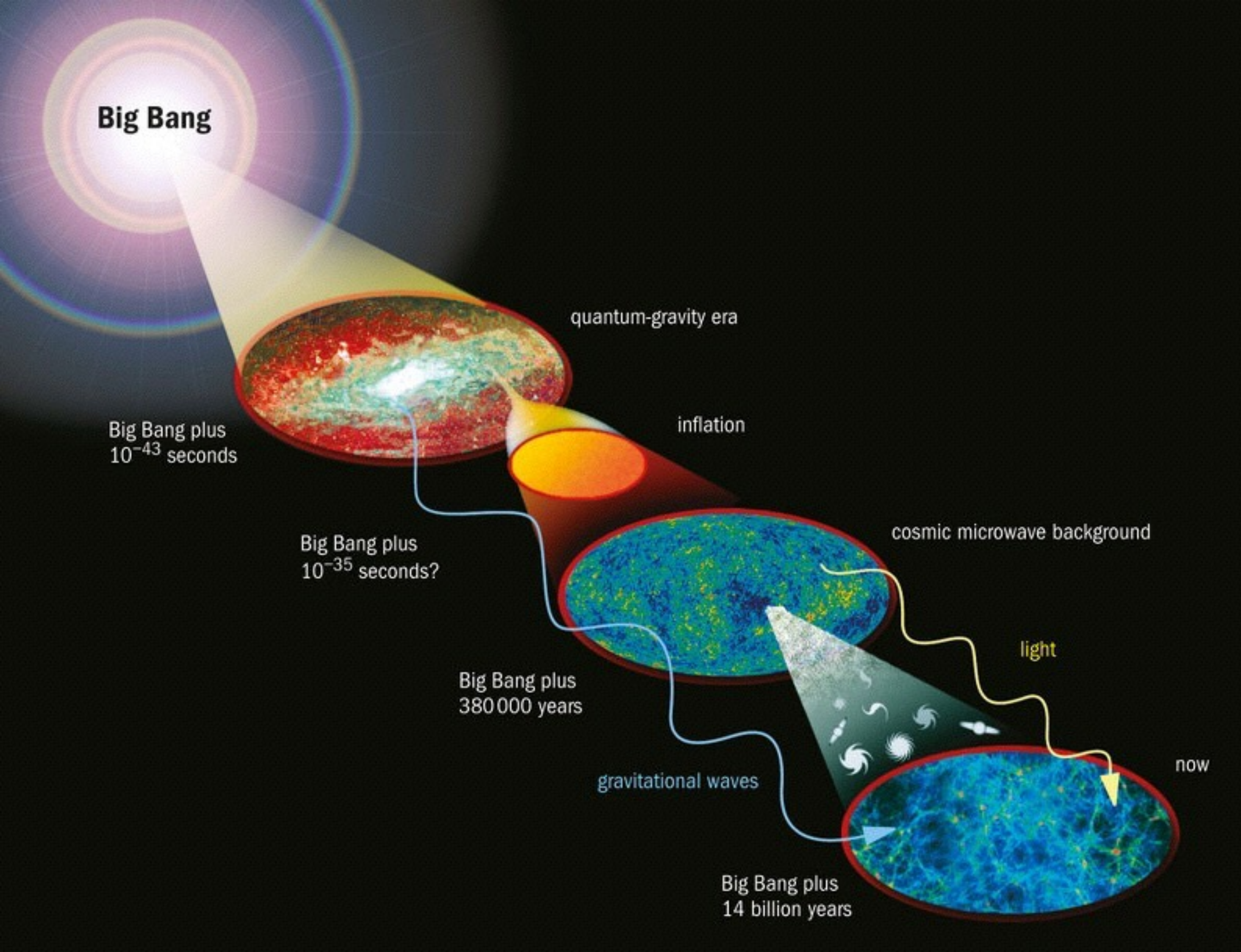
Non-equilibrium random matrix theory and inflation

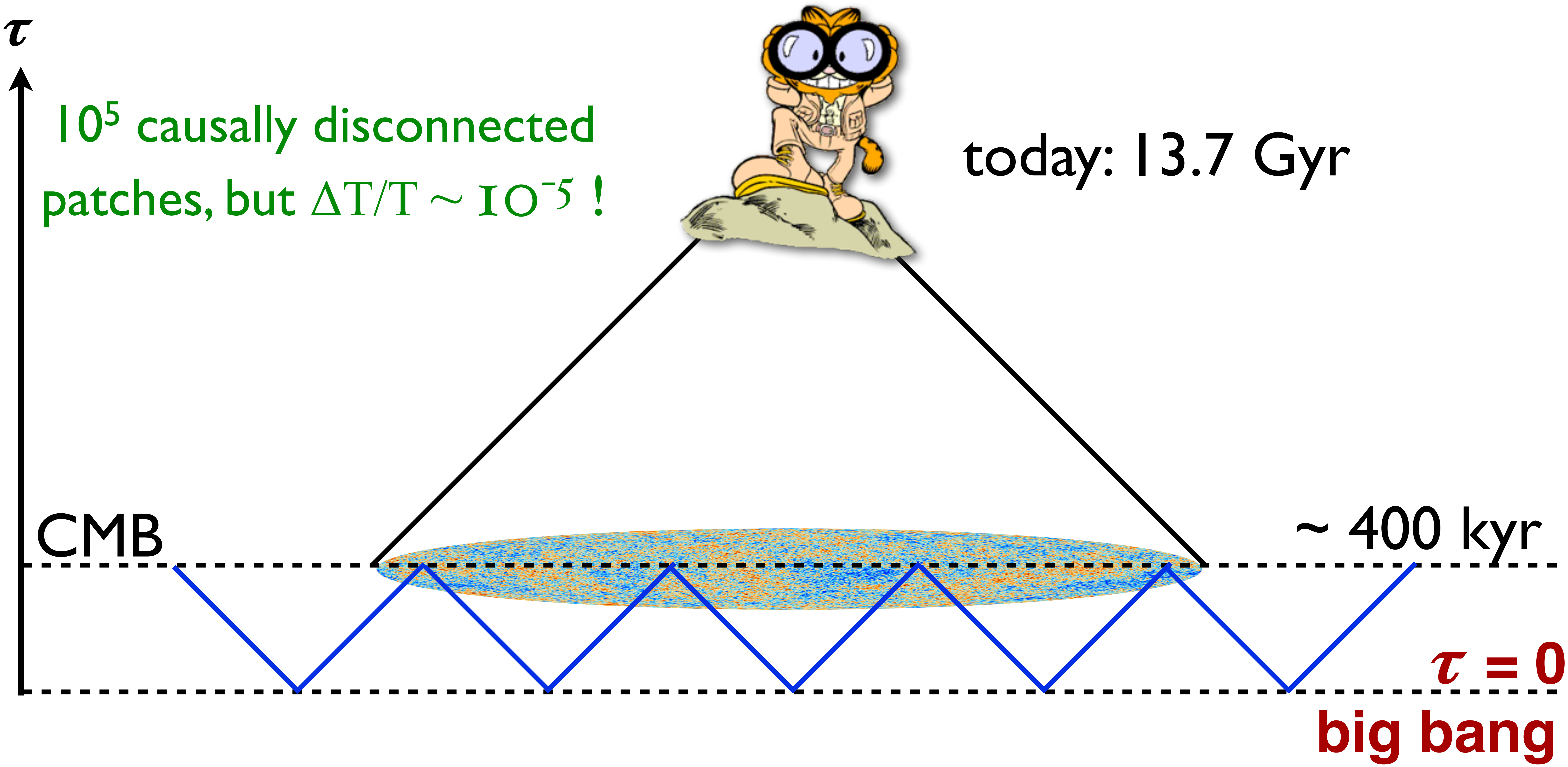
Alexander Westphal
in collaboration with: Francisco Pedro

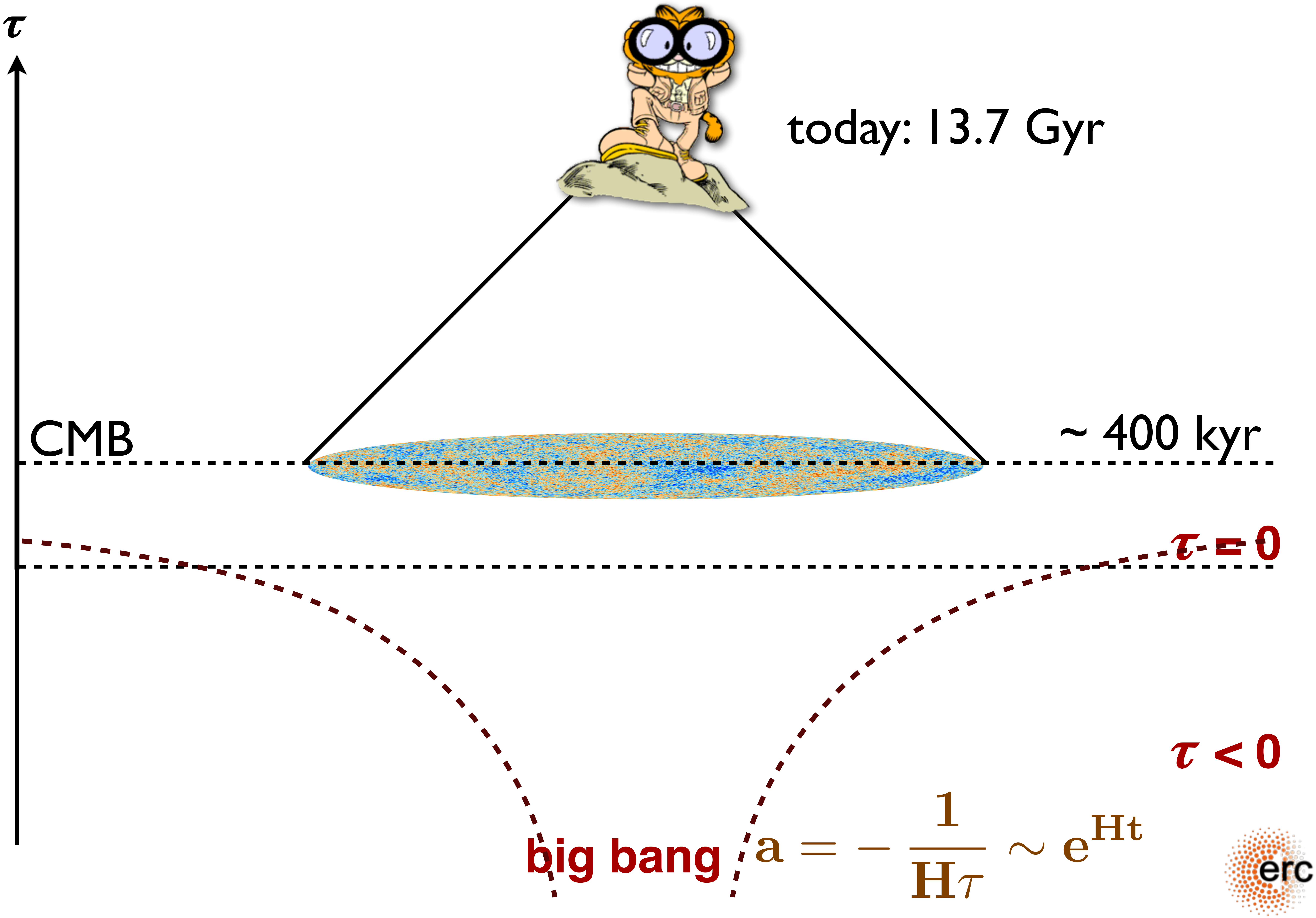
arXiv:1606.07768, 1609.xxxxx



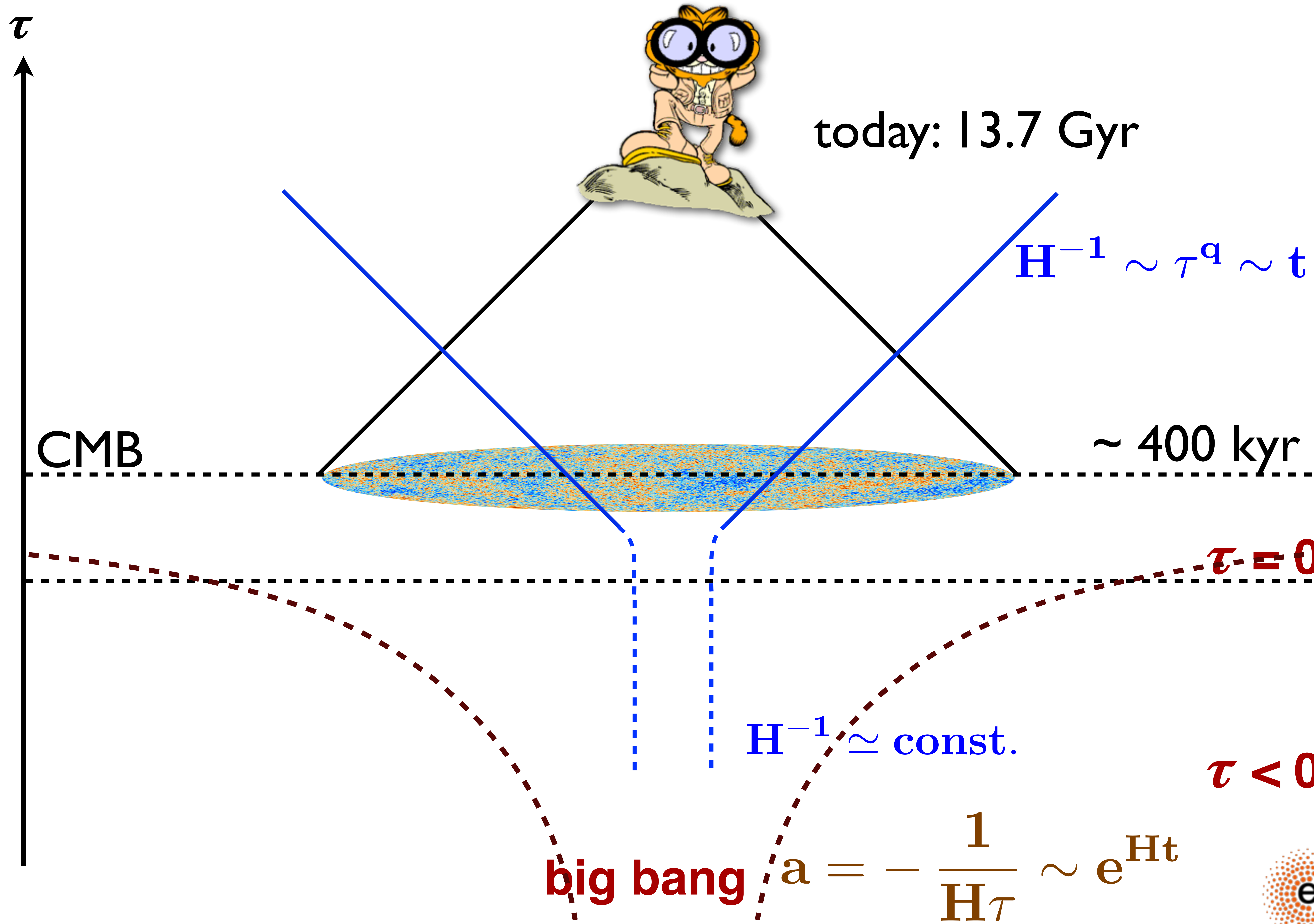

Big Bang
quantum-gravity era
Big Bang plus
10^-43 seconds
inflation
Big Bang plus
10^-35 seconds?
cosmic microwave background
light
Big Bang plus
380 000 years
gravitational waves
now
Big Bang plus
14 billion years

horizon problem of the hot big bang

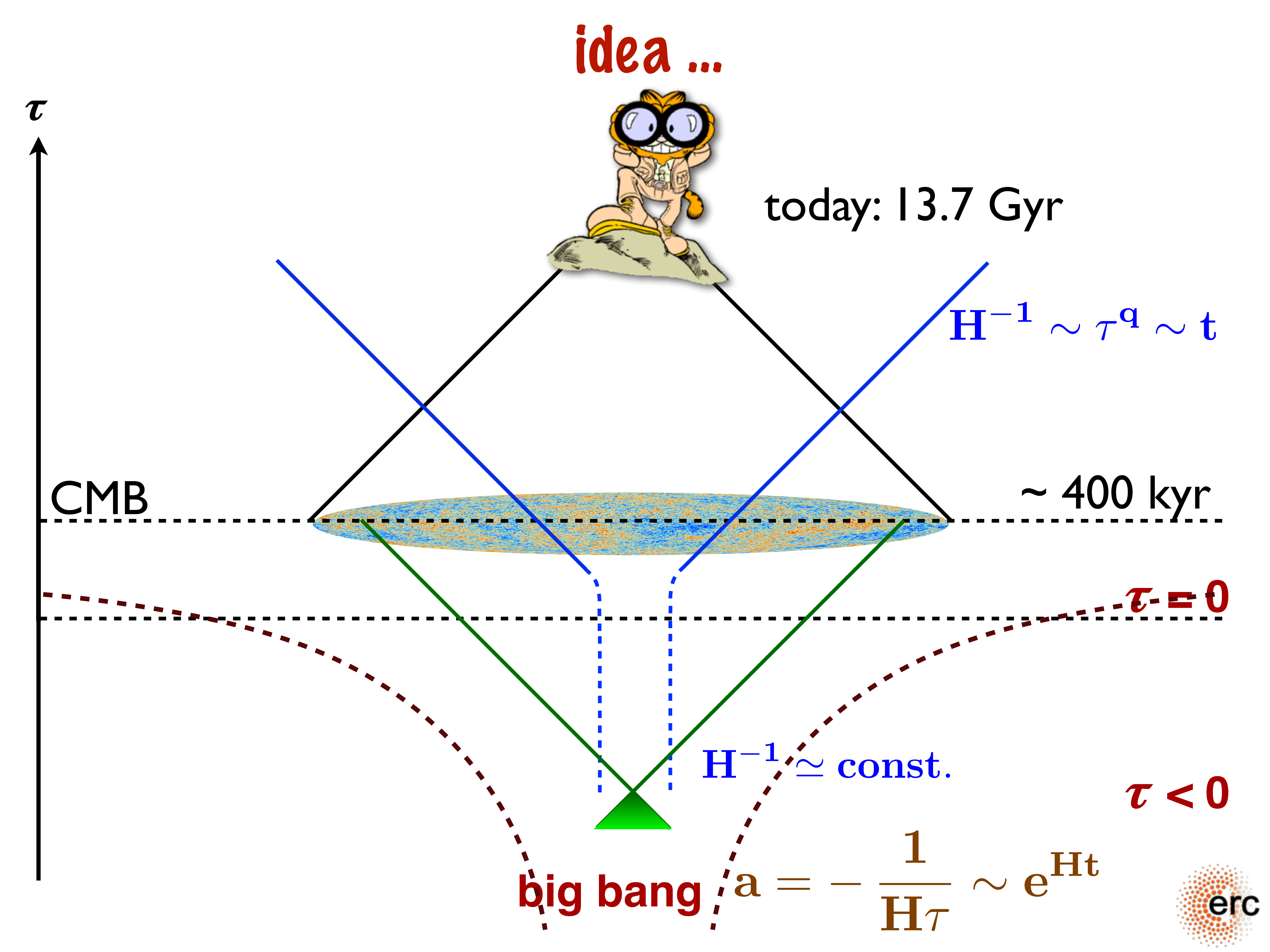



τ
today: 13.7 Gyr
CMB
~ 400 kyr
$\tau = 0$
$\tau < 0$
big bang
$a = -\frac{1}{H\tau} \sim e^{Ht}$

idea ...



τ
today: 13.7 Gyr
$\mathbf{H^{-1} \sim \tau^q \sim t}$
CMB
~ 400 kyr
$\tau = 0$
$\mathbf{H^{-1} \simeq const.}$
$\tau < 0$
big bang
$\mathbf{a = -\frac{1}{H\tau} \sim e^{Ht}}$



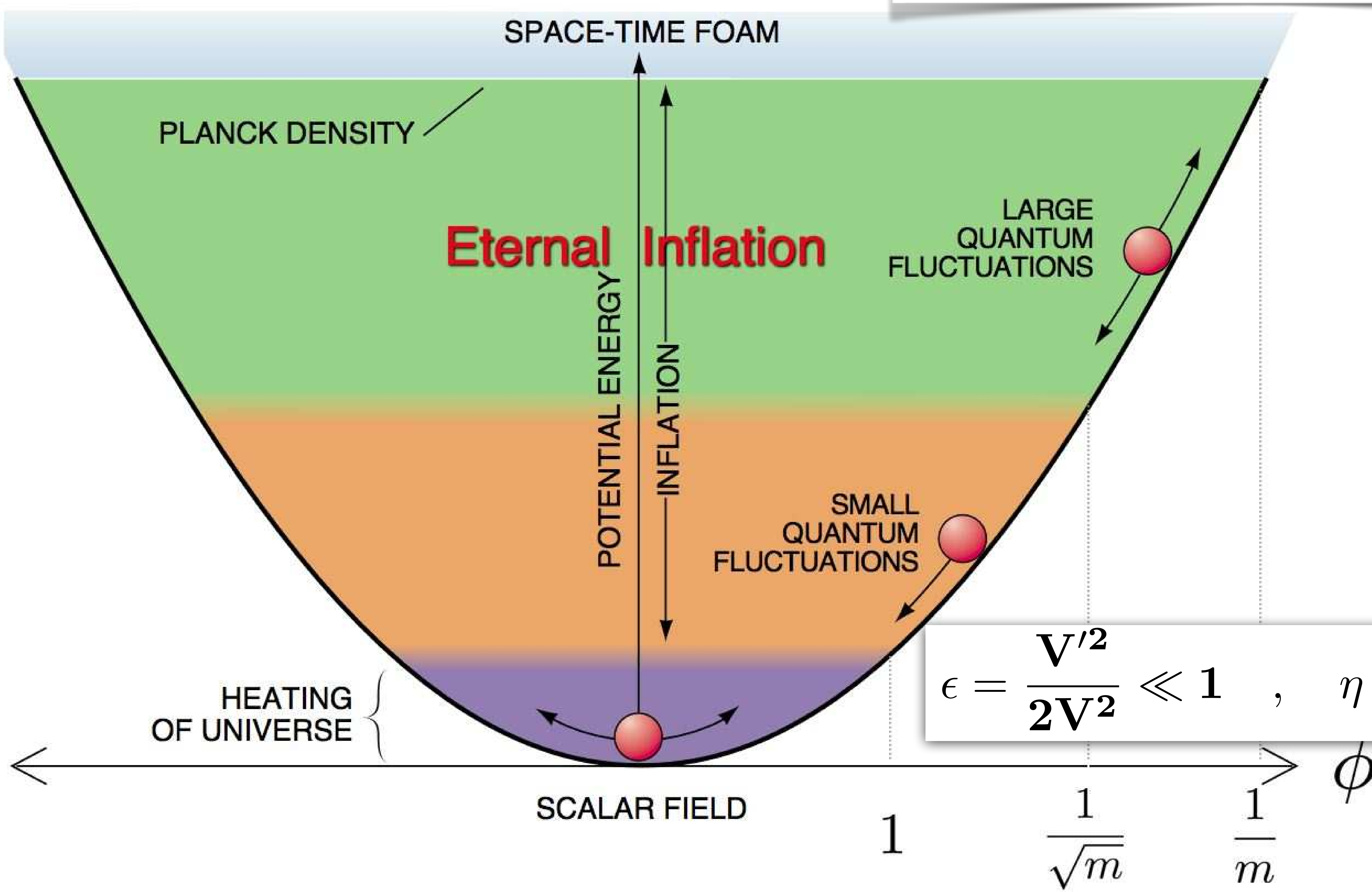
idea ...
τ
today: 13.7 Gyr
$\mathbf{H^{-1} \sim \tau^q \sim t}$
CMB
~ 400 kyr
$\tau = 0$
$\mathbf{H^{-1} \simeq const.}$
$\tau < 0$
big bang
$\mathbf{a = -\frac{1}{H\tau} \sim e^{Ht}}$
erc

slow-roll inflation ...

[Guth, Linde, Albrecht, Steinhardt '80s]

$$V(\phi) = \frac{m^2}{2}\phi^2$$

$$3H\dot{\phi} \simeq -V' \;, \;\; H^2 \simeq \frac{1}{3}V$$



$$\epsilon = \frac{\mathbf{V'^2}}{\mathbf{2V^2}} \ll \mathbf{1} \quad , \quad \eta = \frac{\mathbf{V''}}{\mathbf{V}} \ll \mathbf{1}$$

[picture from lecture notes: Linde '07]

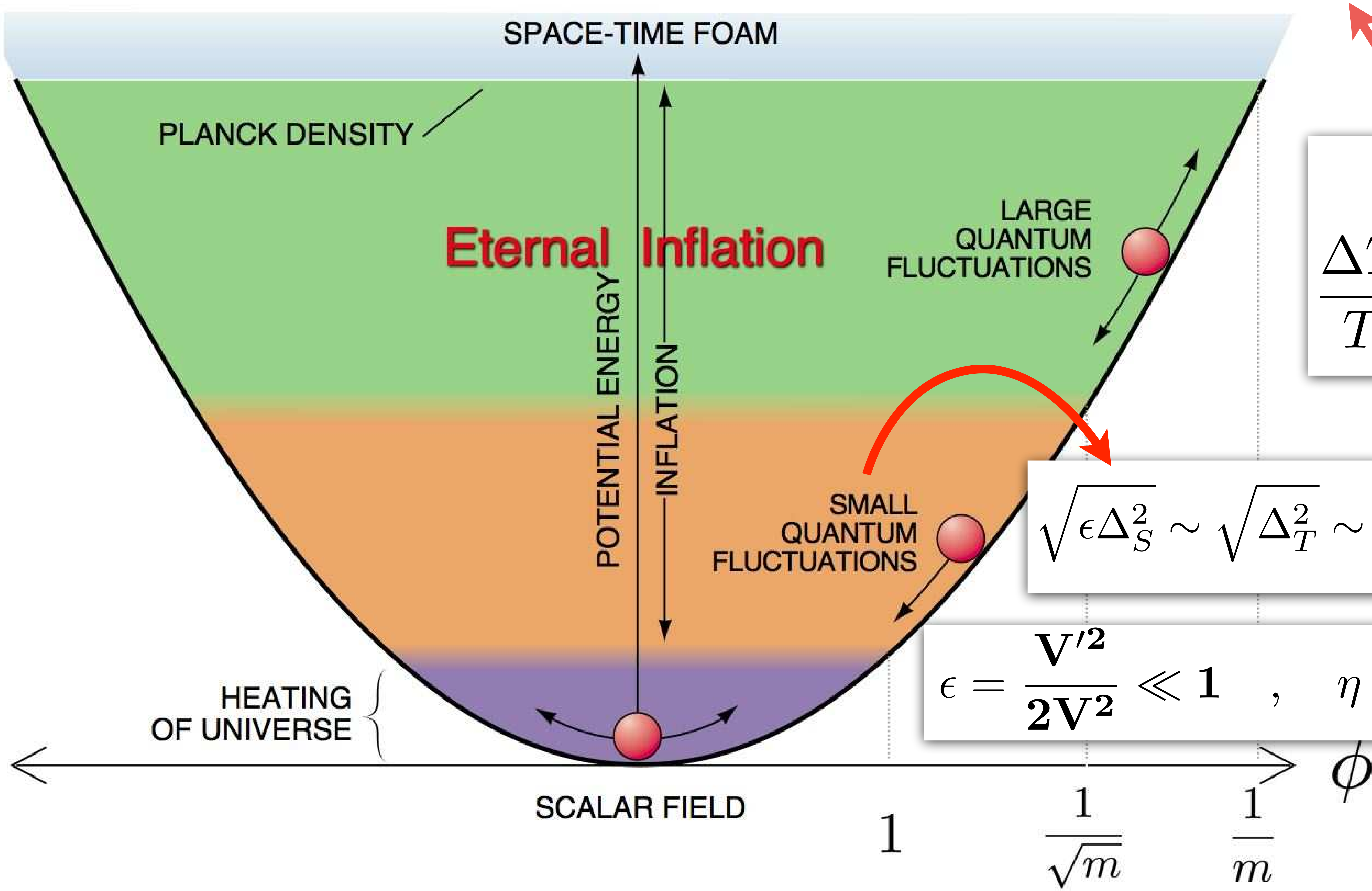
erc

slow-roll inflation ...

[Guth, Linde, Albrecht, Steinhardt '80s]

$$V(\phi) = \frac{m^2}{2}\phi^2$$

$$m \sim 10^{13}\ \mathrm{GeV}$$



CMB:
$$\frac{\Delta T}{T} \sim 10^{-5}$$

$$\sqrt{\epsilon \Delta_S^2} \sim \sqrt{\Delta_T^2} \sim \frac{H}{M_P} \sim 10^{-5}$$

$$\epsilon = \frac{\mathbf{V'^2}}{\mathbf{2V^2}} \ll \mathbf{1} \quad , \quad \eta = \frac{\mathbf{V''}}{\mathbf{V}} \ll \mathbf{1}$$

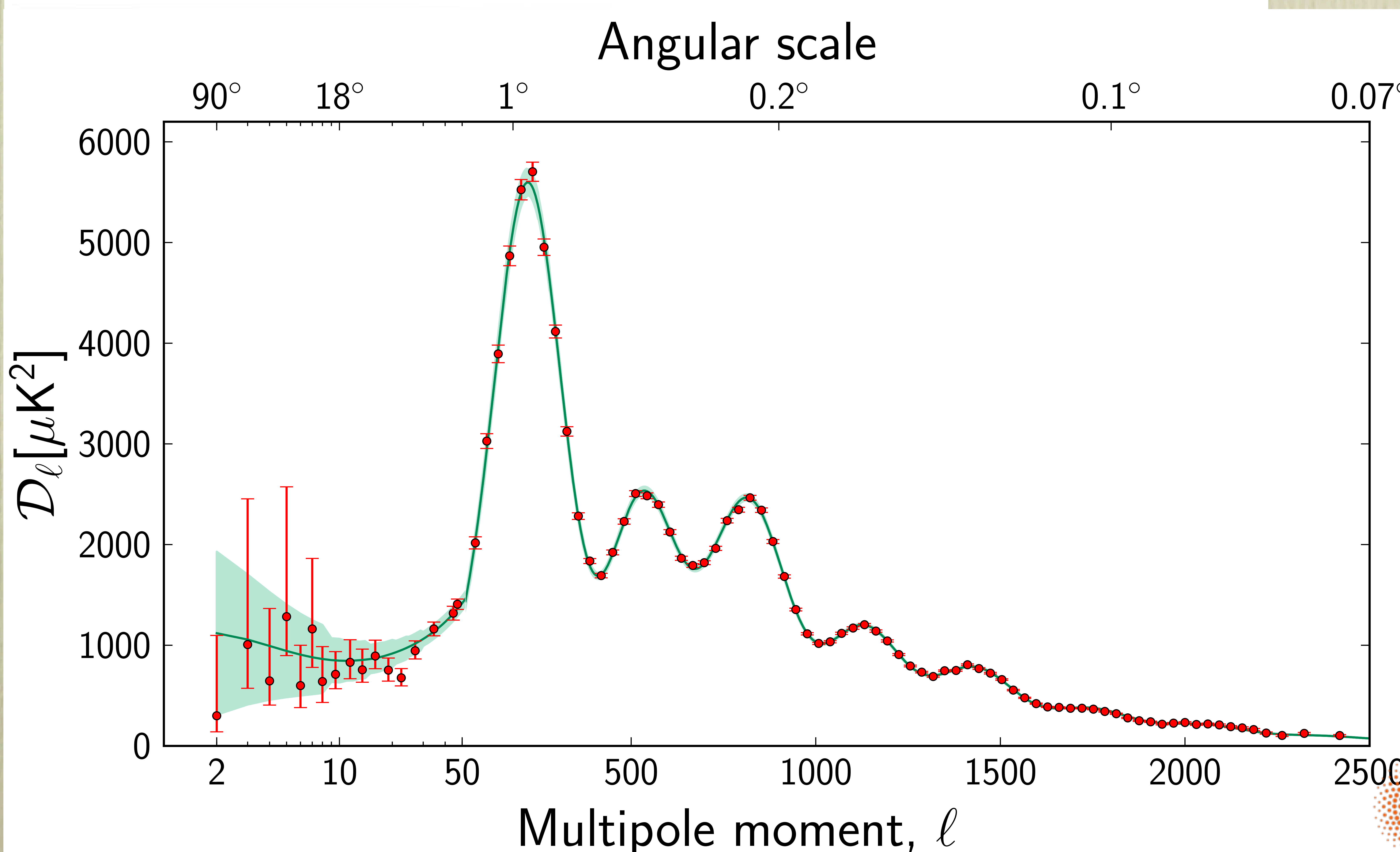
[picture from lecture notes: Linde '07]

erc

slow-roll inflation ...

[Guth, Linde, Albrecht, Steinhardt '80s]

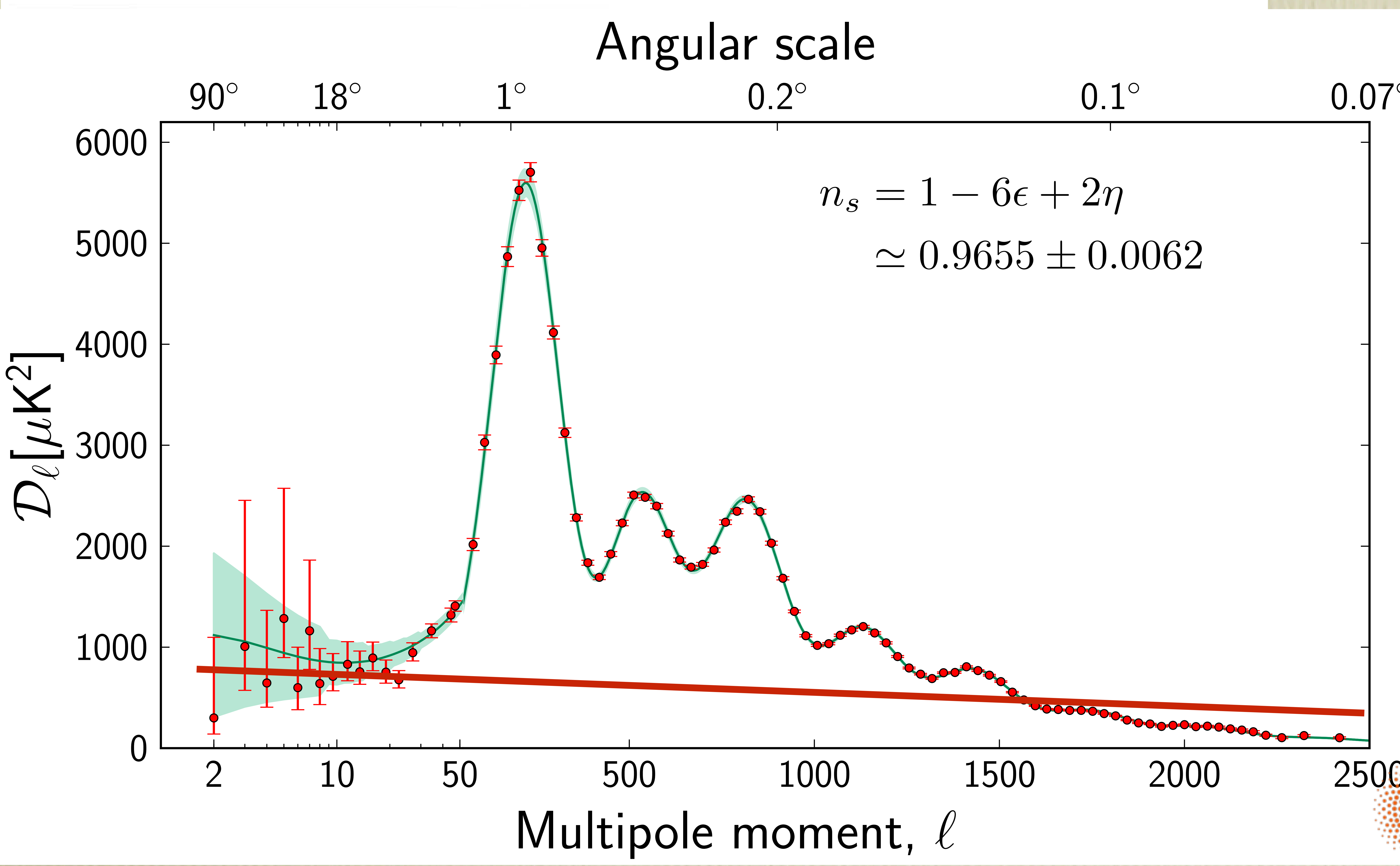
$$V(\phi) = \frac{m^2}{2}\phi^2$$



slow-roll inflation ...

[Guth, Linde, Albrecht, Steinhardt ‘80s]

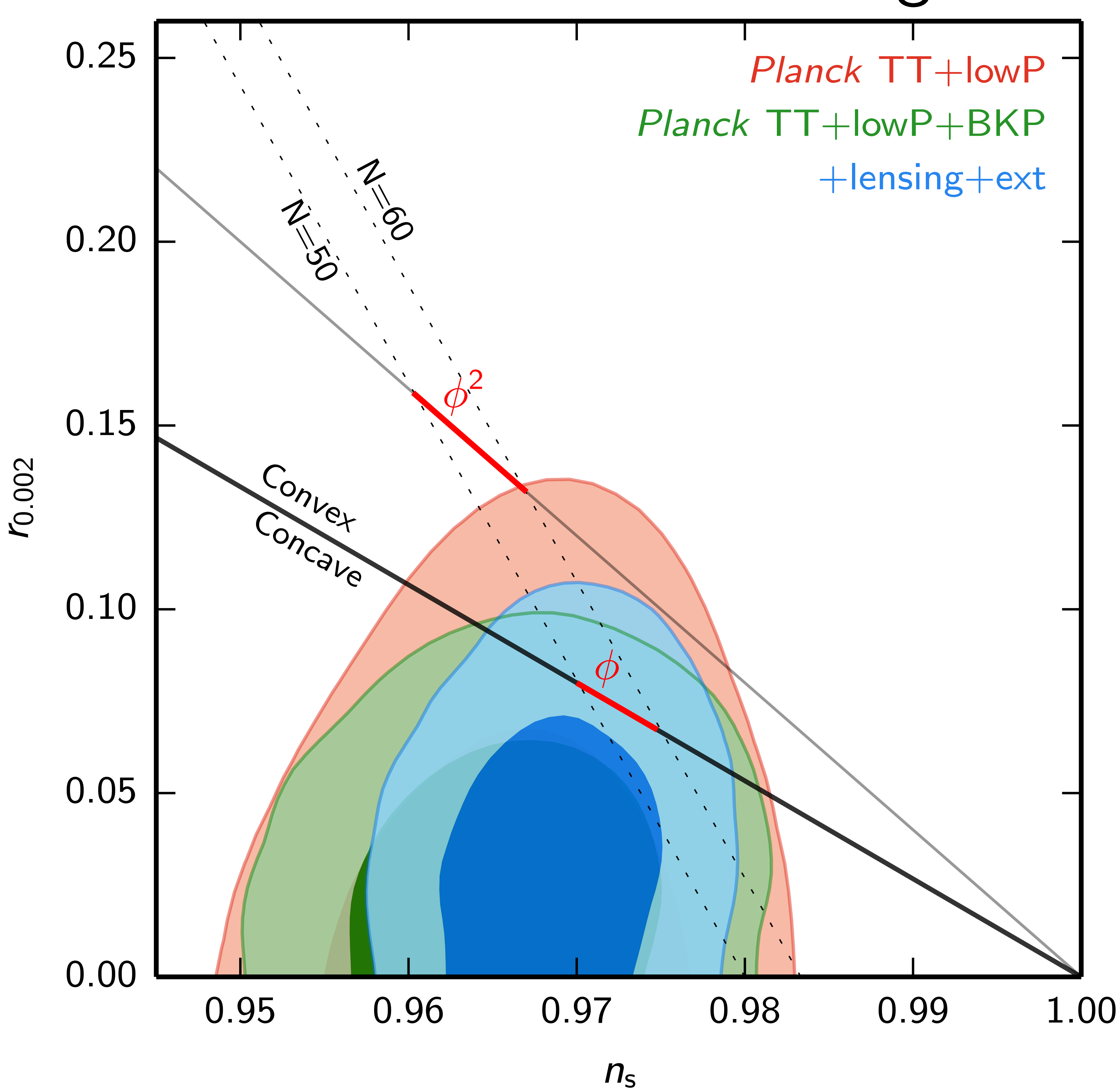
$$V(\phi) = \frac{m^2}{2}\phi^2$$

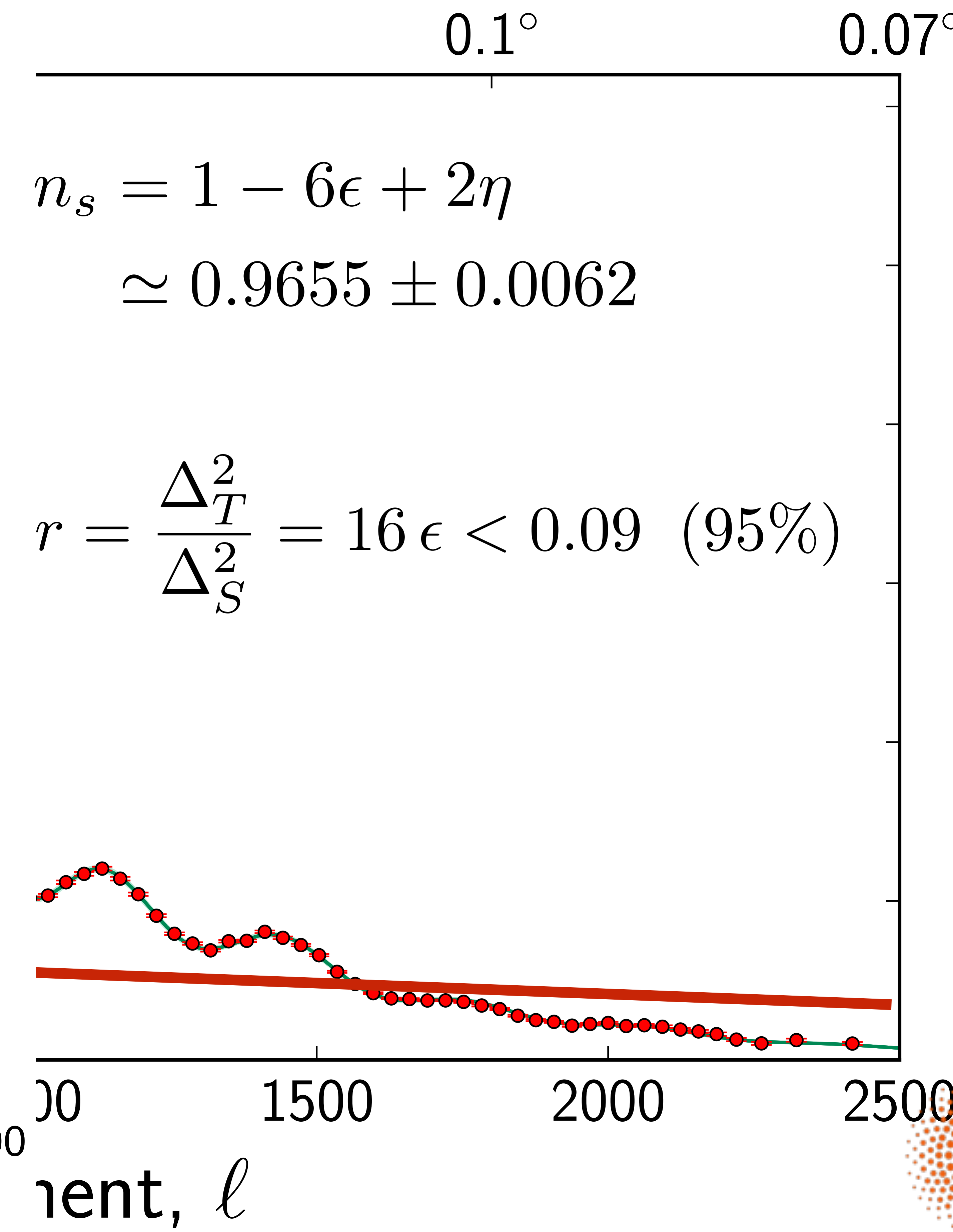


slow-roll inflation ...

[Guth, Linde, Albrecht, Steinhardt '80s]

$$V(\phi) = \frac{m^2}{2}\phi^2$$





$$n_s = 1 - 6\epsilon + 2\eta$$
$$\simeq 0.9655 \pm 0.0062$$

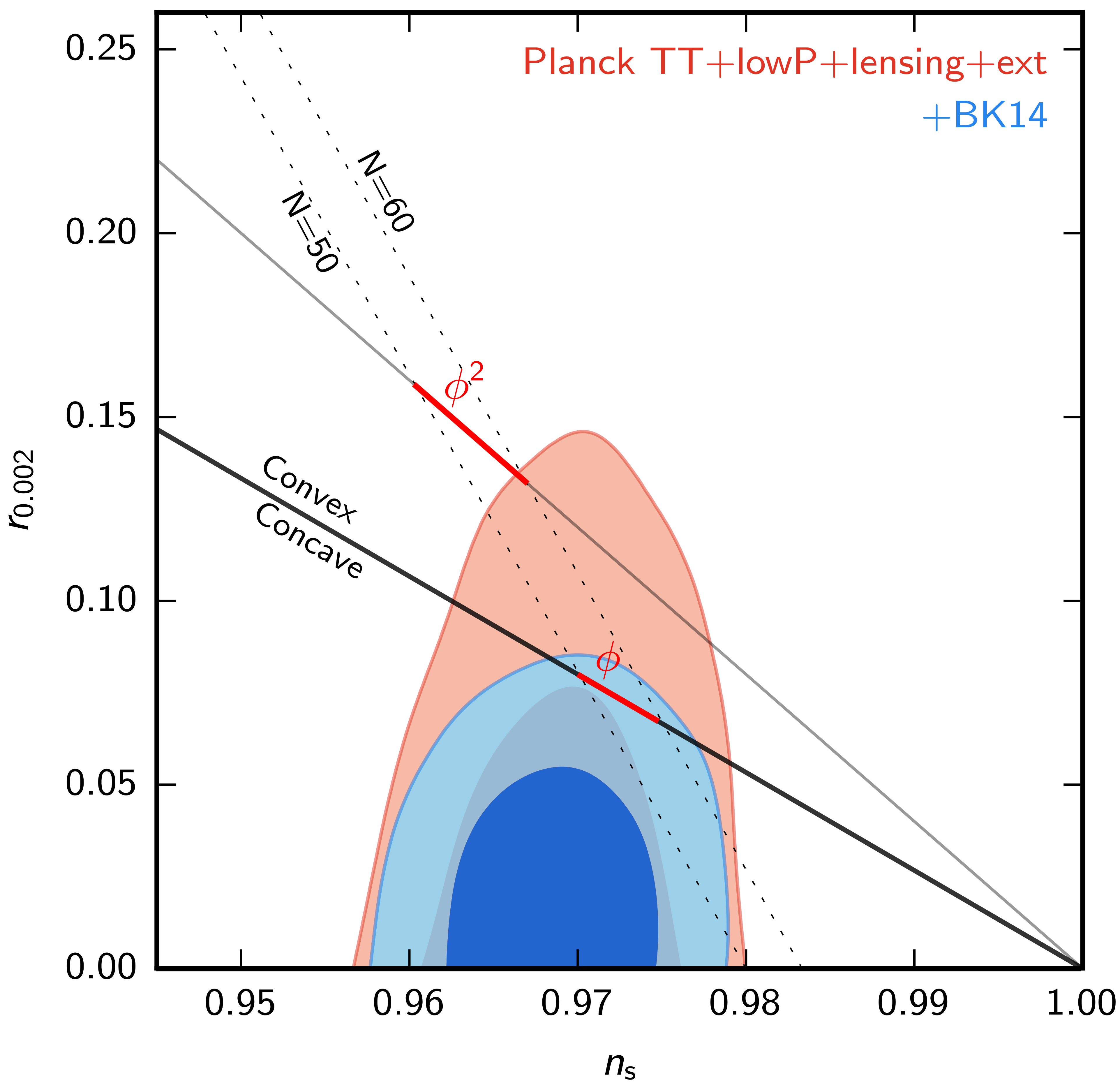
$$r = \frac{\Delta_T^2}{\Delta_S^2} = 16\,\epsilon < 0.09 \quad (95\%)$$

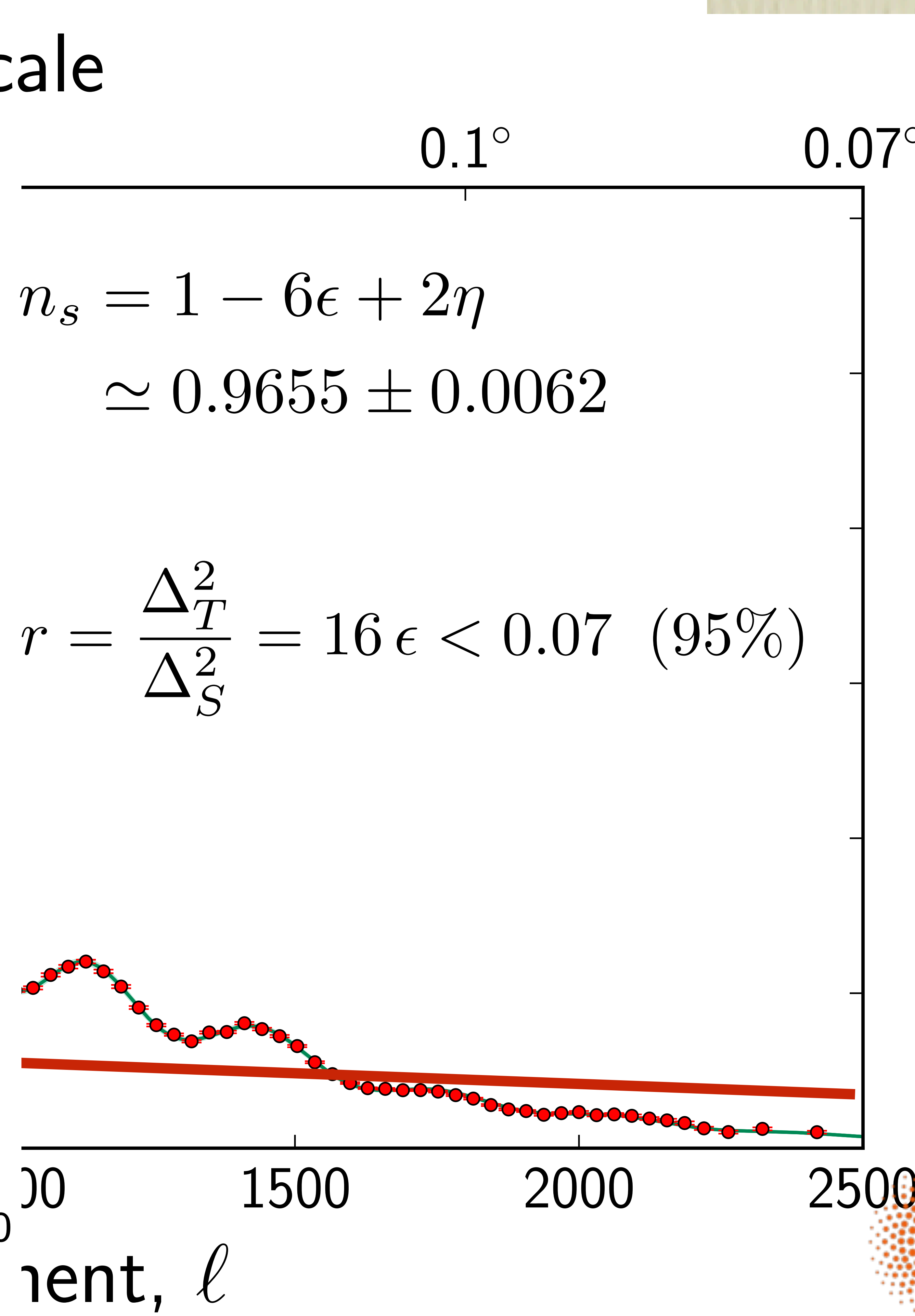
< 1

slow-roll inflation ...

[Guth, Linde, Albrecht, Steinhardt '80s]

$$V(\phi) = \frac{m^2}{2}\phi^2$$



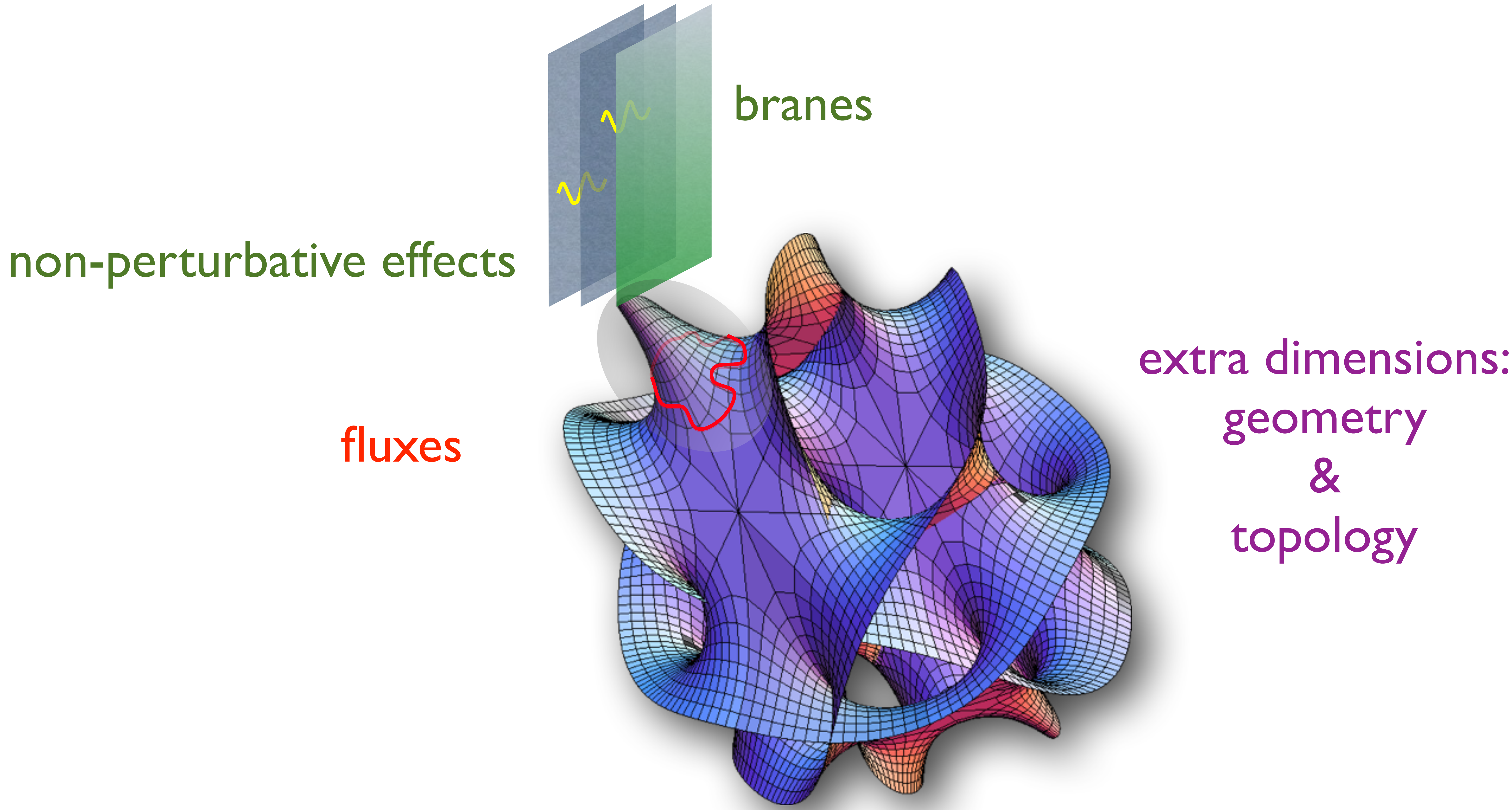


$$n_s = 1 - 6\epsilon + 2\eta$$

$$\simeq 0.9655 \pm 0.0062$$

$$r = \frac{\Delta_T^2}{\Delta_S^2} = 16\,\epsilon < 0.07 \quad (95\%)$$

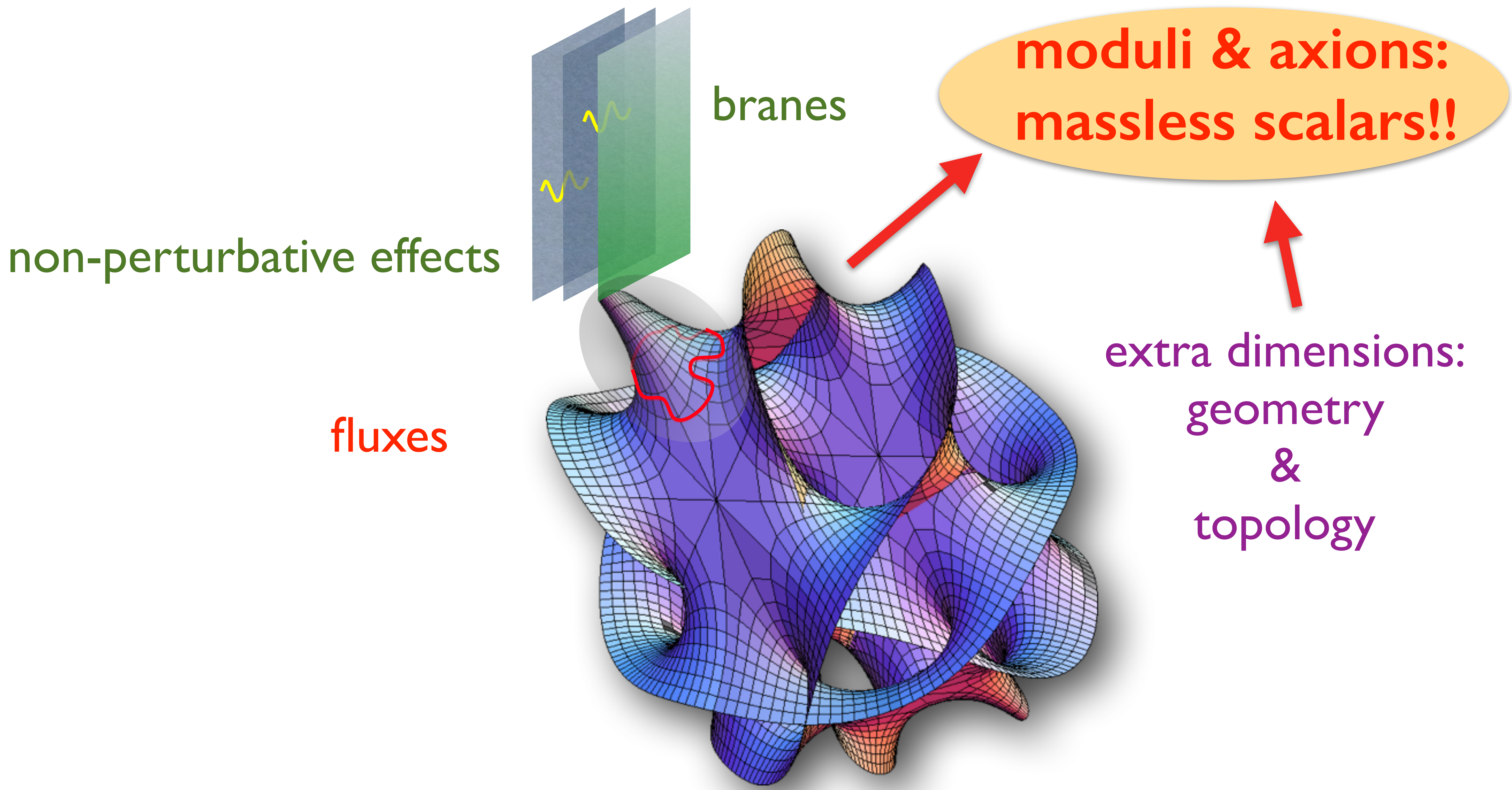
- string theory: many vacua - recipes w/ many ingredients …



- add need for control (weak coupling / large volume):

➥ we are often forced under lamp posts

- string theory: many vacua - recipes w/ many ingredients …



- add need for control (weak coupling / large volume):

➥ we are often forced under lamp posts

erc

varieties of string inflation ...

- tensor-to-scalar ratio linked to field range:

$$\frac{\Delta\phi(N_e)}{M_{\rm P}} \gtrsim \frac{N_e}{50}\sqrt{\frac{r}{0.01}} \quad , \quad r = \frac{\mathcal{P}_T}{\mathcal{P}_S}$$

[Lyth '97]

- $r << O(1/N_e^2)$ models:

$$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$$

- $r = O(1/N_e^2)$ models:

$$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$$

- $r = O(1/N_e)$ models:

$$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$$

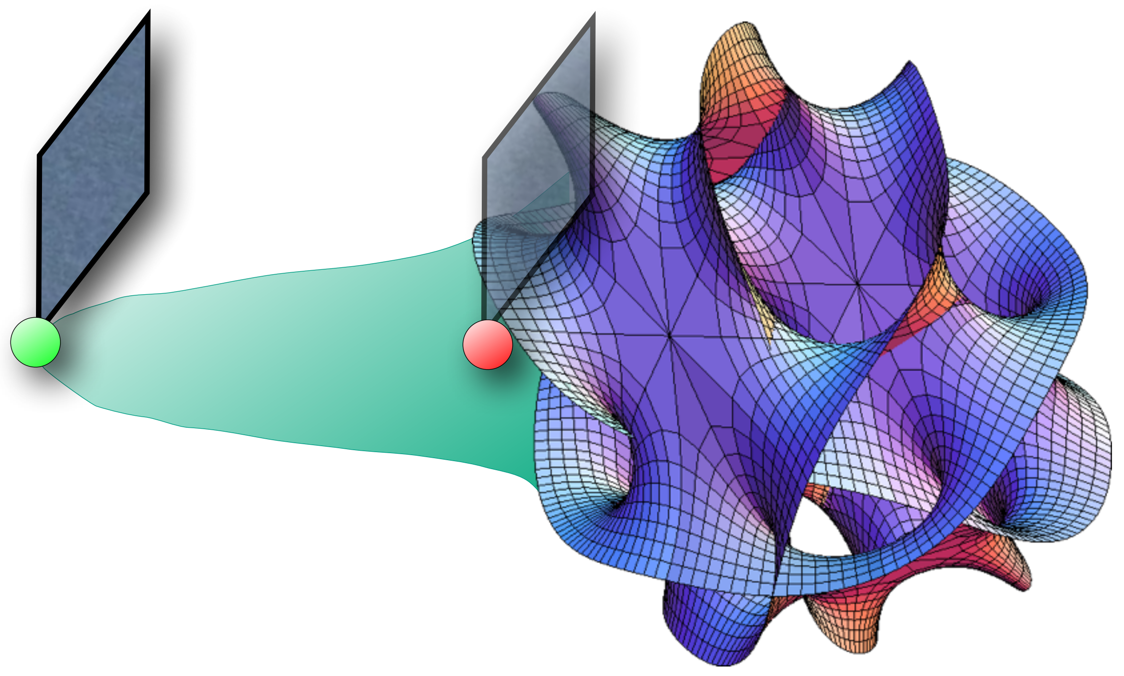
erc

varieties of string inflation ...

- $r << O(1/N_e^2)$ models:

$$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$$

warped D-brane inflation & DBI;



- $r = O(1/N_e^2)$ models:

$$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$$

- $r = O(1/N_e)$ models:

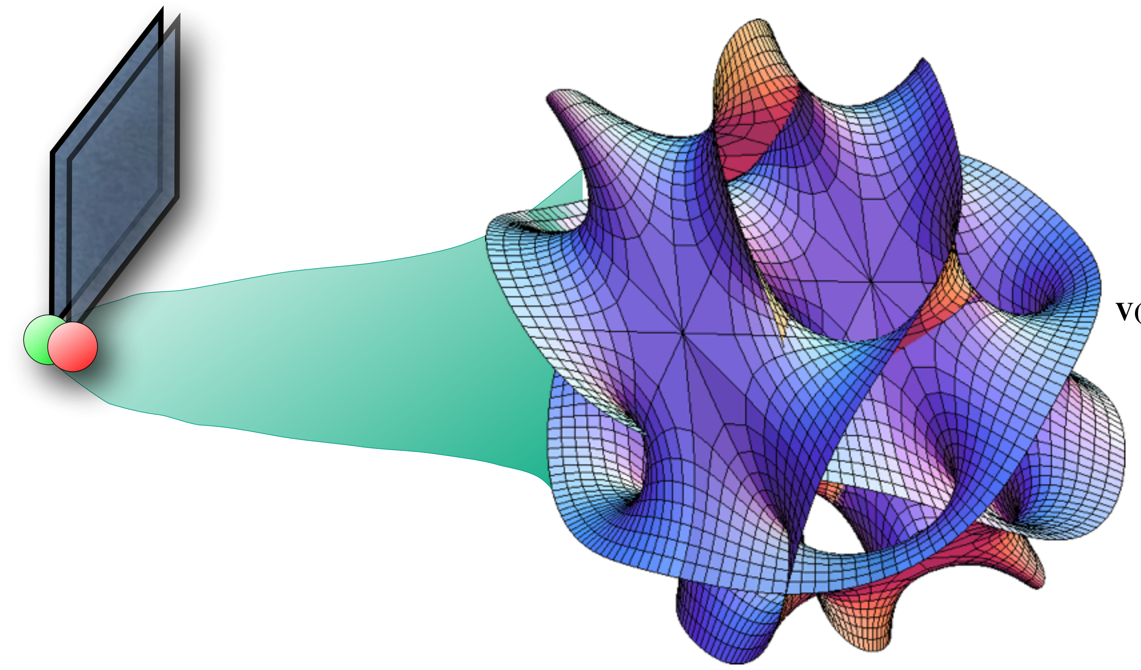
$$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$$

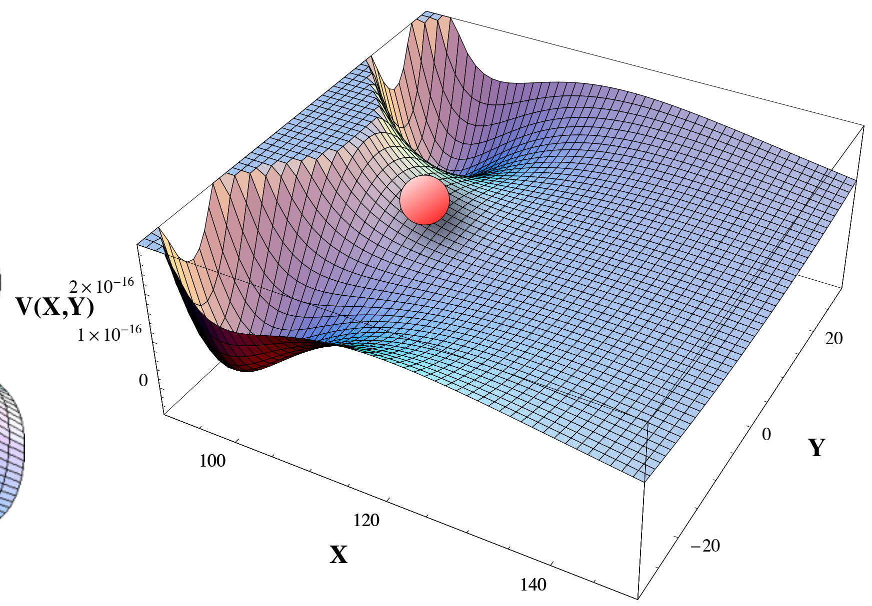
varieties of string inflation ...

- $r << O(1/N_e^2)$ models:

$$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$$

warped D-brane inflation & DBI;
varieties of Kähler moduli inflation





- $r = O(1/N_e^2)$ models:

$$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$$

- $r = O(1/N_e)$ models:

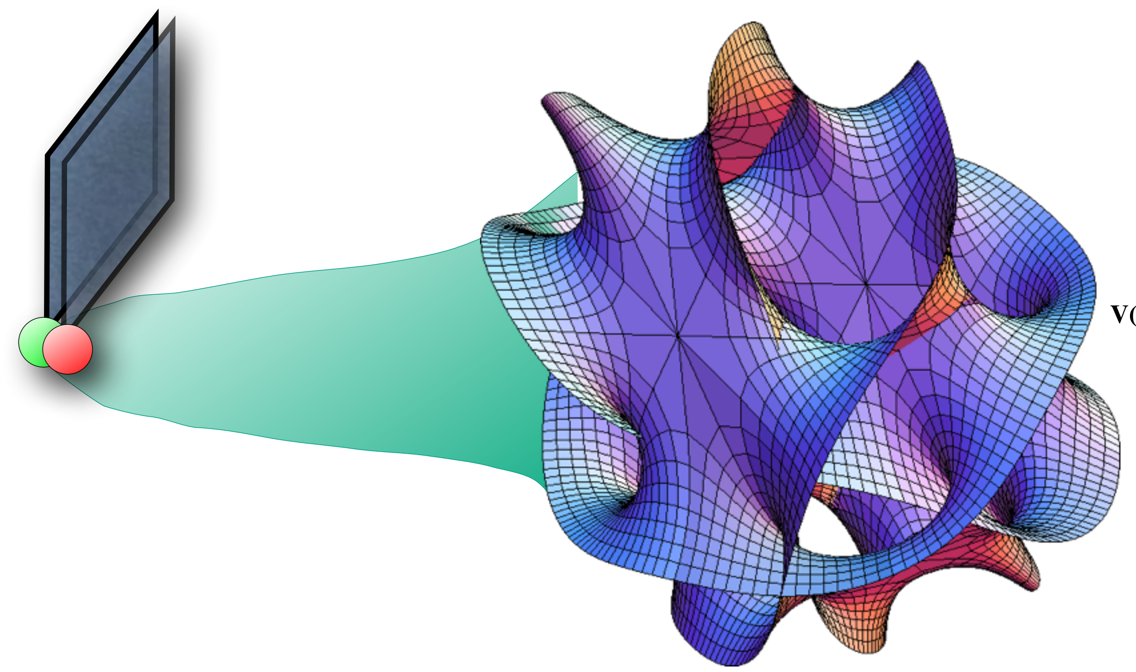
$$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$$

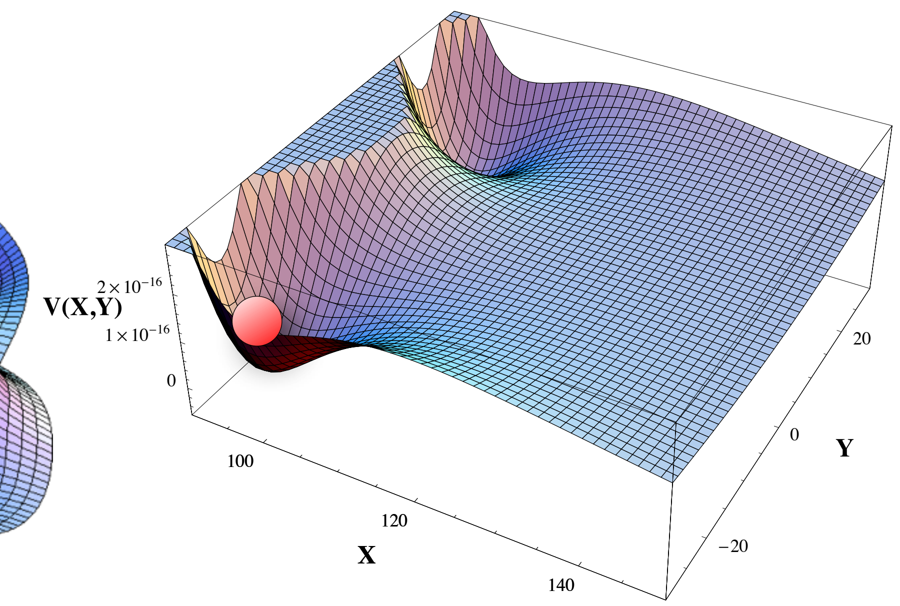
varieties of string inflation ...

- $r << O(1/N_e^2)$ models:

$$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$$

warped D-brane inflation & DBI;
varieties of Kähler moduli inflation

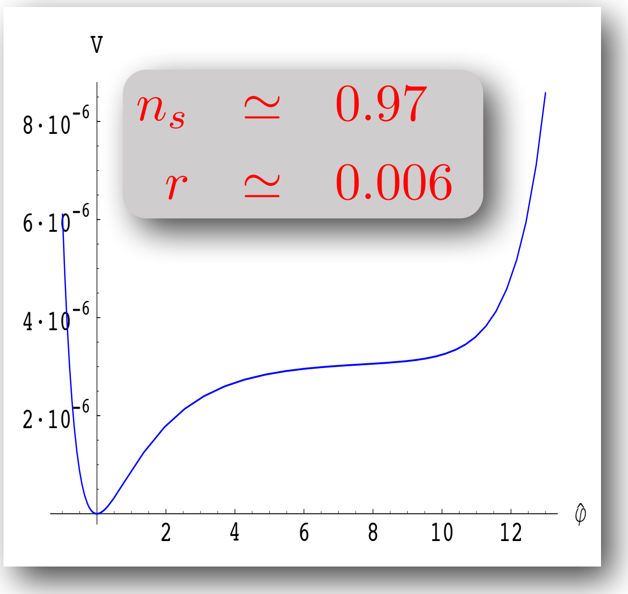




- $r = O(1/N_e^2)$ models:

$$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$$

fibre inflation in LARGE volume scenarios (LVS)



<u>low-*l* suppression generic:</u>
[Cicoli,Downes,Dutta,Pedro & AW]
[Kallosh, Linde & AW]

- $r = O(1/N_e)$ models:

$$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$$

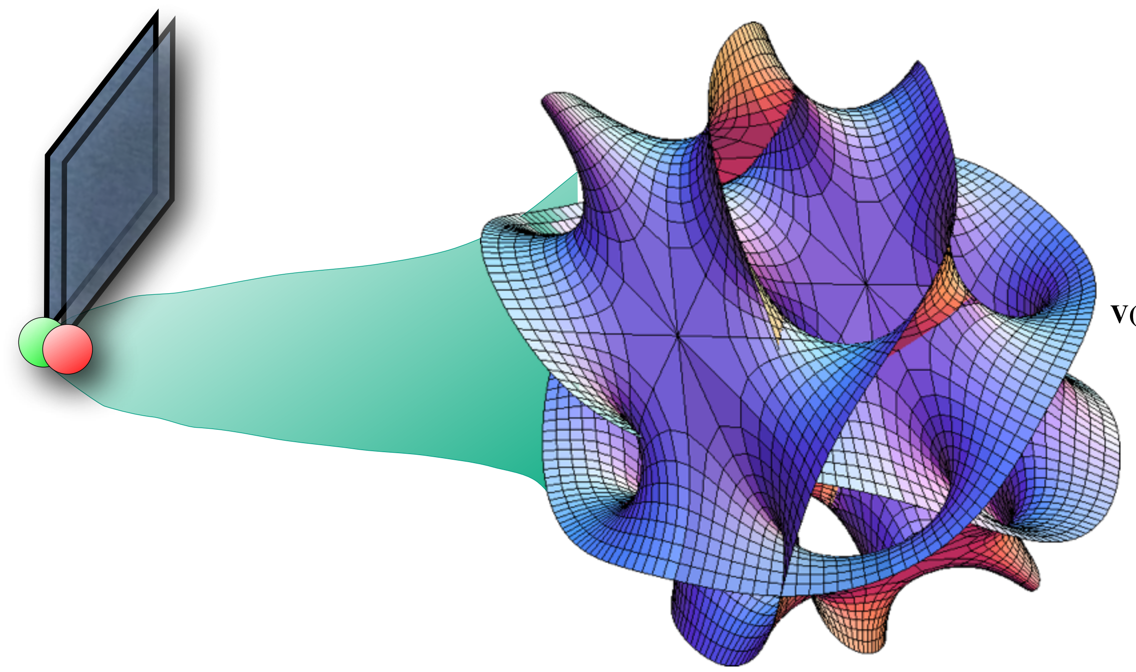
erc

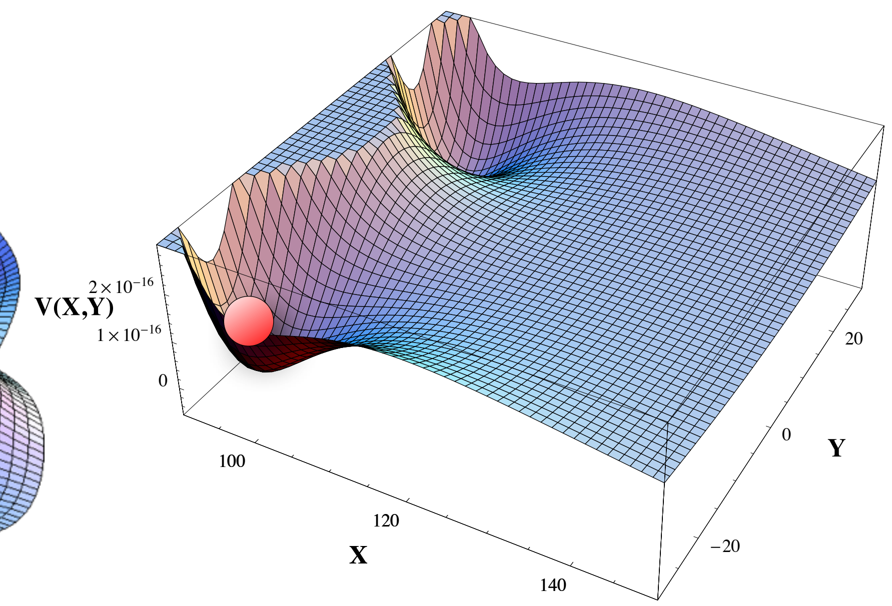
varieties of string inflation ...

- $r \ll O(1/N_e^2)$ models:

$$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$$

warped D-brane inflation & DBI;
varieties of Kähler moduli inflation



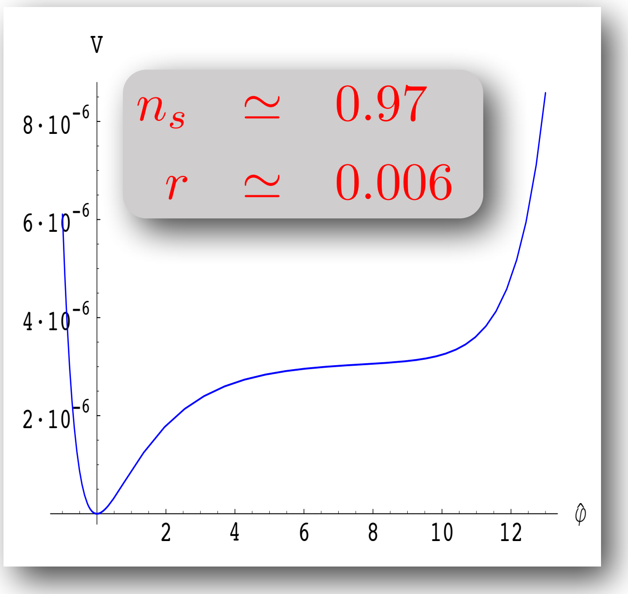


- $r = O(1/N_e^2)$ models:

$$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$$

fibre inflation in LARGE volume scenarios (LVS)

<u>low-*l* suppression generic:</u>
[Cicoli,Downes,Dutta,Pedro & AW]
[Kallosh, Linde & AW]



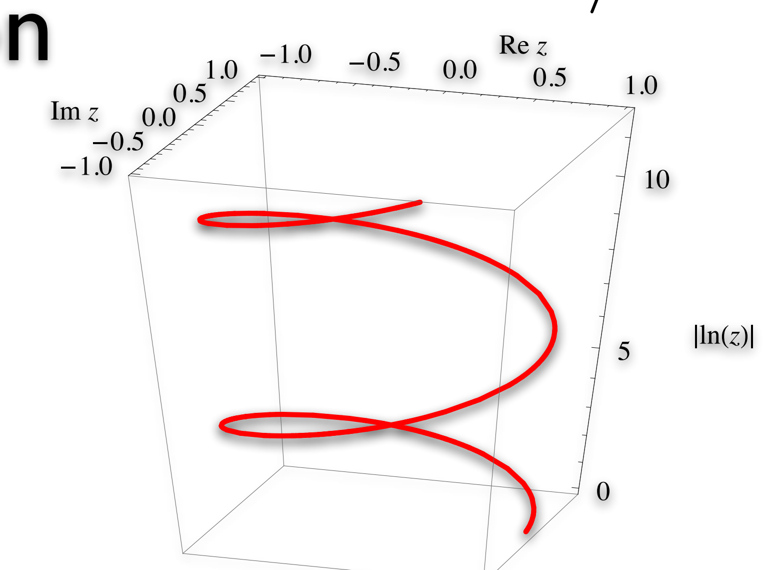
- $r = O(1/N_e)$ models:

$$\Delta\phi \sim \sqrt{N_e}M_P \gg M_P \quad \Rightarrow$$

axion monodromy inflation
2-axion inflation
N-flation

observable $r > 0.01$

$T_{D5/NS5}$



small-field string inflation ...

- Brane-Antibrane Dvali & Tye; Alexander; Dvali, Shafi & Solganik; Burgess, Majumdar, Nolte, Quevedo, Rajesh & Zhang.

no mod. stab.

- D3-D7 Dasgupta, Herdeiro, Hirano & Kallosh; Hsu, Kallosh & Prokushkin; Hsu & Kallosh; Aspinwall & Kallosh; Haack, Kallosh, Krause, Linde, Lüst & Zagermann; ...
- ➡ warped brane-antibrane Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi; Firouzjahi & Tye; Burgess, Cline, Stoica & Quevedo; Iizuka & Trivedi; Krause & Pajer; Baumann, Dymarsky, Klebanov, McAllister & Steinhardt; Baumann, Dymarsky, Kachru, Klebanov & McAllister; ...
- DBI Silverstein & Tong; Alishahiha, Silverstein & Tong; Chen; Chen; Shiu & Underwood; Leblond & Shandera; …
- Racetrack Blanco-Pillado, Burgess, Cline, Escoda, Gomez-Reino, Kallosh, Linde & Quevedo; Linde & AW; ...
- Kähler moduli Conlon & Quevedo; AW; Bond, Kofman, Prokushkin & Vaudrevange; Ben-Dayan, Jing, AW & Zarate …

$r = 0$

---

large-field string inflation ...

- ➡ Fibre inflation ($r < 0.01$) Cicoli, Burgess & Quevedo / + de Alwis; Broy, Pedro & AW; Broy, Ciupke, Pedro & AW; Cicoli, Ciupke, de Alwis & Muia

$r \sim 0.001$

- Single-Axion inflation with $f > M_P$ Grimm; Blumenhagen & Plauschinn;
- 2-Axion inflation Kim, Nilles & Peloso; Berg, Pajer & Sjors; Kappl, Krippendorf & Nilles; Long, McAllister & McGuirk;Tye & Wong; Ben-Dayan, Pedro & AW; Gao, Li & Shukla ...
- N-flation Dimopoulos, Kachru, McGreevy, Wacker; Easther & McAllister; Grimm; Cicoli, Dutta & Maharana; Choi, Kim & Yun; Bachlechner, Dias, Frazer & McAllister
- ➡ axion monodromy Silverstein & AW; McAllister, Silverstein & AW; Flauger, McAllister, Pajer, AW & Xu; Dong, Horn, Silverstein & AW; Shlaer; Gur-Ari; Palti & Weigand; Marchesano, Shiu & Uranga; Blumenhagen & Plauschinn; Hebecker, Kraus & Witkowski; Ibanez & Valenzuela; Kaloper, Lawrence & Sorbo; McAllister, Silverstein, AW & Wrase; Franco, Galloni, Retolaza & Uranga;

$r \sim 0.1$

erc

- inflation in string theory …

many (N) moduli and axions

e.g. axion monodromy …

[McAllister, Silverstein, AW '08]
many since ...

fibre inflation …

[Burgess et al. ’08/’16; Cicoli et al. '16]
[Broy, Ciupke, Pedro & AW ’15]
[Broy, Pedro & AW ’14]

- field spaces:

with structure: axions, approximate no-scale directions

- inflation in string theory …

  many (N) moduli and axions

  e.g. axion monodromy …

  [McAllister, Silverstein, AW '08]
  many since ...

  fibre inflation …

  [Burgess et al. ’08/’16; Cicoli et al. '16]
  [Broy, Ciupke, Pedro & AW ’15]
  [Broy, Pedro & AW ’14]

- field spaces:

  with structure: axions, approximate no-scale directions

  no structure: e.g. flux-stabilized moduli

- statistical description of the landscape:

  ➥ Random Matrix Theory of critical points

- describe system by large-N Gaussian random matrices:

much of the structure described by eigenvalue dynamics

likely eigenvalue configurations	Dyson ⟷ Brownian Motion	equilibrium states of 1D gas of charged particles

[Dyson '62 & '63]

example: *N* scalar fields coupled to Einstein gravity

$$\mathcal{L} = \frac{1}{2}\partial\phi^a\partial\phi^a - \Lambda_v^4\sqrt{N}\left[v_0 + v_a\phi^a + \frac{1}{2}v_{ab}\phi^a\phi^b\right]$$

Let v_{ab} be a **Wigner matrix**

$$W = \frac{A + A^\dagger}{2}$$

$$A \sim \Omega(\mu, \sigma)$$

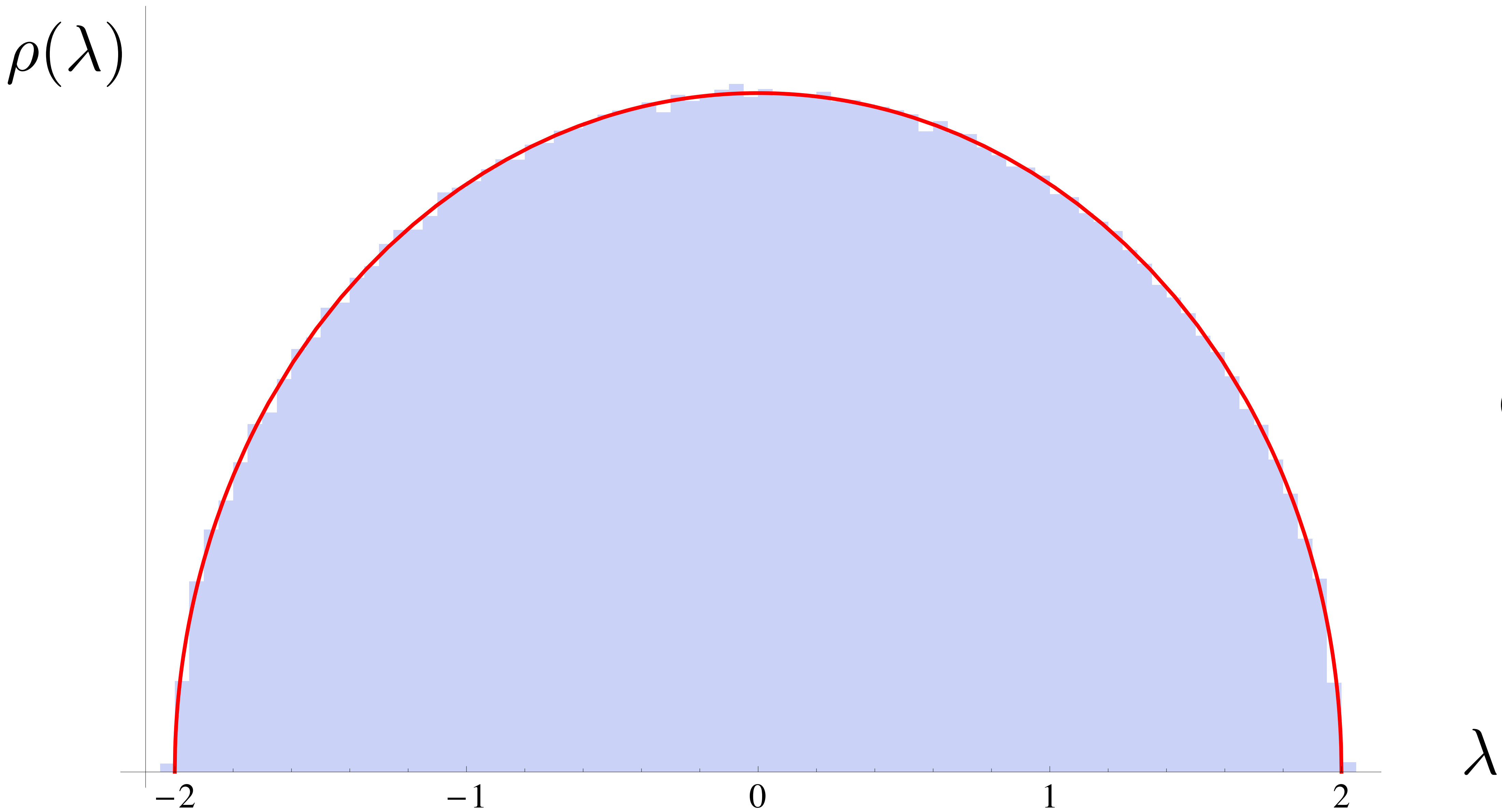
- relevant matrices for inflation - Wigner ensemble:

➥ approximates the tachyonic directions in both random SUGRA and non-SUGRA cases

[Aazami & Easther '05]

[Marsh, McAllister & Wrase '11]

[Chen, Shiu, Sumitomo & Tye '11]



eigenvalue spectrum of a large-N Gaussian/Wigner random matrix

- eigenvalue density - semicircle law:

$$\rho(\lambda) = \frac{1}{\pi N \sigma^2}\sqrt{2N\sigma^2 - \lambda^2} \quad , \quad \sigma = \sqrt{\frac{2}{N}}$$

- two relevant questions:

  - how often do I get a certain eigenvalue distribution in a given equilibrium matrix ensemble (e.g. inflation) ?

  answered by: [Dean & Majumdar ’06/‘08]

  - given a non-equilibrium configuration — how fast do I relax to another configuration (e.g. exit from inflation) ?

  main result here!

many applications in systems with large-N correlation matrices:

condensed matter, nuclear physics, string landscape, computational biology, quantitative finance, …

- eigenvalue probability distribution (pdf) [Dyson '62 & '63]

$$dP = \mathcal{C} \exp\left\{ -\frac{\beta}{2\sigma^2} \mathrm{Tr} M^2 \right\} dM_{ij} = \mathcal{C} e^{-\beta\mathcal{H}} \prod_{i=1}^{N} d\lambda_i$$

$$\mathcal{H} = \frac{1}{2\sigma^2} \sum_{i=1}^{N} \lambda_i^2 - \sum_{i<j} \ln|\lambda_i - \lambda_j|$$

- compute probabilities by integrating pdf:

$$P(\forall\lambda > \zeta) = \int_{\zeta}^{\infty} dP$$

- probabilities can be computed:

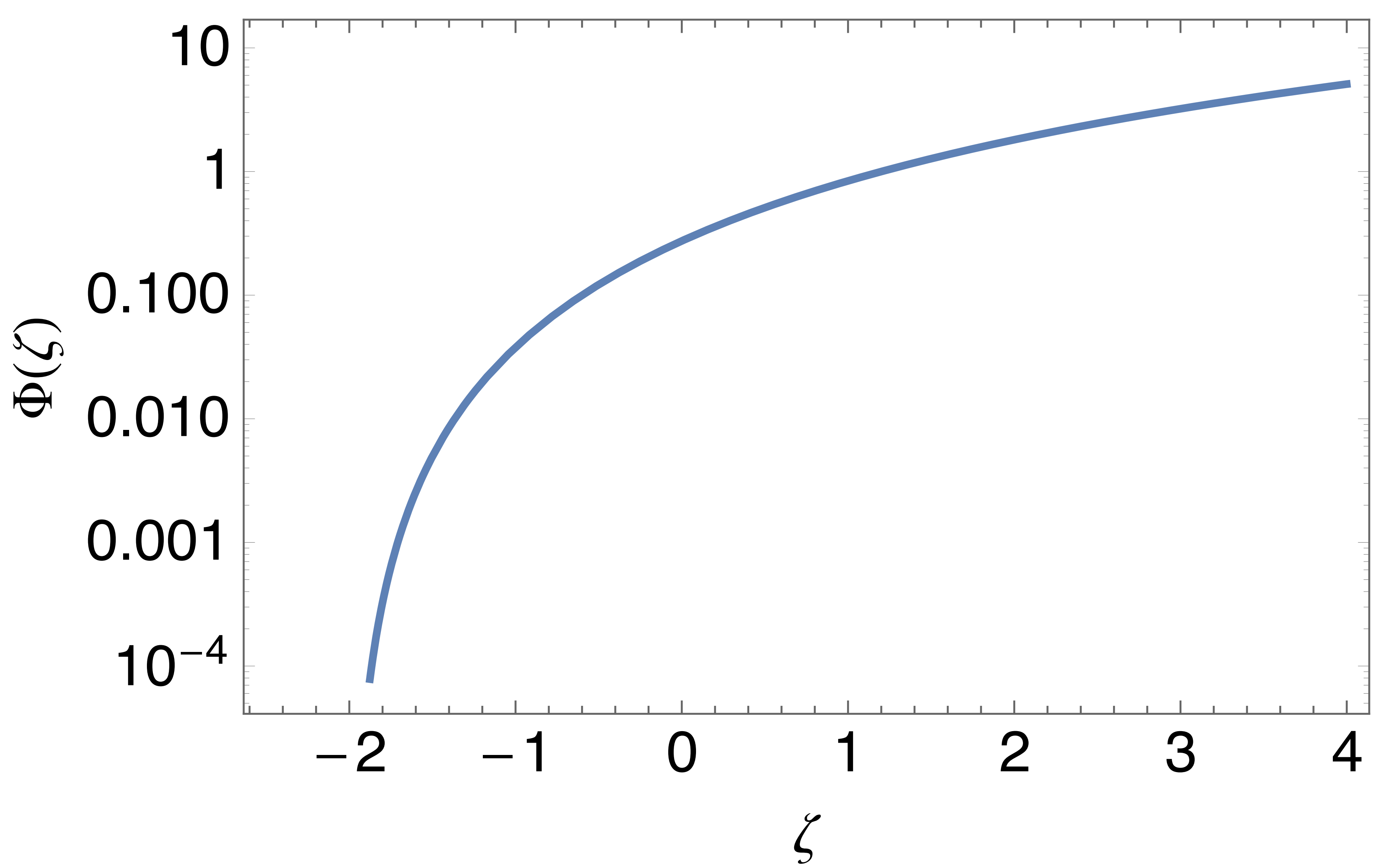
[Aazami & Easther '05]

[FGP & Westphal '13]

- numerically
- analytically via saddle point approximation

[Dean & Majumdar '06/'08]

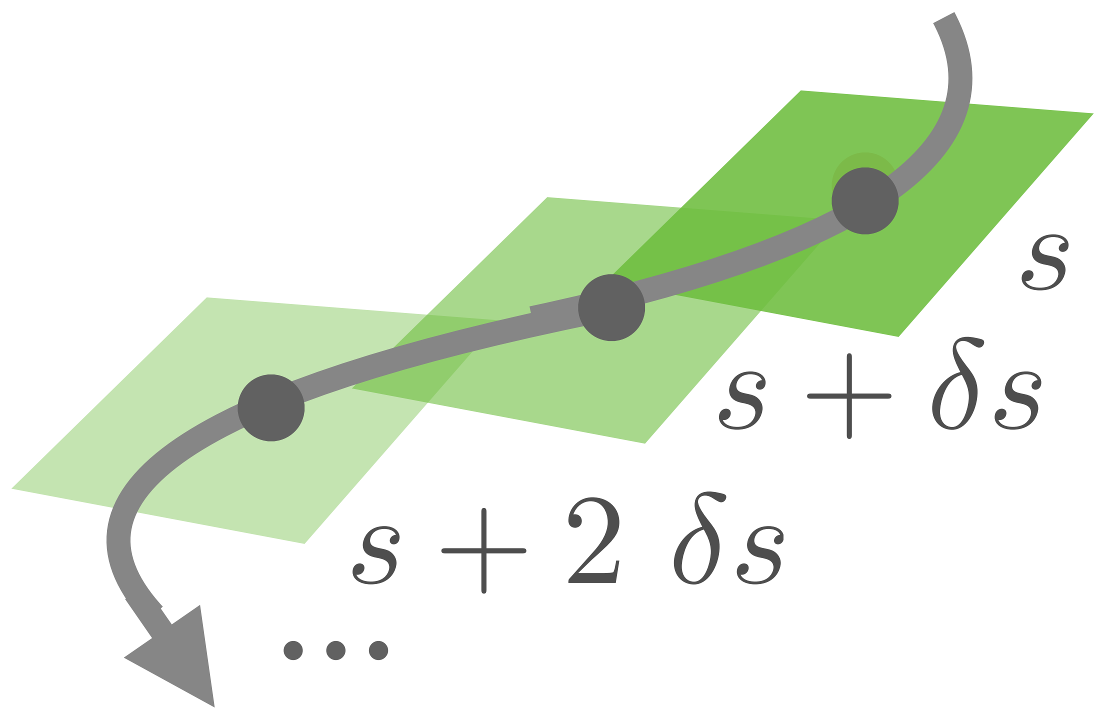
$$P(\forall\lambda > \zeta) = \exp\left(-\beta N^2 \Phi(\zeta)\right)$$



- Move away from the static ensemble

[Dyson '63]
[Marsh et al. '13]
[Dias et al. '16]
[Freivogel et al. '16]

s coordinate along the background trajectory



$$M_{ij}(s+\delta s) = M_{ij}(s) + \delta M_{ij}$$

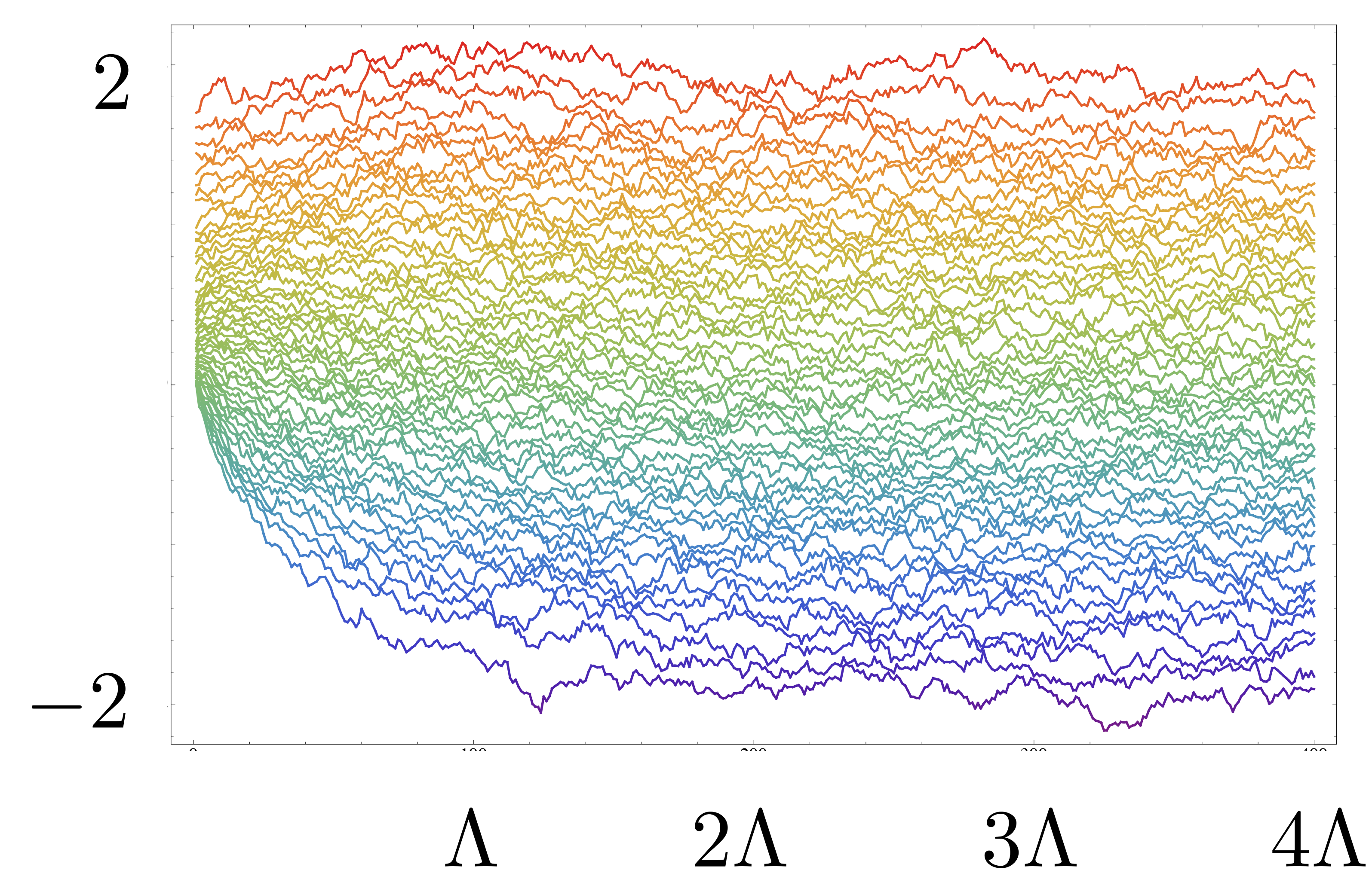
$$\langle \delta M_{ij} \rangle = -M_{ij}\frac{\delta s}{\sigma^2 f}$$

$$\langle (\delta M_{ij})^2 \rangle = (1+\delta_{ij})\frac{\delta s}{\beta f}$$

- "Time" dependence: <u>Dyson Brownian Motion (DBM)</u>

eigenvalues relax in 1 corr. length toward Wigner distrib.

fluctuating toward a minimum *<u>extremely</u>* suppressed



- DBM described by Fokker/Planck eq. - solution:

[Dyson '62 & '63]

$$P(M(s), M_0) = \int dP \qquad q \equiv e^{-\frac{s}{\sigma^2 f}}$$

$$\boxed{dP = \mathcal{C}\ \exp\left\{-\frac{\beta}{2\sigma^2(1-q^2)}\mathrm{Tr}[(M - qM_0)^2]\right\} dM_{ij}}$$

Hamiltonian:

$$\mathcal{H} = \frac{1}{2\sigma^2(1-q^2)}\sum_{i=1}^{N}(\lambda_i^2 - 2q\lambda_i M_0^{ii}) - \sum_{i<j}\ln|\lambda_i - \lambda_j|$$

linear potential for eigenvalues:
memory of init. conds. !

- Estimate exit probability:

  - Numerical evolution of matrices
  - Analytical saddle point integration of dP

[Dean & Majumdar '06/'08]

for time dependent case - our result [Pedro & AW '16]

- Each eigenvalue subject to different linear potential

To get a qualitative description approximate:

$$M_0^{ii} \to m \equiv \frac{1}{N}\sum_{i=1}^{N} M_0^{ii} = \frac{1}{N}\mathrm{Tr}[M_0] \equiv \langle\lambda_{M_0}\rangle.$$

mean eigenvalue of M_0

- time-dependent rate function:

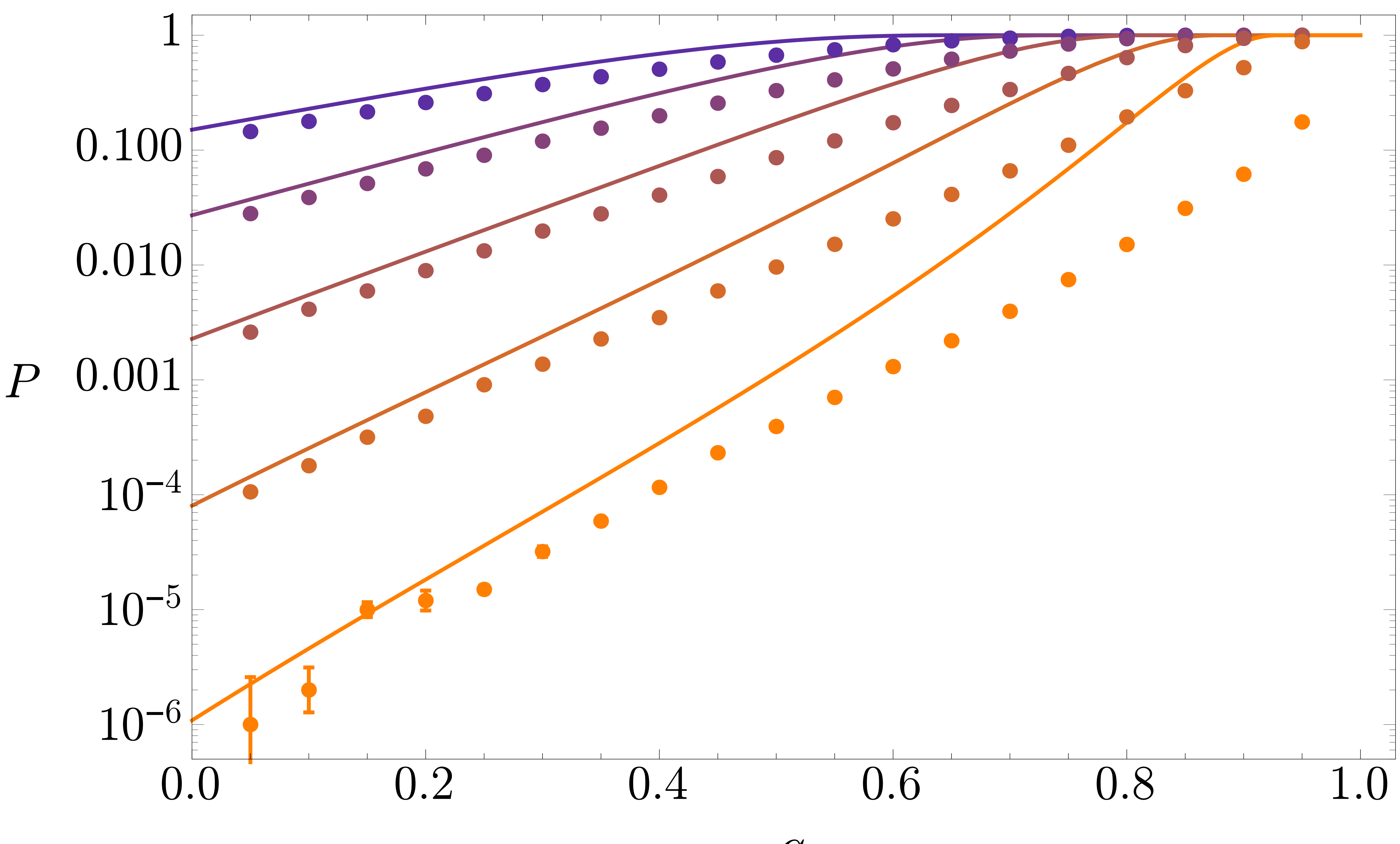
$$\Psi(\tilde{\zeta}) = \frac{1}{108\tilde{a}^2}\left\{36\tilde{a}\tilde{\zeta}^2 - \tilde{\zeta}^4 + (15\tilde{a}\tilde{\zeta} + \tilde{\zeta}^3)\sqrt{6\tilde{a} + \tilde{\zeta}^2} + 27\tilde{a}^2\left[\ln(72\tilde{a}) - 2\ln(2(\sqrt{6\tilde{a} - \tilde{\zeta}} - \tilde{\zeta}))\right]\right\}$$

$$\tilde{a} \equiv 2(1-q^2)$$
$$b \equiv -2qm$$
$$\tilde{\zeta} \equiv \zeta + b/2$$

[Pedro & AW '16]

- What is the probability of: $M(s) : \forall \lambda > \zeta$

$$\boxed{P(M(s), M_0) = \exp\left[-\beta N^2 \Psi(s,\zeta) + \mathcal{O}(N)\right]}$$



$M_0 : \forall \lambda > 0$

$\zeta = \{-1, -0.75, -0.5, -0.25, 0\}$

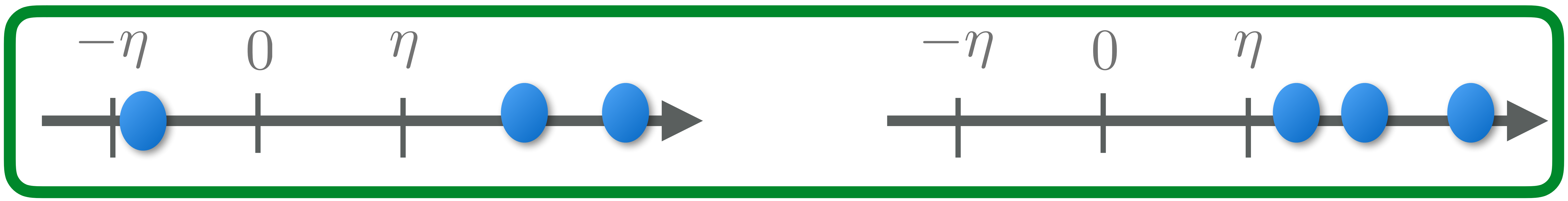
$\beta = 2$

$N = 5$

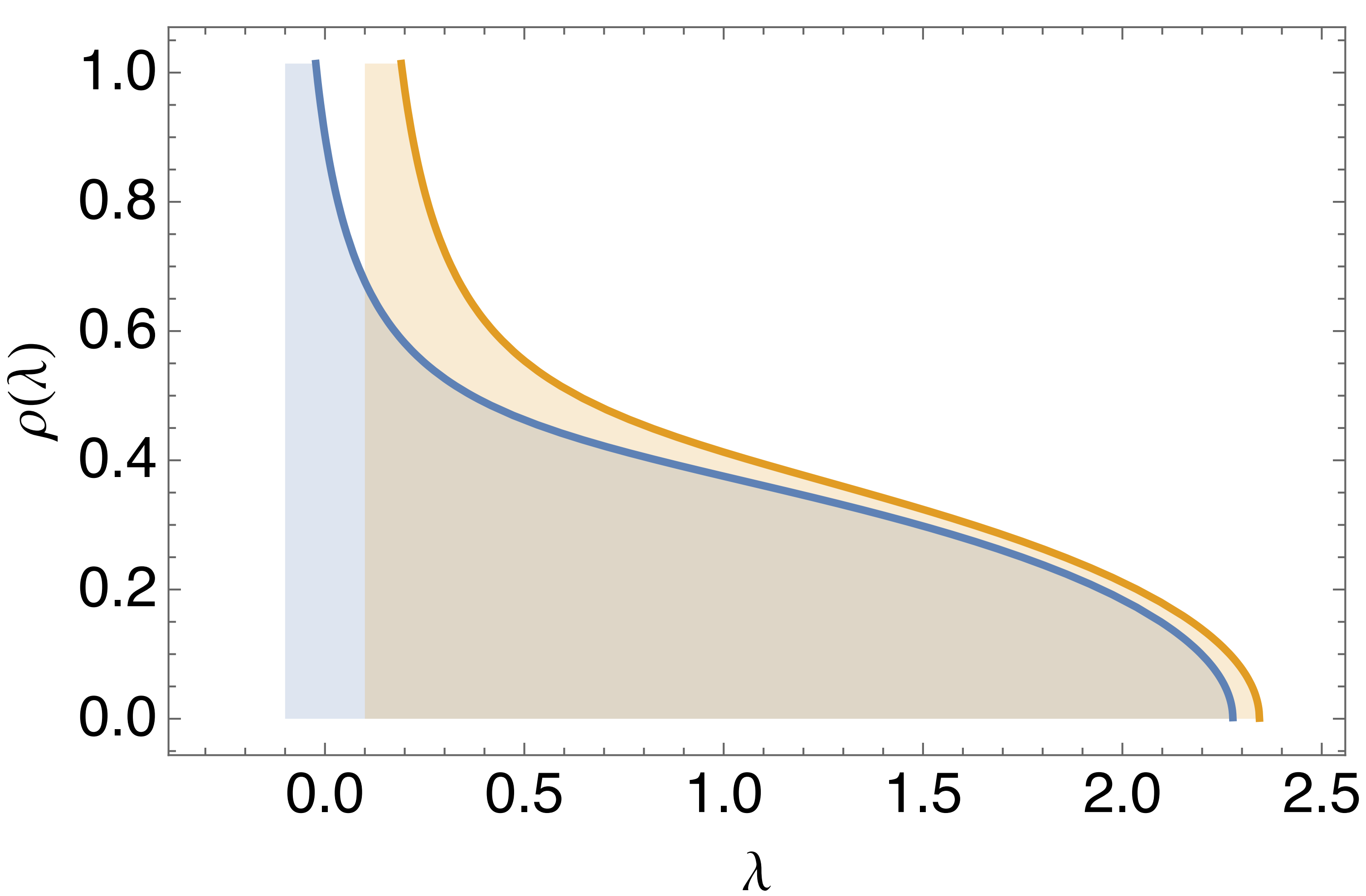
Apply static D&M result to: small-field landscape
how many minima vs inflationary patches?

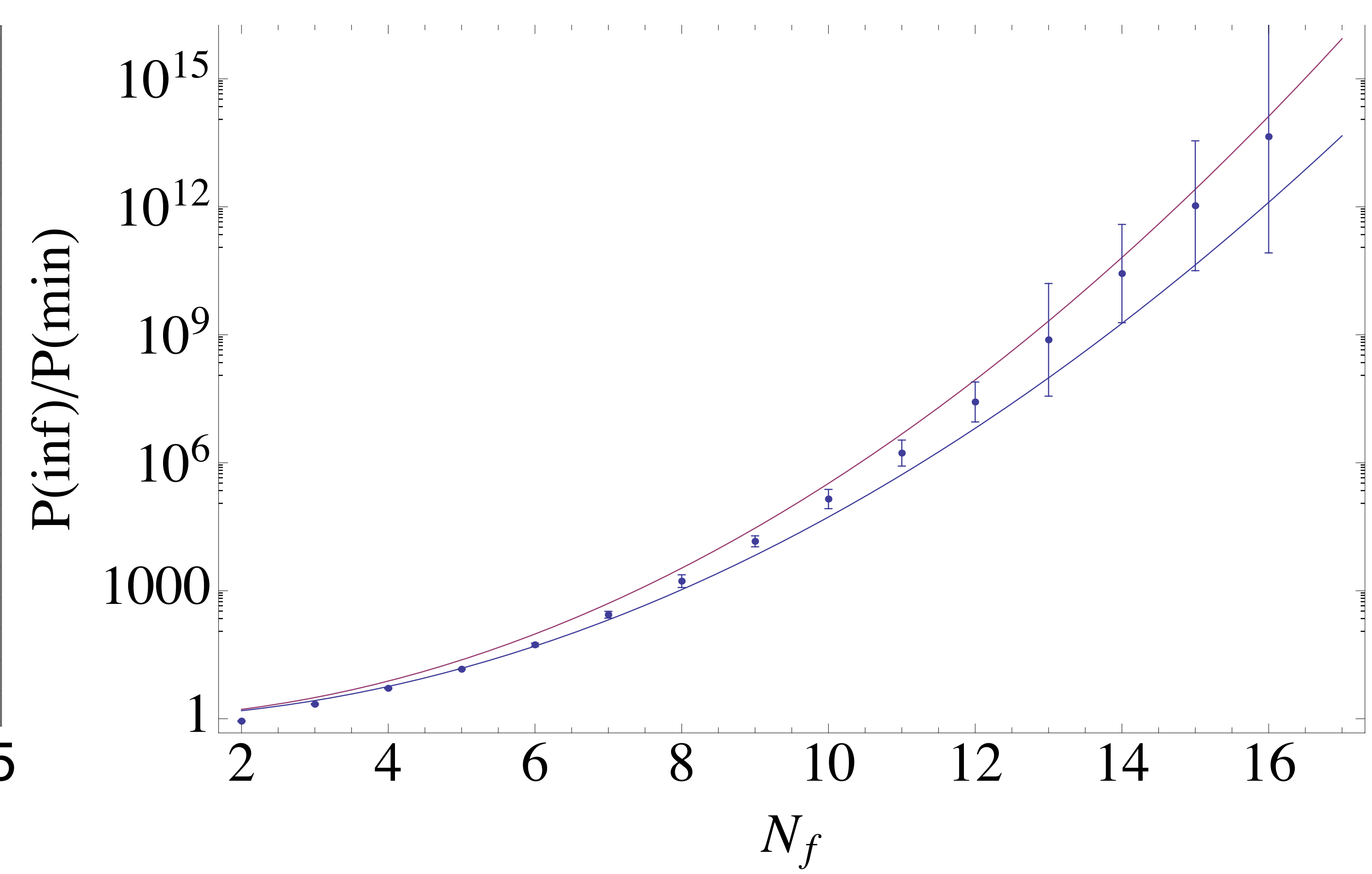
[Pedro & AW '13]

inf **min**



$$\frac{P(\text{inf})}{P(\text{min})} \sim e^{-\beta N^2\{\Phi(-\eta)-\Phi(\eta)\}} = e^{-\beta N^2 \Delta} \quad \Bigg| \quad \Delta = \frac{4}{3\sqrt{3}}\eta + \mathcal{O}(\eta^2)$$

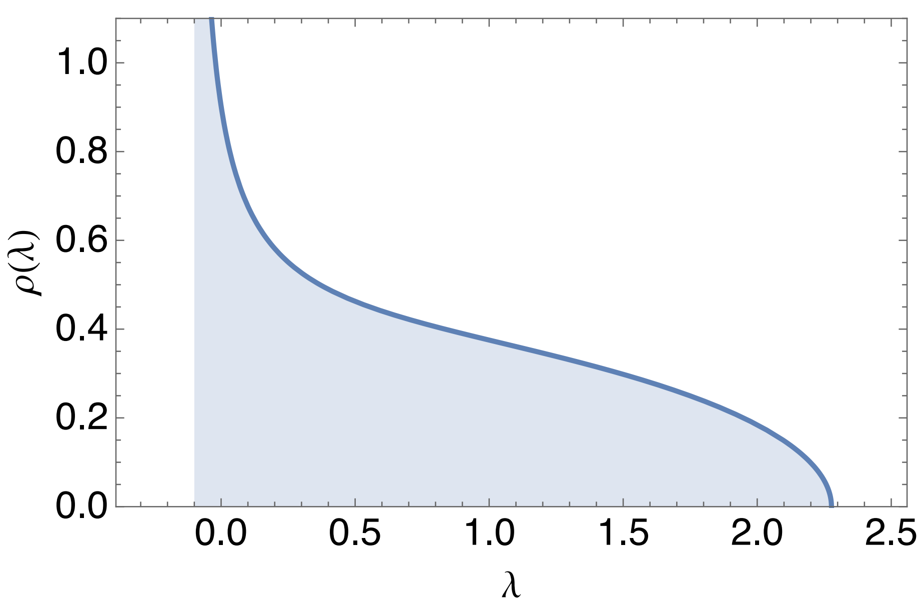




- Inflationary patches MUCH more abundant than minima:

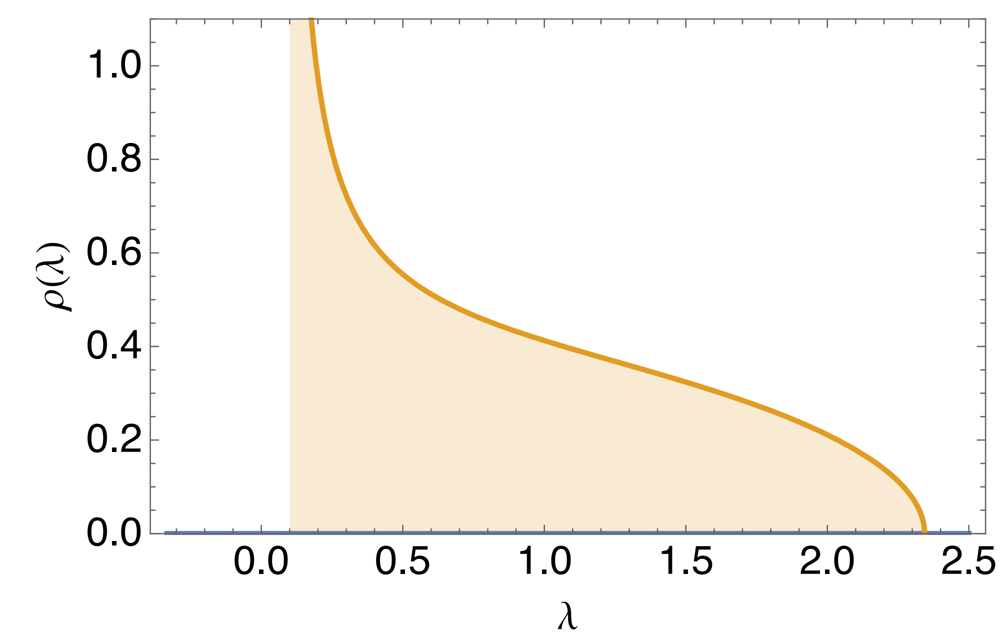
How are they connected in the landscape?

inf.



???

min.

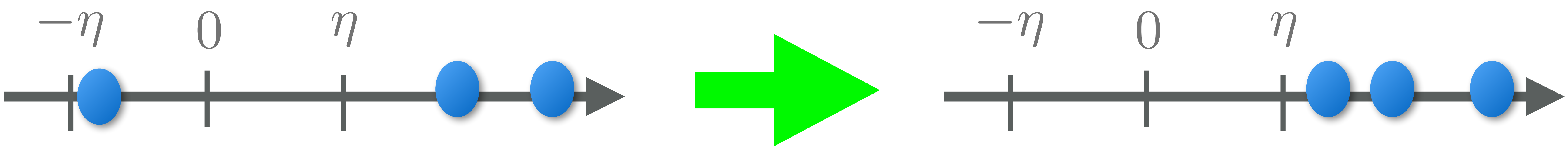


$\{\phi_i\}$ $\{\phi_i + \delta\phi_i\}$

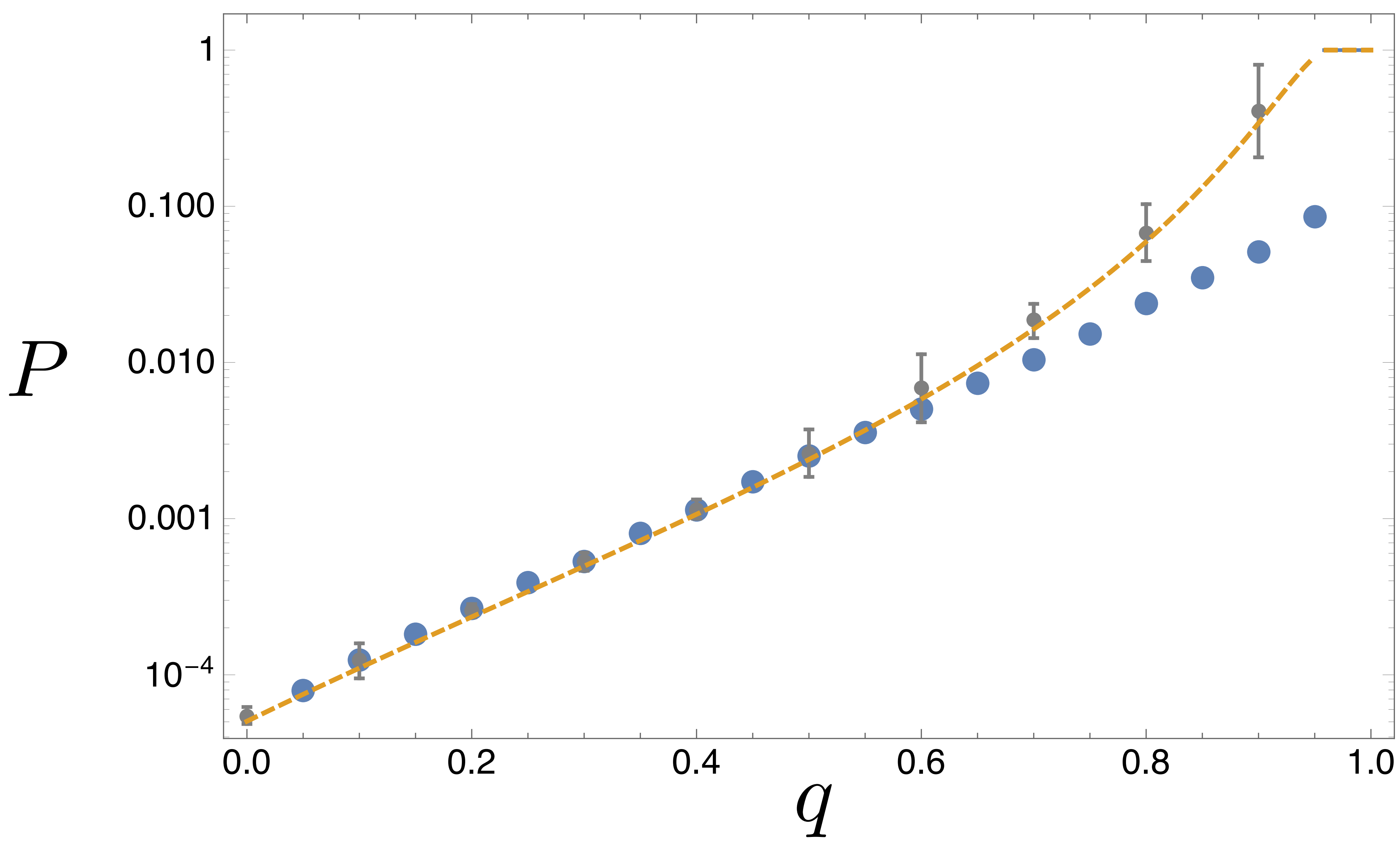
Want to find the **exit probability**

[Pedro & AW ’16]

- Apply analytical relaxation method to: **small-field landscape**



inf **min**



$\beta = 1$

$\eta = 1/10$

$N = 5$

Good agreement after one correlation length: $q < e^{-1} \sim 0.35$

$M_{\rm inf} \gg \Lambda$: interested in small q regime

- Transition probability is exponentially suppressed

? Who wins then ? $\boxed{\dfrac{\mathcal{N}_{inf}}{\mathcal{N}_{min}} \quad \text{vs} \quad \dfrac{\mathcal{N}_{inf \wedge min}}{\mathcal{N}_{min}}}$

Conditional probability: $$P(\min|\inf) = \frac{P(\min \cap \inf)}{P(\inf)}$$

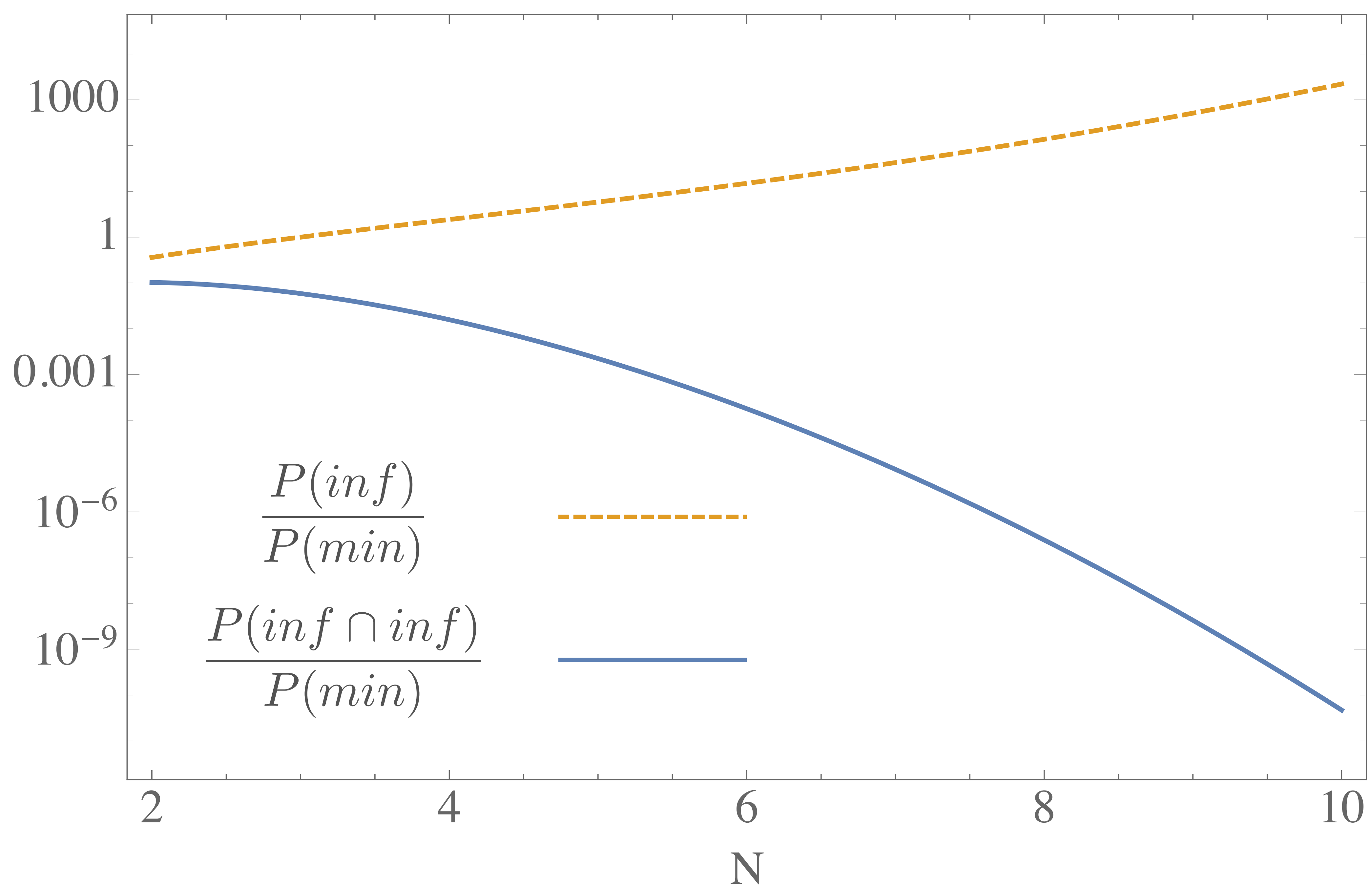
Anthropically relevant trajectories:

$$P(\min \cap \inf) = \underbrace{P(\inf)}_{\text{initial condition}} \; \underbrace{P(\min|\inf)}_{\text{evolution}}$$

- **Globally:** $q \ll 1 \wedge |\eta| \ll 1$

$$P(\min \cap \inf) \simeq \exp\left[-\beta N^2 \left\{\frac{\ln 3}{4} - \frac{2}{3\sqrt{3}}(\eta + m\,q)\right\}\right]$$

$m \simeq 0.7$



Transition probability overcomes number count

In small-field landscapes … Our History is highly unlikely !!!!

[AW '12]

- The landscape '*Drake equations*' of tensor modes

$$\nu_{\rm small} \sim N_{\rm manifolds} \times N_{\rm cr.p.} \times f_{\rm dS-min.} \times \overbrace{P_{\rm small}(\min \cap \inf)}^{\lll 1}$$

$$\nu_{\rm large} \sim N_{\rm manifolds} \times N_{\rm cr.p.} \times f_{\rm dS-min.} \times P_{\rm large}(\min \cap \inf)$$

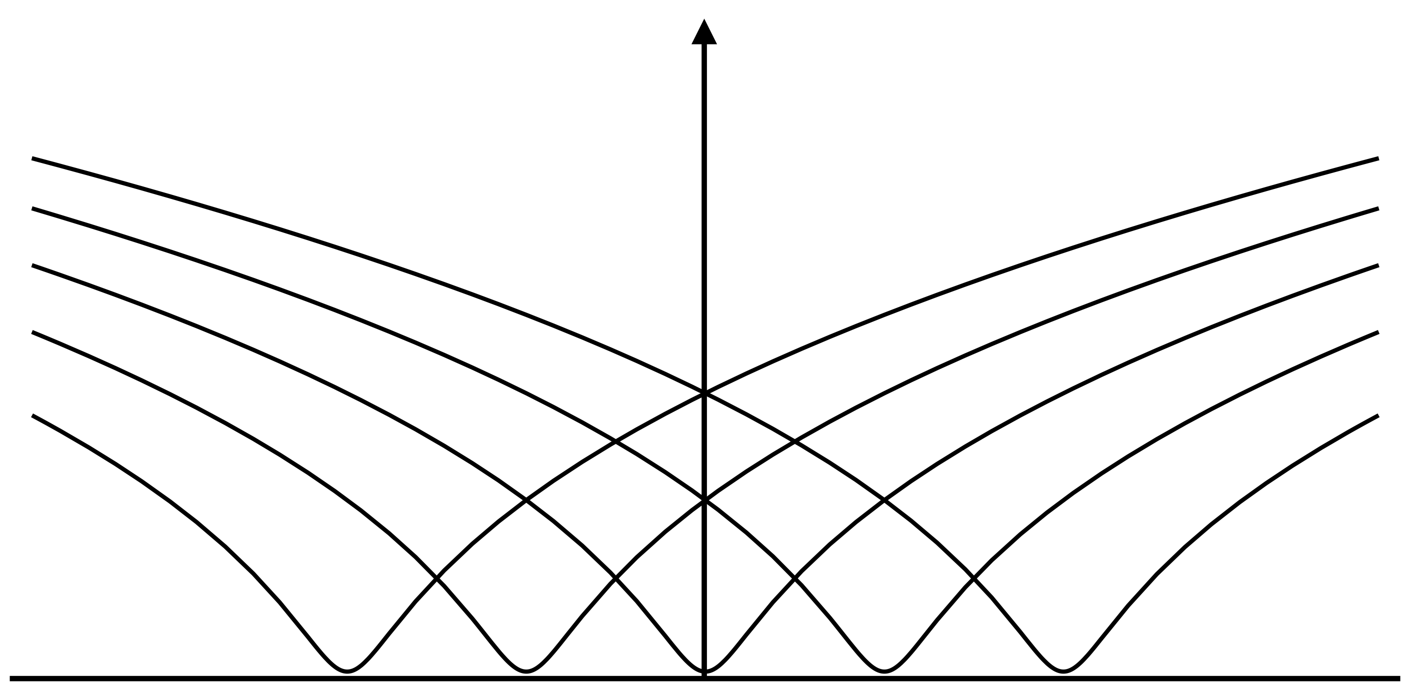
erc

[AW '12]

- The landscape '*Drake equations*' of tensor modes

$$\frac{\nu_{\rm small} \sim \cancel{N_{\rm manifolds} \times N_{\rm cr.p.}} \times f_{\rm dS-min.} \times \overbrace{P_{\rm small}(\min \cap \inf)}^{\lll 1}}{\nu_{\rm large} \sim \cancel{N_{\rm manifolds} \times N_{\rm cr.p.}} \times f_{\rm dS-min.} \times P_{\rm large}(\min \cap \inf)}$$

➥ large-field models: minimum built-in !



$$P_{\rm large}(\min|\inf) = 1$$

$$\Rightarrow \quad P_{\rm large}(\min \cap \inf) \sim 1$$

➥ **maybe string landscape wants larg/ish *r* by preferring structure?**

- compute prior probability for e-folds N_e

$$V = V_0\left(1 - \frac{\eta_0}{2}\phi^2 - \frac{1}{p\Delta\phi^p}\phi^p + \dots\right)$$

$$P(N_e) = \int d\sqrt{H}\, d\Delta\phi\, d\eta\; \delta\left(N_e - \frac{1}{\eta}\right)\, \delta\left(\frac{\delta\rho}{\rho} - f(\sqrt{H},\Delta\phi,\eta))\right) \times \exp(\mathcal{O}(1)N^2/N_e)$$

$$\sim \frac{1}{N_e^{4+2/(p-2)}} e^{\mathcal{O}(1)\frac{N^2}{N_e}}$$

<u>so N < 10</u> to get
$\Omega_K < 0.001$!!

where do we go from here ...

➥ small-field models

➥ large-field models

less-accidental small-field saddle points (e.g. Kahler moduli) or plateaus with structure

accidental small-field saddle points (e.g. complex structure moduli)

axion monodromy

unwinding inflation
... others/unknown ??

where do we go from here ...

➥ small-field models

➥ large-field models

less-*accidental* small-field saddle points (e.g. Kahler moduli) or plateaus with structure

*accidental* small-field saddle points (e.g. complex structure moduli)

*axion monodromy*

covered here ...

*unwinding inflation*

*... others/unknown ??*

exit also often built-in ... P = ??

maybe $P_{large} / P_{small} >> 1$?

P = ??