String Theory, Inflation and Amplitudes

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Rome workshop on String Cosmology,
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Cosmological Concordance Model

Early Universe Inflation: first 10^{-35} sec

Model building to explain data using supergravity motivated by string theory (can't use global susy, have to solve Einstein equations)

Absence of non-gaussianity: preference to a single very light scalar, inflaton, all other moduli have to be stabilized

Tilt of the power spectrum $$n_s \approx 0.96$$

Primordial gravity waves $$r < 0.07$$

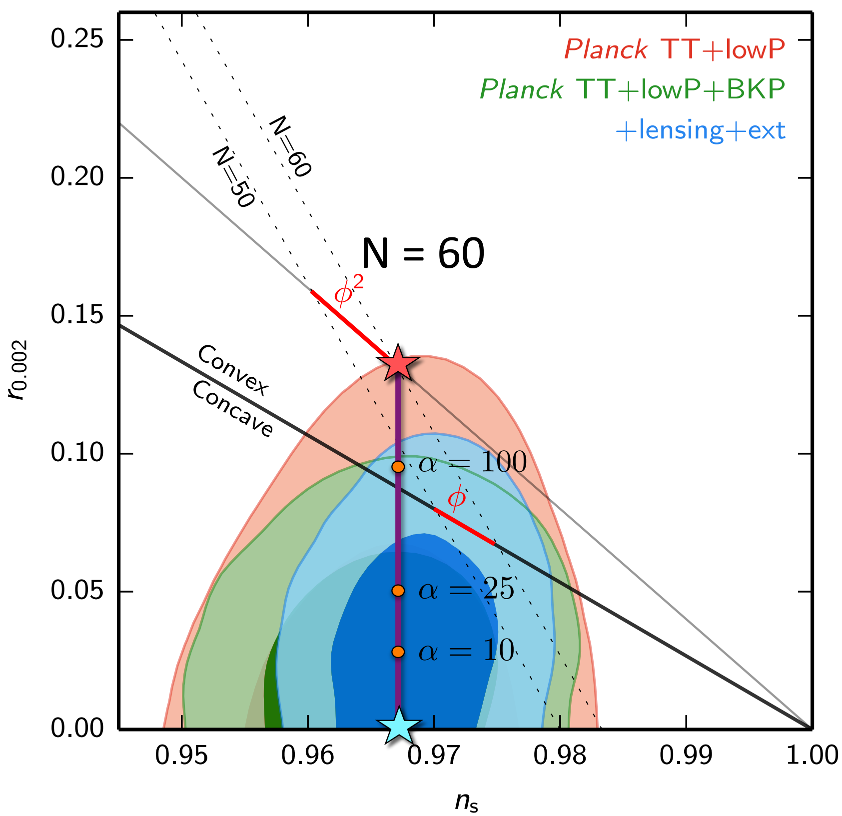
Slow roll inflation, near de Sitter space

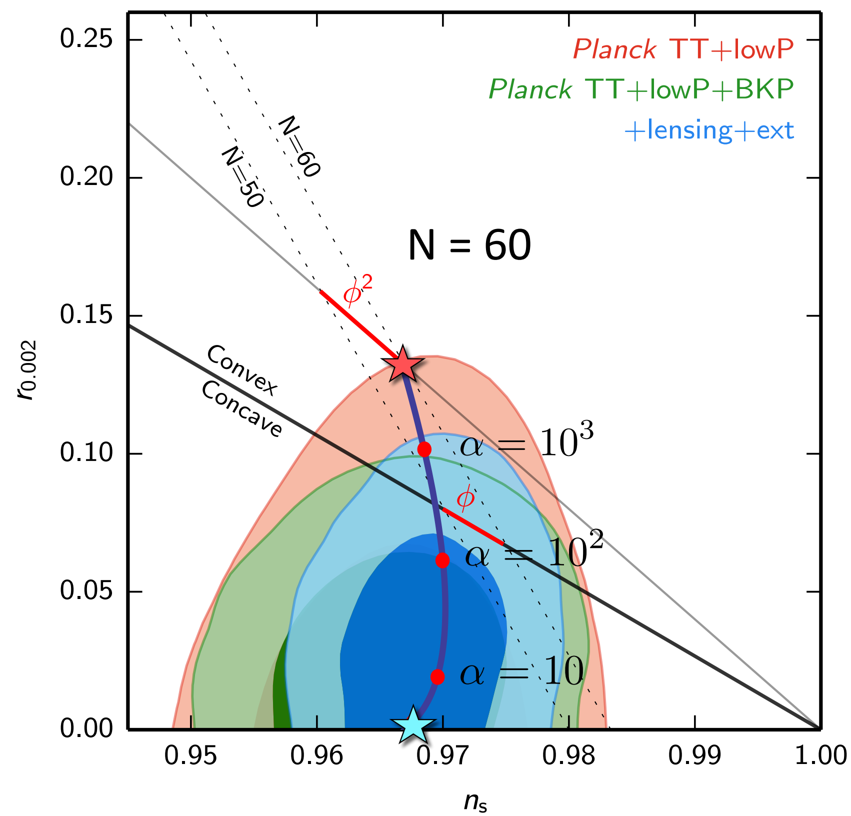
Current Universe acceleration: during the last few billion years

Cosmological constant, de Sitter space, provides a good fit to data

$$\Lambda \approx 10^{-120} M_{Pl}^4$$

Dark Matter ???





Supergravity Models with 2 Superfields: inflaton and stabilizer

Simplest T-models

Simplest E-models

RK, Linde, Roest 2013

Ferrara, RK, Linde, Porrati, 2013,

α-attractors in supergravity, cosmological constant, and SUSY breaking

$$ds^2 = \frac{3\alpha}{(1-Z\bar{Z})^2} dZ d\bar{Z}$$

$$ds^2 = \frac{3\alpha}{(T+\bar{T})^2} dT d\bar{T}$$

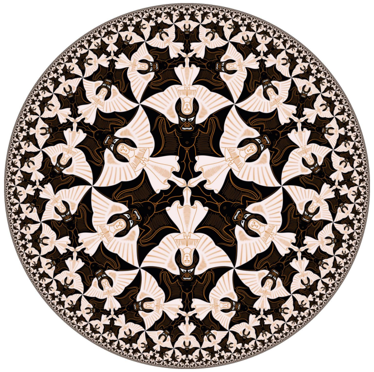
$$\mathcal{R}_K = -\frac{2}{3\alpha}$$

Curvature of the moduli space in Kahler geometry

Hyperbolic geometry
of a Poincaré disk

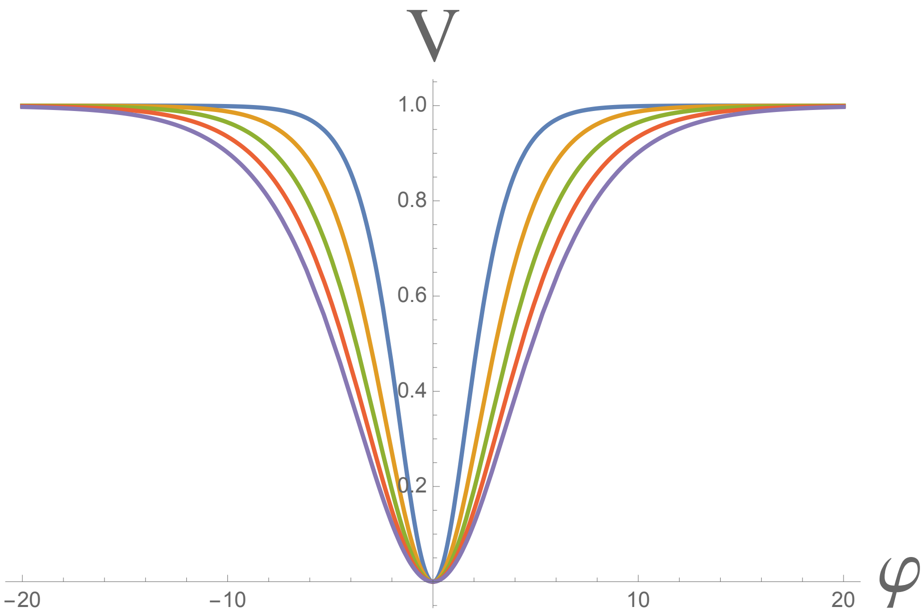
Disk or **half-plane**

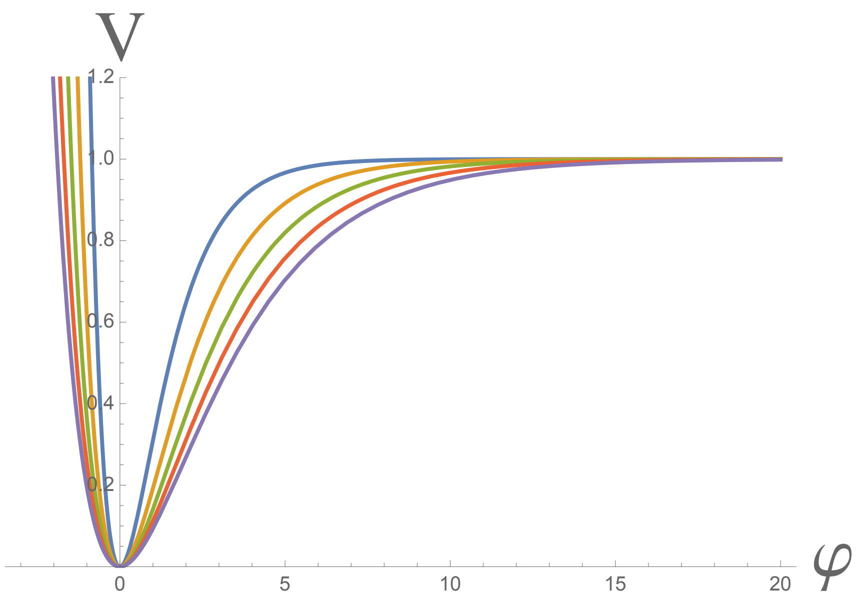
Escher in the Sky, RK, Linde 2015



$$3\alpha = R^2_{\text{Escher}} \approx 10^3 r$$

Plateau potentials α-attractors





$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\Big(\tanh\frac{\varphi}{\sqrt{6\alpha}}\Big)^2$$

Simplest T-model

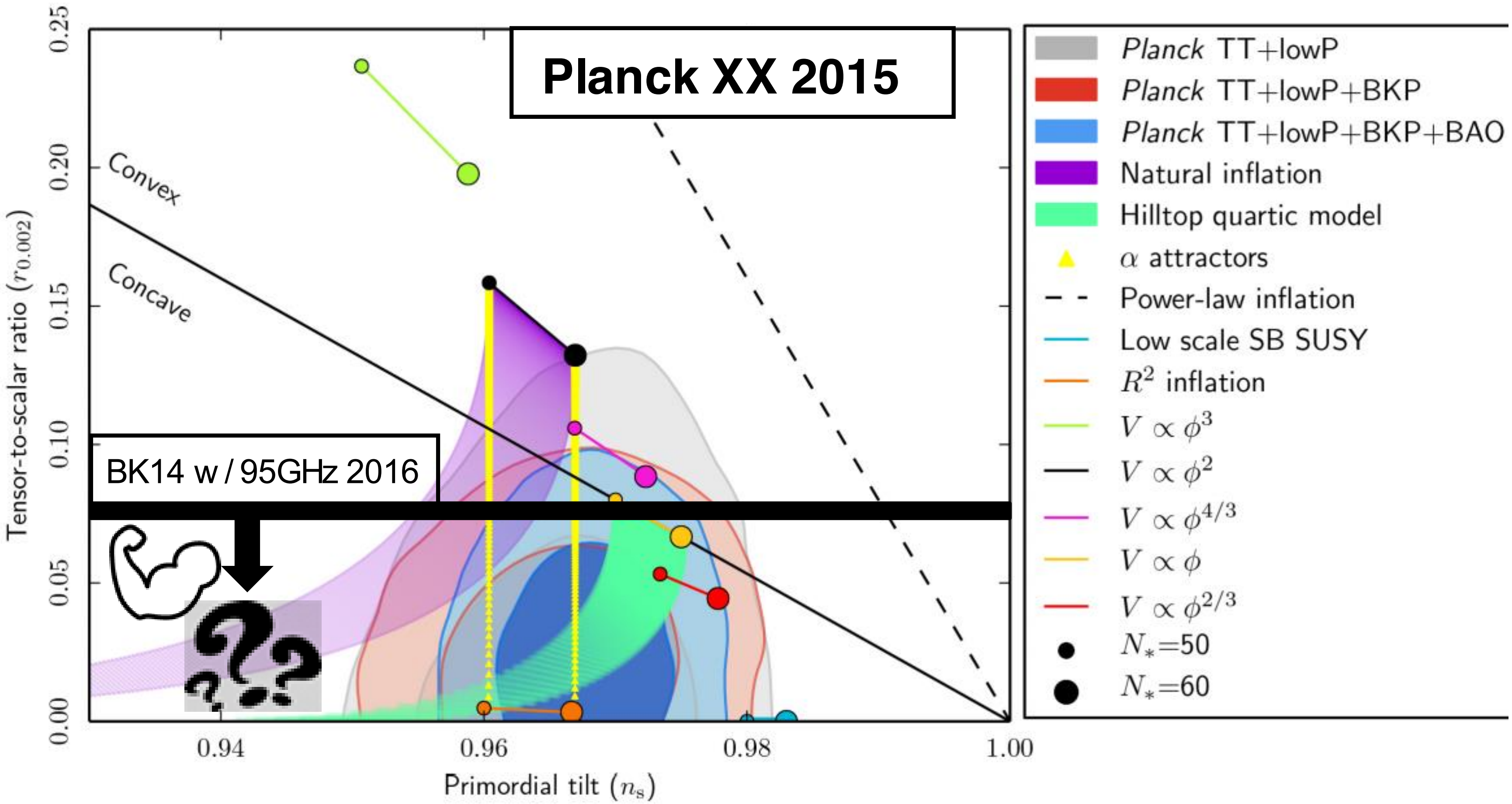
$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\Big(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\Big)^2$$

Simplest E-model

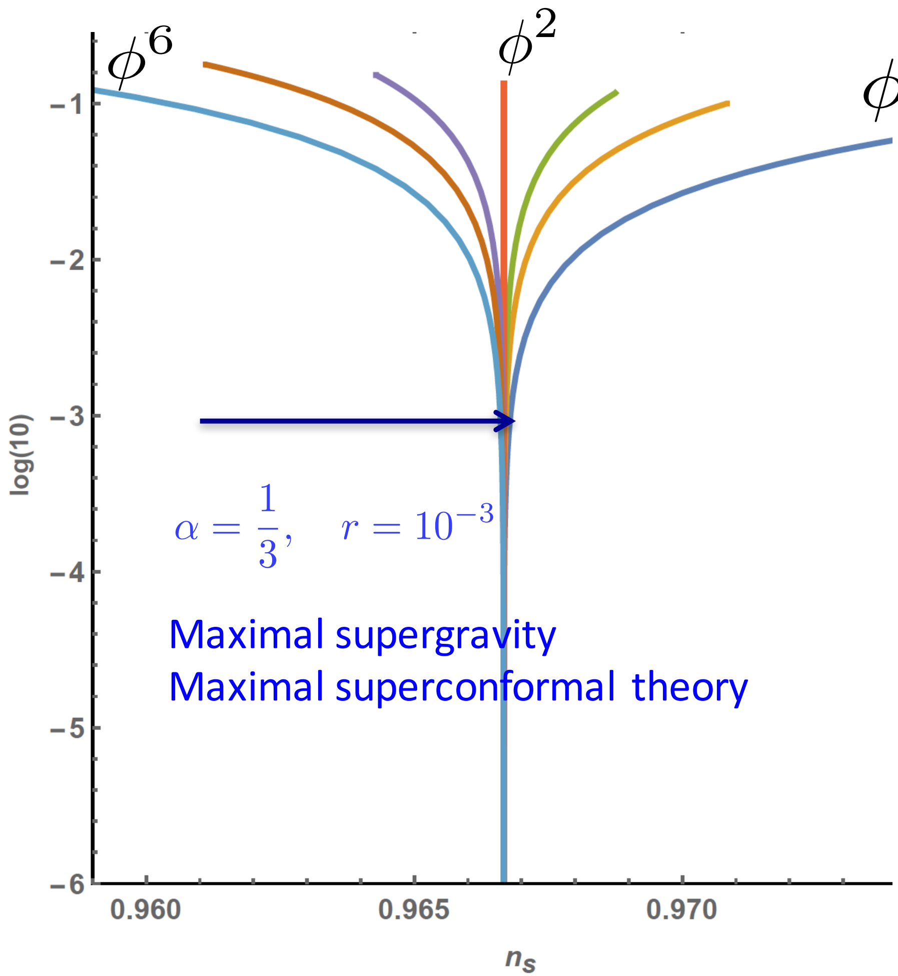
α-attractors, 2 yellow lines in the sweet spot of Planck data

RK, Linde, Roest, 2013

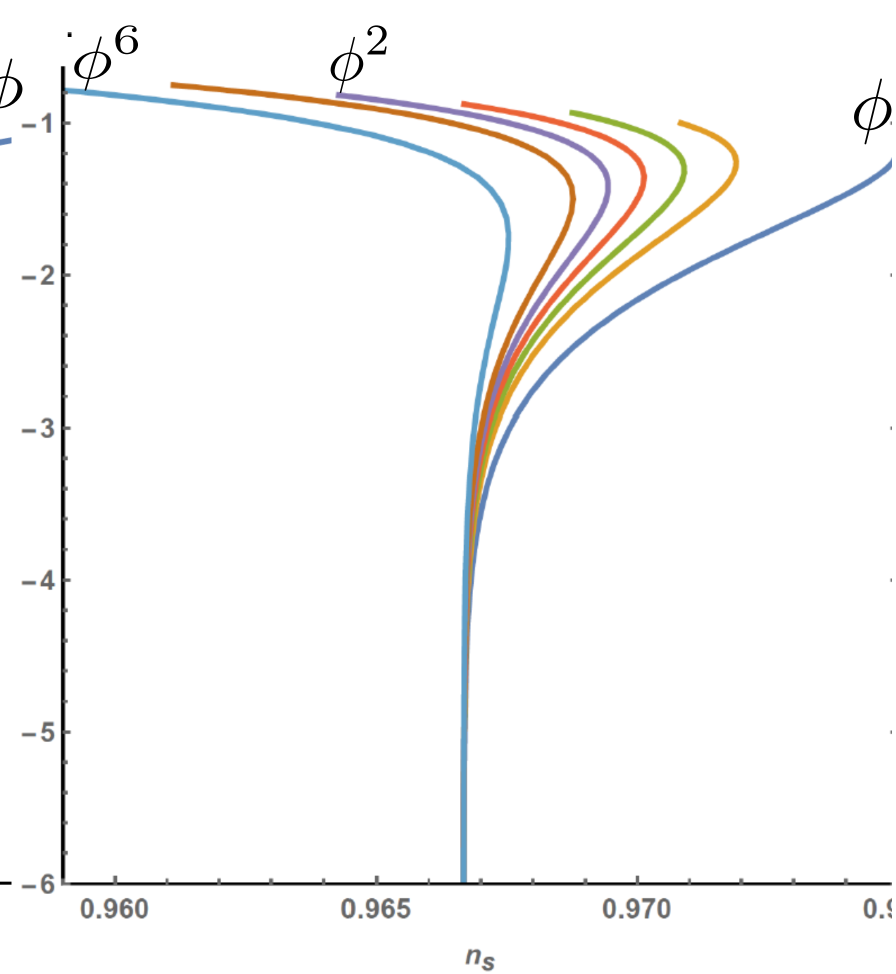
r < 0.07



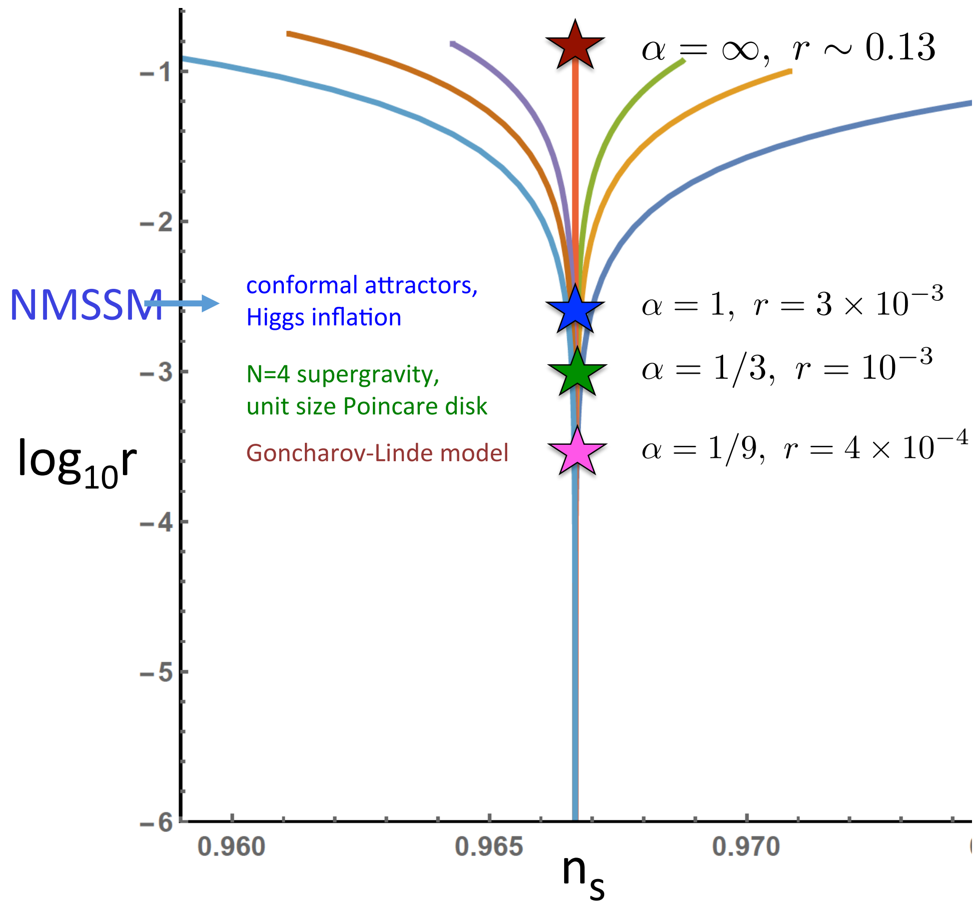
α-attractors

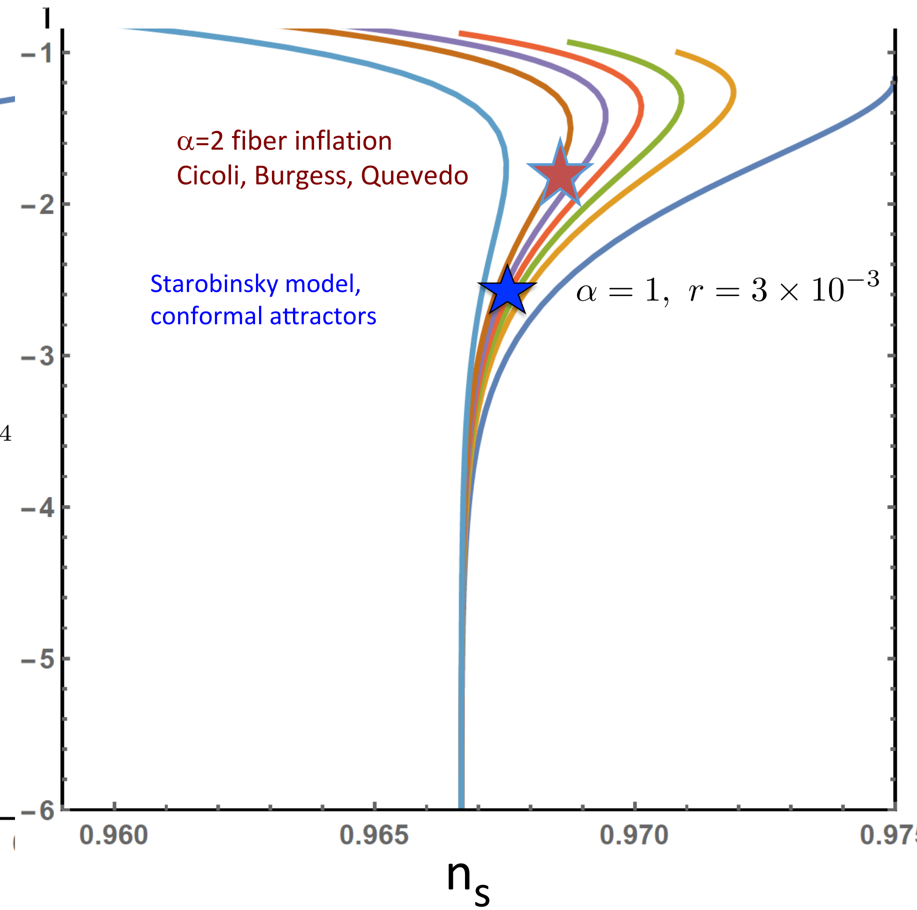


$$\left(\tanh \frac{\varphi}{\sqrt{6\alpha}}\right)^{2n}$$



$$\left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^{2n}$$



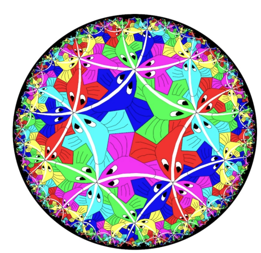


Any $\alpha < 20$ $\qquad$ $r < 0.07$

$$3\alpha = R^2_{\text{Escher}} \approx 10^3 r$$

Generic $\mathcal{N}$=1 supergravity

Any α



Hyperbolic geometry of a Poincaré disk

Special choices of α and future data

$\alpha = 2$	$r \approx 6 \times 10^{-3}$	Fiber inflation
$\alpha = 1$	$r \approx 3 \times 10^{-3}$	Critical point of superconformal $\mathcal{N}$=1 attractors, Higgs inflation, R^2 …
$\alpha = 1/3$	$r \approx 10^{-3}$	Maximal superconformal $\mathcal{N}$=4 model, maximal supergravity $\mathcal{N}$=8
$\alpha = 1/9$	$r \approx 3 \times 10^{-4}$	1984 model of Goncharov-Linde
Any $\alpha < 20$	$r < 0.07$	Generic $\mathcal{N}$=1 supergravity

All of these models fit the current data

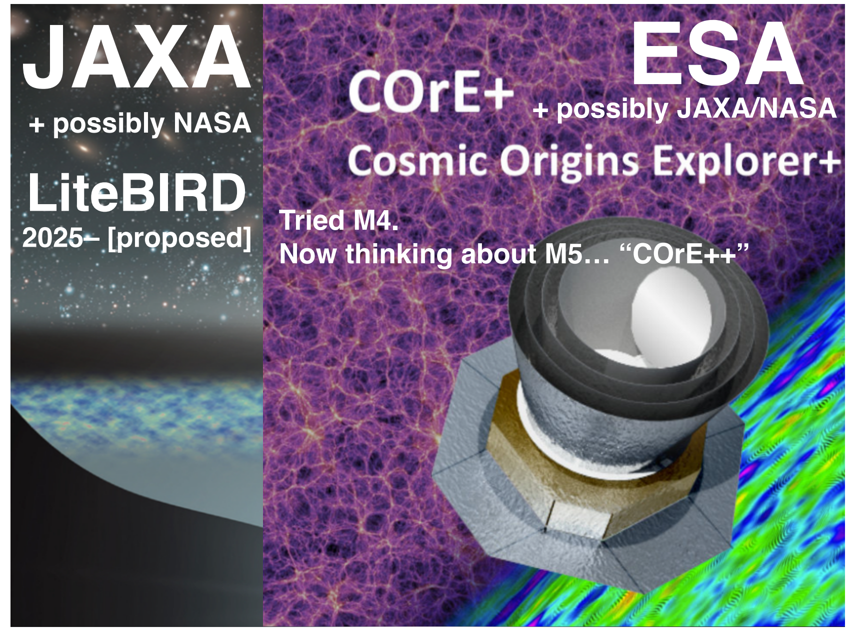
B-mode detection experiments

Ground based,
South pole, Chile

Ali in Tibet ???

Greenland ???

Future space missions



Hunt for Big Bang Gravitational Waves Gets $40-Million Boost

The nonprofit Simons Foundation will fund a new observatory to search for signs of stretching in the very early universe

With linear supersymmetry it is possible to stabilize unwanted moduli in supergravity. This can be achieved by adding stabilization terms to Kahler potential, corresponding to sectional and bisectional curvatures of the moduli space.

However, it is somewhat difficult to find simple models in agreement with the data on inflation and especially to construct de Sitter vacua at the exit stage.

Moreover, the stabilizer superfield which helps to stabilize the sinflaton, did not have a clear Interpretation in string theory.

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Therefore we use constrained superfields, starting from the nilpotent one corresponding to a non-linear realization of the Volkov-Akulov type supersymmetry

$$S^2 = 0$$

This is the simplest way to exit inflation into de Sitter space.

The second useful constrained multiplet in cosmological models is the orthogonal one: no worries about the sinflaton and inflatino

$$S(\Phi - \bar{\Phi}) = 0$$

Cosmological Constant in Supergravity

Known to be negative in pure supergravity, without scalar fields, 1977, Townsend

$\Lambda < 0$ **AdS**

Supergravity with a positive cosmological constant without scalars was not known.

$\Lambda > 0$ **dS**

Constructed 38 years later, in 2015

No-go theorems prohibit linearly realized supersymmetry

$\mathcal{N}$=1 dS supergravity has a non-linearly realized supersymmetry.

Derived using superconformal symmetry and Lagrange multipliers or superspace methods

Alternative derivation: starting with linear supergravity, taking limit to infinite mass of the scalar partner

RK, Karlsson, Murli, 2015

Interaction with arbitrary matter multiplets

RK, Wrase
Schillo, van der Woerd, Wrase 2015

The hints came from inflationary model building: in α-attractor models (yellow lines on Planck r/n_s plot).

Non-linear supersymmetry is a nice feature that allows to stabilize extra moduli and reduce the evolution to the one driven by a single scalar inflaton.

Advanced versions of these models are based on a supersymmetry which is not a standard linear SUSY but a non-linear SUSY.

A feature known in non-perturbative string theory: D-branes with Born-Infeld vectors and Volkov-Akulov spinors

Easy to get rid of unwanted SUSY partners
Can be useful for LHC phenomenology???

The nilpotent chiral superfield

- SUSY 101: supersymmetry relates bosons and fermions

Not necessarily!

- If we break supersymmetry we expect a massless goldstone fermion, the goldstino

- Volkov, Akulov 1972, 1973

The nilpotent chiral superfield

$$S_{VA} = \int E^0 \wedge E^1 \wedge E^2 \wedge E^3 = \int d^4x \,\det(E),$$

$$E^\mu = dx^\mu + \bar{\chi}\gamma^\mu d\chi = dx^\nu\left(\delta_\nu^\mu + \bar{\chi}\gamma^\mu\partial_\nu\chi\right)$$

- Invariant under: $\delta_\epsilon\chi = \epsilon + (\bar{\chi}\gamma^\mu\epsilon)\partial_\mu\,\chi$
- There is only a fermion!
- Supersymmetry is non-linearly realized
- Supersymmetry is spontaneously broken
- The partner of the 1-fermion state is a 2-fermion state

- In $N = 1$ supersymmetry in 4d we can have a so called nilpotent chiral superfield

Volkov, Akulov 1972, 1973
Rocek; Ivanov, Kapustnikov 1978
Lindstrom, Rocek 1979
Casalbuoni, De Curtis, Dominici, Feruglio, Gatto 1989
Komargodski, Seiberg 0907.2441

- This can be thought of as a chiral superfield that squares to zero

$$S = s + \sqrt{2}\theta\chi + \theta^2 F, \qquad S^2 = 0$$

$$S^2 = 0 \quad \Rightarrow \quad s^2 = 2\sqrt{2}s\theta\chi = \theta^2(2sF - \chi\chi) = 0$$

$$s = \frac{\chi\chi}{2F} = \frac{\chi_1\chi_2}{F} \quad \Rightarrow \quad s\chi = 0 \ \text{ and } \ s^2 = 0$$

The nilpotent chiral superfield

$$S = \frac{\chi\chi}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

- These nilpotent chiral superfields consist only of fermions!
- Supersymmetry is non-linearly realized and spontaneously broken ($F \neq 0$)
- There is a variety of different actions but all are related to S_{VA} via non-linear field redefinitions

Kuzenko, Tyler 1009.3298, 1102.3043

- S-matrix is unique!

Early work on string theory SUSY breaking:
Pradisi, Sagnotti, Gimon, Polchinski
Bianchi, Pradisi, Sagnotti, 1992
Antoniadis, Dudas, Sugimoto,Uranga
Pradisi, Riccioni, 2001

D-branes

$$S = \frac{1}{a}\int \omega_0 \times \omega_1 \times \omega_2 \times \omega_3$$

Volkov-Akulov

RK, Wrase, 2014, supersymmetric KKLT

Bergshoeff, Dasgupta, RK, Wrase, Van Proeyen 2015

Antoniadis, Dudas, Ferrara and Sagnotti, 2014

VA-Starobinsky supergravity

$$S^{\mathrm{D3}} = 0\,, \qquad S^{\overline{\mathrm{D3}}} = -2T_3\int d^4\sigma\, \det E$$

Ferrara, RK, Linde, 2014 application to cosmology, generic superconformal case

$$E = dX - \bar{\theta}\Gamma^m d\theta$$

D-brane VA geometric connection

Supersymmetric KKLT uplift

In advanced supergravity inflation, a stabilizer superfield is nilpotent

A universal role of the goldstino multiplet at the minimum of the inflationary potential

$$F_s = e^{\frac{K}{2}} D_S W$$

$$\Lambda = F_s^2 - 3m_{3/2}^2 > 0$$

$$m_{3/2} \equiv e^{\frac{K}{2}} W$$

SUSY breaking parameter

Gravitino mass

Volkov-Akulov non-linearly realized supersymmetry of the purely fermionic multiplet is spontaneously broken. A tiny CC results from an incomplete cancellation of the positive goldstino and negative gravitino contribution to supergravity energy

String theory landscape

sgoldstino is not a fundamental scalar but a bilinear combination of fermionic goldstino's divided by the value of the auxiliary field

String Theory Realizations of the Nilpotent Goldstino

RK, Quevedo, Uranga 2015

Nilpotent superfield $S^2=0$

Stabilizer, helps to stabilize the sinflaton, no need to stabilize the sgoldstino

Allows to build models with the exit from Inflation into de Sitter space, which was practically impossible before in supergravity models

Ferrara, RK, Thaler 1512.00545, Carrasco, RK, Linde 1512.00546,
Dall'Agata, Farakos 1512.02158

Orthogonal nilpotent (degree 3) superfield SB=0, $B^3=0$

$$B = \frac{1}{2i}(\Phi - \bar{\Phi})$$

Sgoldstino, sinflaton and inflatino vanish in the unitary gauge. Inflatino is not mixed with gravitino at the end of inflation!

Ultimate single field inflationary model in supergravity with two superfields (constrained)

Both multiplets live on anti-D3 brane

Vercnocke, Wrase. Vercnocke, RK, Wrase , 2016

Using nilpotent orthogonal fields

Ferrara, RK, Thaler 1512.00545, Carrasco, RK, Linde 1512.00546, Dall'Agata, Farakos 1512.02158

Consider a theory

$$K = -\frac{3}{2}\alpha \log\left[\frac{(1-\Phi\bar{\Phi})^2}{(1-\Phi^2)(1-\bar{\Phi}^2)}\right] + S\bar{S} \qquad W = Sf(\Phi) + g(\Phi)$$

The expression for the inflaton potential in these theories does NOT contain $f'(\Phi)$

$$V = f^2(\phi) - 3g^2(\phi) \qquad \chi^s = 0$$

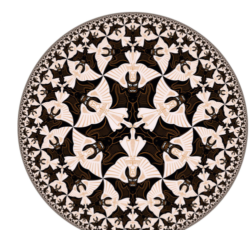
$$F^\phi = 0$$

The cosmological constant and the gravitino mass in the minimum are

$$\Lambda = f^2(0) - 3g^2(0) \qquad m_{3/2} = g(0)$$

The canonical inflaton field φ is related to the original field ϕ as follows:

$$\phi = \tanh\frac{\varphi}{\sqrt{6\alpha}}$$



α-Attractors: Planck, LHC and Dark Energy

Example 1:

Carrasco, RK, Linde 1512.00546

$$f(\phi) = \sqrt{F^2(\phi) + a^2}, \quad g(\phi) = \sqrt{G^2(\phi) + b^2}$$

$$V = F^2(\phi) - 3G^2(\phi) + a^2 - 3b^2, \quad m_{3/2} = \sqrt{G^2(\phi) + b^2}.$$

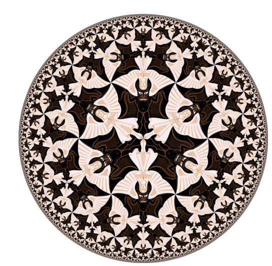
In canonical variables,

$$V = F^2(\tanh \frac{\varphi}{\sqrt{6\alpha}}) - 3G^2(\tanh \frac{\varphi}{\sqrt{6\alpha}}) + a^2 - 3b^2$$

Full functional freedom to chose any α-attractor potential, with any cosmological constant and gravitino mass.

$$\Lambda = a^2 - 3b^2, \quad m_{3/2} = b.$$

T-model with CC



α-Attractors: Planck, LHC and Dark Energy

Example 2:

$$K = -\frac{3}{2}\,\alpha \log\left[\frac{(\Phi+\bar{\Phi})^2}{4\Phi\bar{\Phi}}\right] + S\bar{S} \qquad W = Sf(\Phi) + g(\Phi)$$

$$f(\phi) = \sqrt{(1-\phi)^2 + a^2}, \qquad g(\phi) = b$$

In canonical variables,

$$V = M^2\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2 + \Lambda$$

For $\alpha = 1$, it represents the Starobinsky model, but now it may have an arbitrary cosmological constant and gravitino mass.

$$\Lambda = a^2 - 3b^2, \quad m_{3/2} = b.$$

E-model with CC

The absence of the inflatino also helps us argue that there **there is no problem with the unitarity bound during inflation in our refined class of models.** The effective cutoff in supergravity is the scale at which scattering amplitudes violate unitarity bound.

In the theories with nilpotent fields, during inflation with $H \ll M_{Pl}$, this cutoff is expected at

$$\Lambda_{\text{cut-off}} \simeq \left((H^2 + m_{3/2}^2) M_{\text{Pl}}^2\right)^{1/4} > \sqrt{H M_{\text{Pl}}}$$

This UV cut-off is much higher than the typical energy of inflationary quantum fluctuations O(H).

In general, there could be some additional contributions to scattering due to gravitino-inflatino mixing, but in the theory that we consider there is no inflatino, and therefore no violation of the unitarity bound is expected during inflation at sub-Planckian energy density.

Non-linearly realized supersymmetry

Constrained multiplets are partnerless.

Fermion without a scalar partner, Volkov-Akulov goldstino

Gauge field without a gaugino

Inflaton without sinflaton and without inflatino

These models can be derived by imposing Lagrange multipliers and solving equations of motion consistently.

Alternatively, one can start with a linear supersymmetry model and send the mass of the partners to infinity consistently RK, Karlsson, Mosk, Murli, 2016

Generic constrained superfields can be coupled to gravity
Ferrara, RK, Van Proyen, Wrase, 2016
Dudas, Dall'Agata, Farakos, 2016
Aoki, Yamada, 2016

Dirac-Born-Infeld-Volkov-Akulov theory with 16+16 supersymmetries

Bergshoeff, Coomans, RK, Shahbazi, Van Proeyen 2013

$$S = -\frac{1}{\alpha^2} \int \mathrm{d}^4\sigma \sqrt{-\det(G_{\mu\nu} + \alpha \mathcal{F}_{\mu\nu})} \,,$$

The 16+16-component global supersymmetry of the action consists of 16 supersymmetries corresponding to a deformation of the original supersymmetries of the $\mathcal{N}$=4 Maxwell multiplet

The other 16 supersymmetries correspond to non-linear VA-type supersymmetries.

New fermionic soft theorems, 2014

[Wei-Ming Chen, Yu-tin Huang, Congkao Wen]

Exploring soft constraints on effective actions, 2016

[Massimo Bianchi, Andrea L. Guerrieri, Yu-tin Huang, Chao-Jung Lee, Congkao Wen]

Dirac-Born-infeld-Volkov-Akulov and recent progress in amplitudes.

The amplitudes behave badly at large z in the complex plane, can't use BCFW recursion relation

New recursion relation were discovered using soft limit theorems. In particular in VA sector the 4-f-coupling is unique.

$$\langle 12\rangle\, s_{12}\, [34] \qquad\qquad \chi^2 \Box \bar{\chi}^2$$

All n-point on-shell amplitudes are restored using new recursion relation or some related amplitude methods. 2016

S. He, Z. Liu, J-B. Wu

F. Cachazo, P. Cha, S. Mizera

$\longrightarrow$ double-soft theorems in DBI- VA theory will provide clues for the 'mysterious non-linearly realized (super) symmetries of the theory'.

A single particle soft limit of any 2n-point on-shell amplitude vanishes

A double-soft limit of any (2n+2)-point amplitude is related to 2n-point amplitude in agreement with G/H coset structure of non-linear (super)symmetry of this model

$$[\, G, G] = H \qquad \text{as is known in case of } \frac{E_{7(7)}}{SU(8)}$$

Double-soft limit

The double-soft limit in DBI-VA model relates amplitudes with two soft particles of spin $s = 0, 1/2, 1$ to amplitudes without these 2 particles. The small parameter $t \to 0$ is introduced as follows $\bar{\lambda}(p) \to t\bar{\lambda}(p)$, $\lambda(q) \to t\lambda(q)$

$$\mathcal{M}_{n+2}^{(s)} = t^{1+2s} \sum_{a=1}^{n} \frac{(k_a \cdot (q-p))^{2-2s}}{2k_a \cdot (p+q)} [p|a|q\rangle^{2s} \mathcal{M}_n^s + \mathcal{O}(t^{2+2s})$$

For $s = 1/2$, for soft fermions, one can see

that if either q or p actually vanish so that $\frac{(k_a \cdot (q-p))}{2k_a \cdot (p+q)} = \pm 1$, the amplitude vanishes as the single soft limit theorem is predicting. Also in $s = 1/2$ case

Non-linear supersymmetry algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = P_{\alpha\dot{\alpha}}$$

G-broken symmetry

H - unbroken symmetry

UV completion for constrained superfieds: string theory

1-loop quantum corrections to D3 brane action

Shmakova, 1999 using helicity amplitudes
De Giovanni, Santabrogio, Zanon, 1999, using superfields

Classical VA 4-point fermion helicity amplitude

$\langle 12\rangle \, s_{12} \, [34]$

They found on-shell 1-loop UV divergent amplitudes in Dirac-Born-Infeld-Volkov-Akulov model with 16 linear and 16 non-linear supersymmetries

VA 4-fermion part $\langle 12\rangle \, [34] \, s \, (s^2 + 3/4 \, t^2)$

$\mathcal{N}$=4 partner 4-vector part $\langle 12\rangle^2 \, [34]^2 \, (s^2 + t^2 + u^2)$

This is precisely the beginning of expansion of type I open string theory amplitude

Schwarz, 82

$$\frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} K(\epsilon, p)$$

Excellent behavior at large s, fixed angles, non-analytic in D3brane tension

Nonlinear Supersymmetry and Amplitudes

Original motivation:

RK, work in progress

2008, Arkani-Hamed, Cachazo, Kaplan on nonlinear $E_{7(7)}$ in N=8 supergravity. A double soft limit with two scalar momenta going to zero is defined by the coset space algebra $E_{7(7)}/SU(8)$ (Cremmer, Julia 1979, de Wit, Nicolai 1982)

$$[E_{7(7)}, E_{7(7)}] = SU(8)$$

2014, Chen, Huang, Wen. A double soft limit in Volkov-Akulov nonlinear supersymmetry (1972) is defined by the coset (super) space algebra, super-Poncare/translations.

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = P_{\alpha\dot\alpha}$$

2016, He, Liu, Wu and Cachazo, Cha, Mizera. In Dirac-Born-Infeld-Volkov-Akulov models, (Bergshoeff, Coomans, Kallosh, Shahbazi, Van Proeyen 2013) again double soft limit was defined recently.

In all cases the observation was made based on explicit answers for amplitudes.
The questions was posed by the authors of 2016 papers: could the universality of the double soft limits in these models be explained by nonlinear symmetries, not just observed on amplitudes which were computed?

$\{ G, G] = H$

Where is this rule for double soft limit coming from?

B. De Wit, D. Freedman, 1975

Studied single and double soft limit in Volkov-Akulov theory, found a relation between double-soft limit and commutators

Ward-type Identities for Yang-Mills and Gravity are well known. Using background field method, they take a simple form, and allow to make predictions on local UV divergences. Linear symmetries are

$$\delta_{\rm lin} A^a_\mu = D^{ab}_\mu \xi^b = \partial_\mu \xi^a + f^{abc} A^b_\mu \xi^c$$

$$\delta_{\rm lin} g_{\mu\nu} = D_{(\mu}\xi_{\nu)} = -g_{\mu\nu,\sigma}\xi^\sigma - g_{\nu\mu,\sigma}\xi^\sigma - g_{\mu\sigma}\xi^\sigma{}_{,\nu}$$

Much less is known for nonlinear symmetries, for example in Volkov-Akulov model there is a constant spinor, a translation linear in field and a term quadratic in fields

$$\delta\psi^\alpha = \zeta^\alpha + \xi^\mu \partial_\mu \psi^\alpha - i(\psi\sigma^\rho\bar\zeta - \zeta\sigma^\rho\bar\psi)\partial_\rho\psi^\alpha$$

For $E_{7(7)}$

$$\delta y = \Sigma + y\bar\Lambda - \Lambda y - y\bar\Sigma y$$

Here y is an inhomogeneous coordinate of the $\frac{E_{7(7)}}{SU(8)}$ coset space, Σ is a constant, an off-diagonal part of the element of $E_{7(7)}$ and finally, $\bar\Lambda$ and Λ represent a linear $SU(8)$ part.

The common feature is the constant (fermionic or bosonic) in symmetry rules, a term linear, in fields, and the term quadratic in fields

The answer to these and other questions about nonlinear symmetries will be given in RK, work in progress, using the generalization of de Witt's background field method to the case of nonlinear symmetries, in general. New identities explain

1. conditions for Adler's zero, vanishing single soft limit
2. the relation between nonlinear symmetries and features in a double-soft limit
3. define multiple-soft limit
4. show conditions beyond soft limits

For example, the identity relevant to the double soft limit is

$$\Big(S_{,ji_1i_2}\mathcal{R}^{i_1}_{\alpha}\mathcal{R}^{i_2}_{\beta} + S_{,ji_1}\mathcal{R}^{i_1}_{\gamma}f^{\gamma}_{\alpha\beta} + S_{,i_1}(\mathcal{R}^{i_1}_{\alpha,ji_2}\mathcal{R}^{i_2}_{\beta} - \mathcal{R}^{i_1}_{\beta,i_2}\mathcal{R}^{i_2}_{\alpha,j}\Big)\xi^{\alpha}\xi'^{\beta} = 0$$

Work in progress with Karlsson, Murli

We have tested new identities for the background field in the VA model, and confirmed it

Conclusion

- A nilpotent chiral multiplet and other constrained multiplets with non-linear supersymmetry are useful in cosmological inflationary model building

- Necessary to construct de Sitter supergravity without scalars to describe dark energy and provide a supersymmetric KKLT

- Present on the world-volume of D-branes in string theory and break SUSY spontaneously.

$$S^2 = S\,Y^i = S\,W_\alpha = S\,\overline{D}_{\dot{\alpha}}\overline{H}^{\bar{\imath}} = S(\Phi - \overline{\Phi}) = 0$$

- Fermion without a scalar, vector without a gluino, inflaton without an inflatino and without a sinflaton…