General Features of Spontaneous Baryogenesis

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Outline

- Introduction
- Generalities about SBG
- Evolution of Baryon Asymmetry
- Kinetic Equation in Time-dependent Background
- Conclusions

Standard Cosmological Model (SCM)

The Standard Model of Cosmology:

- the Standard Model of particle physics $\Rightarrow$ matter content
- General Relativity $\Rightarrow$ gravitational interaction
- the inflationary paradigm $\Rightarrow$ to fix a few problems of the scenario

SCM explains a huge amount of observational data:

- Hubble's law
- the primordial abundance of light elements
- the cosmic microwave background

Some Puzzles

Matter Inventory of the Universe:

- **Baryonic matter** (mainly protons and neutrons): **5%** .
- **Dark matter** (weakly interactive particles not belonging to the MSM of particle physics): **25%**.
- **Dark energy: 70%** of the Universe is really a mystery looking like a uniformly distributed substance with an unusual equation of state $\mathbf{P} \approx -\varrho$.
  DE is responsible for the contemporary accelerated expansion of the Universe.

The mechanism of inflation still remains at the level of paradigm. It does not fit the framework of the Standard Model of particle physics.

Matter-antimatter asymmetry around us: the local Universe is clearly matter dominated, but such an asymmetry cannot be created within the MSM of particle physics.

History of the Universe

Age	Temperature	Event
0	$+\infty$	Big bang (prediction of classical GR)
10^{-43} s	10^{19} GeV	Planck era (?)
10^{-35} s	10^{16} GeV	Grand Unified Theories era (?)
?	?	Inflation (?)
?	?	**Baryogenesis**
10^{-11} s	200 GeV	Electroweak symmetry breaking*
10^{-5} s	200 MeV	QCD phase transition*
1 s – 15 min	0.05-1 MeV	Big bang nucleosynthesis
60 kyr	1 eV	Matter-radiation equality
370 kyr	0.3 eV	Recombination and photon decoupling
0.2 – 1 Gyr	15 – 50 eV	Reionization
1 – 10 Gyr	3 – 15 eV	Structure formation
6 Gyr	4 K	Transition to an accelerating Universe
9 Gyr	3 K	Formation of the Solar System
13.8 Gyr	2.7 K	Today

Baryogenesis

Matter dominated Universe:

- The amount of antimatter is very small and it can be explained as the result of high energy collisions in space.
- The existence of large regions of antimatter in our neighborhood would produce high energy radiation as a consequence of matter-antimatter annihilation, which is not observed.

Inflationary paradigm:

- an initially tiny asymmetry cannot help, because the exponential expansion of the Universe during the inflationary period would have washed out any initial asymmetry.

A satisfactory model of our Universe should be able to explain the origin of the matter-antimatter asymmetry.

The term **baryogenesis** is used to indicate the generation of the asymmetry between baryons and antibaryons.

Sakharov Principles

Predominance of matter over antimatter was beautifully explained by Sakharov (1967) as dynamically generated in the early universe due to three conditions called now *Sakharov principles*:

1. Non-conservation of baryonic number.
2. Breaking of C and CP invariance.
3. Deviation from thermal equilibrium.

NB. None of them is strictly necessary.

There are some interesting scenarios of baryogenesis for which one or several of the above conditions are not fulfilled.

A very popular scenario is the so called spontaneous baryogenesis (SBG).

- The term "spontaneous" is related to spontaneous breaking of underlying symmetry of the theory.
- For successful SBG two of the Sakharov's conditions, namely, breaking of thermal equilibrium and a violation of C and CP symmetries, are unnecessary.

Spontaneous Baryogenesis (SBG)

- A. Cohen, D. Kaplan, Phys. Lett. B 199, 251 (1987); Nucl.Phys. B308 (1988) 913. A.Cohen, D.Kaplan, A. Nelson, Phys.Lett. B263 (1991) 86-92
- Review: A.D.Dolgov, Phys. Repts 222 (1992) No. 6; V.A. Rubakov, M.E. Shaposhnikov, Usp. Fiz. Nauk, 166 (1996) 493; A.D. Dolgov, Surveys in High Energy Physics, 13 (1998) 83.

Unbroken phase: the theory is invariant with respect to the global $U(1)$-symmetry, which ensures conservation of total baryonic number.
Spontaneous symmetry breaking: the Lagrangian density acquires the term

$$\mathcal{L}_{\mathbf{SB}} = (\partial_\mu \theta) \mathbf{J}^\mu_{\mathbf{B}}$$

where θ is the Goldstone field and $\mathbf{J}^\mu_{\mathbf{B}}$ is the baryonic current of matter field.

- NB: Due to the SSB this current is not conserved.

Next step: the statement (questionable)

$$\mathcal{H}_{\mathbf{SB}} = -\mathcal{L}_{\mathbf{SB}} = -(\partial_\mu \theta) \mathbf{J}^\mu_{\mathbf{B}}$$

Baryon Asymmetry in Thermal Equilibrium

Spatially homogeneous field $\theta = \theta(t)$:

$$\mathcal{H}_{SB} = -\dot{\theta}\, n_B\,, \quad n_B \equiv J^4_B$$

- n_B is the baryonic number density, so it is tempting to identify $\dot{\theta}$ with the chemical potential, μ, of the corresponding system.

If this is the case, then in thermal equilibrium with respect to the B-nonconserving interactions the baryon asymmetry would evolve to:

$$n_B = \frac{g_S B_Q}{6}\left(\mu\, T^2 + \frac{\mu^3}{\pi^2}\right) \to \frac{g_S B_Q}{6}\left(\dot{\theta}\, T^2 + \frac{\dot{\theta}^3}{\pi^2}\right)$$

- T is the cosmological plasma temperature
- g_S, B_Q are the number of the spin states and the baryonic number of quarks

SSB and Goldstone Mode

Let us consider the theory of complex scalar field Φ interacting with "quarks", **Q**, and "leptons", **L**, with the Lagrangian:

$$\mathcal{L}(\Phi) = g^{\mu\nu}\partial_\mu\Phi^*\partial_\nu\Phi - V(\Phi^*\Phi) + \bar{Q}(i\gamma^\mu\partial_\mu - m_Q)\,Q + \bar{L}(i\gamma^\mu\partial_\mu - m_L)L + \mathcal{L}_{int}(\Phi, Q, L)$$

$\mathcal{L}_{int}$ describes the interaction between Φ and fermionic fields:

$$\mathcal{L}_{int} = \frac{\sqrt{2}}{m_X^2}\frac{\Phi}{f}\,(\bar{L}\gamma_\mu Q)(\bar{Q}^c\gamma_\mu Q) + h.c.$$

- Q^c is charged conjugated quark spinor
- m_X is a parameter with dimension of mass and **f** is related to the vacuum expectation value of Φ defined below

Such an interaction can appear e.g. in *SU*(5) Grand Unified Theory.

Spontaneous Symmetry Breaking

This theory is invariant under the following $\mathbf{U(1)}$ transformations:

$$\mathbf{\Phi \to e^{i\alpha}\Phi, \quad Q \to e^{-i\alpha/3}Q, \quad L \to L}$$

- In the unbroken symmetry phase this invariance leads to the conservation of the total baryonic number of Φ and of quarks.

The global $\mathbf{U(1)}$-symmetry is assumed to be spontaneously broken at the energy scale $\mathbf{f}$ via the potential of the form:

$$\mathbf{V(\Phi^*\Phi) = \lambda\left(\Phi^*\Phi - f^2/2\right)^2}$$

This potential reaches minimum at the vacuum expectation value of Φ equal to $\langle\mathbf{\Phi}\rangle = \mathbf{f e^{i\phi_0/f}/\sqrt{2}}$ with a constant phase $\phi_\mathbf{0}$.

- Heavy radial mode of Φ with the mass $\mathbf{m_{radial} = \lambda^{1/2} f}$ is frozen out
- Light degree of freedom: the variable field ϕ = Goldstone boson of the spontaneously broken $U(1)$ (the angle around the bottom of the Mexican hat potential)
- The dimensionless angular field: $\theta \equiv \phi/\mathbf{f}$, and thus $\Phi = \langle\Phi\rangle \mathbf{e^{i\theta}}$

Lagrangian Densities

As a result the following effective Lagrangian for θ is obtained:

$$\mathcal{L}_1(\theta) = \frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + \bar{Q}_1(i\gamma^\mu\partial_\mu - m_Q)Q_1 + \bar{L}(i\gamma^\mu\partial_\mu - m_L)L + \left(\frac{e^{i\theta}}{m_X^2}(\bar{L}\gamma_\mu Q_1)(\bar{Q}_1^c\gamma_\mu Q_1) + h.c.\right) - U(\theta)$$

If $U(\theta) = 0$, the theory still remains invariant under the global, $\alpha = \text{const}$, transformations:

$$Q \to e^{-i\alpha/3}Q, \quad L \to L, \quad \theta \to \theta + \alpha$$

If we only rotate the quark field with coordinate dependent $\alpha = \theta(t, \mathbf{x})$, introducing the new field $Q_1 = e^{-i\theta/3}Q_2$, then:

$$\mathcal{L}_2(\theta) = \frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + \bar{Q}_2(i\gamma^\mu\partial_\mu - m_Q)Q_2 + \bar{L}(i\gamma^\mu\partial_\mu - m_L)L + \left(\frac{1}{m_X^2}(\bar{Q}_2\gamma_\mu L)(\bar{Q}_2\gamma_\mu Q_2^c) + h.c.\right) + (\partial_\mu\theta)J^\mu - U(\theta)$$

- $J^\mu = (1/3)\bar{Q}\gamma^\mu Q$ is the quark baryonic current

Dirac Equations

EoM for the quark field $\mathbf{Q_1}$ obtained from Lagrangian $\mathcal{L}_1(\theta)$:

$$(i\gamma^\mu \partial_\mu - m_Q)Q_1 + \frac{e^{-i\theta}}{m_X^2}\left[\gamma_\mu L(\bar{Q}_1\gamma_\mu Q_1^c) + 2\gamma_\mu Q_1^c(\bar{Q}_1\gamma_\mu L)\right] = 0$$

EoM for the phase rotated field $\mathbf{Q_2}$ from Lagrangian $\mathcal{L}_2(\theta)$:

$$\left(i\gamma^\mu \partial_\mu - m_Q + \frac{1}{3}\gamma^\mu \partial_\mu \theta\right) Q_2 + \frac{1}{m_X^2}\left[\gamma_\mu L(\bar{Q}_2\gamma_\mu Q_2^c) + 2\gamma_\mu Q_2^c(\bar{Q}_2\gamma_\mu L)\right] = 0$$

According to these equations, the baryonic current is not conserved.

Indeed, its divergence is:

$$\partial_\mu J_B^\mu = \frac{i\, e^{-i\theta}}{m_x^2}(\bar{Q}_1\gamma_\mu Q_1^c)(\bar{Q}_1\gamma^\mu L) + h.c.$$

Similarly for $\mathbf{Q_2}$ but without the factor $\mathbf{exp(-i\theta)}$.

When the symmetry is broken, the non-conservation of the physical baryons ("quarks" in our case) becomes essential and may lead to the observed cosmological baryon asymmetry.

Flat Space-Time

Equations for θ-field derived from two Lagrangians in flat space-time:

$$\mathbf{f}^2(\partial_t^2 - \Delta)\theta + U'(\theta) + \left[\frac{i\, e^{-i\theta}}{m_X^2}(\bar{Q}_1\gamma_\mu L)(\bar{Q}_1\gamma_\mu Q_1^c) + h.c.\right] = 0$$

and

$$f^2(\partial_t^2 - \Delta)\theta + U'(\theta) + \partial_\mu J_B^\mu = 0$$

In the spatially homogeneous case

$$\partial_\mu J_B^\mu = \dot{n}_B\,, \quad \theta = \theta(t)\,, \quad \text{and} \quad \text{if} \quad U(\theta) = 0$$

EoM can be easily integrated giving:

$$f^2\left[\dot{\theta}(t) - \dot{\theta}(t_{in})\right] = -n_B(t) + n_B(t_{in})$$

- It is usually assumed: $n_B(t_{in}) = 0$.

Cosmological FRW background

EoM of θ in cosmological Friedmann-Robertson-Walker background:

$$\mathbf{f^2(\partial_t + 3H)\dot{\theta} - a^{-2}(t)\,\Delta\theta + U'(\theta) = -(\partial_t + 3H)n_B}$$

- $\mathbf{a(t)}$ is the cosmological scale factor
- $\mathbf{H = \dot{a}/a}$ is the Hubble parameter

For the homogeneous theta-field, $\theta = \theta(t)$, this equation turns into:

$$\mathbf{f^2(\partial_t + 3H)\dot{\theta} + U'(\theta) = -(\partial_t + 3H)n_B}$$

- The current divergence in this case: $\mathcal{D}_\mu \mathbf{J}^\mu = \mathbf{\dot{n}_B + 3Hn_B}$, but not just $\mathbf{\dot{n}_B}$.

The kinetic equation $\Longrightarrow$ the evolution of $n_B(t)$ through $\theta(t)$ $\Longrightarrow$ the closed system of integro-differential equations.

Thermal equilibrium: the algebraic relation between $\dot{\theta}$ and $\mathbf{n_B}$

- $\dot{\theta}$ is constant or slowly varying function of time
- the integration over time is sufficiently long

Kinetic Equilibrium: Usual Dispersion Relation

Equilibrium with respect to baryo-conserving interactions $\Rightarrow$ the phase space distribution functions of fermions:

$$\mathbf{f_{eq} = [1 + \exp{(E/T - \xi_B)}]^{-1}}$$

- dimensionless chemical potential $\xi_B = \mu/T$ has equal magnitude but opposite signs for particles and antiparticles.

The baryonic number density:

$$n_B = \int \frac{d^3p}{(2\pi)^3} (f_B - f_{\bar{B}})$$

For normal dispersion relation $E = \sqrt{p^2 + m^2}$ in the massless case:

$$n_B = \frac{g_S B_Q}{6} \xi_B T^3 = \frac{g_S B_Q}{6} \mu_B T^2$$

- Since $n_B(t_{in}) = 0$, so we should also take $\xi_B(t_{in}) = 0$.

Conserved baryonic number: $n_B = 0$ and in the equilibrium $\xi_B \equiv 0$ at any time.
B-nonconserving interaction: from the kinetic equation $\Rightarrow$ $\xi_B = c\,\dot{\theta}/T$ (SBG).

Kinetic Equilibrium: Rotated Quark Field

Dirac equation for the phase rotated field:

$$\left(i\gamma^\mu\partial_\mu - m_Q + \frac{1}{3}\gamma^\mu\partial_\mu\theta\right)Q_2 + \frac{1}{m_X^2}\left[\gamma_\mu L(\bar{Q}_2\gamma_\mu Q_2^c) + 2\gamma_\mu Q_2^c(\bar{Q}_2\gamma_\mu L)\right] = 0$$

Dispersion relation:

$$\mathbf{E = \sqrt{p^2 + m^2} \mp \dot{\theta}/3}$$

The baryonic number:

$$\mathbf{n_B = \int \frac{d^3p}{(2\pi)^3}(f_B - f_{\bar{B}}) = \frac{g_S B_Q}{6}\left(\xi_B + \frac{\dot{\theta}}{3T}\right)T^3}$$

- If initially $\mathbf{n_B(t_{in}) = 0}$, then $\mathbf{\xi_B(t_{in}) = -\dot{\theta}_{in}/(3T_{in})}$.

Conserved baryonic number: $\mathbf{n_B \equiv 0}$ and in the equilibrium $\mathbf{\xi_B + \dot{\theta}/(3T) = 0}$

B-nonconserving interaction: the kinetic equation $\Rightarrow$ equilibr. solution $\mathbf{\xi_B = 0}$.

So $\boldsymbol{\xi_B \neq \dot{\theta}}$ but nevertheless the baryon asymmetry is proportional to $\boldsymbol{\dot{\theta}}$.

Kinetic Equation in (quasi)stationary Background

The temporal evolution of the distribution function of i-th type particle, $\mathbf{f_i(t, p)}$, in an arbitrary process $\mathbf{i + Y \leftrightarrow Z}$ in the FRW background, is governed by:

$$\frac{\mathbf{df_i}}{\mathbf{dt}} = (\partial_t - \mathbf{H\, p_i} \partial_{\mathbf{p_i}}) \mathbf{f_i} = \mathbf{I_i^{coll}}$$

with the collision integral equal to:

$$\mathbf{I_i^{coll}} = \frac{(2\pi)^4}{2E_i} \sum_{Z,Y} \int d\nu_Z \, d\nu_Y \, \delta^4(\mathbf{p_i} + \mathbf{p_Y} - \mathbf{p_Z})$$

$$\left[|\mathbf{A}(\mathbf{Z} \to \mathbf{i} + \mathbf{Y})|^2 \prod_Z f \prod_{i+Y} (1 \pm f) - |\mathbf{A}(\mathbf{i} + \mathbf{Y} \to \mathbf{Z})|^2 f_i \prod_Y f \prod_Z (1 \pm f) \right]$$

- $\mathbf{A(a \to b)}$ is the amplitude of the transition from state $\mathbf{a}$ to state $\mathbf{b}$
- Y and Z are arbitrary, generally multi-particle states, and

$$d\nu_Y = \prod_Y \overline{dp} \equiv \prod_Y \frac{d^3p}{(2\pi)^3 2E}.$$

The signs '+' or '−' in $\prod(1 \pm f)$ are chosen for bosons and fermions respectively.

Time Dependent Interaction: $\mathcal{L}_1^{int} = i\,\mathbf{e}^{-\mathbf{i}\theta}(\bar{Q}_1\gamma_\mu Q_1^c)(\bar{Q}_1\gamma^\mu L)/m_x^2$

The amplitude of the process is given by the matrix element of the Lagrangian between $|\mathbf{in}\rangle$ and $|\mathbf{fin}\rangle$ states which are taken as plane waves, $\exp(-iEt + i\mathbf{px})$, coming from decomposition of the field operators.

- In absence of external field the integration over $dt\, d^3x$ gives the four-momentum conservation delta-function: $\delta^4(\mathbf{P_{in} - P_{fin}})$.
- Now we have an additional factor under the integral: $\mathbf{\exp[\pm i\theta(t)]}$.
- For $\theta(\mathbf{t}) \approx \dot{\theta}\mathbf{t}$ with constant or slowly varying $\dot{\theta}$, the integral is simply taken.

$\mathbf{q_1 + q_2 \leftrightarrow \bar{q} + l}$: the kinetic equation gives the expression for the evolution of the baryonic number density

$$\mathbf{\dot{n}_B + 3Hn_B} \sim \int \mathbf{d\tau_{l\,\bar{q}} d\tau_{q_1\,q_2} |A|^2} \delta(\mathbf{E_{q_1} + E_{q_2} - E_l - E_{\bar{q}}} - \dot{\theta})$$
$$\delta(\mathbf{P_{in} - P_{fin}})\mathbf{e}^{-\mathbf{E_{in}/T}}\left(\mathbf{e}^{\xi_L - \xi_B + \dot{\theta}/\mathbf{T}} - \mathbf{e}^{2\xi_B}\right)$$

$d\tau_{l\,\bar{q}} = d^3p_l d^3p_{\bar{q}}/[4E_q E_{\bar{l}}(2\pi)^6]$.

NB. The energy is non-conserved due to the action of the external field $\theta(\mathbf{t})$:

$$\mathbf{E_{in} = E_{fin}} + \dot{\theta}$$

Kinetic Equation for Time-Varying Amplitude

Two-body inelastic process with baryonic number non-conservation:

$$\mathbf{a + b \leftrightarrow c + d}$$

The kinetic equation:

$$\dot{n}_B + 3Hn_B = -\frac{(2\pi)^3}{t_{max}} \int d\nu_{in} d\nu_{fin}\, \delta(P_{in} - P_{fin})\, |A|^2\, (f_a f_b - f_c f_d)$$

- $d\nu_{in} = d^3 p_a d^3 p_b / [4 E_a E_b (2\pi)^6]$

The amplitude of the process is defined as:

$$A = \left(\int_0^{t_{max}} dt\, e^{i[(E_c + E_d - E_a - E_b)t + \theta(t)]} \right) F(p_a, p_b, p_c, p_d)$$

- **F** is a function of 4-momenta of the participating particles, determined by the concrete form of the interaction Lagrangian.
- We consider two possibilities: $\mathbf{F = const}$ and $\mathbf{F} = \psi^4\, \mathbf{m_X^{-2}}$, where ψ^4 symbolically denotes the product of the Dirac spinors of particles **a, b, c, d**.

Kinetic Equation and Equilibrium Solution

We perform almost all (but one) integration over the phase space and finally obtain:

$$\dot{n}_B + 3Hn_B = -\frac{T^5}{2^5 \pi^6\, t_{max}} \times$$
$$\int_0^\infty dy \left[e^{\xi_a+\xi_b} \left(|A_+|^2 + |A_-|^2 e^{-y} \right) - e^{\xi_c+\xi_d} \left(|A_-|^2 + |A_+|^2 e^{-y} \right) \right]$$

- $y = E_-/T$ with $E_- = E_{in} - E_{fin}$
- A_+ is the amplitude taken at positive E_-, while A_- is taken at negative E_-.
- the only difference between A_+ and A_- is that in $A_-(\theta) = A_+(-\theta)$.

The equilibrium is achieved when the integral vanishes. It takes place at:

$$\xi_a + \xi_b - \xi_c - \xi_d = \frac{\langle |A_+|^2 e^{-y} + |A_-|^2 \rangle}{\langle |A_+|^2 + |A_-|^2 e^{-y} \rangle} - 1$$

where the angular brackets mean integration over **dy**.

$$A = \left(\int_0^{\mathbf{t_{max}}} dt\, \mathbf{e^{i[(E_c+E_d-E_a-E_b)t+\theta(t)]}}\right) F(p_a, p_b, p_c, p_d),\ \dot{\boldsymbol{\theta}} = \mathbf{const}$$

The integral is taken analytically resulting in:

$$|\mathbf{A}|^2 \sim \frac{\mathbf{2 - 2\cos[(\dot{\boldsymbol{\theta}} - E_-)t_{max}]}}{\mathbf{(\dot{\boldsymbol{\theta}} - E_-)^2}}$$

Large $\mathbf{t_{max}}$:

- this expression tends to $\delta(\mathbf{E_-} - \dot{\theta})$ and the equilibrium solution coincides with the standard result:

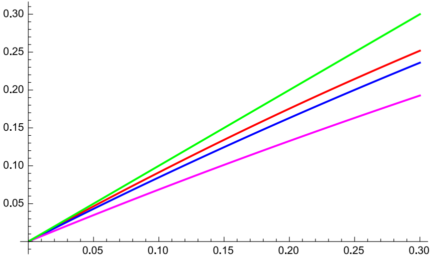
$$\boldsymbol{\xi}_\mathbf{a} + \boldsymbol{\xi}_\mathbf{b} - \boldsymbol{\xi}_\mathbf{c} - \boldsymbol{\xi}_\mathbf{d} - \dot{\boldsymbol{\theta}}/\mathbf{T} = \mathbf{0}$$

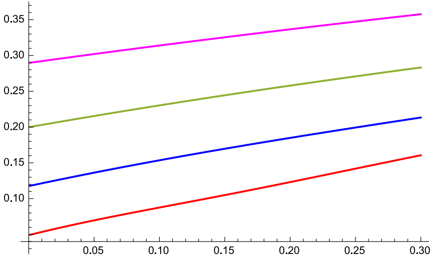
- The limit of $\dot{\boldsymbol{\theta}} = \mathbf{const}$ corresponds to the energy non-conservation by the rise (drop) of the final state energy in reaction $\mathbf{a} + \mathbf{b} \leftrightarrow \mathbf{c} + \mathbf{d}$ exactly by $\dot{\boldsymbol{\theta}}$.

If $\mathbf{t_{max}}$ is not sufficiently large, the non-conservation of energy is not equal to $\dot{\boldsymbol{\theta}}$ and the equilibrium solution would be different.

Numerical Results: $\dot{\theta} = const$, finite t_{max}

Equilibrium solution: $\xi_a + \xi_b - \xi_c - \xi_d = \frac{\langle |A_+|^2 e^{-y} + |A_-|^2 \rangle}{\langle |A_+|^2 + |A_-|^2 e^{-y} \rangle} - 1$



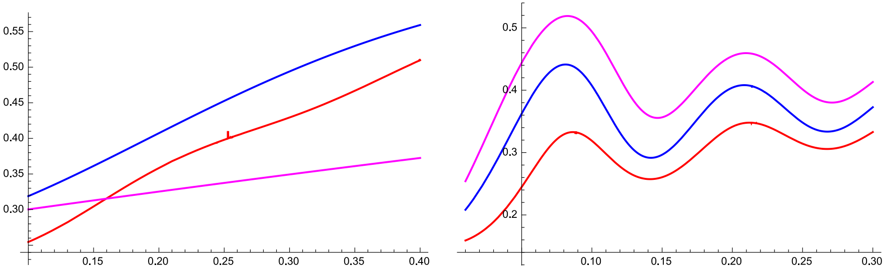


Left panel: $\dot{\theta}/T$ for infinite time integration (green line).
Cut-off in time integration $\tau \equiv t_{max} T = 30$ (red); 10 (blue); 3 (magenta)

Right panel: relative differences between the equilibrium solutions and $\dot{\theta}/T$, normalized to $\dot{\theta}/T$, as functions of $\dot{\theta}$ for different t_{max}.
$\tau \equiv t_{max} T = 30$ (red); 10 (blue); 5 (green); 3 (magenta).

Second order Taylor expansion: $\theta(t) = \dot{\theta}\, t + \ddot{\theta}\, t^2/2$

The real and imaginary parts of the integral $\int_0^{\mathbf{t_{max}}} \mathbf{dt}\,\mathbf{exp}\left[\mathbf{i}\theta(\mathbf{t})\right]$ are expressed through the Fresnel functions.



Relative differences between equilibrium solutions and $\dot{\boldsymbol{\theta}}/\mathbf{T}$, normalized to $\dot{\boldsymbol{\theta}}/\mathbf{T}$:

- *Left:* as functions of $\dot{\boldsymbol{\theta}}$ for different $\mathbf{t_{max}}$; $\ddot{\boldsymbol{\theta}} = \mathbf{0.1}$.
  $\boldsymbol{\tau} \equiv t_{max}\, T$ = 30 (red); 10 (blue); 3 (magenta).
- *Right:* as functions of $\ddot{\boldsymbol{\theta}}$ for different $\dot{\boldsymbol{\theta}}$; $\boldsymbol{\tau} \equiv \mathbf{t_{max}\,T} = \mathbf{10}$.
  $\dot{\boldsymbol{\theta}}$ = 0.1 (red); 0.2 (blue); 0.3 (magenta).

CONCLUSIONS:

- We argue that in the standard description $\dot{\theta}$ is not formally the chemical potential, though in equilibrium with respect to baryo-nonconserving interactions $\dot{\theta}$ may tend to the chemical potential with the numerical coefficient which depends upon the model.
- Moreover, this is not always true but depends upon the representation chosen for the "quark" fields.
- In the theory described by the Lagrangian which appears "immediately" after the spontaneous symmetry breaking, $\theta(t)$ directly enters the interaction term and in equilibrium $\mu_B \sim \dot{\theta}$ indeed.
- If the dependence on θ is shifted to the quartic product of the quark fields, then chemical potential in equilibrium does not tend to $\dot{\theta}$, but to zero.
- However, the magnitude of the baryon asymmetry in equilibrium is always proportional to $\dot{\theta}$.
- To include the effects of an arbitrary variation of $\theta(t)$ as well as the effects of the finite time integration we transformed the kinetic equation in such a way that it becomes operative in the case of non-conserved energy.
- A shift of the equilibrium value of the baryonic chemical potential due to this effect is numerically calculated.

THE END

THANK YOU FOR YOUR ATTENTION!

Due to the quark-lepton transitions the current is non-conserved:

$$\partial_\mu J^\mu_B = \frac{i\, e^{-i\theta}}{m_x^2} (\bar{Q}_1 \gamma_\mu Q_1^c)(\bar{Q}_1 \gamma^\mu L) + h.c. \quad or \quad \partial_\mu J^\mu_B = \frac{i}{m_x^2} (\bar{Q}_2 \gamma_\mu Q_2^c)(\bar{Q}_2 \gamma^\mu L) + h.c.$$

Let us first consider the latter case, with the interaction described by Lagrangian:

$$\mathcal{L}_2^{\mathbf{int}} = \frac{\mathbf{i}}{\mathbf{m_x^2}} (\mathbf{\bar{Q}_2 \gamma_\mu Q_2^c})(\mathbf{\bar{Q}_2 \gamma^\mu L}) + \mathbf{h.c.}$$

An example: the process $\mathbf{q_1 + q_2 \leftrightarrow \bar{q} + l}$.

Since the interaction in this representation does not depend on time:

- The energy is conserved and the collision integral has the usual form with conserved four-momentum.
- Quarks are supposed to be in kinetic equilibrium but probably not in equilibrium with respect to B-nonconserving interactions:

$$f_Q = \exp\left(-E/T + \xi_B\right) \ \text{and} \ f_{\bar{Q}} = \exp\left(-E/T - \xi_B\right)$$

- Dispersion relation: $\mathbf{E = \sqrt{p^2 + m^2} \mp \dot{\theta}/3}$, $\mathbf{n_B = (\xi_B + \dot{\theta}/(3T))T^3}$

Time Independent Interaction: Equilibrium Solution

The kinetic equation takes the form:

$$\frac{g_S B_Q}{6}\frac{d}{dt}\left(\xi_B + \frac{\dot\theta}{3T}\right) = -c_1 \Gamma \xi_B$$

$c_1 \sim 1$ and Γ is the rate of baryo-nonconserving reactions, $\Gamma \sim T^5/m_X^4$.

For constant or slow varying temperature:

- for large Γ the equilibrium solution is $\xi_B = 0$
- the baryon number density is proportional to $\dot\theta$, $n_B = (g_S B_Q/18)\,\dot\theta T^2$, with $\dot\theta$ evolving

$$\dot\theta = \frac{f^2}{f^2 + g_S B_Q T^2/18}\left(\frac{T}{T_{in}}\right)^3 \dot\theta(t_{in}).$$

The equilibrium value of n_B drops down with decreasing temperature as T^5.

However at small temperatures baryon non-conserving processes switch-off and n_B tends to a constant value in comoving volume.

Slow Varying $\dot{\theta}$: The Equilibrium Solution

The conservation of $(B+L)$ implies the following relation: $\xi_L = -\xi_B/3$. Keeping this in mind, we find

$$\dot{n}_B + 3Hn_B \approx -\left(1 - e^{\dot{\theta}/T - 3\xi_B + \xi_L}\right) I \approx \left(\frac{\dot{\theta}}{T} - \frac{10}{3}\xi_B\right) I$$

- factor I comes from the collision integral
- We assumed that ξ_B and $\dot{\theta}/T$ are small.
- In relativistic plasma with temperature T: $I \approx T^8/m^4$.

For a large factor I we expect the equilibrium solution

$$\xi_B = \frac{3}{10}\frac{\dot{\theta}}{T}$$

so, up to the different numerical factor, $\dot{\theta}$ seems to be the baryonic chemical potential, as expected in the canonical version of SBG scenario.

NB: The baryonic chemical potential is not aways proportional to $\dot{\theta}(t)$.