Radiative corrections in e^+e^- collisions with the **BabaYaga** event generator

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in collaboration with

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Outline

- Motivations for precise luminometry
- QED processes & radiative corrections
- The **BabaYaga** and **BabaYaga@NLO** event generators
  - theoretical framework
  - improving theoretical accuracy:
    QED Parton Shower and matching with NLO corrections
- Results, tuned comparisons, theoretical accuracy
- Dark-photon searches through radiative return method
- Conclusions

Relevant references

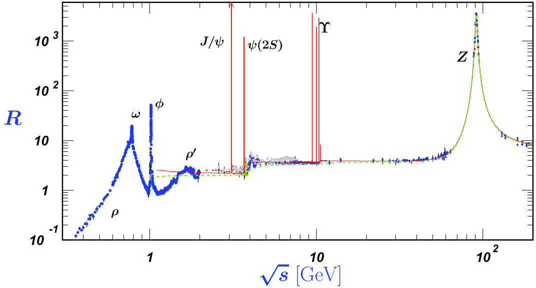
- Website
  http://www.pv.infn.it/hepcomplex/babayaga.html

- **BabaYaga** main references:
  - Barzè et al., Eur. Phys. J. C **71** (2011) 1680 — **BabaYaga** with dark photon
  - Balossini et al., Phys. Lett. **663** (2008) 209 — **BabaYaga@NLO** for $e^+e^- \to \gamma\gamma$
  - Balossini et al., Nucl. Phys. **B758** (2006) 227 — **BabaYaga@NLO** for Bhabha
  - C.M.C.C. et al., Nucl. Phys. Proc. Suppl. **131** (2004) 48 — **BabaYaga@NLO**
  - C.M.C.C., Phys. Lett. B **520** (2001) 16 — improved PS **BabaYaga**
  - C.M.C.C. et al., Nucl. Phys. B **584** (2000) 459 — **BabaYaga**

- Related work:
  - S. Actis et al.
    "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", Eur. Phys. J. C **66** (2010) 585
    Report of the Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies
  - C.M.C.C. et al., JHEP **1107** (2011) 126
    NNLO massive pair corrections

Why precision luminosity generators?

- Precision measurements require a precise knowledge of the machine luminosity
- *e.g.*, the measurement of the $R(s)$ ratio is a key ingredient for the predictions of $a_\mu = (g_\mu - 2)/2$ and $\Delta\alpha_{\rm had}(M_Z)$ and in turn for SM precision tests



$$a_\mu = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} {\rm d}s\, K(s)\frac{R(s)}{s} \qquad \Delta\alpha^{(5)}_{\rm had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\text{Re}\int_{m_\pi^2}^{\infty}\frac{R(s){\rm d}s}{s(s-M_Z^2-i\epsilon)}$$

Reference processes for luminosity

- Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

- Normalization processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ Large-angle QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories to achieve a typical precision at the level of $1 \div 0.1\%$

$\hookrightarrow$ QED RC corrections are mandatory

$\mapsto$ **BabaYaga** has been developed for high-precision simulation of QED processes at flavour factories (primarily for luminosity determination)

Theory of QED corrections into MC generators

- ⋆ The most precise MC generators include exact $\mathcal{O}(\alpha)$ (NLO) photonic corrections matched with higher–order leading logarithmic contributions
  [+ vacuum polarization, using a data driven routine for the calculation of the non–perturbative $\Delta\alpha^{(5)}_{\rm had}(q^2)$ hadronic contribution]
- ⋆ Common methods used to account for multiple photon corrections are the analytical collinear QED Structure Functions (SF), YFS exponentiation and QED Parton Shower (PS)

- The QED PS [implemented in BabaYaga/BabaYaga@NLO] is an exact MC solution of the QED DGLAP equation for the electron SF $D(x, Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t}, Q^2)$$

- The PS solution can be cast into the form

$$D(x, Q^2) \;=\; \Pi(Q^2) \sum_{n=0}^{\infty} \int \frac{\delta(x - x_1 \cdots x_n)}{n!} \prod_{i=0}^{n} \left[\frac{\alpha}{2\pi} P(x_i)\; L\; dx_i \right]$$

  → $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x) dx$, $L \equiv \ln Q^2/m^2$ collinear log, ϵ soft–hard separator and Q^2 virtuality scale
  → the kinematics of the photon emissions can be recovered → exclusive photons generation

- The accuracy is improved by matching exact NLO with higher-order leading log corrections
  - ⋆ theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections, for all QED channels [Bhabha, $\gamma\gamma$ and $\mu^+\mu^-$]

Summary of QED (photonic) radiative corrections

Pictorially, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Blue: Leading-Log PS, Leading-Log YFS, SF

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h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Red: matched PS, YFS, SF + NLO

Matching NLO and PS in BabaYaga@NLO

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma_{LL}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$
- $d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}]\, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{LL}^{SV}(\varepsilon) + d\sigma_{LL}^{H}(\varepsilon)$
- $d\sigma_{\rm NLO}^{\alpha} = [1 + C_{\alpha}]\, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1}|^2 d\Phi_1 \equiv d\sigma_{\rm NLO}^{SV}(\varepsilon) + d\sigma_{\rm NLO}^{H}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,LL}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$

$$d\sigma_{\rm matched}^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for $n+2$ final-state particles

Matching NLO and PS in BabaYaga@NLO

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of LL
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm NLO}$
- resummation of higher orders LL contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV \mid H,i} \times LL$

G. Montagna et al., **PLB** 385 (1996)

- the theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \sim 5 \times 10^{-4}$$

Results with BabaYaga@NLO

- to show the typical size of RC, the following setups and definitions are used (for Bhabha)

  a. $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $20^\circ < \theta_\pm < 160^\circ$, $\xi_{max} = 10^\circ$
  b. $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $55^\circ < \theta_\pm < 125^\circ$, $\xi_{max} = 10^\circ$
  c. $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $20^\circ < \theta_\pm < 160^\circ$, $\xi_{max} = 10^\circ$
  d. $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $55^\circ < \theta_\pm < 125^\circ$, $\xi_{max} = 10^\circ$

$$\delta_{VP} \equiv \frac{\sigma_{0,VP} - \sigma_0}{\sigma_0} \qquad \delta_\alpha \equiv \frac{\sigma_\alpha^{NLO} - \sigma_0}{\sigma_0}$$

$$\delta_{HO} \equiv \frac{\sigma_{matched}^{PS} - \sigma_\alpha^{NLO}}{\sigma_0} \qquad \delta_{HO}^{PS} \equiv \frac{\sigma^{PS} - \sigma_\alpha^{PS}}{\sigma_0}$$

$$\delta_\alpha^{non\text{-}log} \equiv \frac{\sigma_\alpha^{NLO} - \sigma_\alpha^{PS}}{\sigma_0} \qquad \delta_\infty^{non\text{-}log} \equiv \frac{\sigma_{matched}^{PS} - \sigma^{PS}}{\sigma_0}$$

Results with BabaYaga@NLO

setup	(a)	(b)	(c)	(d)
δ_{VP}	1.76	2.49	4.81	6.41
δ_{α}	-11.61	-14.72	-16.03	-19.57
δ_{HO}	0.39	0.82	0.73	1.44
δ_{HO}^{PS}	0.35	0.74	0.68	1.34
$\delta_{\alpha}^{non\text{-}log}$	-0.34	-0.56	-0.34	-0.56
$\delta_{\infty}^{non\text{-}log}$	-0.30	-0.49	-0.29	-0.46

Table: Relative corrections (in per cent) to the Bhabha cross section for the four setups

- ⋆ in short, the fact that $\delta_{\alpha}^{non\text{-}log} \simeq \delta_{\infty}^{non\text{-}log}$ and $\delta_{HO} \simeq \delta_{HO}^{PS}$ means that the matching algorithm preserves both the advantages of exact NLO calculation and PS approach:
  - → it includes the missing NLO RC to the PS
  - → it adds the missing higher-order RC to the NLO

Estimating the theoretical accuracy

S. Actis et al. Eur. Phys. J. C **66** (2010) 585

- It is extremely important to compare independent calculations/implementations/codes, in order to
  - ⋆ asses the technical precision, spot bugs (with the same th. ingredients)
  - ⋆ estimate the theoretical “error” when including partial/incomplete higher-order corrections
- A number of generators are available,
  some of them including QED h.o. and NLO corrections according to different approaches (collinear SF + NLO, YFS exponentiation,. . .)

Generator	Processes	Theory	Accuracy	Web address
BHAGENF/BKQED	$e^+e^-/\gamma\gamma,\mu^+\mu^-$	$\mathcal{O}(\alpha)$	1%	www.lnf.infn.it/˜graziano/bhagenf/bhabha.html
BabaYaga v3.5	$e^+e^-,\gamma\gamma,\mu^+\mu^-$	Parton Shower	$\sim\ 0.5\%$	www.pv.infn.it/˜hepcomplex/babayaga.html
BabaYaga@NLO	$e^+e^-,\gamma\gamma,\mu^+\mu^-$	$\mathcal{O}(\alpha)+$ PS	$\sim 0.1\%$	www.pv.infn.it/˜hepcomplex/babayaga.html
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	0.5%(LEP1)	placzek.home.cern.ch/placzek/bhwide
MCGPJ	$e^+e^-,\gamma\gamma,\mu^+\mu^-$	$\mathcal{O}(\alpha)+$ SF	$< 0.2\%$	cmd.inp.nsk.su/˜sibid

Large angle Bhabha: tuned comparisons & technical precision

Without vacuum polarization, to compare QED corrections consistently

At the Φ and τ–charm factories (cross sections in nb)

By BabaYaga group, Ping Wang and A. Sibidanov

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_\mp \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_\mp \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\lvert\vartheta_+ + \vartheta_- - \pi\rvert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

★ Agreement well below 0.1%! ★

At BaBar (cross sections in nb)

By A. Hafner and A. Denig

angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

★ Agreement at the $\sim 0.1\%$ level! ★

Theoretical accuracy, comparisons with NNLO

- NLO RC being included, the theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO)
  $\hookrightarrow$ anyway large NNLO RC already included by h.o. exponentiation and by $\mathcal{O}(\alpha)$ LL $\times$ finite-NLO

⋆ The full set of NNLO QED corrections to Bhabha scattering has been calculated in the last years

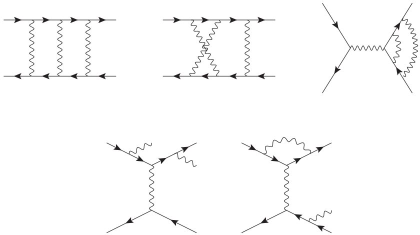
- **BabaYaga** formulae can be truncated at $\mathcal{O}(\alpha^2)$ to be consistently and systematically compared with all the classes of NNLO corrections

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\mathrm{SV}} + \sigma^{\alpha^2}_{\mathrm{SV,H}} + \sigma^{\alpha^2}_{\mathrm{HH}}$$

  - $\sigma^{\alpha^2}_{\mathrm{SV}}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$
    $\mapsto$ compared with the corresponding available NNLO QED calculation
  - $\sigma^{\alpha^2}_{\mathrm{SV,H}}$: one–loop soft+virtual corrections to single hard bremsstrahlung
    $\mapsto$ presently estimated relying on existing (partial) results
  - $\sigma^{\alpha^2}_{\mathrm{HH}}$: double hard bremsstrahlung
    $\mapsto$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

NNLO Bhabha calculations

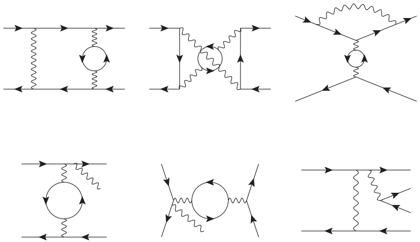
- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- Electron loop corrections

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26

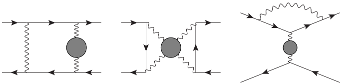


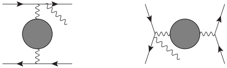
NNLO Bhabha calculations

- Heavy fermion and hadronic loops

R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601
S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

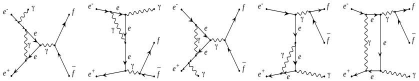
J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300





- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419

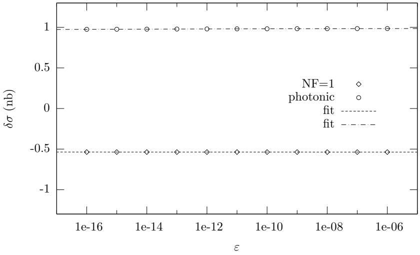


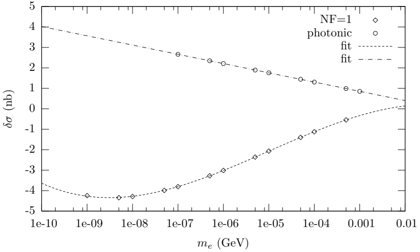
Comparison with NNLO calculation for $\sigma_{\rm SV}^{\alpha^2}$

Using realistic cuts for luminosity @ KLOE

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of **BabaYaga@NLO** with

- Penin (photonic): function of the logarithm of the soft photon cut–off (left plot) and a fictitious electron mass (right plot)





★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\texttt{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

Lepton and hadron loops & pairs at NNLO

- The exact NNLO soft+virtual corrections and $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-)$ $[l = e, \mu, \tau]$, $e^+e^- \to e^+e^-(\pi^+\pi^-)$ are available
- Compared to the *approximation* in **BabaYaga@NLO**, using realistic luminosity cuts ($S_i \equiv \sigma_i^{\text{NNLO}}/\sigma_{BY}$)

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

⋆ The uncertainty due to lepton and hadron pair NNLO corrections is at the level of a few units in 10^{-4}

Carloni, Czyz, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann *et al.*, JHEP **1107** (2011) 126

Error budget for Bhabha luminometry

main conclusion of the Luminosity Section of the WG Report

Putting the sources of uncertainties (in large–angle Bhabha) all together:

Source of error (%)	$\Phi-$factories	$\sqrt{s}$ = 3.5 GeV	$B-$factories
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta^{\rm err}_{{\rm SV},\alpha^2}\rvert$	0.02	0.02	0.02
$\lvert\delta^{\rm err}_{{\rm HH},\alpha^2}\rvert$	0.00	0.00	0.00
$\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$	0.05	0.05	0.05
$\lvert\delta^{\rm err}_{\rm pairs}\rvert$	0.03	0.016	0.03
$\lvert\delta^{\rm err}_{\rm total}\rvert$ linearly	0.12	0.1	0.13
$\lvert\delta^{\rm err}_{\rm total}\rvert$ in quadrature	0.07	0.06	0.06

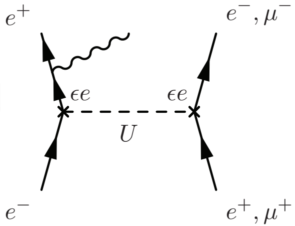
- For the experiments on top of and closely around the narrow resonances (J/ψ, Υ,...), the accuracy deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ parameterizations

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP/SLC

BabaYaga for dark photon searches at low-enegies

- From normalization to "discovery" tool →

$$e^+e^- \to \gamma + \gamma^{\mathbf{dark}} \to \ell^+\ell^-\gamma\ (n\gamma)$$



- dark photon (U-boson) production via radiative return, including ISR (with LL collinear structure-functions)

- Implemented model:
  "secluded" $U(1)_S$ symmetry with a light vector gauge field, heavy DM, complex Higgs field for $U(1)_S$ SSB
  ↪ coupling to SM fields through $\gamma^{\mathrm{dark}}/\gamma$ mixing

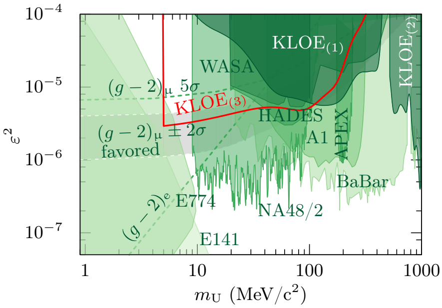
$$\mathcal{L}_{mix} = \frac{\epsilon}{2} F^{\mu\nu}_{\gamma^{\mathrm{dark}}} F^{\gamma}_{\mu\nu}$$

- extremely weak signal → control of background mandatory

KLOE-2 results

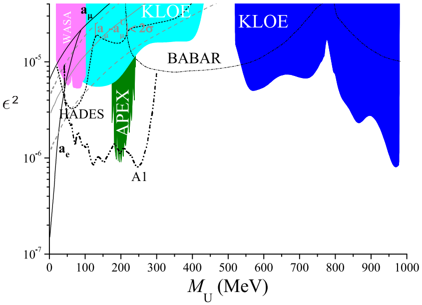
- **`BabaYaga`** with dark-photon production used in KLOE-2 analyses for exclusion plots

$$e^+e^- \to \gamma U, U \to e^+e^-$$



A. Anastasi *et al.*, Phys. Lett. B **750** (2015) 633

$$e^+e^- \to \gamma U, U \to \mu^+\mu^-$$



D. Babusci *et al.* Phys. Lett. B **736** (2014) 459

Conclusions

- ⋆ In the last 15 years **BabaYaga/BabaYaga@NLO** has been developed for high-precision luminometry at flavour factories
- ⋆ It simulates QED processes

  ↪ $e^+e^- \to e^+e^- \quad (+n\gamma)$

  ↪ $e^+e^- \to \mu^+\mu^- \quad (+n\gamma)$

  ↪ $e^+e^- \to \gamma\gamma \quad (+n\gamma)$

  with multiple-photon emission in a QED Parton Shower framework, matched with exact NLO matrix elements
- ⋆ A theoretical precision at the 0.5×10^{-3} level is achieved (at least for Bhabha), with a systematic comparison to independent calculations/codes and missing higher-order corrections
- ⋆ It has been extended to simulate dark-photon production via radiative-return method for search at low-energy e^+e^- colliders
- ⋆ Improving the accuracy of QED processes would imply the inclusion of exact full 2-loop corrections, which is (in principle) feasible if needed by experiments