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Jefferson Lab

On the importance of hadronic τ-decays for Charged Lepton Flavour Violation

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In collaboration with A. Celis (LMU, Munich),
and V. Cirigliano (LANL)
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Outline :

1. Introduction and Motivation
2. Charged Lepton-Flavour Violation
3. Special Role of $\tau \to \mu\pi\pi$: hadronic form factors
4. Conclusion and outlook

1. Introduction and Motivation

1.1 The τ lepton

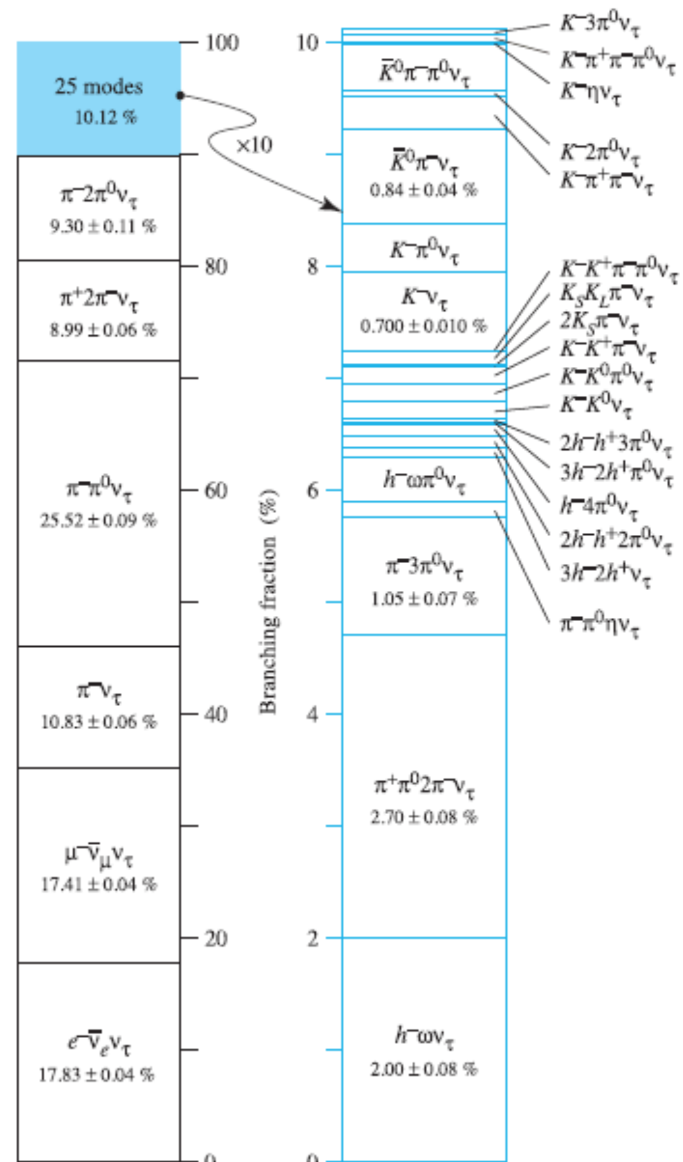
PDG'14

- τ lepton discovered in 1976 by M. Perl et al. (SLAC-LBL group)
  - Mass: $m_\tau = 1.77682(16)$ GeV
  - Lifetime: $\tau_\tau = 2.096(10) \cdot 10^{-13}\, s$

- Enormous progress in tau physics since then (CLEO, LEP, Babar, Belle, BES, VEPP-2M, neutrino experiments,...)

S. Eidelman

Group	$\int L\, dt$, fb^{-1}	$N_{\tau\tau}$, 10^6
LEP (Z-peak)	0.34	0.33
CLEO (10.6 GeV)	13.8	12.6
BaBar (10.6 GeV)	534	492
Belle (10.6 GeV)	854	782
τ-c (4.2 GeV)	10	32
SuperB	50k	45k



1.2 Testing QCD and EW with τ

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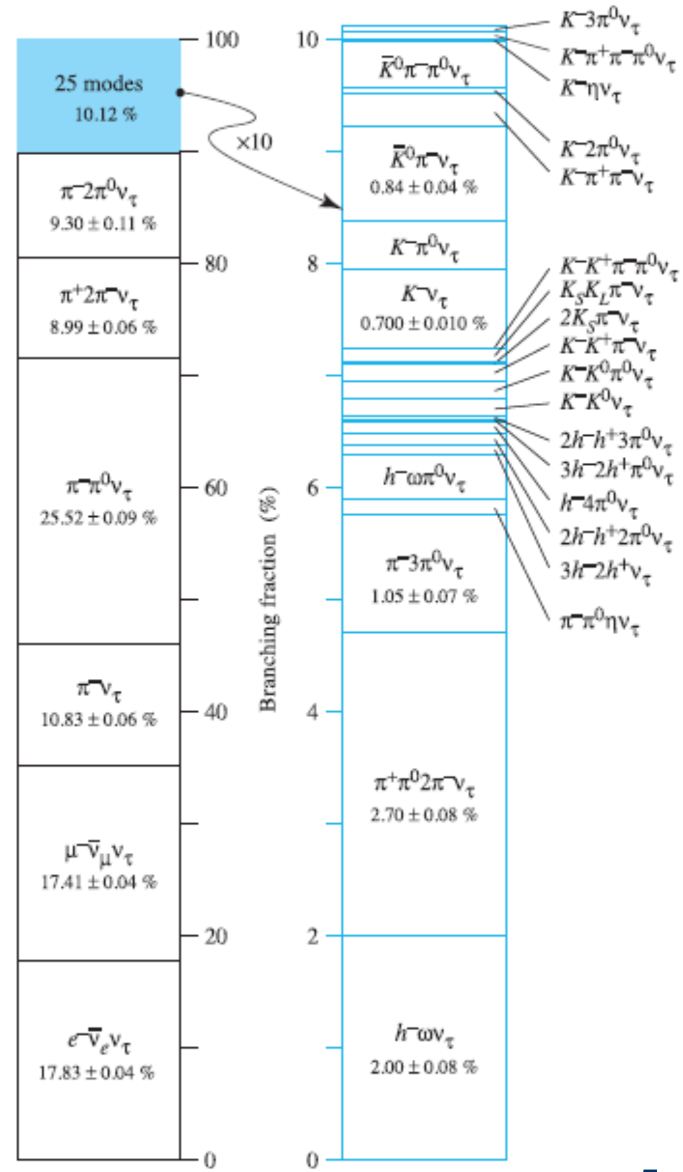
- Enormous progress in tau physics since then (CLEO, LEP, Babar, Belle, BES, VEPP-2M, neutrino experiments,...)
  - Early years: consolidate τ as a standard lepton no invisible decays and standard couplings
  - Better data: determination of fundamental SM parameters and QCD studies
  - More recently: huge number of tau at the B factories: BaBar, Belle:
    - Tool to search for NP: rare decays, final states in hadron colliders
    - Precision physics: ⇨ α_S, $|V_{us}|$ etc



2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

- Lepton Flavour Violation is an « accidental » symmetry of the SM (m_ν=0)

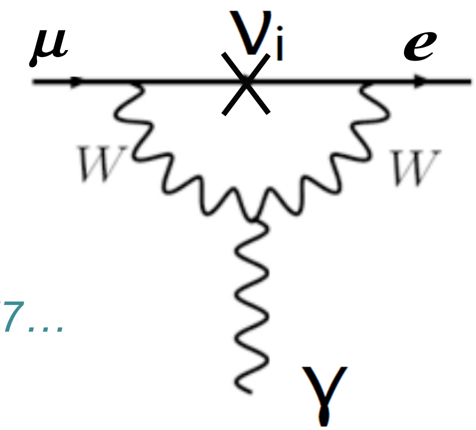
- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

  E.g.: $\mu \to e\gamma$

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2}\right|^2 < 10^{-54}$$

Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$



- Extremely *clean probe of beyond SM physics*

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013

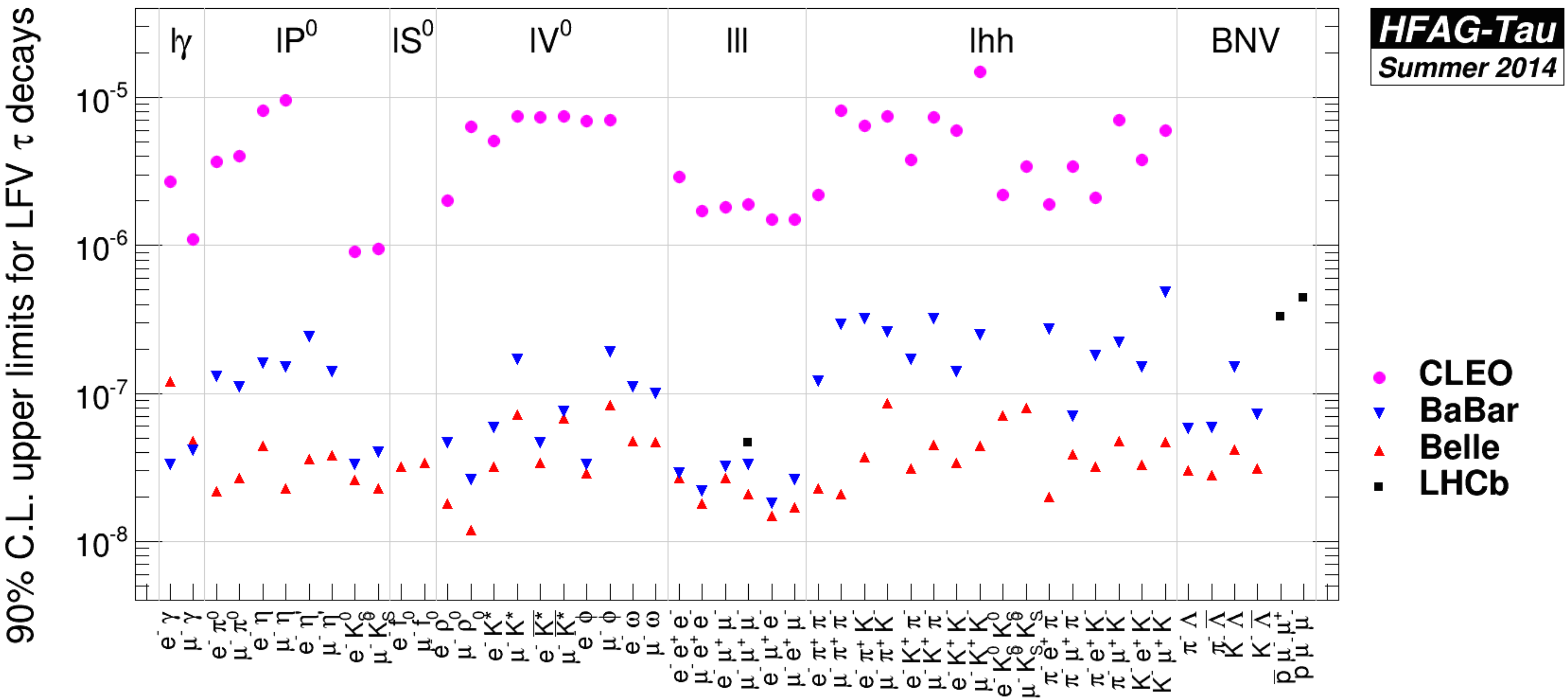
		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent

- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ← $P,\ S,\ V,\ P\bar{P},\ldots$



- 48 LFV modes studied at Belle and BaBar

2.2 CLFV processes: tau decays

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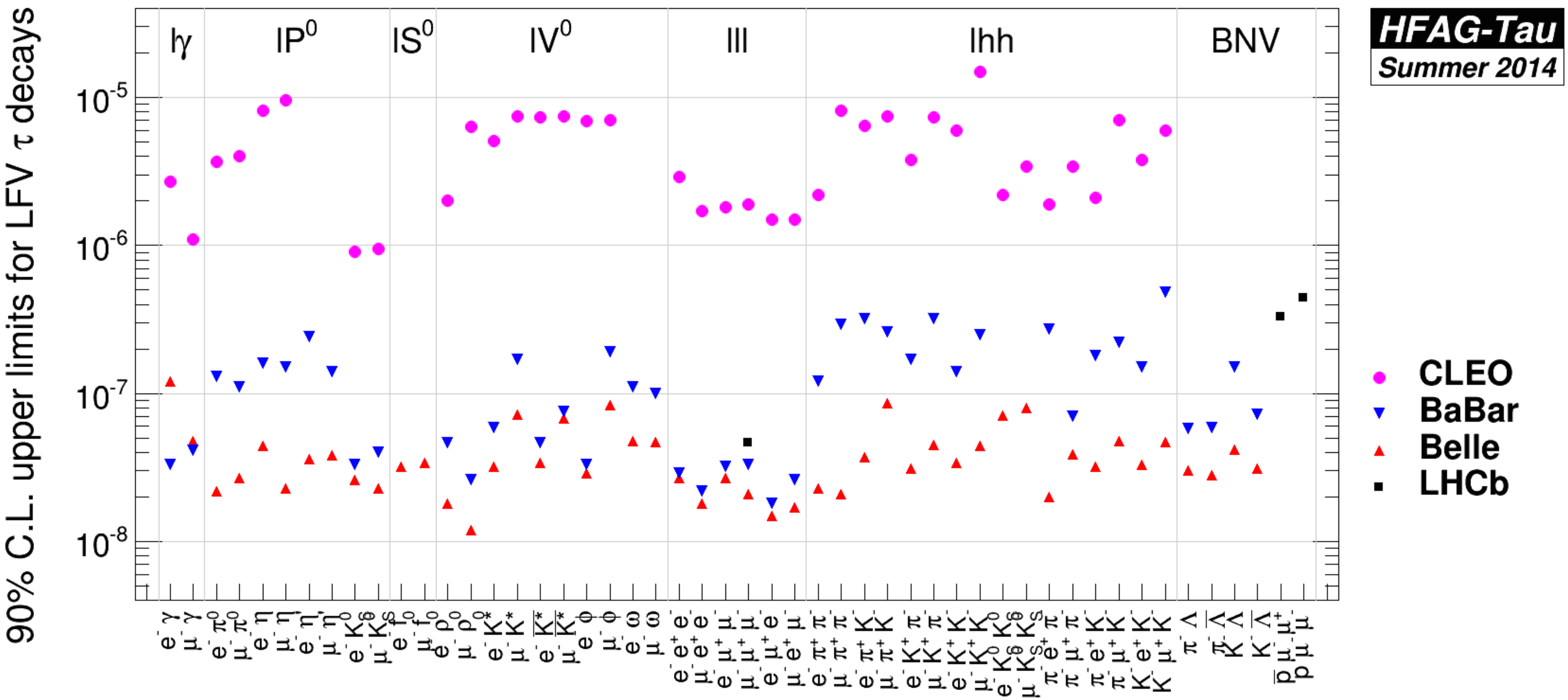

- Expected sensitivity 10^{-9} or better at *LHCb, Belle II?*

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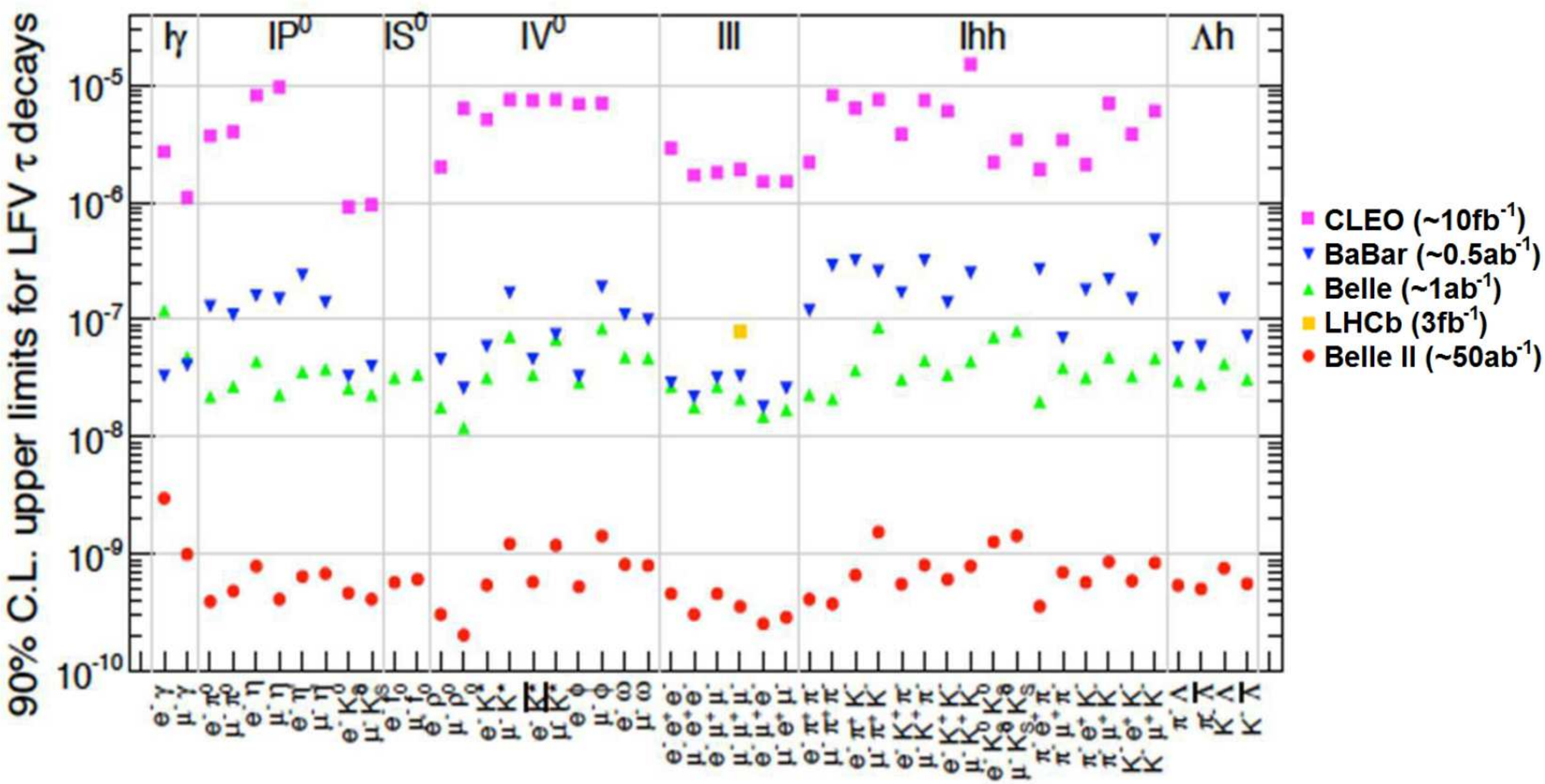
S. Eidelman@Tau2016



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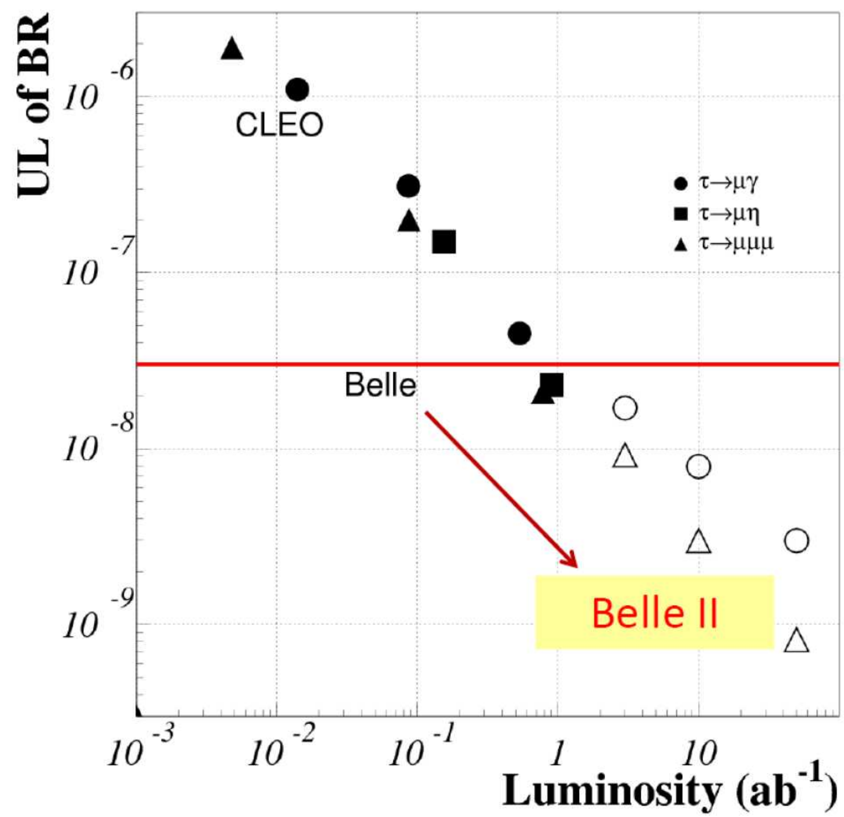
S. Eidelman@Tau2016

- With $5 \times 10^{10}\ \tau^+\tau^-$ and $\epsilon \sim 3\%$: $\mathcal{B} < 10^{-9}$ for $N_{\rm ev} = 0$
- Background suppression needed (PID, higher ϵ, better $\Delta E_\gamma / E_\gamma$)
- $\tau \to l\gamma, \mu\eta(\gamma\gamma), l\rho$: BG $\neq 0$, $\mathcal{B} \propto 1/\sqrt{N}$
- $\tau \to lll, \mu\eta(\pi^+\pi^-\pi^0), \Lambda\pi$: BG $= 0$, $\mathcal{B} \propto 1/N$



2.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{q} \Gamma q$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau\, G_{\mu\nu}^a G_a^{\mu\nu}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu$

$\Gamma \equiv 1, \gamma^\mu$

- Each UV model generates a *specific pattern* of them

2.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

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	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part: form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

2.5 Ex: Non standard LFV Higgs coupling

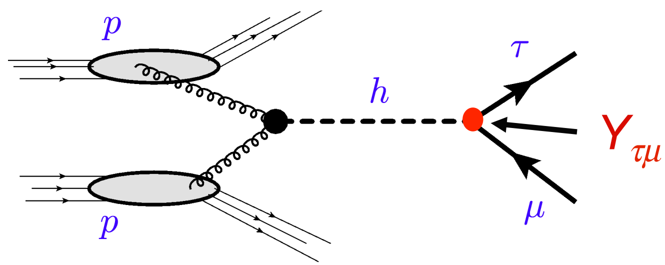
- $$L_Y = -m_i \bar{f}_L^i f_R^i - h\left(Y_{e\mu}\bar{e}_L\mu_R + Y_{e\tau}\bar{e}_L\tau_R + Y_{\mu\tau}\bar{\mu}_L\tau_R\right) + \dots$$

SM BSM

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

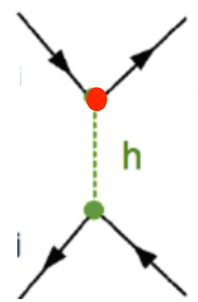
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

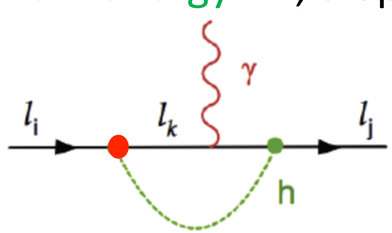
- High energy : LHC

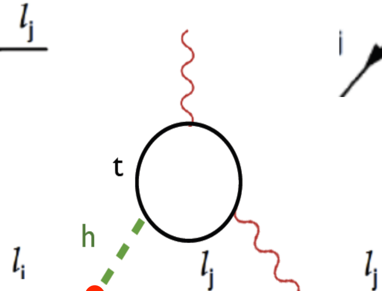


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







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SM BSM

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\text{v}}\delta_{ij}$

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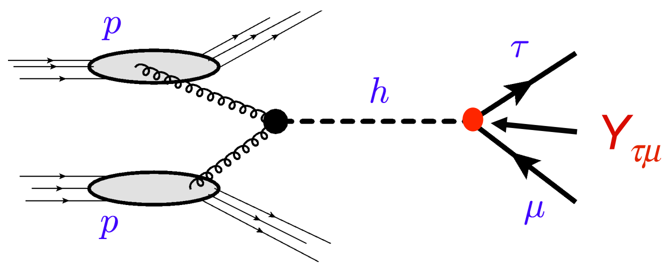
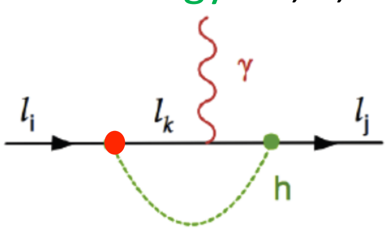
- High energy : LHC

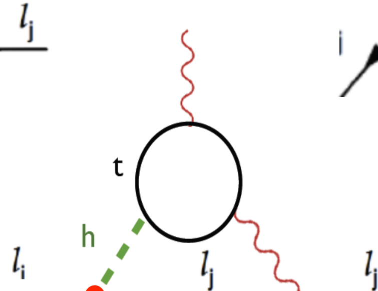


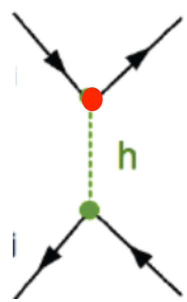
Hadronic part treated with perturbative QCD

Reverse the process

- Low energy : D, S, G operators







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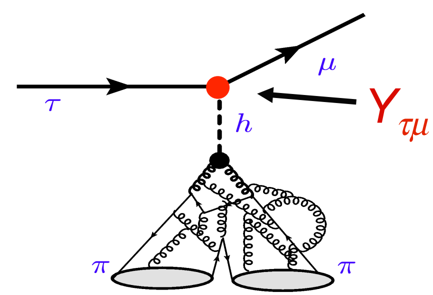
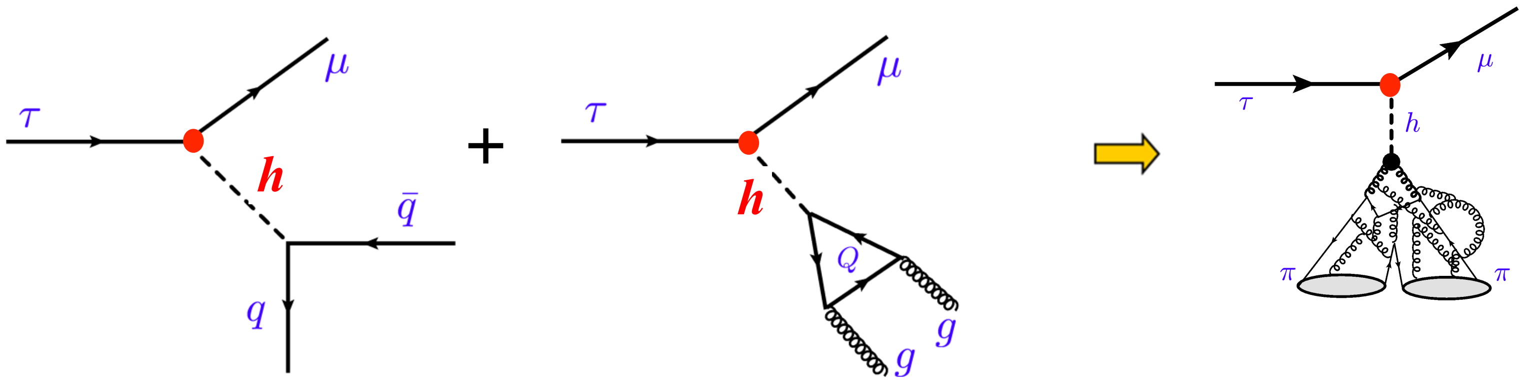


Hadronic part treated with non-perturbative QCD

2.6 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



- Problem : Have the hadronic part under control, ChPT not valid at these energies!

  ⇨ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

  Daub, Dreiner, Hanart, Kubis, Meissner'13

  Celis, Cirigliano, E.P.'14

- Dispersion relations: based on unitarity, analyticity and crossing symmetry

  ⇨ Take *all rescattering* effects into account

  $\pi\pi$ final state interactions important

3. Description of the hadronic form factors

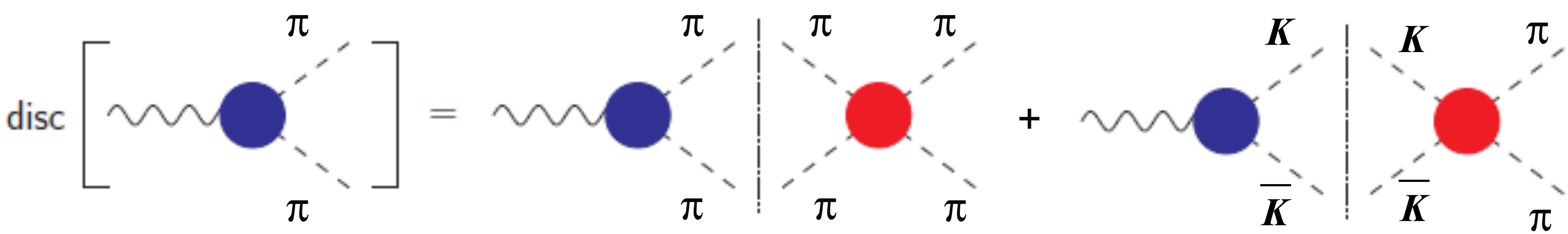
3.1 Unitarity

- *Coupled channel analysis* up to √s ~1.4 GeV: *Mushkhelishvili-Omnès* approach

  Inputs: I=0, S-wave ππ and KK data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub, Dreiner, Hanart, Kubis, Meissner'13
Celis, Cirigliano, E.P.'14

- Unitarity ⇨ the discontinuity of the form factor is known



⇨
$$\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

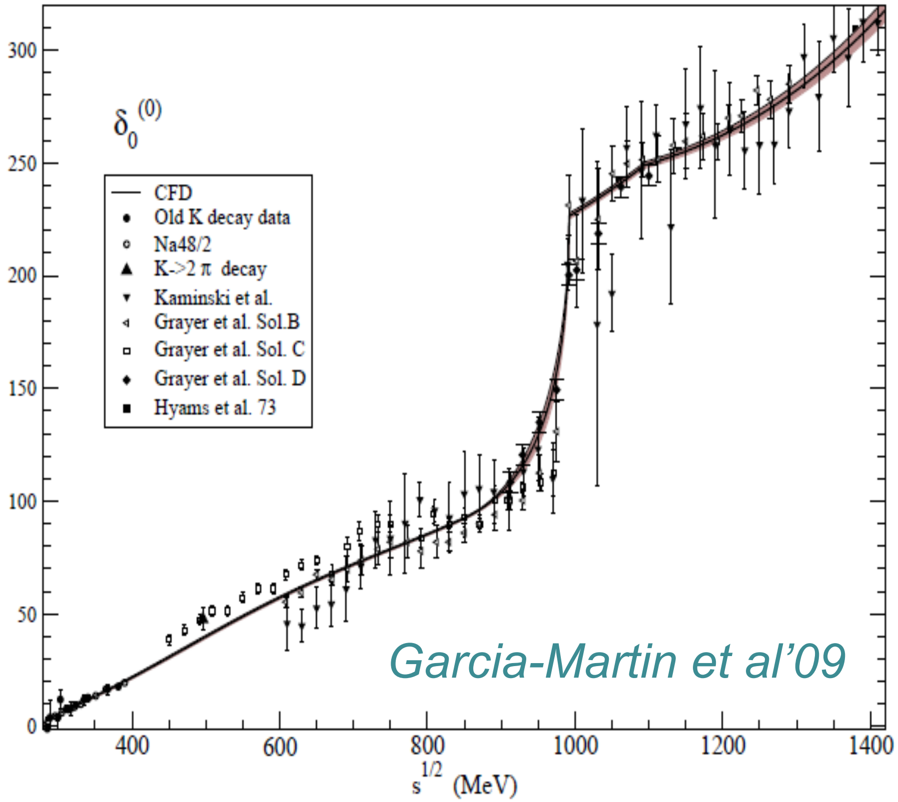
$n = \pi\pi, K\bar{K}$

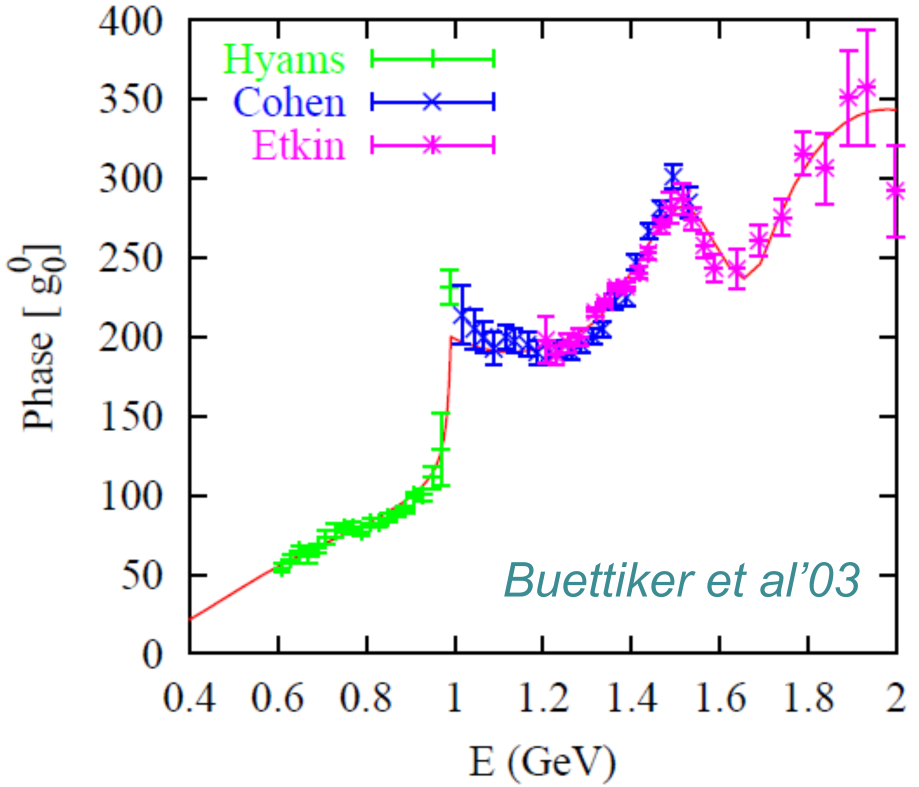
Scattering matrix:

$$\begin{pmatrix} \pi\pi \to \pi\pi, & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi, & K\bar{K} \to K\bar{K} \end{pmatrix}$$

3.2 Inputs for the coupled channel analysis

- Inputs : $\pi\pi \to \pi\pi, K\bar{K}$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buettiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇒ *reconstruct T matrix*

3.3 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

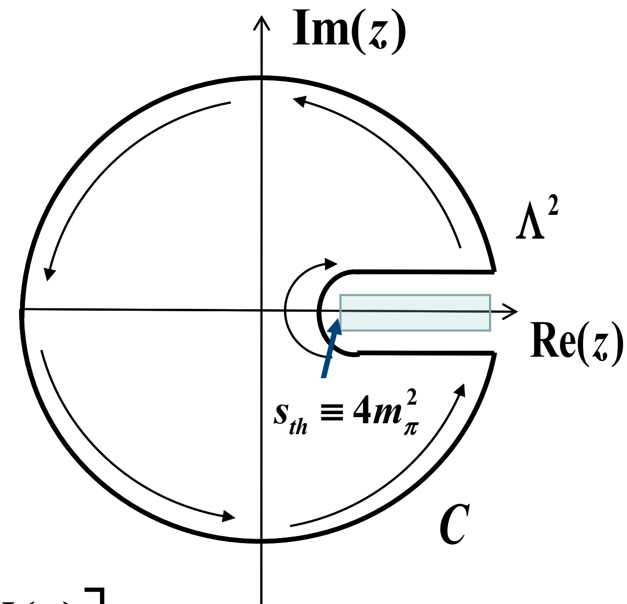
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

Canonical solution $X(s) = C(s), D(s)$:

- Knowing the discontinuity of X(s) ⇒ write a dispersion relation for it
- Analyticity of the FFs: X(z) is
  - real for $z < s_{th}$
  - has a branch cut for $z > s_{th}$
  - analytic for complex z



- Cauchy Theorem and Schwarz reflection principle:

$$X(s) = \frac{1}{\pi} \oint_C dz \frac{X(z)}{z-s}$$

$$= \frac{1}{2i\pi} \int_{|z|=\Lambda^2} dz \frac{X(z)}{z-s} + \frac{1}{2i\pi} \int_{s_{th}=4M_\pi^2}^{\Lambda^2} dz \frac{disc\left[X(z)\right]}{z-s-i\varepsilon}$$

$\Lambda^2 \to \infty$ ⇒

$$X(s) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} dz \frac{\mathrm{Im}\left[X(z)\right]}{z-s-i\varepsilon}$$

X(s) can be reconstructed everywhere from the knowledge of ImX(s)

3.3 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

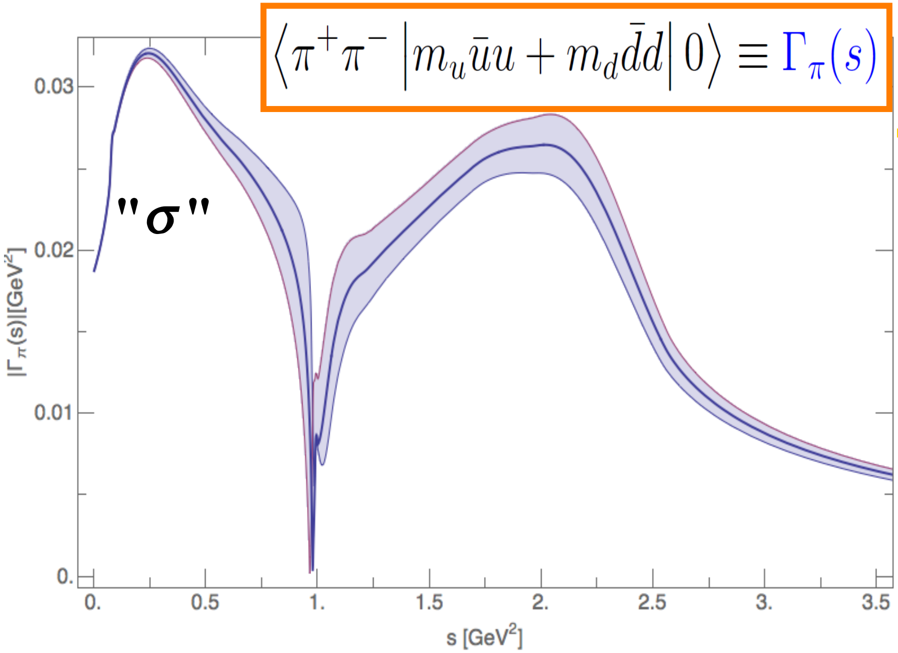
Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

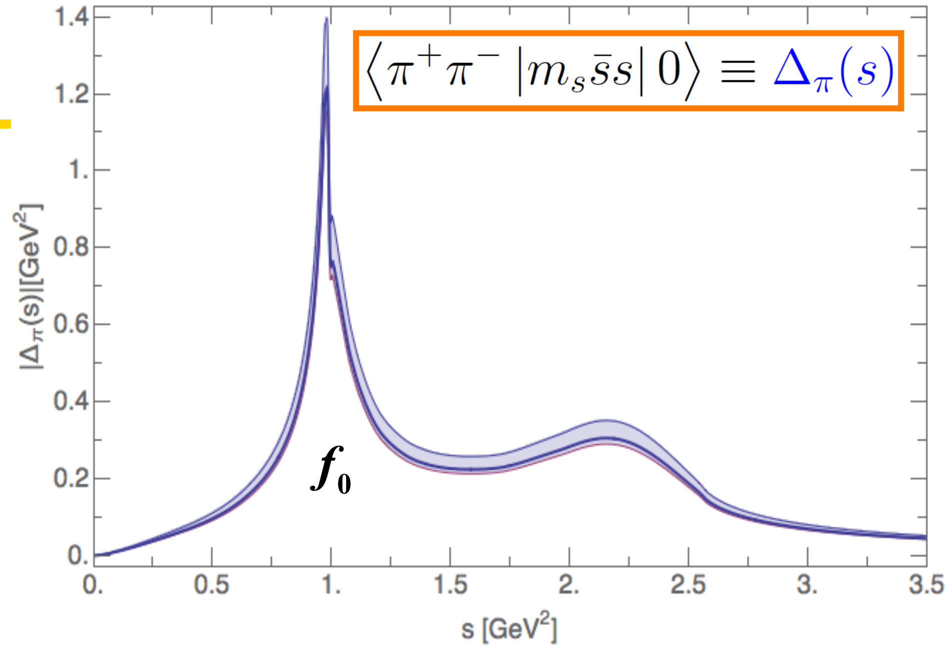
Polynomial determined from a matching to ChPT + lattice

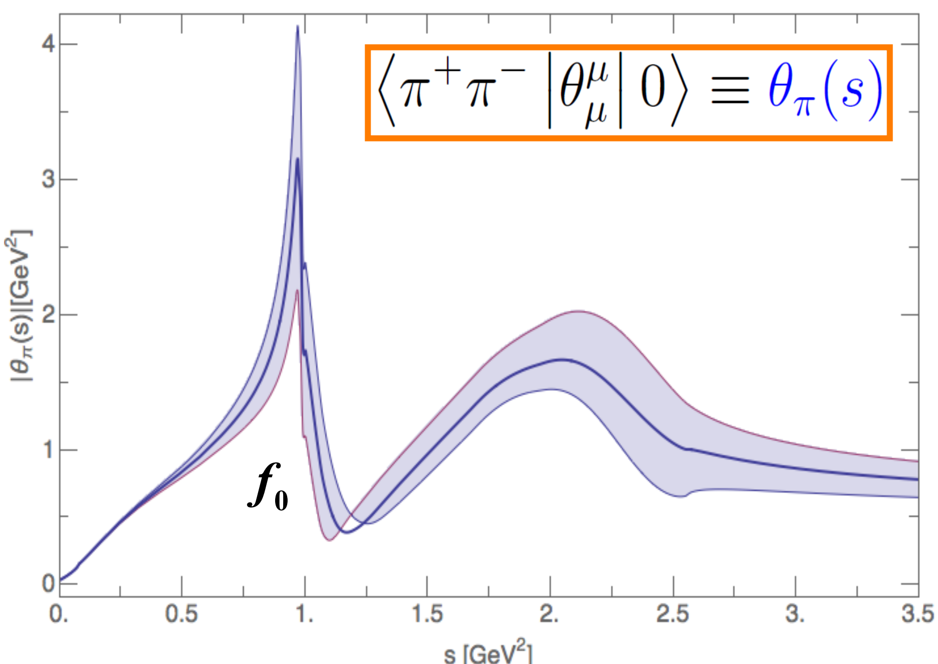
- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} T^*_{mn} \sigma_m(s) X_m^{(N)}(s) \longrightarrow X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im} X_n^{(N+1)}(s')}{s' - s}$$







- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 -1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs

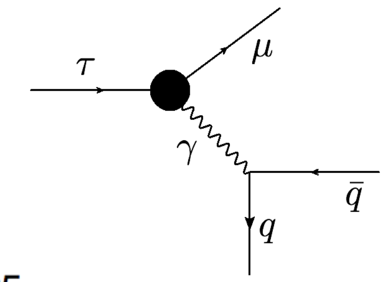
4. Results

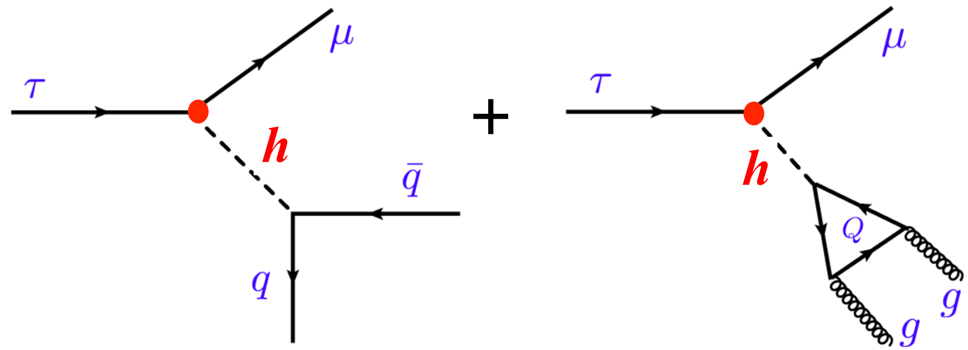
4.1 Spectrum

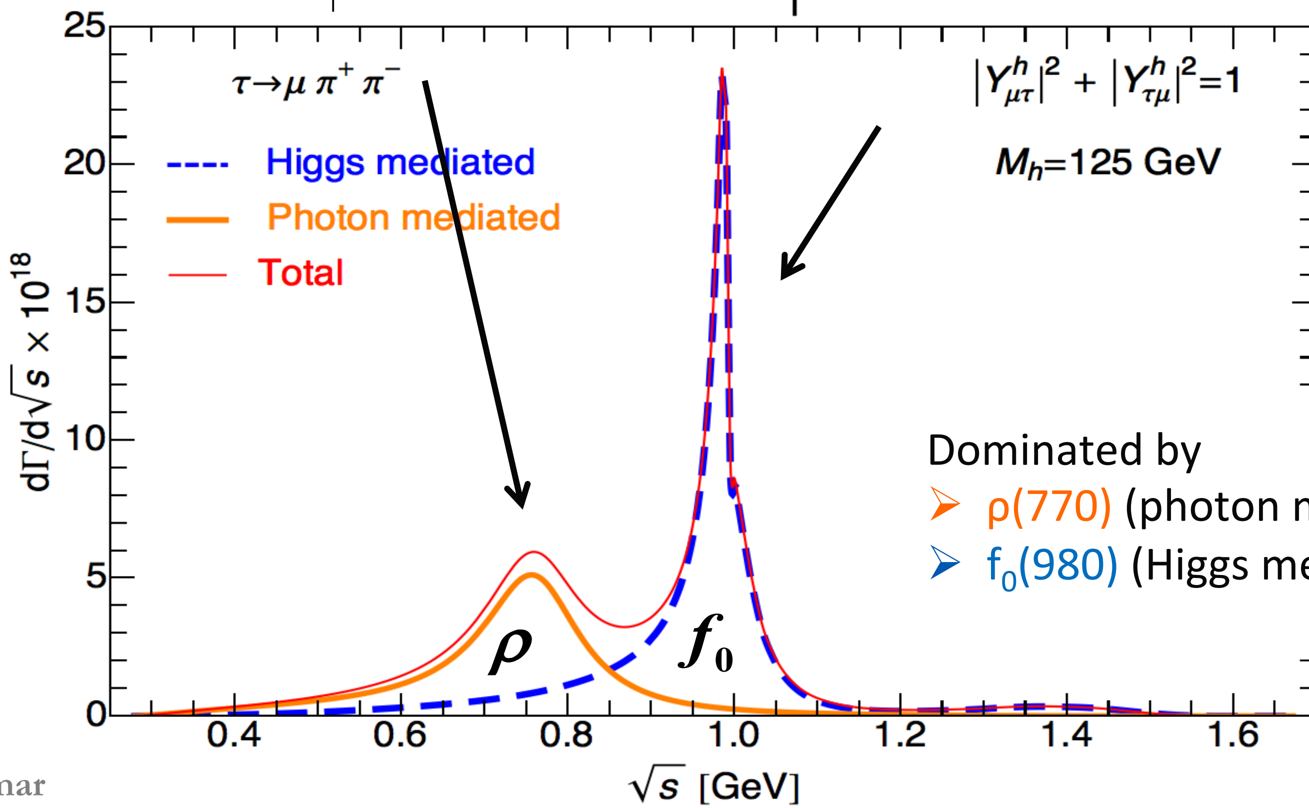
- At low energy

Cirigliano, Celis, E.P.'14

  - $\tau \to \mu\pi\pi$:



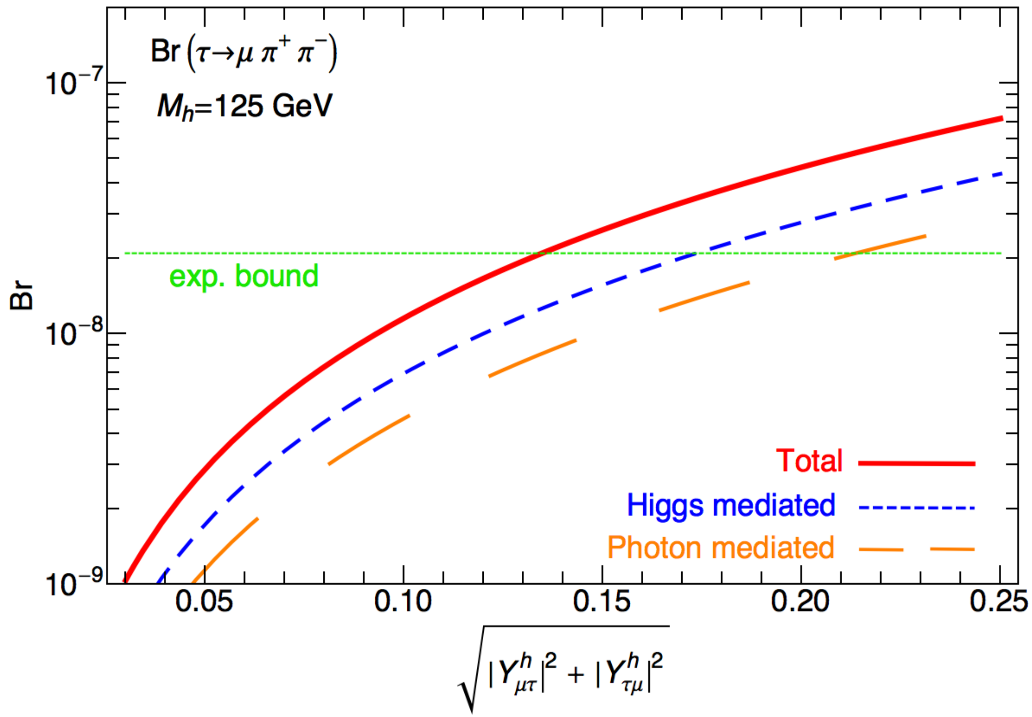




Dominated by
- ρ(770) (photon mediated)
- f_0(980) (Higgs mediated)

4.2 Bounds

Celis, Cirigliano, E.P.'14



Bound:

$$\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \le 0.13$$

Process	$(\mathrm{BR} \times 10^8)$ 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

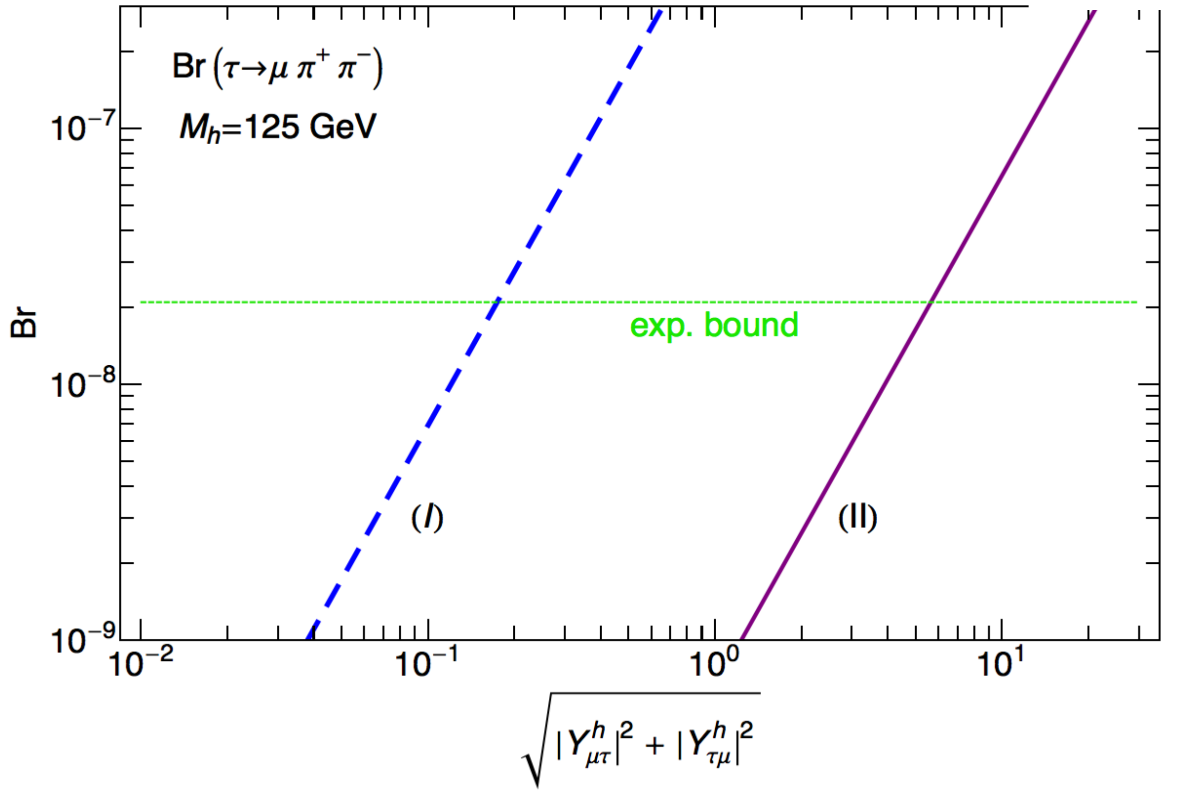
Less stringent but more robust handle on LFV Higgs couplings

?

BaBar'10, Belle'10'11'13 except last from *CLEO'97*

4.3 Impact of our results

Celis, Cirigliano, E.P.'14

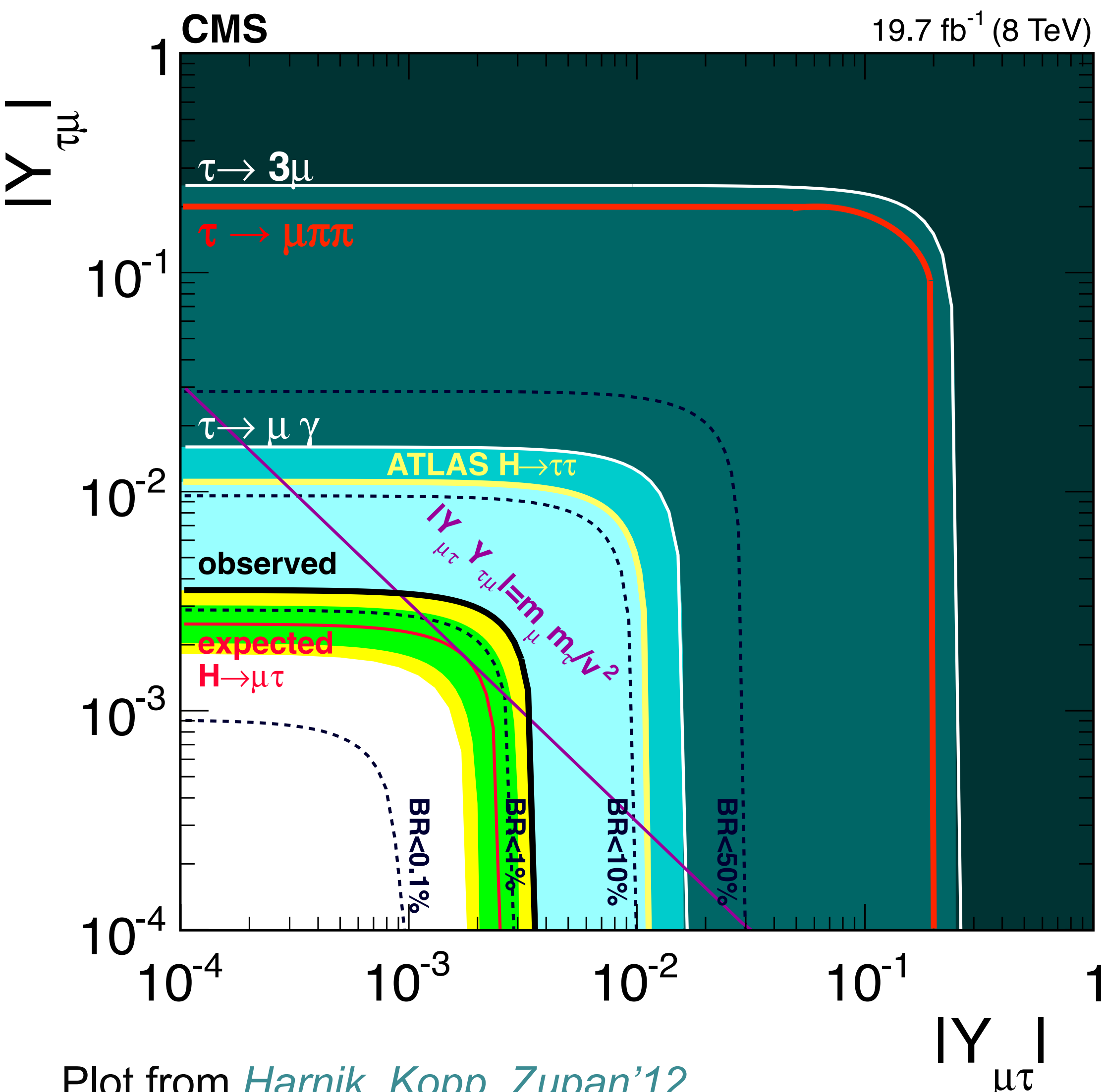


- Dispersive treatment of hadronic part ⇨ bound reduced by one order of magnitude!

- ChPT, EFT only valid at low energy for $\mathbf{p \ll \Lambda = 4\pi f_\pi \sim 1\ GeV}$

  ⇨ *not valid up to* $E = (m_\tau - m_\mu)$!

4.4 Constraints in the τμ sector



Plot from *Harnik, Kopp, Zupan'12* updated *by CMS'15*

- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    ⇨ sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    ⇨ robust handle on LFV
- Constraints from HE: *LHC* wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

⇨

$$BR(\tau \to \mu\gamma) < 2.2 \times 10^{-9}$$

$$BR(\tau \to \mu\pi\pi) < 1.5 \times 10^{-11}$$

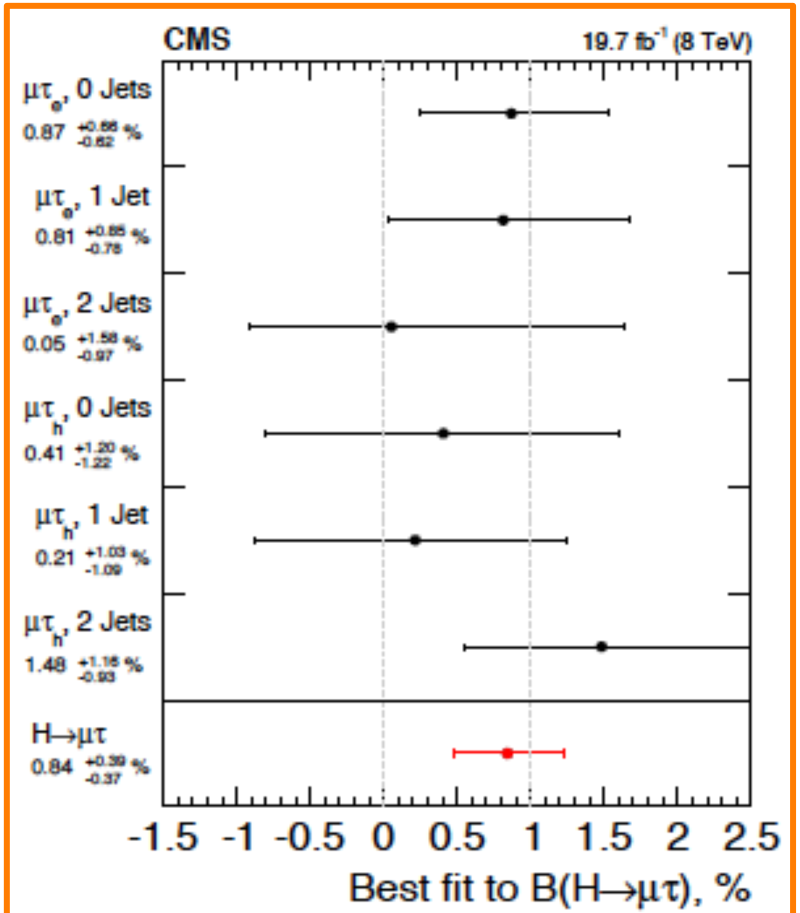
4.5 Hint of New Physics in h → τμ?

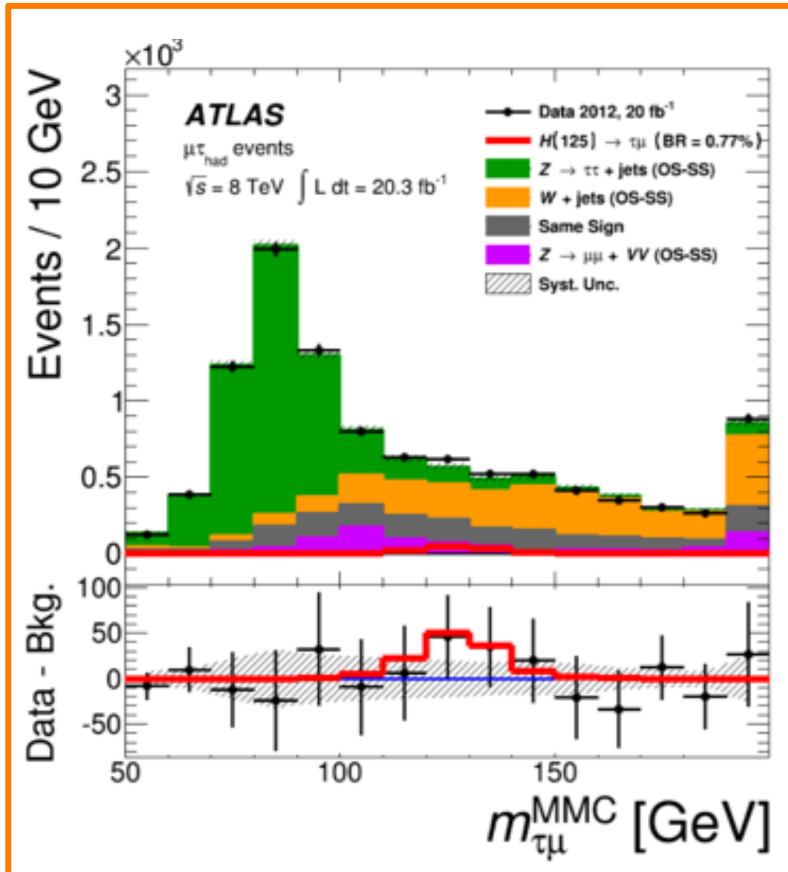
$BR(h \to \tau\mu) = \left(0.84^{+0.39}_{-0.37}\right)\%$ @2.4σ

CMS'15

$BR(h \to \tau\mu) = \left(0.53 \pm 0.51\right)\%$ @1σ

ATLAS'15



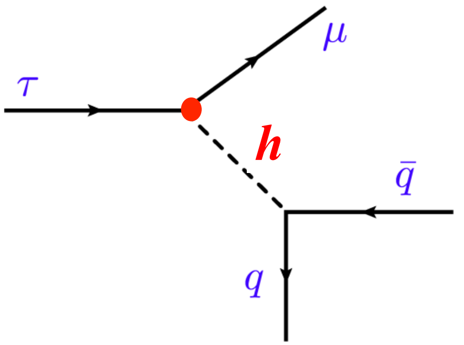


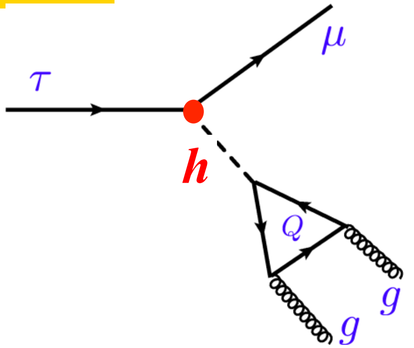
⇨ $BR(h \to \tau\mu) = \left(-0.76^{+0.81}_{-0.84}\right)\%$ 13 TeV@CMS *M. Cepeda@Higgs Tasting'16*

4.6 What if $\tau \to \mu(e)\pi\pi$ is observed?

Talk by J. Zupan
@ KEK-FF2014FALL

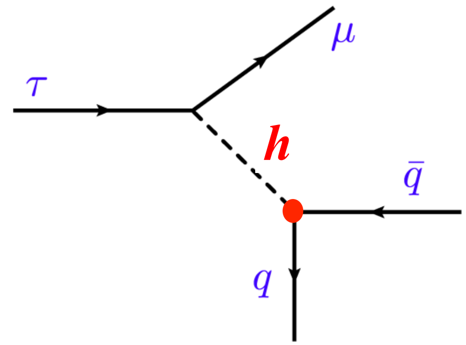
- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$

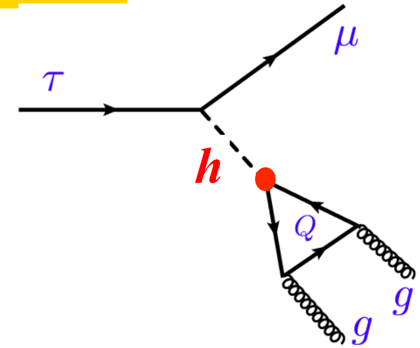




4.6 What if $\tau \to \mu(e)\pi\pi$ is observed?

Talk by J. Zupan @ KEK-FF2014FALL





- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!
- $Y_{u,d,s}$ poorly bounded
- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8}$$
$$Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7}$$

below present experimental limits!

- If discovered ⇒ *upper limit* on $Y_{u,d,s}$*!*
  ⇒ Interplay between high-energy and low-energy constraints!

5. Conclusion and outlook

Conclusion and outlook

- Tau physics is a very rich field: test QCD and EW, new physics, etc..
- In this talk, focus on CLFV:
  - Extremely small SM rates
  - Experimental results at low energy are very precise
    ⇨ very high scale sensitivity
- CLFV decays excellent model discriminating tools especially τ decays
  ⇨ *Hadronic decays* such as $\tau \to \mu(e)\pi\pi$ important!
- To consider hadronic decays, need to control the hadronic uncertainties: need to know hadronic matrix elements, form factors etc.
- For $\tau \to \mu(e)\pi\pi$: need to know the $\pi\pi$ form factors

  ⇨ Use dispersion relations
- Dispersion relations rely on analyticity, unitarity and crossing symmetry
  ⇨ Rigorous treatment of two and three hadronic final state

Conclusion and outlook

- $\tau \to \mu(e)\pi\pi$ gives interesting constraints on LFV new physics operators involving quarks

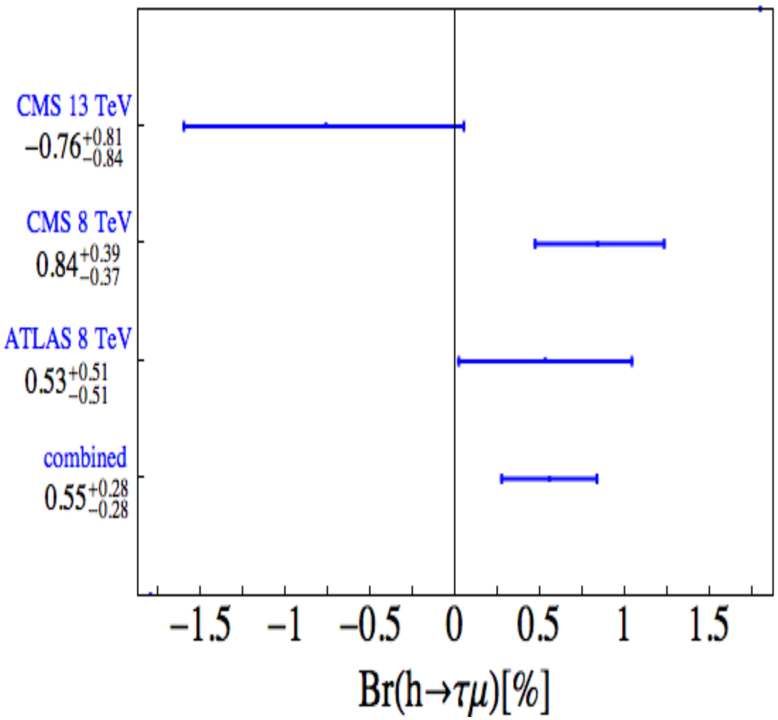
- Interplay low energy and collider physics: LFV of the Higgs boson

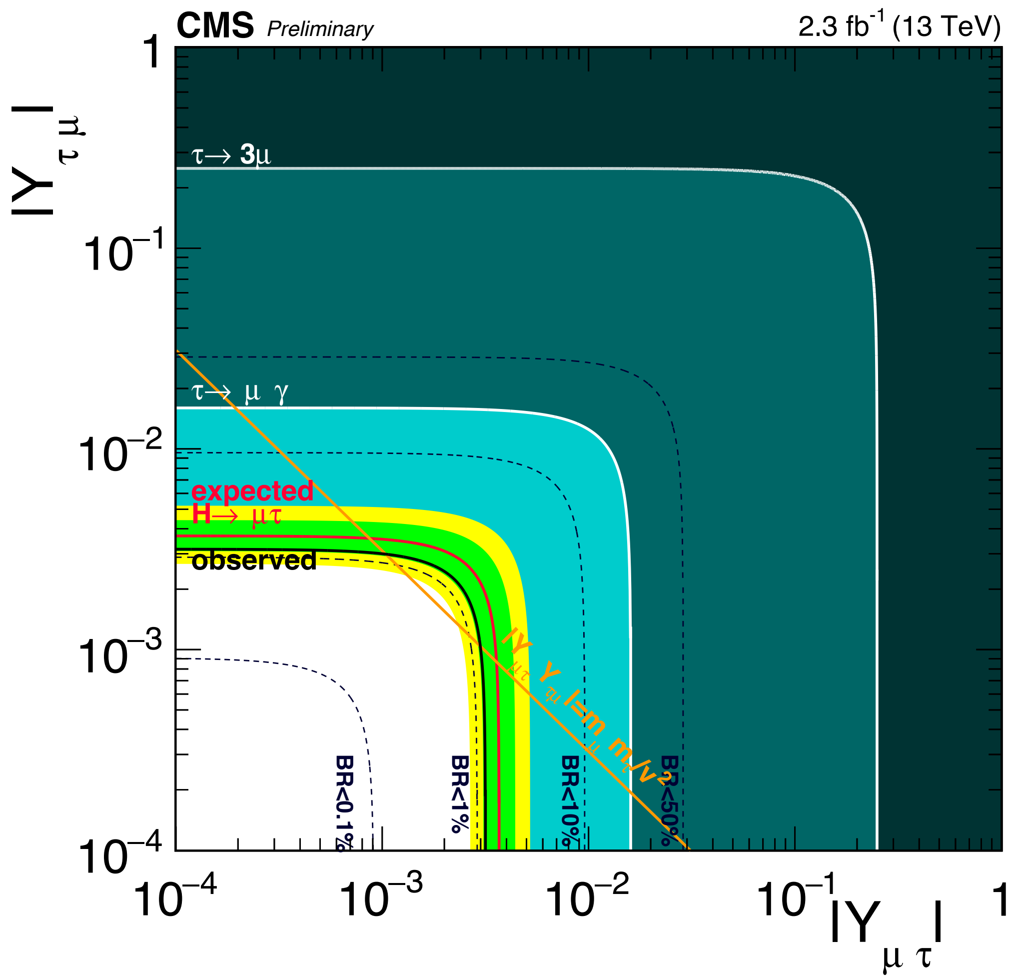
- Complementarity with LFC sector: EDMs, g-2 and colliders:
  ⇒ New physics models usually strongly correlate these sectors

6. Back-up

4.7 Hint of New Physics in h → τμ?

CMS'16



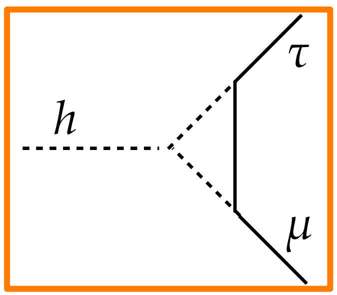


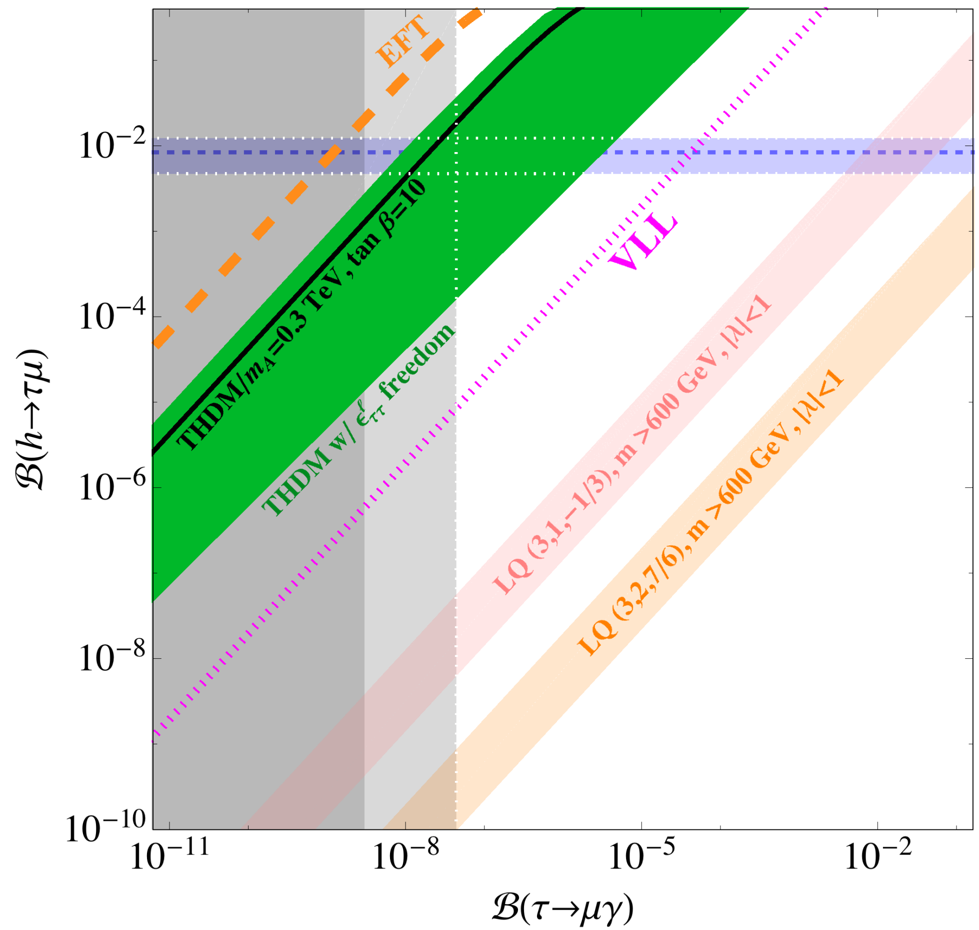
See talks on *Friday*

4.8 Interplay between LHC & Low Energy

Dorsner et al.'15

- If real what type of NP?
- If h → τ μ due to loop corrections:
  - extra charged particles necessary
  - τ → μγ too large





- h → τ μ possible to explain if extra scalar doublet:
  ➡ *2HDM of type III*
- Need other sources of EWSB: *2HDMs, technicolour models* Altmannshofer et al.'15
- Constraints from *τ → μγ* important!

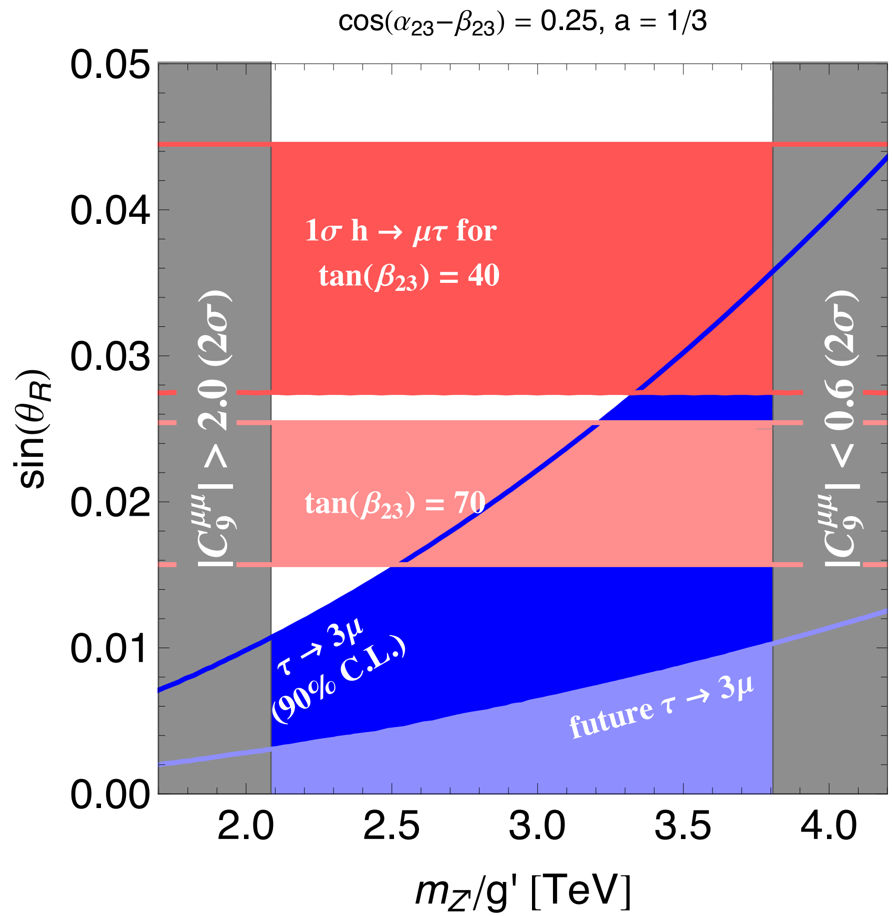
4.8 Interplay between LHC & Low Energy

- *2HDMs* with gauged $L_\mu - L_\tau$
  ⇨ Z', explain anomalies for
  - $h \to \tau\mu$
  - $B \to K^*\mu\mu$
  - $R_K = B \to K\mu\mu / B \to Kee$

- Constraints from $\tau \to 3\mu$
  crucial ⇨ *Belle II, LHC*

- See also, e.g.:
  Aristizabal-Sierra & Vicente'14,
  Lima et al'15, Aloni, Nir, Stamou'15,
  Omhura, Senaha, Tobe '15
  Altmannshofer et al.'15
  Bauer and Neubert'16, Buschmann et al.'16, etc…

Altmannshofer & Straub'14, Crivellin et al'15
Crivellin, D'Ambrosio, Heeck.'15



Determination of the polynomial

- Fix the polynomial with requiring $F_P(s) \to 1/s$ + ChPT:

Brodsky & Lepage'80

- Feynman-Hellmann theorem:

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$

Determination of the polynomial

- Fix the polynomial with requiring $F_P(s) \to 1/s$ + ChPT:

  Brodsky & Lepage'80

- Feynman-Hellmann theorem:

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

$\Rightarrow$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

- For the scalar FFs:

$$\begin{aligned}
P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\
Q_\Gamma(s) &= \frac{2}{\sqrt{3}}\Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\
P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\
Q_\Delta(s) &= \frac{2}{\sqrt{3}}\Delta_K(0) = \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{2}M_\pi^2\right) + \cdots
\end{aligned}$$

- Problem: large corrections in the case of the kaons!

  ⇨ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\ M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05}\,(M_K^2 - 1/2 M_\pi^2)$$

Daub, Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

  ⇨ Relax the constraints and match to ChPT

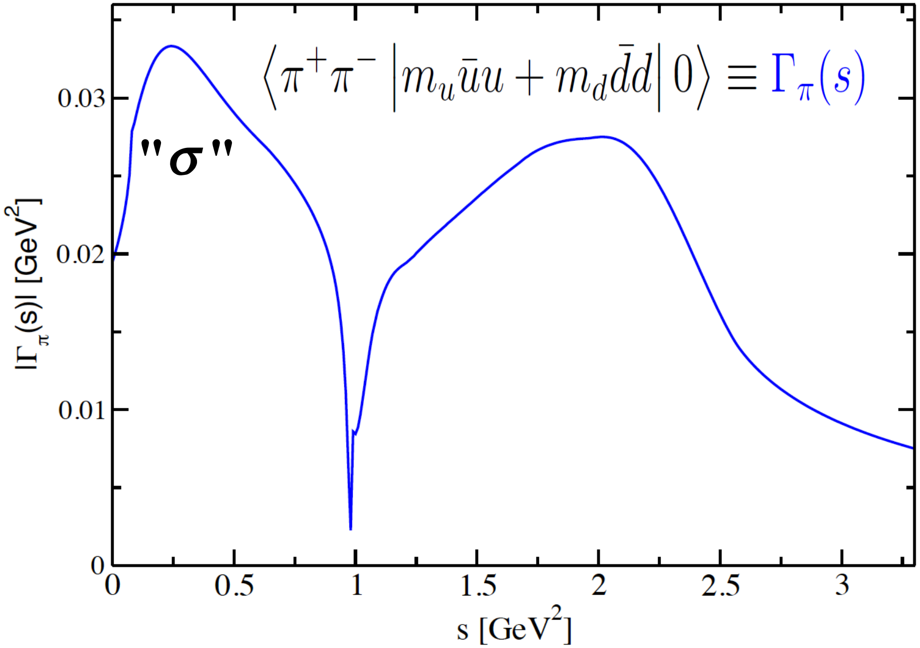
$$P_\theta(s) = 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s$$

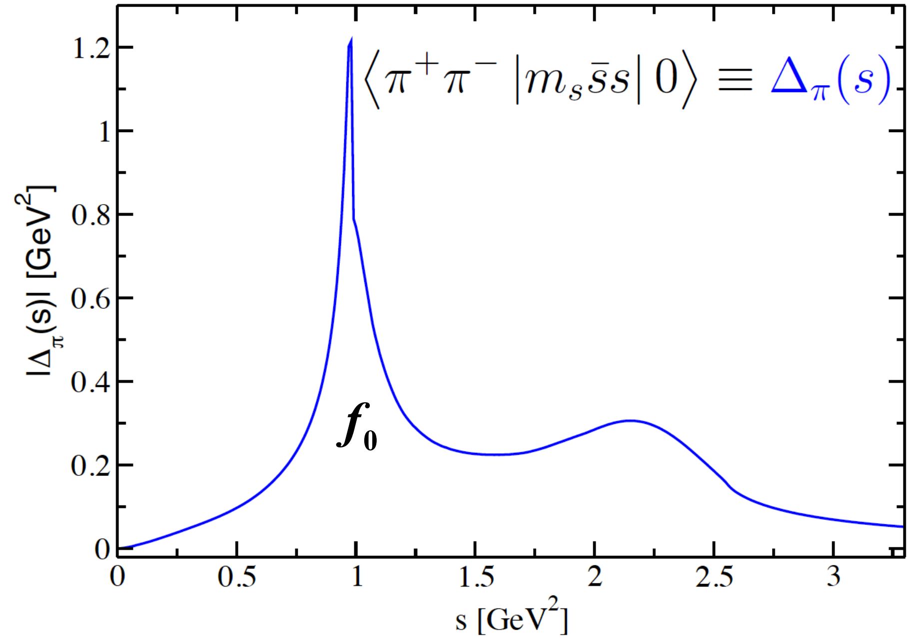
$$Q_\theta(s) = \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s$$

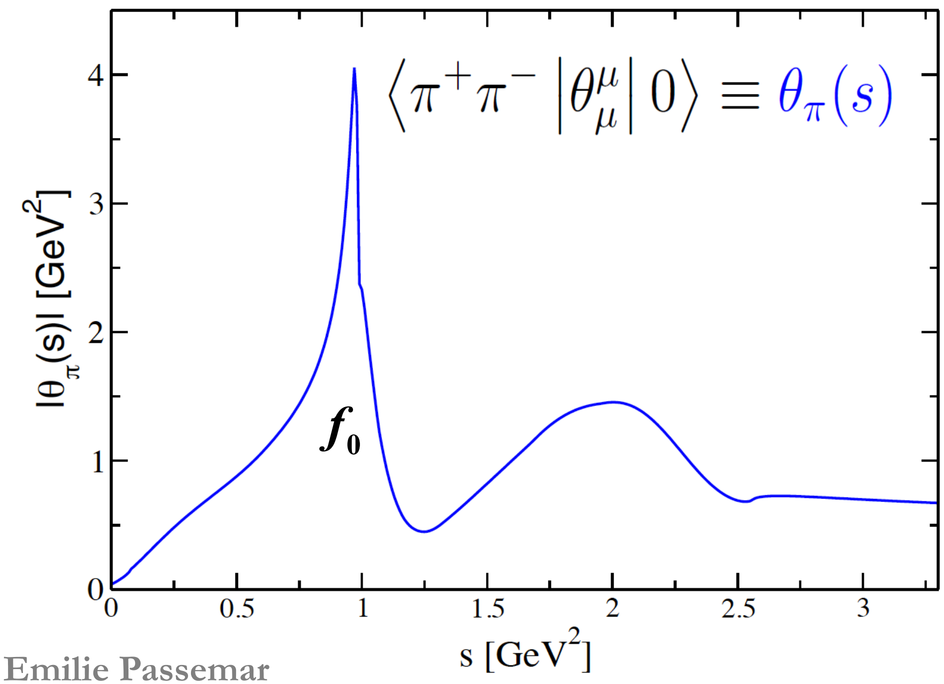
with $\dot{f} = \left(\frac{df}{ds}\right)_{s=0}$

- At LO ChPT: $\dot{\theta}_{\pi,K} = 1$

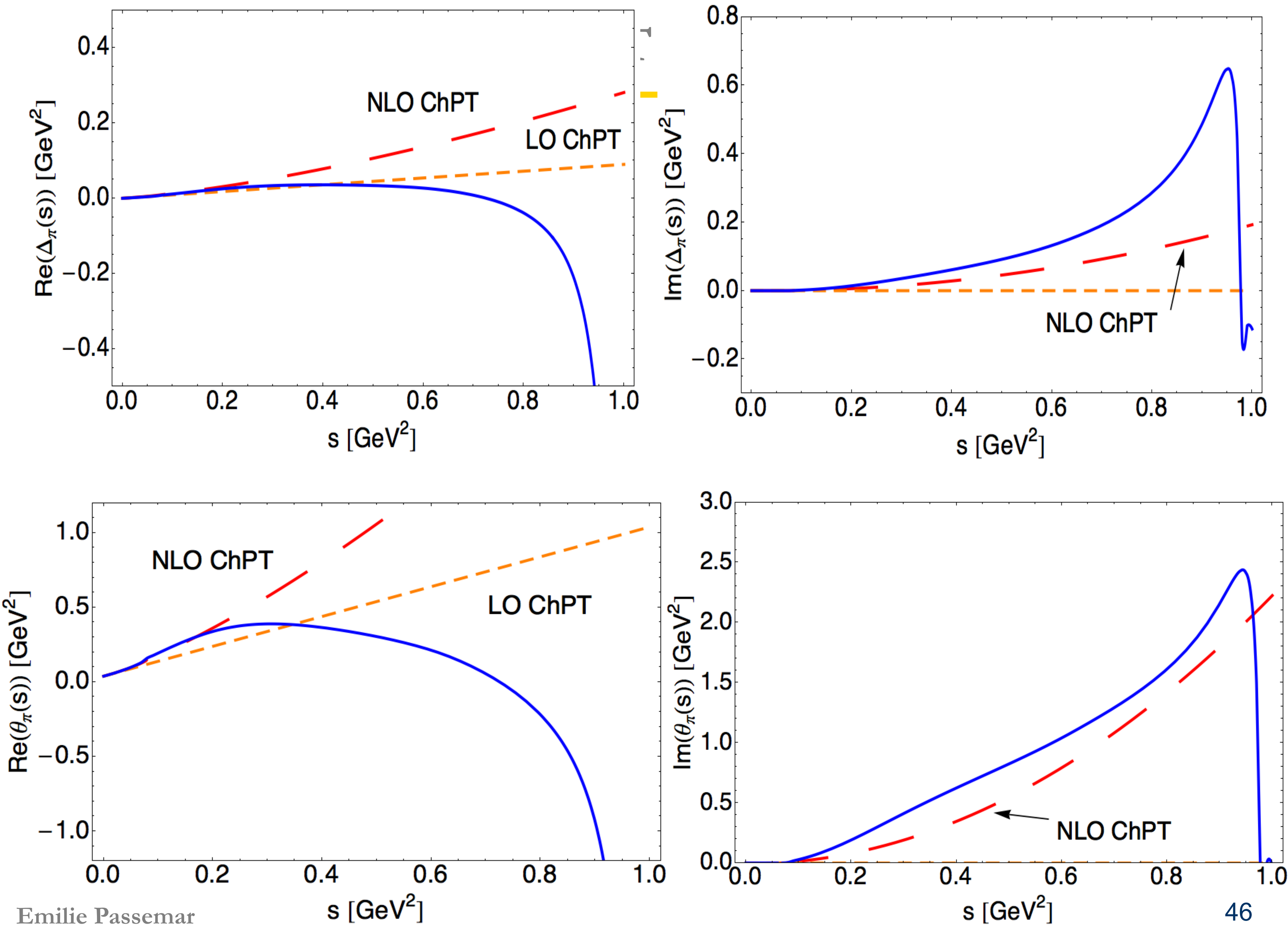
- Higher orders ⇨ $\dot{\theta}_K = 1.15 \pm 0.1$







Dispersion relations:
Model-independent method,
based on first principles
that extrapolates ChPT
based on data



NLO ChPT
LO ChPT
Re(Δ_π(s)) [GeV^2]
s [GeV^2]
NLO ChPT
Im(Δ_π(s)) [GeV^2]
s [GeV^2]
NLO ChPT
LO ChPT
Re(θ_π(s)) [GeV^2]
s [GeV^2]
NLO ChPT
Im(θ_π(s)) [GeV^2]
s [GeV^2]

3.4.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

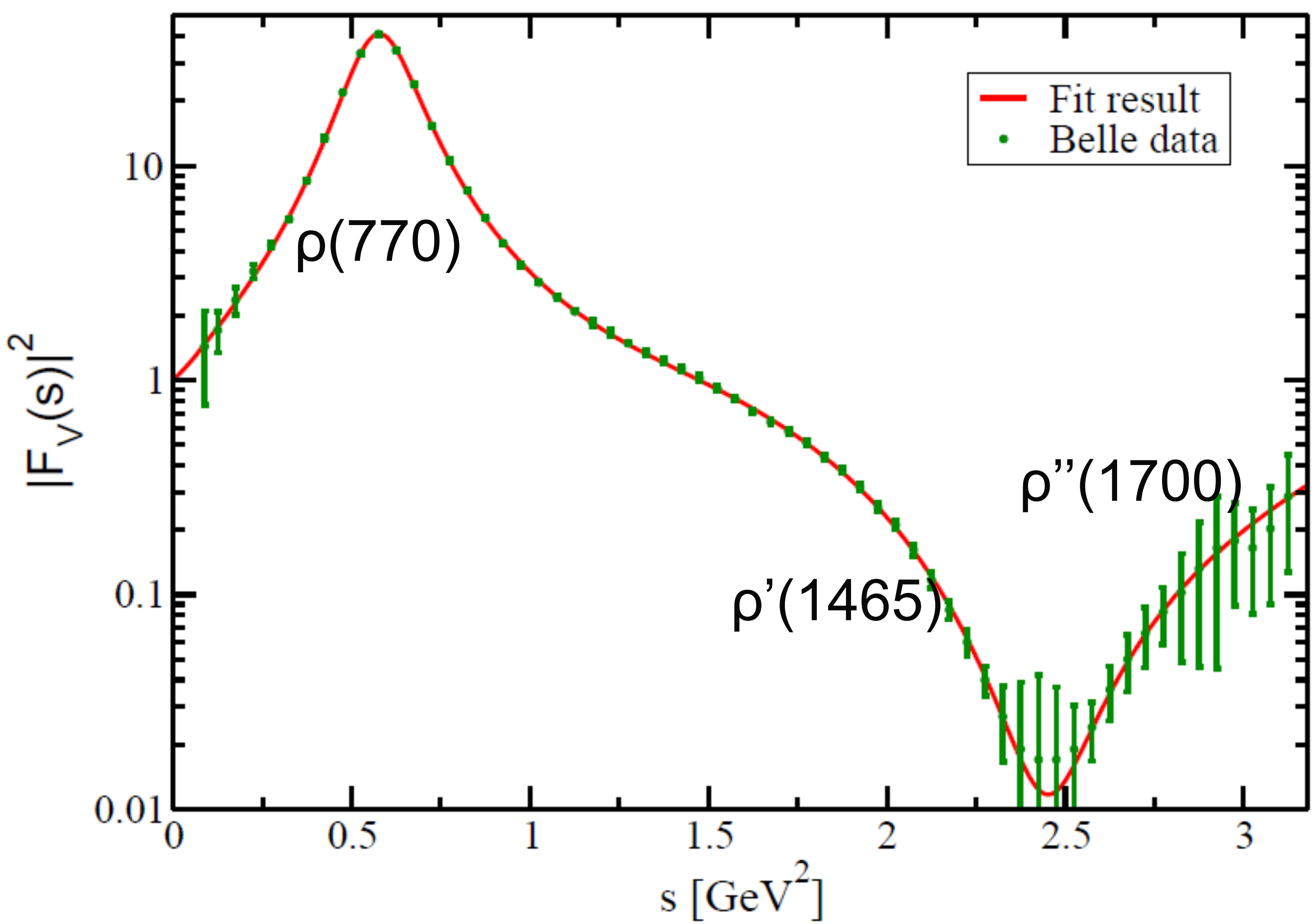
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$F_V(s) = \exp\left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_V'' - \lambda_V'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{\left(s'-s-i\varepsilon\right)}\right]$$

Extracted from a model including 3 resonances ρ(770), ρ'(1465) and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

3.4.3 Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $$R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state:

$$\frac{dBR(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}$$

and

$$dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu\gamma)} \frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}$$

3.6 Model discriminating of BRs

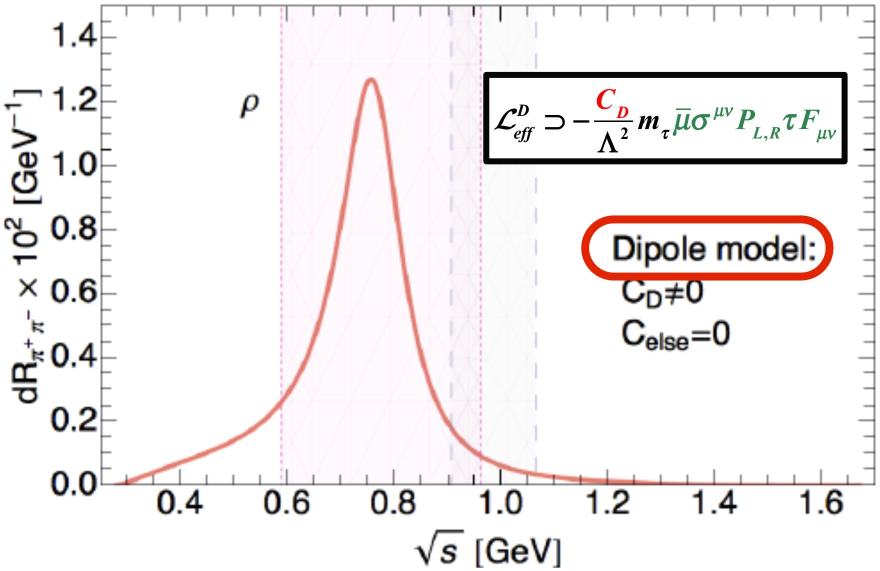
- Studies in specific models *Buras et al.'10*

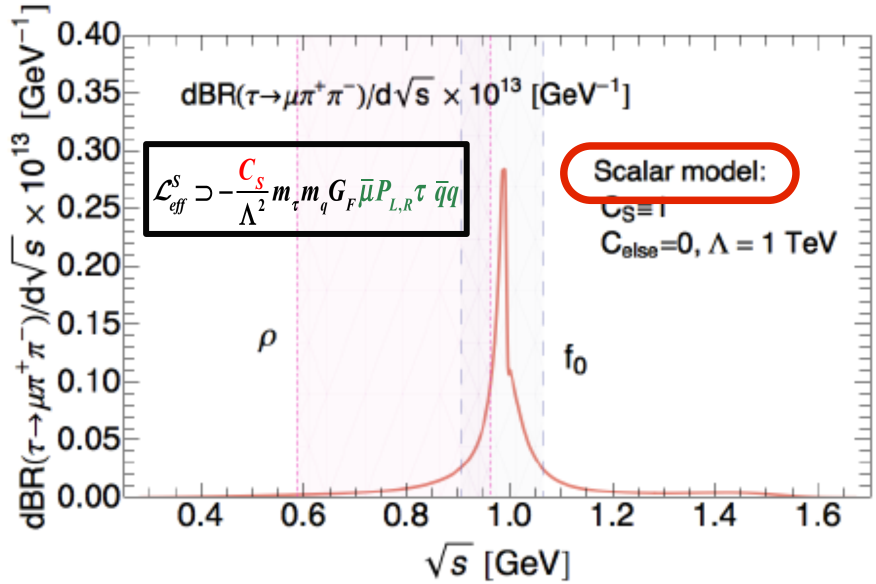
ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^-\to e^-e^+e^-)}{\mathrm{Br}(\mu\to e\gamma)}$	$0.02\ldots1$	$\sim 6\cdot10^{-3}$	$\sim 6\cdot10^{-3}$	$0.06\ldots2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots0.4$	$\sim 1\cdot10^{-2}$	$\sim 1\cdot10^{-2}$	$0.07\ldots2.2$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots0.4$	$\sim 2\cdot10^{-3}$	$0.06\ldots0.1$	$0.06\ldots2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots0.3$	$\sim 2\cdot10^{-3}$	$0.02\ldots0.04$	$0.03\ldots1.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots0.3$	$\sim 1\cdot10^{-2}$	$\sim 1\cdot10^{-2}$	$0.04\ldots1.4$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}$	$0.8\ldots2$	~ 5	$0.3\ldots0.5$	$1.5\ldots2.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}$	$0.7\ldots1.6$	~ 0.2	$5\ldots10$	$1.4\ldots1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{\mathrm{Br}(\mu\to e\gamma)}$	$10^{-3}\ldots10^{2}$	$\sim 5\cdot10^{-3}$	$0.08\ldots0.15$	$10^{-12}\ldots26$

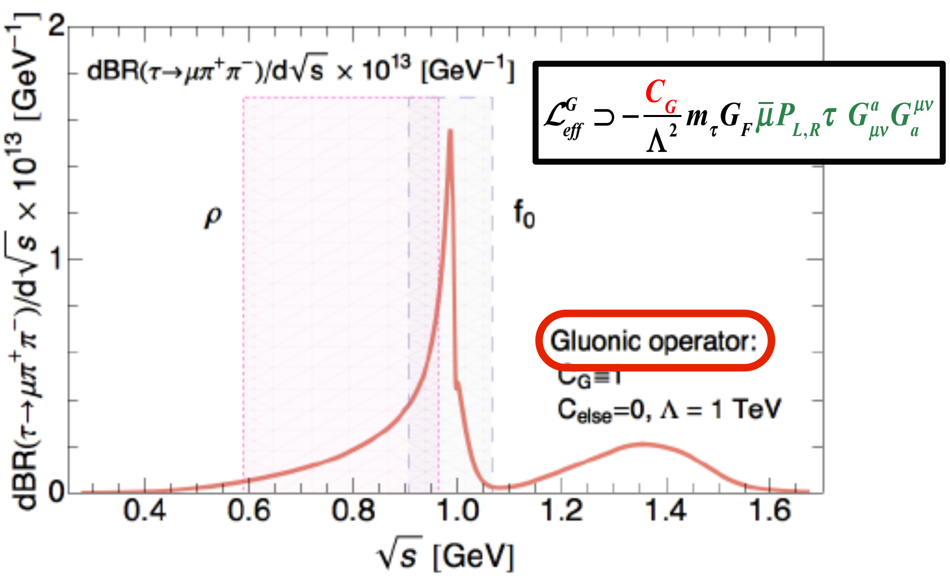
➡ Disentangle the *underlying dynamics* of NP

3.7 Model discriminating of Spectra: τ→μππ

Celis, Cirigliano, E.P.'14







Very different distributions according to the *final hadronic state!*

NB: See also Dalitz plot analyses for τ → μμμ

Dassinger et al.'07

Universality improved $B(\tau \to e\nu\bar{\nu})$ and R_{had} - HFAG 2016 prelim.

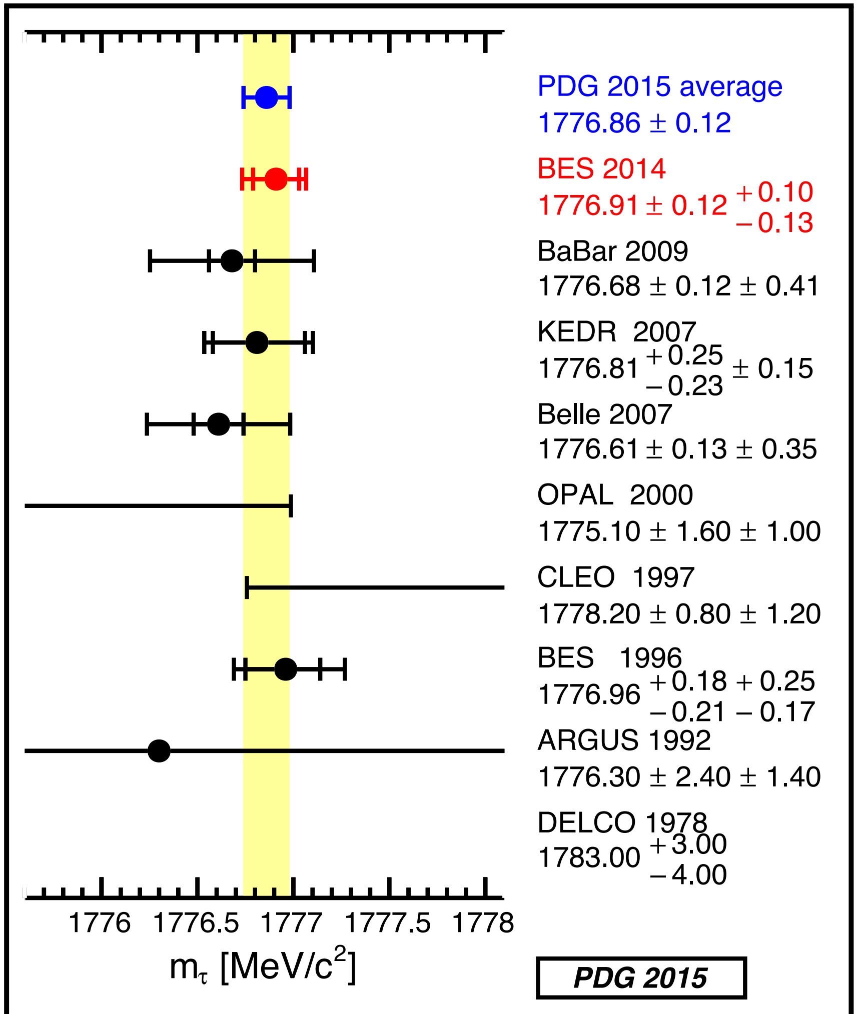
Universality improved $B(\tau \to e\nu\bar{\nu})$

- (M. Davier, 2005): assume SM lepton universality to improve $B_e = B(\tau \to e\bar{\nu}_e\nu_\tau)$ fit B_e using three determinations:
  - $B_e = B_e$
  - $B_e = B_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2)$
  - $B_e = B(\mu \to e\bar{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_e^2/m_\mu^2) \cdot (\delta_\gamma^\tau \delta_W^\tau)/(\delta_\gamma^\mu \delta_W^\mu)$
    [above we have: $B(\mu \to e\bar{\nu}_e\nu_\mu) = 1$]
- $\mathbf{B_e^{\text{univ}} = (17.818 \pm 0.022)\%}$ HFAG-PDG 2016 prelim. fit

$R_{\text{had}} = \Gamma(\tau \to \text{hadrons})/\Gamma_{\text{univ}}(\tau \to e\nu\bar{\nu})$

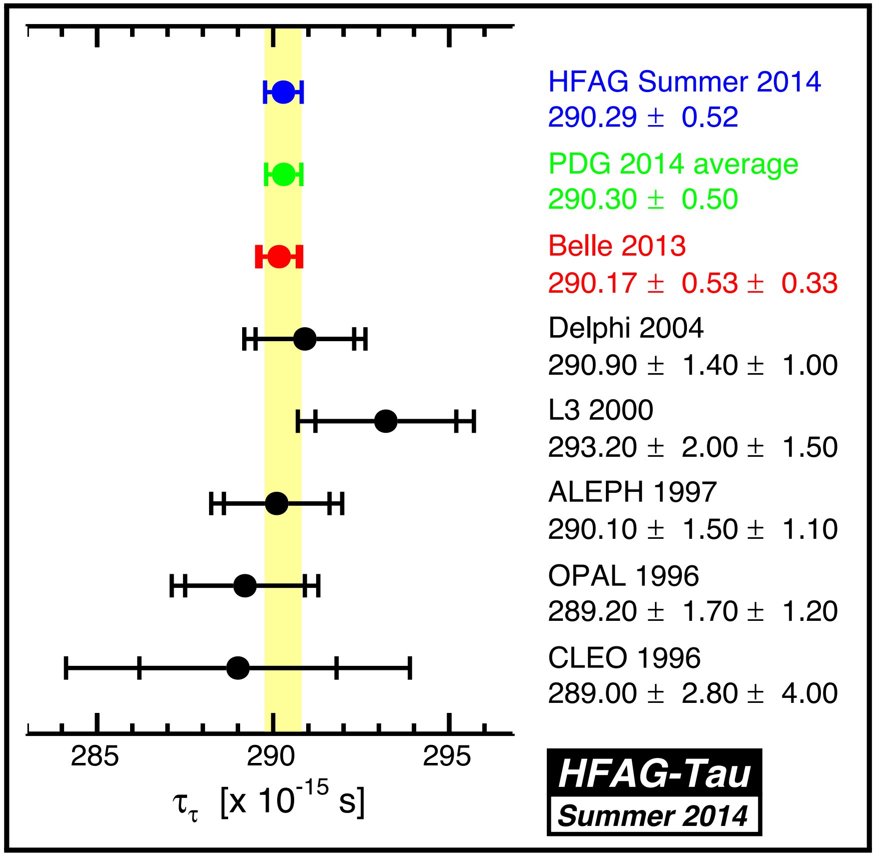
- $$R_{\text{had}} = \frac{\Gamma(\tau \to \text{hadrons})}{\Gamma_{\text{univ}}(\tau \to e\nu\bar{\nu})} = \frac{B_{\text{hadrons}}}{B_e^{\text{univ}}} = \frac{1 - B_e^{\text{univ}} - f(m_\mu^2/m_\tau^2)/f(m_e^2/m_\tau^2) \cdot B_e^{\text{univ}}}{B_e^{\text{univ}}}$$
  - two different determinations, second one not "contaminated" by hadronic BFs
- $\mathbf{R_{\text{had}} = 3.6359 \pm 0.0074}$ HFAG-PDG 2016 prelim. fit
- $\mathbf{R_{\text{had}}(\text{leptonic BFs only}) = 3.6397 \pm 0.0070}$ HFAG-PDG 2016 prelim. fit

Tau mass



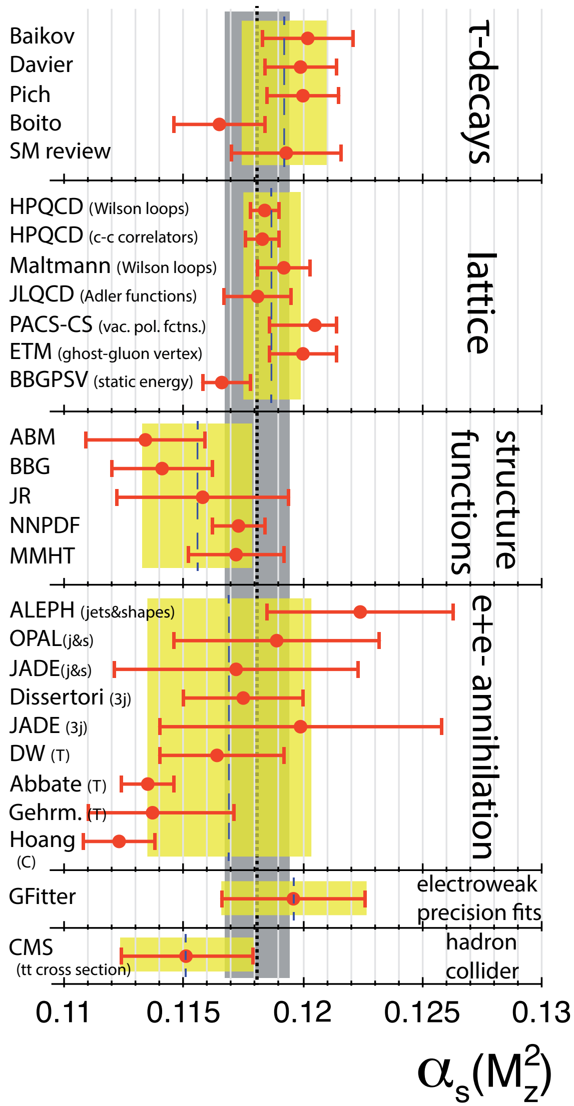
- most precise measurements by e^+e^- **colliders at** $\tau^+\tau^-$ **threshold**
  - few events but very significant

Tau lifetime

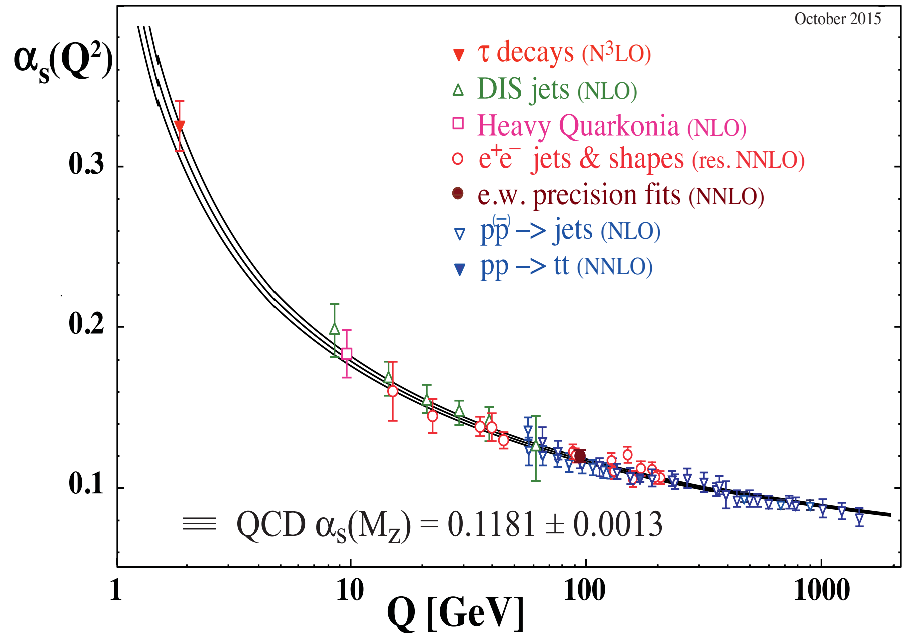


- LEP experiments, many methods
  - impact parameter sum (IPS)
  - momentum dependent impact parameter sum (MIPS
  - 3D impact parameter sum (3DIP)
  - impact parameter difference (IPD)
  - decay length (DL)
- Belle
  - 3-prong vs. 3-prong decay length
  - largest syst. error: alignment

3.4 Extraction of α_s



Bethke, Dissertori, Salam, PDG'15



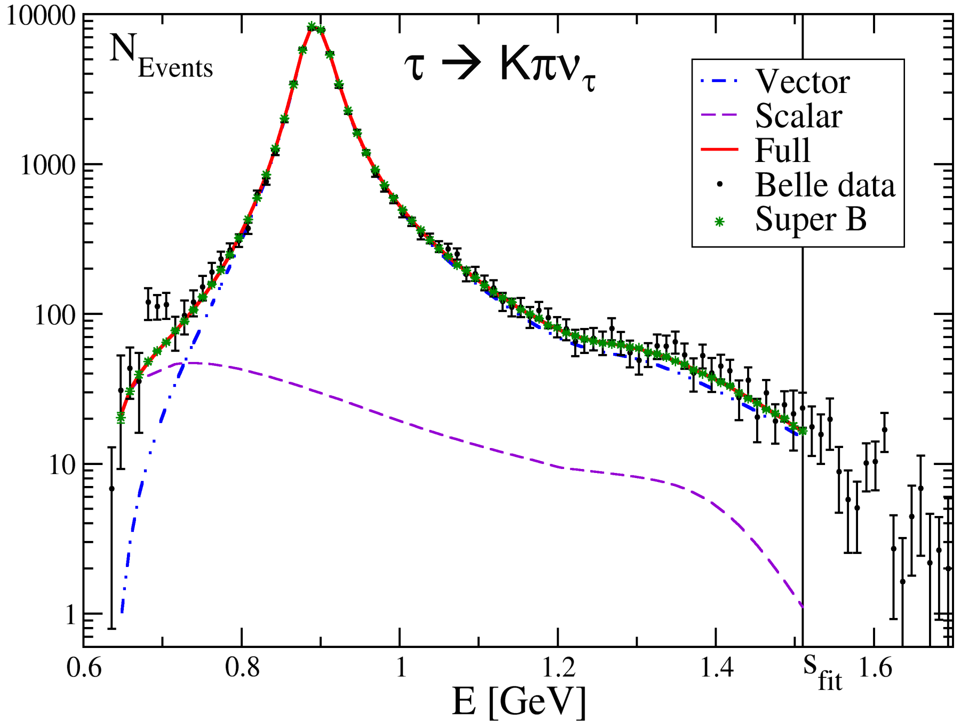
- *Extraction of α_s* from hadronic τ very interesting : Moderate precision at the τ mass ⇒ very good precision at the Z mass

- Beautiful test of the QCD running

3.6 Exclusive Tau decays

- Invariant mass spectra: constraints on FF very important for testing QCD dynamics and the SM and new physics:

$K\pi$ form factors : ⇨ V_{us}



Jamin, Pich, Portolés'08

Boito, Escribano, Jamin'09,'10

Bernard, Boito, E.P'11

Bernard'13,

Escribano, González-Solis, Jamin, Roig'14

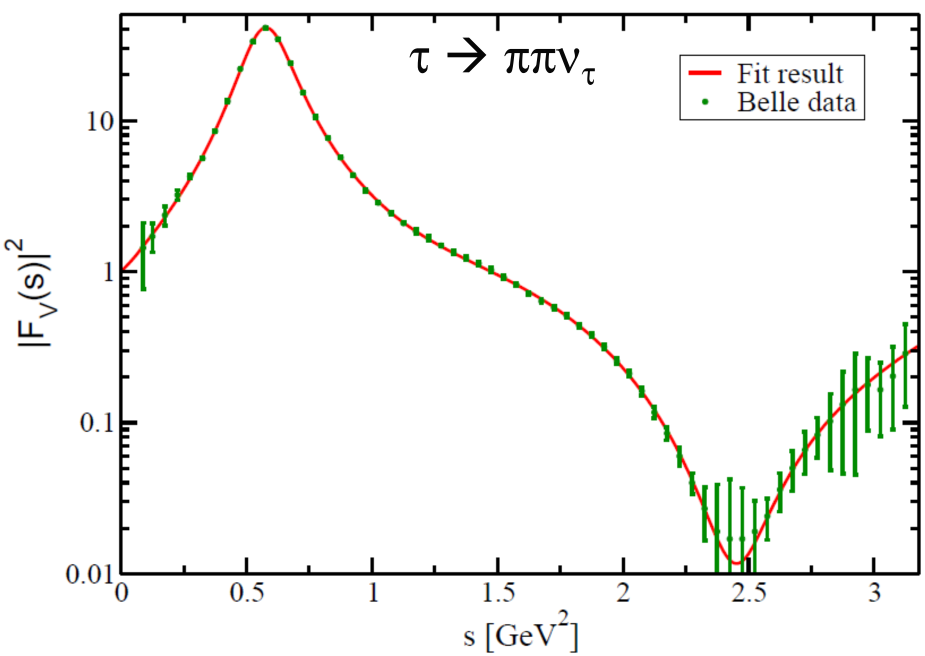
See talk by *S. González-Solis, R. Escribano*

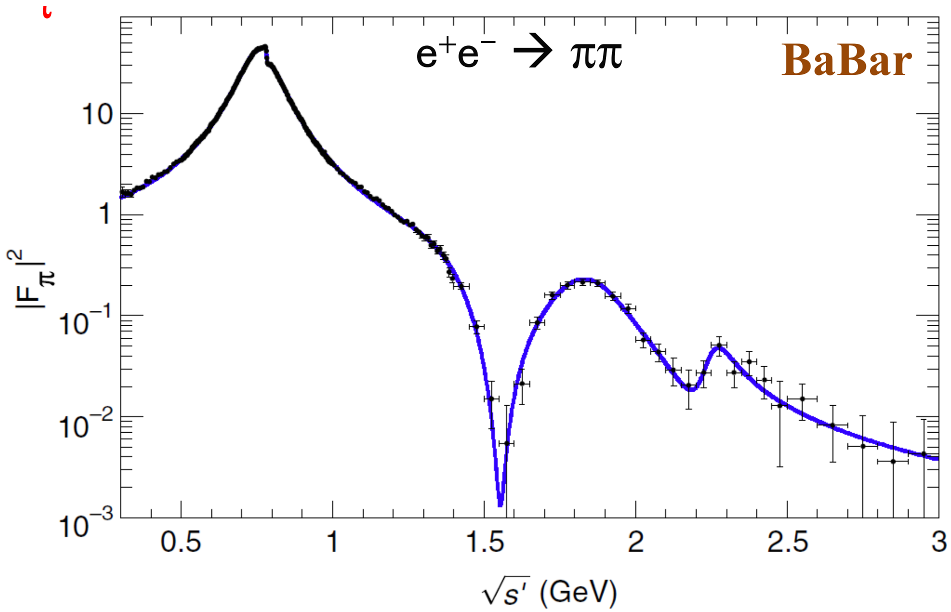
3.6 Exclusive Tau decays

- Invariant mass spectra: constraints on FF very important for testing QCD dynamics and the SM and new physics:

$\pi\pi$ form factors ⇨ g-2 of the muon, LFV hadronic tau decays, proton radius etc

Gómez Dumm – Roig'13 *Cirigliano, Celis, E.P.'14*





- 3 body tau spectra also important: e.g. $\tau \rightarrow \pi\pi\pi\nu_\tau$, $\tau \rightarrow K\pi\pi\nu_\tau$, $\tau \rightarrow \eta^{(')}\pi\pi\nu_\tau$

⇨ in this case Dalitz plots needed

e.g.: Gómez Dumm & Roig'12
Was et al.