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Zhu Xi 朱熹 (1130-1200)

Fundamental Physics using Ultra-Strong Lasers*

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Outline

- 20^{th} vs 21^{st} century fundamental field Theory
- Status of intense lasers & astrophysical sources for supercritical EM fields
- Schwinger mechanism and vacuum birefringence
- Schwinger mechanism in gravity and condensed matter
- QED simulation of quantum cosmology

20^{th} Century Quest for Fundamental Field Theory

Fundamental Field Theory

- Quantum Field Theory for the Standard Model
  - Dirac theory for abelian gauge [PRSL SA 126 ('30)]

$$D_\mu = \partial_\mu - ieA_\mu$$

  - Dirac sea (Vacuum) [MPCPS 30 ('34)]
  - Yang-Mills theory nonabelian gauge [PR 96 ('54)] and QCD for

$$D_\mu = \partial_\mu - iqA_\mu^a T_a$$

  - Klein-Gordon or Higgs-boson theory or higher spin theory

Weak Fields and Perturbations

- Anomalous magnetic moment (four-loop computation of 891 four-loop Feynman diagrams) [Kinoshita, Nio, PRD ('06); Aoyama, Hayakawa, Kinoshita, PRD ('08)]

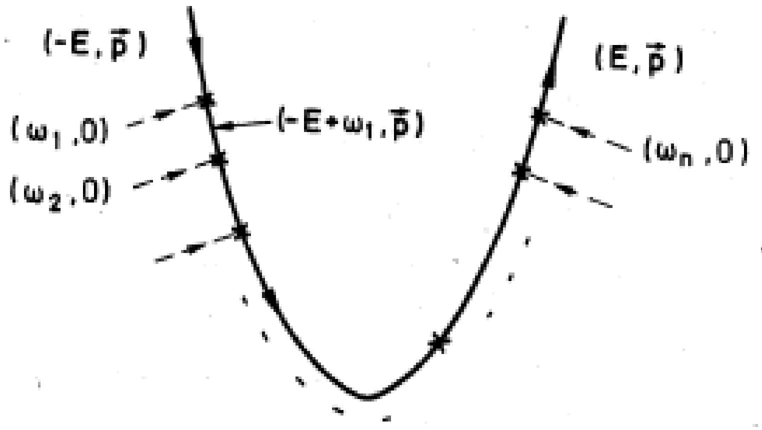
$$\left(\frac{g\text{-}2}{2}\right)_{\text{th}} = \frac{1}{2}\frac{\alpha}{\pi} - 0.32848\cdots\left(\frac{\alpha}{\pi}\right)^2 + 1.18124\cdots\left(\frac{\alpha}{\pi}\right)^3 - 1.9144(35)\left(\frac{\alpha}{\pi}\right)^4 + \cdots$$

- Remarkably agrees with the experimental observation [Hanneke, Fogwell, Gabrielse, PRL ('08)]

$$\left(\frac{g\text{-}2}{2}\right)_{\text{exp}} = 0.00115965218073(28)$$

Diagrammatic Approach to One-Loop
[Chiu, Nussinov, PRD ('79)]

- Even number of perturbations included:



The interaction of a scalar particle with A_μ is

$$V = -2g p_z a(t) + g^2 a^2(t) ,$$

or in momentum space

$$V = -2g p_z \tilde{a}(\omega)\delta^3(\vec{k}) + g^2 \tilde{a}^2(\omega)\delta^3(\vec{k}) ,$$

$$A_n = -(-g^2)^n \int \frac{\prod_{i=1}^{n} d\omega_i \tilde{a}^2(\omega_i)}{\prod_{k=1}^{n-1}\left(2E - \sum_i^k \omega_i\right)\sum_i^k \omega_i} \times \delta\left(\sum_i \omega_i - 2E\right).$$

21st Century Quest for Strong Field Physics

Heisenberg-Euler/Schwinger QED Action

- QED in intense lasers (coherent, multi-photons): PW, EW, ZW

$$D_\mu = \partial_\mu - ieA_\mu \;, \quad |p,N\rangle = e^{-N/2}\sum_{n=0}^{\infty}\frac{N^{n/2}}{\sqrt{n!}}|n\rangle$$

- Gauge invariant Maxwell scalar and pseudo-scalar

$$F = F^{\mu\nu}F_{\mu\nu}/4 = \left(B^2 - E^2\right)/2,\; G = F^{\mu\nu}F^{*}_{\mu\nu}/4 = B\cdot E,\quad X = \sqrt{2(F+iG)} = X_r + iX_i$$

- QED one-loop action (nonlinear QED action) [Heisenberg-Euler, Z. Phys. ('36); Schwinger, Phys. Rev. ('51)]

$$L_{\mathrm{eff}} = -F - \frac{1}{8\pi^2}\int_0^\infty ds\,\frac{e^{-m^2 s}}{s^3}\left[(qs)^2 G\,\frac{\mathrm{Re}\cosh(qXs)}{\mathrm{Im}\cosh(qXs)} - 1 - \frac{2}{3}(qs)^2 F\right]$$

When EM Fields Supercritical?

- Potential energy of an electron across a Compton wavelength in a constant E-field equals to the rest mass of electron

$$eE_c \times \left(\frac{\hbar}{mc}\right) = mc^2 \Rightarrow E_c = \left(\frac{e^2}{\hbar c}\right)\frac{e}{\left(e^2/mc^2\right)^2} = 1.3\times 10^{16}\,(\mathrm{V/cm})$$

- Cyclotron energy of an electron in the Landau level in a constant B-field equals to the rest mass of electron

$$\hbar \times \left(\frac{eB_c}{mc}\right) = mc^2 \Rightarrow B_c = \frac{m^2c^3}{e\hbar} = 4.4\times 10^{13}\,\mathrm{G}$$

Status of Intense Lasers

Towards 100 MeV proton generation using ultrathin targets irradiated with petawatt laser pulses

Chang Hee Nam[1,2],
I J. Kim[1,3], H. T. Kim[1,3] , I. W. Choi[1,3], K. H. Pae[1,3], C. M. Kim[1,3],
S. K. Lee[1,3], J. H. Sung[1,3], and T. M. Jeong[1,3]

[1]*Center for Relativistic Laser Science, Institute for Basic Science (IBS), Korea*
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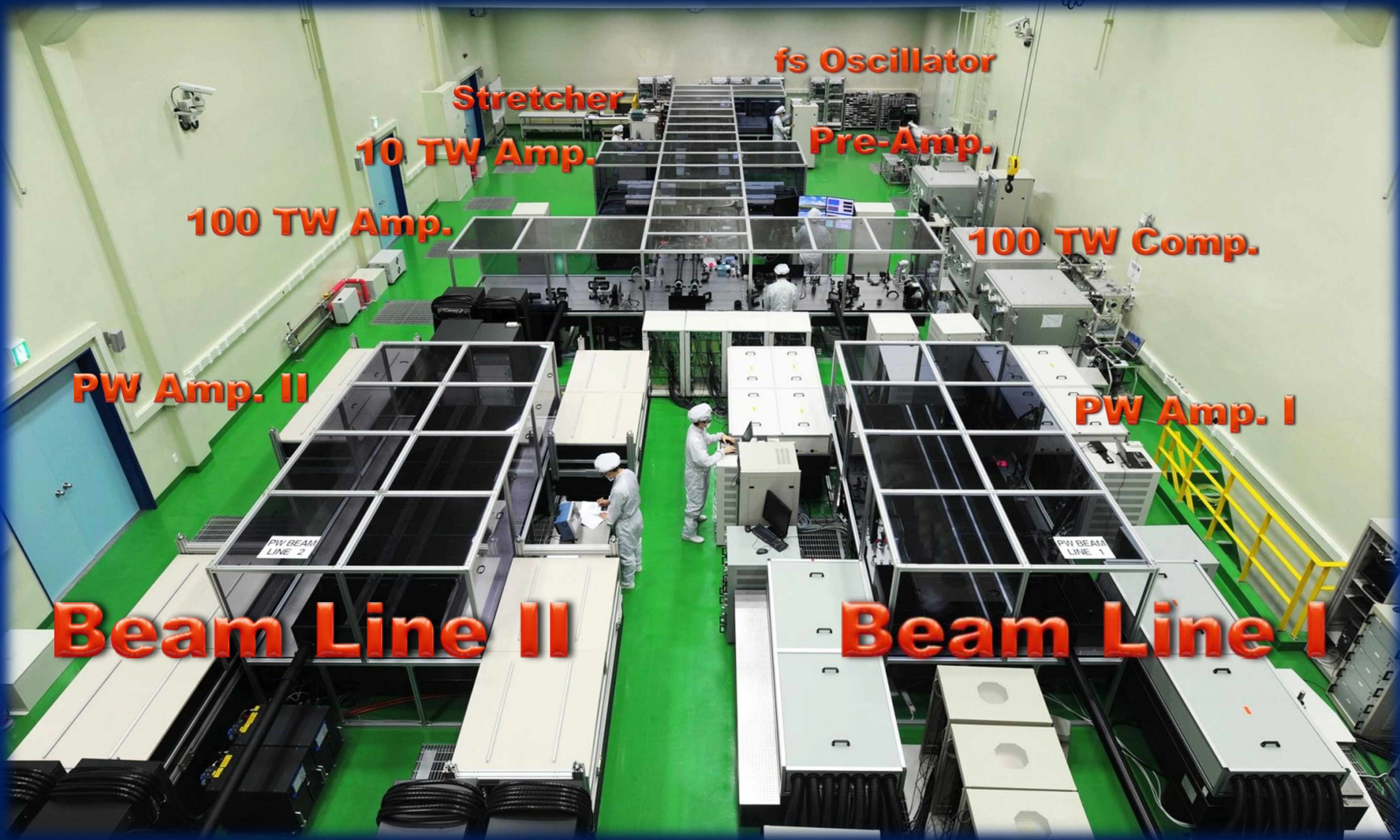
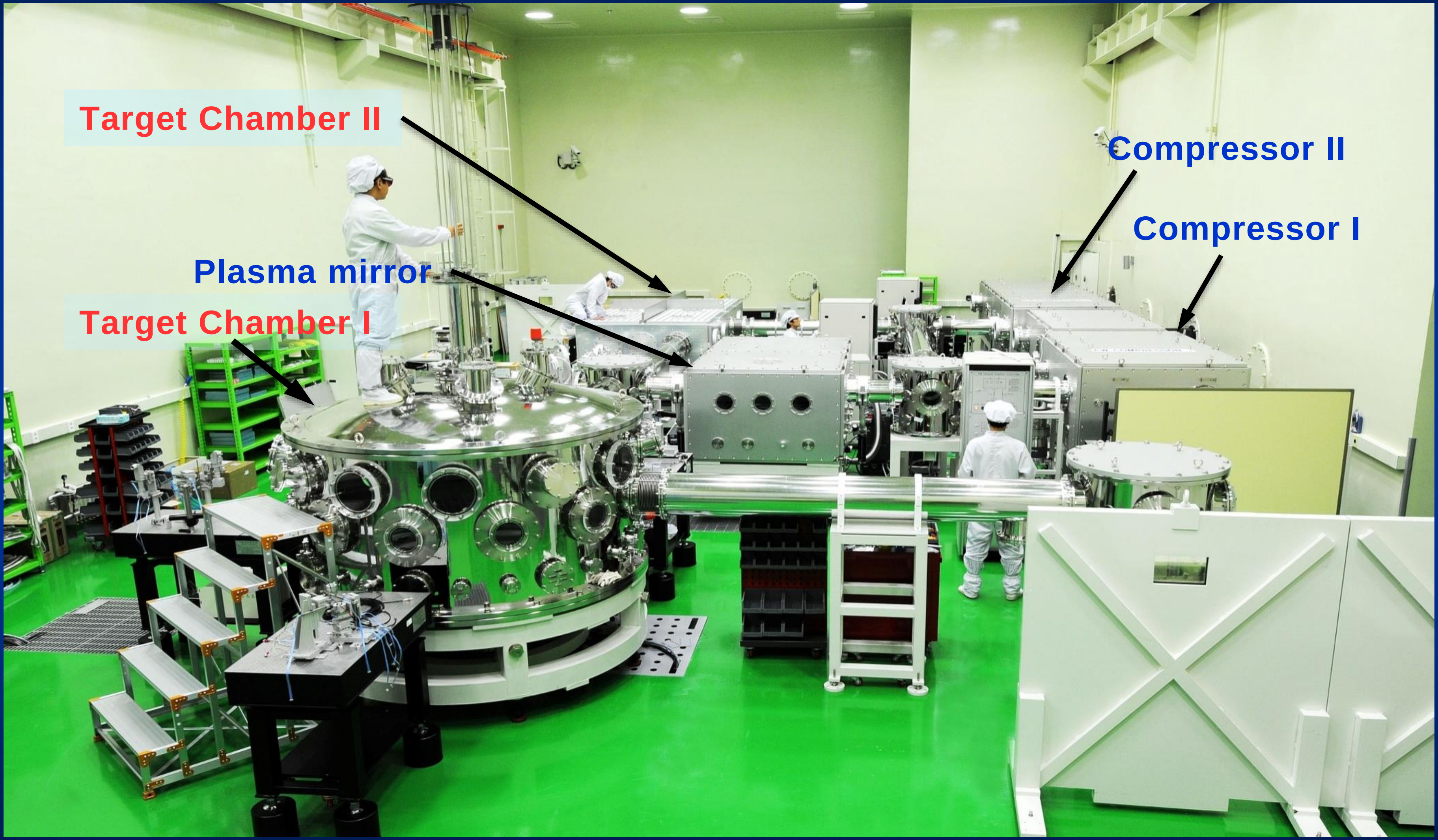
[3]*Advanced Photonics Research Institute, GIST, Gwangju, Korea*

PW Ti:Sapphire Laser

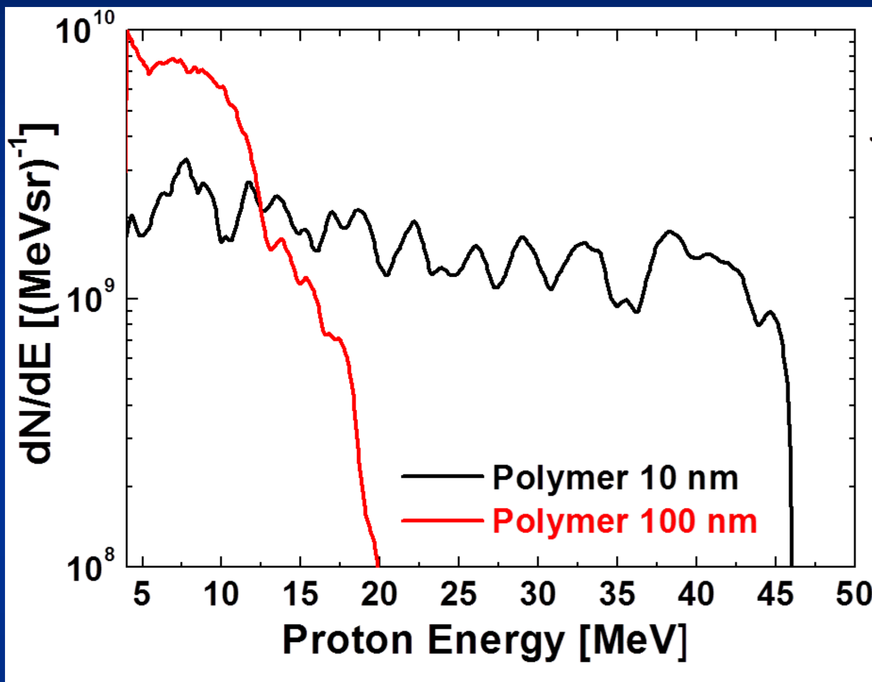


Target Chambers for PW Laser Experiments



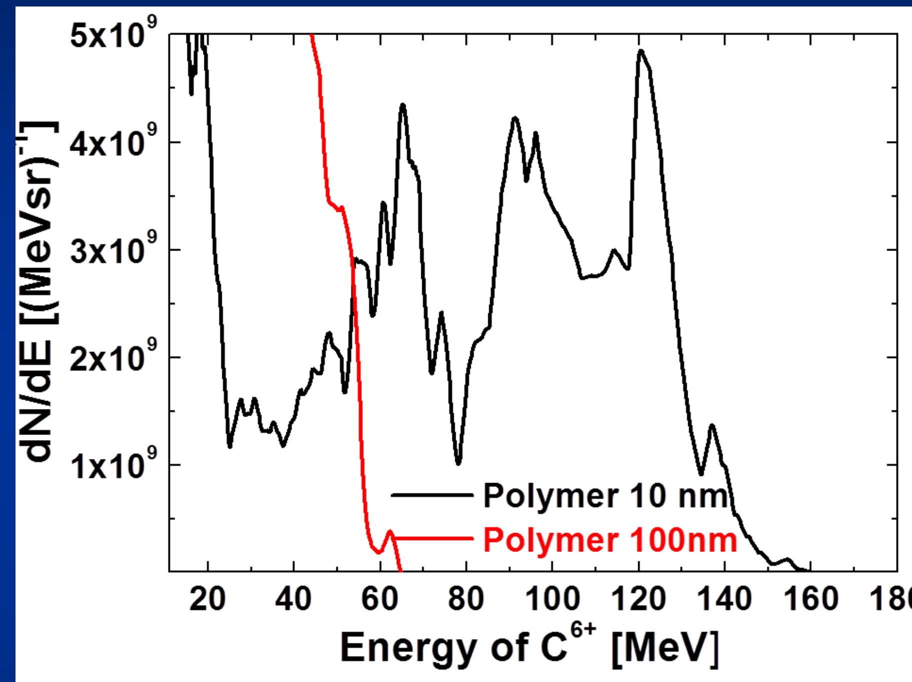
Energy spectra of protons and C^{6+} ions

Laser intensity: 3.3×10^{20} W/cm^2



Proton spectrum

Exponential decay (100 nm) vs. plateau structure (10 nm)



C^{6+} spectrum

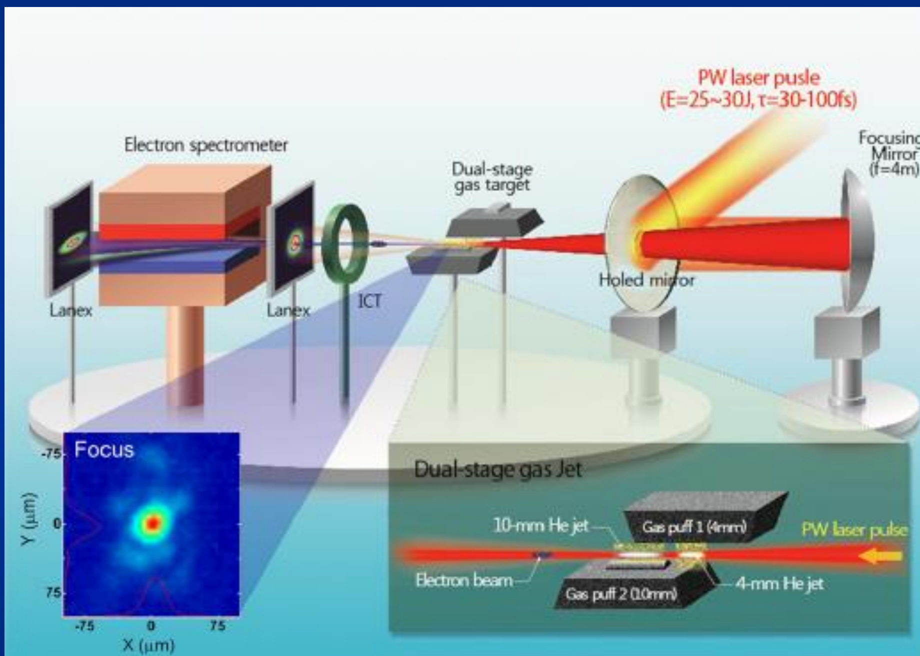
Quasi-monoenergetic peaks (10 nm)!

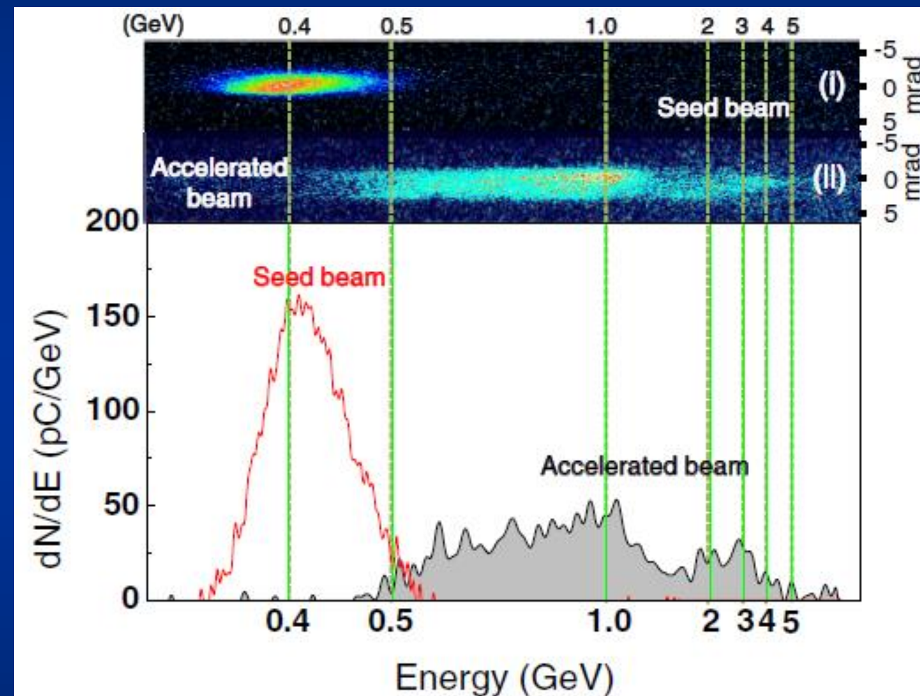
ibS CoReLS

I.J. Kim et al., PRL 111 ('13)

GIST

Multi-GeV Electron by Laser Wake-Field Accelerator





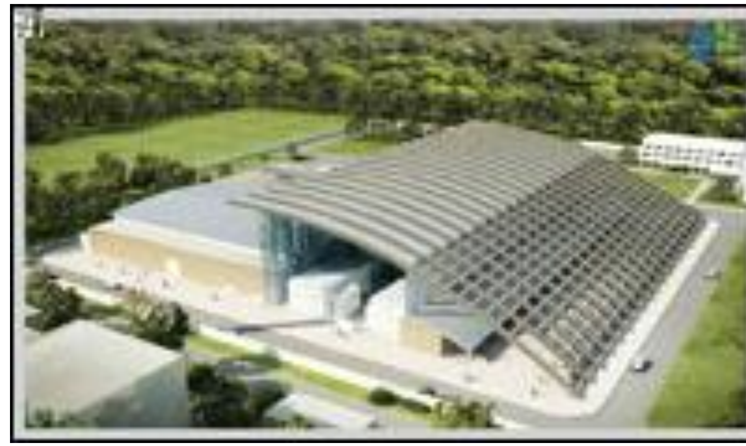
H.T. Kim et al., PRL 111 ('13)

Four Pillars of ELI

Extreme Light Infrastructure [http://www.extreme-light-infrastructure.eu]

ELI-Beamlines Facility: Czech Republic

ELI-Nuclear Physics Facility: Romania



ELI-Attosecond Facility: Hungary

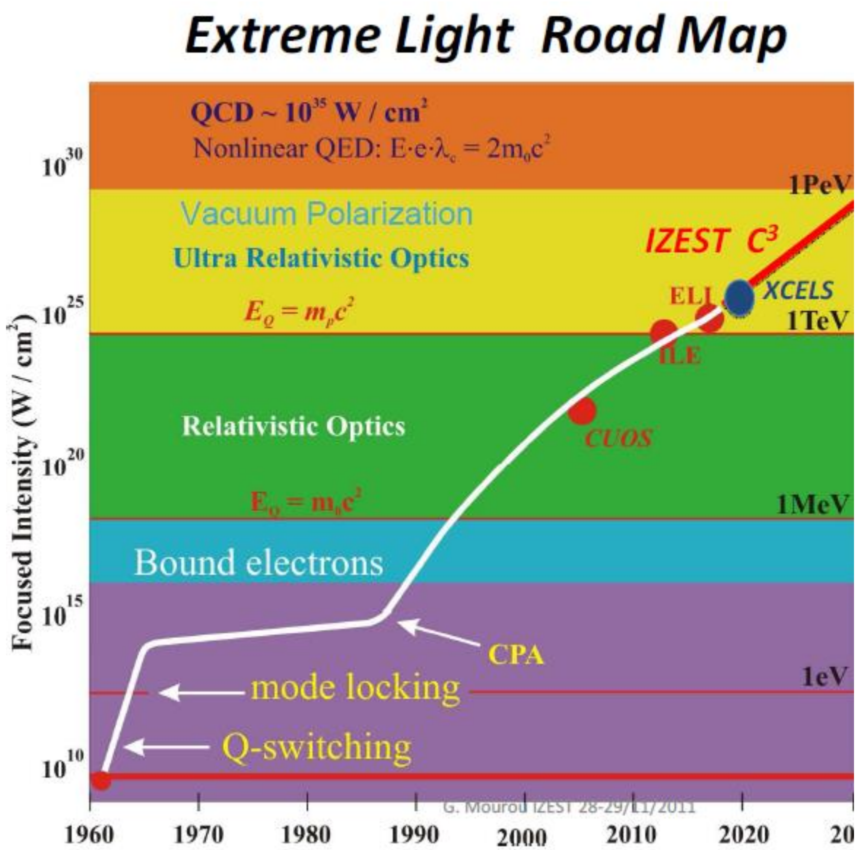
- ELI-Ultra High Energy Field Facility?
  - scheduled in commissioning in 2017.
  - 200 PW (10 beams of 10-20 PW), one shot per min and intensity 10^{25} W/cm^2.

Statistics of 4 Pillars of ELI

Country	Facility focus	Power (PW)	Pulse energy (J)	Pulse width (fs)	Rep rate (Hz)
Romania (2015)	Nuclear physics	10 (x 2)	200	20	0.1
Hungary	Attosecond physics	1/20	5/400	5/20	1000/0.1
Czech Republic	Secondary beam radiation, high-energy particles	1/5/10(x2)	10/50/200	10/10/20	10/10/0.1
To be determined	High intensity	10-20 (x10)	30-40 kJ	15	0.1

[Di Piazza, Muller, Hatsagortsyan, and Keitel, Rev. Mod. Phys. 84 ('12)]

ELI/IZEST & Schwinger Limit



- Schwinger limit (critical strength) for e⁻e+ pair production

$$I_c = \frac{E_c^2}{8\pi} = 2.3\times 10^{29} (W/cm^2)$$

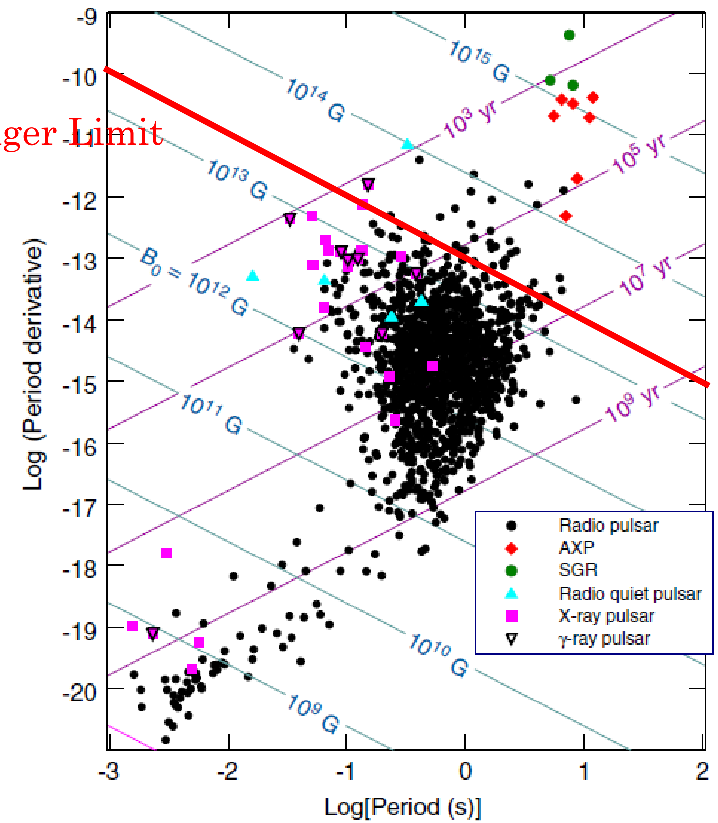
- Energy density is equal to one e⁻e+ pair per unit Compton volume.

[Homma, Habs, Mourou, Ruhl and Tajima, PTP Suppl. 193 ('12)]

Supercritical EM Fields in Astrophysical Sources

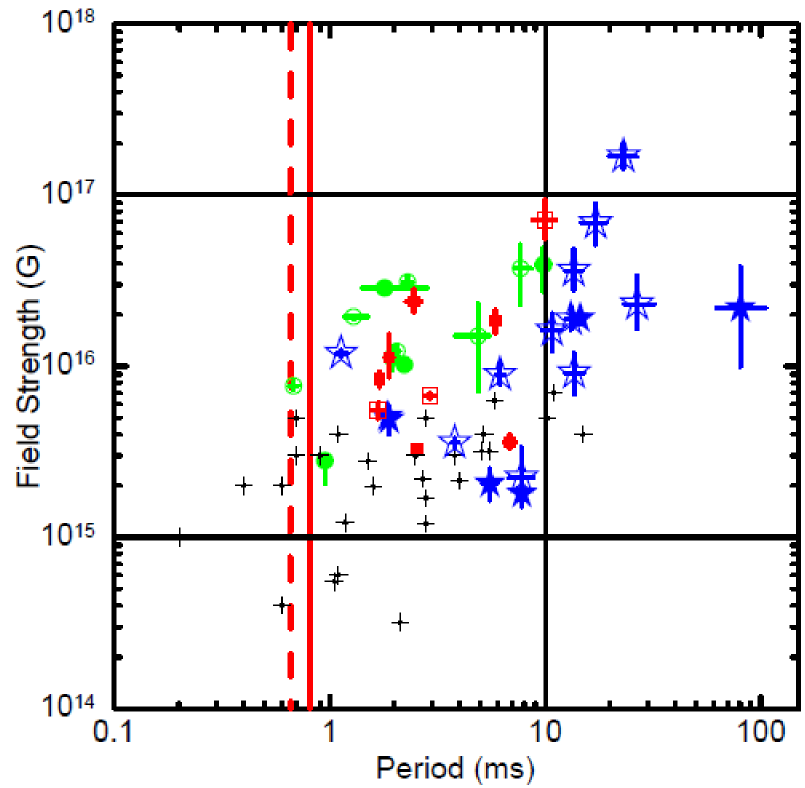
Neutron Stars and Magnetars

Neutron Stars



[Harding, Lai, Rep. Prog. Phys. 69 ('06)]

Strongest Magnetic Fields in the Universe



Gompertz, PhD thesis ('15)
blue: stable magnetars
green: unstable to collapse to black holes

Pair Production & Magnetized Vacuum

- The rotating magnetic field for pulsars or magnetars induces an electric field and changes the Landau levels

$$\vec{B}(t) = B_{\|}\vec{e}_{\|} + B_{\perp}(\cos\omega t\vec{e}_1 + \sin\omega t\vec{e}_2),\ \vec{A} = \frac{1}{2}\vec{B}\times\vec{r} \Rightarrow \vec{E} = -\frac{1}{2}\dot{\vec{B}}\times\vec{r}$$

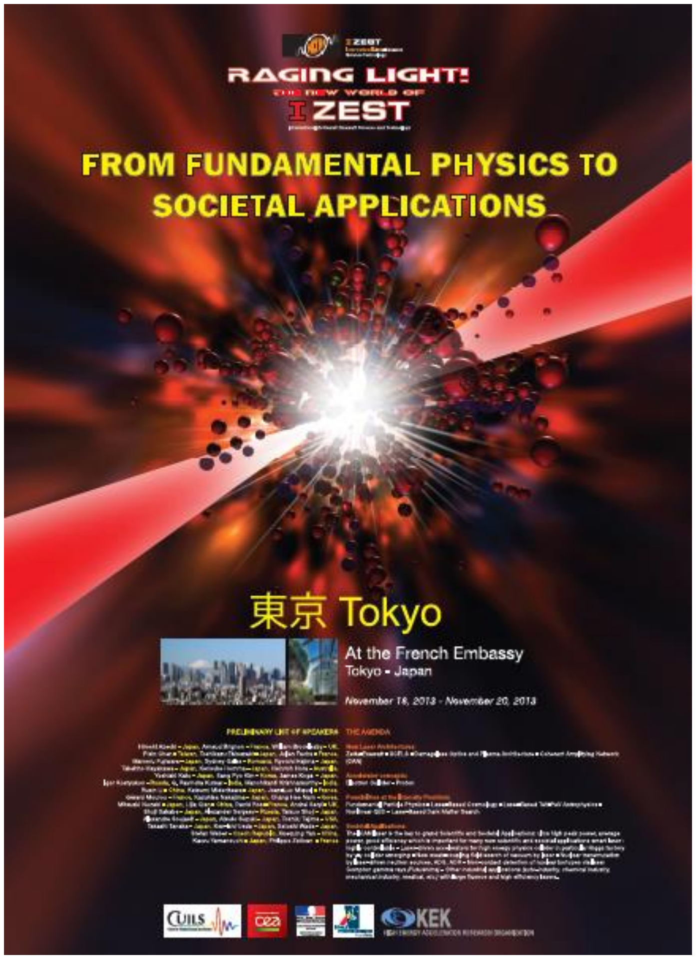
- The rotating magnetic field may produce electron-positron pairs [Di Piazza, Calucci, PRD ('02); Heyl, Hernquist, MNRAS ('05)].
- Dynamical breakdown and pair production: change of magnetic field greater than Landau energy [SPK, AP ('14)]

$$\left| d\ln(B(t))/dt \right| \geq \omega(t)$$

- The birefringence of photon evolution in the magnetic vacuum of QED [Wang, Lai, MNRAS ('09)].

What Fundamental Physics in Strong QED?

Suzuki's Challenge



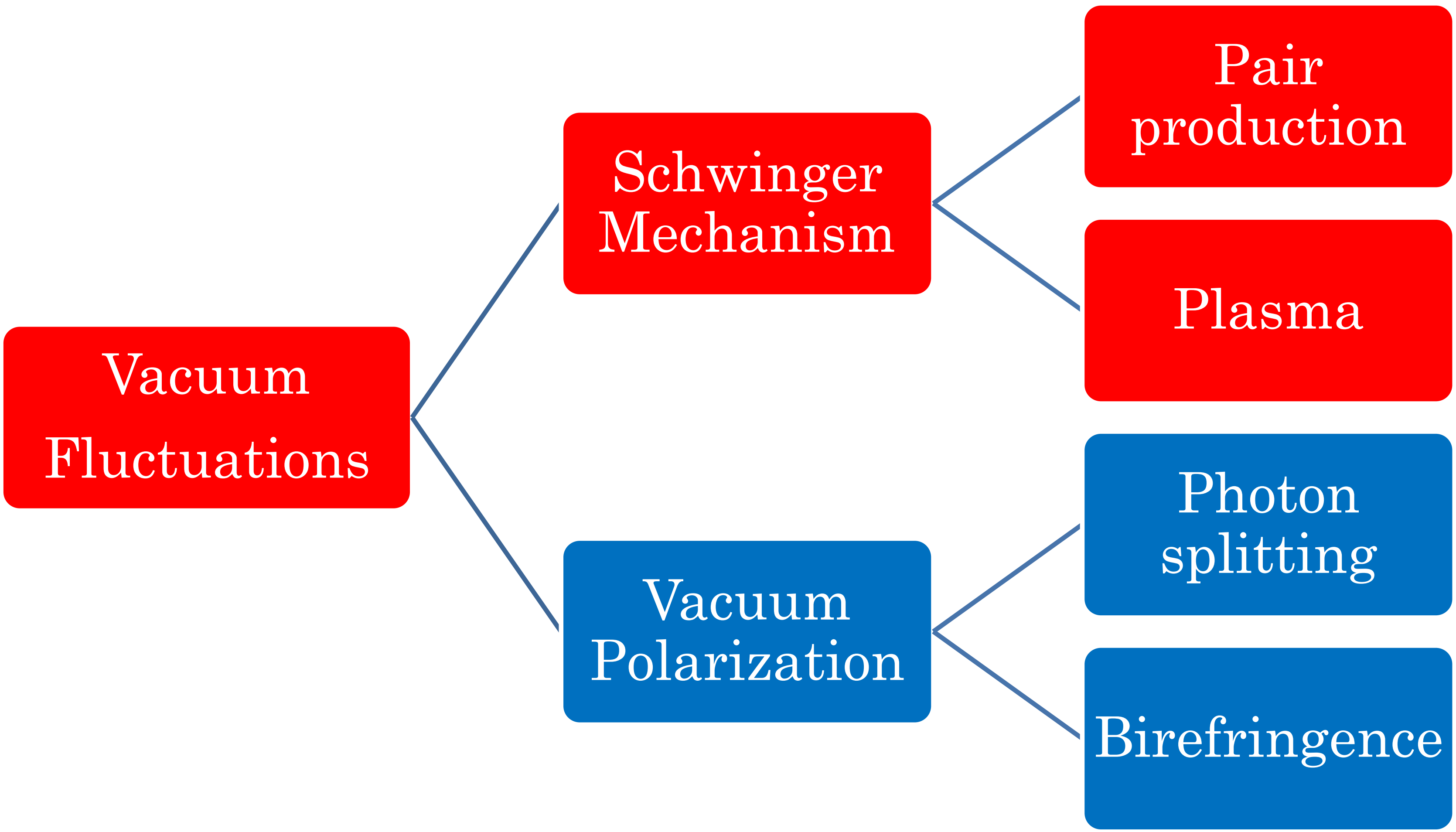
- The most fundamental Physics: Vacuum is not EMPTY!

$$\left|\text{vacuum}\right\rangle = \sum_{i}^{\infty} \left|n_i\right\rangle = \left|0^+\right\rangle$$

- Vacuum? Conceptual definition: Entity which includes everything which we don't know. Unsolved mysteries from particle adventure?
  - Accelerating Universe (Dark Energy)?
  - Why No Antimatter?
  - Dark Matter?
  - Origin of Mass? (BEH mechanism)

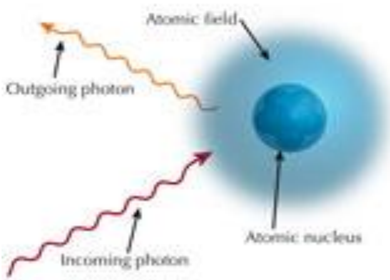
Schwinger Mechanism & Vacuum Birefringence

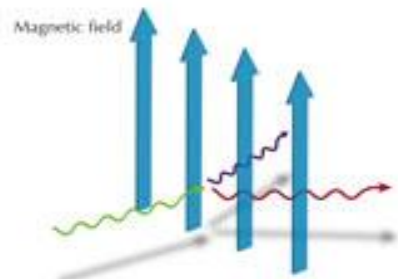
Physics in Strong QED

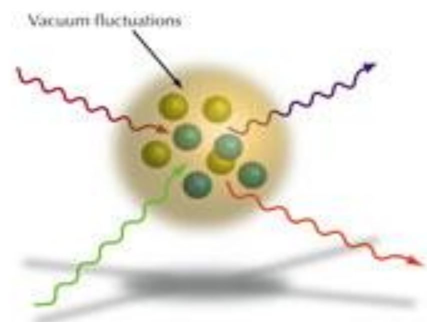


Fundamental Physics with ELI

- Can test strong QED

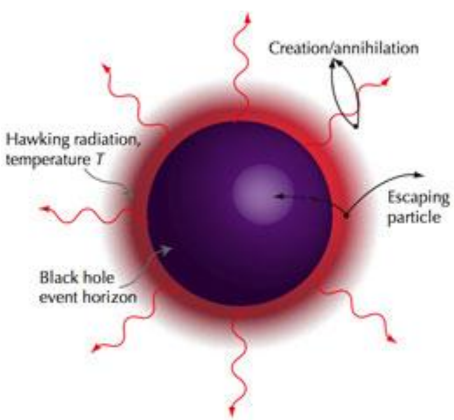


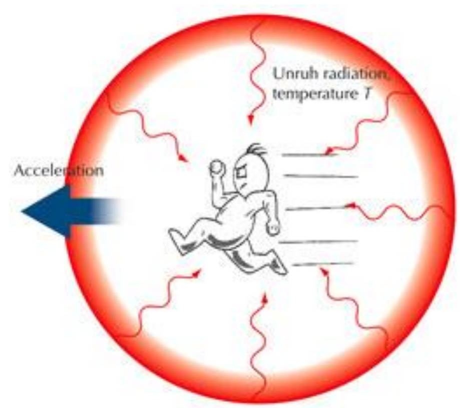




(Delbruck scattering) (Photon splitting) (Pair production)

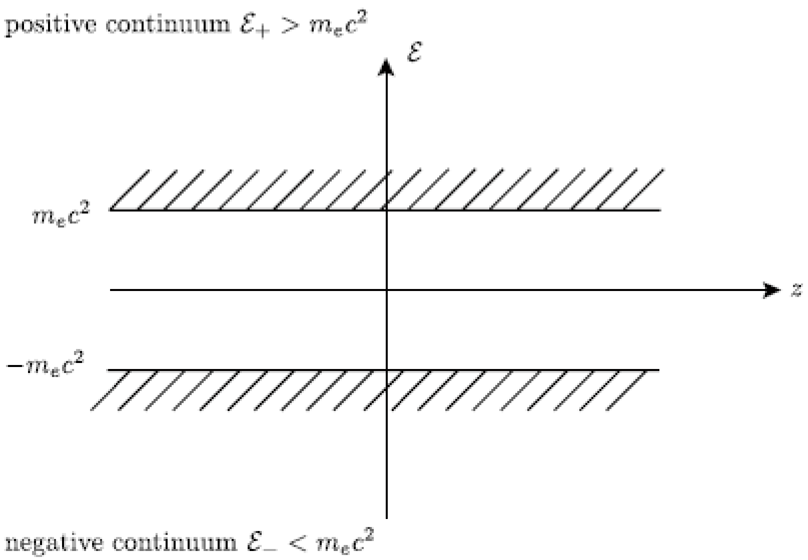
- Can test the Hawking-Unruh radiation $\left(a > 10^{24} g \right)$





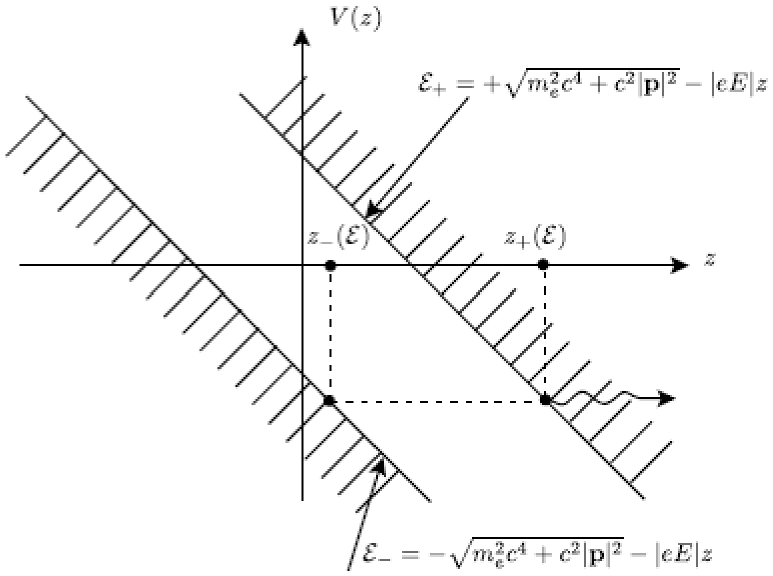
Schwinger Pair Production

Dirac Vacuum (Sea)



[Fig. from Ruffini, Vereshchagin, Xue, Phys. Rep. 487 ('10)]

Dirac Sea in E-Field



Tunneling probability for particle-antiparticle pairs

$$N(\vec{p}_\perp) = \exp\left(-\pi \frac{m^2 + \vec{p}_\perp^{\,2}}{|eE|}\right) = \exp\left(-\pi \frac{\varepsilon^2 / m^2}{E / E_c}\right)$$

Schwinger Mechanism

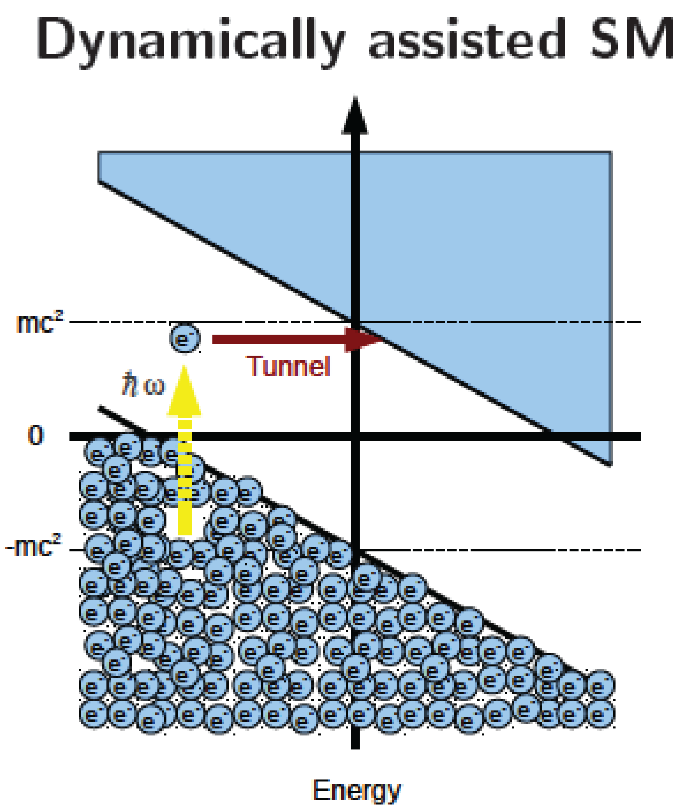
- If F and G not both zero, find a Lorentz frame in which E and B are parallel, $X = B - iE$.
- Spinor QED: a constant EM-field

$$2\,\mathrm{Im}(L_{\mathrm{eff}}^{\mathrm{sp}}) = \frac{(qE)(qB)}{(2\pi)(2\pi)} \sum_{n=1}^{\infty} \frac{1}{n} \coth\left[\frac{n\pi B}{E}\right] \exp\left[-\frac{\pi m^2 n}{qE}\right]$$

- Scalar QED: a constant EM-field

$$2\,\mathrm{Im}(L_{\mathrm{eff}}^{\mathrm{sc}}) = \frac{(qE)(qB)}{2(2\pi)(2\pi)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \operatorname{csch}\left[\frac{\pi B n}{E}\right] \exp\left[-\frac{\pi m^2 n}{qE}\right]$$

Schwinger Mechanism Enhanced



- Dynamically assisted Schwinger mechanism [Schutzhold, Gies, Dunne, PRL ('08)]

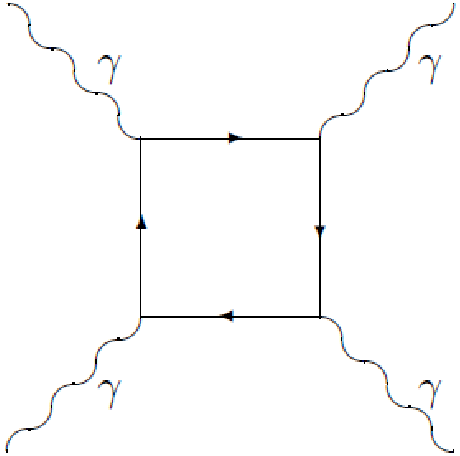
: a strong and slowly varying field (Schwinger effect) superimposed by a weak and rapid varying EM field (dynamic pair production)

- Catalysis mechanism [Dunne, Gies, Schutzhold, PRD ('09)]

: plane-wave X-ray probe beam to strongly focused optical laser pulse

Vacuum Polarization

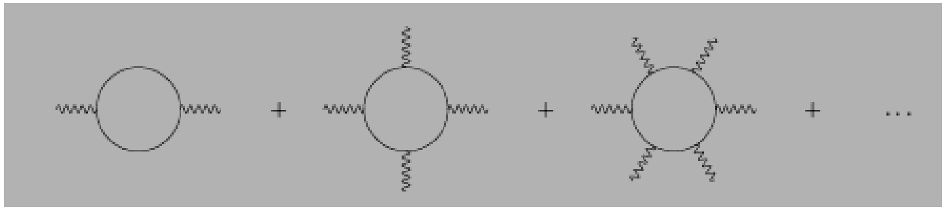
Photon virtual pair interaction



$$\frac{2e^4}{45m^4}\left[4F^2+7G^2\right]$$

$$F=\frac{1}{2}\left(\mathrm{B}^2-\mathrm{E}^2\right),\ G=\mathrm{B}\cdot\mathrm{E}$$

One-Loop Effective Action



- External legs: EM fields or gravitons
- Internal loops: fermions or bosons

One-Loop Vacuum Polarization

- One-loop effective action [Heisenberg, Euler ('36); Schwinger ('51)]

$$L_{\text{eff}} = -F - \frac{1}{8\pi^2}\int_0^\infty ds\, \frac{e^{-m^2 s}}{s^3}\left[(qs)^2 G\, \frac{\operatorname{Re}\cosh(qXs)}{\operatorname{Im}\cosh(qXs)} - 1 - \frac{2}{3}(qs)^2 F\right]$$

$$F = \frac{1}{2}\left(\mathrm{B}^2 - \mathrm{E}^2\right),\ G = \mathrm{B}\cdot\mathrm{E},\ X = \sqrt{2(F+iG)} = X_r + iX_i$$

- Polarization due to one-loop

$$\vec{D} = 4\pi\frac{\partial L_{\text{eff}}}{\partial \vec{E}}\ ,\ \vec{H} = -4\pi\frac{\partial L_{\text{eff}}}{\partial \vec{B}}$$

Pair Annihilation and Pair Production

- Annihilation of electron-positron pair by Dirac ('30)

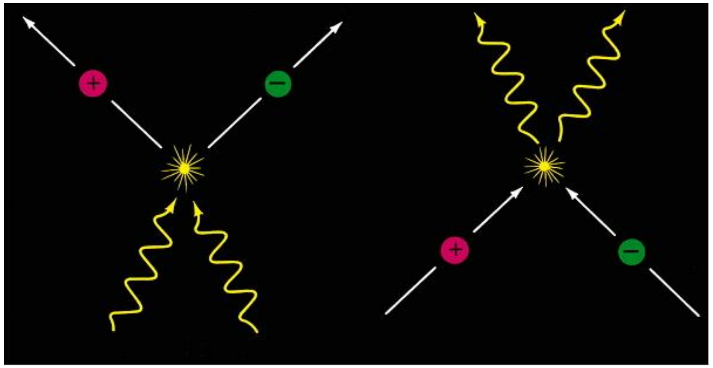
$$e^- + e^+ \to \gamma_1 + \gamma_2$$

$$\sigma_{e^-e^+} \approx \pi\left(\alpha\lambda_C \times \frac{mc^2}{\varepsilon_e}\right)^2 \frac{\varepsilon_e}{mc^2}\left[\ln\left(\frac{2\varepsilon_e}{mc^2}\right) - 1\right], \quad (\varepsilon_e >> mc^2)$$

- Breit-Wheeler process of electron-positron production in collision of two photons ('34) with threshold energy

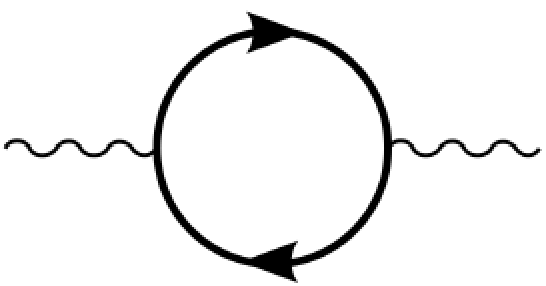
$$\gamma_1 + \gamma_2 \to e^- + e^+$$

$$\sigma_{\gamma\gamma} \approx \pi\left(\alpha\lambda_C \times \frac{mc^2}{\varepsilon_\gamma}\right)^2, \quad (\varepsilon_\gamma >> mc^2)$$



Photon-Photon Scattering

- Classical Maxwell theory is linear and thus prohibits a self-interaction (direct γγ scattering).
- QED permits γ and γ to interact with virtual e-e+ pair from the Dirac sea: the cross section in the low energy limit of the two colliding γ in the center of momentum frame) [Euler ('36); Akhiezer ('37); Karplus, Neuman ('50)]: a vacuum polarization effect



$$\sigma_{\gamma\gamma\rightarrow\gamma\gamma} = \frac{973}{81000\pi}\left(\alpha^2\lambda_C\right)^2\eta^3, \quad (\eta = \frac{2\omega^{*2}}{m^2})$$
$$= 7.4\times10^{-66}(\omega^*[eV])^6$$

- We have NOT seen the photon-photon scattering since the early universe, but ELI is highly likely to detect it.

Testing Fundamental Actions

- Parametrized Post Maxwellian (PPM) Framework [Ni, Mod. Phys. Lett. A ('13)]

$$L_{\mathrm{PPM}} = (1/8\pi)\Big[\big(E^2 - B^2\big) + \xi\Phi\big(\vec{E}\cdot\vec{B}\big) + \big(1/B_{\mathrm{C}}^2\big)\Big\{\eta_1\big(E^2 - B^2\big)^2 + 4\eta_2\big(\vec{E}\cdot\vec{B}\big)^2 + 2\eta_3\big(E^2 - B^2\big)\big(\vec{E}\cdot\vec{B}\big)\Big\}\Big]$$

- Heisenberg-Euler action: birefringence

$$\eta_1 = \alpha/4\pi\ ,\quad \eta_2 = 7\alpha/(180\pi)\ ,\quad \eta_3 = \xi = 0$$

$$\Rightarrow \Delta n = n_{\parallel} - n_{\perp} = 4.0\times10^{-24}\times\big(B_{\mathrm{ext}}/1\mathrm{T}\big)^2$$

- Born-Infeld action: no birefringence

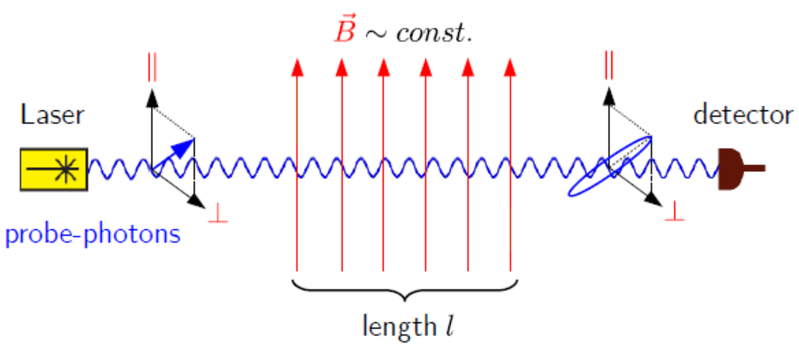
$$L_{\mathrm{Born\text{-}Infeld}} = -\big(b^2/4\pi\big)\sqrt{1 - \big(E^2 - B^2\big)/b^2 - \big(E^2 - B^2\big)^2/b^4}$$

$$\eta_1 = \eta_2 = B_{\mathrm{C}}^2/b^2\ ,\quad \eta_3 = \xi = 0 \Rightarrow \Delta \mathrm{n} = \mathrm{n}_{\parallel} - n_{\perp} = 0$$

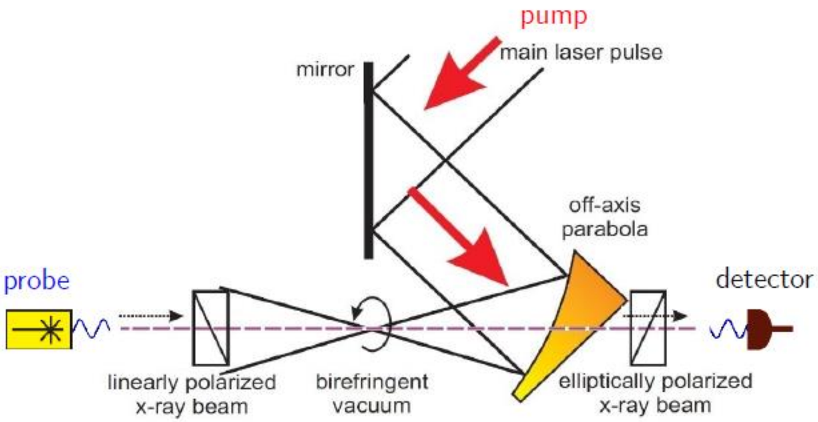
Vacuum Birefringence

[Karbstein, Tomsk Workshop ('06)]

Conventional Scenario



Pumping Intense Lasers



QED Actions in Localized EM Fields

- Pulsed Sauter E-fields [SPK, Lee, Yoon, PRD ('08)]

$$E(t) = E_0 \text{sech}^2 (t / \tau)$$

- Localized Sauter E-fiels [SPK, Lee, Yoon, PRD ('10)]

$$E(z) = E_0 \text{sech}^2 (z / L)$$

- Localized Sauter B-fields [SKP, PRD ('11)]

$$B(z) = B_0 \text{sech}^2 (z / L)$$

Photon-(pseudo)Scalar Interaction

- Quantum anomaly-type interaction between two photon and a (pseudo)scalar field (dark field) [Homma, Habs, Mourou, Ruhl, Tajima, PTP Supl. ('12)]

$$L_\phi = -gM_\phi^{-1} F_{\mu\nu} F^{\mu\nu} \phi$$

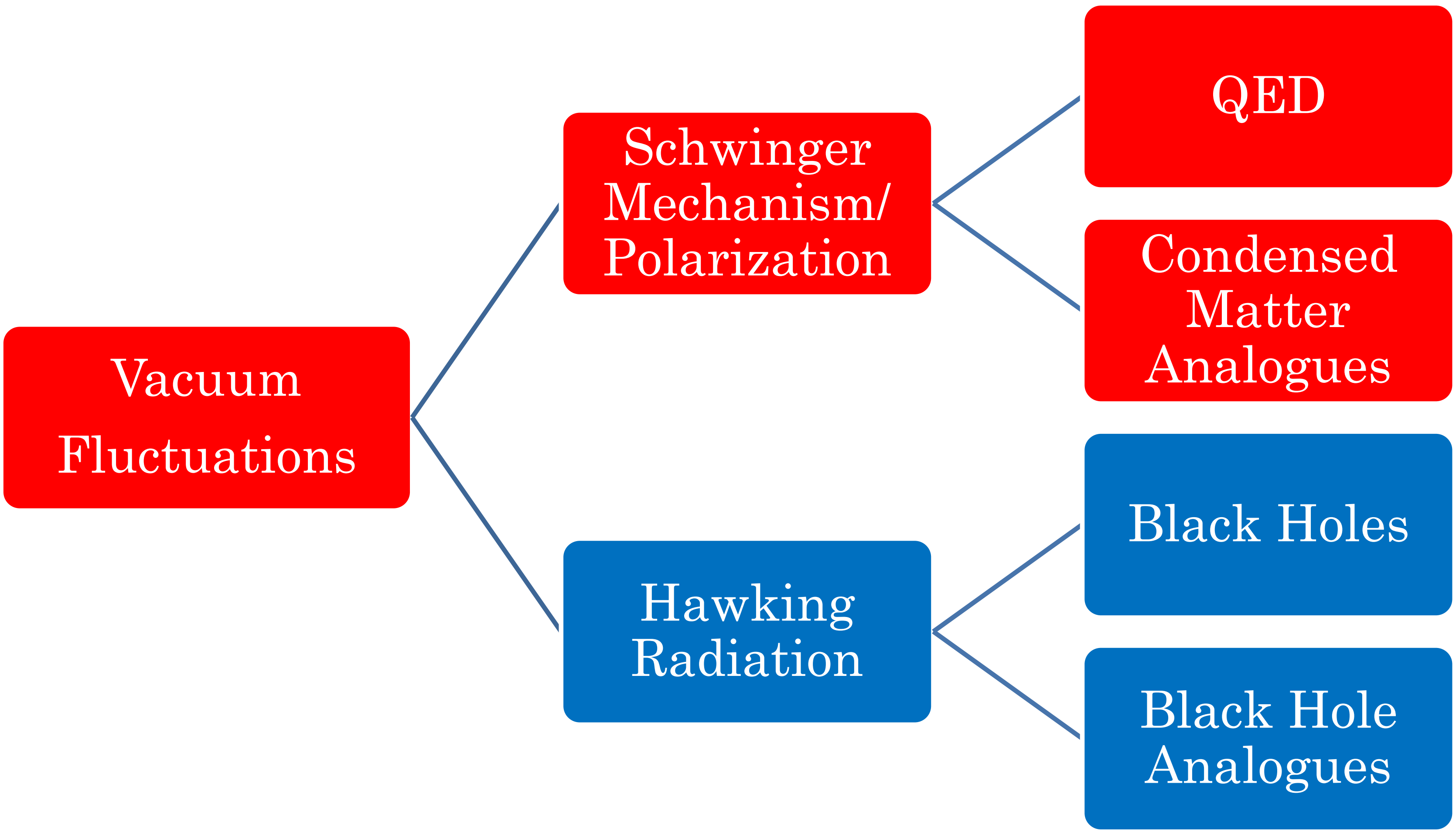
- Cross section for interaction enhanced by coherent (multiparticle) states of intense lasers

$$\left| p, N \right\rangle = e^{-N/2} \sum_{n=0}^{\infty} \frac{N^{n/2}}{\sqrt{n!}} \left| n \right\rangle$$

Schwinger Mechanism in Gravity and Condensed Matter

Quest for Vacuum and Pair Production

[SPK, JHEP ('07)]



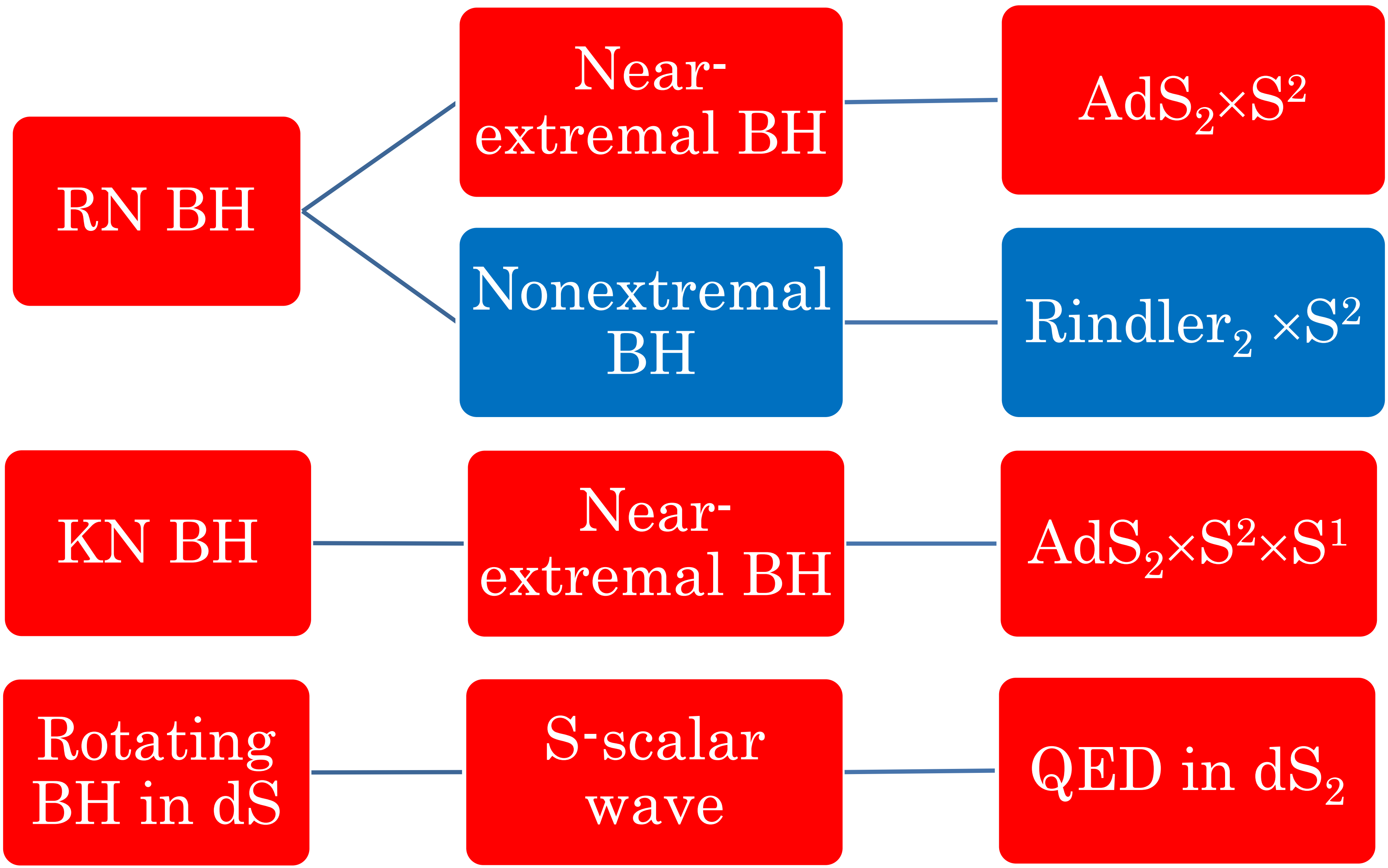
Pair Production

Hawking radiation	Schwinger mechanism
Quantum (Dirac) vacuum	Quantum (Dirac) vacuum
Horizon	Electric field
KG or Dirac or Maxwell Eq	KG or Dirac Eq
Parametric interaction	Gauge interaction
One species of particles (charged or uncharged)	Charged pairs
Vacuum polarization (one-loop effective action)?	Heinsenberg-Euler/Schwinger vacuum polarization

What happens when a black hole has a charge?

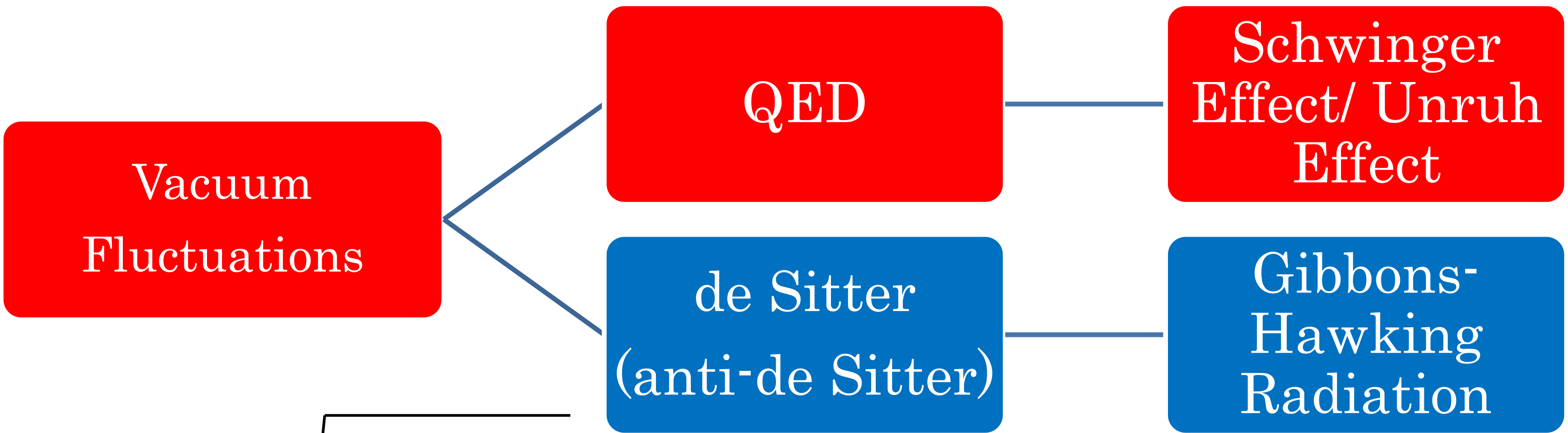
Schwinger Effect in Near-Extremal BHs

[Chen et al, PRD ('12); CQG ('15); ('16)]



Schwinger Effect in (A)dS

[Cai, SPK, JHEP (‘14)]



$$T_{\text{eff}} = T_{\text{U}} + \sqrt{T_{\text{U}}^2 + \frac{R_2}{8\pi^2}}$$

$$R_2 = \begin{cases} 2H^2, \ (\text{dS}_2) \\ -2K^2, \ (\text{AdS}_2) \end{cases}$$

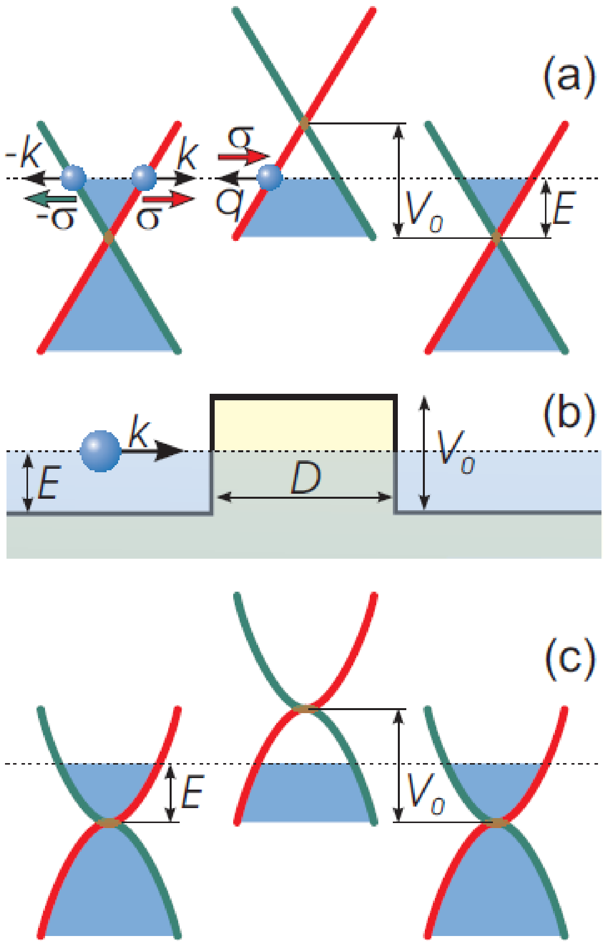
Condensed Matter Analogue of QED

- The relation between the theory of dielectric breakdown in condensed matter and nonlinear QED from the view point of the effective Lagrangian $L_{\mathrm{eff}}(A) = -i \lim_{t\to\infty} \frac{1}{t} \ln\left(\langle \Psi(\mathrm{t}) | \Psi(0) \rangle\right)$

[Oka, Aoki, Lect. Notes Phys. ('09)]

	Condensed Matter	QED
Mechanism Excitation	Landau-Zener tunneling Electron(doublon)-hole pair	Schwinger mechanism Electron-positron pair
Effective action	Nonadiabatic Berry's phase	Heisenberg-Euler/Schwinger
Nonlinear polarization	Photovoltaic Hall effect Floquet picture	γ-γ interaction (birefringence) Furry picture

Graphene Analogue of QED



[Fig. Katsnelson, Novoselov and Geim, Nat. Phys. ('06)]

- Effectively massless Dirac fermions

$$H_0 = -i\hbar v_F \sigma \nabla$$

- The Klein paradox

$$V(x) = \begin{cases} V_0 & (0 < x < D) \\ 0 & (\text{otherwise}) \end{cases}$$

$$T = \frac{\cos^2 \phi}{1 - \cos^2(q_x D)\sin^2 \phi}$$

- $T = 1$ for normal incidence or $q_x D = \pi N$.
- The Klein tunneling was experimentally observed in graphene heterojunctions [Young, P. Kim, Nat. Phys. ('09)].

QED Actions in In-Out Formalism

One-Loop Effective Actions

- In the in-out formalism, the vacuum persistence amplitude gives the effective action [Schwinger, PNAS ('51); DeWitt, Phys. Rep. ('75); *The Global Approach to Quantum Field Theory* ('03)] and is equivalent to the Feynman integral

$$e^{iW} = e^{i\int (-g)^{1/2} d^D x L_{\text{eff}}} = \langle 0, \text{out} \mid 0, \text{in} \rangle$$

- The complex effective action and the vacuum persistence

$$\left| \langle 0, \text{out} \mid 0, \text{in} \rangle \right|^2 = e^{-2\,\text{Im}\,W}$$

$$2\,\text{Im}\,W = \pm VT \sum_{\text{k}} \ln(1 \pm N_k)$$

Bogoliubov Transformation & In-Out Formalism

- The Bogoliubov transformation between the in-state and the out-state, equivalent to the S-matrix,

$$a_{\mathrm{k,out}} = \alpha_{\mathrm{k,in}} a_{\mathrm{k,in}} + \beta^*_{\mathrm{k,in}} b^+_{\mathrm{k,in}} = U_{\mathrm{k}} a_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

$$b_{\mathrm{k,out}} = \alpha_{\mathrm{k,in}} b_{\mathrm{k,in}} + \beta^*_{\mathrm{k,in}} a^+_{\mathrm{k,in}} = U_{\mathrm{k}} b_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

- Commutation relations from quantization rule (CTP):

$$\left[a_{\mathrm{k,out}}, a^+_{\mathrm{p,out}}\right] = \delta(\mathrm{k}-\mathrm{p}),\ \left[b_{\mathrm{k,out}}, b^+_{\mathrm{p,out}}\right] = \delta(\mathrm{k}-\mathrm{p});$$

$$\left\{a_{\mathrm{k,out}}, a^+_{\mathrm{p,out}}\right\} = \delta(\mathrm{k}-\mathrm{p}),\ \left\{b_{\mathrm{k,out}}, b^+_{\mathrm{p,out}}\right\} = \delta(\mathrm{k}-\mathrm{p})$$

- Particle (pair) production

$$N_{\mathrm{k}} = \left|\beta_{\mathrm{k}}\right|^2; \quad \left|\alpha_{\mathrm{k}}\right|^2 \mp \left|\beta_{\mathrm{k}}\right|^2 = 1$$

Effective Actions at T=0 & T

- Zero-temperature effective actions for scalar and spinor via Gamma-function regularization [SPK, Lee, Yoon, PRD ('08); PRD ('10); SPK, PRD ('11)]

$$W = -i \ln \langle 0, \mathrm{out} \mid 0, \mathrm{in} \rangle = \pm i \sum_{\mathrm{k}} \ln \alpha_{\mathrm{k}}^{*}$$

- finite-temperature effective action for scalar and spinor [SPK, Lee, Yoon, PRD ('10)]

$$\exp[i \int d^3 x dt L_{\mathrm{eff}}] = \langle 0, \beta, \mathrm{in} | U^{+} | 0, \beta, \mathrm{in} \rangle = \frac{\mathrm{Tr}(U^{+} \rho_{\mathrm{in}})}{\mathrm{Tr}(\rho_{\mathrm{in}})}$$

Vacuum Polarization & Persistence

- Purely thermal part of the effective action

$$\Delta L_{\rm eff}(T,E) = L_{\rm eff}(T,E) - L_{\rm eff}(T=0,E)$$
$$= \mp i \sum_{k,\sigma} \left[\ln\left(1 \pm e^{-\beta(\omega_k - z_k)}\right) - \ln\left(1 \pm e^{-\beta\omega_k}\right)\right]$$

- Imaginary part of the effective action

$$\mathrm{Im}(\Delta L_{\rm eff}) = \pm \frac{1}{2} i \sum_{k,\sigma} \sum_{j=1} \frac{(\mp n_{FD/BE}(k))^j}{j} \left[(e^{\beta z_k} - 1)^j + (e^{\beta z_k^*} - 1)^j \right]$$

- Real part of the effective action

$$\mathrm{Re}(\Delta L_{\rm eff}(T)) = \mp \sum_{k,\sigma} \arctan\left[\frac{\sin(\mathrm{Re}\, L_{\rm eff}(T=0,k))}{e^{\beta\omega_k}\left(1 + |\beta_k|^2\right)^{(1+2|\sigma|)/2} \pm \cos(\mathrm{Re}\, L_{\rm eff}(T=0,k))} \right]$$

Pair Production at T

- Imaginary part of the effective action (the limit of small mean number of produced pairs)

$$2\,\mathrm{Im}\,\Delta L_{\mathrm{eff}}(T) \approx \mp \sum_{k,\sigma} |\beta_k|^2\, n_{FD/BE}(k)$$

- Consistent with the pair-production rate at T [SPK, Lee, PRD ('07); SPK, Lee, Yoon, PRD ('09)]

$$N^{\mathrm{sp/sc}}(T) = \begin{cases} \sum_k |\beta_k|^2 \tanh(\beta\omega_k/2) \\ \sum_k |\beta_k|^2 \coth(\beta\omega_k/2) \end{cases}$$

Intense Lasers Simulation of Quantum Universe

Quantum Cosmology vs QED

- Wheeler-DeWitt equation for FRW universe with a massive scalar field [SPK, Page, PRD ('92); SPK, PRD ('92)]

$$\left[-\pi_a^2 + \underbrace{V_G(a)}_{\text{electric field E(t)}} + \frac{1}{a^2} \underbrace{\left(\pi_\phi^2 + a^6 m^2 \phi^2 \right)}_{\text{magnetic field B(t)}} \right] \Psi(a, \phi) = 0$$

- Transverse motion of a charged scalar in a time-dependent, homogeneous, magnetic field B(t)

$$\left[\frac{\partial^2}{\partial t^2} + \left(\vec{p}_\perp^{\,2} + \left(\frac{qB(t)}{2} \right)^2 \vec{x}_\perp^{\,2} - qB(t) L_z \right) + m^2 + k_z^2 \right] \Phi_\perp(t, \vec{x}_\perp) = 0$$

1^{st} Quantized QED Formulation

- The two-component, first order wave function of the KG eq expanded by Landau states [SPK, Ann Phys ('14)]

$$\begin{pmatrix} \Psi(t,\vec{x}_\perp) \\ \partial\Psi(t,\vec{x}_\perp)/\partial t \end{pmatrix} = \begin{pmatrix} \vec{\Phi}^T(t,\vec{x}_\perp) & 0 \\ 0 & \vec{\Phi}^T(t,\vec{x}_\perp) \end{pmatrix}$$

$$\times T\exp\left[\int \begin{pmatrix} \Omega(t') & I \\ -\omega^2(t') & \Omega(t') \end{pmatrix} dt'\right] \begin{pmatrix} \vec{\psi}(t_0) \\ d\vec{\psi}(t_0)/dt_0 \end{pmatrix}$$

- The instantaneous Landau energies

$$\omega_n^2(t) = qB(t)(2n+1) + m^2 + k_z^2$$

- The continuous transitions among Landau levels

$$\langle m,t|\Omega(t)|n,t\rangle = \frac{\dot{B}(t)}{4B(t)}\left(\sqrt{n(n-1)}\,\delta_{m,n-2} - \sqrt{(n+1)(n+2)}\,\delta_{m,n+2}\right)$$

2^{nd} Quantized QED Formulation

- Field action Hamiltonian [SPK, Ann Phys. ('14)],

$$H_\perp = \int d^2 x_\perp \left[\Pi_\perp^2 + \Phi_\perp \left(\vec{p}_\perp^{\,2} + \omega_L^2(t) \vec{x}_\perp^{\,2} + m^2 - 2\omega_L(t) L_z \right) \Phi_\perp \right]$$

- QED in the 2^{nd} quantized formulation
  - becomes the relativistic theory of time-dependent, coupled, oscillators due to the angular momentum in a time-dependent magnetic field with/without an electric field.
  - becomes the relativistic theory of time-dependent, decoupled oscillators in a pure electric field.
- QED in general EM fields is an interesting problem as analog of quantum cosmology or Hawking radiation.

Quantum Cosmology vs Scalar QED

Quantum Universe	Scalar QED
Universes	Charged scalars
WDW Equation	KG Equation
Superspace of spacetime & matter	Electromagnetic fields
Massive scalar in early universe ($V_g \approx 0$)	Scalar in homogeneous magnetic field
Coupling of field harmonic wave functions	Coupling of Landau levels
Wave functions of universe	Quantum motion of charge

Summary

- Intense lasers may probe the fundamental property of vacuum:
  - Schwinger mechanism for pair production
  - Vacuum polarization for birefringence etc
- Challenge is that most of untra-strong EM fields are localized in space and/or time.
- Intense lasers simulation of quantum universe.