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Inferring the properties of astrophysical black holes from gravitational wave observations

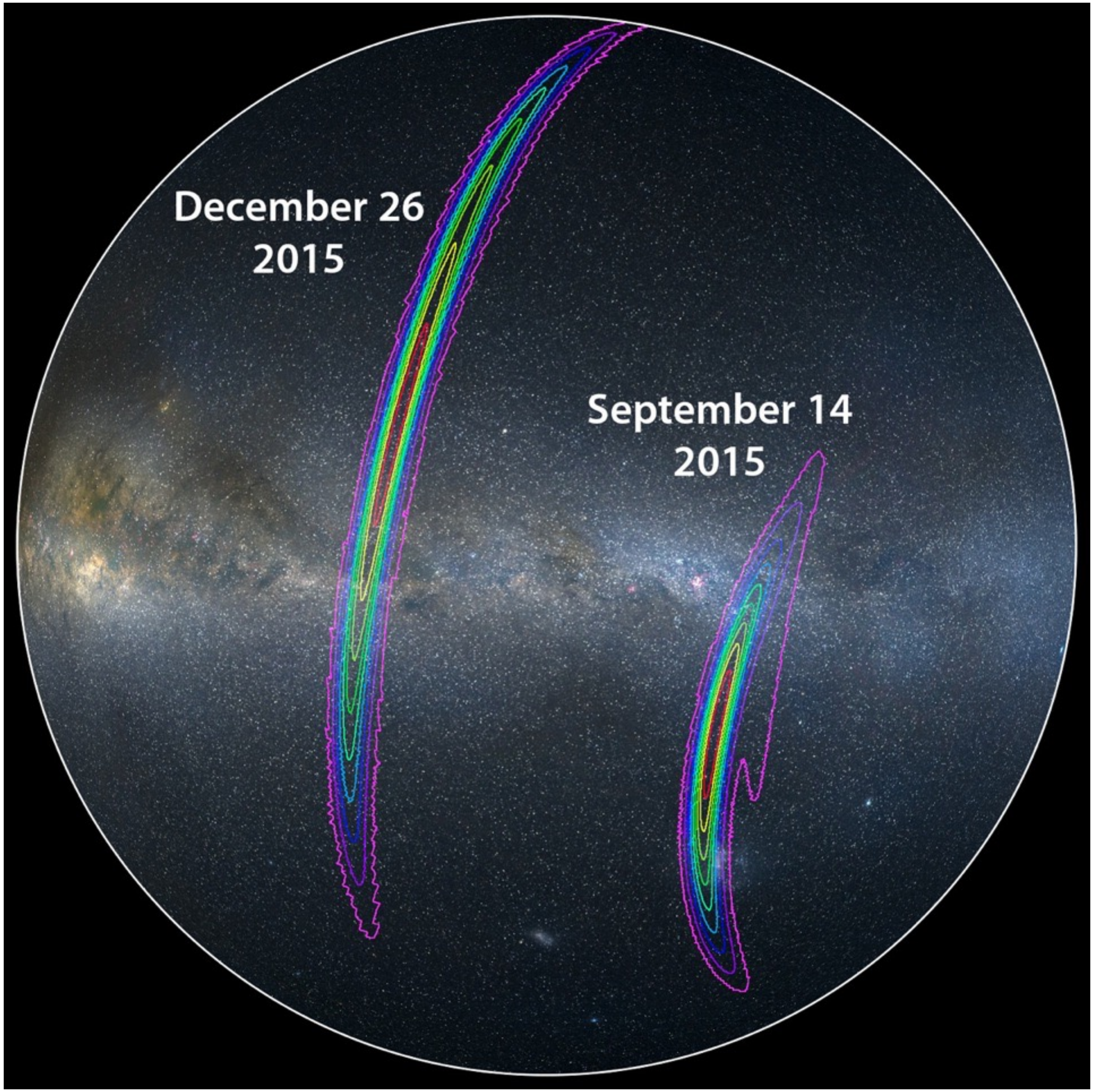
Walter Del Pozzo

Abbott et al., “Observation of Gravitational Waves from a Binary Black Hole Merger”, Phys.Rev.Lett. 116, 061102, 2016
Abbott et al.,”Properties of the Binary Black Hole Merger GW150914”, Phys. Rev. Lett. 116, 241102, 2016
Abbott et al., “Tests of General Relativity with GW150914”, Phys. Rev. Lett. 116, 221101, 2016
Abbott et al., “Binary Black Hole Mergers in the first Advanced LIGO Observing Run”, arXiv:1606.04856

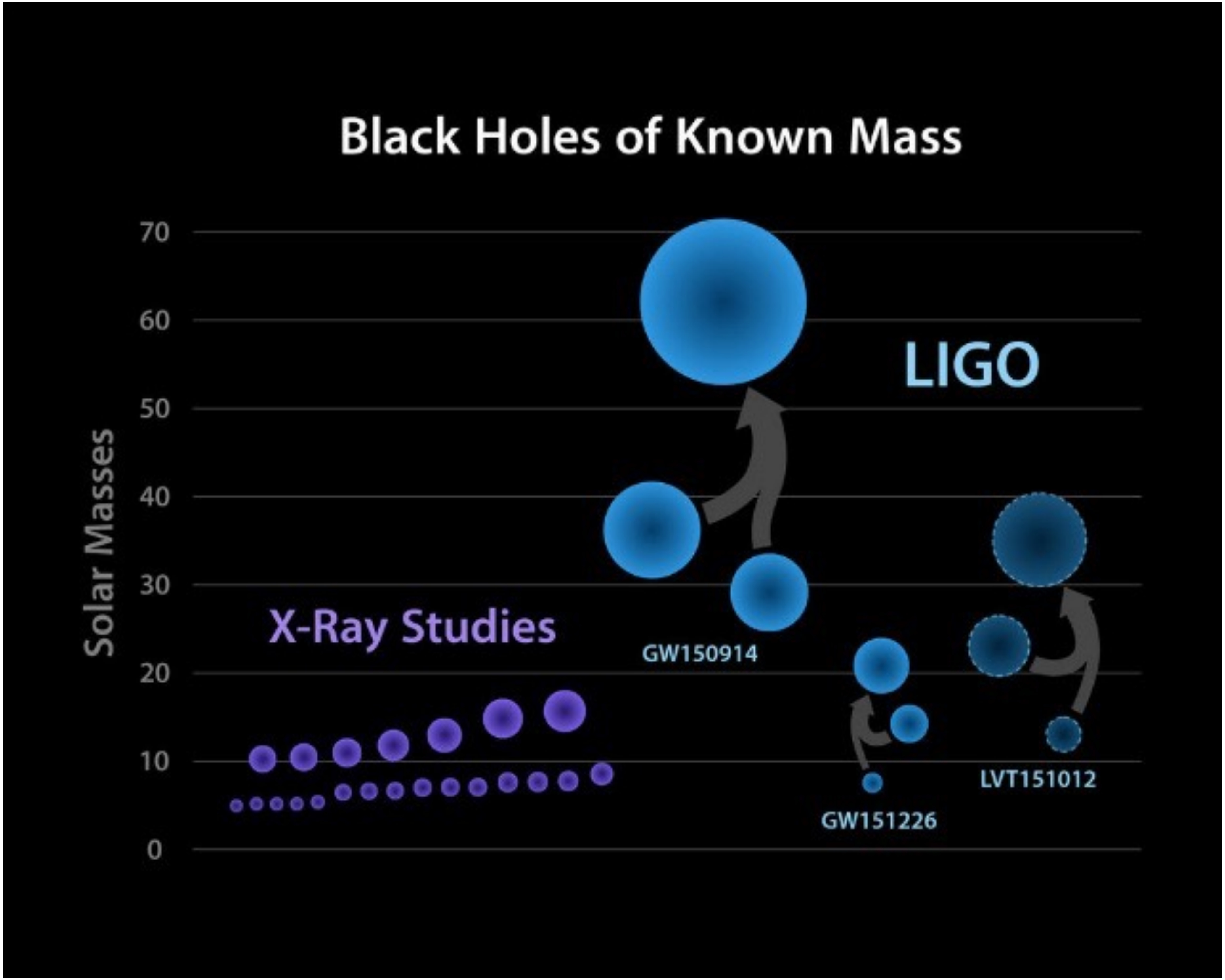
Universita’ di Pisa, 7th July 2016, Pisa

The gravitational wave sky

The gravitational wave sky



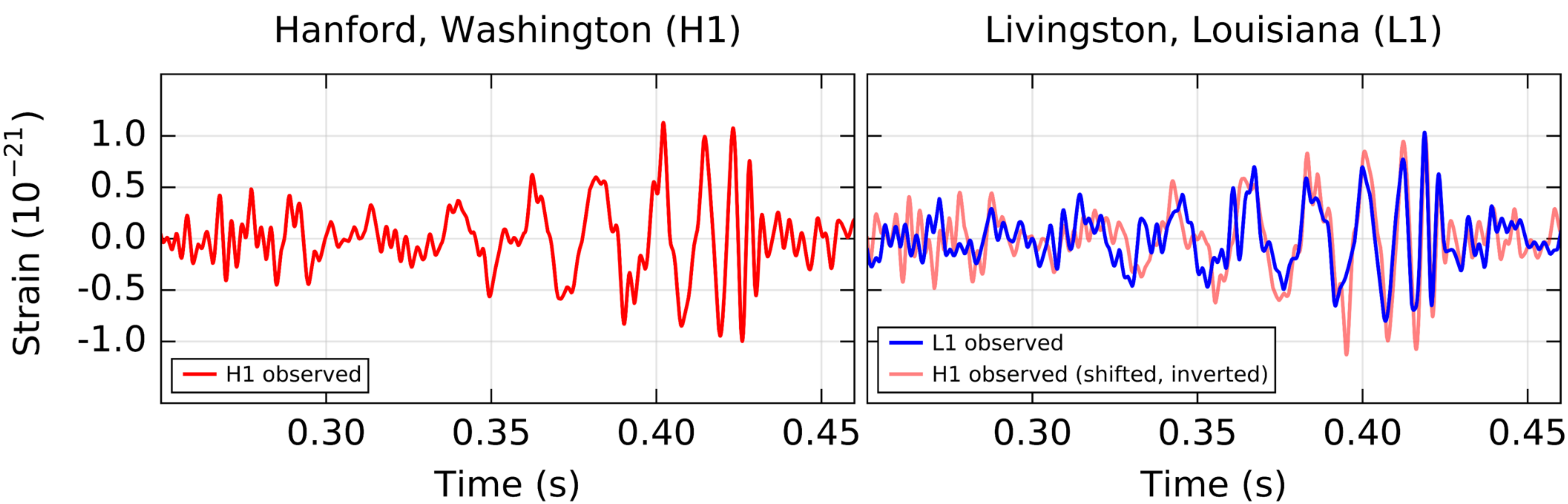
Known black holes



Gravitational wave signal

- Extract physical information from a signal embedded in noise

GW150914

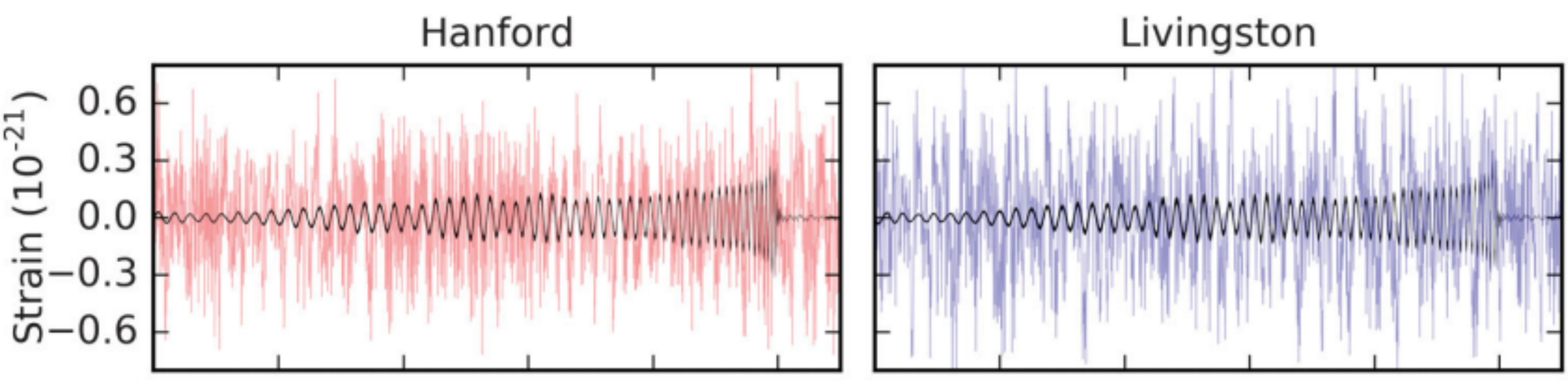


Abbott et al, Phys.Rev.Lett. 116, 061102, 2016

Gravitational wave signal

- Signals do not usually stand above the noise background

GW151226



Abbott et al, arXiv:1606.04856

Challenges

- Detection of weak signals
- Extraction of physical information:
  - fundamental physics
  - cosmology
  - astrophysics

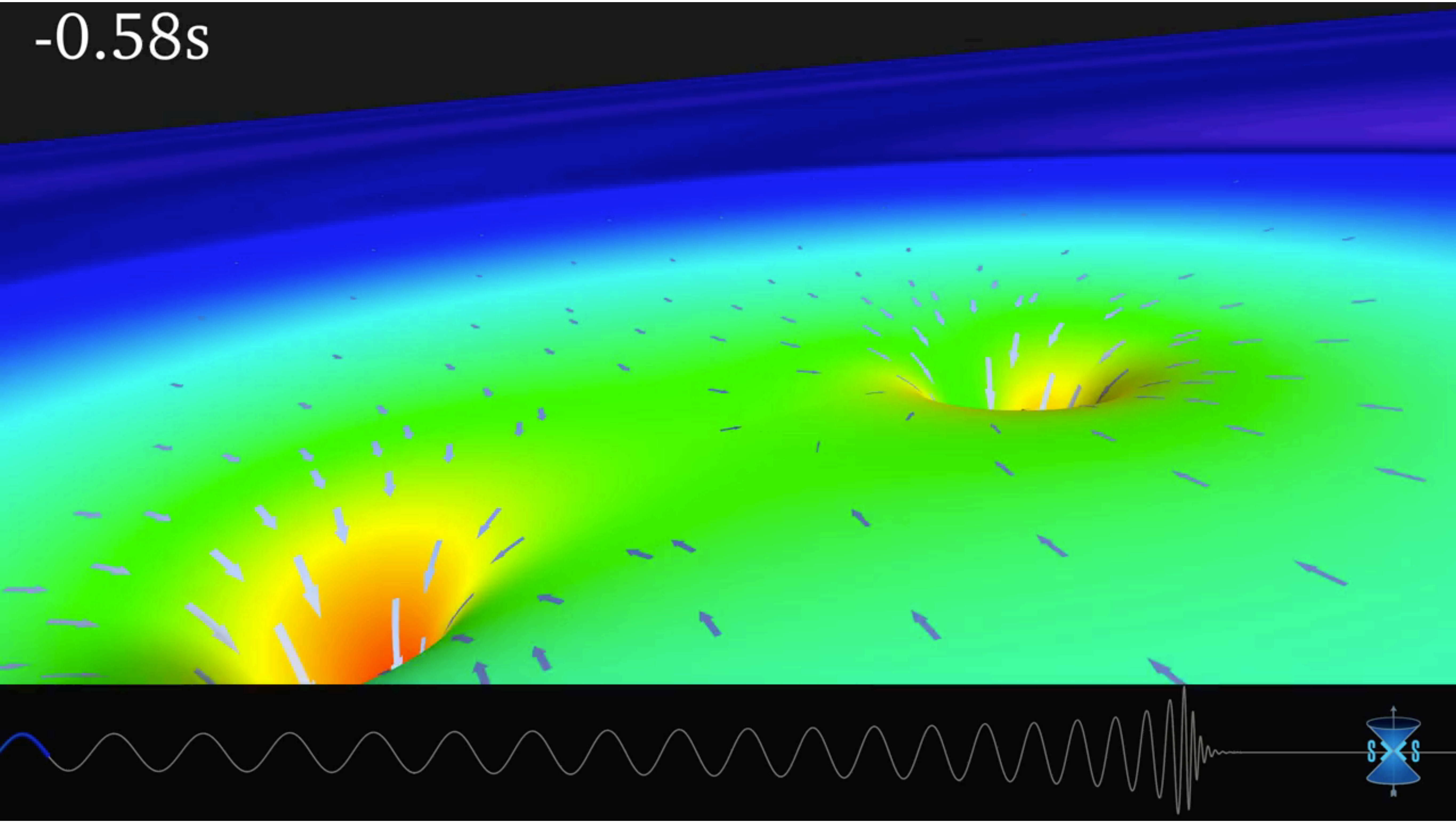
Outline

- The physics in gravitational wave
- Inferring the physical parameters of a source
- The physical properties of GW150914 & GW151226
- Tests of general relativity
- Outlook

The physics in gravitational waves

(from compact binaries coalescences)

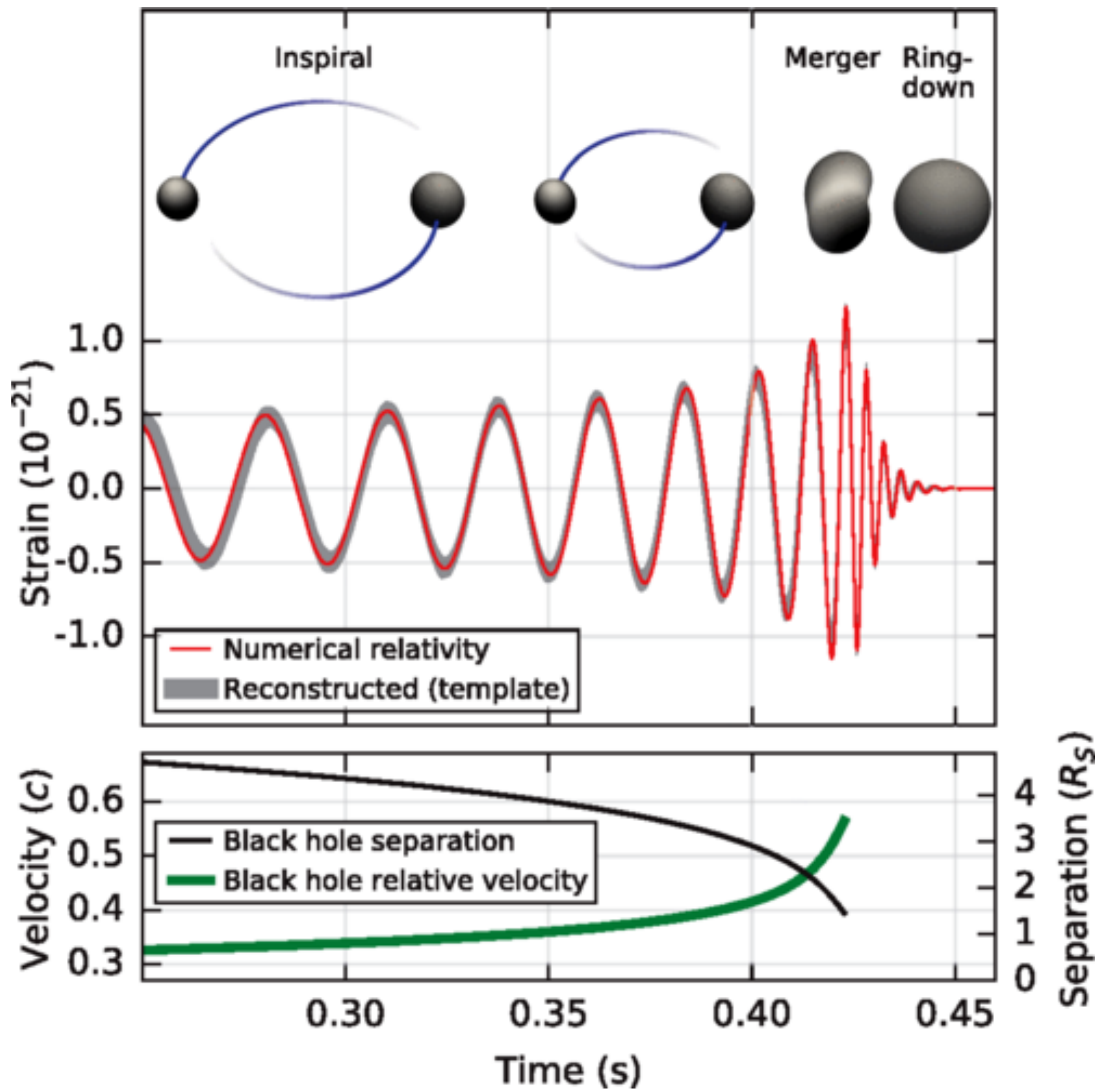
The gravitational wave signal



SXS collaboration

Waveform models

- The physics of the merger process is encoded in the GW signal
- GW signal well understood
  - inspiral —> "analytical"
  - merger —> numerical
  - ringdown —> "analytical"



Waveform models

- Models are parametrised in terms of post-Newtonian and phenomenological coefficients (calibrated on numerical relativity (NR)) solutions

$$h(f) = A(f)e^{i\Phi(f)}$$

$$\Phi(f) = \sum_{k=1}^{7}(\varphi_k + \varphi_k^l \log(f))f^{(5-k)/3} + \sum_{i\neq k}\varphi_i f^i$$

$$\varphi_j \equiv \varphi_j(m_1, m_2, \vec{s}_1, \vec{s}_2) \; \forall j = k, i$$

- All coefficients, in GR, depend on masses and spins
  - Known value in GR

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GW physical parameters

- The GW signal depends on
  - 9 parameters for non-spinning binaries
    - Masses, orientation, sky location, reference time and phase, luminosity distance
  - 15 parameters for the general case
    - Spin vectors
  - More parameters for extra physics (e.g. tests of GR, tidal effects, etc…)
- Inference is done in the Bayesian framework using stochastic samplers

Fundaments of data analysis

- Given a model H, depending on some parameters θ and some data d:

$$p(\theta|d,H,I) = p(\theta|H,I)\frac{p(d|\theta,H,I)}{p(d|H,I)}$$

likelihood

evidence

posterior

prior

- The evidence is

$$p(d|H,I) = \int d\theta\, p(\theta|H,I)p(d|\theta,H,I)$$

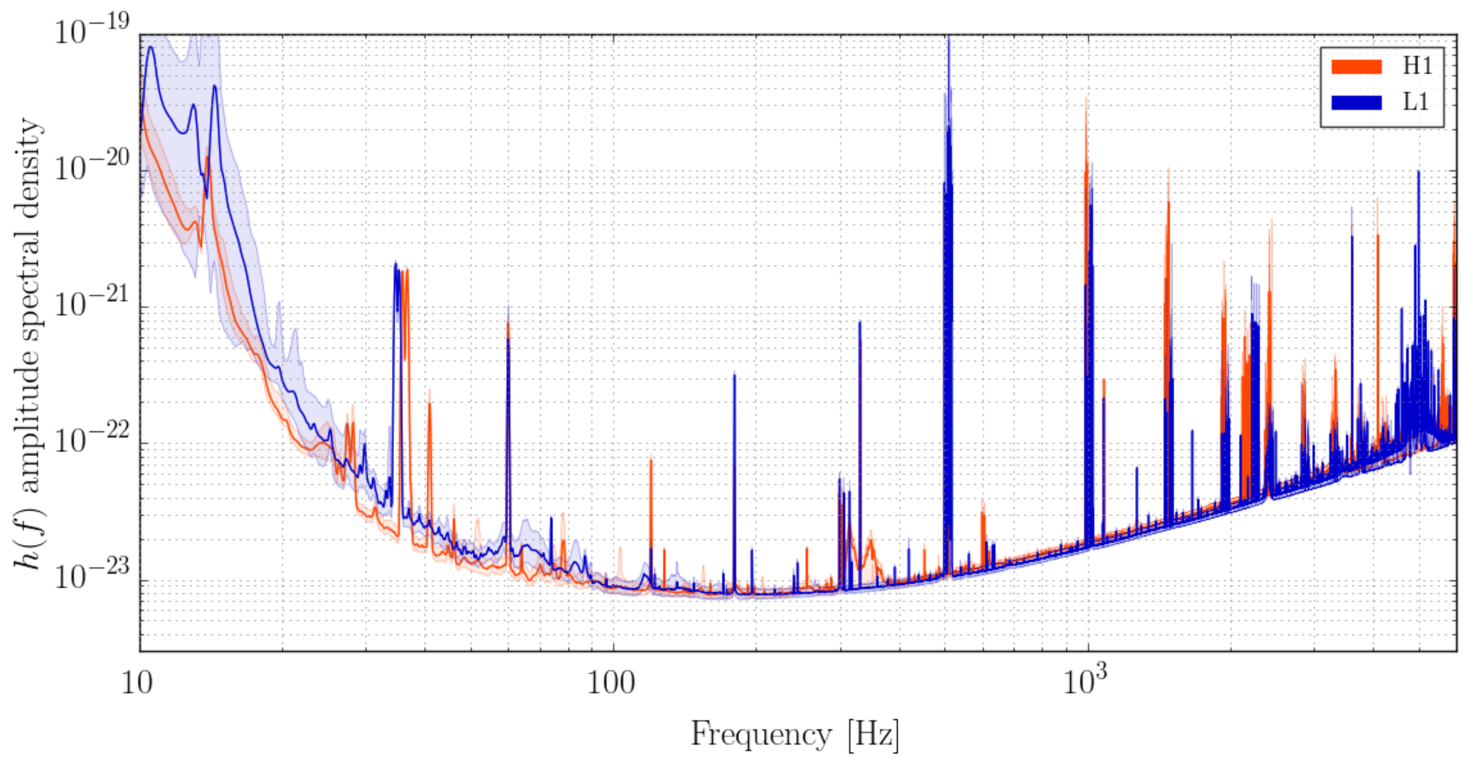
Gravitational waves inference model

- The detector output is linear

$$d(t) = h(t;\theta) + n(t)$$

- where $h(t;\theta)$ is the gravitational wave strain and $n(t)$ is the noise time series

- Gaussian and stationary noise $< n(f)n(f') >= \frac{1}{2}S(f)\delta(f - f')$



Extracting the parameters of a source

- The probability of a given noise realisation is

$$p(n|I) \propto e^{-\frac{(n|n)}{2}}$$

$$(a|b) = 4\mathrm{Re}\left[\int df\, \frac{a^*(f)b(f) + a(f)b^*(f)}{S(f)}\right]$$

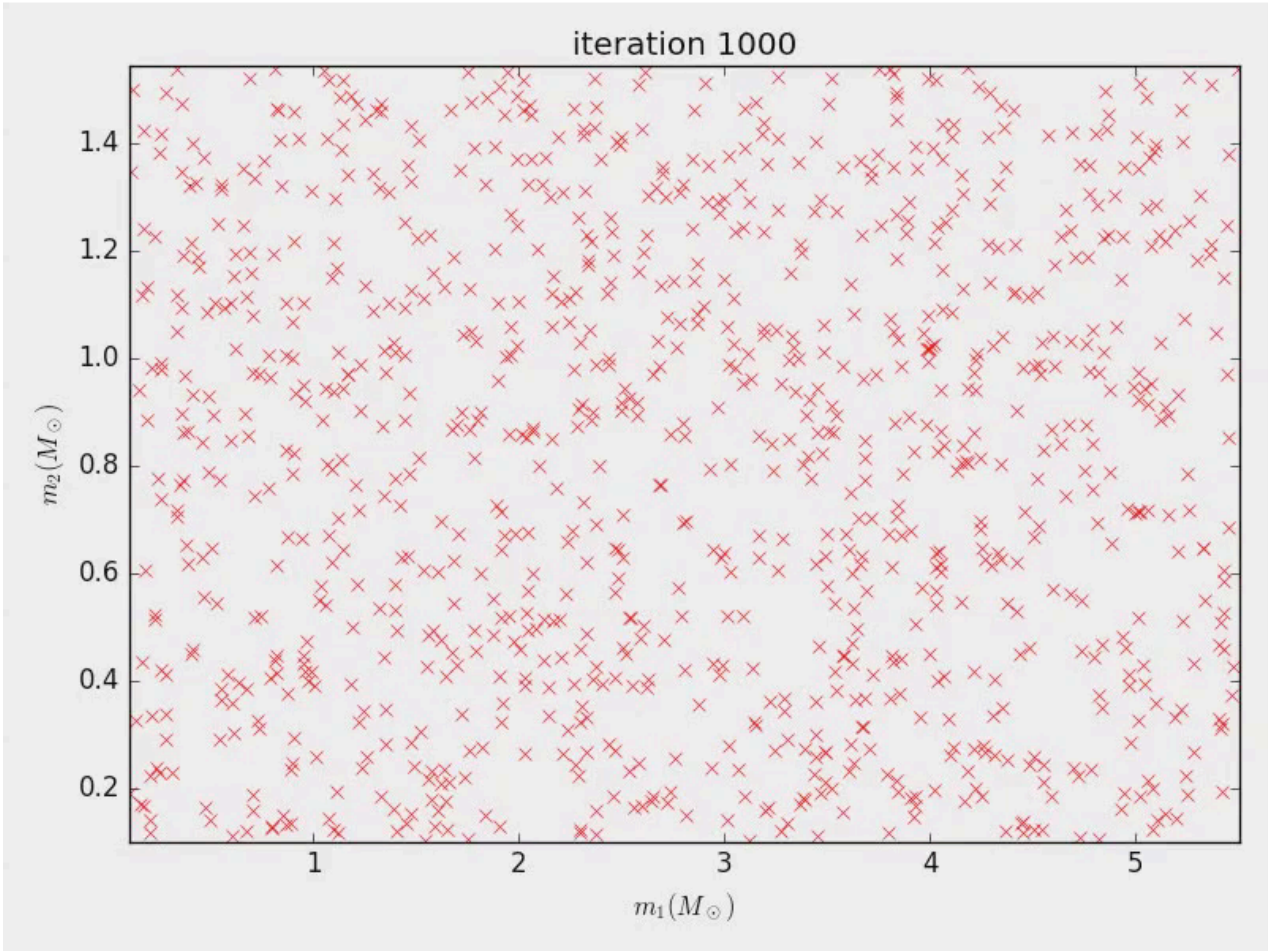
- The noise properties define the *likelihood function*

$$p(d|H, I) \propto e^{-\frac{(d-h|d-h)}{2}}$$

- The posterior is explored using stochastic samplers

Exploring the parameter space

- LALInference nested sampling (e.g. Vetch et al, 2014)

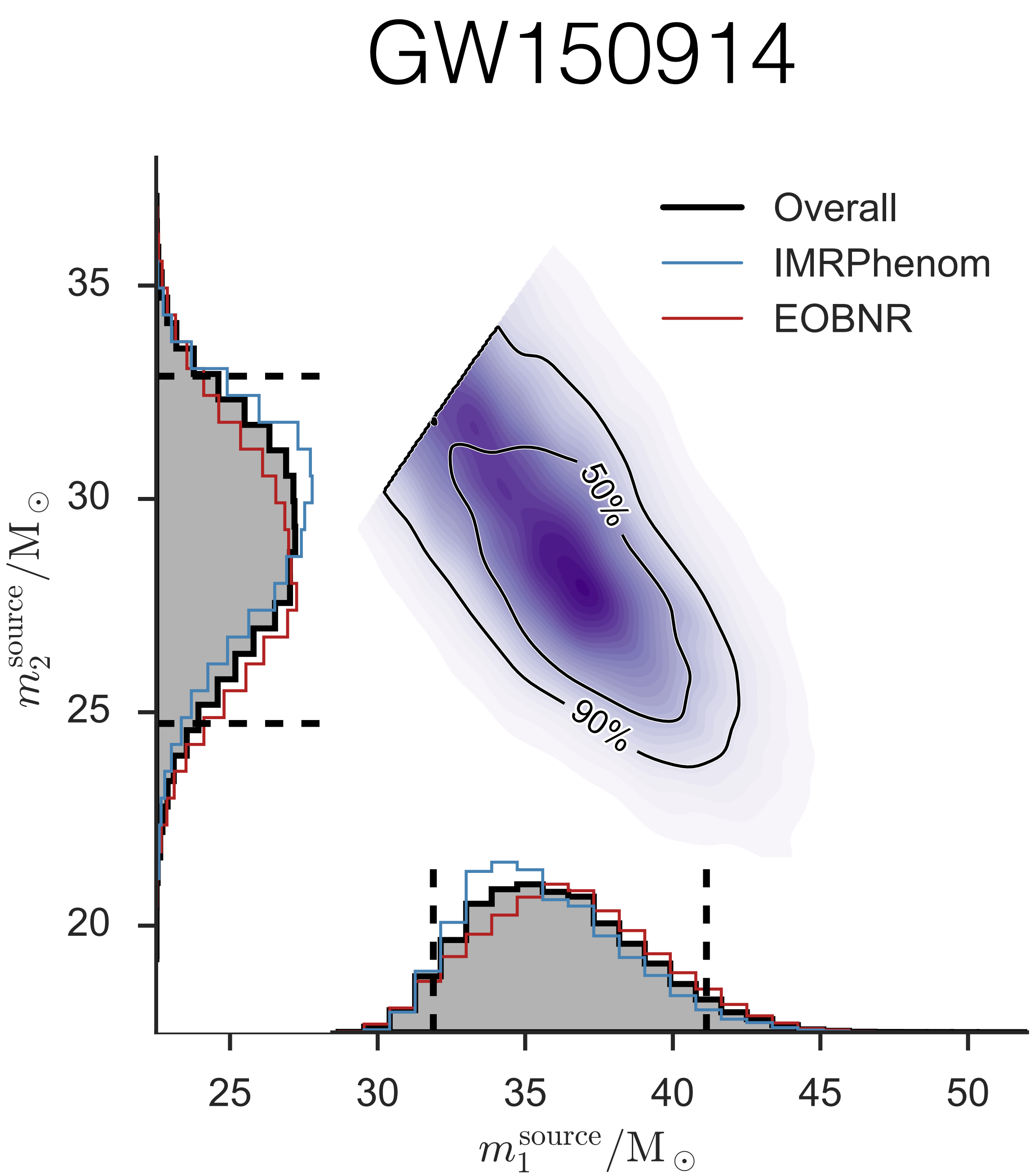


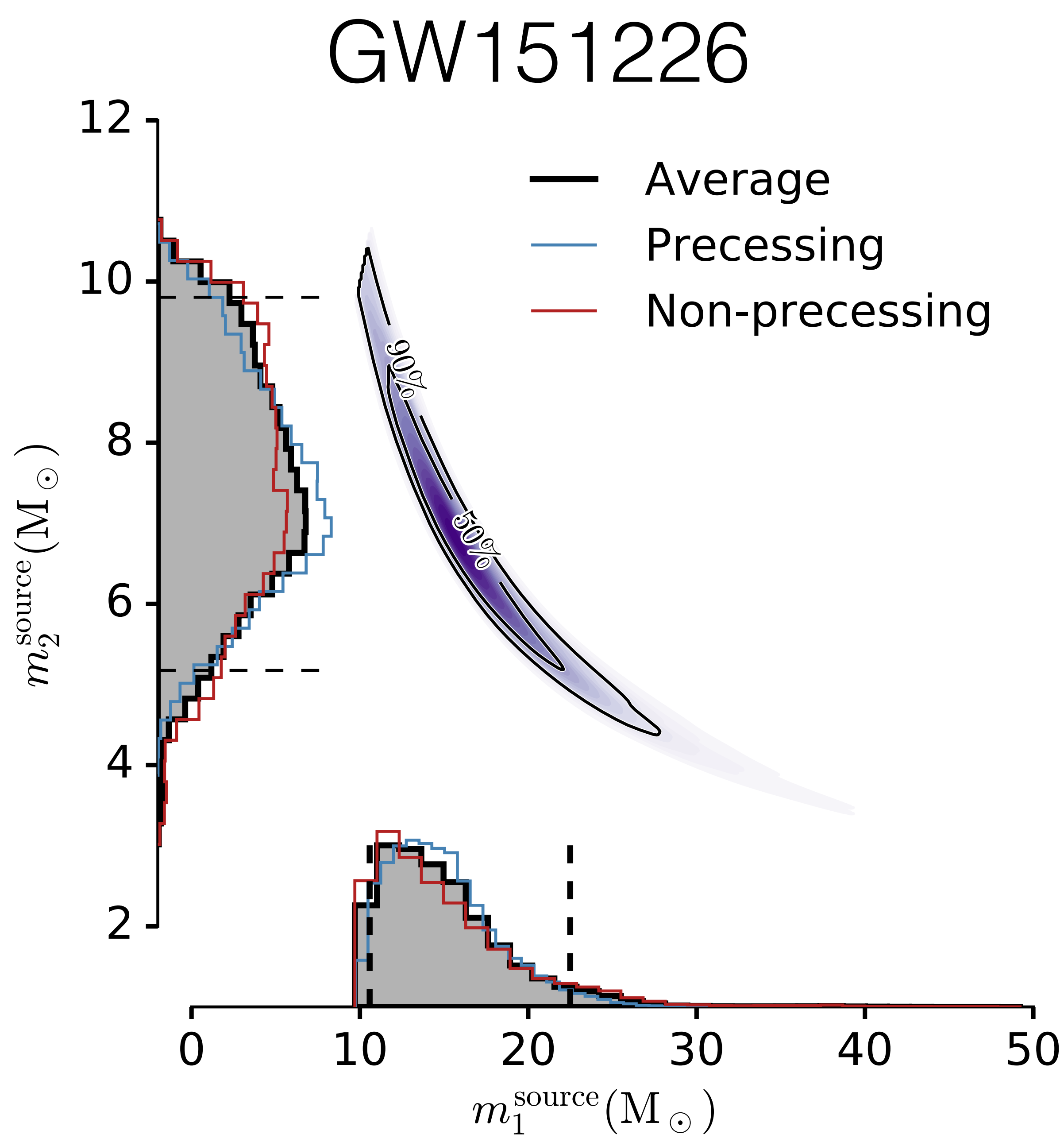
The physical properties of GW150914 & GW151226

Measured BBH parameters

Abbott et al Phys. Rev. Lett. 116, 241102, 2016
Abbott et al, arXiv:1606.04856

- Masses

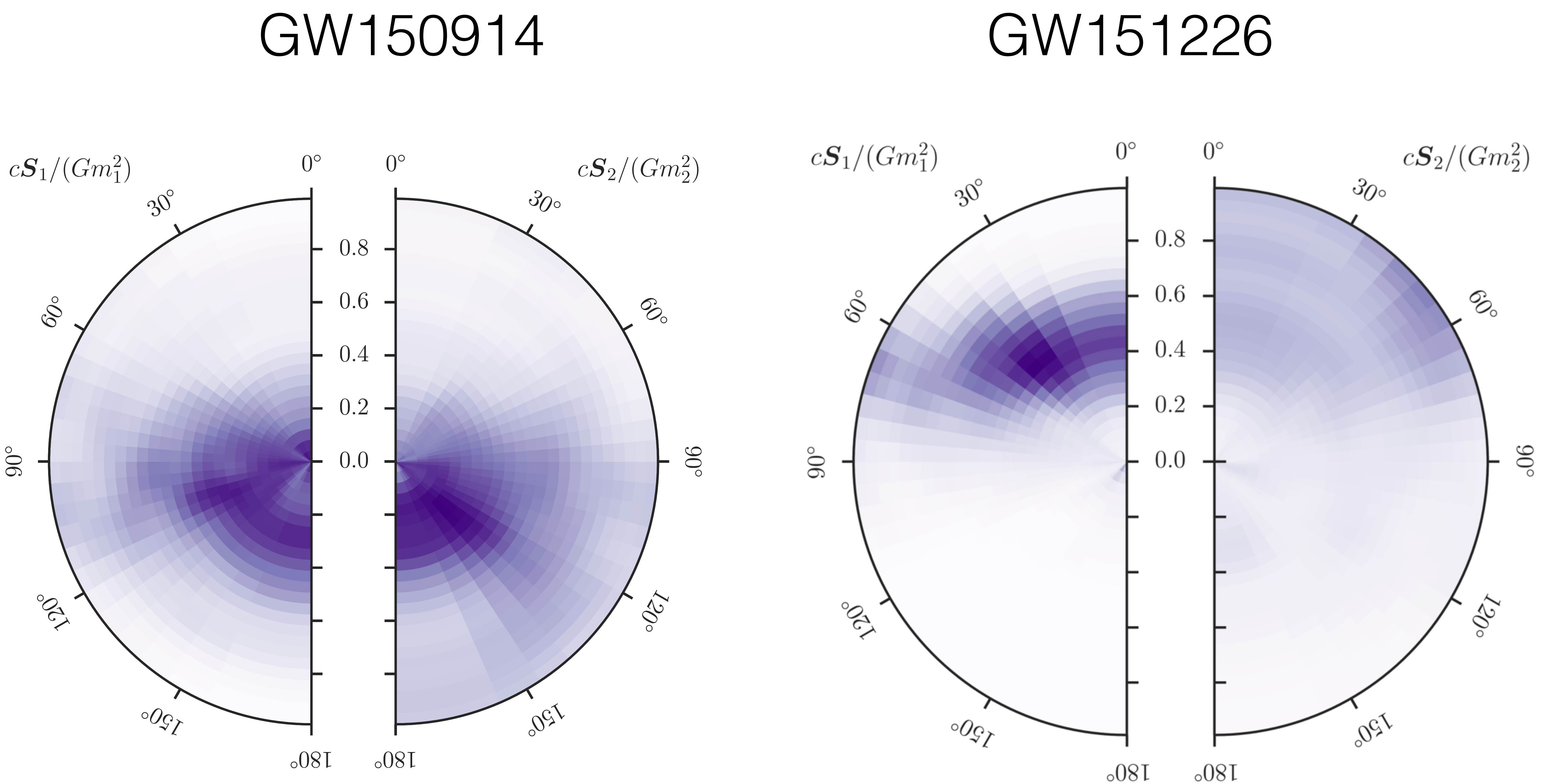




Measured BBH parameters

- Spins

Abbott et al Phys. Rev. Lett. 116, 241102, 2016
Abbott et al, arXiv:1606.04856

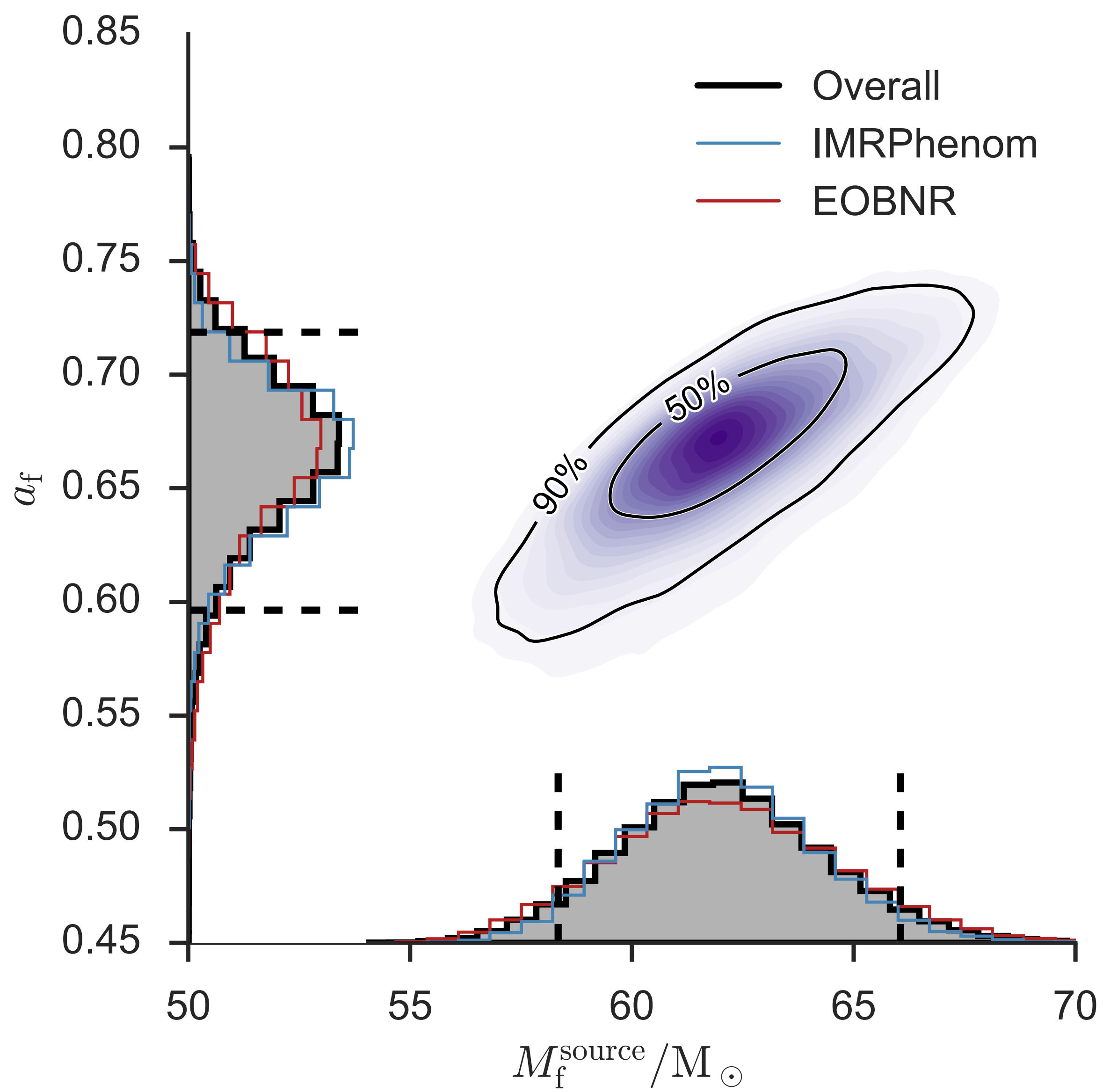


Measured BBH parameters

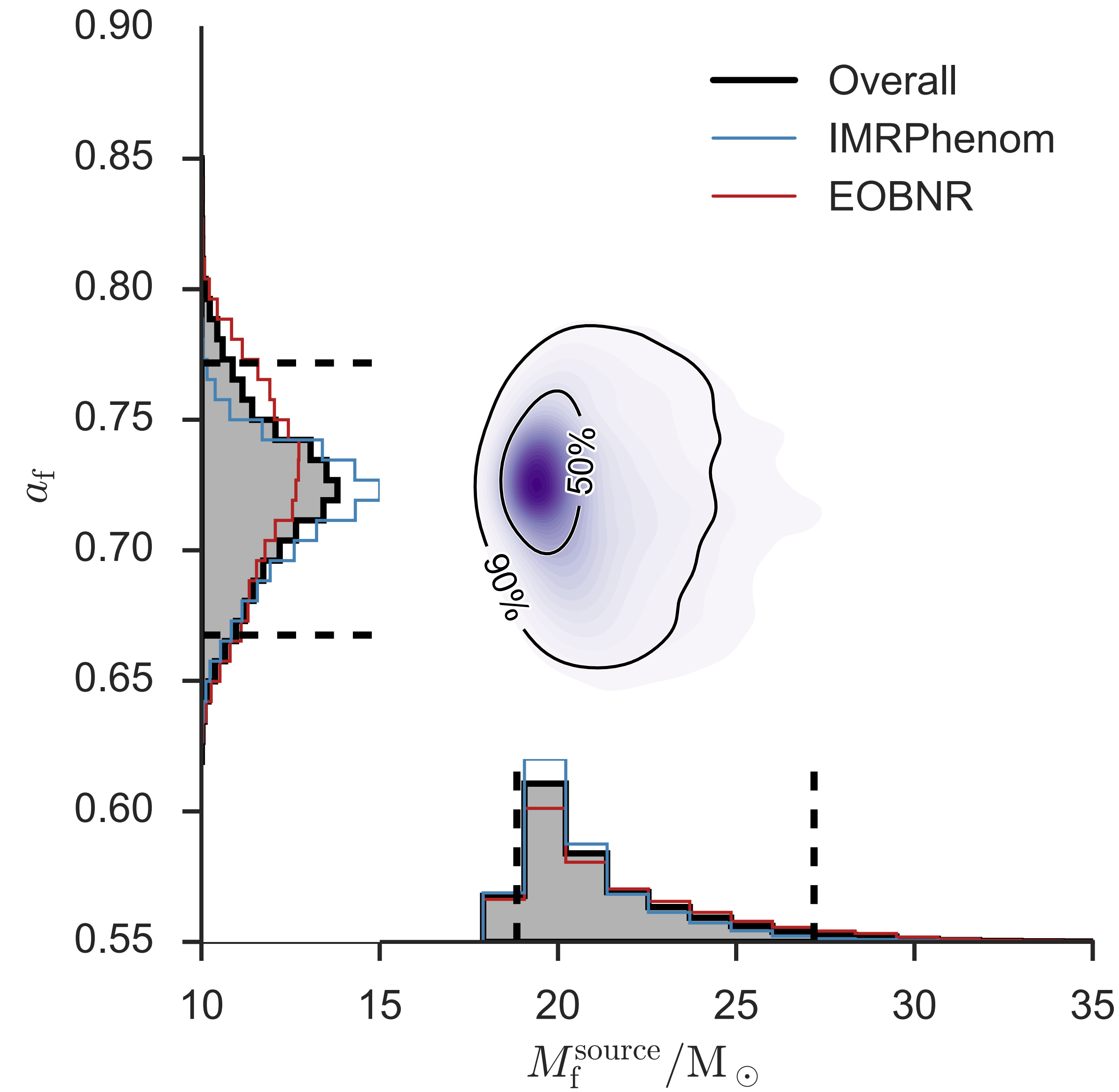
Abbott et al Phys. Rev. Lett. 116, 241102, 2016
Abbott et al, arXiv:1606.04856

- Remnant properties

GW150914



GW151226



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Tests of general relativity

or

Were GW150914 & GW151226 BBH coalescences?

non-GR effects on the waveform

- Alternative theories of gravity modify the waveform
  - change the ϕ_n coefficients by introducing additional parameters
    - e.g. "massive gravity": $\Phi_{MG}(v) = \Phi_{GR}(v) - \frac{\pi^2 DM}{\lambda_g^2(1+z)} v^{-1}$
  - add extra orders not present in the GR waveform
    - e.g. Brans-Dicke: $\Phi_{\mathrm{BD}}(v) = \Phi_{\mathrm{GR}}(v) - \frac{5S^2}{84\omega_{BD}} v^{-2}$
- Non-BHs show different merger and ringdown spectra

Strategies

- Two strategies to test GR

1. Self-consistency tests: perturb around GR and check for evidence of inconsistencies

2. Targeted tests: assume an alternative theory of gravity and constrain its parameters

Parametrised tests of GR

- Waveform models are described by post-Newtonian and phenomenological coefficients
- Allow for fractional changes with respect to the GR value

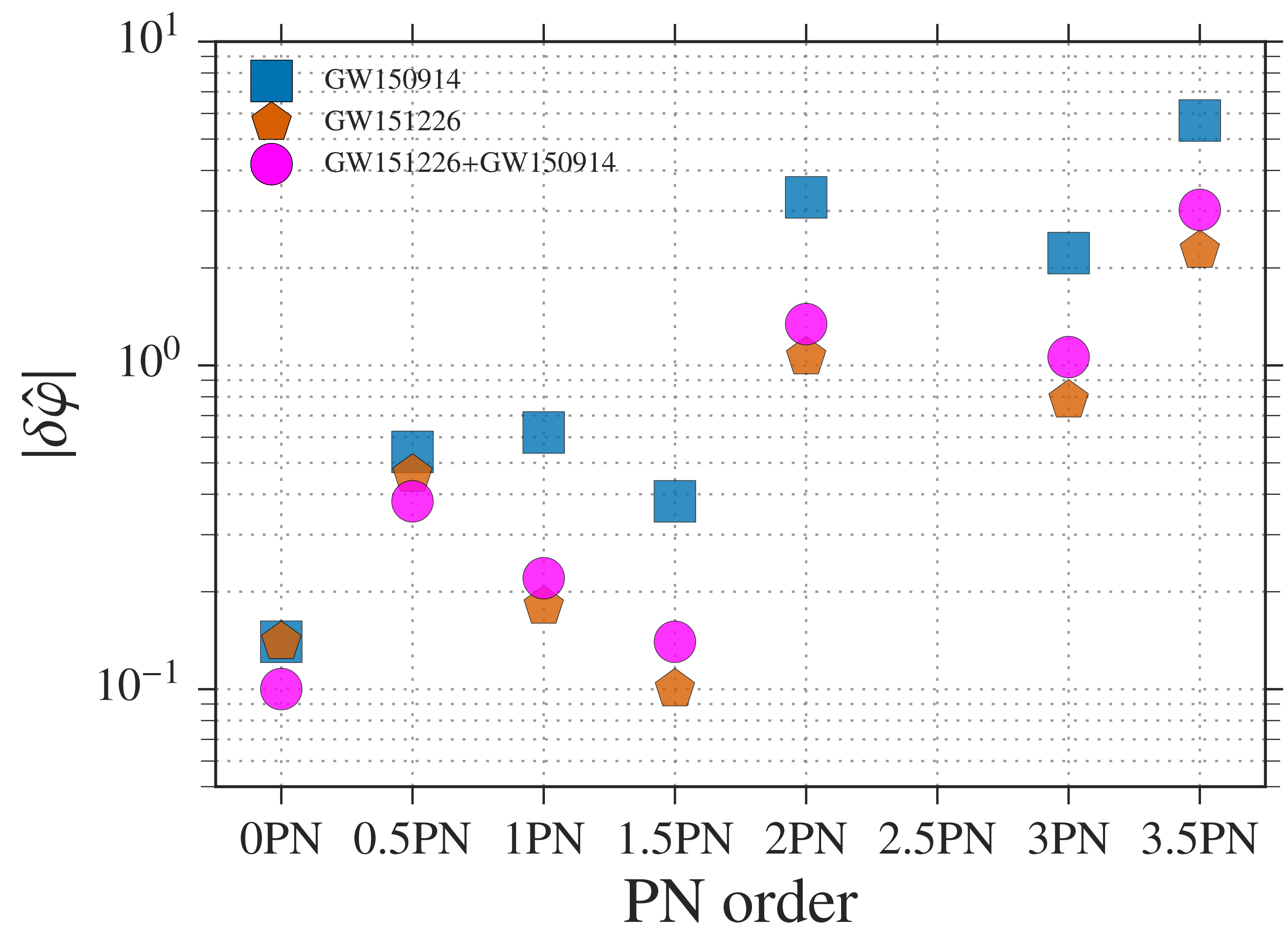
$$\hat{\varphi}_j \equiv \varphi^{\mathrm{GR}}(1 + \delta\hat{\varphi}_j)$$

- Obtain constraints on "generic" deviations from GR (Li, Del Pozzo et al 2012, Agates, Del Pozzo et al 2013)

waveform regime	parameter	f–dependence
early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$
	$\delta\hat{\varphi}_1$	$f^{-4/3}$
	$\delta\hat{\varphi}_2$	f^{-1}
	$\delta\hat{\varphi}_3$	$f^{-2/3}$
	$\delta\hat{\varphi}_4$	$f^{-1/3}$
	$\delta\hat{\varphi}_{5l}$	$\log(f)$
	$\delta\hat{\varphi}_6$	$f^{1/3}$
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$
	$\delta\hat{\varphi}_7$	$f^{2/3}$
intermediate regime	$\delta\hat{\beta}_2$	$\log f$
	$\delta\hat{\beta}_3$	f^{-3}
merger–ringdown regime	$\delta\hat{\alpha}_2$	f^{-1}
	$\delta\hat{\alpha}_3$	$f^{3/4}$
	$\delta\hat{\alpha}_4$	$\tan^{-1}(af + b)$

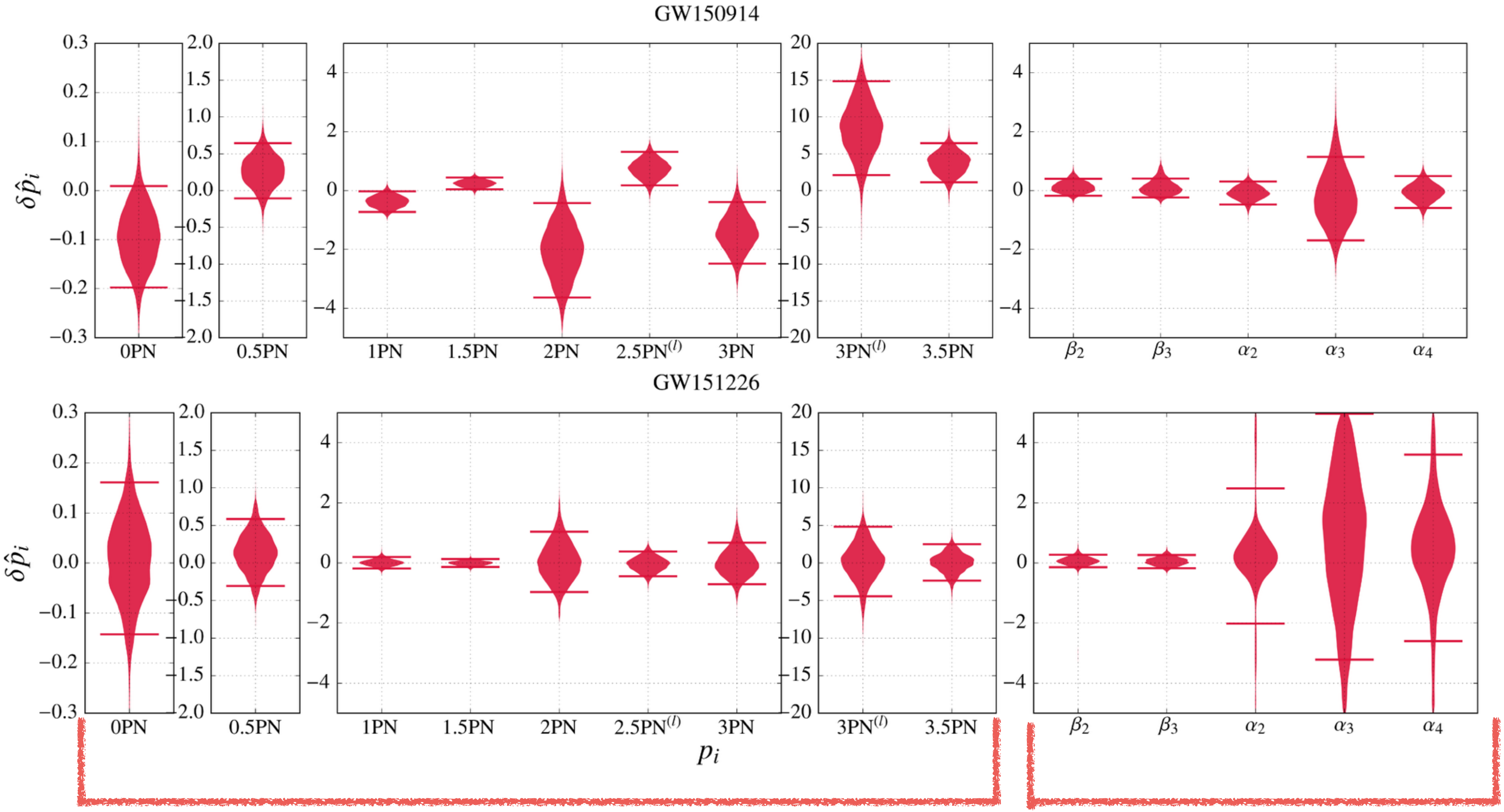
post-Newtonian series

- Dynamical constraints on post-Newtonian series
- First constraints on non-linear dynamics of space-time during the inspiral



Abbott et al, arXiv:1606.04856

Parametrised tests of GR



Inspiral

Merger-ringdown

Energy/Frequency

Abbott et al, arXiv:1606.04856

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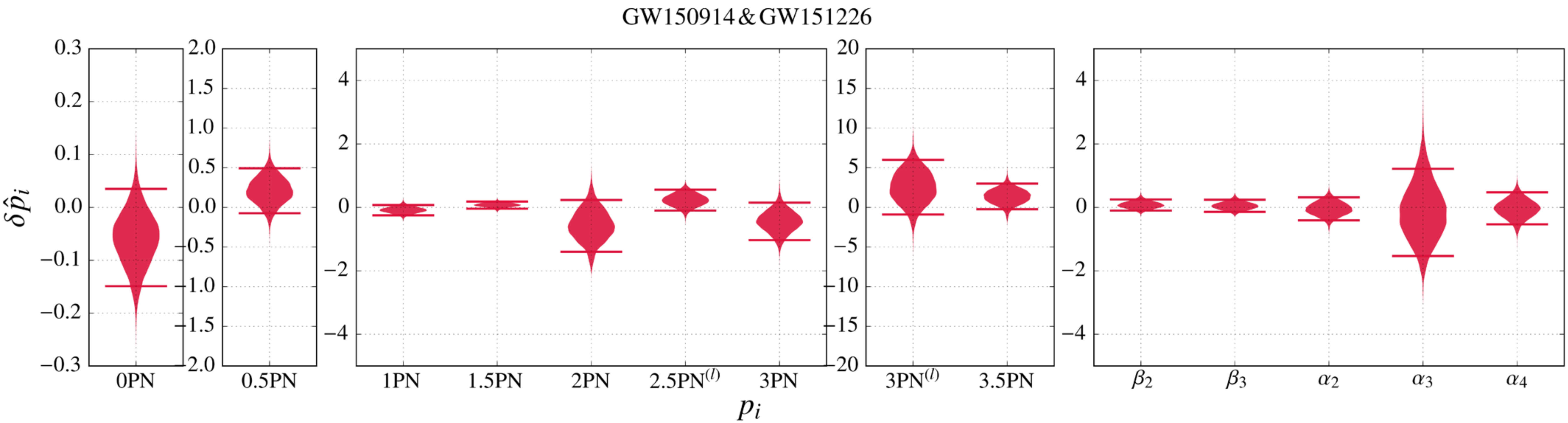
Joint constraints

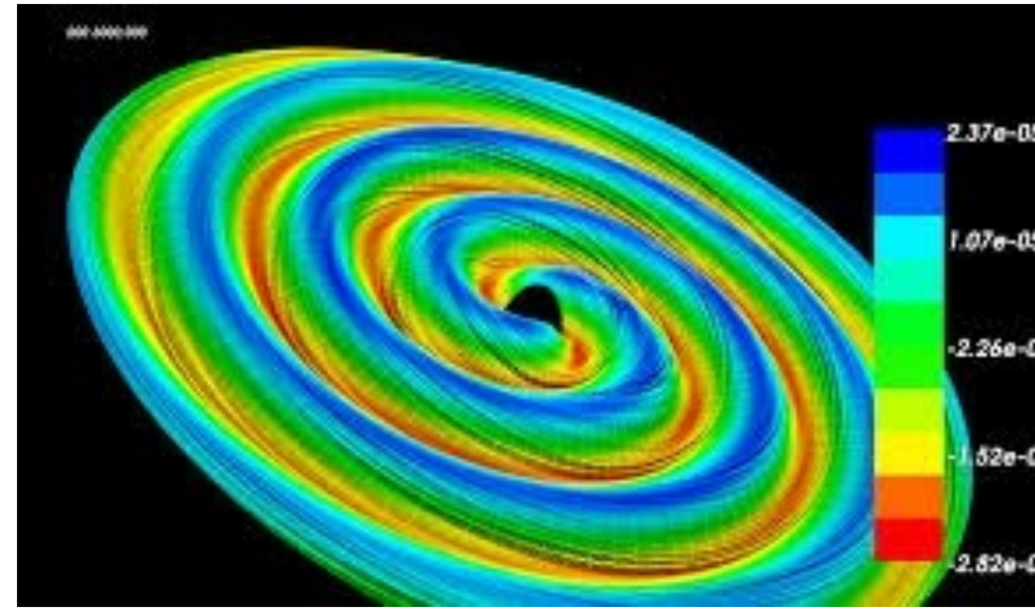


- Tightest constraints on dynamics of space-time yet
- No evidence for violations of GR

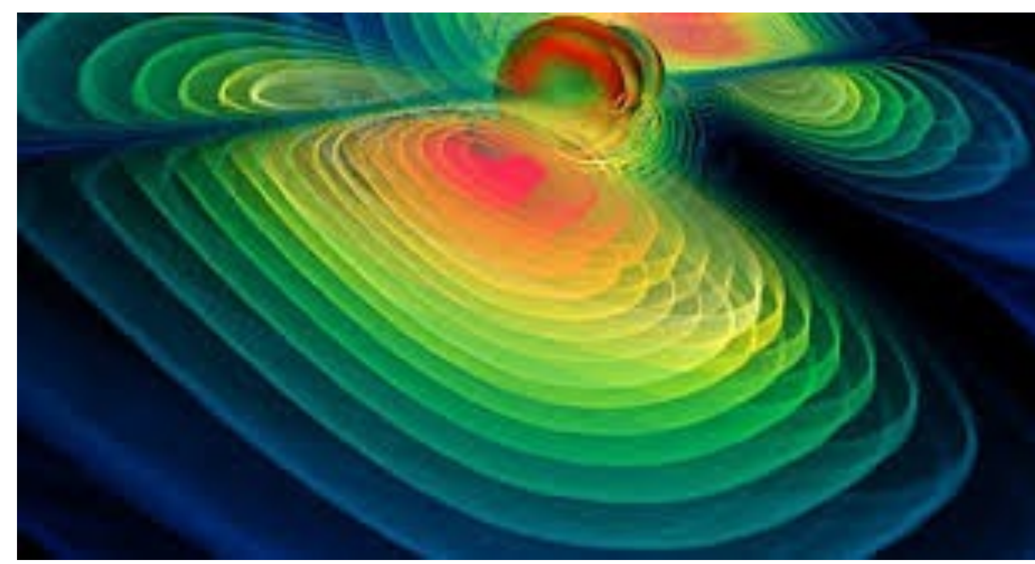
Abbott et al, arXiv:1606.04856

Waveform self-consistency

- For GW150914 a further kind of consistency test is possible since the merger happens in band

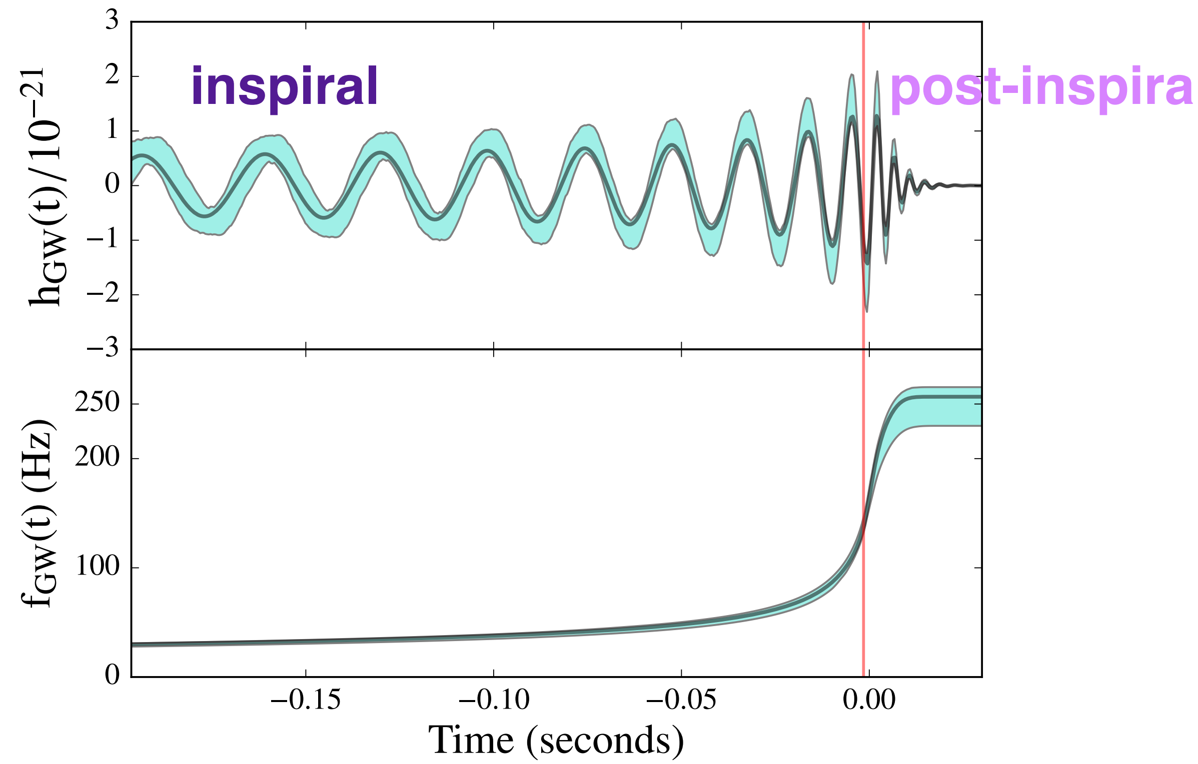


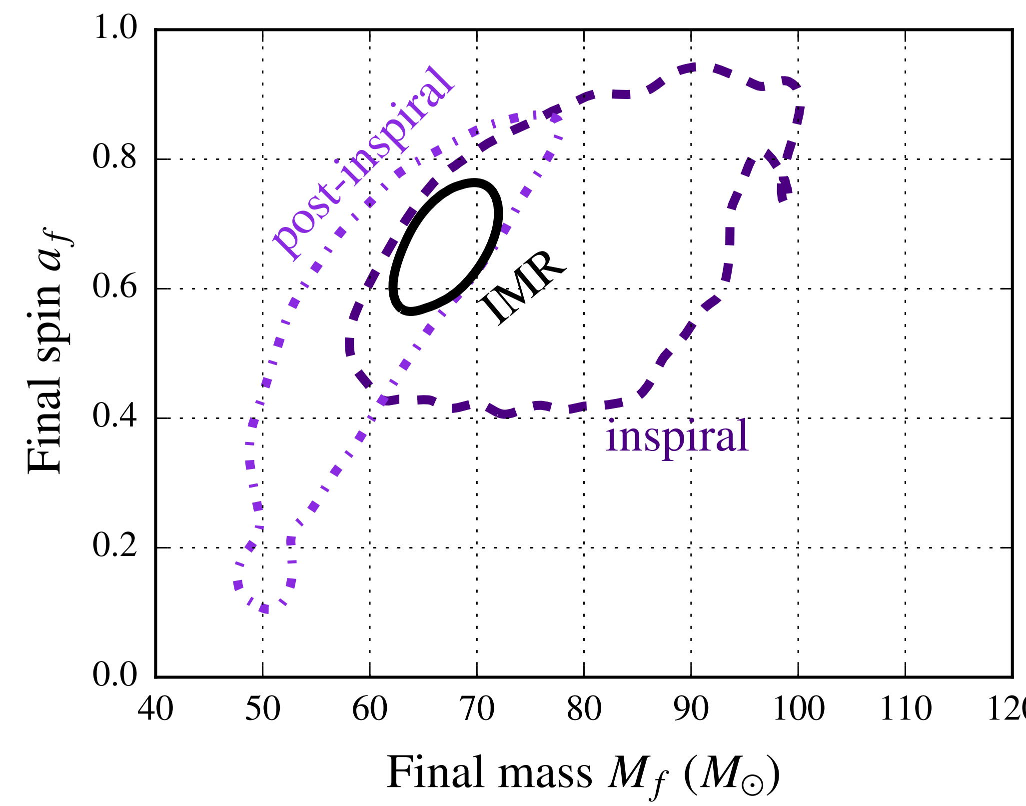
- Phenomenological formulae calibrated on numerical relativity predict spin and mass of the remnant from the inspiralling ones (e.g. Healy et al, 2014)



Waveform self-consistency

- The comparison of the final mass and spin predicted from the "inspiral" with the ones measured directly from the "merger-ringdown" is a consistency test on the waveform (e.g. Ghosh, …, Del Pozzo et al, 2016) and thus, on the corresponding GR solution





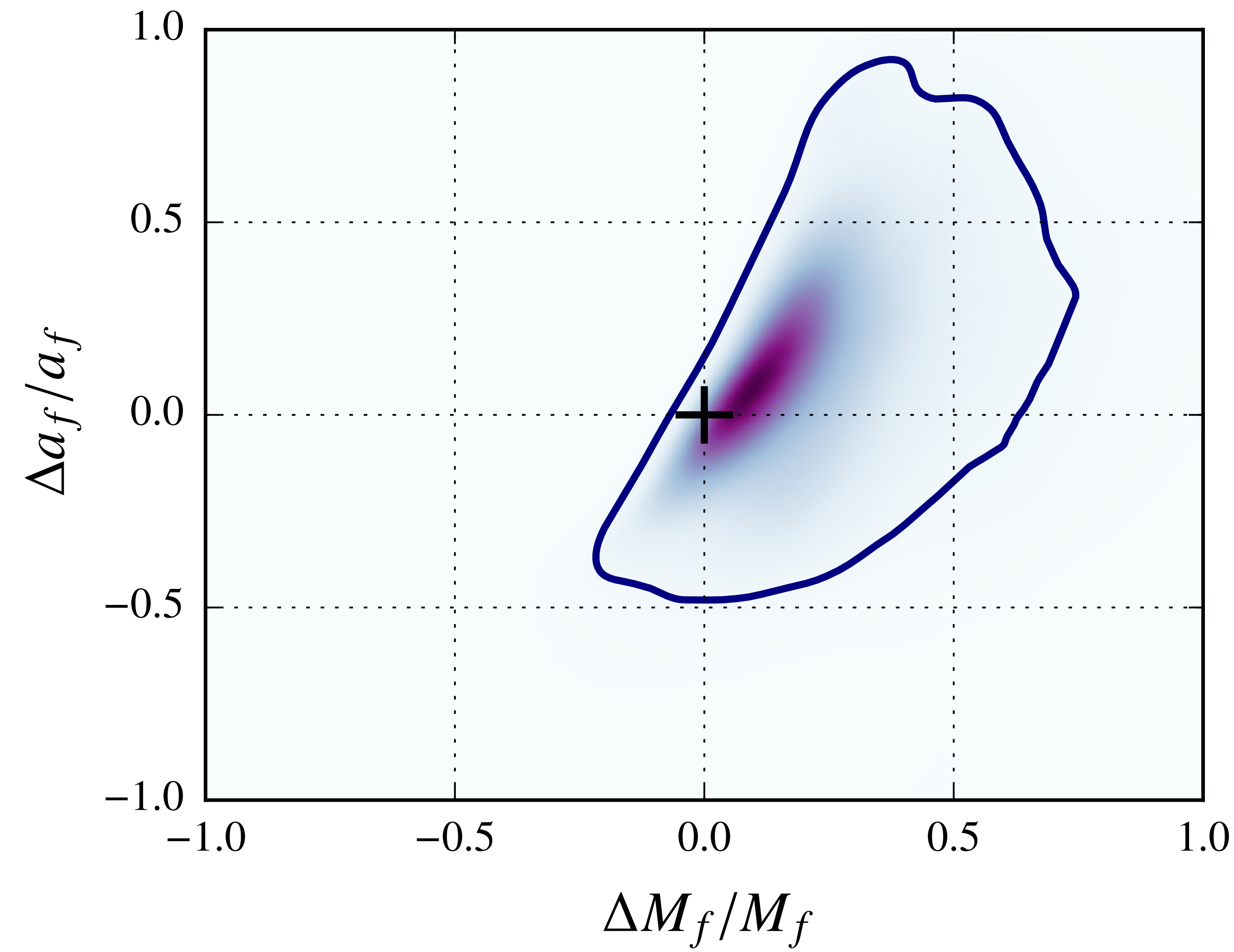
Abbott et al., Phys. Rev. Lett. 116, 221101, 2016

Waveform self-consistency

- Re-parametrising to the relative difference between the inspiral and post-inspiral estimates, one can quantify the confidence level where GR (0,0) lies

- For GW150914, GR lies on the isoprobability contour that encloses 28% of the posterior

- No evidence for deviations from GR



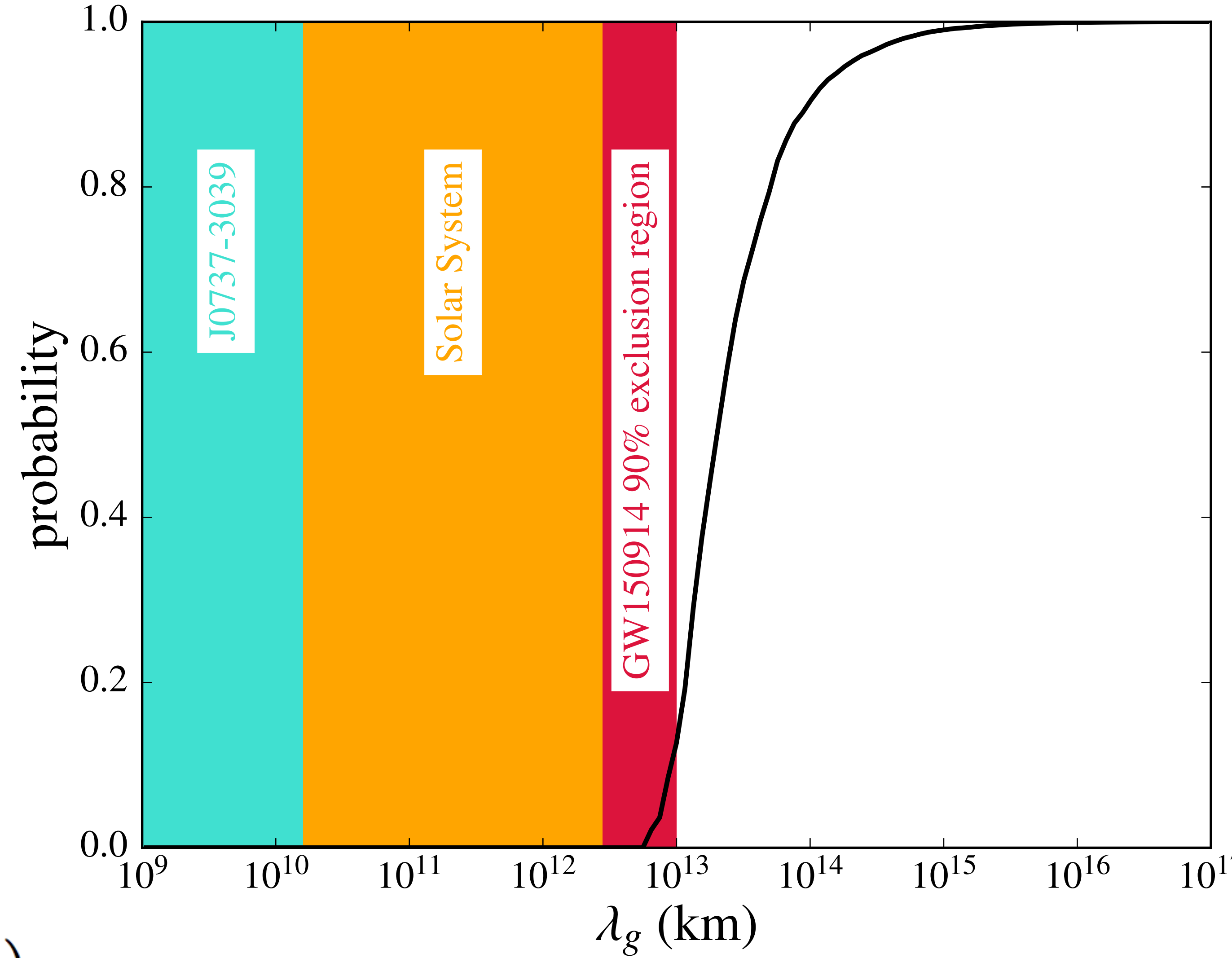
Abbott et al., Phys. Rev. Lett. 116, 221101, 2016

A massive gravity targeted search

- In hypothetical theories where the GW dispersion is modified, the 1PN phase includes the graviton Compton wavelength (e.g. Del Pozzo et al, 2011)

$$\Phi_1 = \Phi_{1,\mathrm{GR}} - \frac{\pi^2 D M}{\lambda_g^2 (1+z)}$$

$$m_g \leq 1.2 \times 10^{-22} \mathrm{eV/c^2}\ (90\%)$$

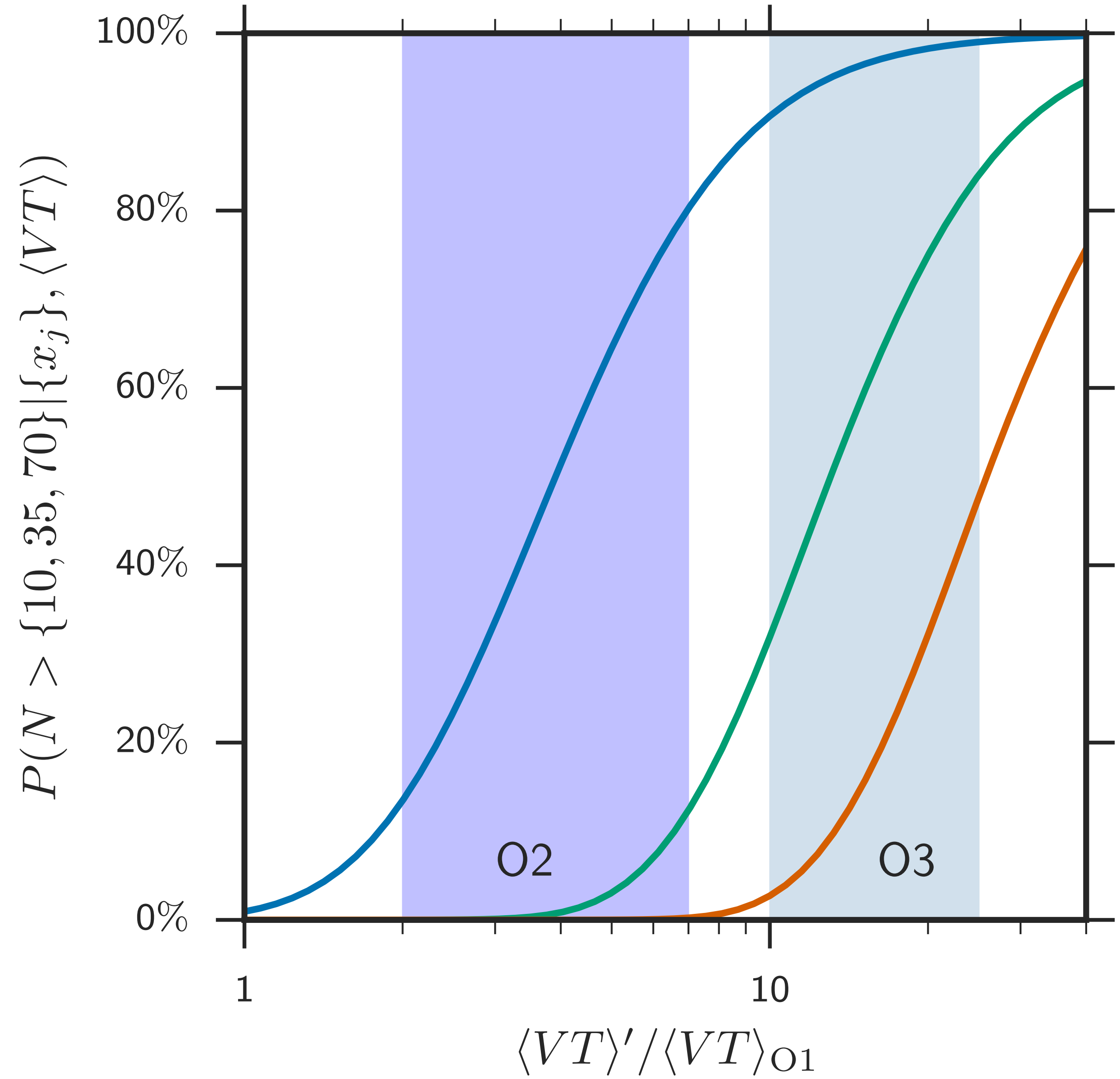


Abbott et al., Phys. Rev. Lett. 116, 221101, 2016

Outlook

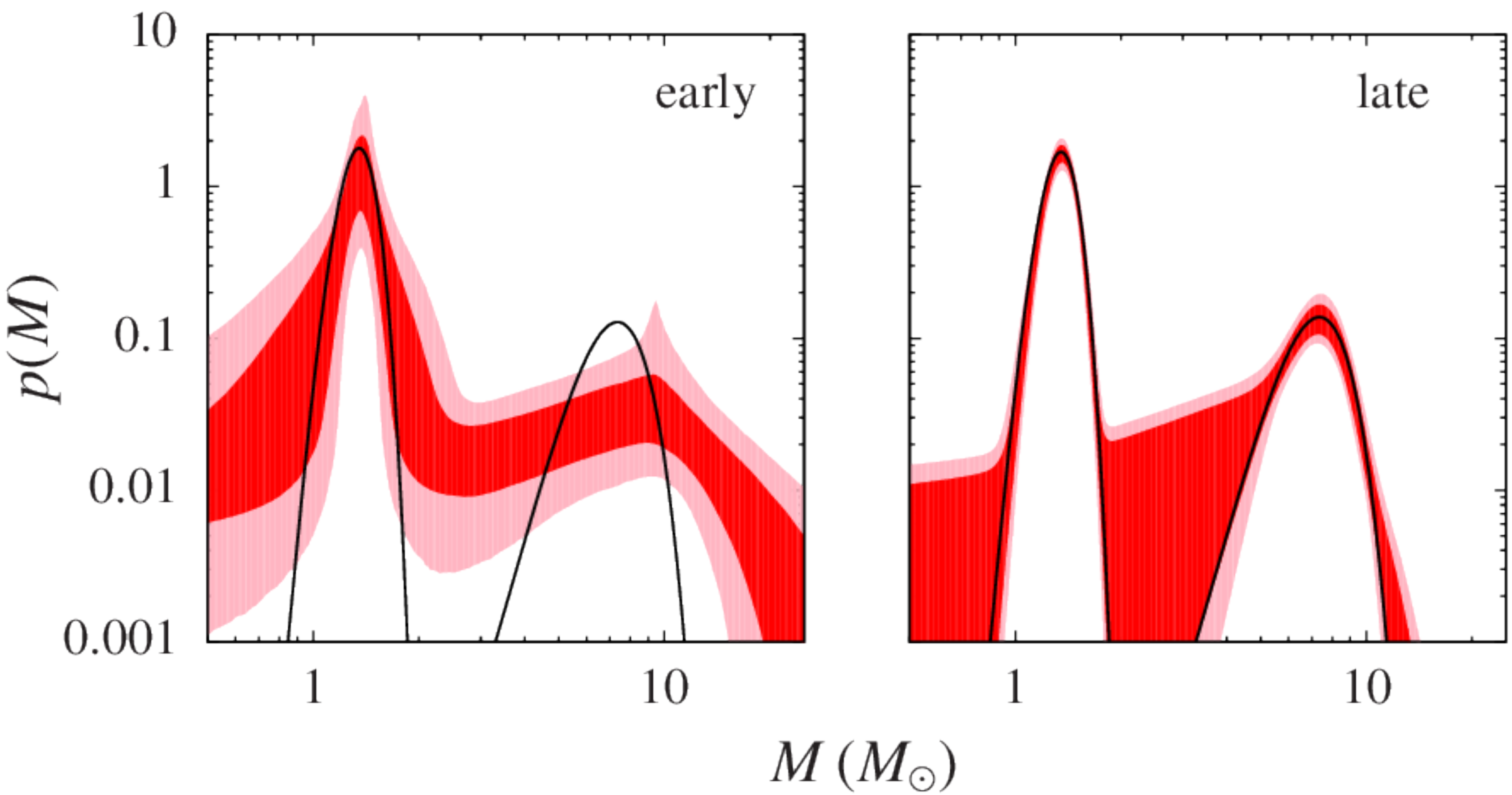
How many BBH do we expect?

- Rate of BBH mergers 9–260 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$
    - Astrophysics
    - Dynamics of space-time
    - Cosmography
- Binary neutron stars?



Abbott et al, arXiv:1606.04856

Astrophysics: the mass distribution



- Observing *O*(100) CBC events will allow the determination of the mass distribution of compact binaries

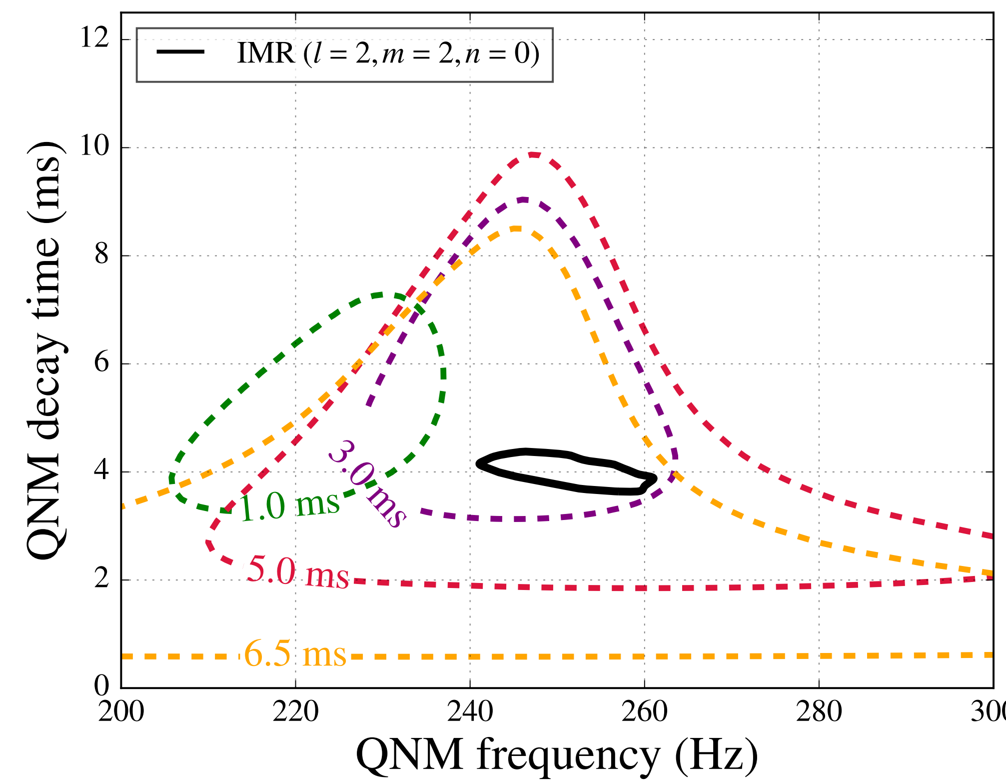
- Strong constraints for stellar evolution models

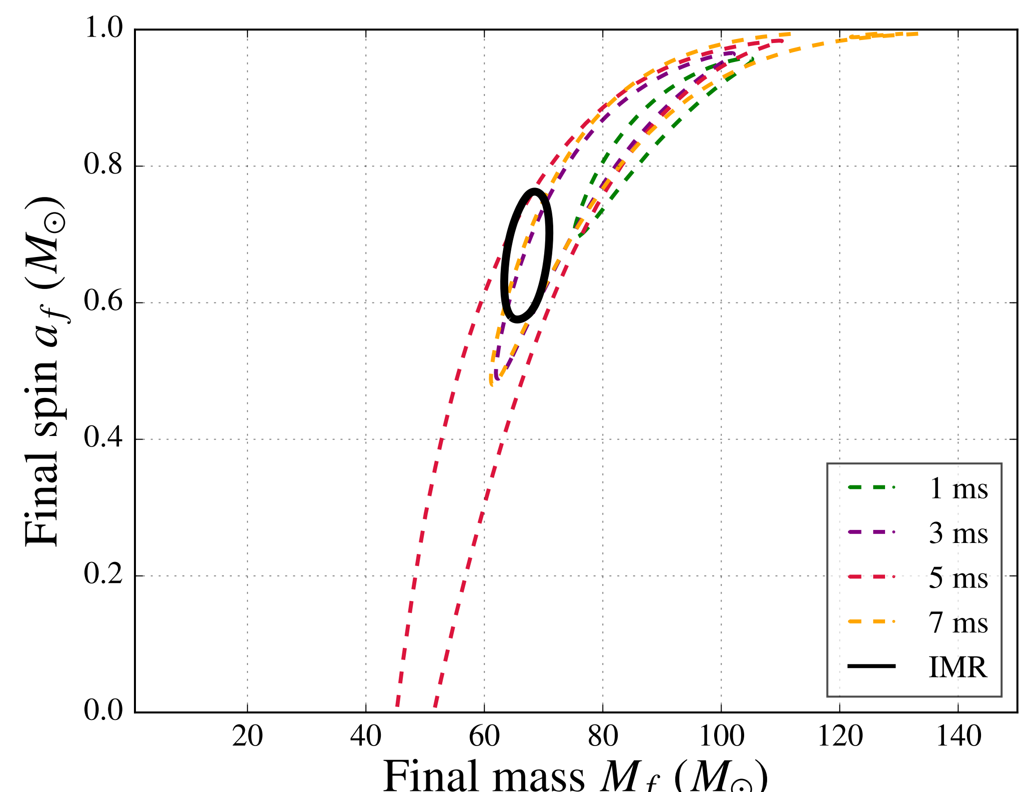
Del Pozzo, Messenger, Veitch, in preparation

Fundamental physics: no-hair theorem

- Detection of more than one ringdown quasi-normal mode allows independent determinations of the remnant mass and spin

- GW150914 was not loud enough to detect more than one QNM

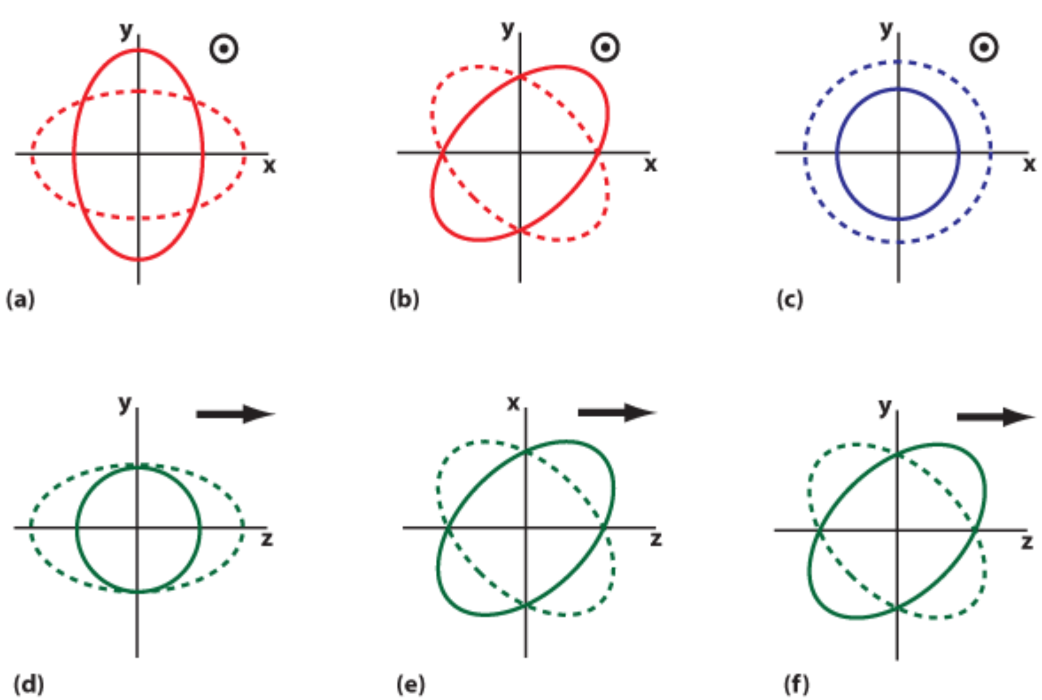




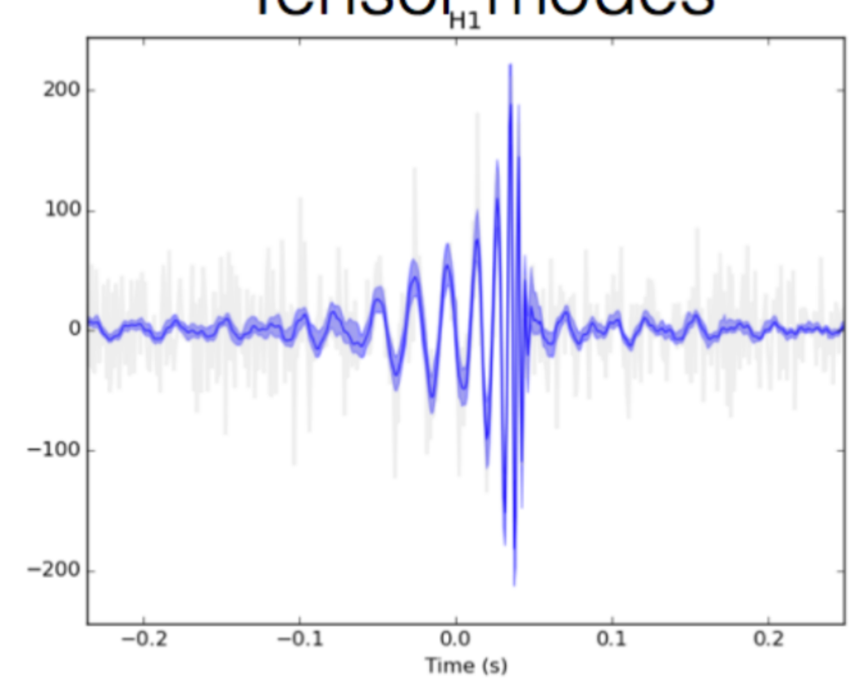
Abbott et al., Phys. Rev. Lett. 116, 221101, 2016

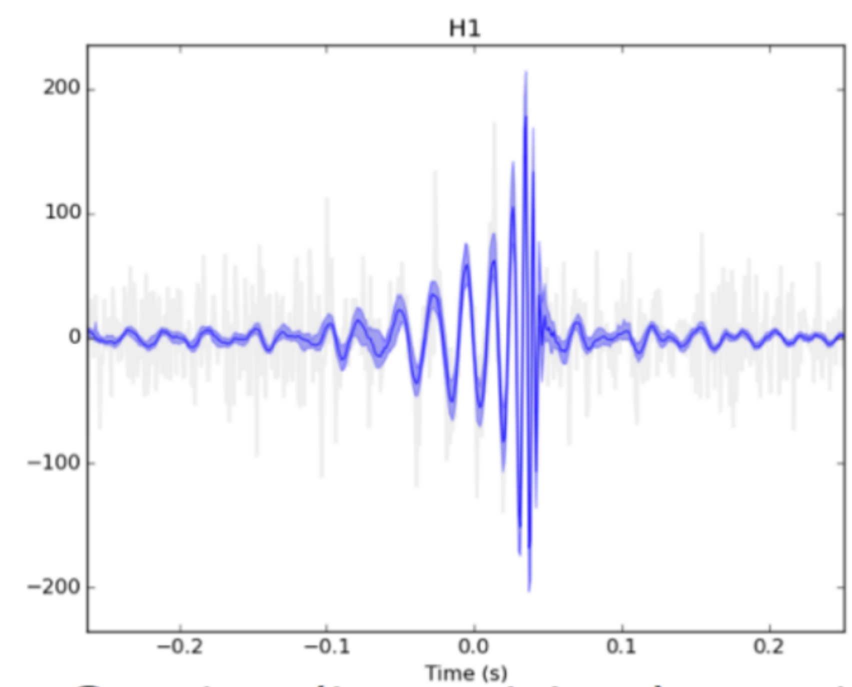
Fundamental physics: polarisation states

- Presence of non-tensor polarisations is a smoking gun for violations of GR
- Virgo (or an electromagnetic counterpart) is necessary to exclude (or detect) non-GR polarisations



Tensor modes

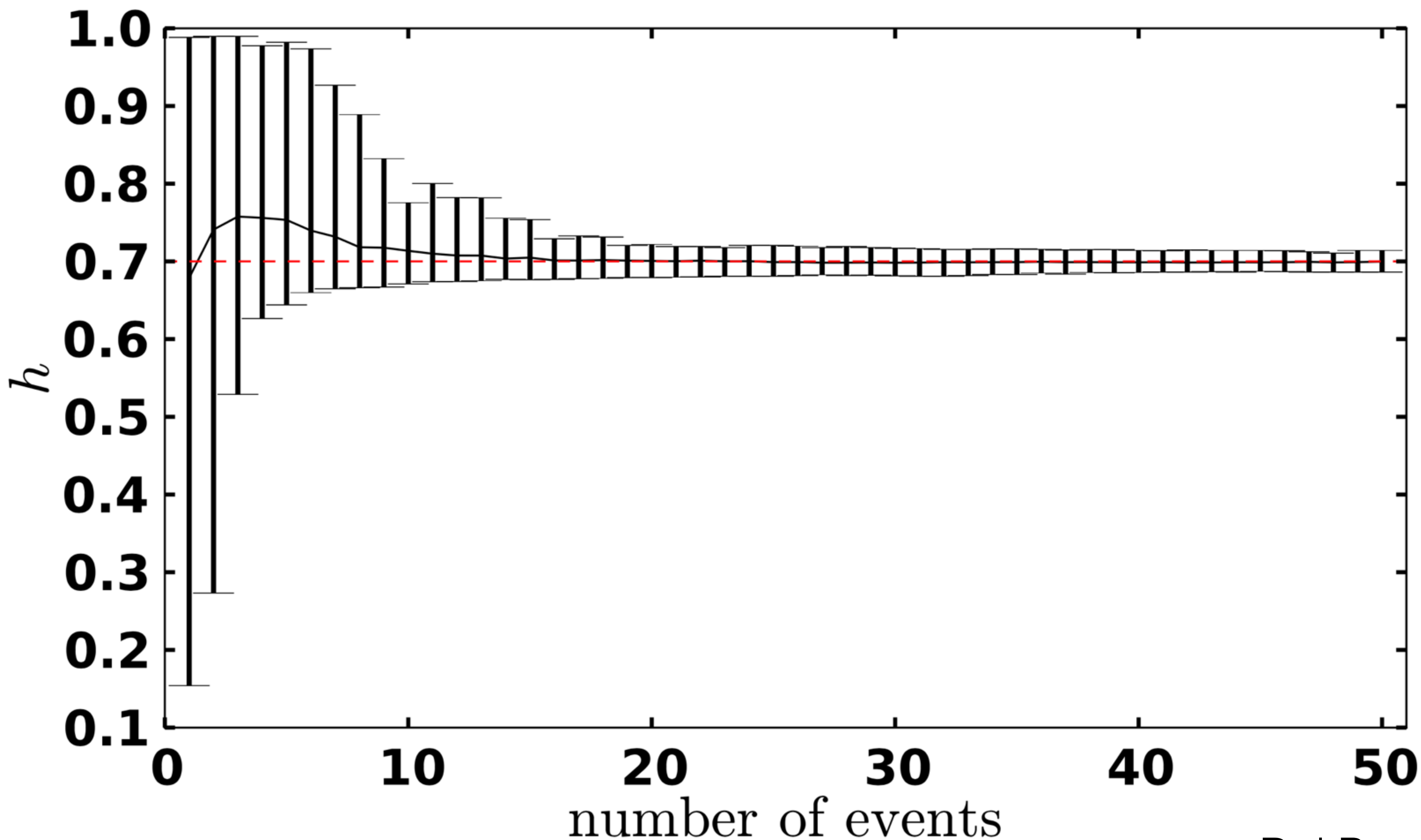




Scalar (breathing) mode

Cosmography: the Hubble constant

- Statistical association with host galaxy allows determination of H_0



Del Pozzo, 2012

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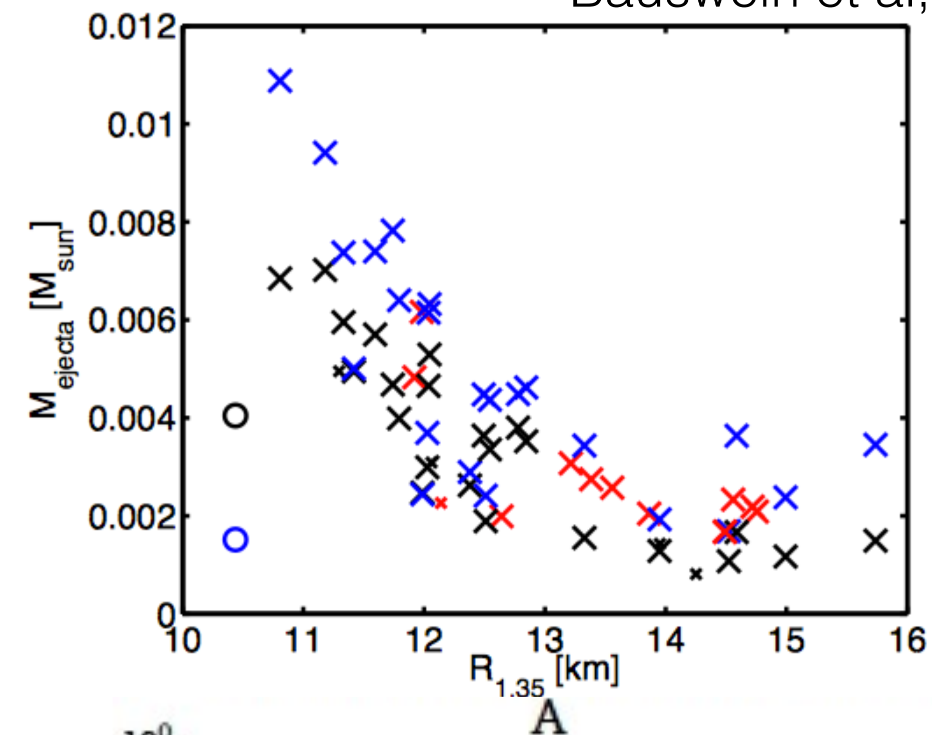
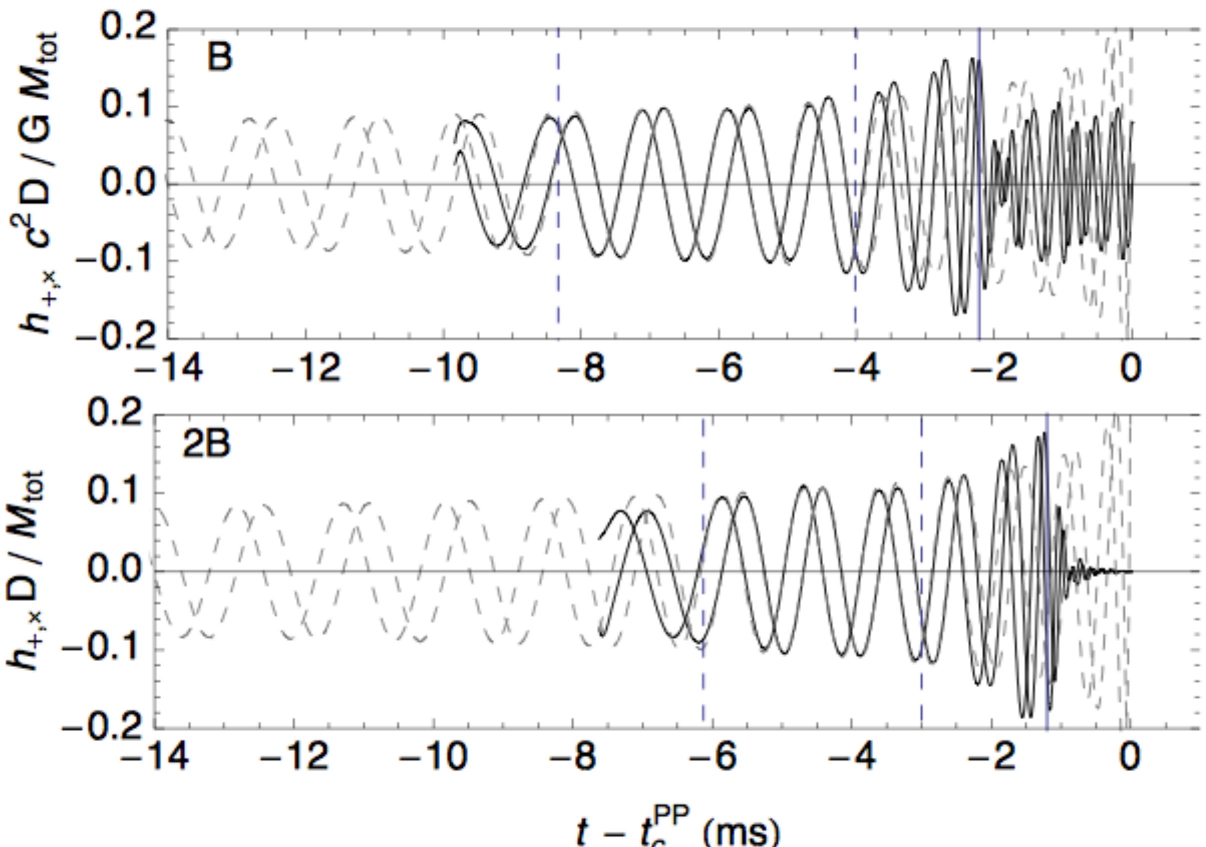
Binary neutron stars

Equation of state of Neutron Stars

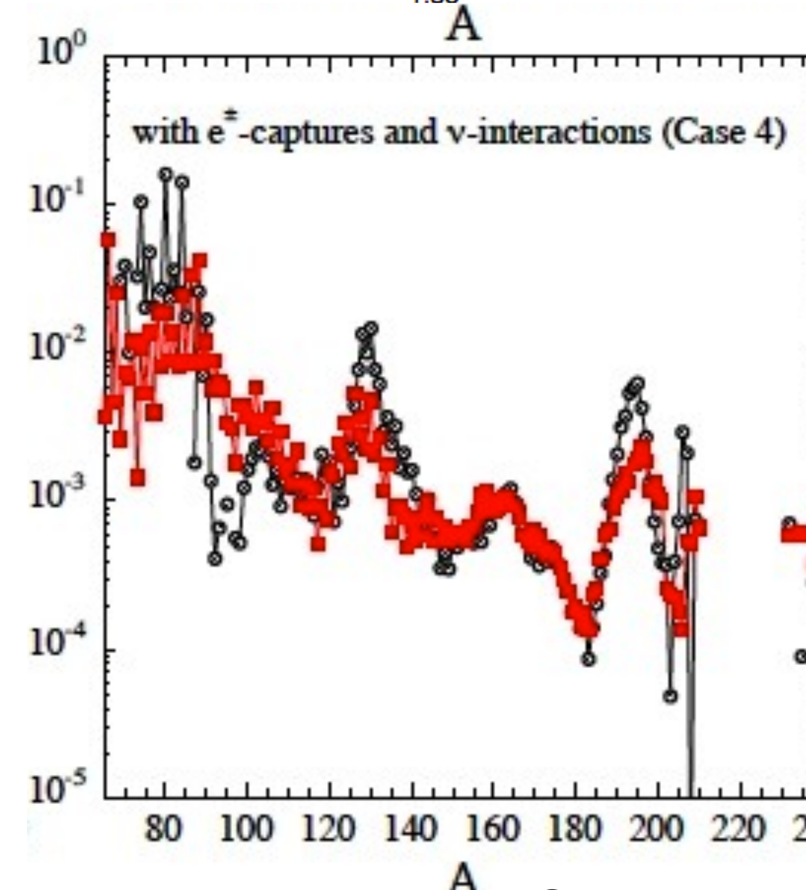
- The merger of a BNS systems yields information about the equation of state (EOS)
    - Gravitational waves
    - Electro-magnetic
- Multi-messenger problem

Bauswein et al, 2013





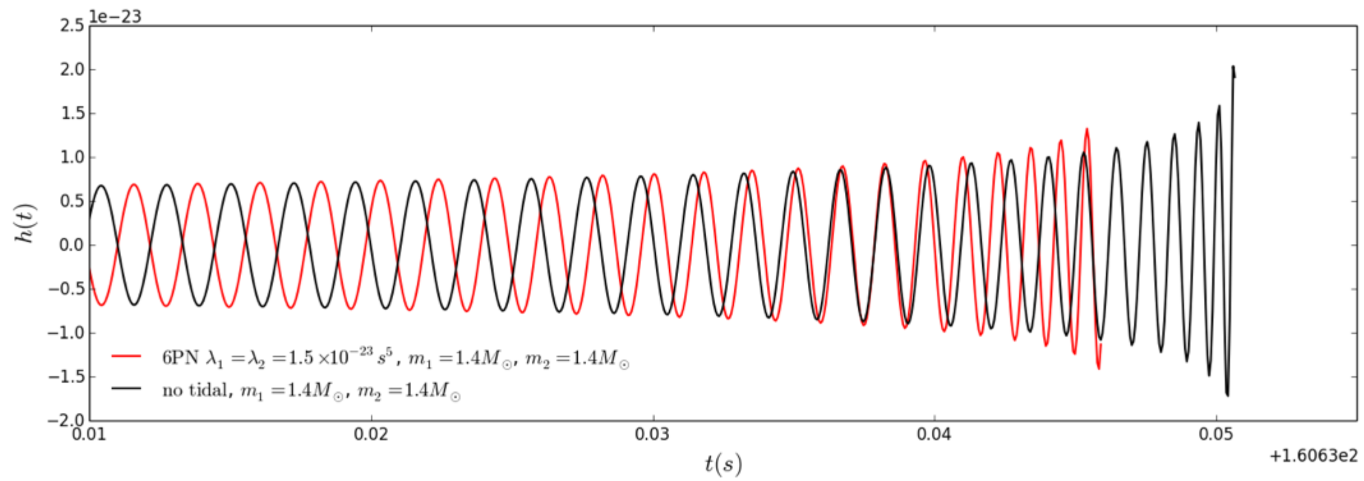
Read et al, 2010

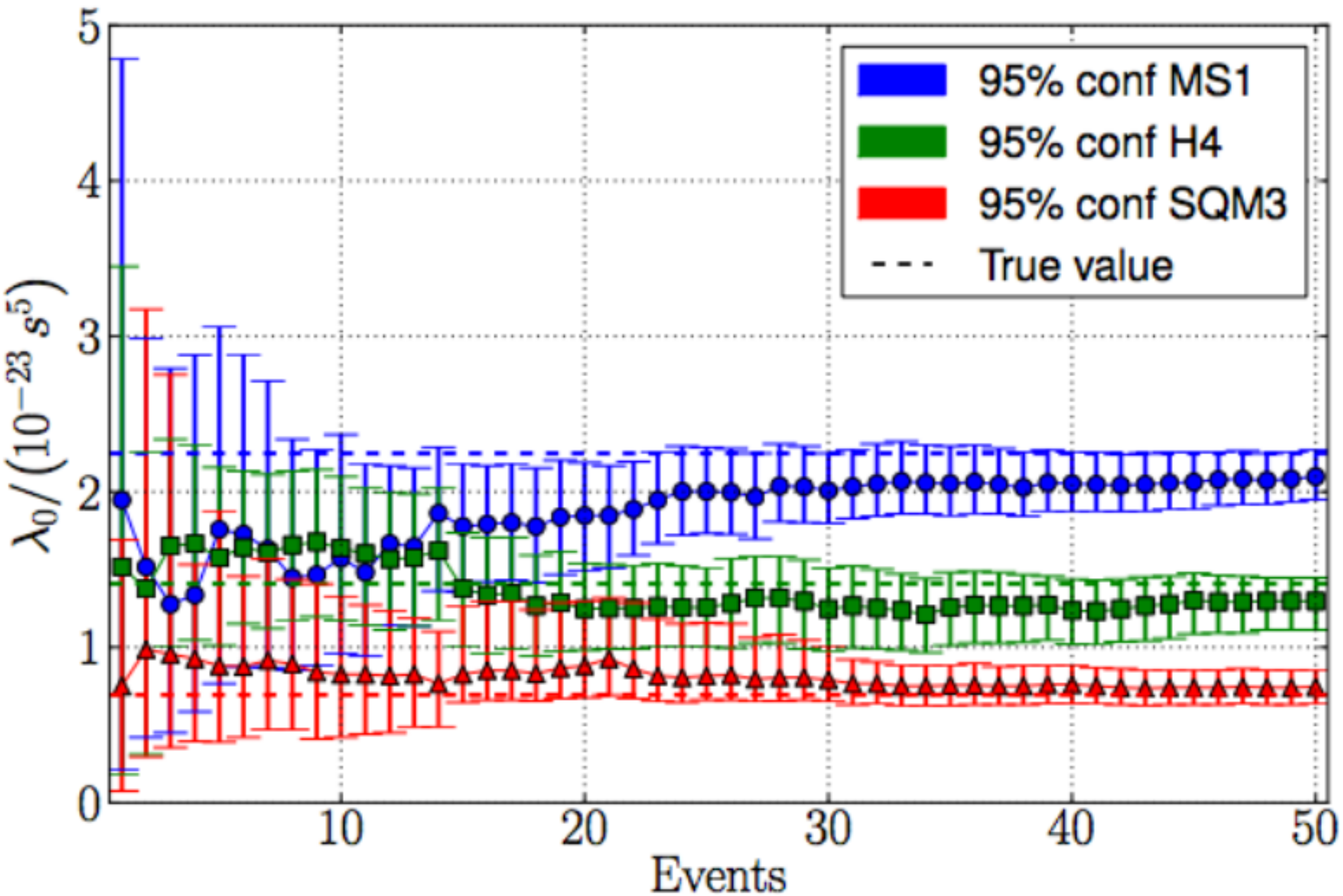


Goriely et al, 2015

Equation of state from tidal deformability

- Tidal deformations show already during the inspired phase





- Discriminate among soft, stiff and intermediate EOS

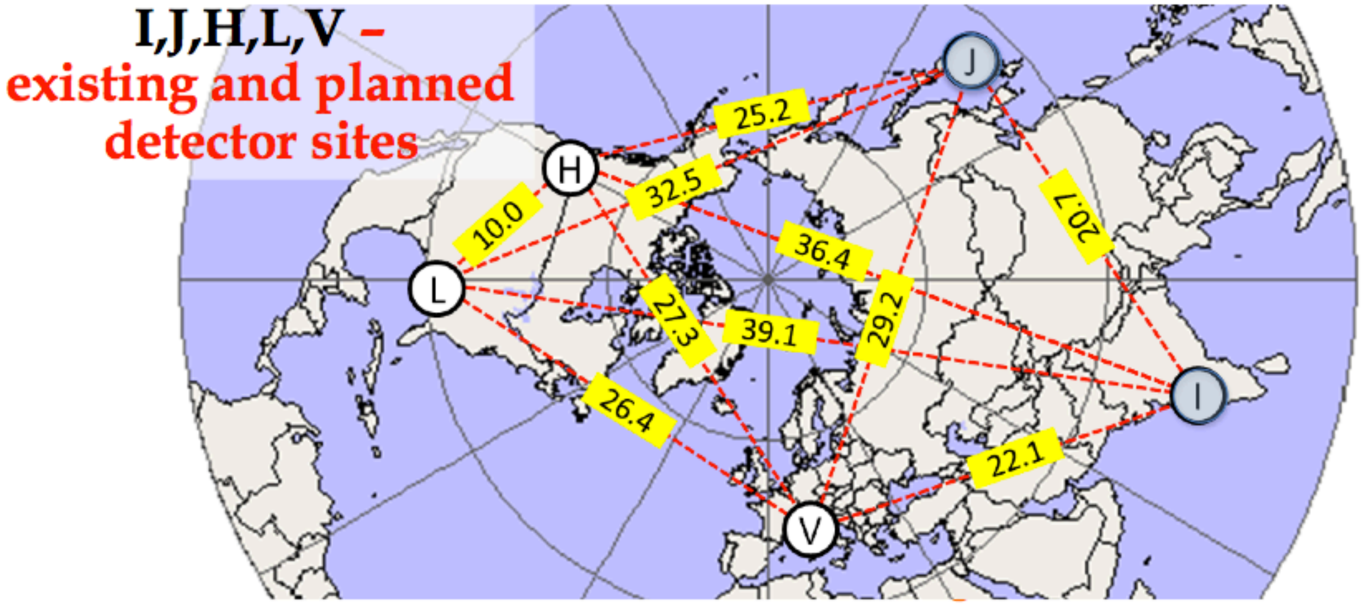
Del Pozzo et al, 2013

Summary

- The era of GW astrophysics is here!
    - Tens of BBH systems in the upcoming years
    - Astrophysical binary black holes (GW150914 & GW151226) behave as expected from GR (within our uncertainties)
- Prospects for a wealth of results in fundamental physics, astrophysics and cosmography

Extra slides

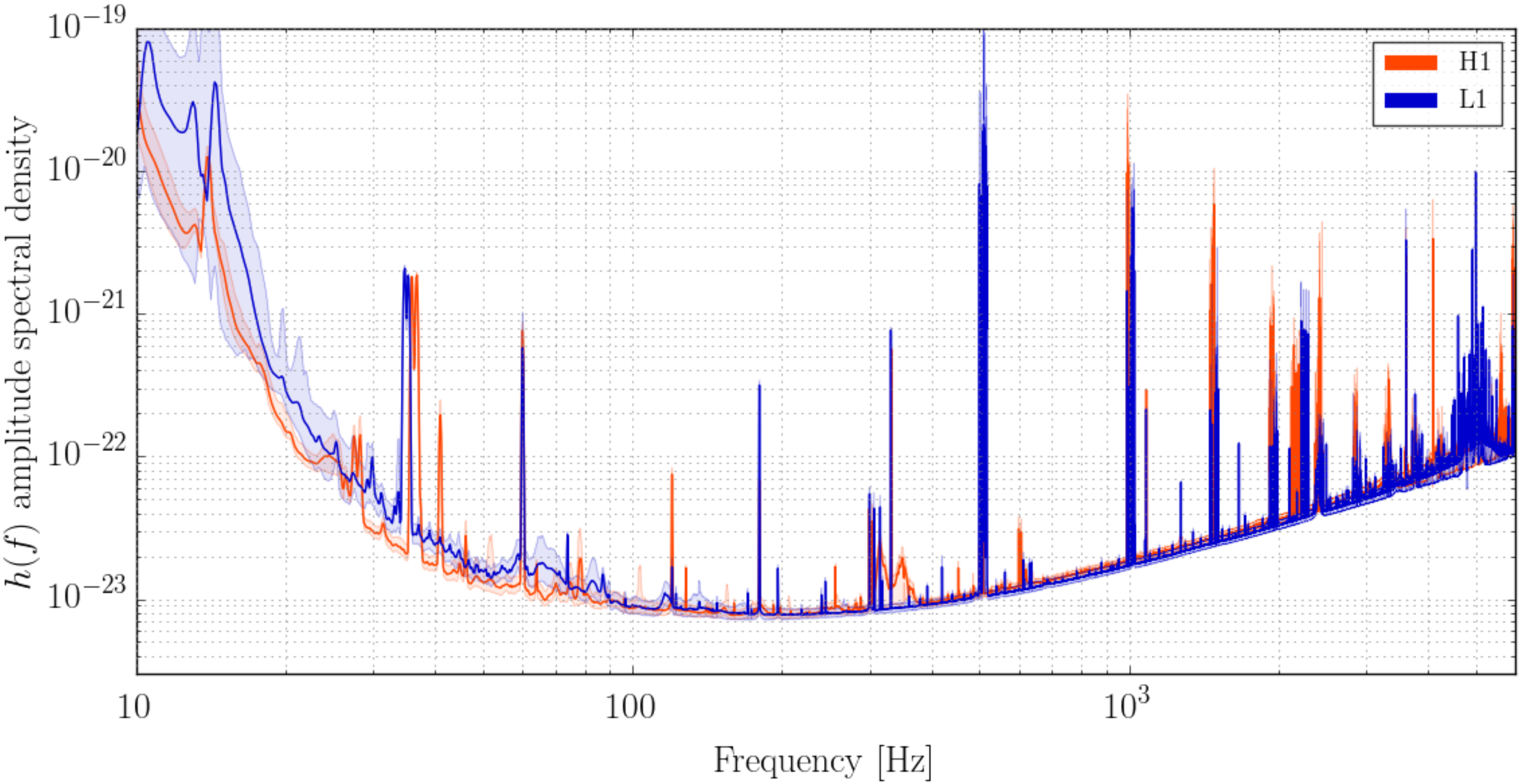
The gravitational wave detectors network



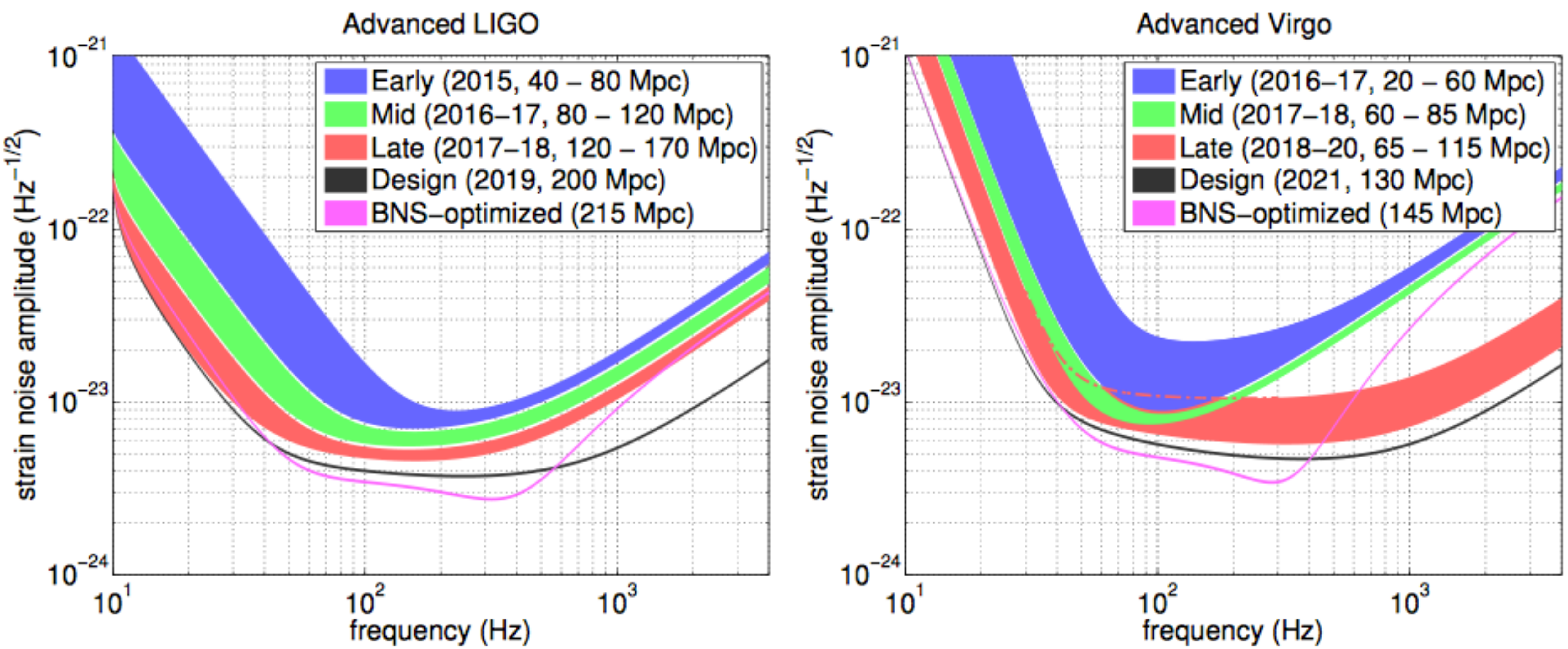
- Two LIGO instruments in Hanford, WA and Livingstone, LO
- Virgo in Cascina, IT
- LIGO India, IN, and KAGRA, JP, will follow

Image Credit: Klimenko, S.

O1 sensitivity



Expected sensitivity



Abadie et al,arxiv:1304.0670

Inferring the physical parameters of a source

Fundaments of data analysis

- Given a model H, depending on some parameters θ and some data d:

$$\underbrace{p(\theta|d,H,I)}_{\text{posterior}} = \underbrace{p(\theta|H,I)}_{\text{prior}} \frac{p(d|\theta,H,I)}{p(d|H,I)} \begin{matrix}\text{likelihood}\\ \\ \text{evidence}\end{matrix}$$

- The evidence is

$$p(d|H,I) = \int d\theta\, p(\theta|H,I) p(d|\theta,H,I)$$

Fundaments of data analysis II

- Bayesian approach allows to compute posteriors from multiple observations (for system independent parameters):

$$p(\theta|d,H,I) = p(\theta|H,I)\prod_i \frac{p(d_i|\theta,H,I)}{p(d_i|H,I)}$$

- Given two models H_1 , H_2 , perform model selection via the odds ratio

$$O_{12} = \frac{p(H_1|I)}{p(H_2|I)}\frac{p(d|H_1,I)}{p(d|H_2,I)}$$

Gravitational waves inference model

- The detector output is linear

$$d(t) = h(t;\theta) + n(t)$$

- where $h(t;\theta)$ is the gravitational wave strain and $n(t)$ is the noise time series

- The noise is assumed to be a Gaussian and stationary stochastic process

$$< n(f)n(f') >= \frac{1}{2}S(f)\delta(f-f')$$

Extracting the parameters of a source

- The probability of a given noise realisation is

$$p(n|I) \propto e^{-\frac{(n|n)}{2}}$$

- Where we introduced the scalar product

$$(a|b) = 4\mathrm{Re}\left[\int df\, \frac{a^*(f)b(f) + a(f)b^*(f)}{S(f)}\right]$$

- The noise properties define the *likelihood function*

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Extracting the parameters of a source

- The integral is high dimensional
  - 9 parameters for non-spinning binaries
    - masses, orientation, sky location, reference time and phase, luminosity distance
  - 15 parameters for the general case
    - Spin vectors
  - More parameters for extra physics (e.g. tests of GR, tidal effects, etc…)
- The problem is tackled using stochastic samplers

Extracting the parameters of a source

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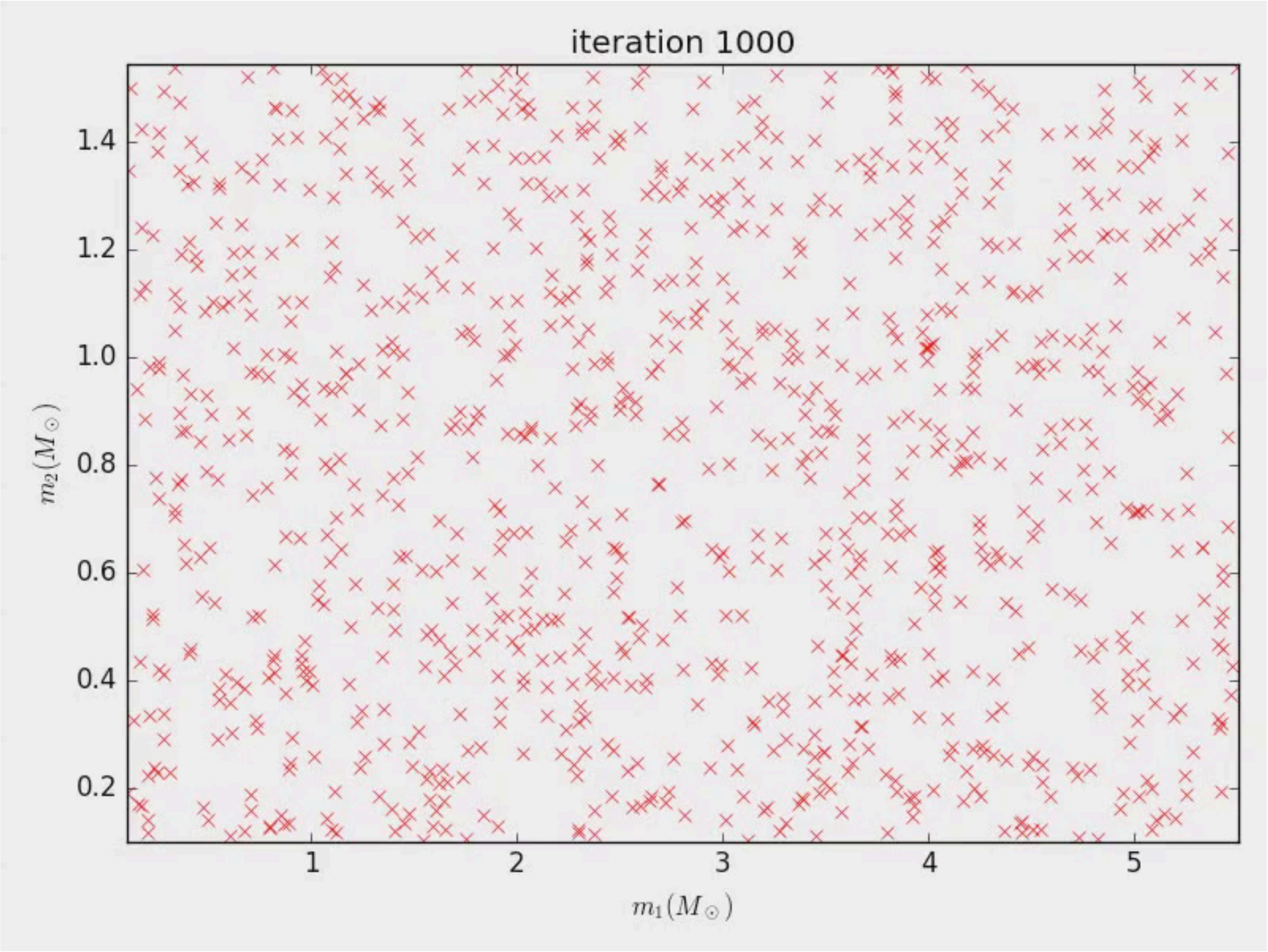
Implementations

- The parameter estimation algorithms and necessary infrastructure are implemented in the LALInference package as part of the LIGO Algorithm Library (LAL)
    - Parallel tempering Markov Chain Monte Carlo
    - Nested Sampling
- Documented in Veitch et al, 2014

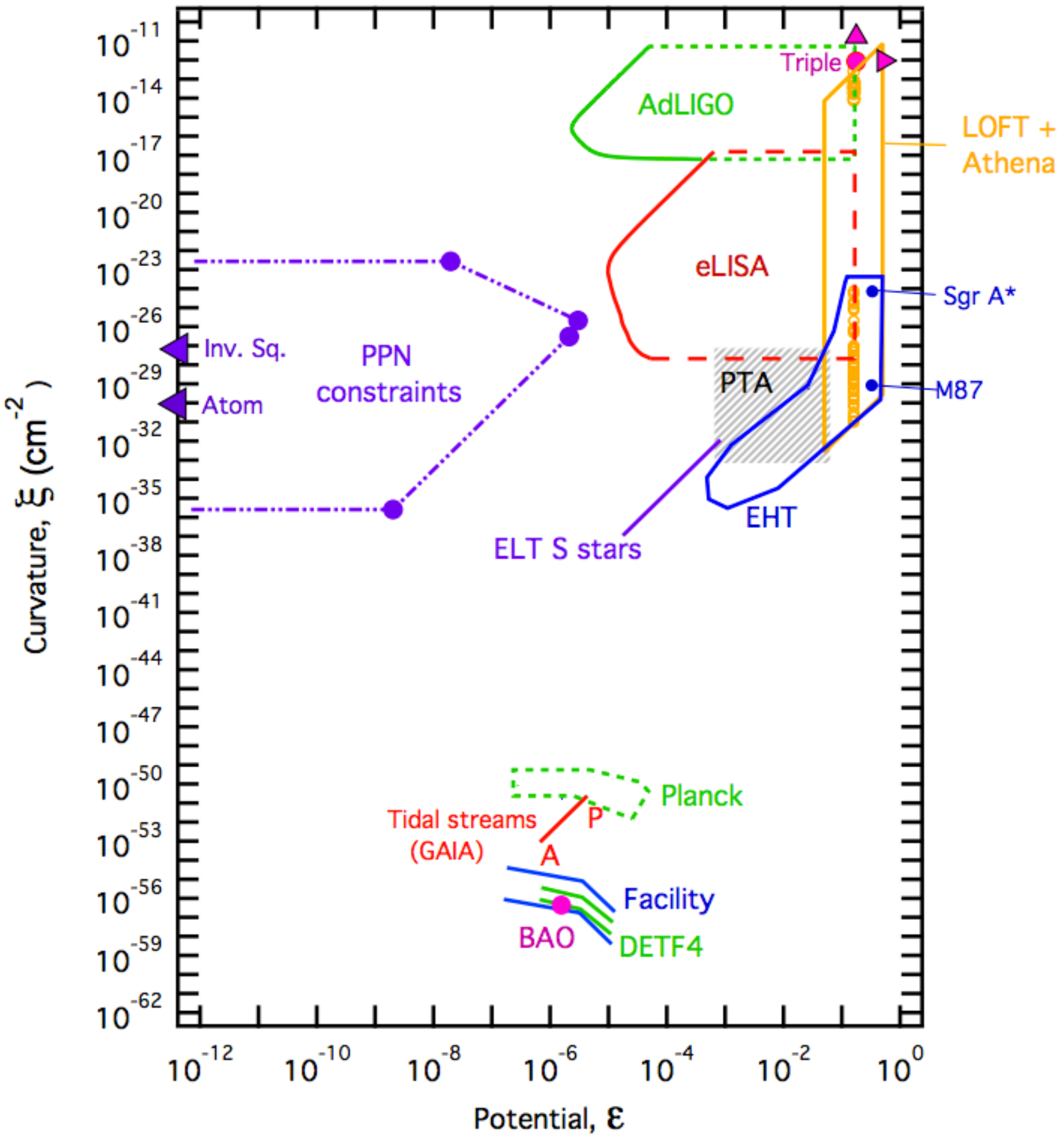
Exploring the parameter space

- nested sampling algorithm



Probing extreme curvature

- Gravitational waves observations opened a new window on the dynamics of space-time in extreme curvature
- double pulsar compactness $GM/(c^2R) \sim 2 \times 10^{-6}$, $v/c \sim 4 \times 10^{-3}$
- BNS (BBH) : $GM/(c^2R) \sim 0.2$, $v/c \sim 0.4$
- Clean systems
  - Contamination from absorptions/scattering negligible



Baker et al, 2015