Relativistic hydrodynamics

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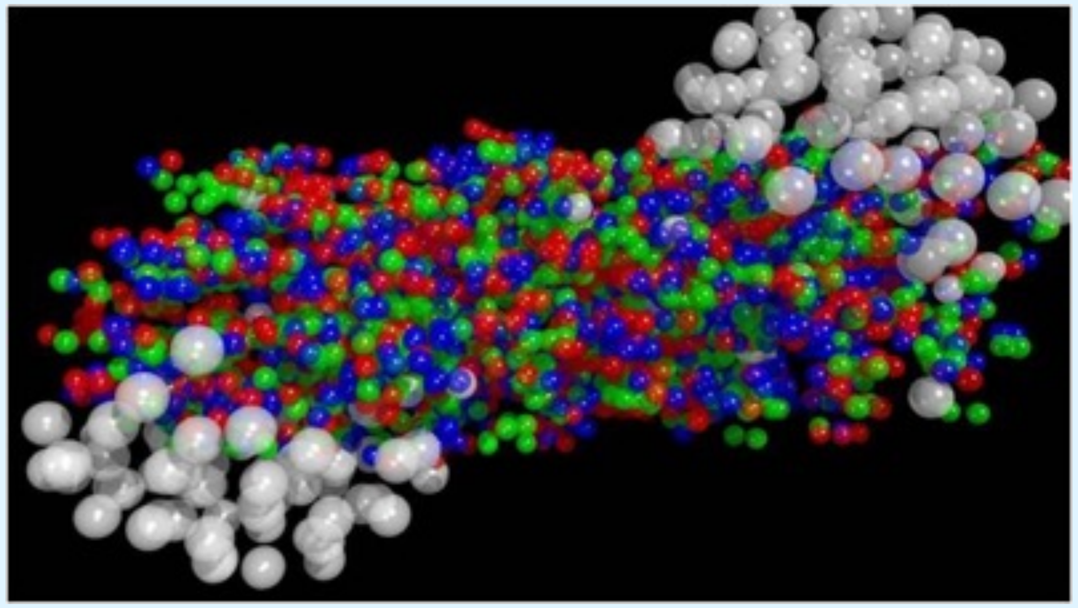
Why Relativistic Hydrodynamics?

Hydrodinamics is an effective theory of matter reliable when the evolution of the system can be describe in terms of few function (energy density, pressure,…) and evolve over scale much larger than the microscopic one.

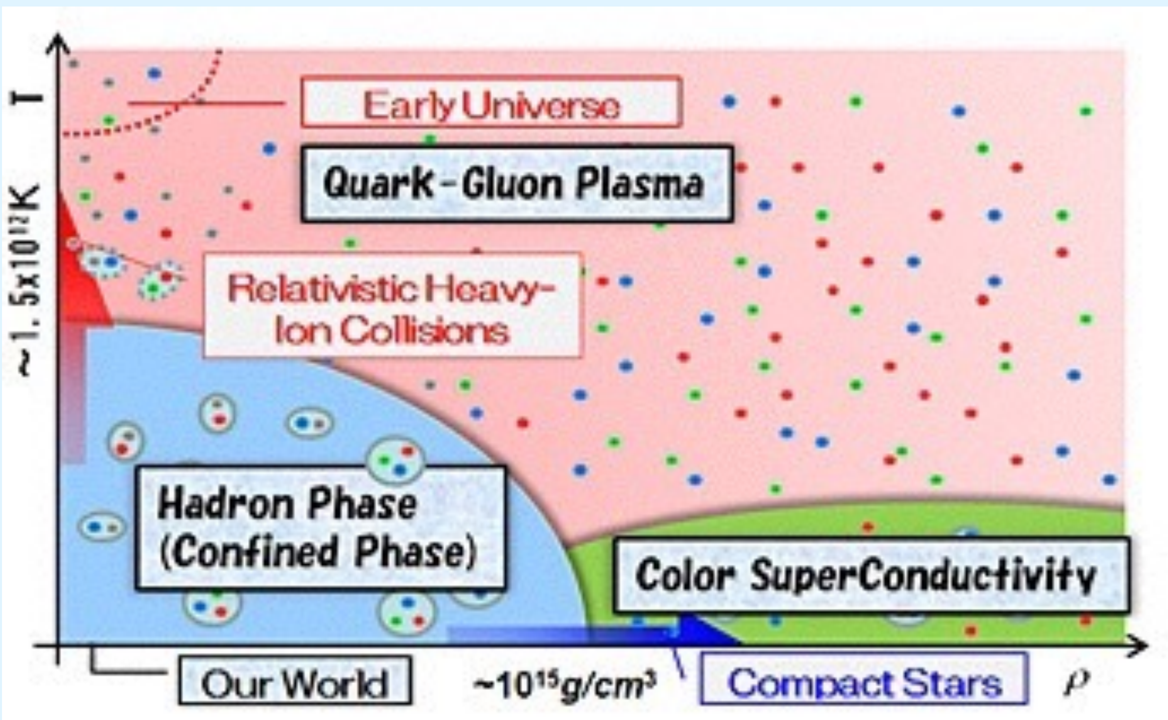
Relativistic hydrodynamics is used to describe the evolution of a system when is to difficult to do in another way :

- ★Astrophysics
- ★Cosmology
- ★Heavy ion collision



Heavy Ion Collision and Quark Gluon Plasma

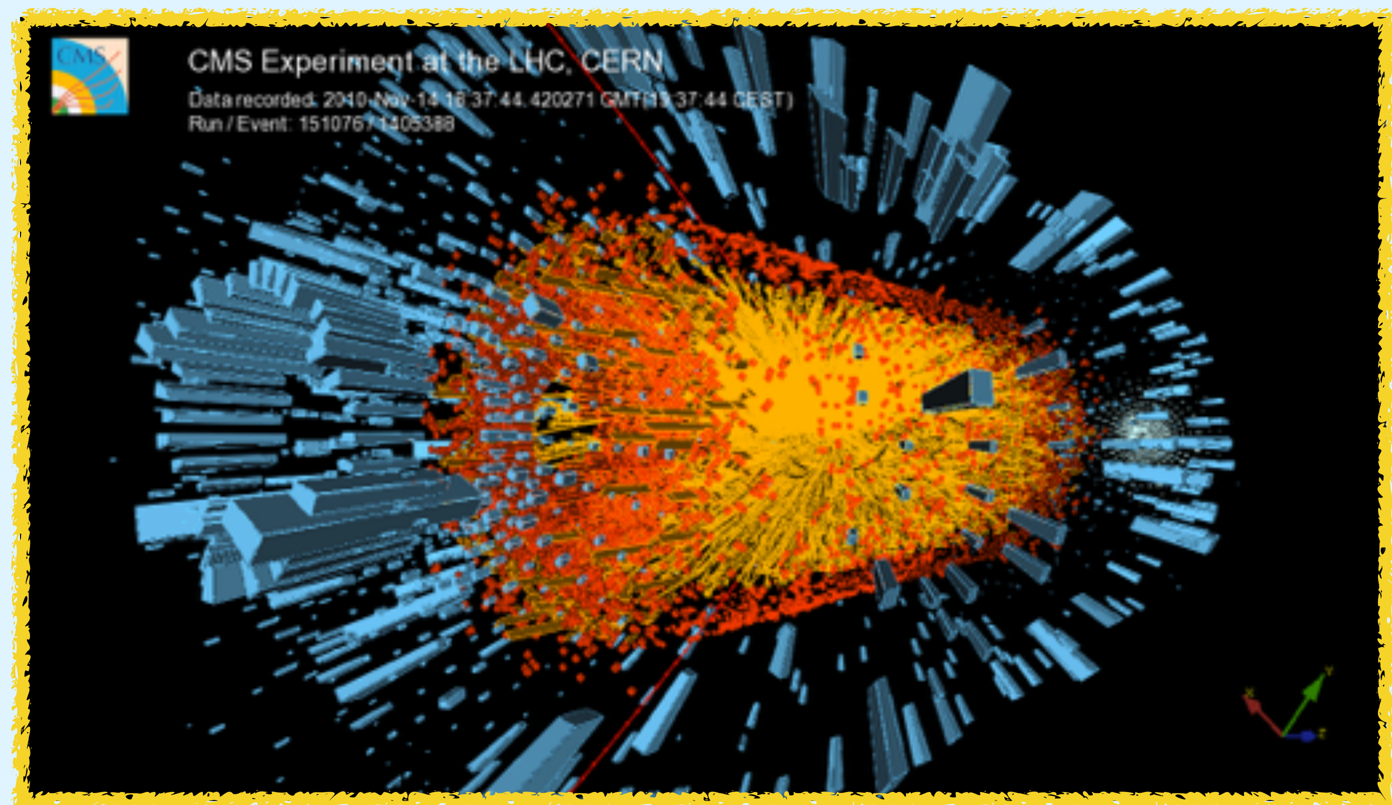
The goal of the Heavy Ion Collision program is study how the matter (quark and gluons) behaves at very high temperature.



- T>165 MeV the quarks and gluons are de-confined
- T<165 MeV the quarks and gluons are confined into the hadrons

Those temperature can be reach in a heavy ion collision at high energy (Au@RICHPb@LHC)

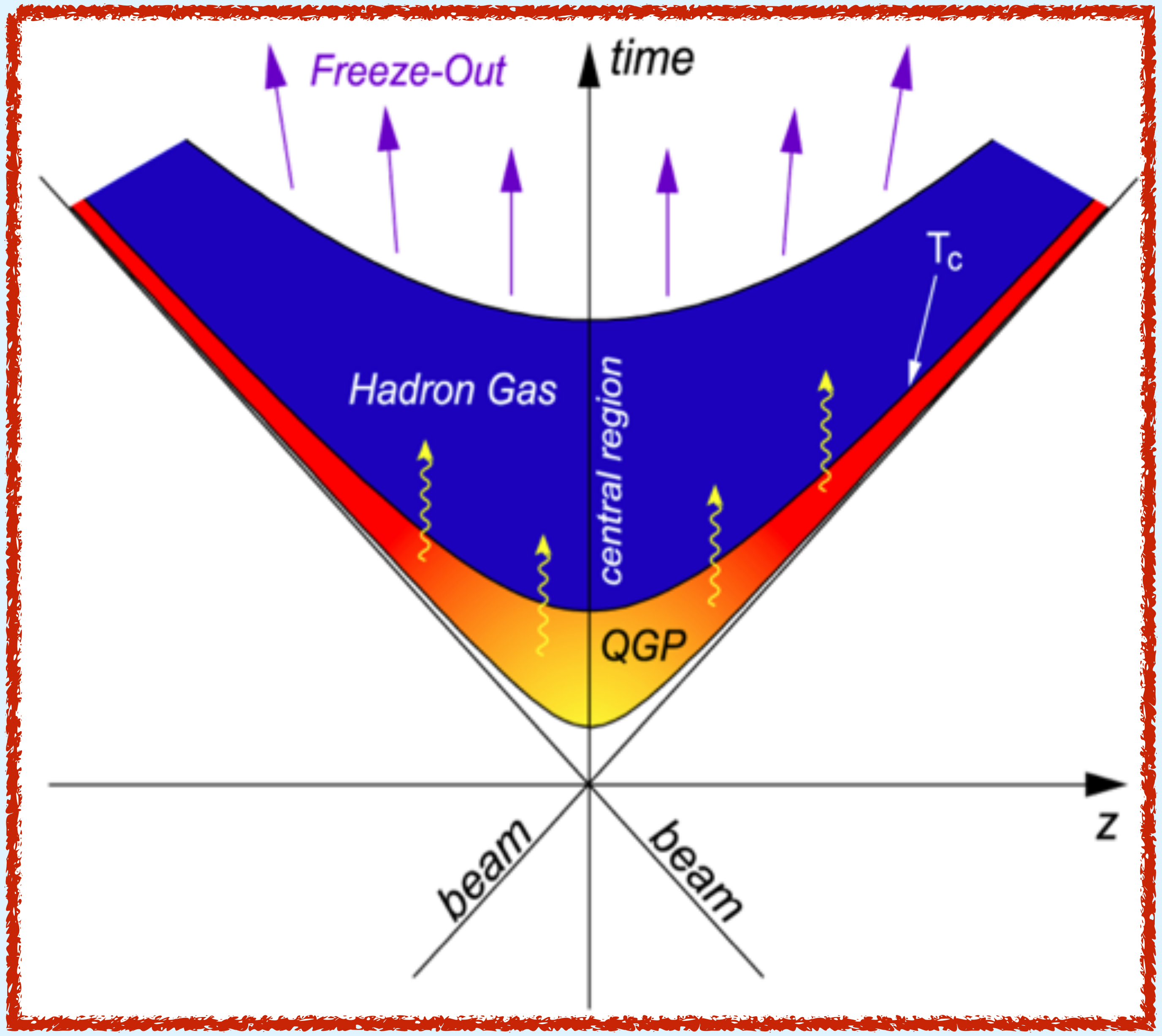
$$\sqrt{s} \sim 200\mathrm{GeV} : 2.76\mathrm{Tev}$$



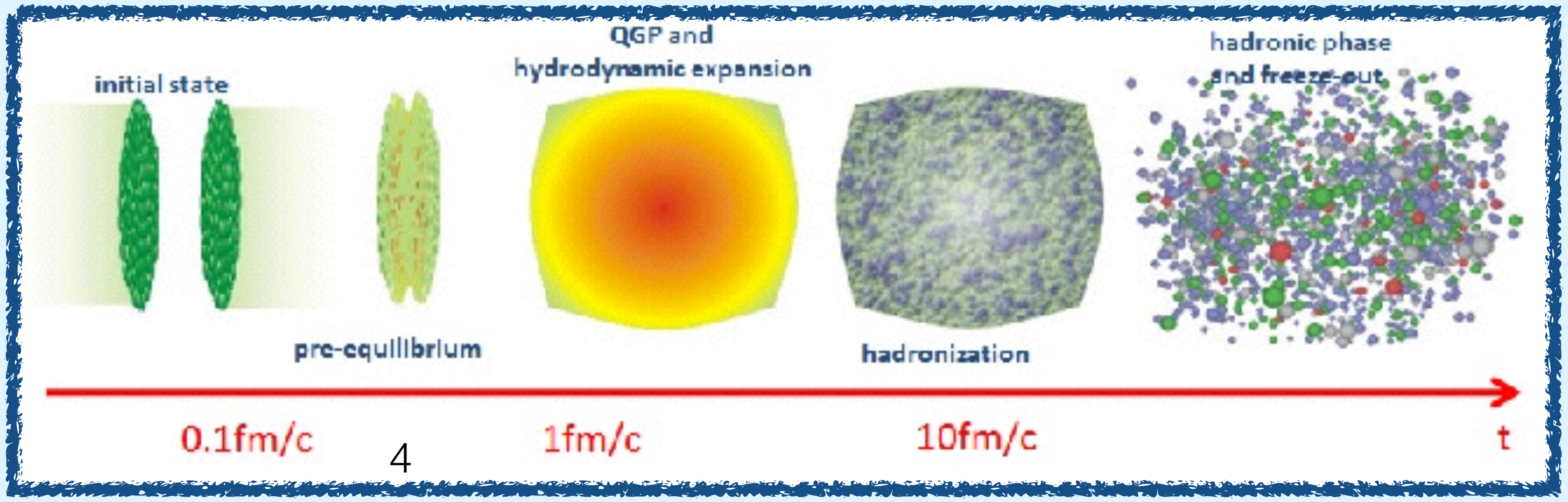
Heavy Ion Collision and quark Gluon Plasma II

The evolution after the collision can be divided in 3 phases

- Pre-equilibrium phase
- Hydrodynamical evolution
- Freeze-Out



The hydro phase is an essential ingredients in order to describe the evolution of the system



Ideal evolution

The equation of relativistic hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu j^\mu = 0$$

At global and homogeneous equilibrium the energy momentum tensor and charge current are:

$$T^{\mu\nu}(x) = (\rho + p) u^\mu u^\nu - g^{\mu\nu} p$$

$$j^\mu = n u^\mu$$

The basic assumption of hydrodynamics is the local equilibrium condition

$$\rho(x) = \rho_{eq}(T(x), \mu(x)) \quad p(x) = p_{eq}(T(x), \mu(x))$$
$$n(x) = n_{eq}(T(x), \mu(x))$$

Navier-Stokes equations

The viscosity can be add as extra term of the energy momentum tensor:

$$T^{\mu\nu} = \rho u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + u^\mu q^\nu + \Pi^{\mu\nu} - \Pi\Delta^{\mu\nu}$$

$$\Pi^{\mu\nu} \equiv 2\eta\left[\frac{1}{2}(\Delta^\mu_\alpha\Delta^\nu_\beta + \Delta^\mu_\beta\Delta^\nu_\alpha) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}\right]\partial^\alpha u^\beta$$

$$\Pi \equiv \zeta\partial\cdot u$$

- The Navier-Stokes theory can not be used in a practical way since it is unstable and acausal.

- This problem can be solved adding more terms, made by second order in gradients of the velocity

Israel-Stewart Theory $\rightarrow$

$$\tau_\Pi D\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \Pi^{\mu\nu} + \cdots$$

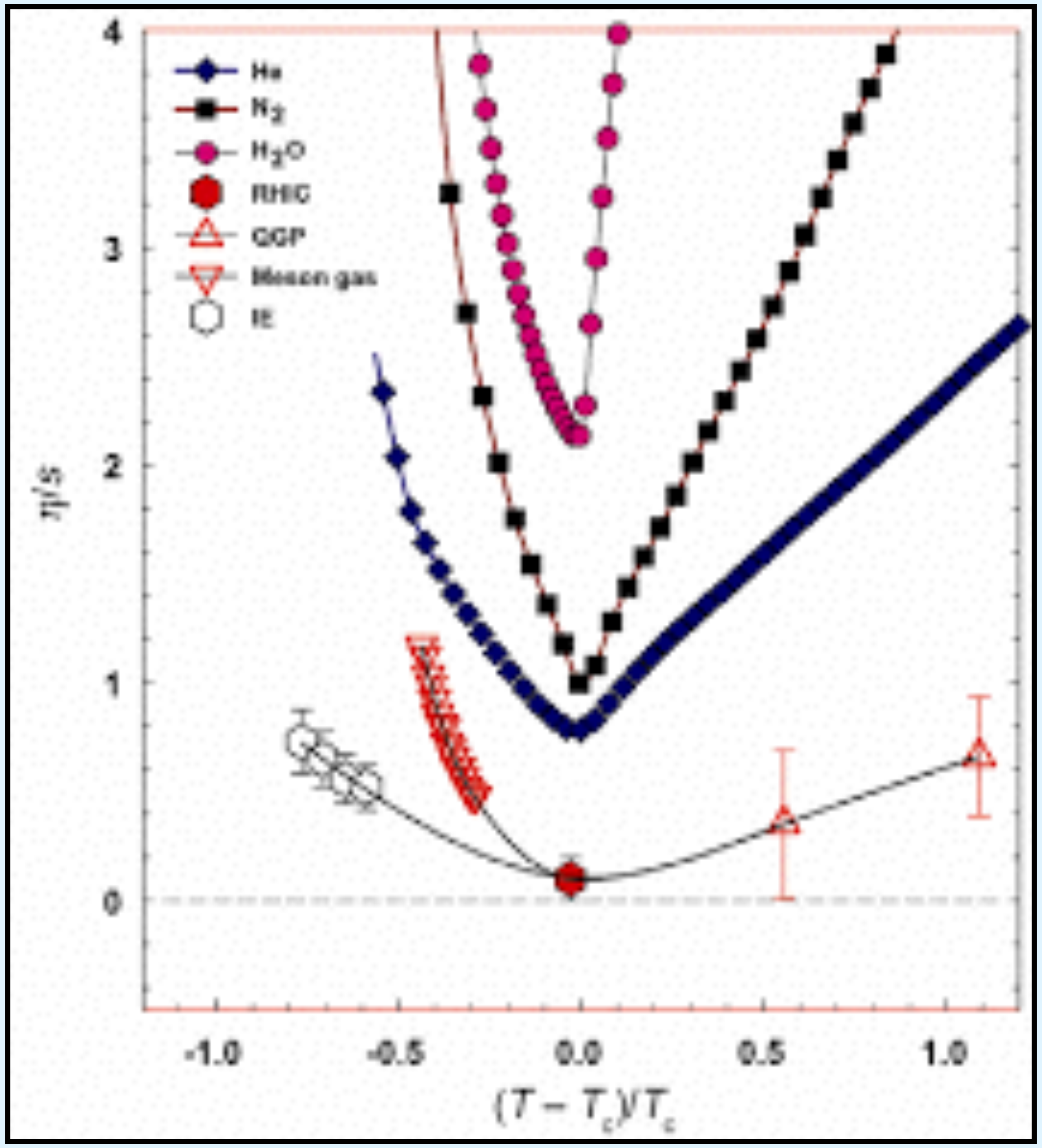
[Muller 1967, Israel Stewart 1976]

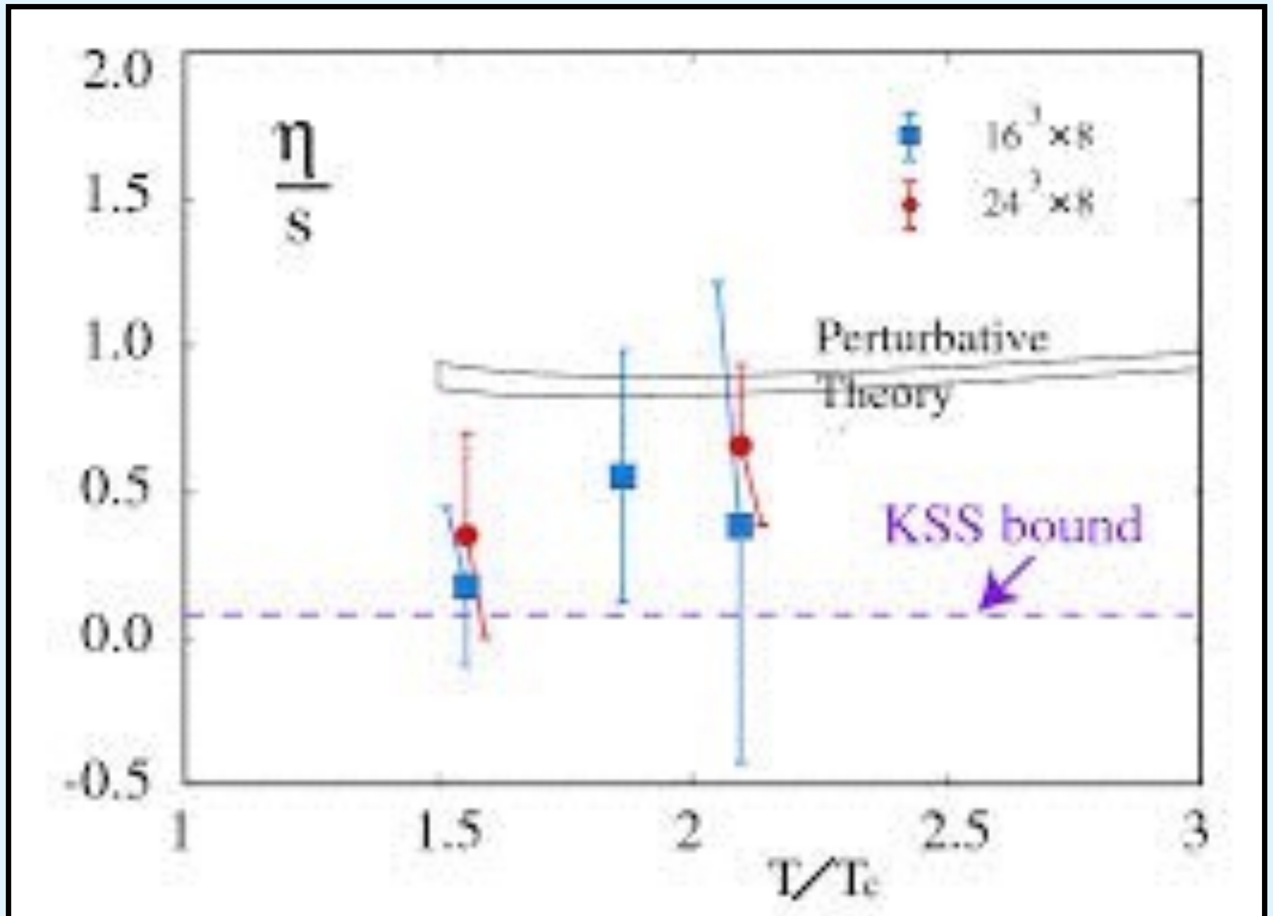
Viscosity of QGP

The QGP is almost a perfect fluids

$$\frac{\eta}{s} \sim 2.5 \frac{1}{4\pi} \sim 0.2$$

Very close to the holographic bound $\quad \dfrac{\eta}{s} = \dfrac{1}{4\pi}$





Low viscosity means strongly coupled fluid, and no quasiparticles

Beyond viscous Hydrodynamics

The general structure up to second order in gradients contains many terms:

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\pi\left[\langle D\sigma^{\mu\nu}\rangle + \frac{\nabla\cdot u}{3}\sigma^{\mu\nu}\right] + \kappa\left[R^{<\mu\nu>} - 2u_\alpha u_\beta R^{\alpha<\mu\nu>\beta}\right]$$
$$+\lambda_1\sigma^{<\mu}{}_\lambda\sigma^{\nu>\lambda} + \lambda_2\sigma^{<\mu}{}_\lambda\Omega^{\nu>\lambda} + \lambda_3\Omega^{<\mu}{}_\lambda\Omega^{\nu>\lambda}$$
$$+\kappa^* 2u_\alpha u_\beta R^{\alpha<\mu\nu>\beta} + \eta\tau_\pi^*\frac{\nabla\cdot u}{3}\sigma^{\mu\nu} + \lambda_4\nabla^{<\mu}\ln s\nabla^{\nu>}\ln s\,.$$

vorticity

acceleration

- The most important is the relaxation time $\eta\tau_\pi$
- The coefficient involving acceleration and vorticity (angular velocity) are non dissipative, and correspond to equilibrium situation.

Equilibrium with rotation and acceleration

The condition to have an equilibrium situation is that the inverse four temperature must be a Killing vector field

four temperature

$$\beta^\mu = b^\mu + \varpi^{\mu\nu} x_\nu = \frac{u^\nu}{T}$$

four velocity

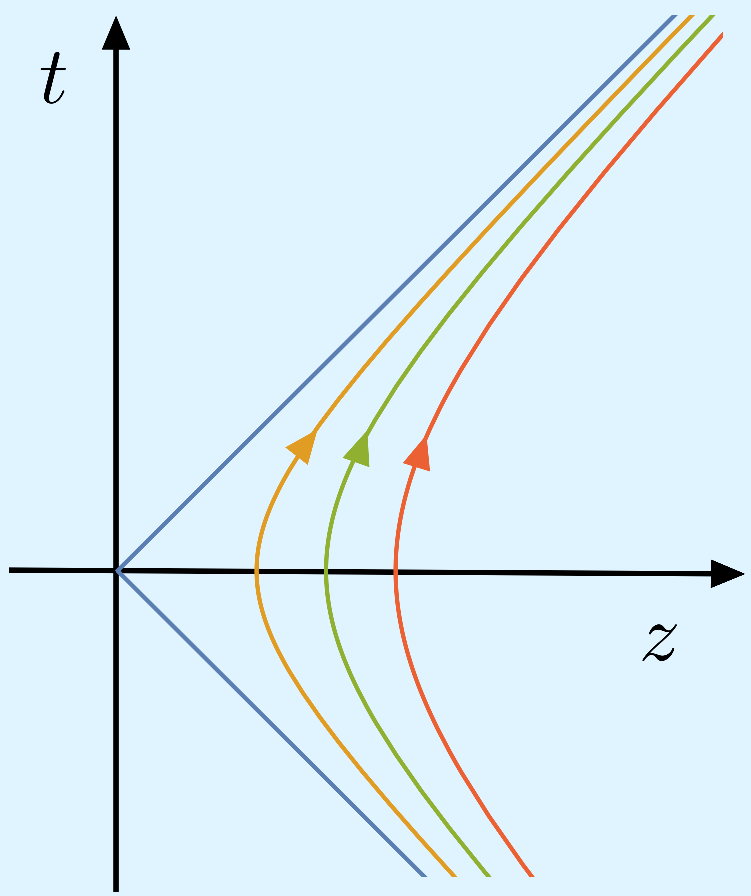
proper temperature

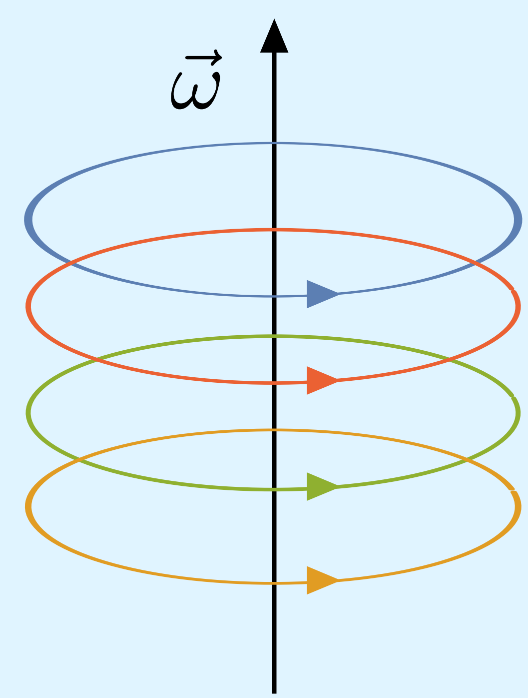
$$\beta^\mu = \frac{1}{T_0}(1 + az, 0, 0, at)$$

acceleration

$$\beta^\mu = \frac{1}{T_0}(1, \boldsymbol{\omega} \times \mathbf{x})$$

angular velocity





Energy momentum with rotation and acceleration

$$T^{\mu\nu}(x) = \left[\rho + \left(\frac{\hbar|a|}{cKT}\right)^2 U_\alpha + \left(\frac{\hbar|\omega|}{KT}\right)^2 U_w\right] u^\mu u^\nu - \left[p + \left(\frac{\hbar|a|}{cKT}\right)^2 D_\alpha + \left(\frac{\hbar|\omega|}{KT}\right)^2 D_w\right] \Delta^{\mu\nu}$$
$$+ A\left(\frac{\hbar|a|}{cKT}\right)^2 \hat{a}^\mu \hat{a}^\nu + W\left(\frac{\hbar|\omega|}{KT}\right)^2 \hat{\omega}^\mu \hat{\omega}^\nu + G\frac{\hbar^2|\omega||a|}{c(KT)^2}(u^\mu \hat{\gamma}^\nu + \hat{\gamma}^\mu u^\nu) + o(\varpi^2)$$

Free Boson

$$U_w = \frac{1}{12\beta^4}$$
$$U_\alpha = \frac{1}{12\beta^4}$$
$$W = -\frac{1}{12\beta^4}$$
$$A = \frac{1}{12\beta^4}$$
$$G = \frac{1}{36\beta^4}$$
$$D_\alpha = -\frac{1}{18\beta^4}$$
$$D_w = 0$$

Free Fermion

$$U_w = \frac{1}{8\beta^4}(1 + \frac{3\beta^2\mu^2}{\pi^2})$$
$$U_\alpha = \frac{1}{24\beta^4}(1 + \frac{3\beta^2\mu^2}{\pi^2})$$
$$W = 0$$
$$A = 0$$
$$G_1 = \frac{2}{9\beta^4}(1 + \frac{3\beta^2\mu^2}{\pi^2}) =$$
$$D_\alpha = \frac{1}{72\beta^4}(1 + \frac{3\beta^2\mu^2}{\pi^2})$$
$$D_w = \frac{1}{24\beta^4}(1 + \frac{3\beta^2\mu^2}{\pi^2})$$

$|a|$: Acceleration

$|\omega|$: Angular velocity

The ideal form is modified by quantum correction

Some numbers

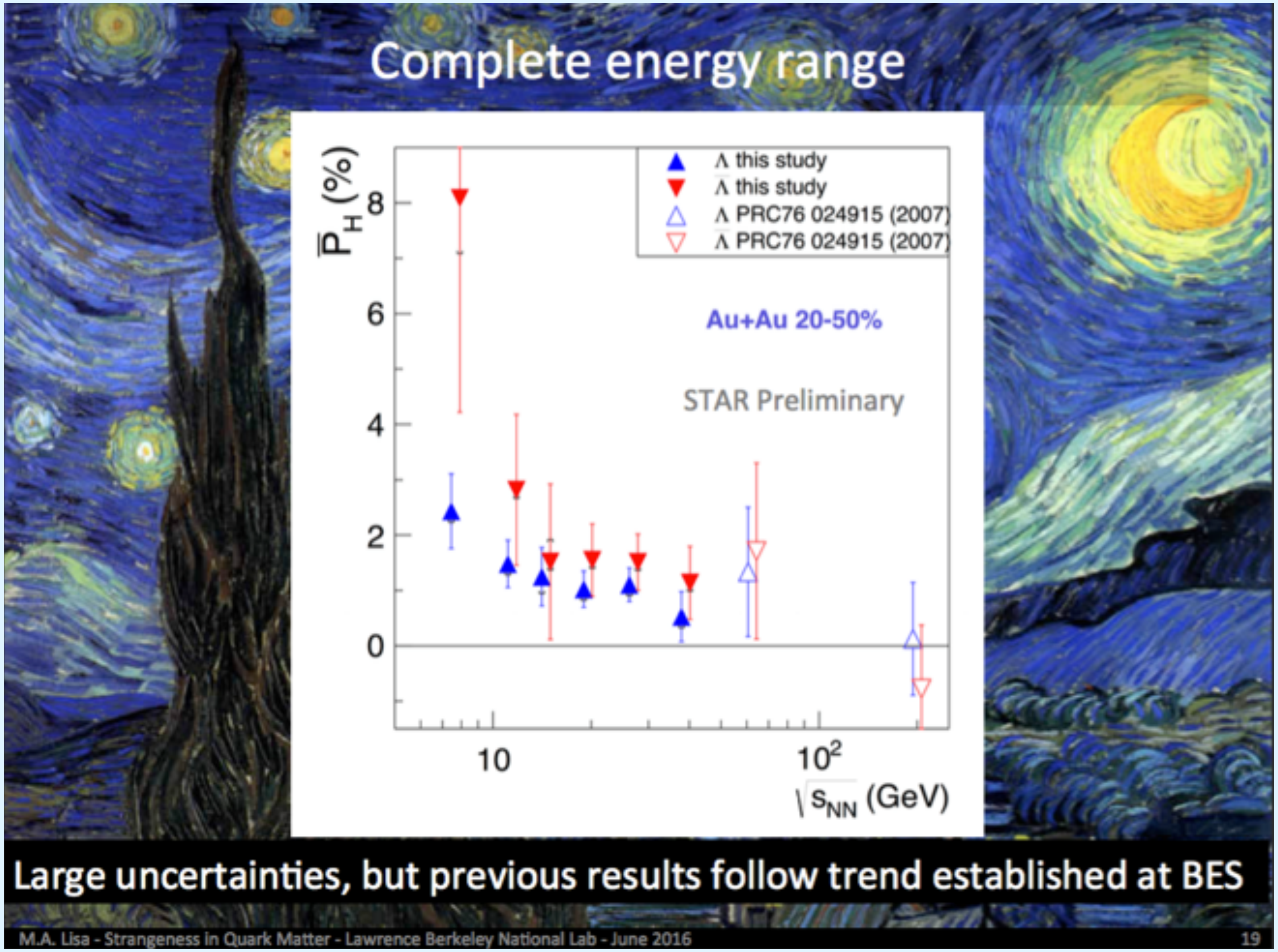
- At room temperature those coefficient are ridiculous tiny.

- In a Heavy ion collision the temperature is T>400 MeV (@ LHC) and the initial acceleration is a= 10^30 g

$$\frac{\hbar a}{cKT} \sim 0.04$$

- These effect could be seen if the coefficient are sufficiently big in a more realistic calculation (including the interaction).

Polarization of Lambda



Polarization of lambda observed in heavy ion collision due to acceleration and vorticity. The particle and anti-particle have the same sign of the polarization