Long-range interacting systems with collisions, MPC approach

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Outline

Many particle systems with long-range forces

- N-body systems with $r^{-\alpha}$ forces in the limit of large N are typically collisionless, and described in terms of their one particle phase space distribution f through the CBE

$$\partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \mathbf{F} \cdot \nabla_{\mathbf{v}} f = 0$$

- For the numerical modeling particle-mesh, PIC or tree- codes are used

Many particle systems with long-range forces

- A galaxy hosts $\sim 10^{12}$ stars, collisional lifetime larger than lifetime $t_H \sim 13\mathrm{Gyr}$

$$t_{\mathrm{relax}} \propto \frac{v^3}{(Gm)^2 n \ln \Lambda} > t_H$$

- In a charged particle beam $\sim 10^{10}$ ions, relaxation time minutes, lifetime few seconds.

$$t_{\mathrm{relax}} \propto \frac{v^3}{q^2 n \ln \Lambda} > t_{\mathrm{cross}}$$

looks like collisions, in general, can be neglected...

However there are systems of interest that are in some parts, or at some time collisional, e.g. :

- Frictional cooling of ion beams, Csonka, Phys.Rev.A **46**, 2101 (1992)
- In cores of elliptical galaxies $t_{relax} << t_H$ Merritt, Rept.Prog.Phys. **69**, 2513 (2006)
- Dense plasma cores in fusion devices, strongly coupled ultra-cold plasmas, dynamical friction...

How to model collisional & weakly collisional systems

- Direct force calculation, but time scales with $O(N^2)$
- Hybrid PIC-Montecarlo methods: Cartwright, Verboncoeur & Birdsall, Phys.Plasm. **7**, 3252 (2000); Vasiliev, MNRAS **446**, 3150 (2015)
- Hybrid Particle-Mesh Direct force cell by cell. (see Hockney & Eastwood 1988)
- Multi-particle collision scheme plus standard PIC or particle-mesh

The multi-particle collision method, in a nutshell

- It actually comes from fluid dynamics: Malevanets & Kapral J.Chem.Phys. **112**, 7260 (2000)
- Collision are *stochastic* but preserve total momentum, kinetic energy and number of particles, i.e.:

$$P_i = \sum_{j=1}^{N_i} m_j v_{j,\mathrm{old}} = \sum_{j=1}^{N_i} m_j v_{j,\mathrm{new}} = \sum_{j=1}^{N_i} m_j (a_i w_j + b_i);$$

$$K_i = \sum_{i=1}^{N_j} m_j \frac{v^2_{j,\mathrm{old}}}{2} = \sum_{j=1}^{N_i} m_j \frac{v^2_{j,\mathrm{new}}}{2} = \sum_{j=1}^{N_i} m_j \frac{(a_i w_j + b_i)^2}{2},$$

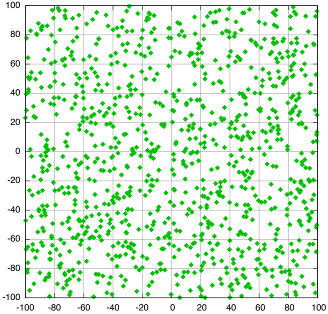
- It is a grid based method scaling as $N \log N$

The multi-particle collision method, in a nutshell

The system is coarse-grained on a grid, then in each collision cell a stochastic rotation of the velocity vectors takes place:

$$\mathbf{v}_i(t+\Delta t) = \mathbf{u}_i(t) + \delta\mathbf{v}_{i,\perp}(t)\cos(\alpha) + (\delta\mathbf{v}_{i,\perp}(t)\times\hat{\mathbf{R}})\sin(\alpha) + \delta\mathbf{v}_{i,\parallel}(t),$$

where $\mathbf{R}$ is a random axis and $\mathbf{u}$ the c.o.m. speed and



$$\sin(\alpha) = -2AB/(A^2+B^2)$$
$$\cos(\alpha) = (A^2-B^2)/(A^2+B^2)$$
$$A = \sum_{i=1}^{N_c}[\mathbf{r}_\times(\mathbf{v}_i-\mathbf{u})]\,|_z$$
$$B = \sum_{i=1}^{N_c}\langle\mathbf{r}_i;(\mathbf{v}_i-\mathbf{u})\rangle$$

standard propagation: $\mathbf{r}_i(t+\Delta t) = \mathbf{r}_i(t) + \mathbf{v}_i(t)\Delta t$

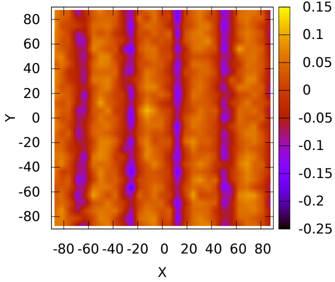
The MPC method for Plasma Physics

- Long-range part of Coulomb (or gravitational) interaction treated with Particle-mesh or PIC (mean field)
- Collisions implemented on the sub-mesh scale with MPC with a cell-dependent collision probability (Bufferand, Ciraolo et al. Phys.Rev.E. **87**, 23101 (2013), Di Cintio et al. Phys.Rev.E. in press, ArXix:150908796D)

$$\omega_c = \frac{8\pi Q^4 n \ln\Lambda}{m^2|\mathbf{v}|^3}; \quad \mathcal{P}_c = \Delta t\langle\omega_c\rangle; \quad \text{or} \quad \mathcal{P}_c = \frac{1}{1+(\tilde{K}_i/\mathcal{E}_{\text{int}})^2}.$$

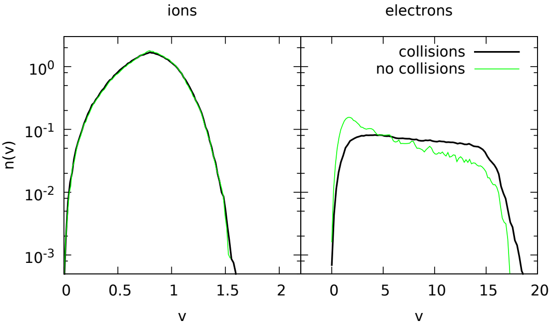
- In inhomogeneous systems collision happen only where and when ω_c is large, provided that the cell size is smaller than the mean free path.

- Decoupled ion-electron plasma with laser-driven electrons ($T_i \neq T_e$)



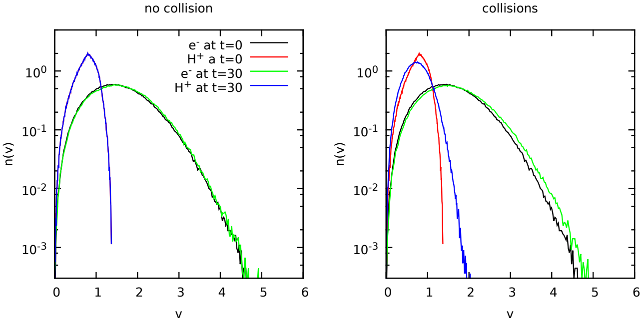
The space-time modulated external field compresses the initially thermal electrons enhancing their collisions, ions are almost unaffected (see fig. on the left).

We simulate this set-up with and without accounting for the collisions (via MPC) using $\mathcal{E}_{int} = n^{1/3}\langle Q^2\rangle/\langle m\rangle$



Ion dynamics is (consistently) unaffected, while electron velocity distributions $n(v)$ in the two cases are rather different!

Analogous situation is observed for simulations of equilibrating H^+ - e^- plasmas



Using only the mean-field underestimates the relaxation rate.

Low dimensional systems (and systems interacting with long-range forces) are usually associated with anomalous diffusion

$$J(x) = \int w(x, x')\kappa(x')\nabla T(x')dx'$$

Diffusion and transport in 1d systems

- 1d systems with 3 conservation laws (e.g. total energy, momentum and number of particles) $\kappa \sim N^{\gamma}$. Non-integrable models have $\gamma = 1/3$, anomalous diffusion.
- There are counter-examples, finite size effects?
- The statistical properties of these systems are essentially described by the fluctuating Burgers equation for the field $\Psi(x,t)$ with white noise $\mathcal{Z}$

$$\partial_t \Psi + c\Psi\partial_x\Psi = \nu\partial^2_{xx}\Psi + \sqrt{2\nu}\partial_x\mathcal{Z},$$

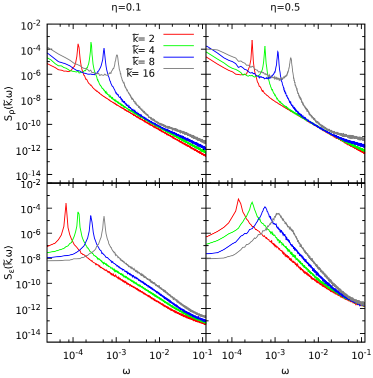
- The structure factors $S(k,\omega) = \langle|\hat{u}(k,\omega)|^2\rangle$ obey KPZ scaling function

Diffusion and transport in 1d systems: model case

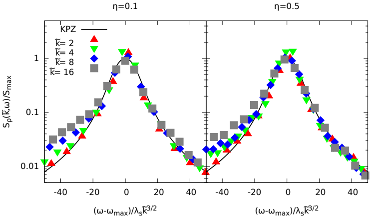
We study the fluctuation in 1d one component plasmas at equilibrium.

- We consider a system of $N = 10^6$ particles in a homogeneous neutralizing background (i.e. the one component plasma model)
- Initial conditions are sampled from a thermal distribution
- Density is fixed so the only free parameter is the temperature

$$S(k,\omega) \sim h_{\mathrm{KPZ}}\left(\frac{\omega \pm \omega_{\max}}{\lambda_s k^{3/2}}\right).$$



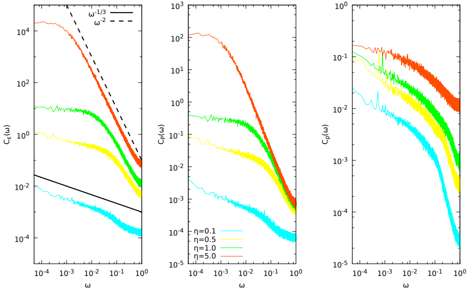
In the regime of high collisionality, we retrieve the predictions of Non-linear Fluctuating Hydrodynamic for the scaling of the structure factors, simular results are found for FPU chains as well (Di Cintio et al. Phys.Rev.E **92** 062108, 2015).



Assuming Green-Kubo

$$\kappa = \frac{D}{k_B T^2 N} \int_0^\infty c_J(t)\mathrm{d}t,$$

we relate κ to γ through $c_J(t) = \langle J(t')J(t'-t)\rangle$. In MPC-1d plasmas we observe the cross-over between anomalous an ballistic transport (Di Cintio et al. Phys.Rev.E **92** 062108, 2015)



Perspectives and applications

- Merging or collisional cooling of charged particle beams
- Scattering of runaway electrons in tokamaks (Di Cintio et al., in preparation 2016)
- Dynamics of galaxy cores, diffusion of stars by central BH
- Dynamics in Globular clusters
- Test of heat transport models in complex plasmas
- Evolution of supra-thermal populations

Summary

- We have a novel way to implement collisions in systems with Coulomb interactions
- Faster than MC, no need to evaluate f
- Easy to incorporate into mesh-based codes
- Accounts for mass or charge spectra

THANK YOU FOR THE ATTENTION!