Topology, η', θ-angle, axion & the like

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Outline of the talk

1. Motivation & Introduction
   - QCD is the theory of strong interactions
2. $\mathrm{U}_A(1)$ anomaly
   - η'-mass
   - lattice checks of the Witten–Veneziano formula
3. θ angle & strong CP problem
   - axion
4. Low energy effective action

Motivation & Introduction - I

- QCD is supposed to be the theory of strong interactions
  - Perturbation theory $\leftrightarrow$ asymptotic freedom for $p^2 \gg \Lambda^2_{QCD}$
    - DIS structure functions moments (anomalous dimensions)
    - Drell–Yan process
  - Exact non-perturbative results related to topology & anomalies
    - $U_A(1)$-anomaly $\rightarrow \eta'$-mass
    - Flavour anomalies $\rightarrow \pi^0 \rightarrow \gamma\gamma$
  - Numerical non-perturbative results from lattice simulations
    - Spontaneous chiral symmetry breaking $\rightarrow$ pions as NG-bosons
    - Hadron spectrum
    - Pseudoscalar meson decay constants
    - Form factors: e.m. & semileptonic
    - $\mathcal{H}_W^{eff}$ hadronic matrix elements
    - Topological susceptibility
    - QCD at $T \neq 0$ and/or $\mu \neq 0$
    - ...

Motivation & Introduction - II

- At a deeper look we see that we need to
  - investigate more carefully
    - the issue of the η'-mass *vs.* the WV formula $\Longleftarrow$
    - the QCD phase diagram
  - understand/solve the strong CP problem in the SM $\Longleftarrow$
    - why θ-angle is small (apparently vanishing)?
    - experimental bound $\theta < 10^{-9}$ (from the neutron EDM)
    - is there a "natural" way to have $\theta = 0$?
- I'll try to briefly set the stage as for the two items marked by $\Longleftarrow$

QCD
$U_A(1)$-anomaly & η' mass

QCD

$$\mathcal{L}_{QCD} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{\psi}(i\gamma^\mu\partial_\mu + g\gamma^\mu A_\mu + M)\psi$$

At $M = 0$, $\mathcal{L}_{QCD}$ is formally invariant under the (global) flavour group

$$G = SU_L(N_f) \otimes SU_R(N_f) \otimes U_A(1) \otimes U_V(1)$$

$$\begin{aligned} \psi_L \to \psi'_L &= U_L\psi_L \qquad U_L \in U_L(N_f) \\ \psi_R \to \psi'_R &= U_R\psi_R \qquad U_R \in U_R(N_f) \end{aligned}$$

$$\psi_L = \frac{1-\gamma_5}{2}\psi \qquad \bar{\psi}_L = \bar{\psi}\frac{1+\gamma_5}{2}$$

$$\psi_R = \frac{1+\gamma_5}{2}\psi \qquad \bar{\psi}_R = \bar{\psi}\frac{1-\gamma_5}{2}$$

$$U_{L/R} = \exp[i\sum_{f=0}^{N_f^2-1}\alpha^f_{L/R}\lambda^f]\,, \quad \mathrm{tr}\,[\lambda^f,\lambda^g] = \frac{1}{2}\delta_{fg}$$

The fate of flavour symmetries

- $SU_L(N_f) \otimes SU_R(N_f)$ softly broken by mass terms (WTIs)
  - WTIs $\to$ PCVC & PCAC
- $U_A(1)$ broken by quantum effects Fujikawa
  - anomalous axial singlet WTIs, at $M = 0$

$$\langle \partial_\mu \mathcal{A}^0_\mu(x) \hat{\mathcal{O}}(y) \rangle = 2N_f \langle Q(x) \hat{\mathcal{O}}(y) \rangle + \langle \delta^x_A \hat{\mathcal{O}}(y) \rangle$$

$$\mathcal{A}^0_\mu = \bar{\psi} \gamma_\mu \gamma_5 \psi \quad \text{singlet axial current}$$

$$Q = \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \quad \text{topological charge}$$

- $U_V(1)$ exact (fermion number conservation)

Low energy effective action of QCD - Γ_{eff}^{QCD}

- Low energy ($< \Lambda_{QCD}$) physics can be encoded in a LEEA

$$\Gamma_{eff}^{QCD}(U,M) = \int dx \left[\frac{F_\pi^2}{4} \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + v \mathrm{Tr}(UM^\dagger + MU^\dagger) + \dots \right], U = e^{2i\lambda^f \pi^f / F_\pi}$$

- Formal construction of Γ_{eff} ($f = 0, 1, \dots, N_f^2 - 1$)

1) $e^{iW(j_\sigma, j_\pi)} \equiv \int D\mu \exp \int dx \left[i\mathcal{L}_{QCD} + \bar{\psi}\lambda^f \psi j_\sigma^f + i\bar{\psi}\lambda^f \gamma_5 \psi j_\pi^f \right]$

2) $i\Gamma(\Sigma^f, \Pi^f) = \min_{j_\sigma, j_\pi} \left[iW(j_\sigma, j_\pi) - \int dx (\Sigma^f j_\sigma^f + \Pi^f j_\pi^f) \right]$ Legendre

with $\frac{\delta\Gamma}{\delta\Sigma^f(x)} = ij_\sigma^f(x) \qquad \frac{\delta\Gamma}{\delta\Pi^f(x)} = ij_\pi^f(x)$

3) assume a non-trivial solution at $j_\sigma = j_\pi = 0 \leftrightarrow$ SχSS

$(\Sigma^f, \Pi^f) = (0, 0) \qquad (\Sigma^0, \Pi^0) = (\sqrt{2N_f} s_0, 0), \quad s_0 \neq 0$

$\Sigma + i\Pi = hU, \quad \Sigma - i\Pi = U^\dagger h, \quad U = e^{2i\pi/F_\pi}, \quad h = s_0 e^{2i\sigma}$

4) heavy σ^f eliminated from $\delta\Gamma/\delta\sigma^f(x) = 0$, light π^f NG-bosons, π^S?

5) chiral symmetry constrains the form of $\Gamma_{eff}^{QCD}(U, M)$

The Witten–Veneziano formula - Theory

- A simplified argument (ignoring renormalization subtleties)
  - Anomalous singlet axial WTI, taking $\mathcal{O} = Q$, reads

$$i \int e^{ipx} p_\mu \langle \hat{\mathcal{A}}^0_\mu(x) \hat{Q}(0) \rangle = 2N_f \int e^{ipx} \langle \hat{Q}(x) \hat{Q}(0) \rangle$$

  - In the limit $u = N_f/N_c \to 0$, $m^2_{\eta'} \to 0$ (see back-up), then

$$\frac{i}{2N_f} \int e^{ipx} p_\mu \langle \hat{\mathcal{A}}^0_\mu(x) \hat{Q}(0) \rangle \Big|_{u=0} = \lim_{u\to 0} \frac{m^2_{\eta'} F^2_{\eta'}}{2N_f} \frac{p^2}{p^2 + m^2_{\eta'}} + \mathrm{O}(p^2) =$$

$$= \frac{m^2_{\eta'} F^2_{\eta'}}{2N_f} \Big|_{u=0} + \mathrm{O}(p^2)$$

  - $$\frac{m^2_{\eta'} F^2_{\eta'}}{2N_f} \Big|_{u=0} = \lim_{p\to 0} \lim_{u\to 0} \int e^{ipx} \langle \hat{Q}(x) \hat{Q}(0) \rangle = \int \langle \hat{Q}(x) \hat{Q}(0) \rangle \Big|_{YM}$$

- The issue is how to define finite operators and correlators
  Di Vecchia Fabricius Rossi Veneziano; Di Giacomo ...; Giusti Testa ...

The Witten–Veneziano formula - Numerics

- taking $m_S^2 \sim m_{\eta'}^2 + m_\eta^2 - 2m_K^2$
  as the mass of the singlet in the chiral limit,
  the WV formula $m_S^2 = 2AN_f/F_\pi^2$ yields
  the phenomenological estimate $A = (180 \text{ MeV})^4$ Veneziano
- first lattice simulation (1981) Di Vecchia Fabricius Rossi Veneziano
  wrong by a missed renormalization factor Di Giacomo *et al.*
- the lastest estimate (2015) gives $A = (180 \text{ MeV})^4$
  Cé Consonni Engel Giusti
- an impressive agreement between theory and phenomenology

θ-angle, strong CP problem & the axion

QCD and the θ-angle

- $Z(\theta) = \int \mathcal{D}\mu \exp i \int dx\, \mathcal{L}_{QCD}(\theta)$
- $\mathcal{L}_{QCD}(\theta) = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{\psi}(i\gamma^\mu\partial_\mu + g\gamma^\mu A_\mu + M)\psi + \frac{g^2}{16\pi^2}\theta\mathrm{Tr}(\tilde{F}_{\mu\nu}F^{\mu\nu})$
- $Z(\theta) = Z(\theta + 2\pi) \quad \leftrightarrow \quad Z(\theta) = \sum_{n=-\infty}^{n=\infty} e^{i\theta n} Z_n$
- $n = \int dx\, Q = \frac{g^2}{16\pi^2}\int dx\, \mathrm{Tr}(\tilde{F}_{\mu\nu}F^{\mu\nu})$, aka "winding" or Pontryagin number

Notes

- $\theta = 0$ means summing over all Pontryagin numbers
- $\mathrm{Tr}(\tilde{F}_{\mu\nu}F^{\mu\nu}) = \partial_\mu K_\mu \quad \rightarrow \quad \int dx\, Q = 0$ in PT
- $Z(\theta)$ actually only depends upon $\bar{\theta} = \theta + Arg\, det\, M$

The QCD LEEA in the presence of a θ-angle

$$\Gamma_{eff}(U,M;\theta) = \int dx \Big[\frac{F_\pi^2}{4} \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + v\mathrm{Tr}(UM^\dagger + MU^\dagger) +$$

$$-\frac{A}{2}\Big(\theta - \frac{i}{2}\mathrm{Tr}\log(U/U^\dagger)\Big)^2 + \dots \Big]$$ Di Vecchia, Veneziano, Witten

$$U = \exp\Big[\frac{2i}{F_\pi}\Big(\sum_{f=1}^{N_f^2-1} \xi^f \lambda^f + \frac{1}{\sqrt{2N_f}}\eta_0\Big)\Big], \quad 2v = F_\pi^2 m_\pi^2/(m_u+m_d)$$

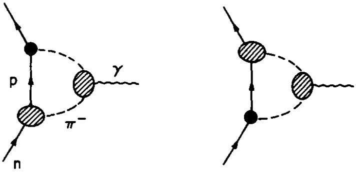
It incorporates the key features of the theory

- the θ-dependence only on upon $\bar{\theta} = \theta + Arg\ det\ M$
- the η'-mass formula
$$\frac{\partial^2 \Gamma_{eff}}{\partial \theta^2}\Big|_{\theta=0} = A = \int dx\, \langle Q(x) Q(0)\rangle \Big|_{YM} = m_{\eta'}^2 \frac{F_\pi^2}{2N_f}$$
- pseudoscalar mass spectrum (expand U up to quadratic terms)
- Γ_{eff} is seen to be θ independent, if one of the quark is massless
- Γ_{eff} + baryons allows computing the neutron EDM

The strong CP problem

- In χPT from Γ_{eff} + baryons, one finds the (almost model independent) result $|D_n| \sim 10^{-16}\theta$
  Crewther, Di Vecchia, Veneziano, Witten
- experimentally one gets the bound $|D_n| < 10^{-25}$ Dress *et al.* 1977
- from which one derives $\theta < 10^{-9}$
- then the strong CP problem can be formulated as follows
  - CP is not a symmetry of the SM
  - then why θ is so small?
  - or (*á la* 't Hooft) is there a symmetry that is recovered at $\theta = 0$?

Figure : Typical diagrams contributing to neutron EDM in the chiral limit



The Peccei-Quinn proposal - I

- consider an extension of the SM with two Higgses
- coupled to *up* and *down* quarks, respectively
- the relevant terms in the SM read

$$\mathcal{L}_{Yuk} = -\frac{m_u}{\langle\Phi_u\rangle}\bar{Q}_L^T \Phi_u u_R - \frac{m_d}{\langle\Phi_d\rangle}\bar{Q}_L^T \Phi_d d_R + \text{leptons} + \text{h.c.}$$

- $\mathrm{U}_u(1)\times\mathrm{U}_d(1)$ spontaneously broken symmetry, parametrized as

$$\Phi_u \to e^{i\alpha x_u}\Phi_u\,, \quad \Phi_d \to e^{i\alpha x_d}\Phi_d\,, \quad u_R \to e^{i\alpha x_u} u_R\,, \quad d_R \to e^{-i\alpha x_u} d_R$$

- 1) $x_u = -x_d \to$ non anomalous U(1) transformation
  - the corresponding Goldstone boson is $[\langle\Phi_u\rangle\Phi_u - \langle\Phi_d\rangle\Phi_d]\langle\Phi\rangle^{-1}$
  - eaten up by the $W \;\to\; M_W^2 = g^2(\langle\Phi_u\rangle^2 + \langle\Phi_d\rangle^2) \equiv g^2\langle\Phi\rangle^2$

The Peccei-Quinn proposal - II

- 2) $U^{PQ}(1)$: $x_u = x, x_d = 1/x$ with $x = \langle\Phi_u\rangle/\langle\Phi_d\rangle$
  anomalous and spontaneously broken
  - $J_\mu^{PQ} = \frac{x}{2}\Phi_u^*\partial_\mu\Phi_u + \frac{1}{2x}\Phi_d^*\partial_\mu\Phi_d + \frac{x}{2}\bar{u}\gamma_\mu\gamma_5 u + \frac{1}{2x}\bar{d}\gamma_\mu\gamma_5 d =$
    $= \langle\Phi\rangle\partial_\mu a + \frac{x}{2}\bar{u}\gamma_\mu\gamma_5 u + \frac{1}{2x}\bar{d}\gamma_\mu\gamma_5 d$
  - $a = [\langle\Phi_d\rangle\Phi_u + \langle\Phi_u\rangle\Phi_d]\langle\Phi\rangle^{-1}$ is the associated Goldstone boson
    $a \perp [\langle\Phi_u\rangle\Phi_u - \langle\Phi_d\rangle\Phi_d]\langle\Phi\rangle^{-1}$
  - $\partial_\mu J_\mu^{PQ} = c^{PQ} Q\,, \quad c^{PQ} = N_f(x^2+1)/2x$
  - the axion a gets a mass from the anomaly similarly to the η'

The LEEA in the presence of a θ-angle and the axion

- The LEEA

$$\Gamma_{eff}(U, M, N; \theta) = \int dx \Big\{ \frac{F_\pi^2}{4} \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + v\mathrm{Tr}(UM^\dagger + MU^\dagger) +$$

$$+ \frac{F_a^2}{4} \partial_\mu N^\dagger \partial^\mu N + \frac{q^2}{2A} - \theta q +$$

$$+ \frac{iq}{2}\Big(\mathrm{Tr}[\log U - \log U^\dagger] + \alpha_{PQ} [\log N - \log N^\dagger]\Big)\Big\}$$

$$U = \exp\Big[\frac{2i}{F_\pi}\Big(\sum_{f=1}^{N_f^2-1} \xi^f \lambda^f + \frac{1}{\sqrt{2N_f}}\eta_0\Big)\Big] \qquad N(x) = \exp\frac{2ia(x)}{F_a}$$

- Expanding up to quadratic terms and diagonalizing mass matrix, one gets Di Vecchia, Veneziano, Sannino
  - axion mass $F_a^2 m_a^2 = 2\alpha_{PQ}^2 \left(\frac{1}{\chi^{YM}} + \sum_i \frac{1}{m_i |\langle \bar{q} q \rangle_i|} \right)^{-1} \qquad A = \chi^{YM}$
  - and mixings within the pseudoscalar meson singlet
  - α_{PQ} is model-dependent, e.g. $\alpha_{PQ} = c^{PQ} = N_f(x^2+1)/2x$

The axion mass as a function of T

- Summarizing (recall GMOR relation $F_\pi^2 m_\pi^2 = 2m_q\langle\bar\psi\psi\rangle$)

$$F_a^2 m_a^2\Big|_T = \frac{2\alpha_{PQ}^2}{\frac{1}{\chi_T^{YM}} + \sum_i \frac{1}{m_i|\langle\bar q q\rangle_i|_T}} = 2\alpha_{PQ}^2\,\chi_T^{QCD} \qquad (1)$$

$$F_{\eta'}^2 m_{\eta'}^2 = 2N_f\,\chi^{YM}$$

- Since $\chi_{T=0}^{YM} \neq 0$, light mass terms dominate and we get

$$F_a^2 m_a^2\Big|_{T=0} = 2\alpha_{PQ}^2 m_\pi^2 F_\pi^2 \frac{m_1 m_2}{(m_1+m_2)^2} \qquad \text{Weinberg}$$

- If at $T \neq 0$ the topological susceptibility becomes very small below the deconfinement and χSB temperature, we obtain

$$F_\alpha^2 m_\alpha^2\Big|_T = 2\alpha_{PQ}^2\,\chi_T^{YM}$$

- Questions are
  - behaviour of χ_T^{QCD} as function of T: who wins in (1)?
  - validity of the chiral LEEA analysis, based on large N

Thank you for your attention

The Witten–Veneziano formula - Theory

- Theorem
  - in the limit $u = N_f/N_c \to 0$, $m_S^2 \to 0$
- A simplified Proof (ignoring renormalization subtleties) Witten
  - $V\, E(\theta) = \int \mathcal{D}\mu \exp i \int dx\, \mathcal{L}_{QCD}(\theta)$
  - $E(\theta)$ does not depend on θ, if (at least) one quark is massless
  - Recall $d^2 E(\theta)/d\theta^2|_{\theta=0} = A = \int dx\, \langle Q(x) Q(0)\rangle$
  - Define $A(k) = \int dx\, e^{ikx} \langle Q(x) Q(0)\rangle = \mathrm{O}(N_c^2)$
    - $A(k=0) \neq 0$ in the absence of quarks (pure YM)
    - $A(k=0) = 0$ in QCD with a massless quark
  - Quark loop expansion
    - $A(k) = A_0(k) + A_1(k) + A_2(k) + \ldots$
    - $A(k) = \mathrm{O}(N_c^2) + N_f \mathrm{O}(N_c) + N_f^2 \mathrm{O}(1) + \ldots$
  - The puzzle: how can chiral quark loops make $A(k=0)=0$, since
    - $A_0(k=0) \neq 0$ without quarks
    - and each quark loop is down by a factor N_f/N_c
  - The solution
    - $A(k) = \sum_{glueballs} \frac{N_c^2 a_n}{k^2 - M_n^2} + \sum_{mesons} \frac{N_f N_c b_n}{k^2 - m_n^2} + \ldots$
    - there must be a meson singlet with $m_S^2 = \mathrm{O}(N_f/N_c)$ as $N_f/N_c \to 0$