

Review of $g - 2$ theory

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KLOE-2 Workshop on e^+e^- collision physics at 1 GeV – Frascati, October 26 - 28, 2016

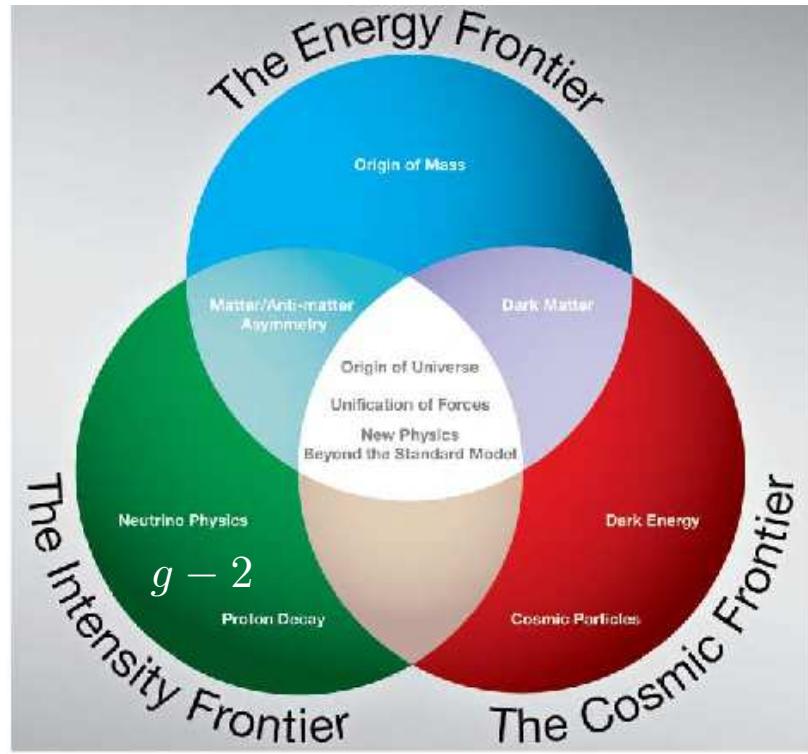


OUTLINE

- Introduction
- QED contributions
- Weak contributions
- Hadronic contributions
 - hadronic vacuum polarization (HVP) —→ talk by F. Jegerlehner
 - hadronic light-by-light (HLxL)
- Summary - Conclusion

Introduction

General context: looking for new physics by exploring the “three frontiers”



The anomalous magnetic moment of the muon explores the intensity (or precision) frontier (possible evidence of new physics in quantum effects)

Requires:

- high precision measurement
- equally precise prediction of the SM value

a_μ is experimentally measured to very high precision

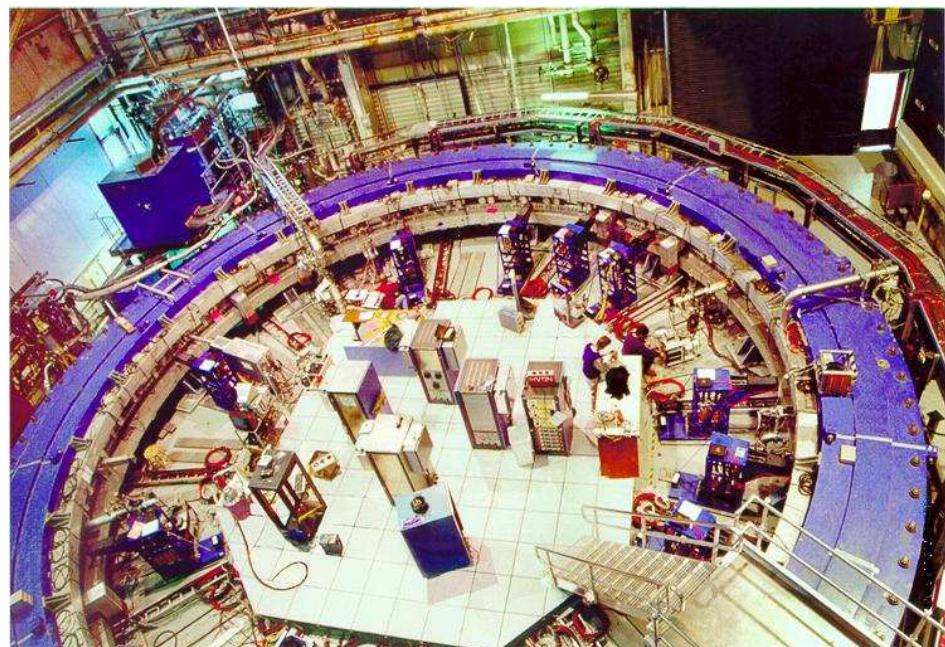
$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

$$\gamma \sim 29.3, p \sim 3.094 \text{ GeV/c}$$

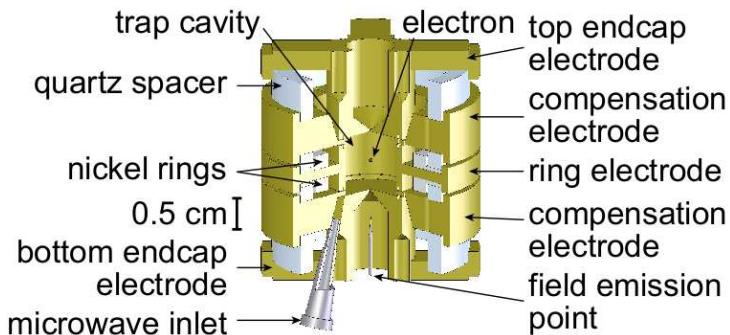
$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \text{ [0.54 ppm]}$$

G. W. Bennett et al, Phys Rev D 73, 072003 (2006)



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G. W. Bennett et al, Phys Rev D 73, 072003 (2006)

Note: $\tau_\tau = (290.6 \pm 1.1) \times 10^{-15} \text{ s}$

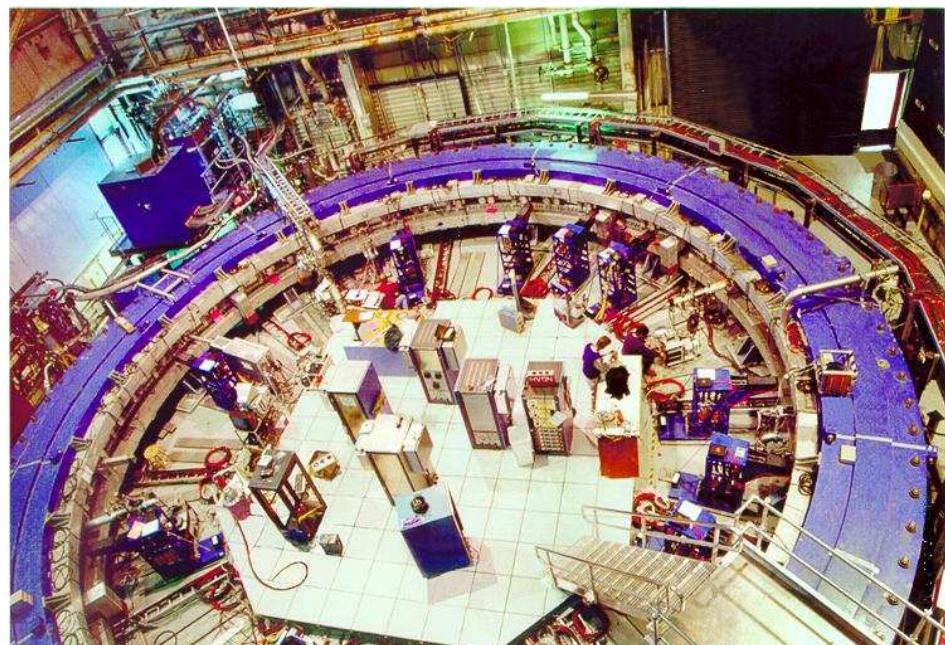
$$-0.052 < a_\tau^{\exp} < +0.013 \text{ (95% CL)} \quad [e^+e^- \rightarrow e^+e^-\tau^+\tau^-] \quad \text{DELPHI, Eur. Phys. J. C 35, 159 (2004)}$$

theory: $a_\tau = 117721(5) \cdot 10^{-8}$

$$a_e^{\exp} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$$

$$\Delta a_e^{\exp} = 2.8 \cdot 10^{-13} \text{ [0.24ppb]}$$

D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)



S. Eidelman, M. Passera, Mod. Phys. Lett. A 22, 159 (2007)
 S. Narison, Phys Lett B 513 (2001); err. B 526, 414 (2002)

QED contributions

QED contributions : loops with only photons and leptons

→ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \dots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)}$ → mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}), A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \rightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

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$A_2^{(2n)}(m_\ell/m_{\ell'}), A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$ →

mass-dependent (non-universal) contributions (multi-flavour QED)

- a_ℓ is finite (no renormalization needed) and dimensionless
 - QED is decoupling
 - Massive degrees of freedom with $M \gg m_\ell$ contribute to a_ℓ through powers of m_ℓ^2/M^2 times logarithms (*) → for a_e the $A_1^{(2n)}$ matter
 - Light degrees of freedom with $m \ll m_\ell$ give logarithmic contributions to a_ℓ , e.g. $\ln(m_\ell^2/m^2) \left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$
 - for a_μ $A_2^{(2n)}(m_\ell/m_{\ell'})$ and $A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$ matter
- (*) also applies to BSM physics to the extent that it is decoupling!

QED contributions : loops with only photons and leptons

→ can be computed in perturbation theory:

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$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

→ requires an input for the fine structure constant α that matches the experimental accuracy on a_ℓ

$$\alpha^{-1}[\text{Rb 11}] = 137.035\,999\,037(91) \quad [0.66\text{ppb}]$$

R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)

QED contributions : loops with only photons and leptons

→ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \dots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

order $(\alpha/\pi)^4$: 891 diagrams

only a few diagrams are known analytically \longrightarrow numerical evaluation

Automated generation of diagrams, systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

$$A_1^{(8)} = -1.912\,98(84)$$

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373) \cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118) \cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28) \cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)

A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, Phys. Rev. D 92, 073019 (2015)]

Agreement at the level of accuracy required by present and future experiments for a_μ
 a_e : $A_1^{(8)}$ remains unchecked so far!

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)

J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\dots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\dots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.9096(20)$	$130.879\,6(63)$
$C_\ell^{(10)}$	$9.16(58)$	$753.29(1.04)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\dots\cdot 10^{-3}$	$5.39\dots\cdot 10^{-6}$	$1.25\dots\cdot 10^{-8}$	$2.91\dots\cdot 10^{-11}$	$6.76\dots\cdot 10^{-14}$

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\begin{aligned} \Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 &\sim 0.9 \cdot 10^{-13} & \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 &\sim 0.04 \cdot 10^{-13} \\ \Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 &\sim 1.8 \cdot 10^{-13} & \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 &\sim 0.7 \cdot 10^{-13} & \Delta a_\mu^{\text{exp}} &= 6.3 \cdot 10^{-10} \end{aligned}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9} \quad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50!$]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \text{LxL}) \left[\frac{2}{3} \ln \frac{m_\mu}{m_e} - \frac{5}{9} \right]^3 \cdot 10 \left(\frac{\alpha}{\pi} \right)^6 \sim 0.6 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi} \right)^6 \sim 1 \cdot 10^{-12}$$

- No sign of substantial contribution to a_μ from higher order QED

$$a_\mu^{\text{QED}}(Rb) = 1\,165\,847\,189.51(9)_{\text{mass}}(19)_{\alpha^4}(7)_{\alpha^5}(77)_{\alpha(Rb11)} \cdot 10^{-12}$$

$$a_\mu^{\text{QED}}(a_e) = 1\,165\,847\,188.46(9)_{\text{mass}}(19)_{\alpha^4}(7)_{\alpha^5}(30)_{\alpha(a_e)} \cdot 10^{-12}$$

Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)

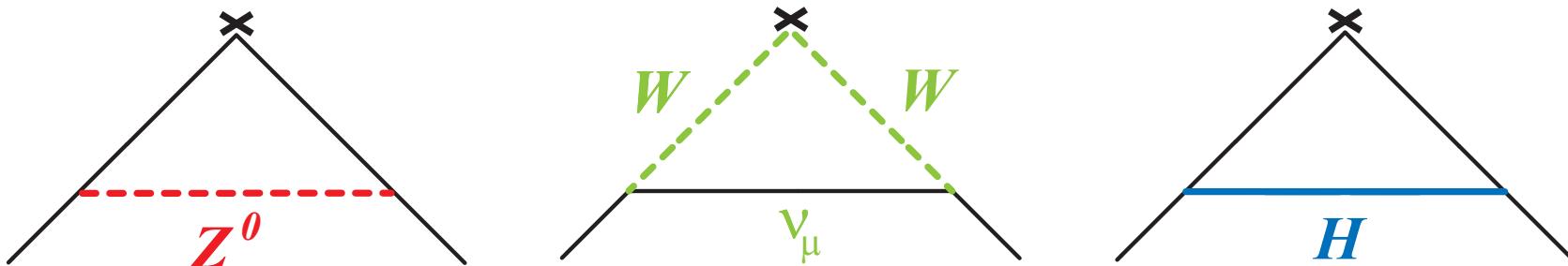
$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} = 737.0(6.3) \cdot 10^{-10}$$

QED provides more than 99.99% of the total value, without uncertainties at this level of precision

The missing part has to be provided by weak and strong interactions

Contributions from weak interactions

- Weak contributions : W , Z ,... loops



$$\begin{aligned}
 a_\mu^{\text{weak}(1)} &= \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3} (1 - 4 \sin^2 \theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \log \frac{M_Z^2}{m_\mu^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{M_H^2} \log \frac{M_H^2}{m_\mu^2}\right) \right] \\
 &= 19.48 \times 10^{-10}
 \end{aligned}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabibbo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

$$a_{\mu}^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_{\mu}^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_{\mu}^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_{\mu}^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \cdot (-79.3)$$

A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)

$$a_{\mu}^{\text{weak}} = (154 \pm 1) \cdot 10^{-11}$$

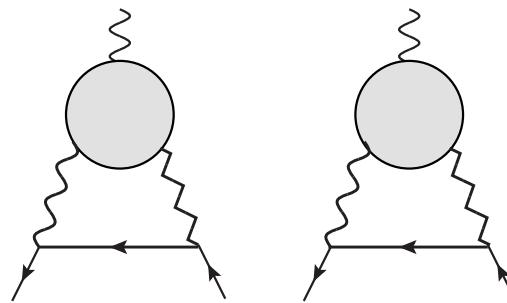
$$a_e^{\text{weak}} = (0.0297 \pm 0.0005) \cdot 10^{-12}$$

Recent update: $a_{\mu}^{\text{weak}} = (153.6 \pm 1.0) \cdot 10^{-11}$

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

A few remarks about the weak contributions

- The knowledge of the mass of the EW scalar boson has reduced the uncertainty in the one-loop EW correction
- The EW correction also includes the complete three-loop short-distance leading logarithms
- There are also hadronic contributions at two-loops, e.g. those [enhanced by $\ln(m_Z/m_\mu)$ and not suppressed by $1 - 4 \sin^2 \theta_w$] involving the $\langle VVA \rangle$ three-point function



$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{weak}} = 721.65(6.38) \cdot 10^{-10}$$

Hadronic contributions

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as

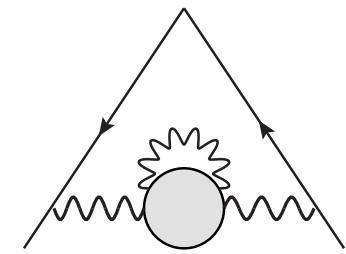
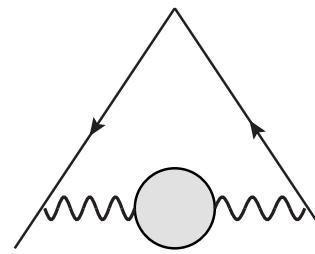
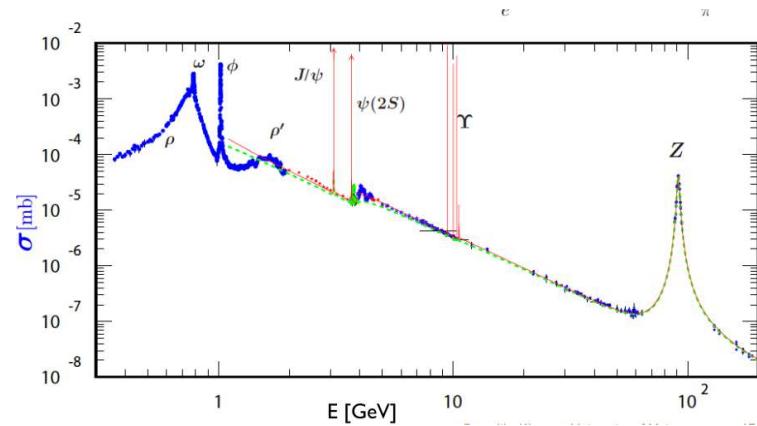
$$a_\ell^{\text{HVP-LO}} = \frac{1}{3} \left(\frac{\alpha}{\pi} \right)^2 \int_{4M_\pi^2}^\infty \frac{dt}{t} K(t) R^{\text{had}}(t) \quad K(t) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x) \frac{t}{m_\ell^2}}$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \implies a_\ell^{\text{HVP-LO}} > 0$
- $K(s) \sim m_\ell^2/(3s)$ as $s \rightarrow \infty \implies$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization

- Can be evaluated using available experimental data



- Some order $\mathcal{O}(\alpha^3)$ corrections included
 - exchange of virtual photons between final state hadrons
 - some radiative exclusive modes, e.g. $\pi^0\gamma$

$$a_\mu^{\pi^0\gamma}(600 \text{ MeV} - 1030 \text{ MeV}) = 4.4(1.9) \cdot 10^{-10}$$

Latest results

$$a_\mu^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10} \quad [\text{M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)}]$$

$$a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10} \quad [\text{K. Hagiwara et al., J. Phys. G 38, 085003 (2011)}]$$

$$a_\mu^{\text{HVP-LO}} = 686.99 \pm 4.21 \cdot 10^{-10} \quad [\text{F. Jegerlehner, EPJ Web Conf. 118 (2016)}]$$

$$a_\mu^{\text{HVP-NLO}} = \frac{1}{3} \left(\frac{\alpha}{\pi} \right)^3 \int_{4M_\pi^2}^\infty \frac{dt}{t} K^{(2)}(t) R^{\text{had}}(t)$$

J. Calmet, S. Narison, M. Perrottet, E. de Rafael, Phys. Lett. B 61, 283 (1976)

B. Krause, Phys. Lett. B 390, 392 (1997)

$$a_\mu^{\text{HVP-NLO}} = -9.84 \pm 0.07 \cdot 10^{-10} \quad [\text{K. Hagiwara et al., J. Phys. G 38, 085003 (2011)}]$$

$$a_\mu^{\text{HVP-NLO}} = -9.934 \pm 0.091 \cdot 10^{-10} \quad [\text{F. Jegerlehner, EPJ Web Conf. 118 (2016)}]$$

$$a_\mu^{\text{HVP-NNLO}} = 1.24 \pm 0.01 \cdot 10^{-10} \quad [\text{A. Kurz et al., Phys. Lett. B 734, 144 (2014)}]$$

$$a_\mu^{\text{HVP-NNLO}} = 1.226 \pm 0.012 \cdot 10^{-10} \quad [\text{F. Jegerlehner, EPJ Web Conf. 118 (2016)}]$$

Hadronic vacuum polarization

- Possibility to extract HVP from Bhabha scattering?

C. M. Carloni-Calame, M. Passera, L. Trentadue, G. Venanzoni, Phys. Lett. B 476, 325 (2015)

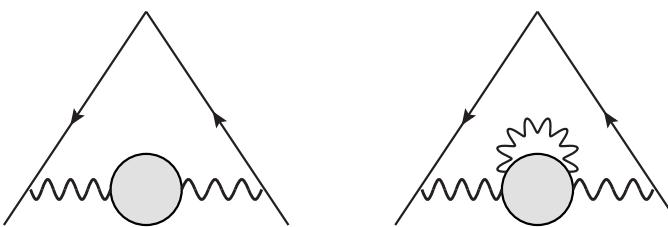
- Constraints on the pion electromagnetic form factor from analyticity and unitarity

H. Leutwyler, arXiv-ph/0212324

B. Ananthanarayan, I. Caprini, D. Das, I. S. Imsong, Phys. Rev. D 93, 116007 (2016)

- Alternative for the (near?) future: **Lattice QCD**:
several contributions at recent ICHEP and LATTICE conferences

comparison with data at the sub-percent level:
isospin breaking effects (radiative corrections)



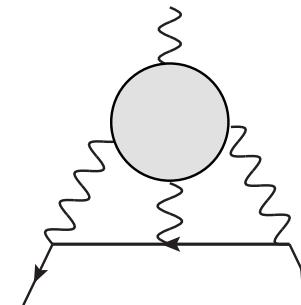
(Experimentalists don't live in the theoretician's paradise)

Hadronic light-by-light: the really complicated thing

- Occurs at order $\mathcal{O}(\alpha^3)$
- Not related, as a whole, to an experimental observable...

?

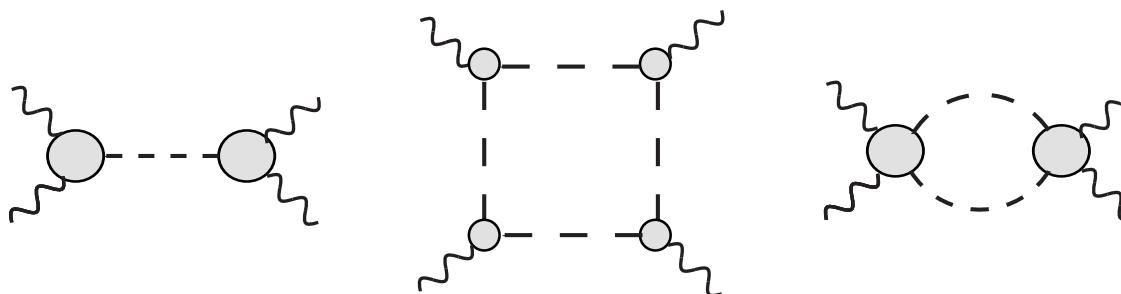
→



- Involves the fourth-rank vacuum polarization tensor

$$\text{F.T. } \langle 0 | T\{VVVV\} | 0 \rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4) \quad q_1 + q_2 + q_3 + q_4 = 0$$

- Many individual contributions have been identified...



Hadronic light-by-light

- Need some organizing principle: ChPT, large- N_c (turns out to be most relevant in practice)

E. de Rafael, Phys. Lett. B 322, 239 (1994)

$$a_\mu^{\text{HLxL}} = N_c \left(\frac{\alpha}{\pi} \right)^3 \frac{N_c}{F_\pi^2} \frac{m_\mu^2}{48\pi^2} \left[\ln^2 \frac{M_\rho}{M_\pi} + c_\chi \ln \frac{M_\rho}{M_\pi} + \kappa \right] + \mathcal{O}(N_c^0)$$

M. Knecht, A. Nyffeler, Phys. Rev. D 65, 073034 (2002)

M. Knecht, A. Nyffeler, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002)

M. J. Ramsey-Musolf, M. B. Wise, Phys. Rev. Lett. 89, 041601 (2002)]

J. Prades, E. de Rafael, A. Vainshtein, Glasgow White Paper (2008)

- Impose QCD short-distance properties

$$\lim_{\lambda \rightarrow \infty} \Pi_{\mu\nu\rho\sigma}(\lambda q_1, q_2 - \lambda q_1, q_3, q_4) = \frac{1}{\lambda} \frac{2i}{3} \frac{q_1^\gamma}{q_1^2} \epsilon_{\mu\nu\gamma}{}^\tau W_{\rho\sigma\tau}(q_3, q_4, q_2) + \mathcal{O}(1/\lambda^2)$$

q_1 euclidian, $q_2 + q_3 + q_4 = 0$, F.T. $\langle 0 | T\{VVA\} | 0 \rangle \longrightarrow W_{\rho\sigma\tau}(q_3, q_4, q_2)$

K. Melnikov, A. Vainshtein, Phys. Rev. D, 113006 (2004)

$$\lim_{\lambda \rightarrow \infty} \Pi_{\mu\nu\rho\sigma}(\lambda q_1, \lambda q_2, q_3 - \lambda q_1 - \lambda q_2, -q_3) = \dots$$

$$\lim_{\lambda \rightarrow \infty} \Pi_{\mu\nu\rho\sigma}(\lambda q_1, \lambda q_2, \lambda q_3, \lambda q_4) = \Pi_{\mu\nu\rho\sigma}^{\text{PQCD}}(q_1, q_2, q_3, q_4) + \dots$$

Hadronic light-by-light

- Present estimates rely mainly on two model-dependent calculation

$$a_\mu^{\text{HLxL}} = + (8.3 \pm 3.2) \cdot 10^{-10}$$

J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)

$$a_\mu^{\text{HLxL}} = + (89.6 \pm 15.4) \cdot 10^{-11}$$

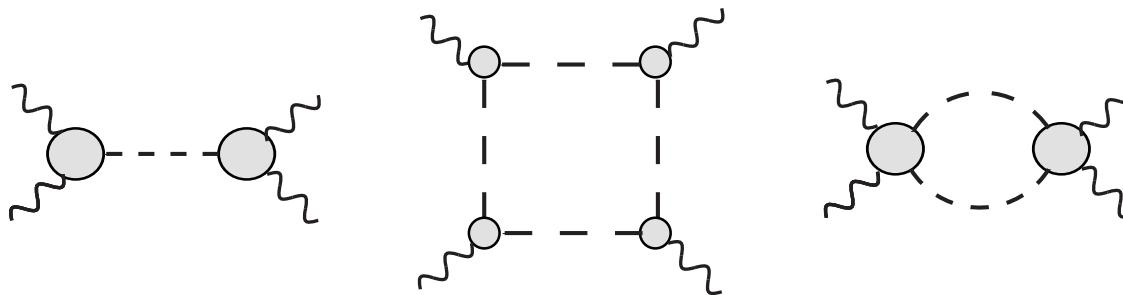
M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D 54, 3137 (1996)
M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)]

that turn out to be positive [M.K. and A. Nyffeler, Phys. Rev. D 65, 073034 (2002)]

Hadronic light-by-light

- More recently: dispersive approaches

— for $\Pi_{\mu\nu\rho\sigma}$ 



$$\Pi = \Pi^{\pi^0, \eta, \eta' \text{ poles}} + \Pi^{\pi^\pm, K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

Needs input from data (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)

[A. Nyffeler, arXiv:1602.03398 [hep-ph]]

— for $F_2^{\text{HLxL}}(k^2)$

only pion pole with VMD form factor (two-loop graph) reconstructed this way so far

V. Pauk and M. Vanderhaeghen, Phys. Rev. D 90, 113012 (2014)

Hadronic light-by-light: the really complicated thing

- Other recent approaches
 - Dyson-Schwinger/Bethe-Salpeter equations

G. Eichman, HC2NP Workshop, Teneriffe, Sept. 2016

- Non-local quark model
 - A. E. Dorokhov, A. E. Radzhabov and A. S. Zhevlakov, Eur.Phys.J. C75, 417 (2015)

Goal: evaluation of HLxL with a reliable uncertainty of $\sim 10\%$

Open issues:

- how will short-distance constraints be imposed?
- how will Π^{residual} be estimated? Cf. axial vectors (leading in large- N_c) $\rightarrow 3\pi$ channel
- how will remaining model-dependence be estimated?

units: 10^{-11}

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	—	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	—	—	—	-19 ± 19	-19 ± 13
π, K l. + subl. in Nc	—	—	—	0 ± 10	—	—	—
axial vectors	2.5 ± 1.0	1.7 ± 1.7	—	22 ± 5	—	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	—	—	—	—	-7 ± 7	-7 ± 2
quark loops	21 ± 3	9.7 ± 11.1	—	—	—	2.3	21 ± 3
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

BPP: J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75 (1995) 1447 [Erratum-ibid. 75 (1995) 3781]; Nucl. Phys. B 474 (1996) 379; [Erratum-ibid. 626 (2002) 410]

HKS: M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75 (1995) 790; Phys. Rev. D 54 (1996) 3137

KN: M. Knecht, A. Nyffeler, Phys. Rev. D 65 (2002) 073034

MV: K. Melnikov, A. Vainshtein, Phys. Rev. D 70 (2004) 113006

BP: J. Bijnens, J. Prades, Acta Phys. Polon. B 38 (2007) 2819; Phys. Proc. Suppl. 181-182 (2008) 15; Mod. Phys. Lett. A 22 (2007) 767

BdRV: J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306 [hep-ph]

N/NJ: A. Nyffeler, Phys. Rev. D 79, 073012 (2009); F. Jegerlehner, A. Nyffeler, Phys. Rep. (2009)

Recent reevaluation of single meson exchanges...

$$a_\mu(f_1, f'_1) = 6.4(2.0) \cdot 10^{-11} \quad a_\mu(f_0, f'_0, a_0) = (-1 \text{ to } -4) \cdot 10^{-11} \quad a_\mu(f_2, f'_2, a_2, a'_2) = 1.1(0.1) \cdot 10^{-11}$$

V. Pauk, M. Vanderhaeghen, Eur. Phys. J C 74, 3008 (2014)

$$a_\mu(a_1, f_1, f'_1) = 7.51(2.71) \cdot 10^{-11} \quad F. Jegerlehner, EPJ Web Conf. 118 (2016)$$

... or of pion box contribution

$$a_\mu^\pi \text{ box} \sim -15.9 \cdot 10^{-11} \quad M. Procura et al., EPJ Web Conf. 118 (2016)$$

Summary - Conclusions

- The anomalous magnetic moments of the muon is among the most precisely measured observables of the standard model

$$a_{\mu}^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]$$

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- At present, the standard model value for a_{μ} misses the experimental determination by about 3.5 standard deviations

Theory (in units of 10^{-10})

QED	$+116\,584\,71.9$	[T. Aoyama et al. (2015)]
HVP-LO	$\begin{cases} +692.3(4.2) \\ +694.9(4.3) \\ +686.99(4.21) \end{cases}$	[M. Davier et al. (2011)] [K. Hagiwara et al. (2011)] [F. Jegerlehner (2015)]
HVP-NLO	$-9.84(7)$	[K. Hagiwara et al. (2011)]
HVP-NNLO	$+1.24(1)$	[A. Kurz et al. (2014)]
HLxL	$\begin{cases} +10.5(2.6) \\ +11.5(4.0) \\ +10.6(3.9) \end{cases}$	[J. Prades et al. (2009)] [F. Jegerlehner, A. Nyffeler (2009)] [F. Jegerlehner (2015)]
EW 1 loop	$+19.48(1)$	[(1972)]
EW 2 loops	$-4.12(10)$	[C. Genniger et al. (2013)]

- The anomalous magnetic moments of the muon is among the most precisely measured observables of the standard model

$$a_{\mu}^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]$$

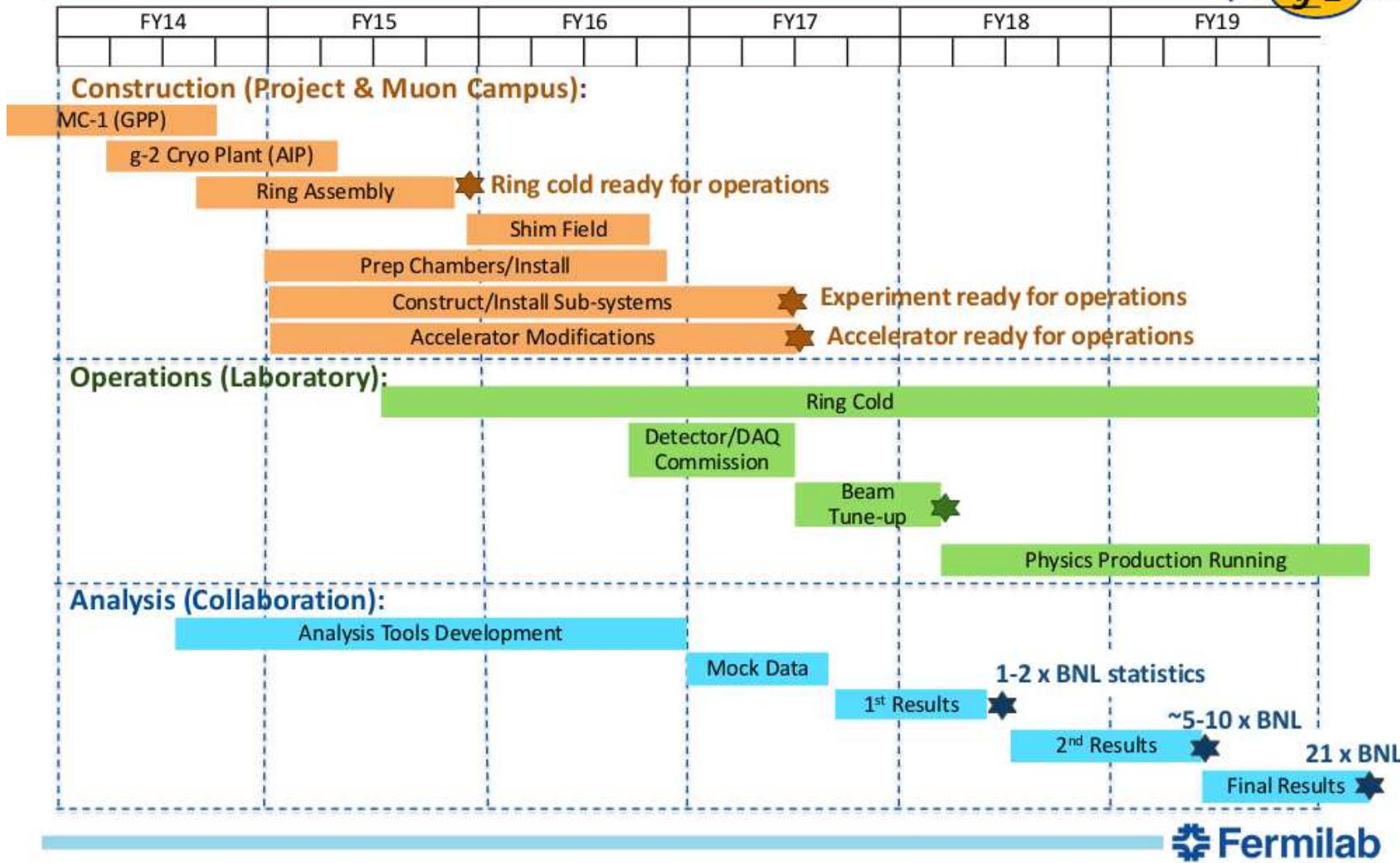
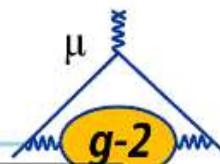
- At present, the standard model value for a_{μ} misses the experimental determination by about 3.5 standard deviations
- It is not obvious to find a straightforward explanation for this persistent discrepancy:

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (27.4 \pm 8.0) \cdot 10^{-10} \quad (\sim 2 \cdot a_{\mu}^{\text{weak}}, \sim a_{\mu}^{\text{QED}}(\alpha^4), \sim 60 \cdot a_{\mu}^{\text{QED}}(\alpha^5), \sim 3 \cdot a_{\mu}^{\text{HLxL}}, \dots)$$

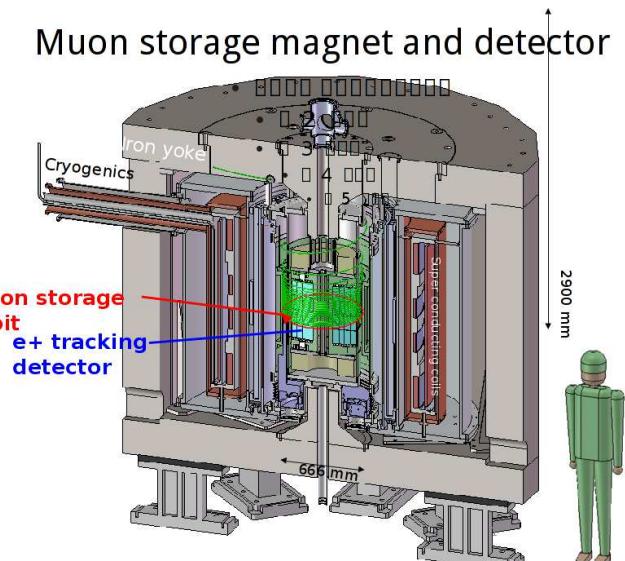
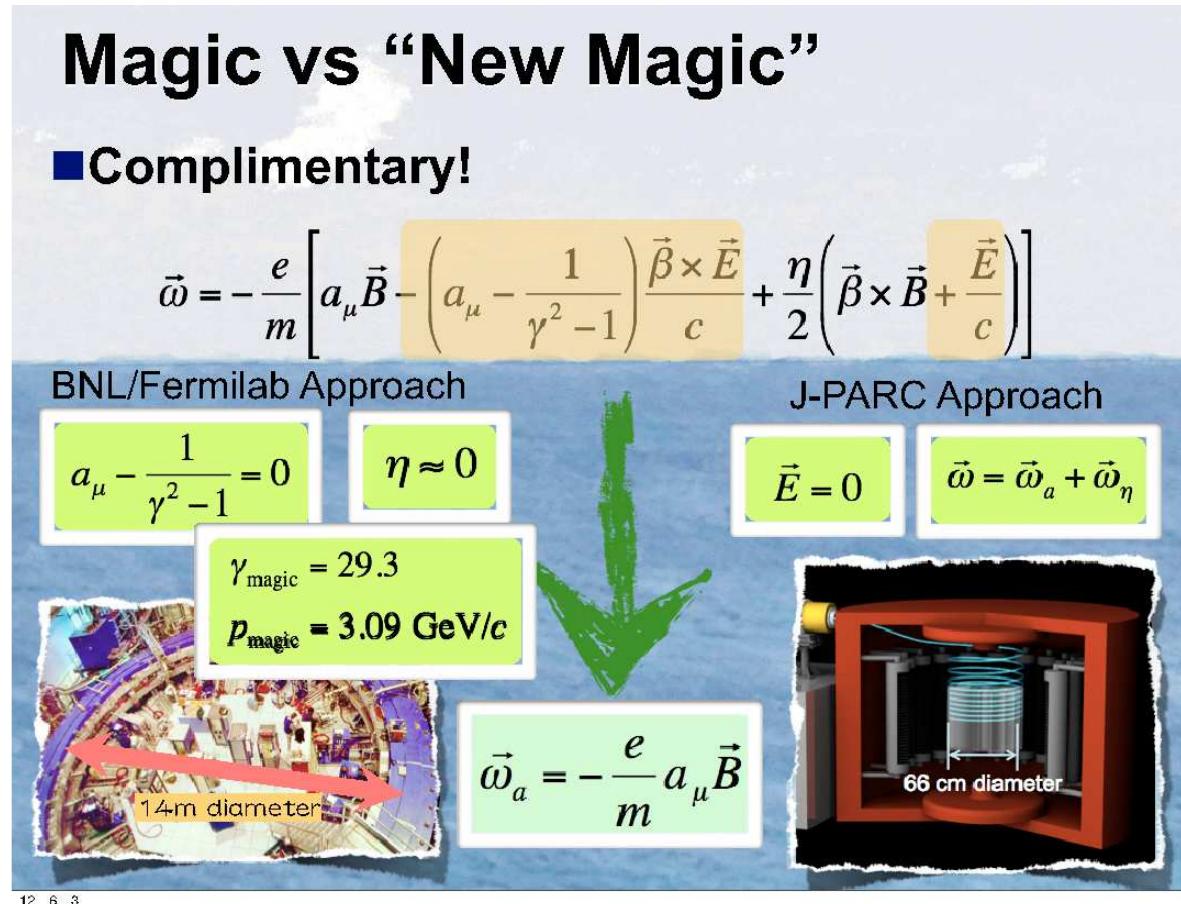
$$\Delta a_{\mu}^{\text{exp}} = 6.3 \cdot 10^{-10}$$

- Two new experiments, with the aim of reducing the experimental uncertainty by a factor of 4, are being prepared, at FNAL (first results expected in \sim 2 years)

Project Timeline



- Two new experiments, with the aim of reducing the experimental uncertainty by a factor of 4, are being prepared, at FNAL (first results expected in ~ 2 years) and at J-PARC



- Two new experiments, with the aim of reducing the experimental uncertainty by a factor of 4, are being prepared, at FNAL (first results expected in ~ 2 years) and at J-PARC
- New high-precision data from VEPP-2000, BESIII,... will reduce the uncertainties on HVP (tensions between data need to be resolved, contributions from lattice QCD to be expected)
- Uncertainty on HLxL will become the limiting factor on theory precision
- New input from theory+experiment+lattice required

Thanks for your attention!