

g – 2 theory: the hadronic part

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Outline of Talk:

- ❖ Introduction
- ❖ Hadronic Vacuum Polarization (HVP) – Data & Status
- ❖ NLO and NNLO HVP effects
- ❖ A problem in DR for HVP and a first direct measurement of $\Pi'_\gamma(s)$
- ❖ Effective field theory: the Resonance Lagrangian Approach
- ❖ HVP from lattice QCD
- ❖ Alternative method: measure space-like $\alpha_{\text{QED,eff}}(t) \rightarrow a_\mu^{\text{had}}$
- ❖ Theory vs experiment: do we see New Physics?

Introduction

Review Theory Marc Knecht, Massimiliano Procura

Review hadronic cross sections Graziano Venanzoni, Simon Eidelman, Achim Denig

$$a_{\mu}^{\text{exp}} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10} \text{ BNL updated}$$

To come – :

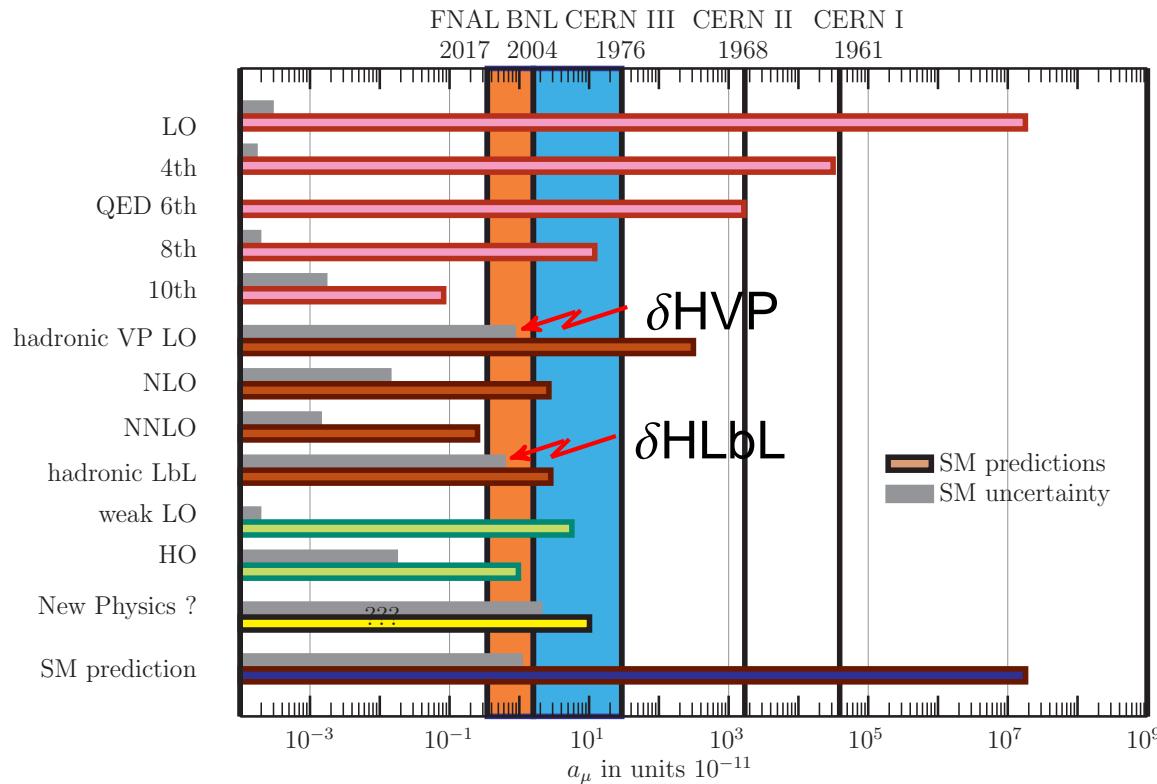
New muon $g - 2$ experiments at Fermilab and J-PARC: improve error by factor 4

⇒ new muon $g - 2$ experiment: $\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{the}} = 4.3 \sigma$ theory as today

Reduction of hadronic uncertainty by factor 2 ⇒ $\Delta a_{\mu} = 7.7 \sigma$

That's what we hope to achieve!

And here we are:



Past and future $g - 2$ experiments testing various contributions.

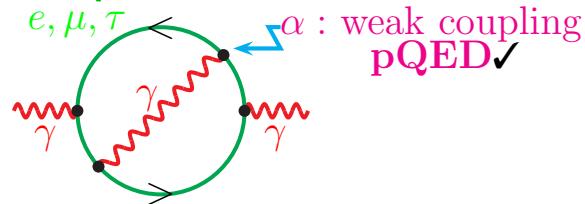
New Physics $\stackrel{?}{=}$ deviation $(a_\mu^{\text{exp}} - a_\mu^{\text{the}})/a_\mu^{\text{exp}}$.

Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

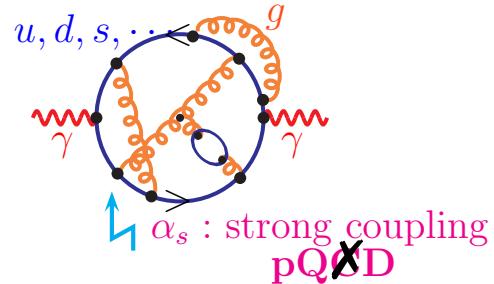
❑ Hadronic stuff: the limitation to theory

General problem in electroweak precision physics:
contributions from hadrons (quark loops) at low energy scales

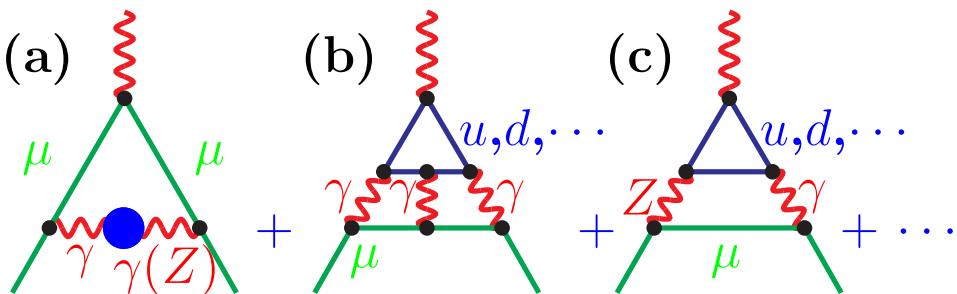
Leptons



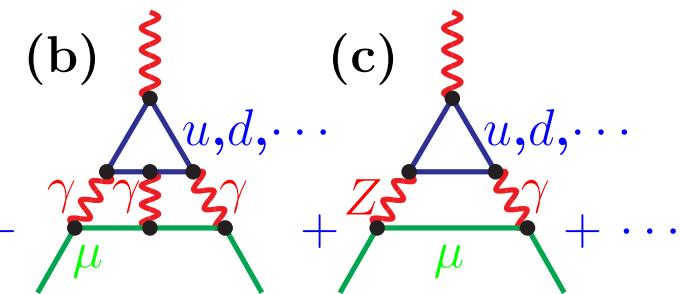
Quarks



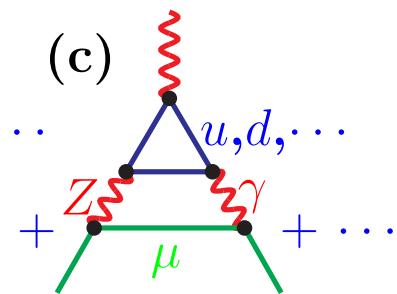
(a)



(b)



(c)



(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$

Light quark loops

(b) Hadronic light-by-light scattering $O(\alpha^3)$



(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$

Hadronic “blobs”

Evaluation of non-perturbative effects:

- ❑ data in conjunction with Dispersion Relations (DR),
- ❑ low energy effective modeling, RLA, HLS, ENJL
- ❑ lattice QCD

(a) HVP via dispersion integral over $e^+e^- \rightarrow \text{hadrons}$ -data

(1 independent amplitude to be determined by one specific data set),
HLS, lattice QCD

(b) HLbL via Resonance Lagrangian Approach (RLA) (CHPT extended by VDM
in accord with chiral structure of QCD), $\gamma\gamma \rightarrow \text{hadrons}$ -data dispersive
approach (28 independent amplitudes to be determined by as many
independent data sets), lattice QCD Blum et al, Wittig et al, ...

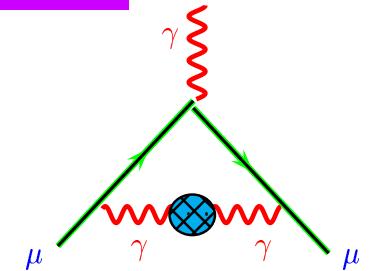
(c) quark and lepton triangle diagrams: $VVV = 0$ by Furry \Rightarrow only VVA
(of $f\bar{f}Z$ -vertex) contributes \Rightarrow ABJ anomaly is perturbative
and non-perturbative simultaneously i.e. leading effects calculable
(anomaly cancellation) de Rafael, Knecht, Perrottet, Melnikov, Vainshtein

☐ Evaluation of a_μ^{had}

Leading **non-perturbative** hadronic contributions a_μ^{had} can be obtained in terms of

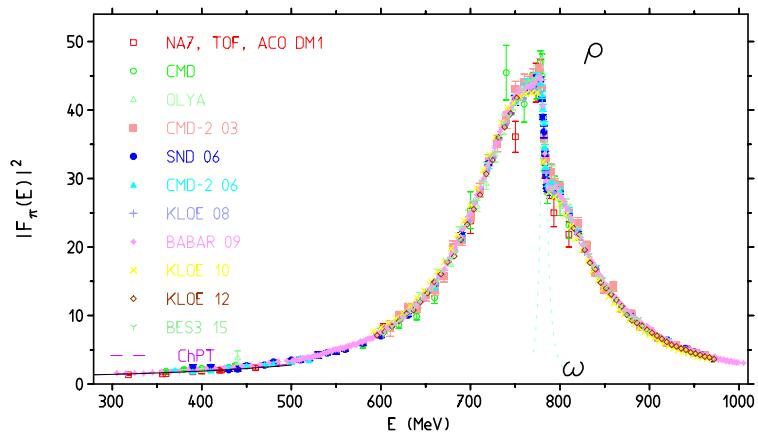
$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via **Dispersion Relation (DR):**

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{\frac{4m_\pi^2}{E_{\text{cut}}^2}}^{E_{\text{cut}}^2} ds \frac{R_\gamma^{\text{data}}(s) \hat{K}(s)}{s^2} + \int_{E_{\text{cut}}^2}^\infty ds \frac{R_\gamma^{\text{pQCD}}(s) \hat{K}(s)}{s^2} \right)$$



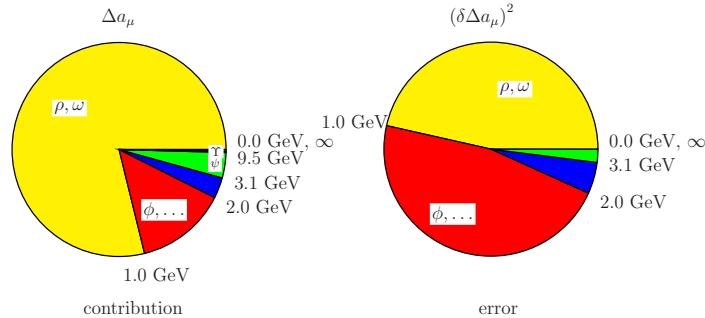
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

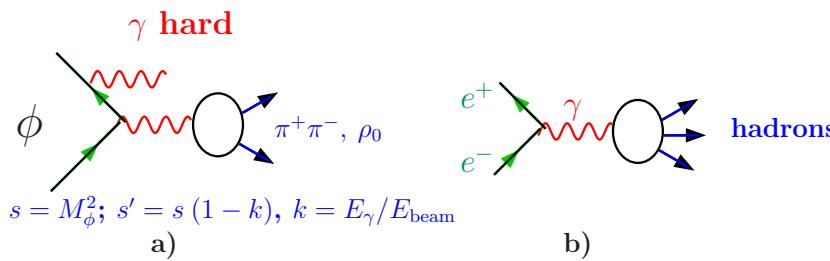
Data: **NSK, KLOE, BaBar, BES3**



$$a_\mu^{\text{had}(1)} = (686.99 \pm 4.21)[687.19 \pm 3.48] 10^{-10}$$

e^+e^- -data based [incl. τ]

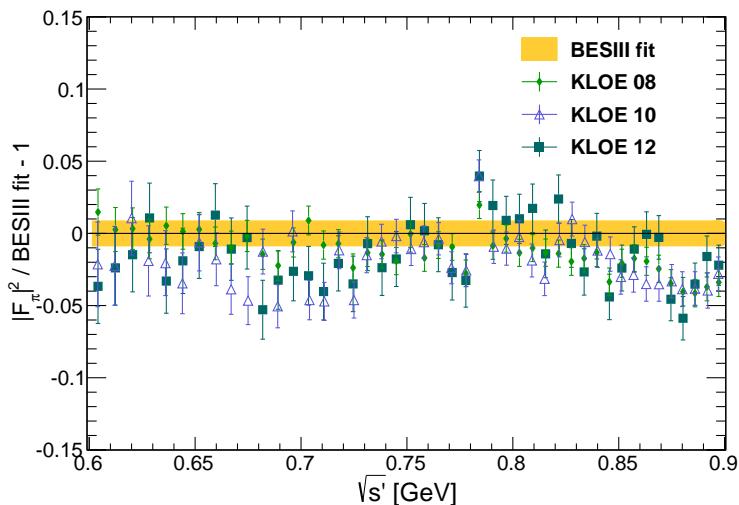
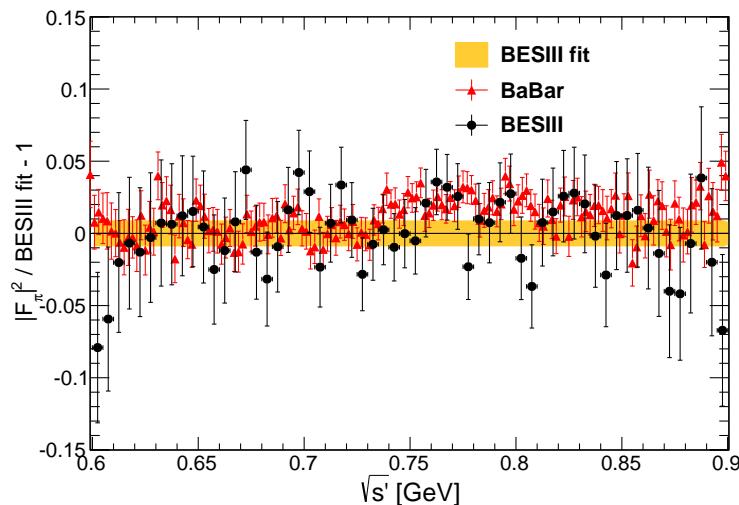




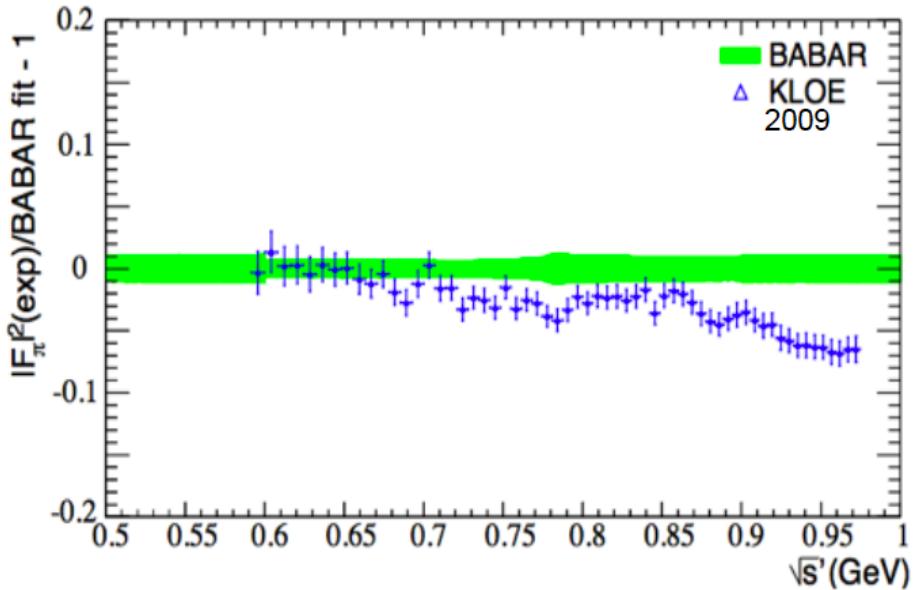
a) Initial state radiation (ISR), b) Standard energy scan.

Experimental input for HVP: VEPP-2000, BESIII-ISR

Talks Achim Denig, Simon Eidelman



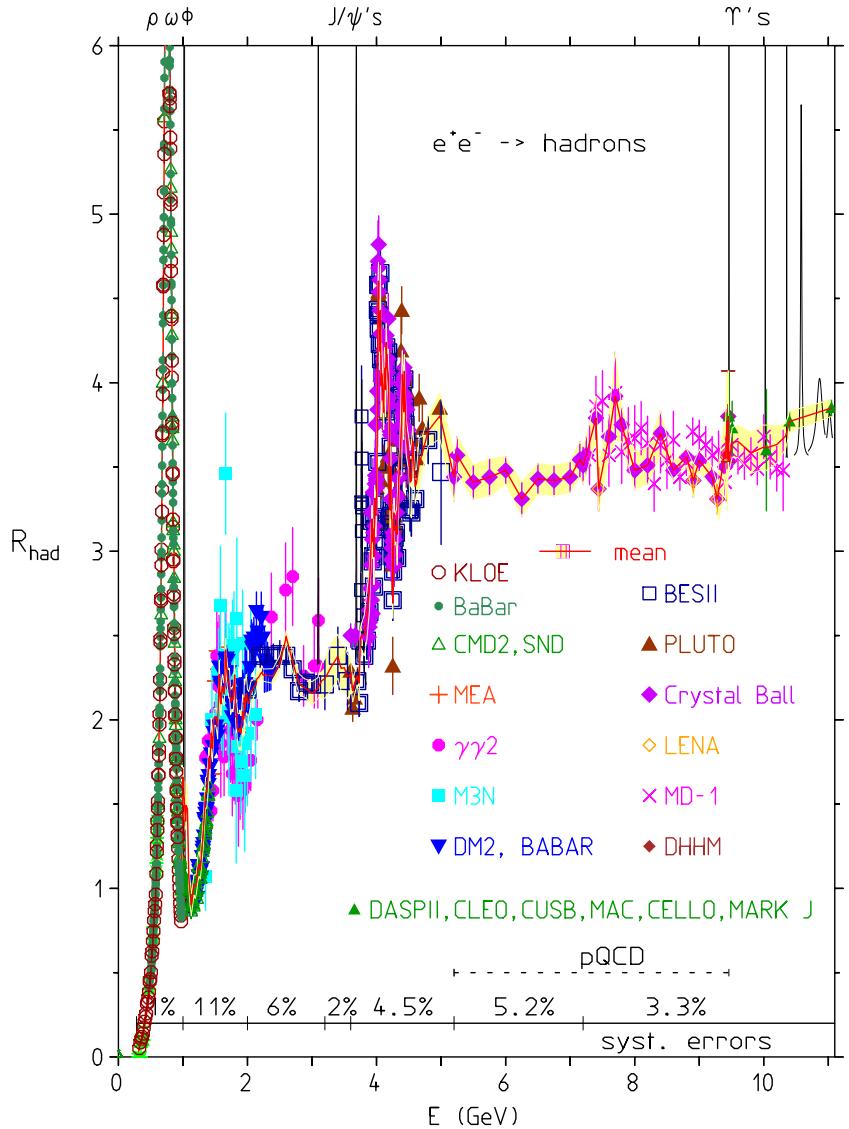
Recent BES-III vs BaBar and KLOE



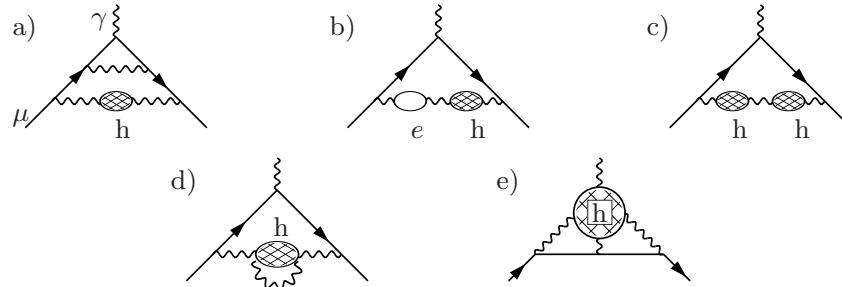
Most precise ISR measurements in conflict.
BESIII may resolve this

Recent/preliminary results:

- $e^+e^- \rightarrow \pi^+\pi^-$ from CMD-3
- $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ from Belle
- $e^+e^- \rightarrow K^+K^-$ from CMD-3
- $e^+e^- \rightarrow K^+K^-$ from SND
- $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma$ from SND
- $e^+e^- \rightarrow \pi^+\pi^-$ from BES-III



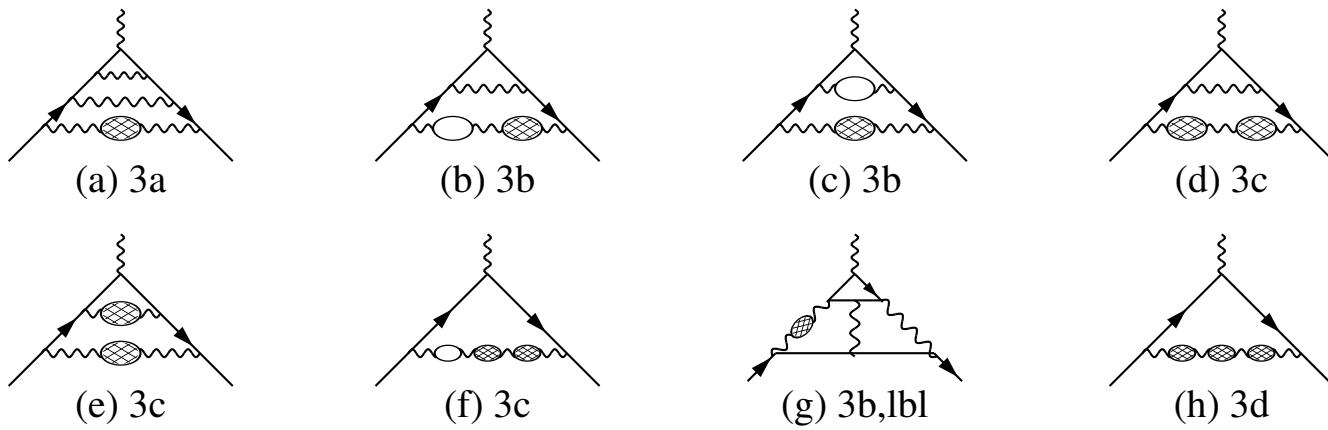
NLO and NNLO HVP effects



Hadronic higher order contributions: a)-c) involving LO vacuum polarization, d) involving HO vacuum polarization and e) involving light-by-light scattering

Higher order contributions from diagrams a) - c) (in units 10^{-11})

$a_\mu^{(2a)}$	$a_\mu^{(2b)}$	$a_\mu^{(2c)}$	$a_\mu^{had(2)}$	Ref.
-199 (4)	107 (3)	2.3 (0.6)	- 90 (5)	KNO84
-211 (5)	107 (2)	2.7 (0.1)	- 101 (6)	Krause96
-209 (4)	106 (2)	2.7 (1.0)	- 100 (5)	ADH98
-207.3 (1.9)	106.0 (0.9)	3.4 (0.1)	- 98 (1)	HMNT03
-207.5 (2.0)	104.2 (0.9)	3.0 (0.1)	- 100.3 (2.2)	FJ06
-205.72(1.36)	103.33(0.64)	3.35(0.05)	- 99.04(0.72)	JS11,FJ16



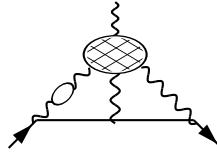
Class	results Kurz et al	my evaluation
$a_\mu^{(3a)}$	$= 0.80 \times 10^{-10}$	$0.782(77) \times 10^{-10}$
$a_\mu^{(3b)}$	$= -0.41 \times 10^{-10}$	$-0.403(37) \times 10^{-10}$
$a_\mu^{(3b,\text{lbl})}$	$= 0.91 \times 10^{-10}$	$0.900(77) \times 10^{-10}$
$a_\mu^{(3c)}$	$= -0.06 \times 10^{-10}$	$-0.0544(7) \times 10^{-10}$
$a_\mu^{(3d)}$	$= 0.0005 \times 10^{-10}$	$5.22(15) \times 10^{-14}$
$a_\mu^{\text{had,NNLO}}$	$= 12.4(1) \times 10^{-11}$	$12.25(12) \times 10^{-11}$

Kurz et al. 2014

NNLO HLBL effects

New NNLO HLBL

Colangelo et al. 2014



A hadronic light-by-light next to leading order correction, which is of the same order as the NNLO corrections.

is estimated to yield

$$a_{\mu}^{\pi^- \text{-pole, NLO}} = 1.5 \times 10^{-11}$$

using a simple VMD form-factor, which yields $a_{\mu}^{\pi^- \text{-pole, LO}} = 57.2 \times 10^{-11}$. Including other contributions gives an estimate:

$$a_{\mu}^{\text{HLbL, NLO}} = (3 \pm 2) \times 10^{-11}$$

as a correction to $a_{\mu}^{\text{HLbL, LO}} = (116 \pm 39) \times 10^{-11}$.

A problem in DR for HVP?

Full photon propagator Dyson resummation of 1PI part (blue blob)

$$\text{Diagram: } \gamma \text{ (wavy line)} \rightarrow \text{blob} \rightarrow \gamma = \text{blob} + \text{blob} \text{ (blue blob)} + \text{blob} \text{ (blue blob)} \text{ (blue blob)} + \dots$$

$$\begin{aligned} i D'_\gamma(q^2) &\equiv \frac{-i}{q^2} + \frac{-i}{q^2} (-i\Pi_\gamma) \frac{-i}{q^2} + \frac{-i}{q^2} (-i\Pi_\gamma) \frac{-i}{q^2} (-i\Pi_\gamma) \frac{-i}{q^2} + \dots \\ &= \frac{-i}{q^2} \left\{ 1 + \left(\frac{-\Pi_\gamma}{q^2} \right) + \left(\frac{-\Pi_\gamma}{q^2} \right)^2 + \dots \right\} \\ &= \frac{-i}{q^2} \left\{ \frac{1}{1 + \frac{\Pi_\gamma}{q^2}} \right\} = \frac{-i}{q^2 + \Pi_\gamma(q^2)} = \frac{-i}{q^2} \frac{1}{1 + \Pi'_\gamma(q^2)}. \end{aligned}$$

Including external e.m. coupling

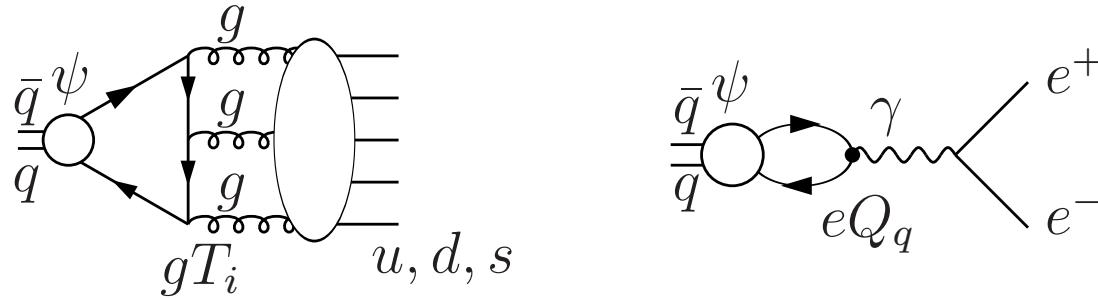
$$i e^2 D'_\gamma(q^2) = \frac{-i}{q^2} \frac{e^2}{1 + \Pi'_\gamma(q^2)}$$

Effective charge

$$\frac{e^2}{1 + \Pi'_\gamma(s)} = \frac{e^2}{1 - \Delta\alpha(s)} = e^2(s)$$

Usually, $\Delta\alpha(s)$ is a correction i.e $\Delta\alpha(s) \ll 1$ and the Dyson series converges well.

Exceptions: narrow OZI suppressed resonances (below $q\bar{q}$ -thresholds)



Γ_{ee} not much smaller than Γ_{QCD} (i.e strong decays): $J/\psi, \psi_2, \Upsilon_1, \Upsilon_2, \Upsilon_3$

Note: imaginary parts from narrow resonances, $\text{Im } \Pi'(s)) = \frac{\alpha}{3} R(s) = \frac{3}{\alpha} \frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

$$|1 - \Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\text{Im } \Pi'(s))^2$$

at $\sqrt{s} = M_R$ is given by

ρ	1.23	$\times 10^{-3}$
ω	2.76	$\times 10^{-3}$
ϕ	1.56	$\times 10^{-2}$
J/ψ	594.81	
ψ_2	9.58	
ψ_3	2.66	$\times 10^{-4}$
Υ_1	104.26	
Υ_2	30.51	
Υ_3	55.58	

- What is measured in an experiment is the full propagator, corresponding to

$$\frac{1}{1-x} ; \quad x \text{ irreducible part}$$

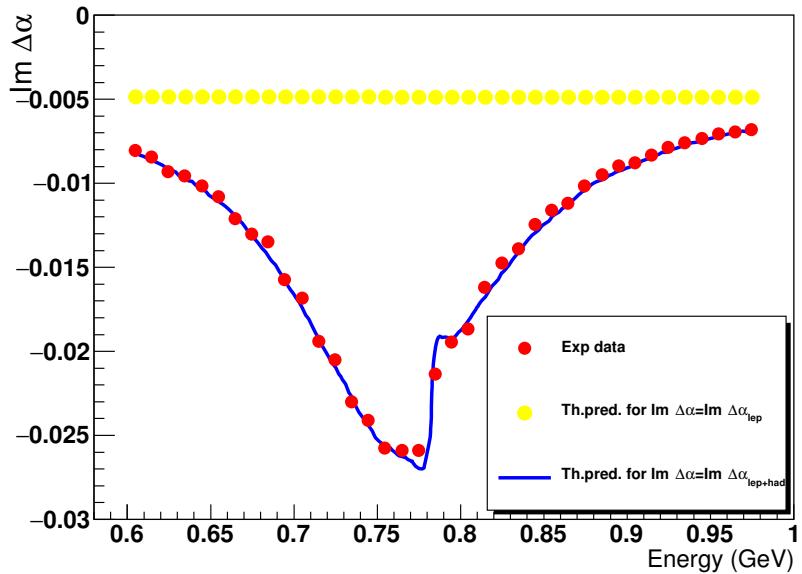
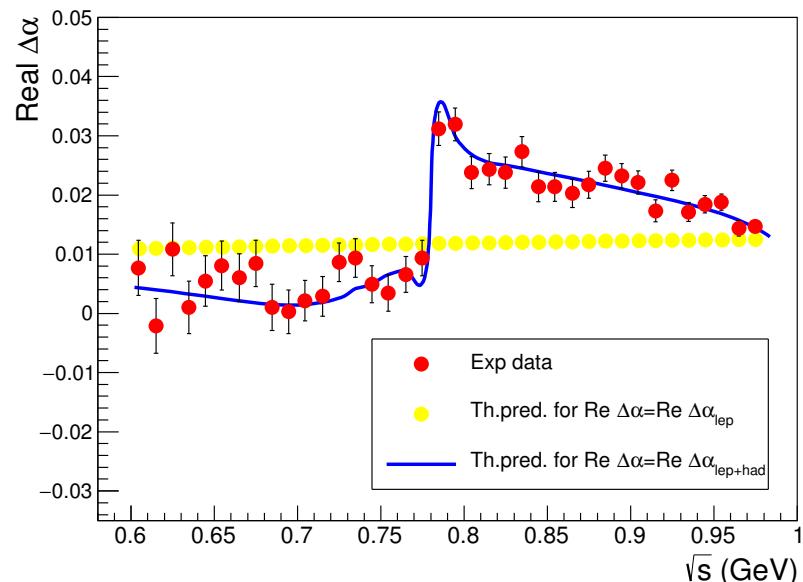
Object required in the DR:

$$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}) / \frac{4\pi\alpha^2}{3s} R^{\text{undressed}} = R^{\text{observed}} * |(1-x)|^2$$

VP subtraction is iterative procedure: does not converge!

News on VP subtraction

First measurement of complex VP function in ρ resonance region \Leftrightarrow complex $\Delta\alpha_{\text{QED}}(s) = -[\Pi'_\gamma(s) - \Pi'_\gamma(0)]$
 KLOE 2016, arXiv:1609.06631, Graziano Talk



$$\square \left| \frac{\alpha(s)}{\alpha(0)} \right|^2 = \frac{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)_{\text{pt}}} \quad \square R(s) = \frac{\sigma(e^+e^- \rightarrow \pi^+\pi^-)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \Rightarrow \text{Re } \alpha(s), \text{Im } \alpha(s)$$

Effective field theory: the Resonance Lagrangian Approach

HVP dominated by spin 1 resonance physics! need theory of $\rho, \omega, \phi, \dots$

NC: $\gamma, \rho^0, \omega, \phi, \dots$ mix e^+e^- -spectra complicated

CC: no mixing of ρ^\pm simple τ -decay spectrum

- Principles to be included: Chiral Structure of QCD, VMD & electromagnetic gauge invariance.
- ❖ General framework: resonance Lagrangian extension of chiral perturbation theory (CHPT), i.e. implement VMD model with Chiral structure of QCD. Specific version Hidden Local Symmetry (HLS) effective Lagrangian. First applied to HLbL of muon $g - 2$ Hayakawa, Kinoshita, Sanda.

Global Fit strategy:

Data below $E_0 = 1.05 \text{ GeV}$ (just above the ϕ) constrain effective Lagrangian couplings, using 45 different data sets (6 annihilation channels and 10 partial width decays).

- Effective theory predicts cross sections:

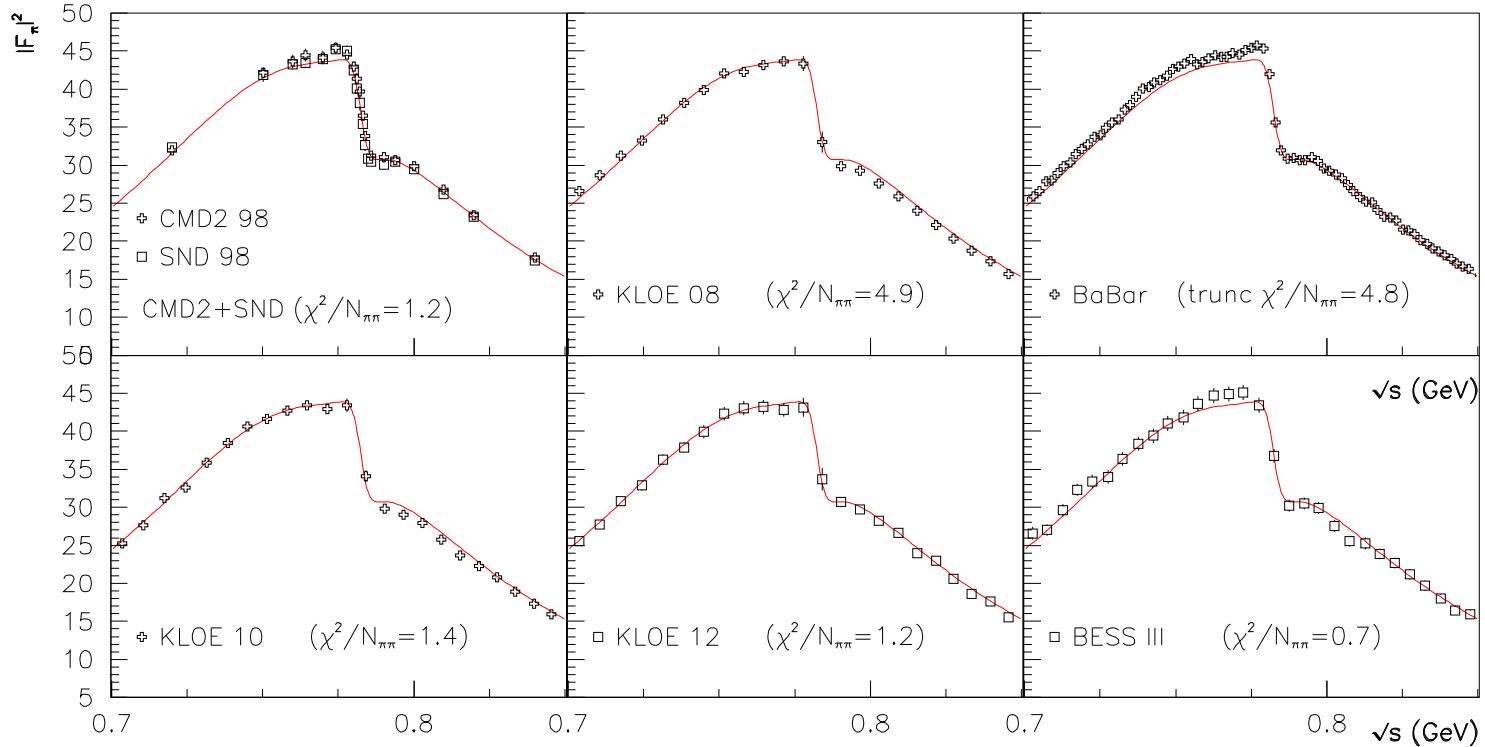
$$\pi^+\pi^-, \pi^0\gamma, \eta\gamma, \eta'\gamma, \pi^0\pi^+\pi^-, K^+K^-, K^0\bar{K}^0 \quad (83.4%),$$

- Missing part: $4\pi, 5\pi, 6\pi, \eta\pi\pi, \omega\pi$ and regime $E > E_0$ evaluated using data directly and pQCD for perturbative region and tail
- Including **self-energy effects** is mandatory ($\gamma\rho$ -mixing, $\rho\omega$ -mixing ... , decays with proper phase space, energy dependent width etc)
- Method works in reducing uncertainties by using **indirect constraints**
- Able to reveal inconsistencies in data, e.g. KLOE vs BaBar

Main goal:

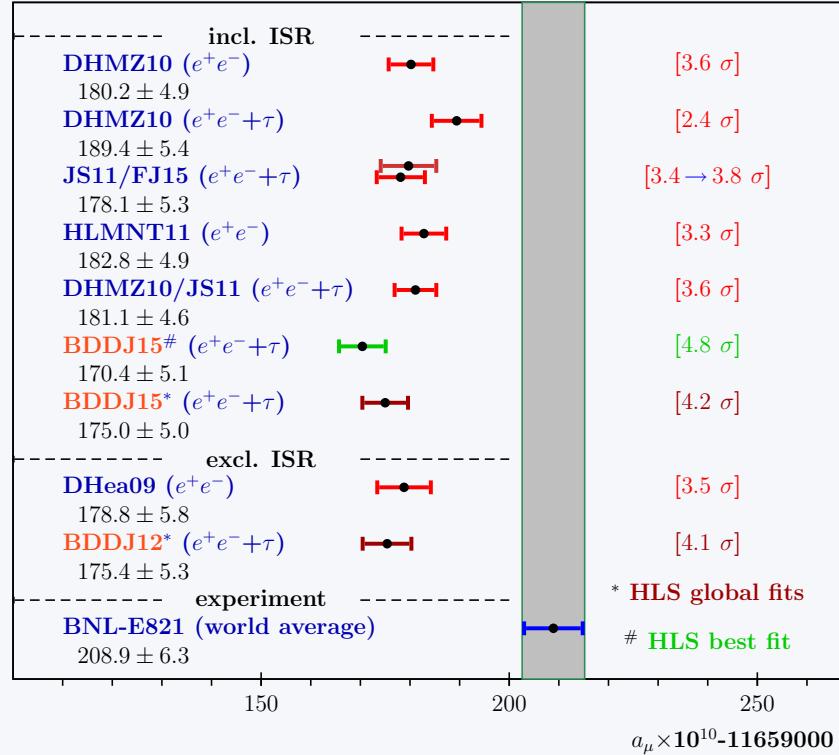
- Single out representative effective resonance Lagrangian by global fit is expected to help in improving EFT calculations of **HLbL**
- could help improving uncertainty on hadronic VP (besides e^+e^- and τ decay data other experimental information)

Fit of τ +PDG vs $\pi^+\pi^-$ -data



Comparing the τ +PDG prediction (red curve) of the pion form factor in e^+e^- annihilation in the $\rho - \omega$ interference region.

Present leading uncertainty: hard to improve by direct $R(s)$ measurements



Comparison with other Results. Note: results depend on which value is taken for HLbL. JS11 and BDDJ13 includes $116(39) \times 10^{-11}$ [JN], DHea09, DHMZ10, HLMNT11 and BDDJ12 use $105(26) \times 10^{-11}$ [PdRV].

HVP from lattice QCD

The need for ab initio calculation of a_μ^{had} is well motivated:

- the problems to determine non-perturbative contributions to the muon $g - 2$ from experimental data at sufficient precision persists and is not easy to improve,
- a model-independent extension of CHPT to the relevant energies ranges up to 2 GeV is missing while the new experiments E989 FNAL and E34 J-PARC

require an improvement of the hadronic uncertainties by a factor of four.

The hope is that LQCD can deliver estimates of accuracy

$$\delta a_\mu^{\text{HVP}} / a_\mu^{\text{HVP}} < 0.5\% , \quad \delta a_\mu^{\text{HLbL}} / a_\mu^{\text{HLbL}} \lesssim 10\%$$

in the coming years.

Primary object for HVP in LQCD: e.m. current correlator in configuration space

$$\langle J_\mu(\vec{x}, t) J_\nu(0, 0) \rangle, \quad J_\mu = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s + \dots$$

In principle, a Fourier transform

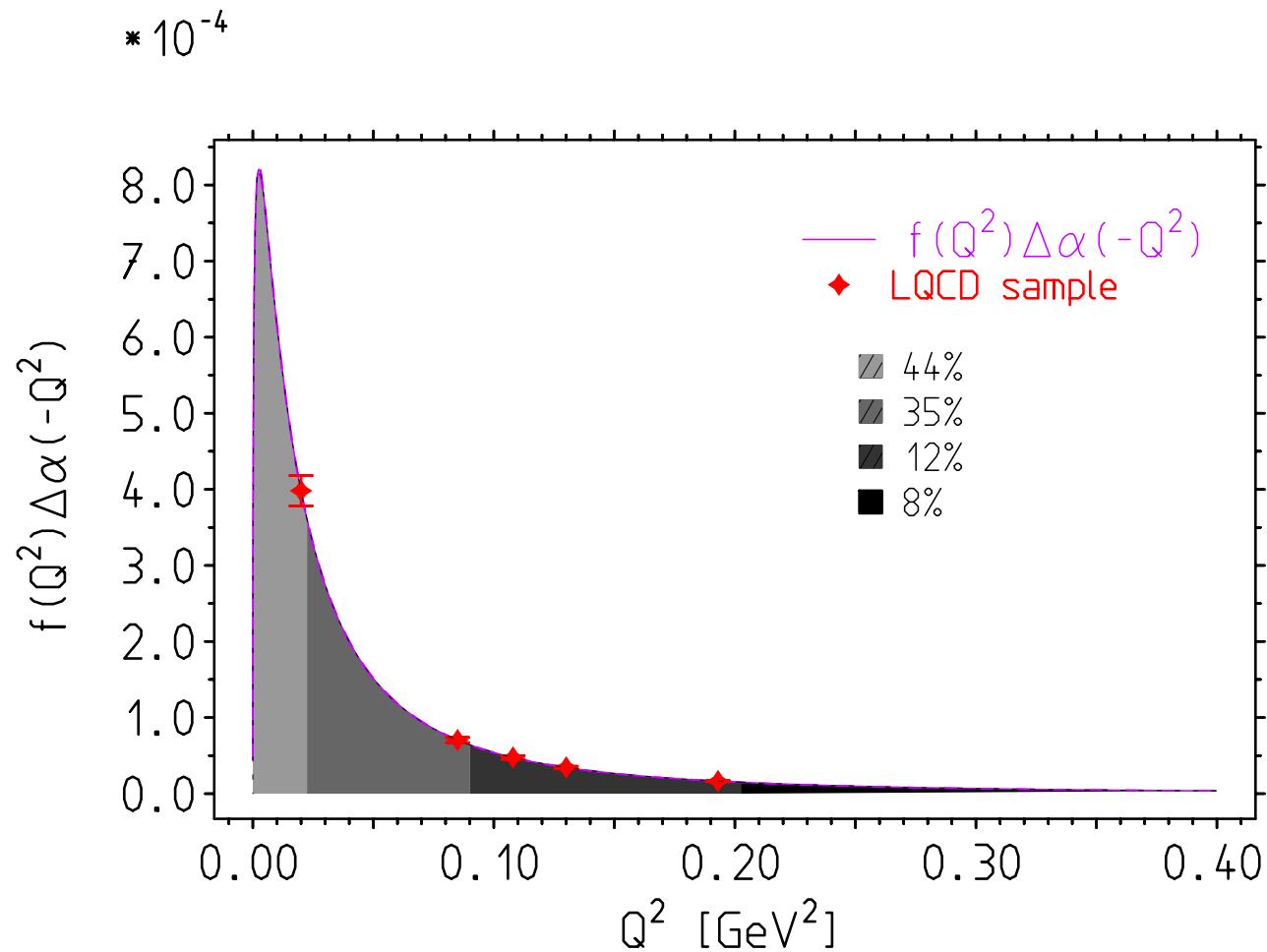
$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQx} \langle J_\mu(x) J_\nu(0) \rangle = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2) \Pi(Q^2)$$

yields the vacuum polarization function $\Pi(Q^2)$ needed to calculate

$$a_\mu^{\text{HVP}} = 4\alpha^2 \int_0^\infty dQ^2 f(Q^2) \left\{ \Pi(Q^2) - \Pi(0) \right\}$$

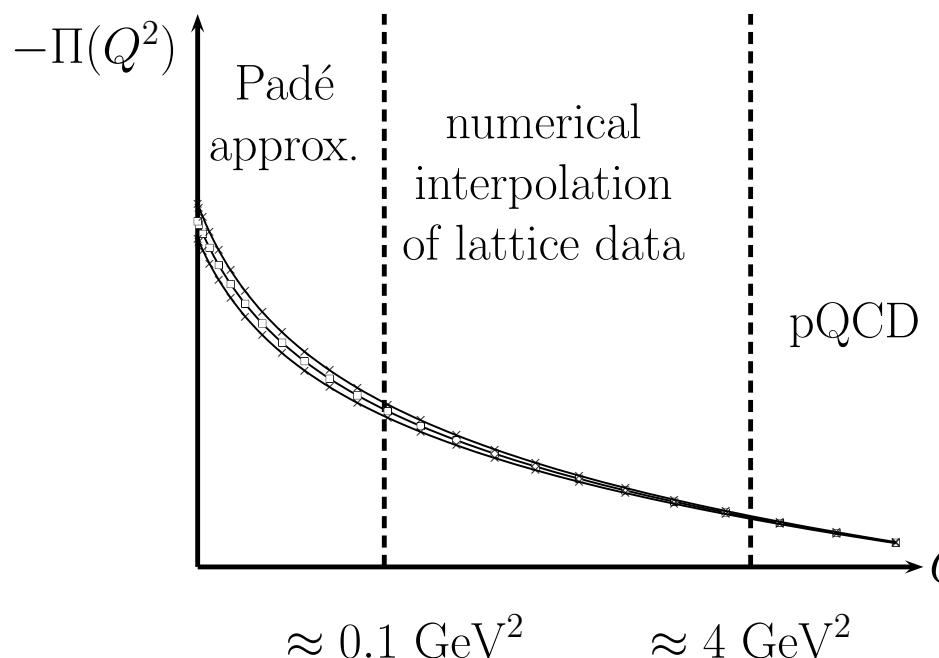
The integration kernel in this representation is

$$f(Q^2) = w(Q^2/m_\mu^2)/Q^2; \quad w(r) = \frac{16}{r^2 (1 + \sqrt{1+4/r})^4 \sqrt{1+4/r}}.$$

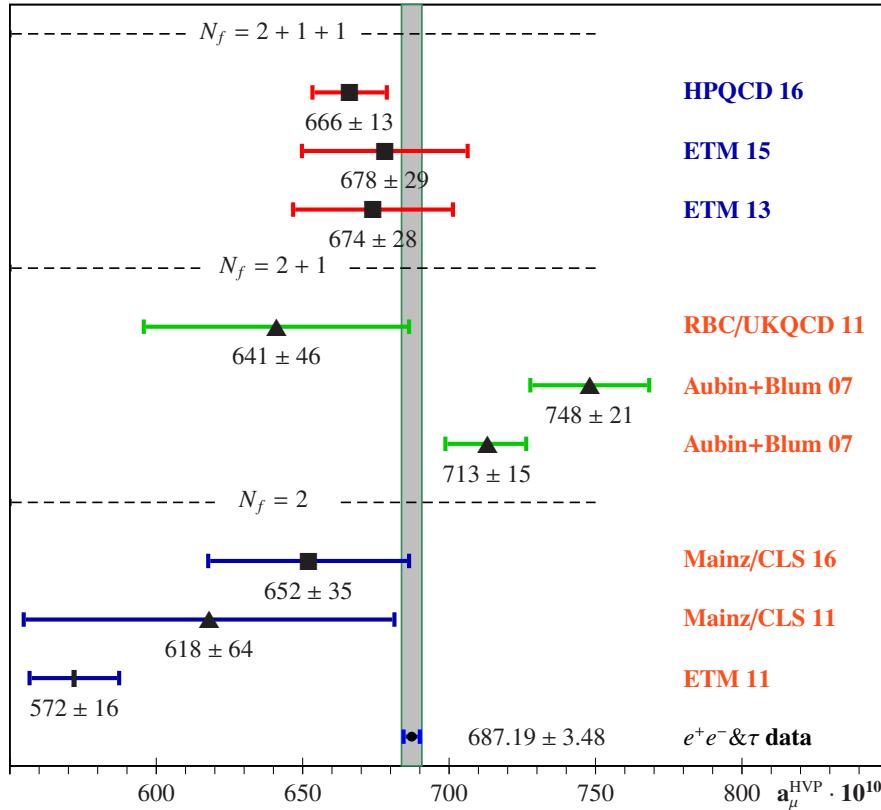


The integrand Q^2 . Ranges between $Q_i = 0.00, 0.15, 0.30, 0.45$ and 1.0 GeV and their percent contribution to a_μ^{had} and the “LQCD sample”

- LQCD lattice in finite box: momenta are quantized $Q_{\min} = 2\pi/L$
where L is the lattice box length. $Q_{\min} \rightarrow 0 \Leftrightarrow L \rightarrow \infty$ infinite volume limit
- $Q_{\min} = 2\pi/L$ with $m_\pi aL \gtrsim 4$ for $m_\pi \sim 200$ MeV, such that $Q_{\min} \sim 314$ MeV
- about 44% of the low x contribution to a_μ^{had} is not covered by data yet



- ❖ lattice data: $Q^2 > (2\pi/L)^2$
- ❖ extrapolate to $Q^2 = 0$ via Padé's
- ❖ Note need $\Pi(0)$!
- ❖ required accuracy: needed LQCD data down to $Q_{\min}^2 \approx 0.1 \text{ GeV}^2$



Summary of recent LQCD results for the leading order a_μ^{HVP} , in units 10^{-10} . Labels:
 ■ marks u, d, s, c , ▲ u, d, s and | u, d contributions. Individual flavor contributions
 from light (u, d) amount to about 90%, strange about 8% and charm about 2%.

Brookhaven, Zeuthen, Mainz, Edinburgh, ...

Alternative method: measure space-like $\alpha_{\text{QED,eff}}(t)$

Newly proposed recently: [arXiv:1504.02228, 1609.08987]

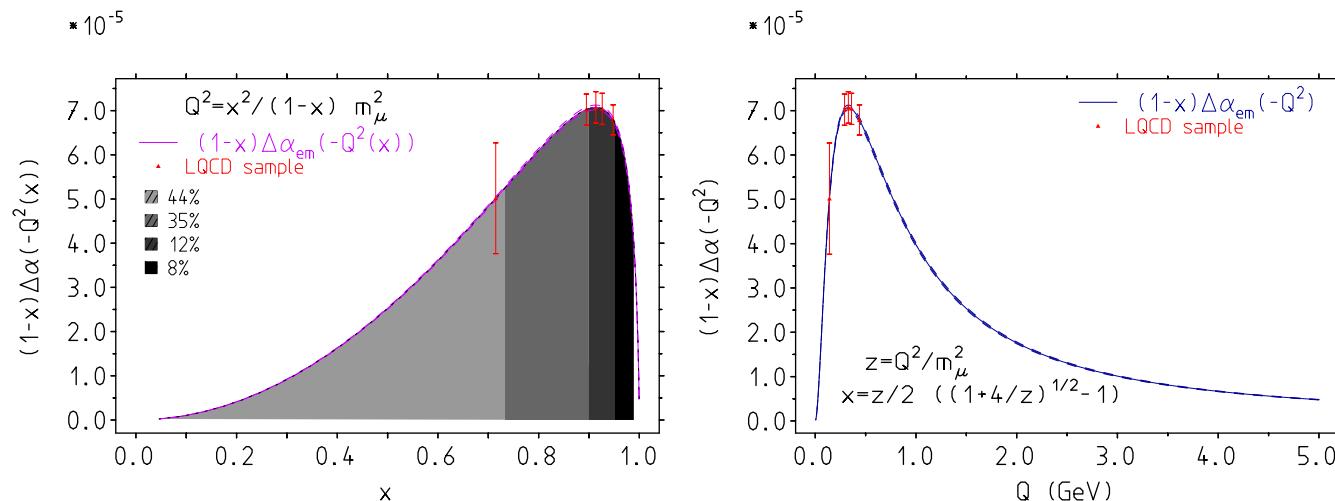
“A new approach to evaluate the leading hadronic corrections to the muon g-2”

Carloni Calame, Passera, Trentadue, Venanzoni 2015; Abbiendi et al. 2016

□ space-like $\Delta\alpha^{\text{had}}(-Q^2) = 1 - \frac{\alpha}{\alpha(-Q^2)} - \Delta\alpha^{\text{lep}}(-Q^2)$ determines a_μ^{had} via

$$a_\mu^{\text{had}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha^{\text{had}}(-Q^2(x))$$

where $Q^2(x) \equiv \frac{x^2}{1-x} m_\mu^2$ is the space-like square momentum-transfer. Also in the Euclidean region the integrand is highly peaked, now around half of the ρ meson mass scale.

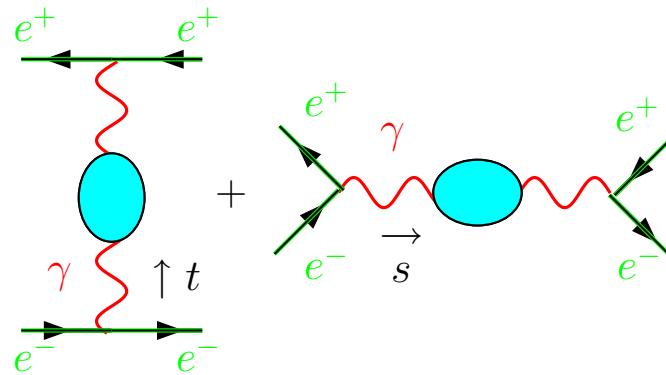


The integrand of a_μ^{had} integral as functions of x and Q . Strongly peaked at about 330 MeV.

- measuring directly low energy running $\alpha_{\text{QED}}(s)$ in space-like region via
- very different paradigm: no VP subtraction issue!
- no exclusive channel collection
- even 1% level measurement can provide important independent information

Bhabha scattering

$$e^+(p_+) \ e^-(p_-) \rightarrow e^+(p'_+) \ e^-(p'_-)$$



VP dressed tree level Bhabha scattering in QED

has two tree level diagrams the t – and the s –channel. With the positive c.m. energy square $s = (p^+ + p^-)^2$ and the negative momentum transfer square

$$t = (p_- - p'_-)^2 = -\frac{1}{2} (s - 4m_e^2) (1 - \cos \theta),$$

θ the e^- scattering angle, there are two very different scales involved

The VP dressed lowest order cross-section is

$$\frac{d\sigma}{d \cos \Theta} = \frac{s}{48\pi} \sum_{ik} |A_{ik}|^2$$

where A_{ik} tree level helicity amplitudes, $i, k = L, R$ left– and right–handed electrons.

Dressed transition amplitudes: ($m_e \approx 0$)

$$|A_{LL,RR}|^2 = \frac{3}{8} (1 + \cos \theta)^2 \left| \frac{e^2(s)}{s} + \frac{e^2(t)}{t} \right|^2$$

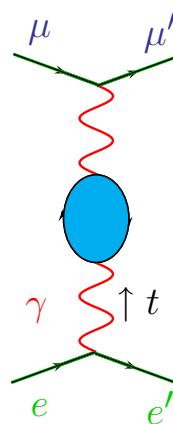
$$|A_{LR,RL}|^2 = \frac{3}{8} (1 - \cos \theta)^2 \left| \frac{e^2(s)}{s} + \frac{e^2(t)}{t} \right|^2.$$

Preferably one uses small angle Bhabha scattering (small $|t|$) as a normalizing process which is dominated by the t –channel $\sim 1/t$, however, detecting electrons and positrons along the beam axis often has its technical limitations.

Care also is needed concerning the ISR corrections because cuts for the Bhabha process ($e^+e^- \rightarrow e^+e^-$) typically are different from the ones applied to $e^+e^- \rightarrow \text{hadrons}$. Usually, experiments have included corresponding uncertainties in their systematic errors, if they not have explicitly accounted for all appropriate radiative corrections.

μ^-e^- scattering

$$\mu^-(p_-) e^-(q_-) \rightarrow \mu^-(p'_-) e^-(q'_-)$$



Get a_μ^{had} from $\mu^-e^- \rightarrow \mu^-e^-$ process

$$\frac{d\sigma_{\mu^- e^- \rightarrow \mu^- e^-}^{\text{unpol.}}}{dt} = 4\pi \alpha(t)^2 \frac{1}{\lambda(s, m_e^2, m_\mu^2)} \left\{ \frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right\}$$

- The primary goal of [arXiv:1504.02228, 1609.08987]: determining a_μ^{had} in an alternative way
- $\Pi'_\gamma(Q^2) - \Pi'_\gamma(0) = -\Delta\alpha^{\text{had}}(-Q^2) = \frac{\alpha}{\alpha(-Q^2)} + \Delta\alpha^{\text{lep}}(-Q^2) - 1$
directly checks lattice QCD data
- My proposal here: determine very accurately

$$\Delta\alpha^{\text{had}}(-Q^2) \text{ at } Q \approx 2.5 \text{ GeV}$$

by this method (one single number!) as the non-perturbative part of $\Delta\alpha^{\text{had}}(M_Z^2)$ as in “Adler function” approach.

Theory vs experiment: do we see New Physics?

Contribution	Value	Error	Reference
QED incl. 4-loops+5-loops	11 658 471.8851	0.036	Remiddi, Kinoshita ...
Leading hadronic vac. pol.	688.60	4.24	HLS driven
Subleading hadronic vac. pol.	-9.832	0.082	2012 update
NNLO hadronic vac. pol.	1.240	0.010	2014 KLMS
Hadronic light-by-light	10.6	3.9	evaluation (J&N 09/J 14)
Weak incl. 2-loops	15.40	0.10	CMV06/FJ12/BSS13
Theory	11 659 177.89	5.76	–
Experiment	11 659 209.1	6.3	BNL Updated
Exp.- The. 3.7 standard deviations	31.21	8.54	–

Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 4σ deviation: new physics? a statistical fluctuation?
 underestimating uncertainties (experimental, theoretical)?
❖ **do experiments measure what theoreticians calculate?**

New evaluations included:

New contribution	Reference	$a_\mu \cdot 10^{11}$		
NNLO HVP	Kurz et al. 2014	12.4	\pm	0.1
NLO HLBL	Colangelo et al. 2014	3	\pm	2
New axial exchange HLBL	Pauk, Vanderhaeghen, F.J. 2014	7.55	\pm	2.71
Old axial exchange HLBL	Melnikov, Vainshtein 2004	22	\pm	5
Tensor exchange HLBL	Pauk, Vanderhaeghen 2014	1.1	\pm	0.1
Total change		+2.1	\pm	3.4 [\leftarrow 5]

The big challenge: two complementary experiments: Fermilab with ultra hot muons and J-PARC with ultra cold muons (very different radiation) to come

Provided deviation is real $3\sigma \rightarrow 9\sigma$ possible? Provided theory and needed cross section data improves the same as the muon $g - 2$ experiments!

Key: more/better data and/or progress in non-perturbative QCD

For muon $g - 2$:

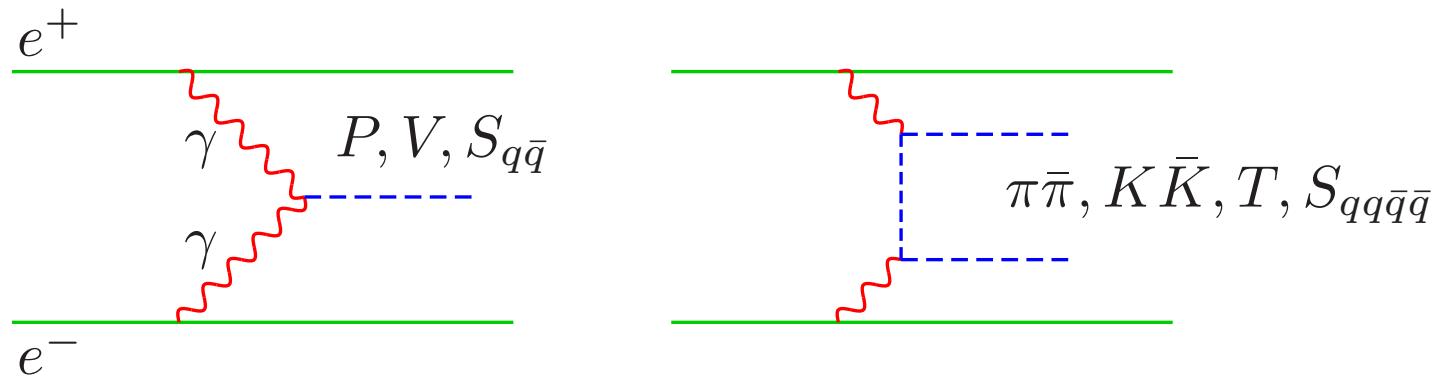
❖ main obstacle: hadronic light-by-light [data, lattice QCD, RLA]

- ❖ progress in evaluating HVP: more data (BaBar, Belle, VEPP 2000, BESIII,...),
lattice QCD in reach (recent progress Jansen et al, Wittig et al, Blum et al)

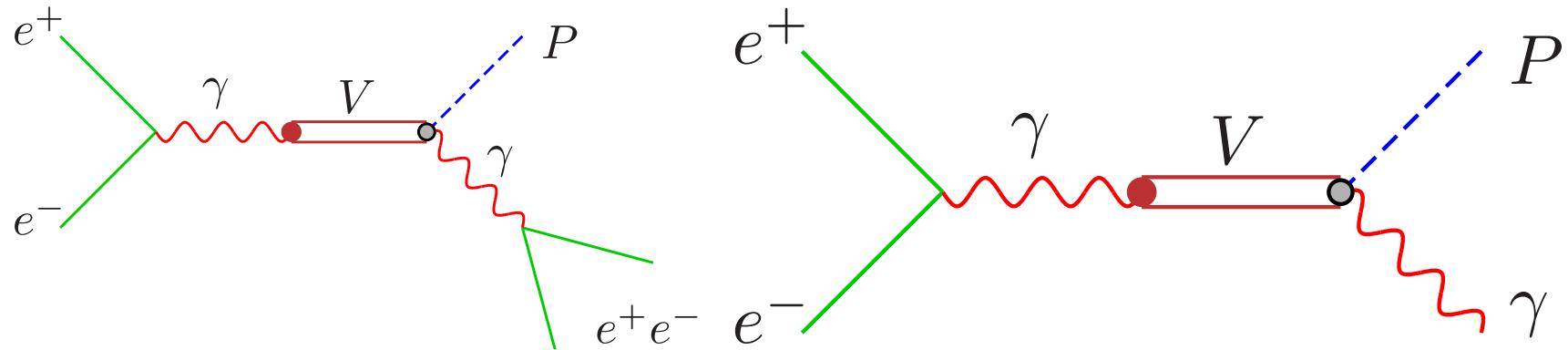
in both cases **lattice QCD will be the answer one day**,

- ❖ also low energy effective RL and DR approach need be further developed.

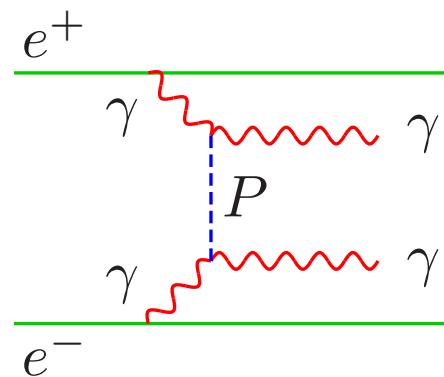
For future improvements one desperately needs more information from $\gamma\gamma \rightarrow$ hadrons in order to have better constraints on modeling of the hadronic amplitudes. The goal is to exploit possible new experimental constraints from $\gamma\gamma \rightarrow$ hadrons



mostly single-tag events: KLOE, KEDR (taggers), BaBar, Belle, BES III



Dalitz-decays: $\rho, \omega, \phi \rightarrow \pi^0(\eta) e^+ e^-$ Novosibirsk, NA60, JLab, Mainz, Bonn, Jülich, BES



would be interesting, but is buried in the background.

The dispersive approach is able to allow for real progress since contributions which we have treated so far as separate contributions will be treated in a integral manner. Remember: 28 independent amplitudes contribute to $g - 2$ vs. data (HVP 1 amplitude vs data)

A lot remains to be done! while new a_μ^{exp} is approaching us!

Thanks you for your attention!

Supplementary Slides

Summary HVP:

- ❖ Dominating $\pi\pi$ channel measured with < 1% accuracy
 - ➡ most precise ISR measurements (KLOE, BABAR) in conflict with each other
 - ➡ cross check by BESIII - ISR
 - ➡ VEPP-2000 aims for unprecedented accuracy 0.3%
- ❖ Higher multiplicities dominated by BABAR ISR measurements
 - ➡ cross check and improvement expected by VEPP-2000, BESIII
- ❖ BELLE-II in intermediate future ?!
- ❖ Issues:
 - Radiative corrections
 - Precise formfactor models in MC generators
 - FSR modeling

- ❖ Lattice: Lots of interest, work on hadronic contributions, esp. HVP

- ➡ Statistical errors (sub) 1%
- ➡ Several groups done/doing physical m_π (m_{quark}) simulations
- ➡ Much effort on understanding systematics
- ➡ 2-3% total error on connected HVP in 2 years possible
- ➡ May be achievable for disconnected too

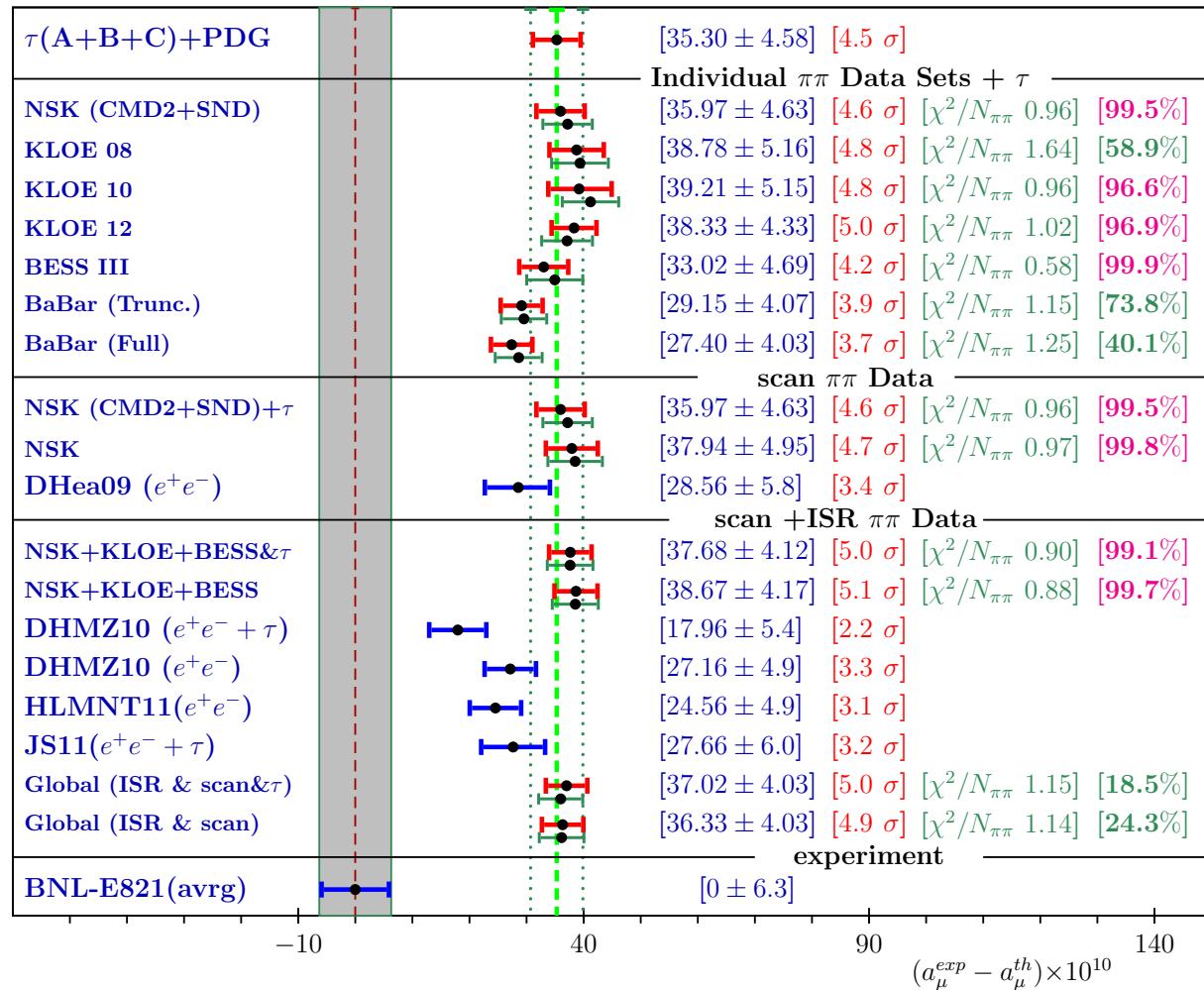
Summary HLBL:

- ❖ Huge experimental progress in all kinematic ranges
- ❖ KLOE-II and BESIII will measure pseudoscalar TFF in low Q₂ range
- ❖ Hadronic models need to be validated by data
 - ➡ exptl. accuracy in most cases not yet precise enough

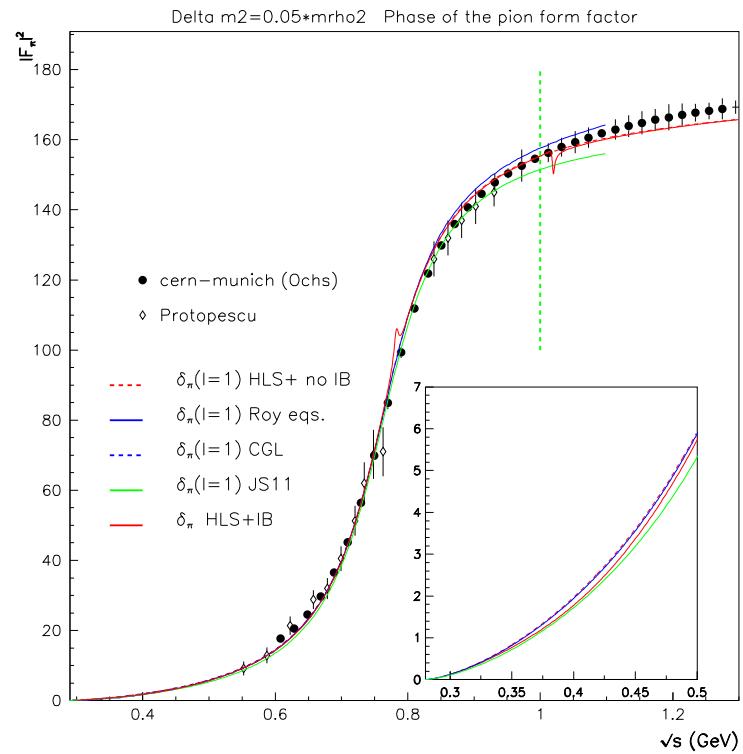
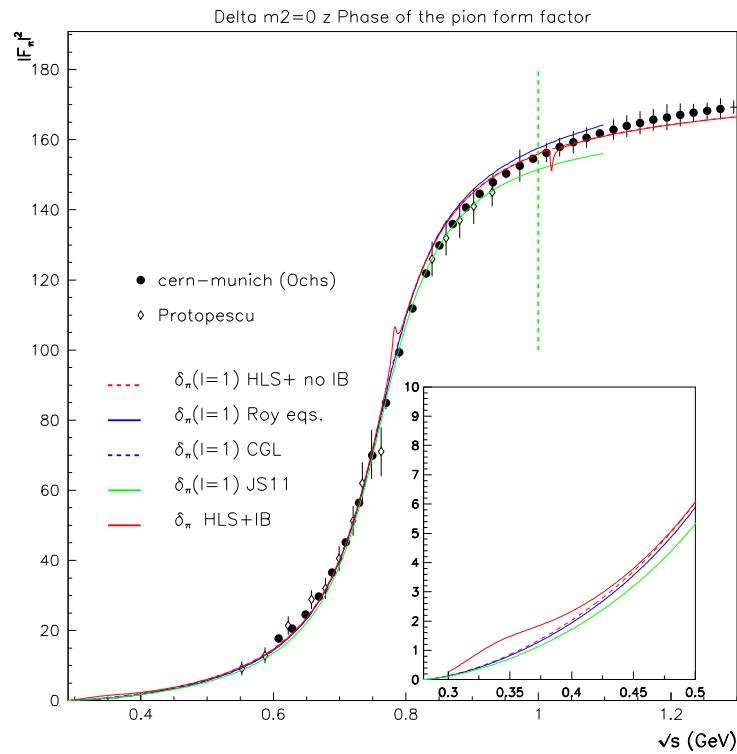
- ❖ Dispersion relations for HLbL calculation
 - ➡ close interplay between theory and experiment
- ❖ Lattice: QCD+QED promising, but significant systematics.
Present running with $m_\pi = 170 \text{ MeV}$ and investigating excited state contamination
- ❖ Dynamical QED+QCD is coming too
- ❖ need more groups working on it!
- ❖ Interest in 4pt function, $\pi \rightarrow \gamma^* \gamma^*$, other simpler quantities

Table 1: Results for $a_\mu^{\text{had}} \times 10^{10}$ from different energy ranges. Given are statistical, systematic and the total error, the relative precision in % [rel] and the contribution to the final error² in % [abs].

final state	range (GeV)	result	(stat)	(syst)	[tot]	rel	abs
ρ	(0.28, 1.05)	505.96	(0.77)	(2.47)	[2.59]	0.5%	37.8%
ω	(0.42, 0.81)	35.23	(0.42)	(0.95)	[1.04]	3.0%	6.1%
ϕ	(1.00, 1.04)	34.31	(0.48)	(0.79)	[0.92]	2.7%	4.8%
J/ψ		8.94	(0.42)	(0.41)	[0.59]	6.6%	1.9%
Υ		0.11	(0.00)	(0.01)	[0.01]	6.8%	0.0%
had	(1.05, 2.00)	60.45	(0.21)	(2.80)	[2.80]	4.6%	44.4%
had	(2.00, 3.20)	21.63	(0.12)	(0.92)	[0.93]	4.3%	4.9%
had	(3.10, 3.60)	3.80	(0.02)	(0.03)	[0.04]	1.1%	0.0%
had	(3.60, 5.20)	7.50	(0.04)	(-0.00)	[0.04]	0.0%	0.0%
pQCD	(5.20, 9.46)	6.27	(0.04)	(0.01)	[0.01]	0.1%	0.0%
had	(9.46, 11.50)	0.87	(0.00)	(0.05)	[0.05]	5.7%	0.0%
pQCD	(11.50, ∞)	1.96	(0.00)	(0.00)	[0.00]	0.0%	0.0%
data		678.81	(1.12)	(4.06)	[4.21]	0.6%	0.0%
F. Jegerlehner		KLOE-2 Workshop, INFN-LNF, Frascati, 26-28 October 2016					39
total		687.04	(1.12)	(4.06)	[4.21]	0.6%	100.0%



The deviation $\Delta a_\mu = a_\mu^{exp} - a_\mu^{th}$ in units of 10^{-10} . In red we display Δa_μ corresponding to the iterated solution and in green those corresponding to the $A = m$ (non-iterated) solution. In blue results from other studies are given.



P-wave $\pi^+\pi^-$ phase–shift data and predictions from CGL and JS11 together with the BHLS phase–shift. The insets magnify the various behaviors close to threshold.

News on $\pi^0 \rightarrow \gamma^* \gamma^*$

Large- N_c QCD inspired approach Knecht,Nyffeler 2003
 Vector Meson Dominance (VMD) ansatz

$$\mathcal{F}_{\pi^0 \gamma^* \gamma^*}^{\text{VMD}}(q_3^2, q_1^2, q_2^2) = \frac{N_c}{12\pi^2 F_\pi} \frac{M_V^2}{(M_V^2 - q_1^2)(M_V^2 - q_2^2)}.$$

- ✖ $\mathcal{F}_{\pi^0 \gamma^* \gamma^*}(m_\pi^2, -Q^2, -Q^2) \sim 1/Q^4$, instead of $\sim 1/Q^2$ (deduced from the OPE)
 - ✖ FF cannot factorize: structure $\mathcal{F}_{\pi^0 \gamma^* \gamma^*}(m_\pi^2, q_1^2, q_2^2) = F(q_1^2)F(q_2^2)$ is excluded
- Leading Meson Dominance (LMD) ansatz

$$\mathcal{F}_{\pi^0 \gamma^* \gamma^*}^{\text{LMD}}(m_\pi^2, q_1^2, q_2^2) = \frac{F_\pi}{3} \frac{c_V - q_1^2 - q_2^2}{(M_V^2 - q_1^2)(M_V^2 - q_2^2)},$$

On-shell vertex condition fixes $c_V = \frac{N_c}{4\pi^2} \frac{M_V^4}{F_\pi^2}$.

- ✖ $\mathcal{F}_{\pi^0 \gamma^* \gamma^*}(m_\pi^2, -Q^2, 0) \sim F_\pi/3M_V^2$, instead of $\sim 1/Q^2$ (deduced from the OPE)

The LMD+V ansatz

$$\begin{aligned}
 \mathcal{F}_{\pi^0*\gamma^*\gamma^*}^{\text{LMD+V}}(p_\pi^2, q_1^2, q_2^2) &= \frac{F_\pi}{3} \frac{\mathcal{P}(q_1^2, q_2^2, p_\pi^2)}{Q(q_1^2, q_2^2)}, \\
 \mathcal{P}(q_1^2, q_2^2, p_\pi^2) &= h_0 q_1^2 q_2^2 (q_1^2 + q_2^2 + p_\pi^2) + h_1 (q_1^2 + q_2^2)^2 + h_2 q_1^2 q_2^2 + h_3 (q_1^2 + q_2^2) p_\pi^2 \\
 &\quad + h_4 p_\pi^4 + h_5 (q_1^2 + q_2^2) + h_6 p_\pi^2 + h_7, \\
 Q(q_1^2, q_2^2) &= (M_{V_1}^2 - q_1^2)(M_{V_2}^2 - q_1^2)(M_{V_1}^2 - q_2^2)(M_{V_2}^2 - q_2^2),
 \end{aligned}$$

with $p_\pi^2 \equiv (q_1 + q_2)^2$. Parameters for pole approximation $p_\pi^2 = m_\pi^2$, now well constrained.

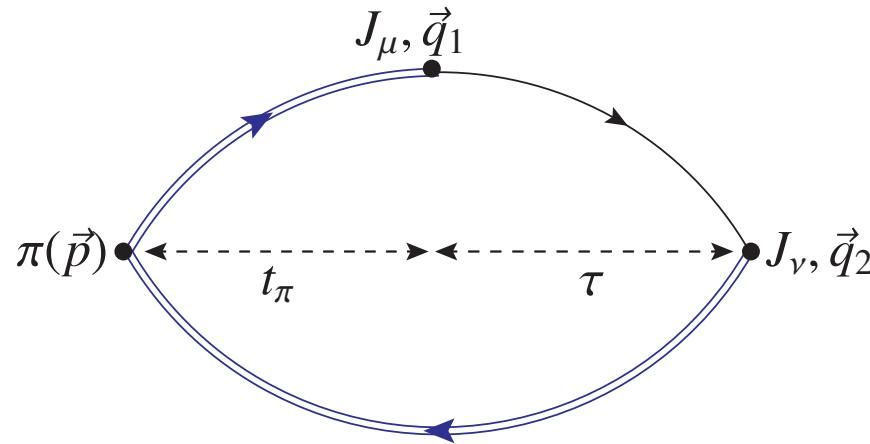
As discussed earlier the dominating contribution to HLbL is related to pseudoscalar meson exchange. The leading matrix element is

$$M_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{\pi^0*\gamma\gamma}(m_\pi^2; q_1^2, q_2^2),$$

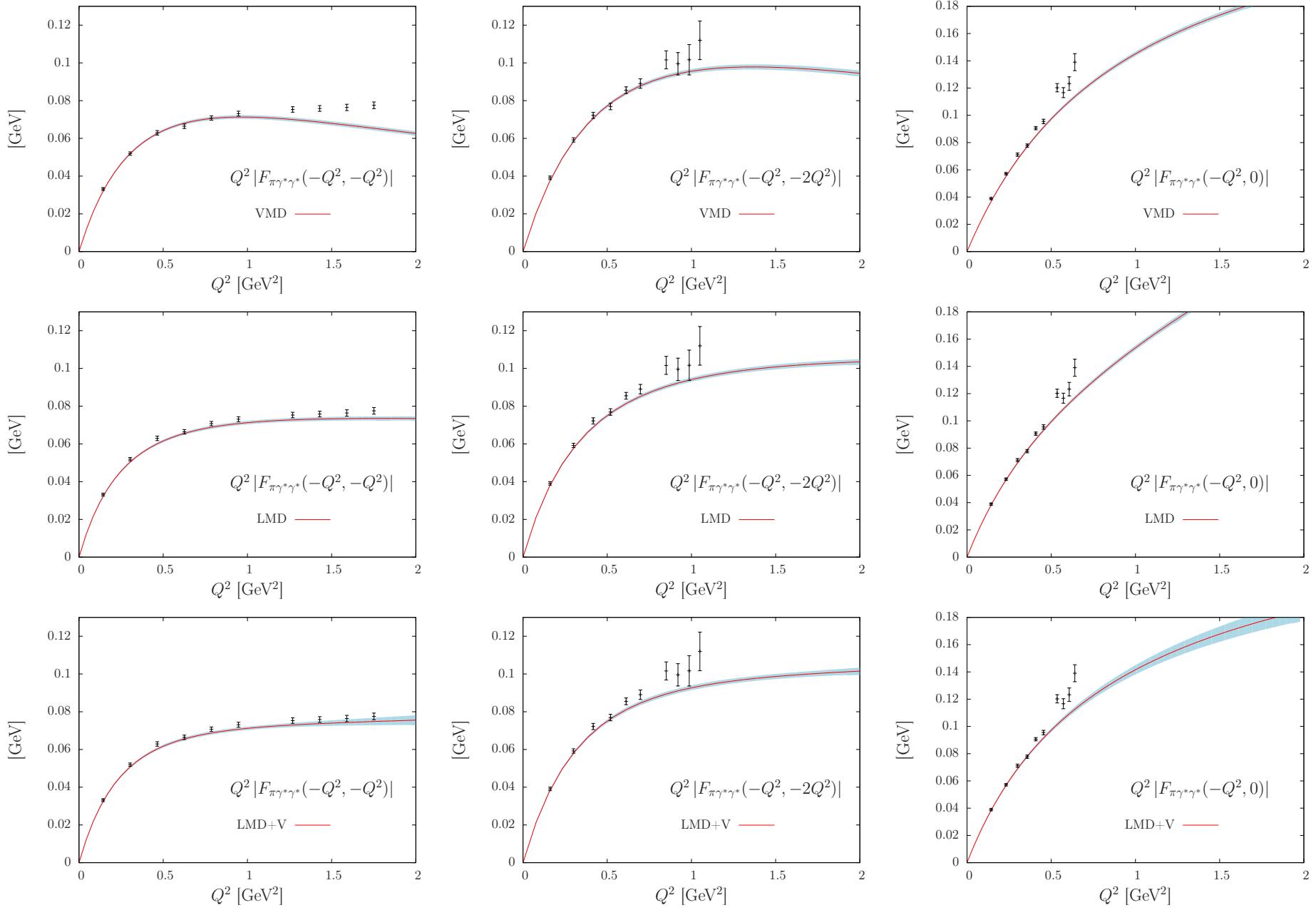
which can be evaluated in LQCD. Gérardin, Meyer, Nyffeler arXiv:1607.08174.

Simulation of $M_{\mu\nu}$ for $N_f = 2$ flavors with $\mathcal{O}(a)$ improved Wilson action

$$\begin{aligned} M_{\mu\nu} &\sim C_{\mu\nu}^{(3)}(\tau, t_\pi; \vec{p}, \vec{q}_1, \vec{q}_2) \\ &= \sum_{\vec{x}, \vec{z}} \langle T \left\{ J_\nu(\vec{0}, \tau + t_\pi) J_\mu(\vec{z}, t_\pi) P(\vec{x}, 0) \right\} \rangle e^{i \vec{p} \cdot \vec{x}} e^{-i \vec{q}_1 \cdot \vec{z}} \end{aligned}$$



Three point correlator defining the $\pi^0 \rightarrow \gamma^* \gamma^*$ off-shell form factor. The three-momenta of the pion and the two photons are \vec{p} , \vec{q}_1 and \vec{q}_2 , t_π and τ the relative time arguments. Kinematics: $\vec{p} = 0$, $q_1^2 = \omega_2^2 - |\vec{q}_1|^2$, $q_2^2 = (m_\pi - \omega_1)^2 - |\vec{q}_1|^2$



Parametrized by LMD+V form factor yields

$$\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2; 0, 0) = 0.273(24) \text{ GeV}^{-1}$$

- ✓ well in agreement with $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2; 0, 0) = 1/(4\pi^2 F_\pi) = 0.274 \text{ GeV}^{-1}$.
- ✓ first time constraint from $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2; -Q^2, -Q^2)$ in the range $0 < Q^2 < 2 \text{ GeV}^2$
- ✓ best fit with LMD+V ansatz, in clear conflict with VMD ansatz.

Results for

$$a_\mu^{\text{HLbL}}(\pi^0) = (65.0 \pm 8.3) \times 10^{-11}$$

well in agreement with phenomenological estimates.

Axial exchanges

Landau-Yang Theorem: \mathcal{A} (axial meson $\rightarrow \gamma\gamma$)=0

e.g. $Z^0 \not\rightarrow \gamma\gamma$, while $Z^0 \rightarrow \gamma e^+ e^- \checkmark$

Why $a_\mu[a_1, f'_1, f_1] \sim 25 \times 10^{-11}$ so large?

- untagged $\gamma\gamma \rightarrow f_1$ no signal!
- single-tag $\gamma^*\gamma \rightarrow f_1$ strong peak is $Q^2 \gg m_{f_1}^2$

Contribution from axial mesons: symmetric Melnikov-Vainshtein MV (2004)
form-factors violate Landau-Yang, \Rightarrow antisymmetrize \Rightarrow contribution reduced by
factor 3, agrees with previous findings by BPP (1995) and Pauk, Vanderhaeghen
2013, FJ 2014

Consequently:

$$a_\mu^{\text{HLbL, LO}} = (106 \pm 39) \times 10^{-11}$$

replacing $(116 \pm 39) \times 10^{-11}$ used in JN 2009 Phys. Rep.