Latest studies of the reaction $e^+ e^- \rightarrow K^+ K^- \gamma$

Leonard Leśniak

Institute of Physics, Jagiellonian University, Kraków

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Motivation

1. The $K^+ K^-$ threshold lies close to the DAΦNE accelerator energy, so the **low energy $K^+ K^-$ interactions** can be studied here.
2. Masses of the scalar resonances **f_0(980)** and **a_0(980)** are also close to the $K^+ K^-$ threshold.
3. The parameters of the scalar resonances are still not well known. According to the **PDG (2016) estimations**:
   the **f_0(980)** mass = 990 $\pm$ 20 MeV and width= 10 to 100 MeV, and
   the **a_0(980)** mass = 980 $\pm$ 20 MeV and width= 50 to 100 MeV.

4. Parameters of the scalar resonances found in experimental analyses are very much **model dependent**.

5. Example: reaction $\mathbf{e^+ e^- \to \pi^+ \pi^- \gamma}$ studied by KLOE in 2006 using two models: kaon-loop (KL) and no-structure (NS). The results are:

		KL	NS
f_0(980) mass (MeV)		980-987	973-981
$g(f_0 K^+ K^-)$	(GeV)	5.0 – 6.3	1.6 – 2.3
$g(f_0 \pi^+ \pi^-)$	(GeV)	3.0 – 4.2	0.9 – 1.1

Model dependence in experimental analyses

Some specific **problems** frequently encountered in data analyses:

1. presence of the thresholds $K^+ K^-$ and $K^0 K^0$ bar,
2. application of the Breit-Wigner formula with a constant width,
5. reduction of a number of scalar resonances to one (for example, sometimes only $f_0(980)$ state is included),
4. simplified treatment of the final-state strong interactions,
5. different description of the reaction production mechanism.

Example: the reaction $\mathbf{e^+ e^- \to \pi^+ \pi^- \gamma}$

Two factors: $\mathbf{e^+ e^-} \to f_0(980)\ \boldsymbol{\gamma}$ production amplitude **P(m)**,

$f_0(980) \to \boldsymbol{\pi^+ \pi^-}$ resonant decay amplitude **D(m)**.

Reaction amplitude: **A(m) = P(m) D(m)**, m= $\pi^+ \pi^-$ effective mass.

Results:

if $|A(m)|^2$ is fitted to data with two **different** functions **P(m)** then the **parameters** of the **resonant amplitude D(m)** are also **different**.

Further arguments to study $e^+ e^- \to K^+ K^- \gamma$

1. The **ϕ(1020)** $\rightarrow$ **$\pi^+ \pi^- \gamma$** branching fraction has been measured but the branching fraction for the ϕ(1020) meson decay into the **$K^+ K^- \gamma$ channel** is yet **unknown**.

2. There are data for the radiative decay of the ϕ(1020) meson into two **scalar resonances**:

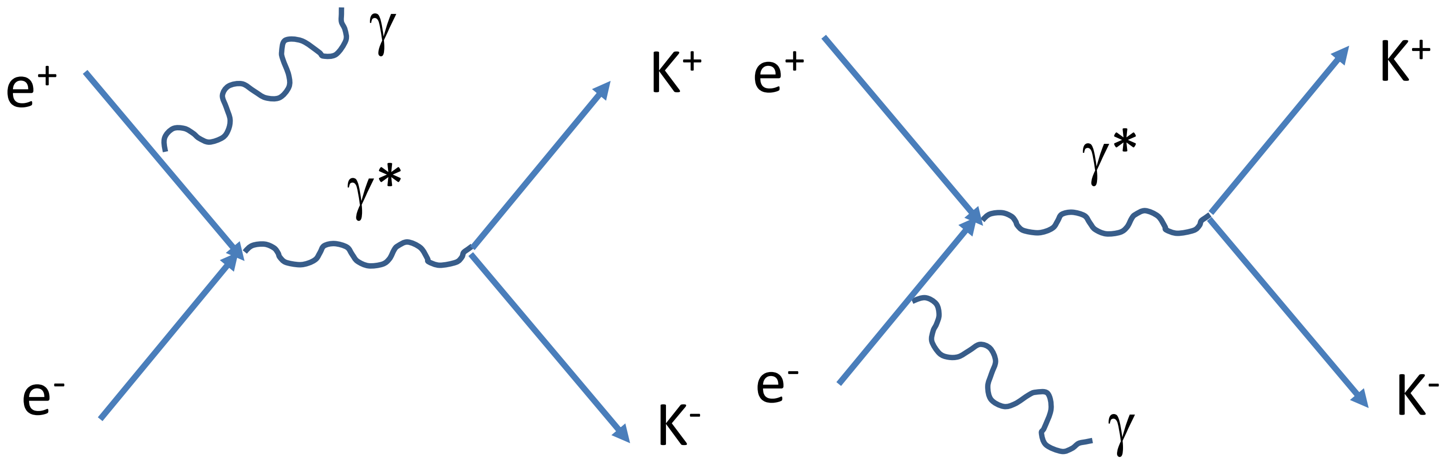
   $\Gamma(f_0(980)\,\gamma)/\,\Gamma_{total} = (3.22 \pm 0.19)\ 10^{-4}$,

   $\Gamma(a_0(980)\,\gamma)/\,\Gamma_{total} = (7.6 \pm 0.6)\ 10^{-5}$.

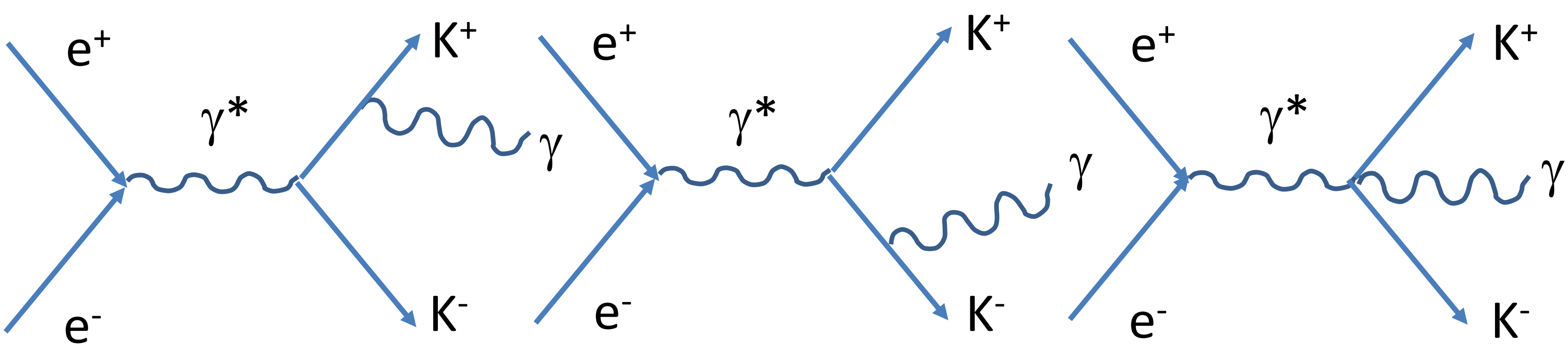
3. Both scalar resonances decay to the $K^+ K^-$ pairs, so one should observe the reaction $e^+ e^- \rightarrow K^+ K^- \gamma$. For the decay $\phi \rightarrow K^0 K^0$bar γ only the upper limit 1.9 10^{-8} is known (KLOE 2009).

4. Measurement of the $e^+ e^- \rightarrow \phi \rightarrow K^+ K^- \gamma$ transition could provide a new information about the **$K^+ K^-$ strong interactions near threshold**.

Reaction mechanisms (1)

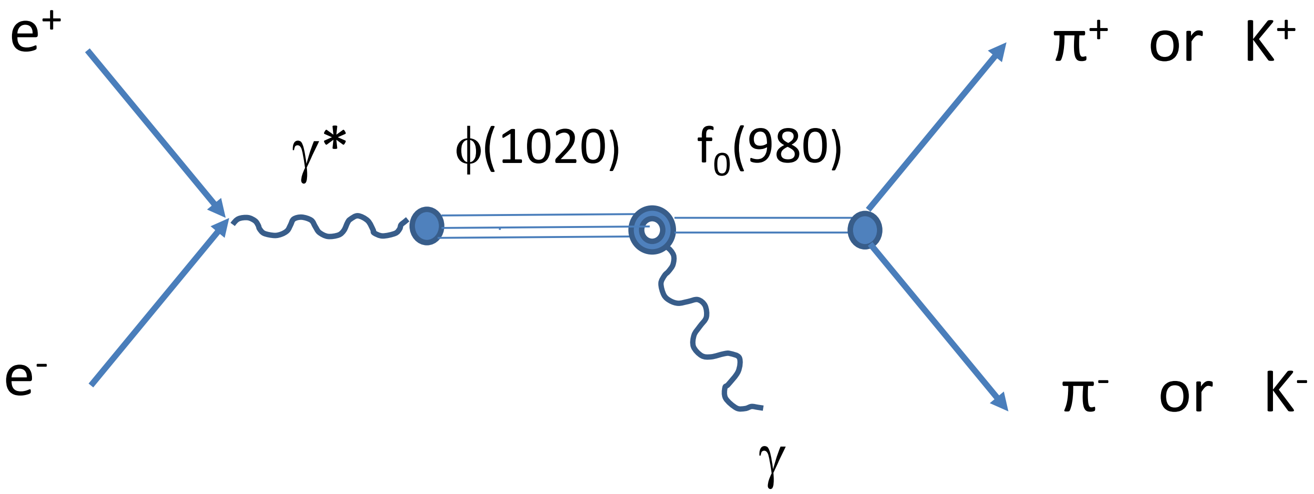


ISR – initial state radiation



FSR – final state radiation

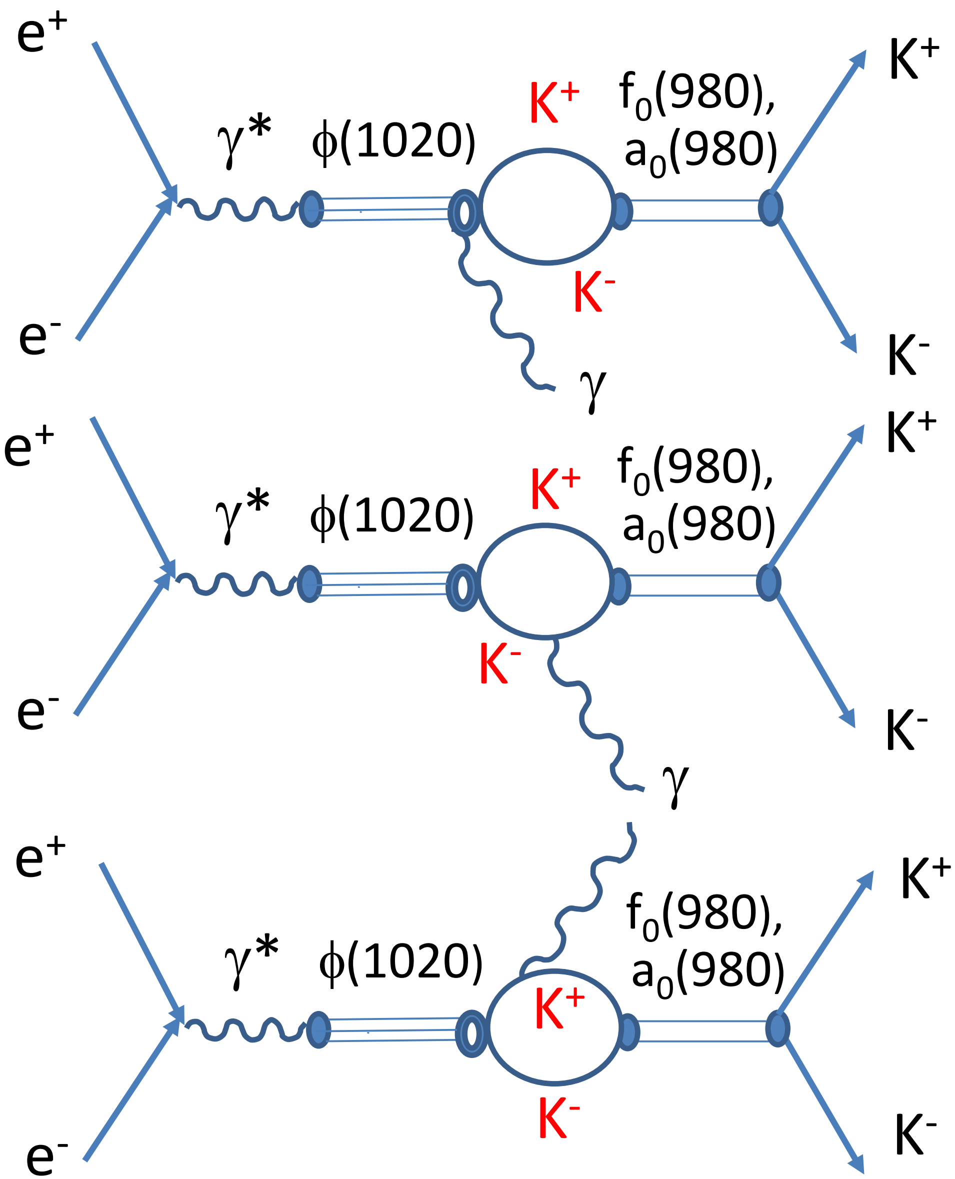
Reaction mechanisms (2)



NS – no-structure model

Ref.: G. Isidori, L. Maiani, M. Nicolaci, S. Pacetti, JHEP 0605 (2006) 049.

Reaction mechanisms (3)



kaon-loop model

Ref.: N. N. Achasov, V. V. Gubin and V. I. Shevchenko, PRD 56(1997)203.

Amplitude of the reaction $e^+ e^- \to K^+ K^- \gamma$

Example of the kaon-loop model:

$$M = A(ISR) + A(FSR) + A(KL) \qquad \text{total amplitude}$$

$$\boxed{\begin{aligned}|M|^2 = {} & |A(ISR)|^2 + |A(FSR)|^2 + |A(KL)|^2 + \\ & 2\,\mathrm{Re}[A^*(ISR)\,A(FSR)] + 2\,\mathrm{Re}[A^*(ISR)\,A(KL)] + 2\,\mathrm{Re}[A^*(FSR)\,A(KL)]\end{aligned}}$$

$$\boxed{\begin{aligned}d\sigma\,(total) = {} & d\sigma\,(ISR) + d\sigma\,(FSR) + d\sigma\,(KL) + \\ & int\,(ISR\text{-}FSR) + int\,(ISR\text{-}KL) + int\,(FSR\text{-}KL)\end{aligned}} \qquad \text{interference terms}$$

If experimental **cuts** are chosen **symmetrical**ly with respect to the interchange of K^+ and K^- then

$$int\,(ISR\text{-}FSR) = int\,(ISR\text{-}KL) = \mathbf{0}.$$

Differential cross-section

Reaction: $e^+(p_{e^+})\, e^-(p_{e^-}) \to K^+(p_{K^+})\, K^-(p_{K^-})\, \gamma$ (q)

$$\boxed{d\sigma = \frac{(2\pi)^4}{2\sqrt{s(s-4m_e^2)}} |\mathrm{M}|^2 d\Phi_3}$$

M -amplitude

Φ_3 – phase space

5 invariants:

2 momentum transfers:

$\boldsymbol{s} = (p_{e^+} + p_{e^-})^2$

$\mathbf{t} = (p_{e^-} - q)^2$

$\boldsymbol{t_1} = (p_{e^-} - p_{K^-})^2$

$\boldsymbol{m^2} = (p_{K^+} + p_{K^-})^2$

$\boldsymbol{m^2_{K^-\gamma}} = (p_{K^-} + q)^2$

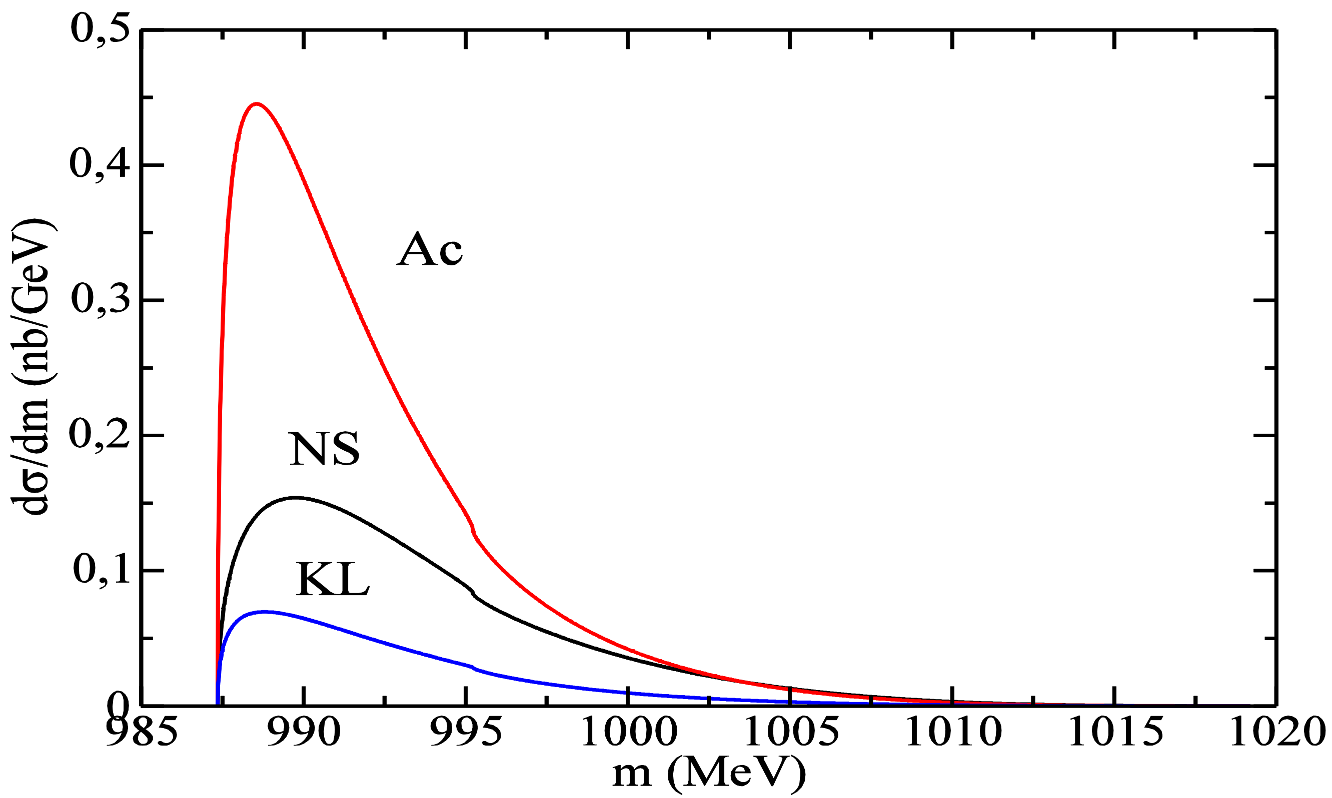
$$\boxed{\frac{d\sigma}{dm^2 dm^2_{K^-\gamma} dt\, dt_1} = \frac{1}{(2\pi)^4} \frac{|\mathrm{M}|^2}{16\, s\, (s-4m_e^2)(s-m^2) r}}$$

$$r = \sqrt{-(t_1 - t_{1min})(t_1 - t_{1max})}$$

m – K^+K^- effective mass, $m_{K^-\gamma}$ – K^- photon effective mass

K^+K^- effective mass distributions

Comparison of three models extended to describe $e^+ e^- \rightarrow K^+ K^- \gamma$



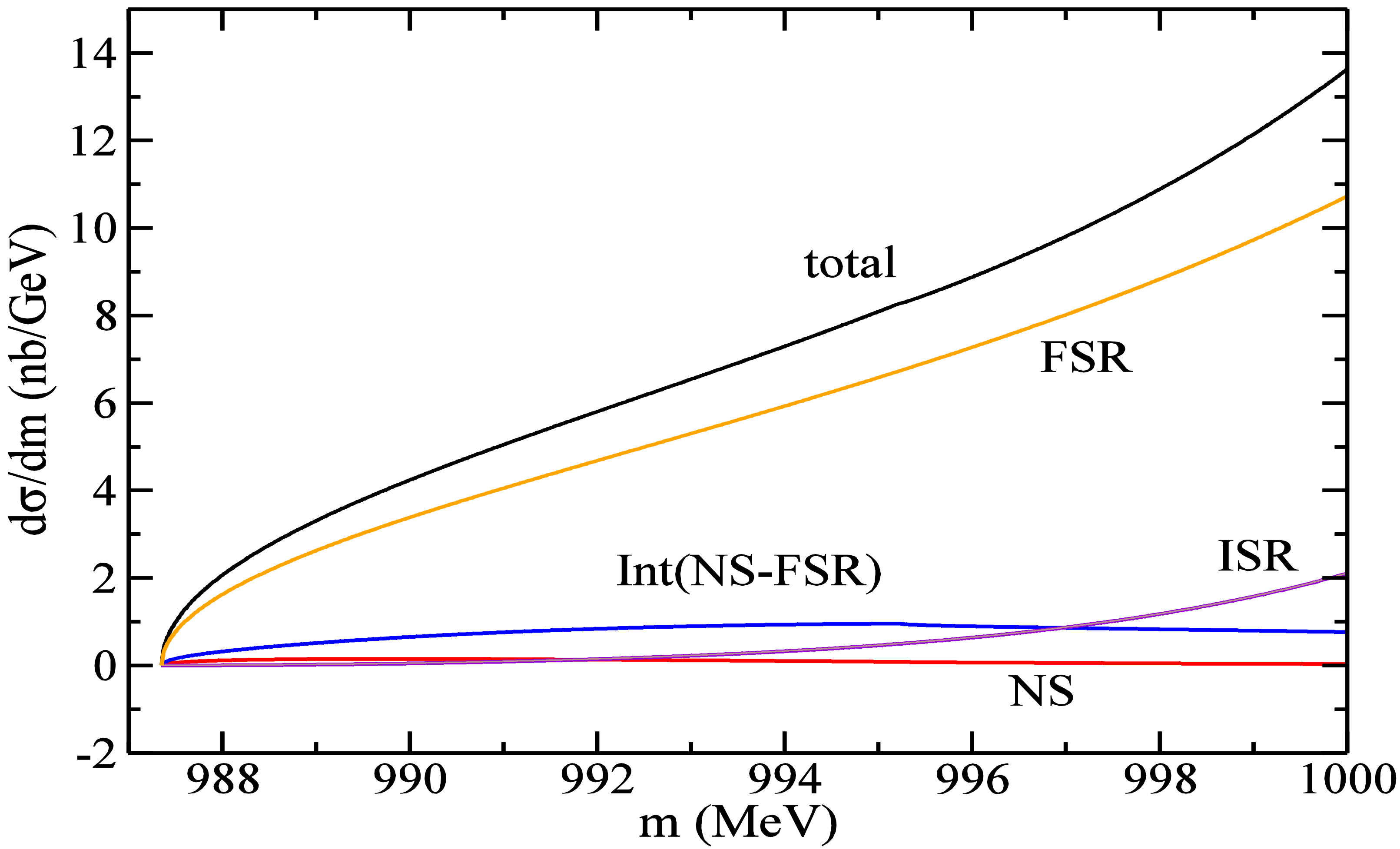
Ac – **Achasov** model,

NS – **no-structure** model

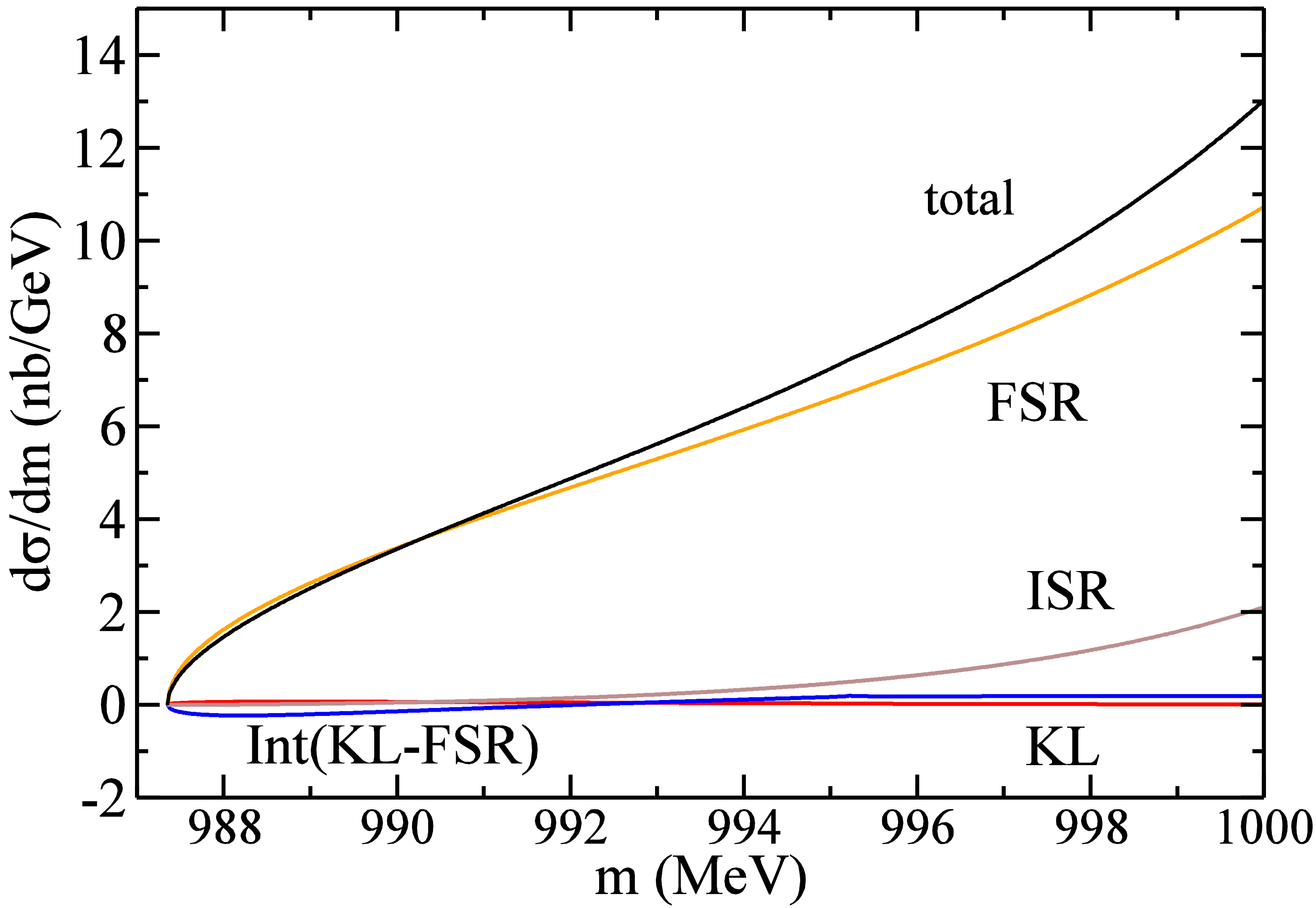
KL – **kaon-loop** model

Cuts for $45^0 < \theta_\gamma < 135^0$

K^+K^- effective mass distributions for the no-structure model ($45^0 < \theta_\gamma < 135^0$)

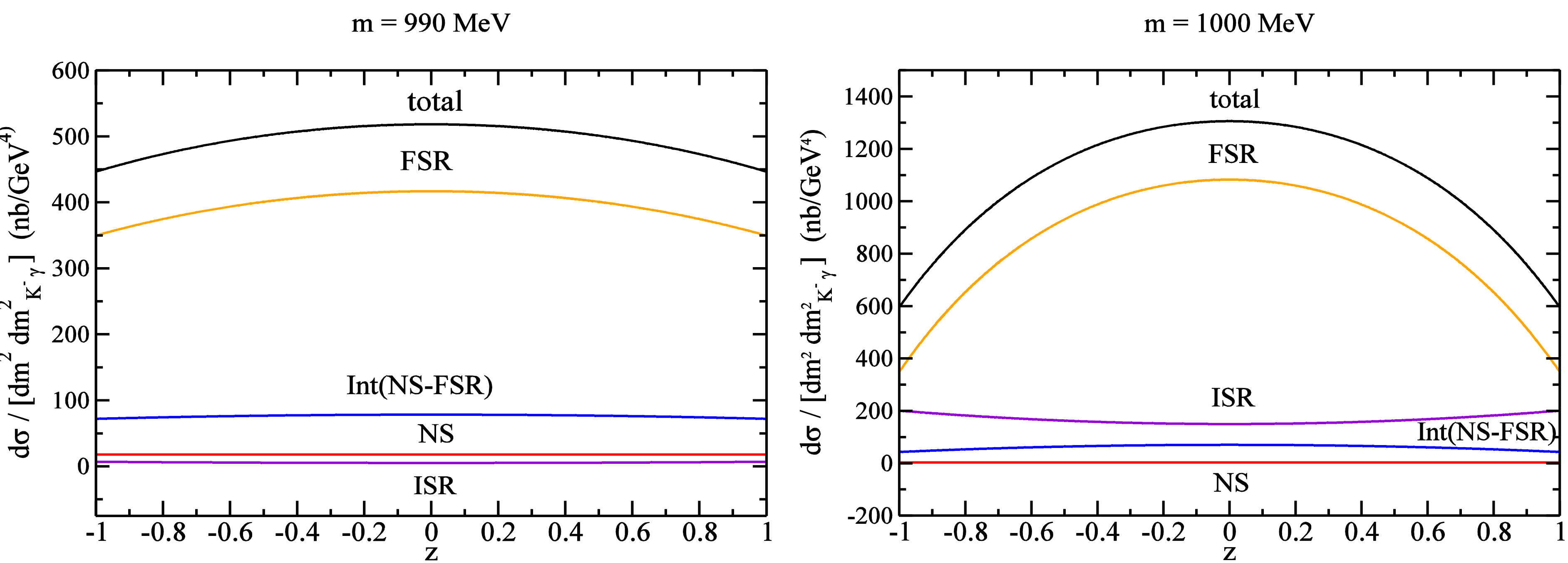


K^+K^- effective mass distributions for the kaon-loop model ($45^0 < \theta_\gamma < 135^0$)



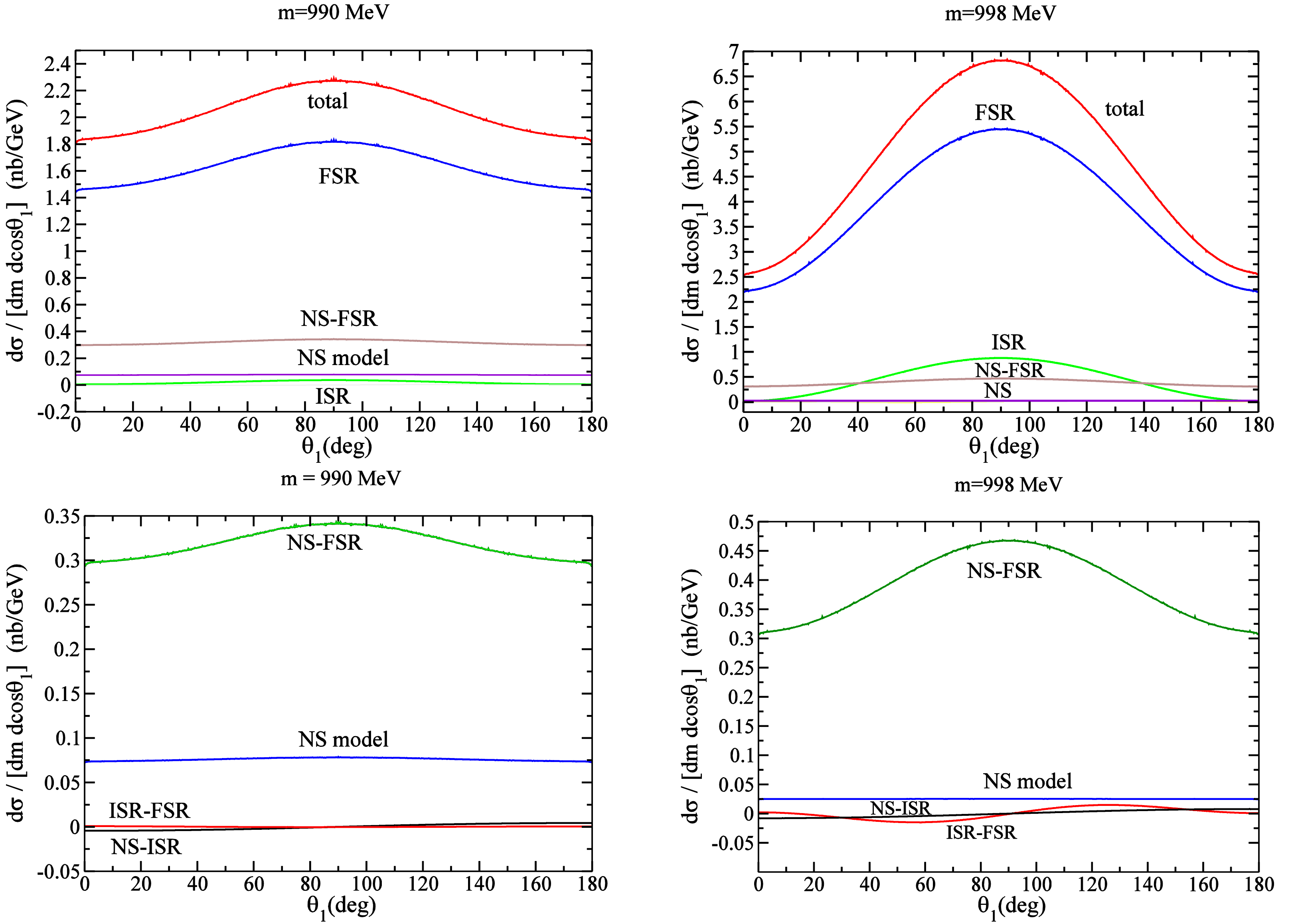
Distributions of the polar angle of K^- with respect to the photon axis in the $K^+ K^-$ center of mass frame

no-structure model $(45^0 < \theta_\gamma < 135)$



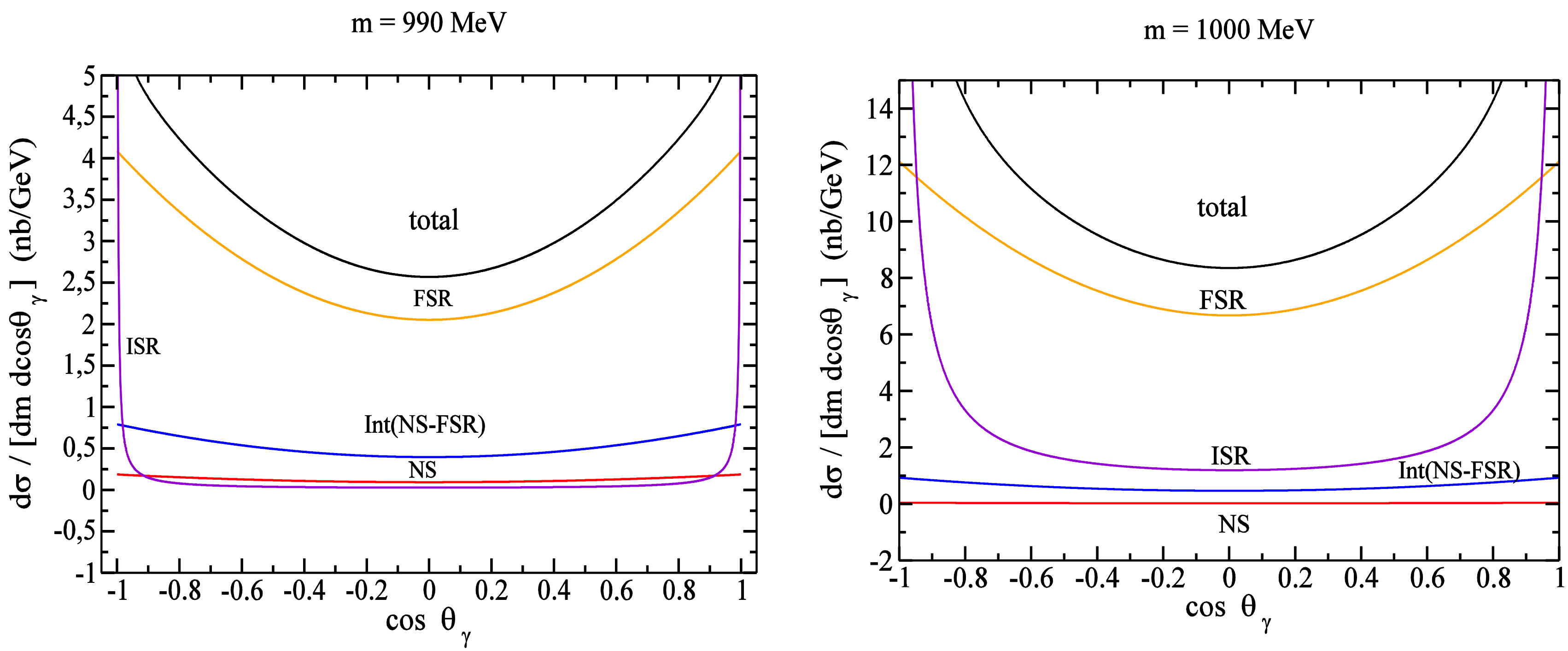
$z = \cos\theta_1^*$

K^- angular distributions at fixed m in the $e^+ e^-$ c.m. frame $(45^0 < \theta_\gamma < 135^0)$

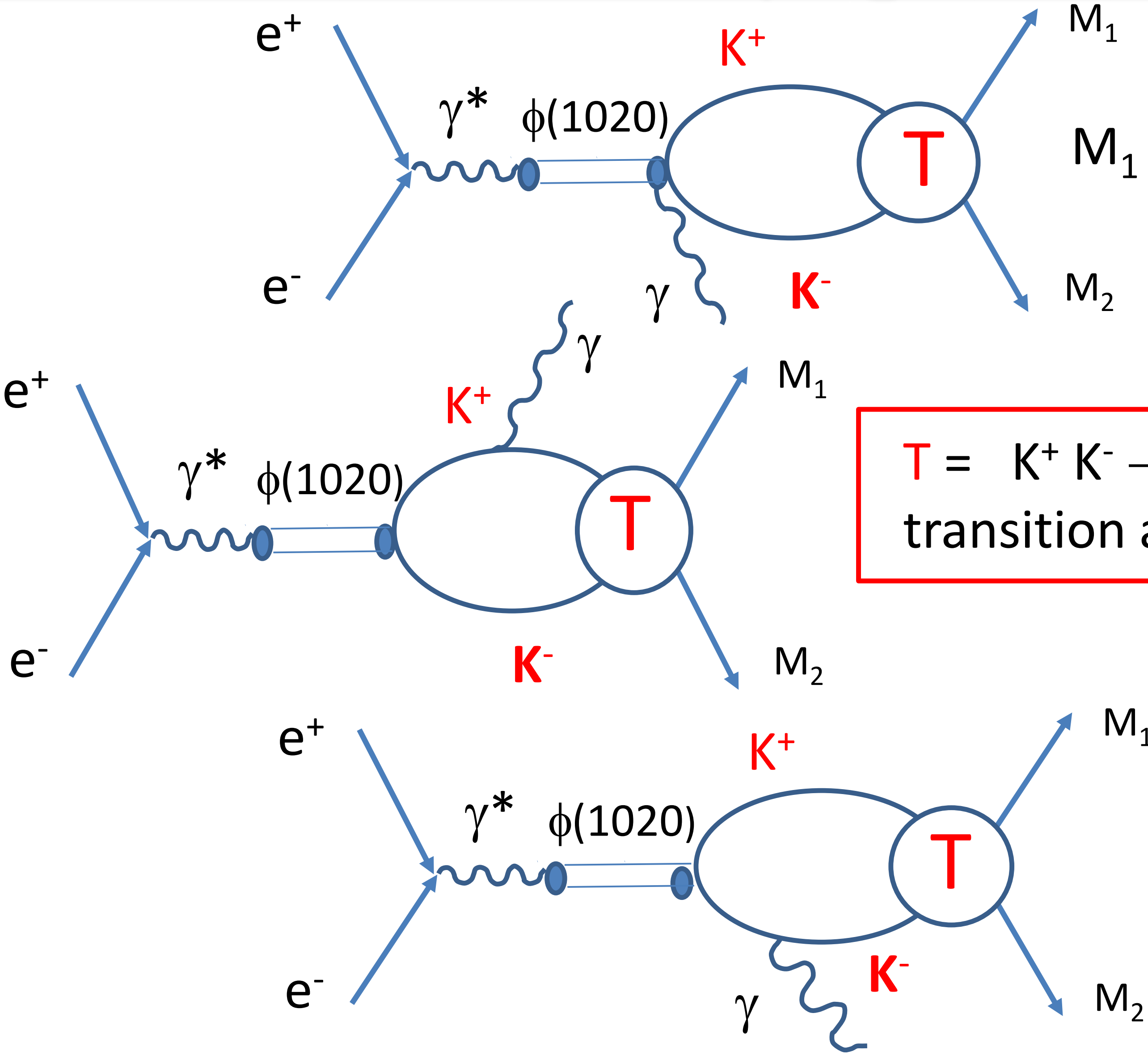


Photon angular distributions

no-structure model (NS)



Proposed multichannel model of the $e^+ e^- \to M_1 M_2 \gamma$ reactions



M_1 , M_2 - mesons

T = $K^+ K^- \to M_1 M_2$
transition amplitude

Features of the proposed model

a) **Unitary** description of the coupled channels represented by a set of amplitudes T.

b) **Analiticity** of the amplitudes T in which all the relevant poles corresponding to the scalar resonances are present.

c) Possible application of the model in the combined analyses of many meson-meson final states like:

1. $e^+ e^- \to \pi^+\pi^-\gamma$,
2. $e^+ e^- \to \pi^0\pi^0\gamma$,
3. $e^+ e^- \to \pi^0\ \eta\ \gamma$,
4. $e^+ e^- \to K^0_S K^0_S \gamma$,
5. $e^+ e^- \to K^+ K^- \gamma$.

Experimental implications

Expected number of events integrated over the K^+K^- effective mass up to 1009 MeV for two photon angle ranges and for the integrated luminosity of 1.7 fb^{-1}:

reaction mechanism	$24^0 < \theta_\gamma < 156^0$	$45^0 < \theta_\gamma < 135^0$
FSR	$5.6\ 10^5$	$4.0\ 10^5$
NS	$3.4\ 10^3$	$2.4\ 10^3$
Int(NS-FSR)	$3.6\ 10^4$	$2.5\ 10^4$
ISR	$3.1\ 10^5$	$1.8\ 10^5$
total	$9.1\ 10^5$	$6.1\ 10^5$

Summary

1. Three models have been extended in order to describe the reaction $e^+ e^- \to K^+ K^- \gamma$.
2. The theoretical results for the effective mass and angular distributions can be used in future experimental data analyses.
3. A new model of the multichannel coupled reactions $e^+ e^- \to M_1 M_2 \gamma$ has been outlined.
4. The model can be applied in a **combined analysis** of the radiative **ϕ(1020)** meson decays into pairs of pseudoscalar mesons.
5. It can also serve in determination of:

a) the **$K^+ K^-$ threshold parameters** of the strong interaction amplitudes and

b) in a better specification of the **properties** of the **scalar meson resonances** **f_0(980)** and **a_0(980)**.

6. Our recent reference: Leonard Lesniak and Michal Silarski, arXiv:1610.01514 [hep-ph].

Kinematical relations (1)

θ_1, θ_γ - K$^-$ and photon **polar angles** in the **e$^+$e$^-$ center-of-mass frame**, z-axis along the e$^-$ momentum

Relations:

$$\boxed{\begin{aligned} t_1 &\approx m_K^2 - \sqrt{s}\,E_1^l\,(1 - v_1 \cos \Theta_1) \\ t &\approx -\sqrt{s}\,\omega^l\,(1 - \cos \Theta_\gamma) \end{aligned}}$$

E_1^l, ω^l are K$^-$ and photon energies, v_1 is K$^-$ velocity.

$$E_1^l = \frac{m^2 + m_{K^-\gamma}^2 - m_K^2}{2\sqrt{s}} \qquad \omega^l = \frac{s - m^2}{2\sqrt{s}}$$

Kinematical relations (2)

Definitions: θ_1^*- polar angle of K^- with respect to the photon axis in the **K^+ K^- center- of- mass frame**,

z = cos θ_1^*

Relation to the $K^-\gamma$ effective mass squared $m^2_{K^-\gamma}$:

$$\boxed{m^2_{K^-\gamma} = m_K^2 + \frac{1}{2}(s - m^2)(1 - \boldsymbol{v}\, z)}$$

$\boldsymbol{v}$ = K^- velocity in the K^+ K^- center of mass frame, $v = \sqrt{1 - \frac{4m_K^2}{m^2}}$

z=+1 corresponds to minimum of $m^2_{K^-\gamma}$, **z= -1** to **maximum** of $m^2_{K^-\gamma}$.

Integrated cross sections

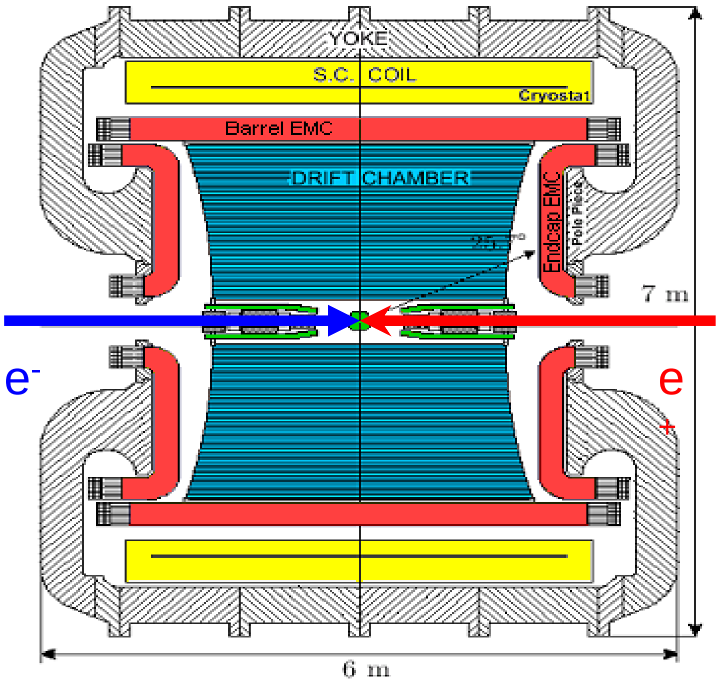
The **cross sections integrated** over the K^+K^- effective mass up to 1009 MeV for two photon angle ranges (units are nanobarns):

reaction mechanism	$24^0 < \theta_\gamma < 156^0$	$45^0 < \theta_\gamma < 135^0$
FSR	0.330 nb	0.238 nb
NS	0.0020 nb	0.0014 nb
Int(NS-FSR)	0.021 nb	0.015 nb
ISR	0.183 nb	0.104 nb
total	0.536 nb	0.358 nb

Experimental search for the reaction $e^+ e^- \rightarrow K^+ K^- \gamma$

- This can be done with the **KLOE data** (analysis has already started)

- **Advantages**:
  - very good kaon momentum determination
  - high statistics
- **Problems**:
  - low energy photons (< 32 MeV) $\Rightarrow$ lower efficiency and energy resolution
  - slow transverse momentum tracks for low K^+K^- effective masses



- Challenging measurement