

Kloe-2 Workshop on  $e^+e^-$   
collision physics at 1 GeV

INFN Laboratori Nazionali di Frascati  
26-28 Ottobre 2016

INFN - Laboratori Nazionali di Frascati, Italy

The Workshop will be devoted to a thoroughly discussion of the KLOE-2 Physics program at D  
and more in general about topics related to  $e^+e^-$  Physics at 1 GeV c.m. energy



- CP and T violation, CPT and QM tests
- Ks decays, eta decays and chiral lagrangians
- Phi decays, light hadron spectroscopy and transition form factors
- ... Physics

# A PROPOSAL TO MEASURE THE HADRONIC CONTRIBUTION TO THE G-2 IN THE SPACE-LIKE REGION

Luca TRENTADUE  
Università di Parma and INFN  
Sezione di Milano Bicocca

## Organising Committee

Caterina Bloise (LNF)  
Eryk Czerwiński (Krakow)  
Erika De Lucia (LNF)  
Antonio Di Domenico (Roma) - **Chair**  
Paolo Gauzzi (Roma)  
Simona Giovannella (LNF)  
Enrico Graziani (Roma Tre)  
Andrzej Kupsc (Uppsala)  
Luca Trentadue (Parma)  
Graziano Venanzoni (LNF)

**Support**  
damato@lnf.infn.it

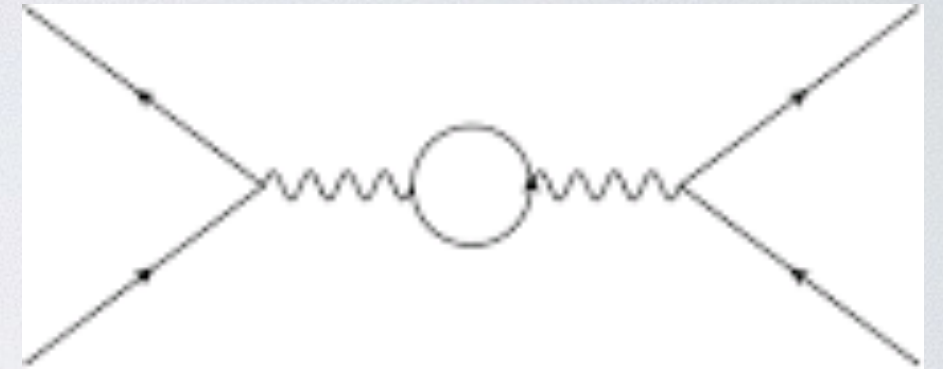
work done in collaboration with:

G. Abbiendi, C. Carloni Calame, U. Marconi, C. Matteuzzi,  
G. Montagna, O. Nicrosini,  
M. Passera, F. Piccinini, R. Tenchini, and G. Venanzoni

C. M. Carloni Calame, M. Passera, L.T. and G. Venanzoni,  
"A new approach to evaluate the leading hadronic corrections to the muon  $g-2$ "  
Phys. Lett. B 746 (2015) 325.

G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna,  
O. Nicrosini M. Passera, F. Piccinini, R. Tenchini, L.T. and G. Venanzoni,  
"Measuring the leading hadronic contribution to the muon  $g-2$  via  $\mu$ -e  
scattering", arXiv:1609.08987 [hep-ex].

Vacuum Polarization makes  $\alpha_{\text{em}}$  running  
assuming a well defined “effective” value at  
any scale



vacuum polarization and the “effective  
charge” are defined by:

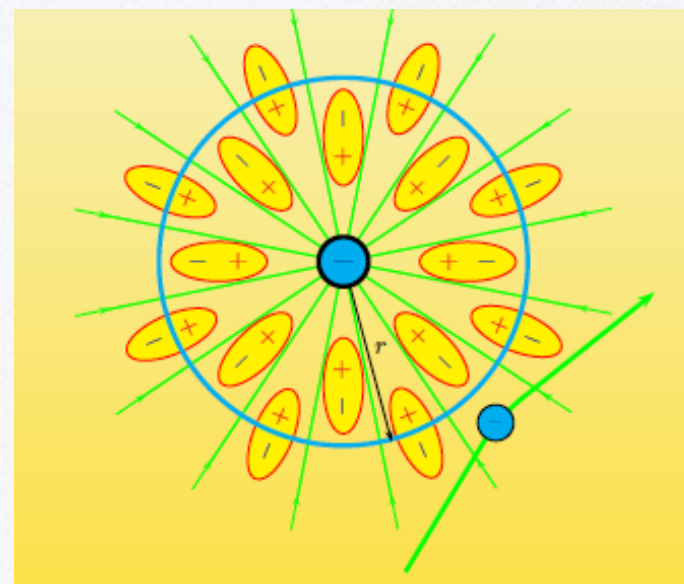
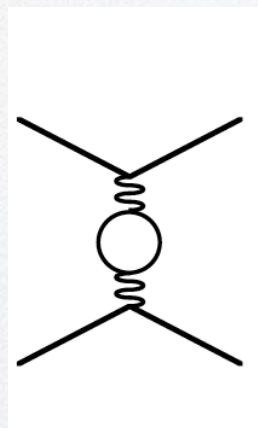
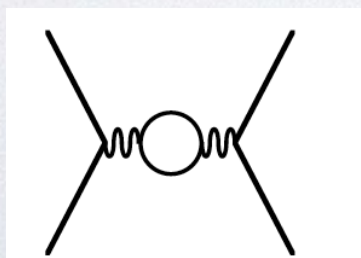
$$e^2 \rightarrow e^2(q^2) = \frac{e^2}{1 + (\Pi(q^2) - \Pi(0))} \quad \alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha}; \quad \Delta\alpha = -\Re e(\Pi(q^2) - \Pi(0))$$

$\Delta\alpha$  takes contributions from leptonic and hadronic and gauge bosons  
elementary states

Among these the non-perturbative  $\Delta\alpha_{\text{had}}$

$$\Delta\alpha = \Delta\alpha_{\text{leptonic}} + \Delta\alpha_{\text{gb}} + \Delta\alpha_{\text{had}} + \Delta\alpha_{\text{top}}$$

$\alpha$



The numerical prediction of electroweak observables involves the knowledge of  $\alpha(q^2)$  usually for  $q^2 \neq 0$

for example the knowledge of  $\alpha(m_Z^2)$  is relevant to the evaluation of quantities measured by the LEP experiments.

This is achieved by evolving from  $q^2 = 0$  to the  $Z$  mass.

The evolution is expressed by the quantity  $\Delta\alpha(q^2)$  receives contributions from leptons, hadrons and the gauge bosons.

The hadronic contribution to the vacuum polarization, which cannot be calculated from first principles, is estimated with the help of a dispersion integral and evaluated by using total cross section measurements of  $e^+e^- \rightarrow$  hadrons at low energies.

Therefore, any evolved value of  $\Delta\alpha(q^2)$  and particularly for  $q^2 \geq m_\pi^2$  is affected by uncertainties originating from hadronic contributions.

F. Jegerlehner: hep-ph/0308117

M. Davier and A. Hoecker: Phys. Lett. **B435** (1998) 427;

M. Davier, S. Eidelman, A. Hoecker and Z. Zhang: Eur. Phys. J. **C27** (2003) 497

D. Karlen and H. Burkhardt: Eur. Phys. J. **C22** (2001) 39; hep-ex/0105065 (2001)

A.A. Pankov and N. Paver, Eur. Phys. J. **C29** (2003) 313

Within the framework of low-energy high precision measurements  
the long-standing (3-4)  $\sigma$   
discrepancy between the experimental value of the muon  
anomalous magnetic moment and the Standard Model prediction

$$a_\mu = \frac{g - 2}{2} \quad \Delta a_\mu (Exp - SM) \simeq 28 \pm 8 \cdot 10^{-10}$$

The accuracy of the SM prediction  $5 \cdot 10^{-10}$

is limited by **strong interactions** effects

The present error on the leading order hadronic  
contribution to muon  $g - 2$

$$\delta a_\mu^{HLO} \simeq 4 \cdot 10^{-10}$$

constitutes the main uncertainty of the SM predictions

Theory (in units of  $10^{-10}$ ) from M. Knecht talk at Capri Workshop on FCCP2015

|            |                                                        |                                                                                                               |
|------------|--------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| QED        | + 116 584 71.9                                         | [T. Aoyama et al. (2015)]                                                                                     |
| HVP-LO     | $\begin{cases} +692.3(4.2) \\ +694.9(4.3) \end{cases}$ | $\begin{matrix} \text{[M. Davier et al. (2011)]} \\ \text{[K. Hagiwara et al. (2011)]} \end{matrix}$          |
| HVP-NLO    | -9.84(7)                                               | [K. Hagiwara et al. (2011)]                                                                                   |
| HVP-NNLO   | +1.24(1)                                               | [A. Kurz et al. (2014)]                                                                                       |
| HLxL       | $\begin{cases} +10.5(2.6) \\ +11.5(4.0) \end{cases}$   | $\begin{matrix} \text{[J. Prades et al. (2009)]} \\ \text{[F. Jegerlehner, A. Nyffeler (2009)]} \end{matrix}$ |
| EW 1 loop  | +19.48(1)                                              | [(1972)]                                                                                                      |
| EW 2 loops | -4.12(10)                                              | [C. Gnendiger et al. (2013)]                                                                                  |

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (27.4 \pm 8.0) \cdot 10^{-10} \quad [3.4\sigma] \quad \text{for } a_{\mu}^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}, \quad a_{\mu}^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (23.7 \pm 8.6) \cdot 10^{-10} \quad [2.8\sigma] \quad \text{for } a_{\mu}^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10}, \quad a_{\mu}^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$$

- QED provides more than 99.99% of the total value, without uncertainties at this level of precision
- hadronic corrections provide the second important contribution, and the bulk of the uncertainty
- theory and experiment give comparable contributions to the total error on  $a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}}$
- the present situation remains inconclusive as to the presence of BSM degrees of freedom
- forthcoming experiments at FNAL (E989) and at J-PARC (E34) plan to increase the experimental precision by a factor of 4  $\longrightarrow$  talk by D. Hertzog

|            |                                                        |                                                                                                               |
|------------|--------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| QED        | + 116 584 71.9                                         | [T. Aoyama et al. (2015)]                                                                                     |
| HVP-LO     | $\begin{cases} +692.3(4.2) \\ +694.9(4.3) \end{cases}$ | $\begin{matrix} \text{[M. Davier et al. (2011)]} \\ \text{[K. Hagiwara et al. (2011)]} \end{matrix}$          |
| HVP-NLO    | -9.84(7)                                               | [K. Hagiwara et al. (2011)]                                                                                   |
| HVP-NNLO   | +1.24(1)                                               | [A. Kurz et al. (2014)]                                                                                       |
| HLxL       | $\begin{cases} +10.5(2.6) \\ +11.5(4.0) \end{cases}$   | $\begin{matrix} \text{[J. Prades et al. (2009)]} \\ \text{[F. Jegerlehner, A. Nyffeler (2009)]} \end{matrix}$ |
| EW 1 loop  | +19.48(1)                                              | [(1972)]                                                                                                      |
| EW 2 loops | -4.12(10)                                              | [C. Gnendiger et al. (2013)]                                                                                  |

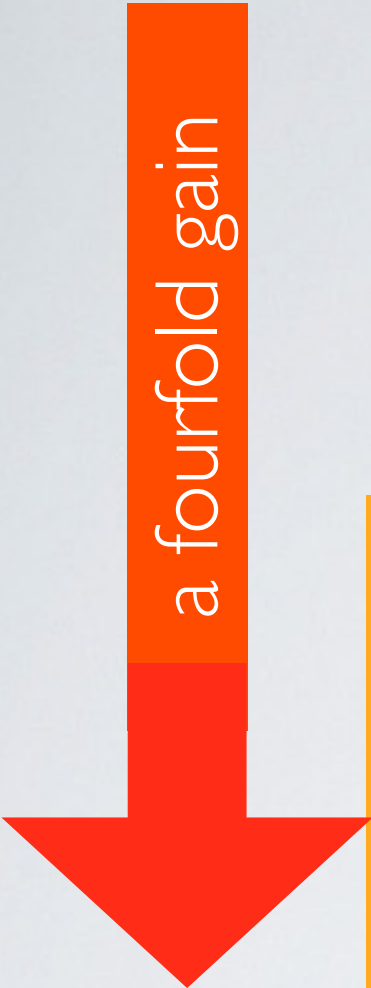
$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (27.4 \pm 8.0) \cdot 10^{-10} \quad [3.4\sigma] \quad \text{for } a_{\mu}^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}, \quad a_{\mu}^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (23.7 \pm 8.6) \cdot 10^{-10} \quad [2.8\sigma] \quad \text{for } a_{\mu}^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10}, \quad a_{\mu}^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$$

### Two issues (at least)

- are the values given in the table reliable (central values and uncertainties)?
- is it possible, in view of the planned experiments at FNAL and J-PARC, to reduce the dominant theoretical uncertainties (HVP-LO and HLxL)?

# How this will possibly change in the next few years ?



a fourfold gain

The present experimental error as from the **BNL E821** is

$$\delta a_{\mu}^{Exp} \simeq 6.3 \cdot 10^{-10} [0.54 \text{ ppm}]$$

The new experiments in preparation at **Fermilab** and **J-PARC** are aiming to a precision of \*

$$\delta a_{\mu}^{Exp-FL/J-PARC} \simeq 1.6 \cdot 10^{-10} [0.14 \text{ ppm}]$$

(\*assuming the same central value as today's one)

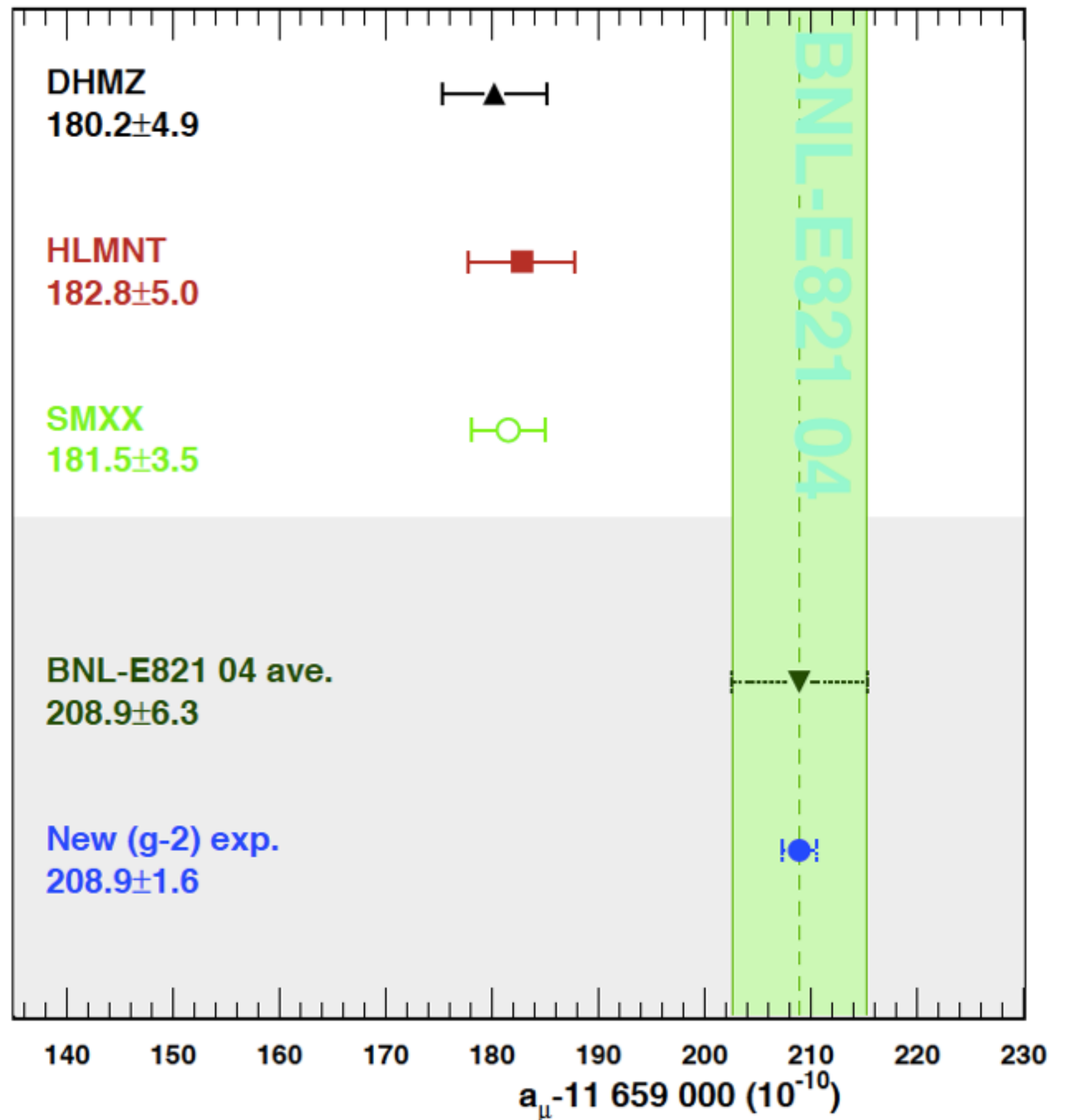
The question is how to cope with such an improvement from the theory side

Comparison between  
the SM predictions and  
the experimental  
determinations

Theory  
parametrizations  
DHMZ ( M.Davier et al.  
) , HLMNT ( K.  
Hagiwara et al. )  
SMXX is the average  
of the two previous  
values

BNL-E821 04 average  
is the current  
experimental value of  
 $a_\mu$

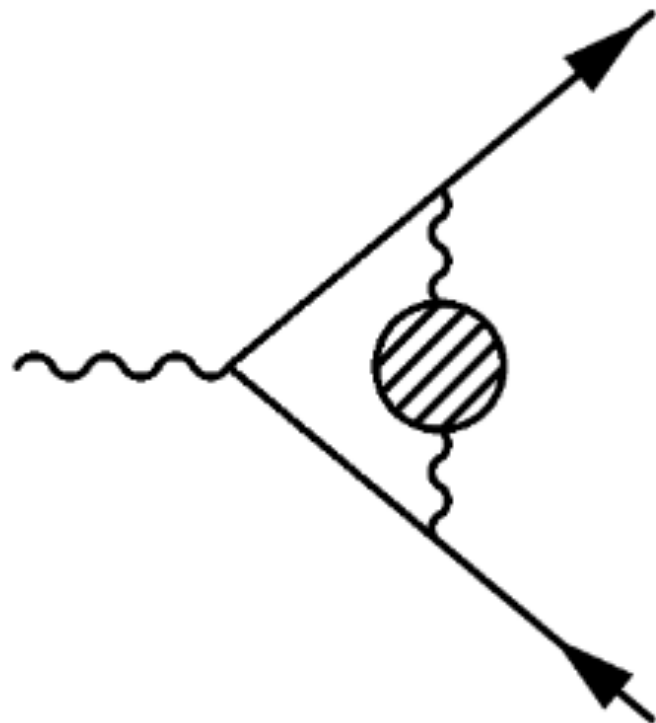
New (g-2) exp. is the  
same central value with  
a fourfold improved  
precision of future g-2  
experiments at  
Fermilab and J-PARC.



# THE STANDARD DISPERSIVE APPROACH

## Hadronic contribution $a_\mu^{\text{had}}$

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}$$



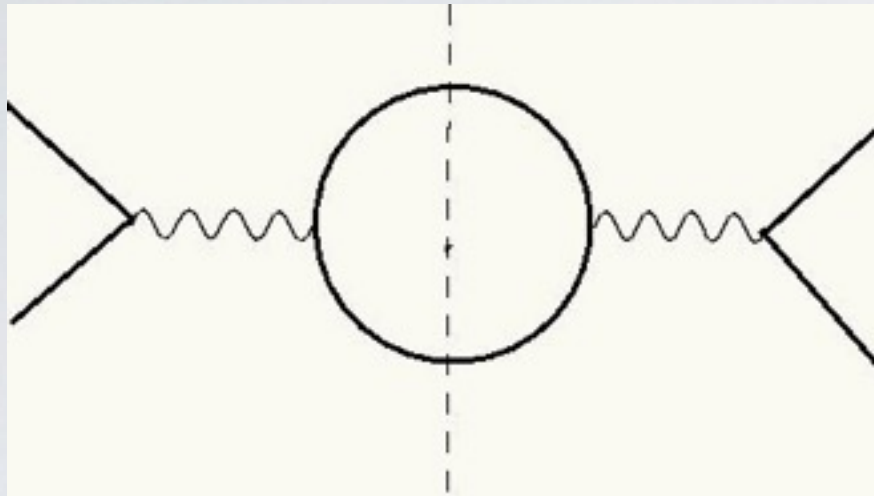
$$a_\mu^{\text{had,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{R(s) \hat{K}(s)}{s^2},$$

C. Bouchiat, L. Michel, Bouchiat, 1961;  
M. Gourdin, E. de Rafael, 1969

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)},$$

$\hat{K}(s)$  grows from 0.63 at  $s = 4m_\pi^2$  to 1 at  $s \rightarrow \infty$ ,  
 $1/s^2$  emphasizes low energies, particularly  $e^+e^- \rightarrow \pi^+\pi^-$ .  
 $a_\mu^{\text{had,LO}} \sim 700 \cdot 10^{-10} \Rightarrow$  accuracy better than 1% needed

$$a_{\mu}^{HLO} = \left(\frac{\alpha}{\pi^2}\right) \int_0^{\infty} \frac{ds}{s} K(s) \operatorname{Im} \Pi_{had}(s + i\epsilon) \quad \hat{K}(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_{\mu}^2}}$$

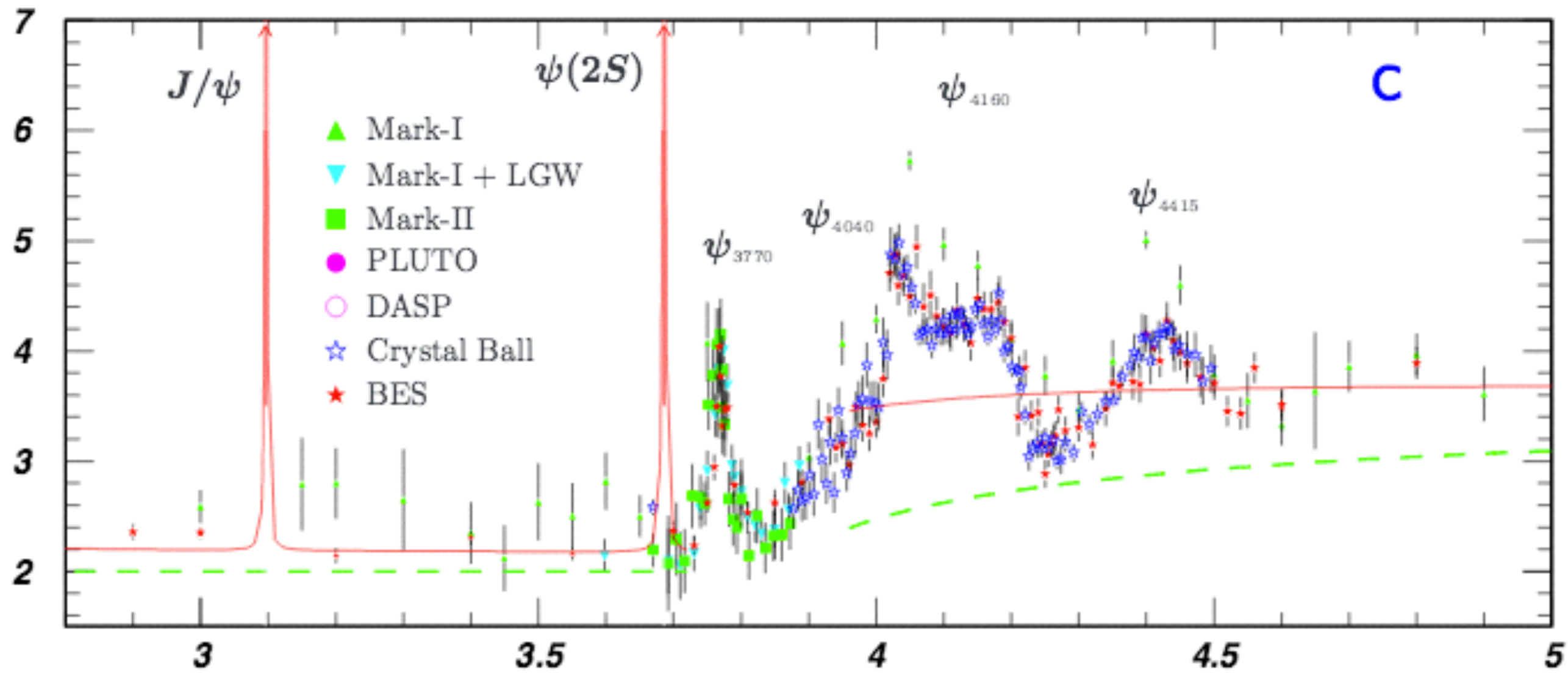


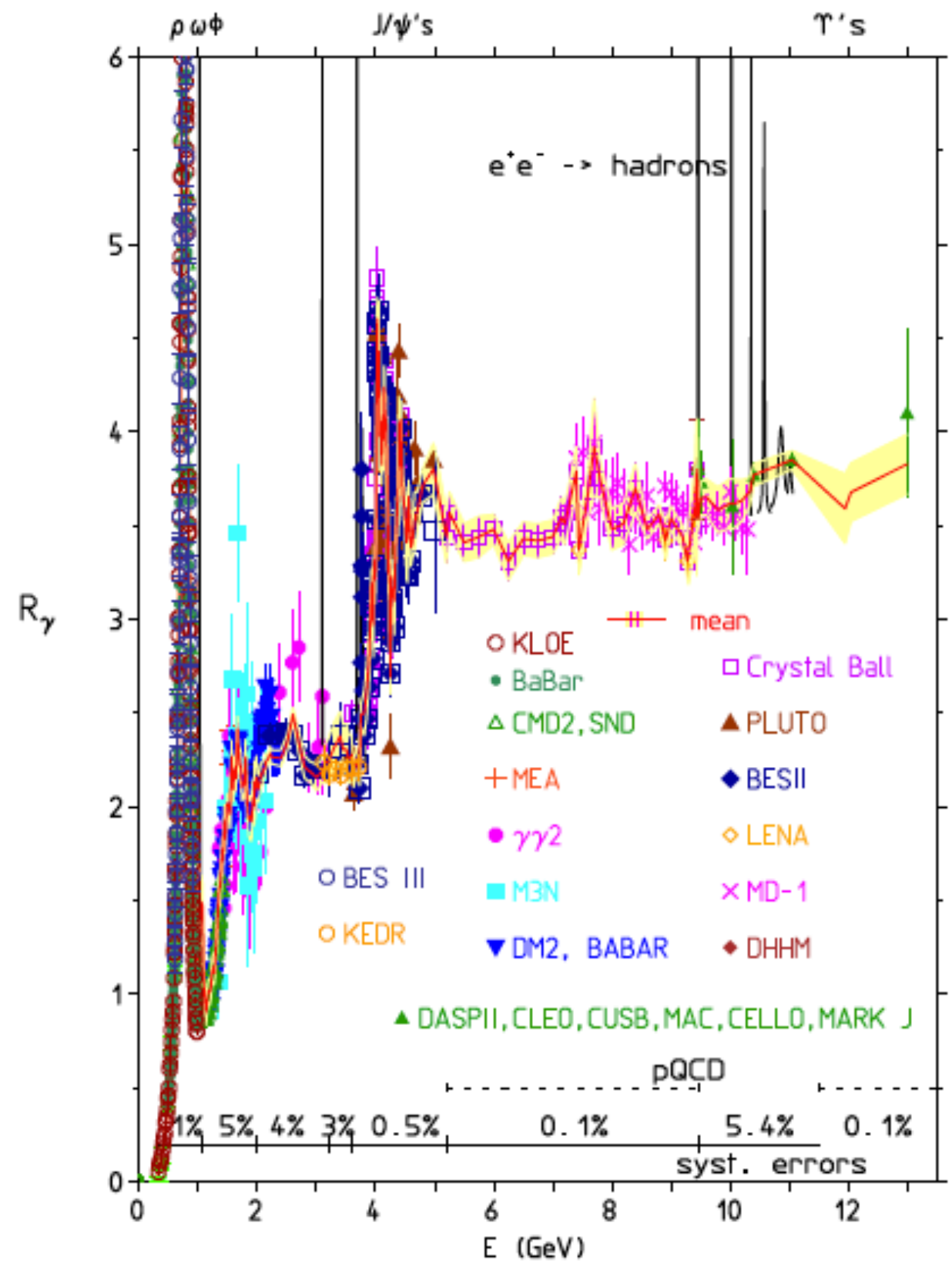
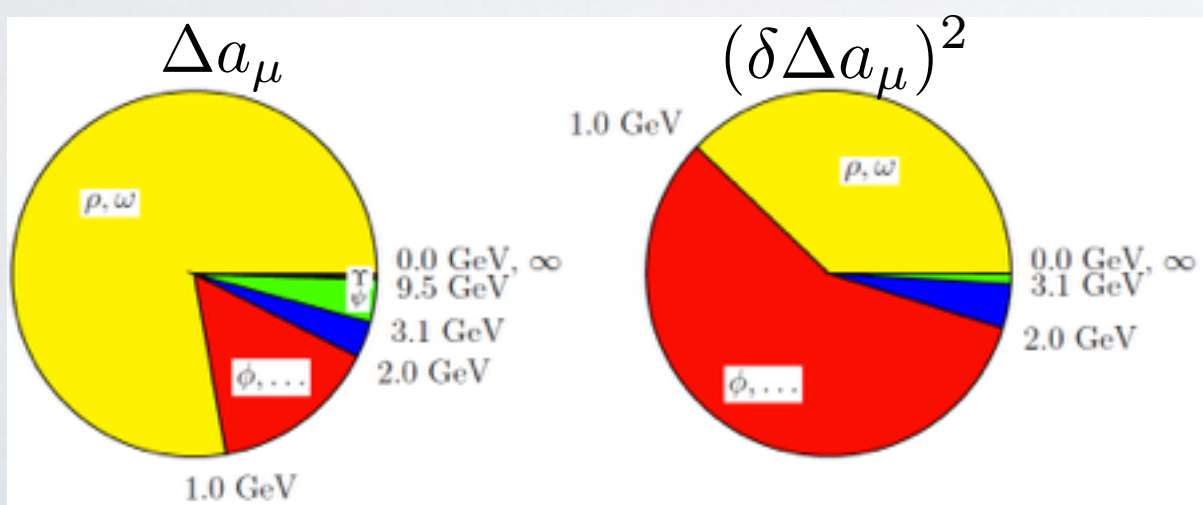
via the optical theorem

$$\operatorname{Im} \hat{\Pi}_{had}(s) \rightarrow \sigma_{tot}^{had}(s)$$

$$a_{\mu}^{HLO} = \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \int_{4m_{\pi}^2}^{\infty} ds \frac{\hat{K}(s) R_{had}(s)}{s^2}$$

$$R_{had}(s) = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$R$ 



from F. Jegerlehner talk in Frascati March 23, 2016

# Measurement of the running of $\alpha_{em}$

A direct measurement of  $\alpha_{em}(s/t)$  in space/  
time-like regions can show the running of  
 $\alpha_{em}(s/t)$

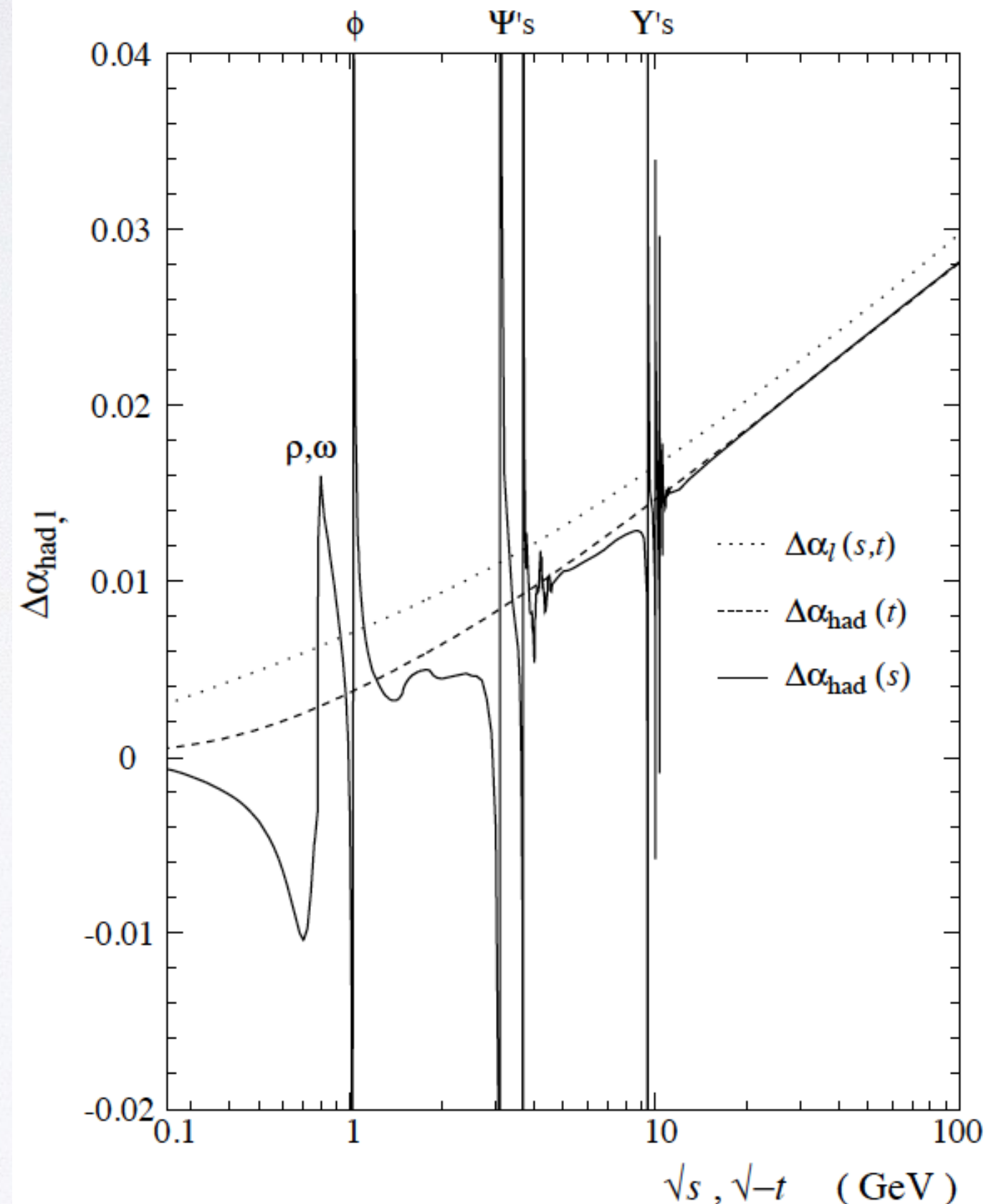
It can provide a test of “duality” (fare way  
from resonances)

It has been done in past by few experiments  
at  $e^+e^-$  colliders by comparing a “well-  
known” QED process with some  
reference (obtained from data or MC)

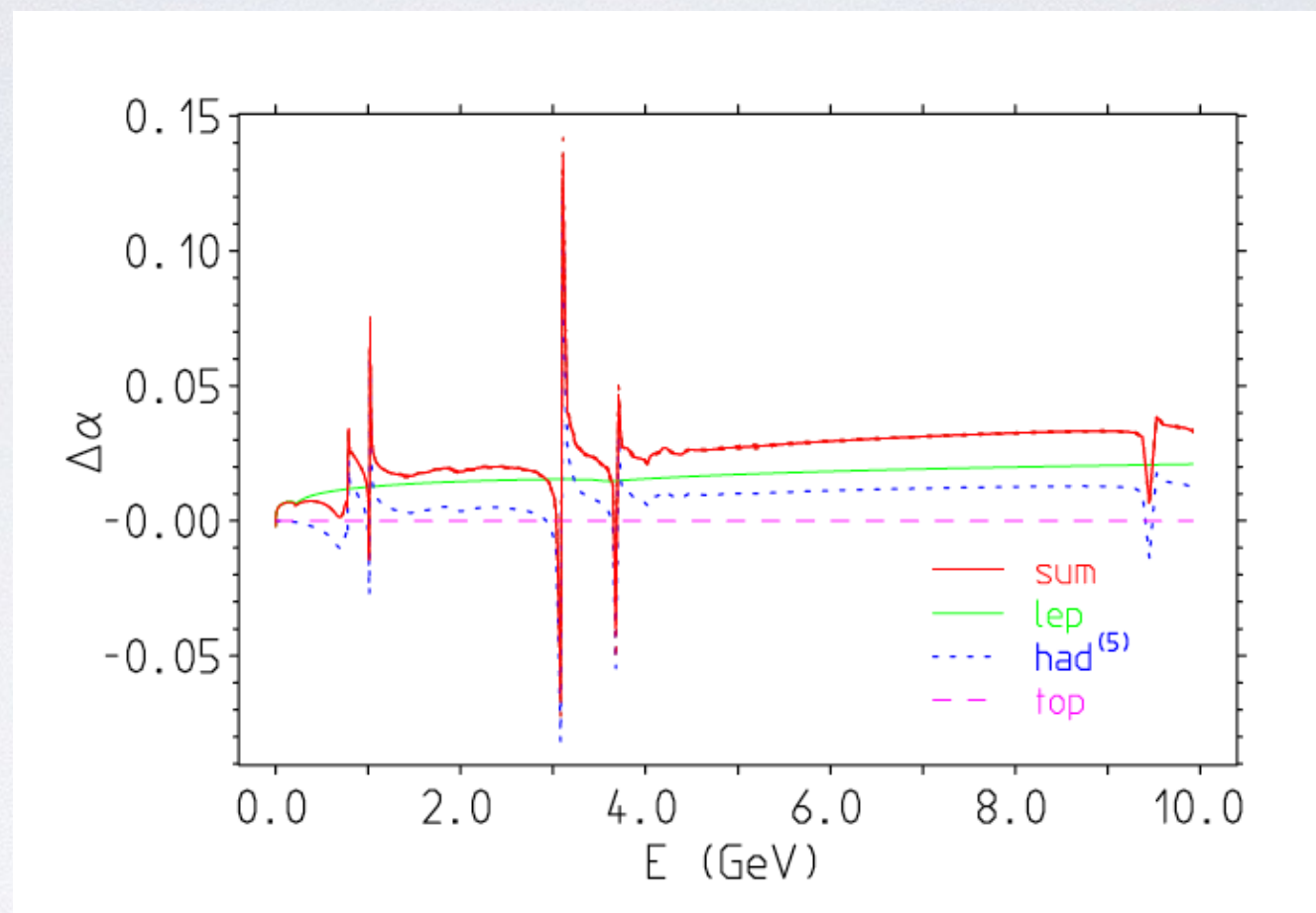
$$\left( \frac{\alpha(q^2)}{\alpha(q_0^2)} \right)^2 \sim \frac{N_{signal}(q^2)}{N_{norm}(q_0^2)}$$

$N_{signal}$  can be any QED process, muon pairs, etc...

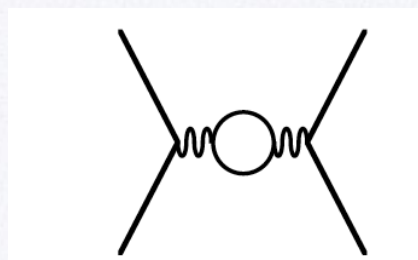
$N_{norm}$  can be Bhabha process, pure QED as  $\gamma\gamma$   
pair production, a well as theory, or any other  
reference process.



# Running of $\alpha_{\text{em}}$



time-like



$$\Delta\alpha_{had}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)}$$

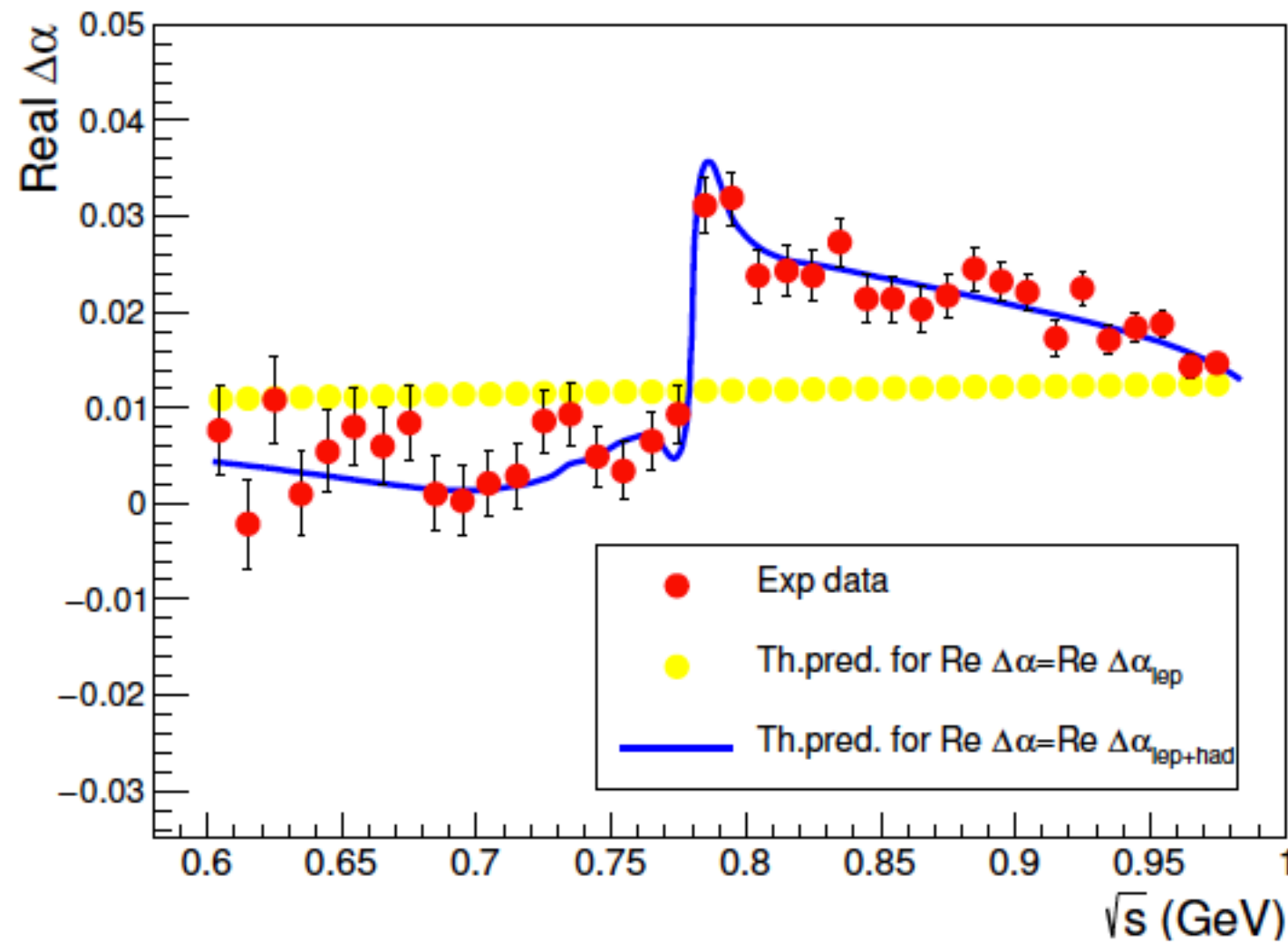


Fig. 6.  $\text{Re } \Delta\alpha$  extracted from the experimental data with only the statistical error included compared with the **alphaQED** prediction (without the KLOE data) when  $\text{Re } \Delta\alpha = \text{Re } \Delta\alpha_{\text{lep}}$  (yellow points) and  $\text{Re } \Delta\alpha = \text{Re } \Delta\alpha_{\text{lep+had}}$  (blue solid line).

from The KLOE-2 Collaboration: A. Anastasi et al. “Measurement of the running of the fine structure constant below 1 GeV with the KLOE Detector, arXiv:1609.06631v1 [hep-ex] 21 Sep 2016

talk of G.Venanzoni at this workshop

# AN ALTERNATIVE APPROACH

We propose an alternative approach by using a space-like formula

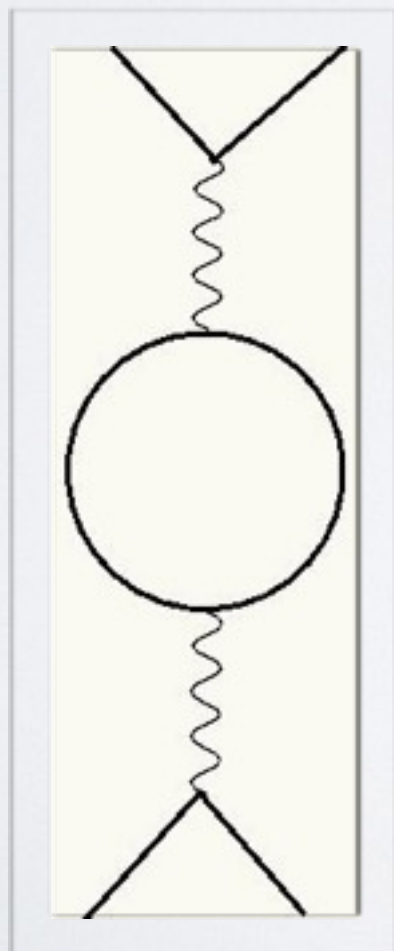
$$a_{\mu}^{HLO} = \left(\frac{\alpha}{\pi}\right) \int_0^1 dx (x-1) \bar{\Pi}_{had}(t(x)) = \left(\frac{\alpha}{\pi}\right) \int_0^1 dx (1-x) \Delta\alpha_{had}(t(x))$$

$$a_{\mu}^{HLO} = \left(\frac{\alpha}{\pi}\right) \int_{-\infty}^0 \frac{dt}{\beta t} \left(\frac{1-\beta}{1+\beta}\right)^2 \bar{\Pi}_{had}(t) = -\left(\frac{\alpha}{\pi}\right) \int_{-\infty}^0 \frac{dt}{\beta t} \left(\frac{1-\beta}{1+\beta}\right)^2 \Delta\alpha_{had}(t(x))$$

$$\beta = \sqrt{1 - \frac{4m_{\mu}^2}{t}}$$

$$t(x) = -\frac{x^2 m_{\mu}^2}{1-x}$$

$$\alpha(t) = \frac{\alpha(0)}{1 - \Delta\alpha(t)}$$



$$t = -|q|^2$$

$\Delta\alpha_{had}(t)$  is the hadronic contribution to the running of  $\alpha$

$$\Delta\alpha_{had}(t) = \Delta\alpha(t) - \Delta\alpha_{lep}(t)$$

## 2. Theoretical framework

The leading-order hadronic contribution to the muon  $g-2$  is given by the well-known formula [4,15]

$$a_{\mu}^{\text{HLO}} = \frac{\alpha}{\pi^2} \int_0^{\infty} \frac{ds}{s} K(s) \text{Im}\Pi_{\text{had}}(s + i\epsilon), \quad (1)$$

where  $\Pi_{\text{had}}(s)$  is the hadronic part of the photon vacuum polarization,  $\epsilon > 0$ ,

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_{\mu}^2)} \quad (2)$$

is a positive kernel function, and  $m_{\mu}$  is the muon mass. As the total cross section for hadron production in low-energy  $e^+e^-$  annihilations is related to the imaginary part of  $\Pi_{\text{had}}(s)$  via the optical theorem, the dispersion integral in Eq. (1) is computed integrating experimental time-like ( $s > 0$ ) data up to a certain value of  $s$  [2,18,19]. The high-energy tail of the integral is calculated using perturbative QCD [20].

Alternatively, if we exchange the  $x$  and  $s$  integrations in Eq. (1) we obtain [21]

$$a_{\mu}^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (x-1) \bar{\Pi}_{\text{had}}[t(x)], \quad (3)$$

where  $\bar{\Pi}_{\text{had}}(t) = \Pi_{\text{had}}(t) - \Pi_{\text{had}}(0)$  and

$$t(x) = \frac{x^2 m_\mu^2}{x - 1} < 0 \quad (4)$$

is a space-like squared four-momentum. If we invert Eq. (4), we get  $x = (1 - \beta) (t/2m_\mu^2)$ , with  $\beta = (1 - 4m_\mu^2/t)^{1/2}$ , and from Eq. (3) we obtain

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_{-\infty}^0 \bar{\Pi}_{\text{had}}(t) \left( \frac{\beta - 1}{\beta + 1} \right)^2 \frac{dt}{t\beta}. \quad (5)$$


Eq. (5) has been used for lattice QCD calculations of  $a_\mu^{\text{HLO}}$  [22]; while the results are not yet competitive with those obtained with the dispersive approach via time-like data, their errors are expected to decrease significantly in the next few years [23].

The effective fine-structure constant at squared momentum transfer  $q^2$  can be defined by

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \quad (6)$$

To summarize

$$a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_0^1 (1-x) \Pi_{had} \left( -\frac{x^2}{1-x} m_{\mu}^2 \right) dx$$

$$t = \frac{x^2 m_{\mu}^2}{x-1} \quad 0 \leq -t < +\infty$$

$$a_{\mu} = (g-2)/2$$

$$x = \frac{t}{2m_{\mu}^2} \left( 1 - \sqrt{1 - \frac{4m_{\mu}^2}{t}} \right); \quad 0 \leq x < 1;$$

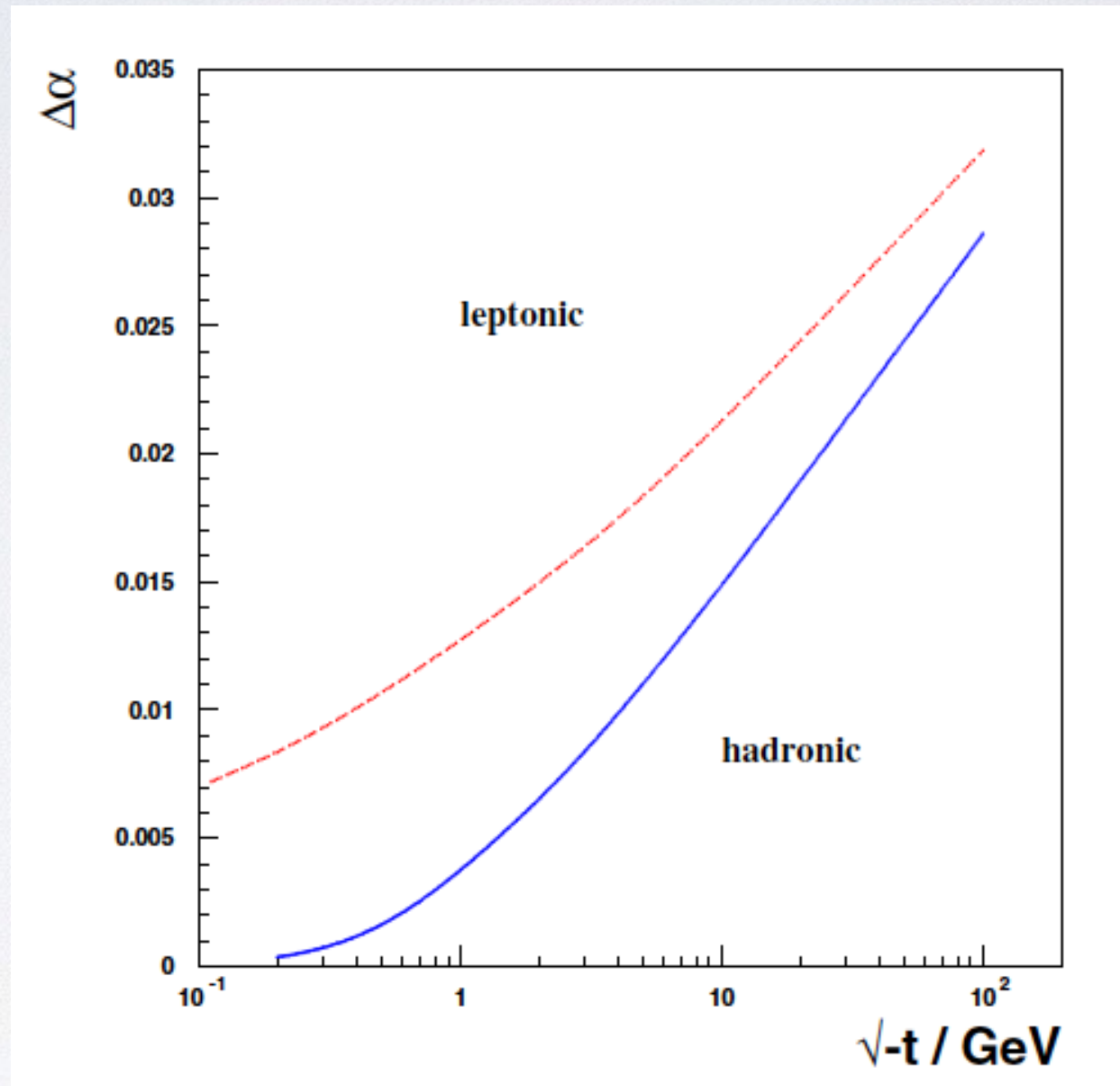
$$t = -s \sin^2 \left( \frac{\vartheta}{2} \right)$$

$$\Delta \alpha_{had}(t) = -\Pi_{had}(t) \quad \text{for } t < 0$$

with the “t”  
kernel

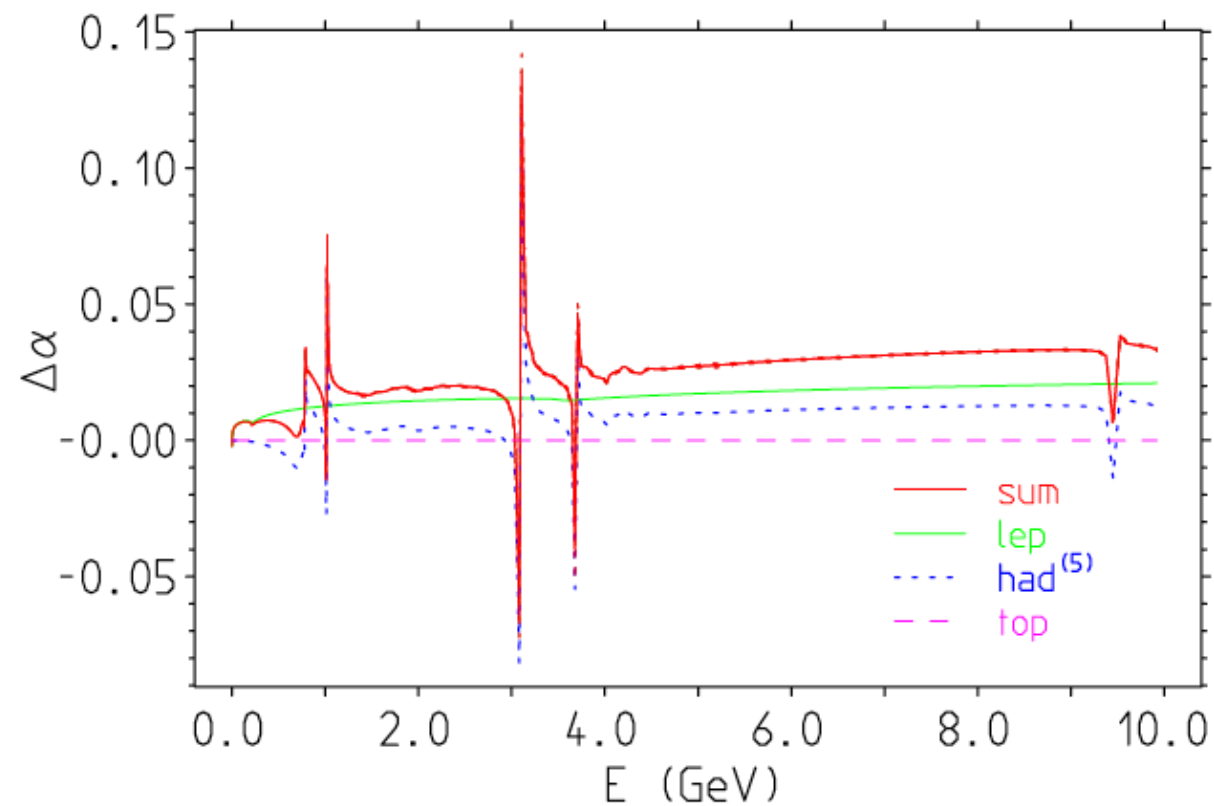
$$a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_0^1 (1-x) \Delta \alpha_{had} \left( -\frac{x^2}{1-x} m_{\mu}^2 \right) dx$$

functional form of the kernel

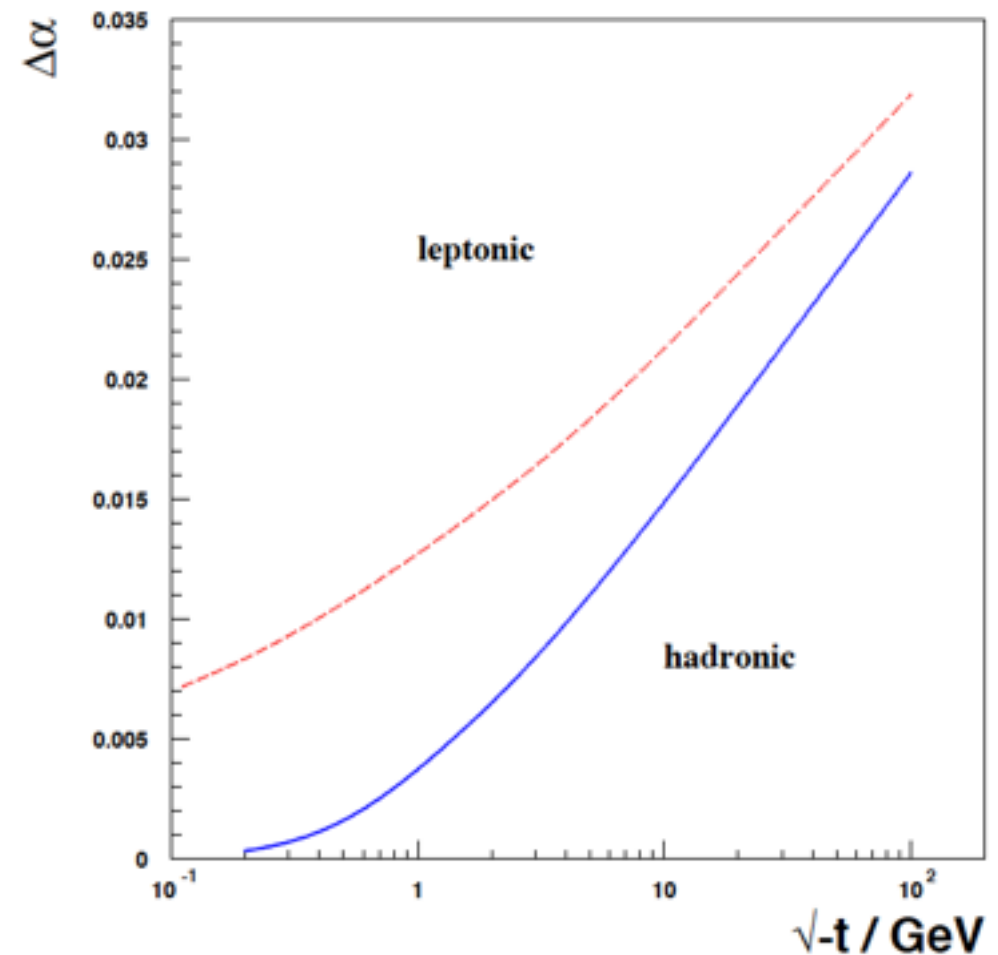
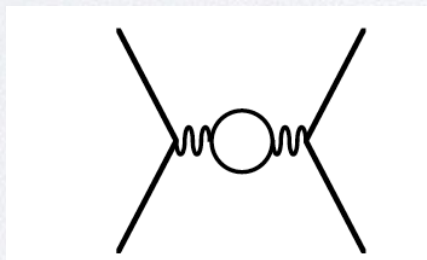


$\Delta\alpha$  is dominated at low  $t$  by leptonic contributions

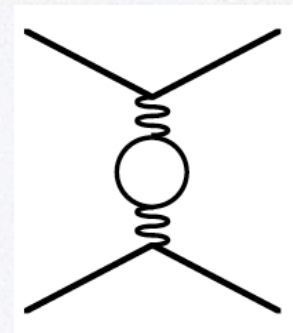
# Running of $\alpha_{\text{em}}$



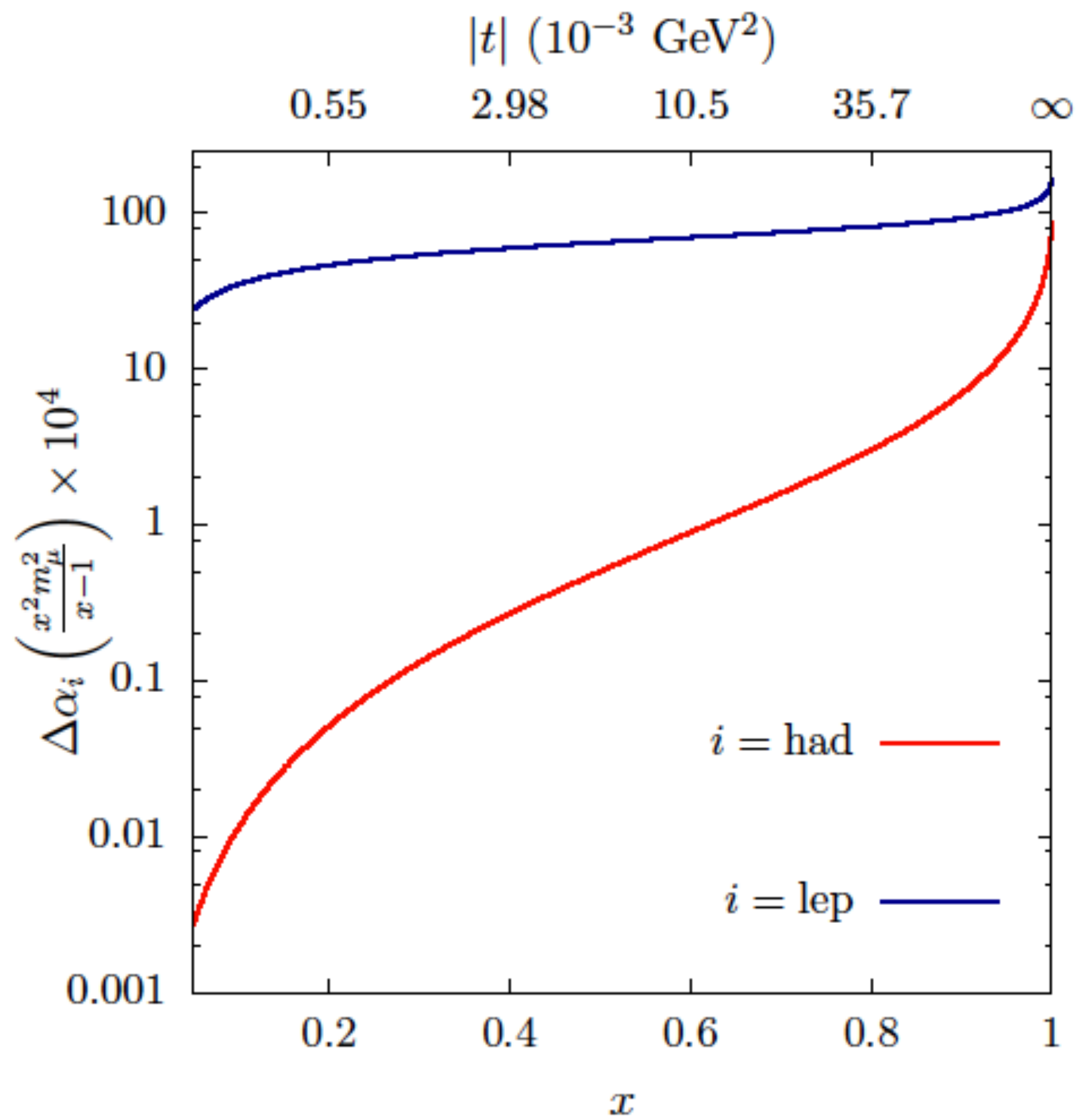
time-like

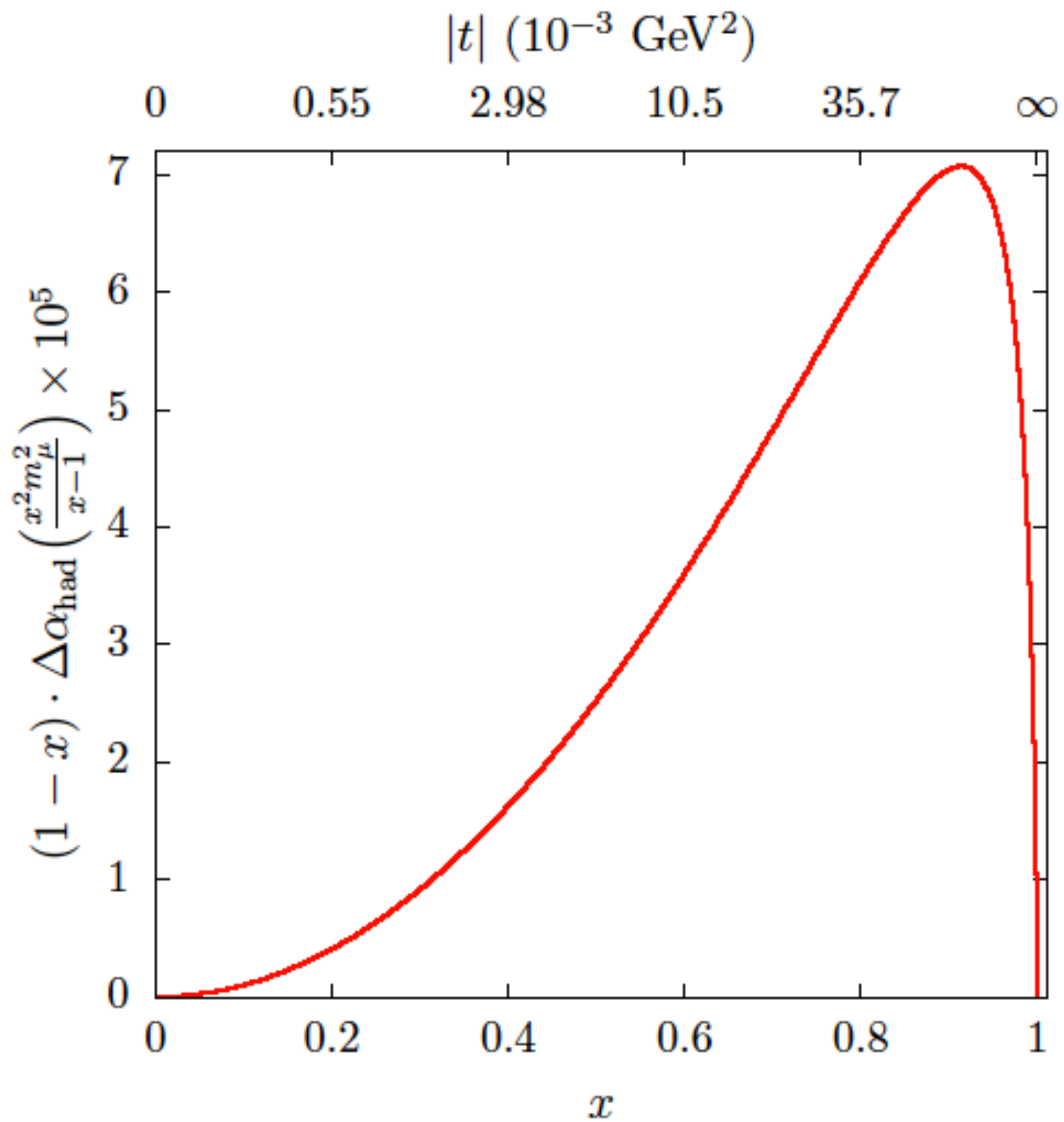


space-like



$$\Delta\alpha_{had}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)}$$





$$x_{peak} = 0.914 \quad t_{peak} = -0.108 \text{ GeV}^2$$

# AN EXAMPLE OF SPACE-LIKE APPROACH

# The running of the electromagnetic coupling $\alpha$ in small-angle Bhabha scattering

A.B. Arbuzov

BLTP, Joint Institute for Nuclear Research, Dubna, 141980, Russia

D. Haidt

DESY, Notkestrasse 85, D-22603 Hamburg, Germany

C. Matteuzzi M. Paganoni

Dipartimento di Fisica, Università di Milano-Bicocca

and

INFN-Milano, Piazza della Scienza 3, I-20126 Milan, Italy

L. Trentadue\*

Department of Physics, CERN Theory Division, 1211 Geneva 23, Switzerland

## Abstract

A method to determine the running of  $\alpha$  from a measurement of small-angle Bhabha scattering is proposed and worked out. The method is suited to high statistics experiments at  $e^+e^-$  colliders, which are equipped with luminometers in the appropriate angular region. A new simulation code predicting small-angle Bhabha scattering is also presented.

The method to measure the running of  $\alpha$  exploits the fact that the cross section for the process  $e^+e^- \rightarrow e^+e^-$  can be conveniently decomposed into three factors :

$$\frac{d\sigma}{dt} = \frac{d\sigma^0}{dt} \left( \frac{\alpha(t)}{\alpha(0)} \right)^2 (1 + \Delta r(t)) \quad (3)$$

each one of them known with an accuracy of at least 0.1%

## 1st factor

$$\frac{d\sigma^0}{dt} = \frac{d\sigma^B}{dt} \left( \frac{\alpha(0)}{\alpha(t)} \right)^2 .$$

The Born cross section contains all the soft and virtual corrections

Bhabha is a pure QED process  
Quarks enter only in loops

$$\frac{d\sigma^B}{dt} = \frac{\pi\alpha_0^2}{2s^2} \text{Re}\{B_t + B_s + B_i\},$$

$$\begin{aligned} B_t &= \left(\frac{s}{t}\right)^2 \left\{ \frac{5 + 2c + c^2}{(1 - \Pi(t))^2} + \xi \frac{2(g_v^2 + g_a^2)(5 + 2c + c^2)}{(1 - \Pi(t))} \right. \\ &\quad \left. + \xi^2 \left( 4(g_v^2 + g_a^2)^2 + (1 + c)^2(g_v^4 + g_a^4 + 6g_v^2 g_a^2) \right) \right\} \\ B_s &= \frac{2(1 + c^2)}{|1 - \Pi(s)|^2} + 2\chi \frac{(1 - c)^2(g_v^2 - g_a^2) + (1 + c)^2(g_v^2 + g_a^2)}{1 - \Pi(s)} \\ &\quad + \chi^2 [(1 - c)^2(g_v^2 - g_a^2)^2 + (1 + c)^2(g_v^4 + g_a^4 + 6g_v^2 g_a^2)] \\ B_i &= 2\frac{s}{t}(1 + c)^2 \left\{ \frac{1}{(1 - \Pi(t))(1 - \Pi(s))} \right. \\ &\quad + (g_v^2 + g_a^2) \left( \frac{\xi}{1 - \Pi(s)} + \frac{\chi}{1 - \Pi(t)} \right) \\ &\quad \left. + (g_v^4 + 6g_v^2 g_a^2 + g_a^4) \xi \chi \right\} \end{aligned}$$

2nd factor

$$\left( \frac{\alpha(t)}{\alpha(0)} \right)^2$$

Vacuum polarization effects  
gives the running of alpha

3rd factor

$$(1 + \Delta r(t))$$

with all the real and virtual effects not incorporated in the running of  
alpha

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(q^2)},$$

$\alpha(0)$  is the Sommerfeld  
fine structure constant  
measured with a precision of  
 $O(10^{-9})$

$\Delta\alpha(q^2)$  from loop contributions to the photon propagator

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH

CERN-PH-EP/2005-014

21 February 2005

Revised 28 June 2005

# Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

OPAL Collaboration

## Abstract

Using the OPAL detector at LEP, the running of the effective QED coupling  $\alpha(t)$  is measured for space-like momentum transfer from the angular distribution of small-angle Bhabha scattering. In an almost ideal QED framework, with very favourable experimental conditions, we obtain:

$$\Delta\alpha(-6.07 \text{ GeV}^2) - \Delta\alpha(-1.81 \text{ GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5},$$

where the first error is statistical, the second is the experimental systematic and the third is the theoretical uncertainty. This agrees with current evaluations of  $\alpha(t)$ . The null hypothesis that  $\alpha$  remains constant within the above interval of  $-t$  is excluded with a significance above  $5\sigma$ . Similarly, our results are inconsistent at the level of  $3\sigma$  with the hypothesis that only leptonic loops contribute to the running. This is currently the most significant direct measurement where the running  $\alpha(t)$  is probed differentially within the measured  $t$  range.

This has been made possible by a very accurate determination of the Luminosity by the OPAL collaboration

# A measurement of the Luminosity at $10^{-4}$ at LEP

Giovanni Abbiendi

INFN - Bologna

Eur. Phys. J. C 45, 1–21 (2006)  
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02389-3

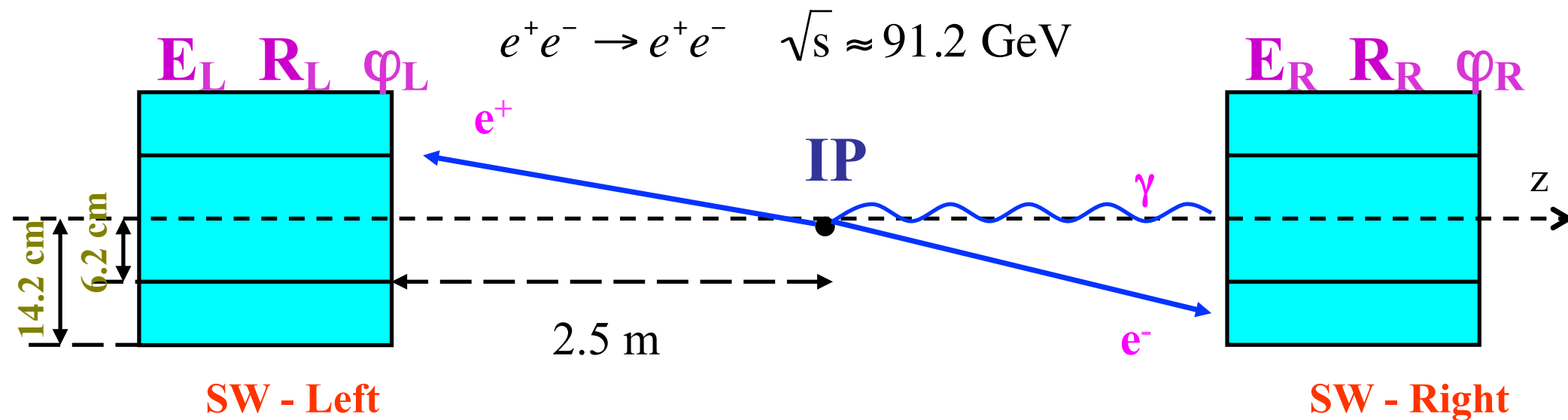
THE EUROPEAN  
PHYSICAL JOURNAL C

## Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

The OPAL Collaboration

G. Abbiendi<sup>2</sup>, C. Ainsley<sup>5</sup>, P.F. Åkesson<sup>3,y</sup>, G. Alexander<sup>22</sup>, G. Anagnostou<sup>1</sup>, K.J. Anderson<sup>9</sup>, S. Asai<sup>23</sup>, D. Axen<sup>27</sup>, I. Bailey<sup>26</sup>, E. Barberio<sup>8,p</sup>, T. Barillari<sup>32</sup>, R.J. Barlow<sup>16</sup>, R.J. Batley<sup>5</sup>, P. Bechtel<sup>25</sup>, T. Behnke<sup>25</sup>, K.W. Bell<sup>20</sup>, P.J. Bell<sup>1</sup>, G. Bella<sup>22</sup>, A. Bellerive<sup>6</sup>, G. Benelli<sup>4</sup>, S. Bethke<sup>32</sup>, O. Biebel<sup>31</sup>, O. Boeriu<sup>10</sup>, P. Bock<sup>11</sup>, M. Boutemour<sup>31</sup>, S. Braibant<sup>2</sup>, R.M. Brown<sup>20</sup>, H.J. Burckhart<sup>8</sup>, S. Campana<sup>4</sup>, P. Capiluppi<sup>2</sup>, R.K. Carnegie<sup>6</sup>, A.A. Carter<sup>13</sup>, J.R. Carter<sup>5</sup>, C.Y. Chang<sup>17</sup>, D.G. Charlton<sup>1</sup>, C. Ciocca<sup>2</sup>, A. Csilling<sup>29</sup>, M. Cuffiani<sup>2</sup>, S. Dado<sup>21</sup>, G.M. Dallavalle<sup>2</sup>, A. De Roeck<sup>8</sup>, E.A. De Wolf<sup>8,s</sup>, K. Desch<sup>25</sup>, B. Dienes<sup>30</sup>, J. Dubbert<sup>31</sup>, E. Duchovni<sup>24</sup>, G. Duckeck<sup>31</sup>, I.P. Duerdoth<sup>16</sup>, E. Etzion<sup>22</sup>, F. Fabbri<sup>2</sup>, P. Ferrari<sup>8</sup>, F. Fiedler<sup>31</sup>, I. Fleck<sup>10</sup>, M. Ford<sup>16</sup>, A. Frey<sup>8</sup>, P. Gagnon<sup>12</sup>, J.W. Gary<sup>4</sup>, C. Geich-Gimbel<sup>3</sup>, G. Giacomelli<sup>2</sup>, P. Giacomelli<sup>2</sup>, R. Giacomelli<sup>2</sup>, M. Giunta<sup>4</sup>, J. Goldberg<sup>21</sup>, E. Gross<sup>24</sup>, J. Grunhaus<sup>22</sup>, M. Gruwé<sup>8</sup>, P.O. Günther<sup>3</sup>, A. Gupta<sup>9</sup>, C. Hajdu<sup>29</sup>, M. Hamann<sup>25</sup>, G.G. Hanson<sup>4</sup>, A. Harel<sup>21</sup>, M. Hauschild<sup>8</sup>, C.M. Hawkes<sup>1</sup>, R. Hawkings<sup>8</sup>, R.J. Hemingway<sup>6</sup>, G. Herten<sup>10</sup>, R.D. Heuer<sup>25</sup>, J.C. Hill<sup>5</sup>, D. Horváth<sup>29,c</sup>, P. Igo-Kemenes<sup>11</sup>, K. Ishii<sup>23</sup>, H. Jeremie<sup>18</sup>, P. Jovanovic<sup>1</sup>, T.R. Junk<sup>6,i</sup>, J. Kanzaki<sup>23,u</sup>, D. Karlen<sup>26</sup>, K. Kawagoe<sup>23</sup>, T. Kawamoto<sup>23</sup>, R.K. Keeler<sup>26</sup>, R.G. Kellogg<sup>17</sup>, B.W. Kennedy<sup>20</sup>, S. Kluth<sup>32</sup>, T. Kobayashi<sup>23</sup>, M. Kobel<sup>3</sup>, S. Komamiya<sup>23</sup>, T. Krämer<sup>25</sup>, P. Krieger<sup>6,1</sup>, J. von Krogh<sup>11</sup>, T. Kuhl<sup>25</sup>, M. Kupper<sup>24</sup>, G.D. Lafferty<sup>16</sup>, H. Landsman<sup>21</sup>, D. Lanske<sup>14</sup>, D. Lellouch<sup>24</sup>, J. Letts<sup>o</sup>, L. Levinson<sup>24</sup>, J. Lillich<sup>10</sup>, S.L. Lloyd<sup>13</sup>, F.K. Loebinger<sup>16</sup>, J. Lu<sup>27,w</sup>, A. Ludwig<sup>3</sup>, J. Ludwig<sup>10</sup>, W. Mader<sup>3,b</sup>, S. Marcellini<sup>2</sup>, A.J. Martin<sup>13</sup>, T. Mashimo<sup>23</sup>, P. Mättig<sup>m</sup>, J. McKenna<sup>27</sup>, R.A. McPherson<sup>26</sup>, F. Meijers<sup>8</sup>, W. Menges<sup>25</sup>, F.S. Merritt<sup>9</sup>, H. Mes<sup>6,a</sup>, N. Meyer<sup>25</sup>, A. Michelini<sup>2</sup>, S. Mihara<sup>23</sup>, G. Mikenberg<sup>24</sup>, D.J. Miller<sup>15</sup>, W. Mohr<sup>10</sup>, T. Mori<sup>23</sup>, A. Mutter<sup>10</sup>, K. Nagai<sup>13</sup>, I. Nakamura<sup>23,v</sup>, H. Nanjo<sup>23</sup>, H.A. Neal<sup>33</sup>, R. Nisius<sup>32</sup>, S.W. O’Neale<sup>1,\*</sup>, A. Oh<sup>8</sup>, M.J. Oreglia<sup>9</sup>, S. Orito<sup>23,\*</sup>, C. Pahl<sup>32</sup>, G. Pásztor<sup>4,g</sup>, J.R. Pater<sup>16</sup>, J.E. Pilcher<sup>9</sup>, J. Pinfold<sup>28</sup>, D.E. Plane<sup>8</sup>, O. Pooth<sup>14</sup>, M. Przybycień<sup>8,n</sup>, A. Quadt<sup>3</sup>, K. Rabbertz<sup>8,r</sup>, C. Rembser<sup>8</sup>, P. Renkel<sup>24</sup>, J.M. Roney<sup>26</sup>, A.M. Rossi<sup>2</sup>, Y. Rozen<sup>21</sup>, K. Runge<sup>10</sup>, K. Sachs<sup>6</sup>, T. Saeki<sup>23</sup>, E.K.G. Sarkisyan<sup>8,j</sup>, A.D. Schaile<sup>31</sup>, O. Schaile<sup>31</sup>, P. Scharff-Hansen<sup>8</sup>, J. Schieck<sup>32</sup>, T. Schörner-Sadenius<sup>8,z</sup>, M. Schröder<sup>8</sup>, M. Schumacher<sup>3</sup>, R. Seuster<sup>14,f</sup>, T.G. Shears<sup>8,h</sup>, B.C. Shen<sup>4</sup>, P. Sherwood<sup>15</sup>, A. Skuja<sup>17</sup>, A.M. Smith<sup>8</sup>, R. Sobie<sup>26</sup>, S. Söldner-Rembold<sup>16</sup>, F. Spano<sup>9</sup>, A. Stahl<sup>3,x</sup>, D. Strom<sup>19</sup>, R. Ströhmer<sup>31</sup>, S. Tarem<sup>21</sup>, M. Tasevsky<sup>8,s</sup>, R. Teuscher<sup>9</sup>, M.A. Thomson<sup>5</sup>, E. Torrence<sup>19</sup>, D. Toya<sup>23</sup>, P. Tran<sup>4</sup>, I. Trigger<sup>8</sup>, Z. Trócsányi<sup>30,e</sup>, E. Tsur<sup>22</sup>, M.F. Turner-Watson<sup>1</sup>, I. Ueda<sup>23</sup>, B. Ujvári<sup>30,e</sup>, C.F. Vollmer<sup>31</sup>, P. Vannerem<sup>10</sup>, R. Vértesi<sup>30,e</sup>, M. Verzocchi<sup>17</sup>, H. Voss<sup>8,q</sup>, J. Vossebeld<sup>8,h</sup>, C.P. Ward<sup>5</sup>, D.R. Ward<sup>5</sup>, P.M. Watkins<sup>1</sup>, A.T. Watson<sup>1</sup>, N.K. Watson<sup>1</sup>, P.S. Wells<sup>8</sup>, T. Wengler<sup>8</sup>, N. Wermes<sup>3</sup>, G.W. Wilson<sup>16,k</sup>, J.A. Wilson<sup>1</sup>, G. Wolf<sup>24</sup>, T.R. Wyatt<sup>16</sup>, S. Yamashita<sup>23</sup>, D. Zer-Zion<sup>4</sup>, L. Zivkovic<sup>24</sup>

# Small-angle Bhabha scattering in OPAL



2 cylindrical calorimeters encircling the beam pipe at  $\pm 2.5 \text{ m}$  from the Interaction Point

19 Silicon layers

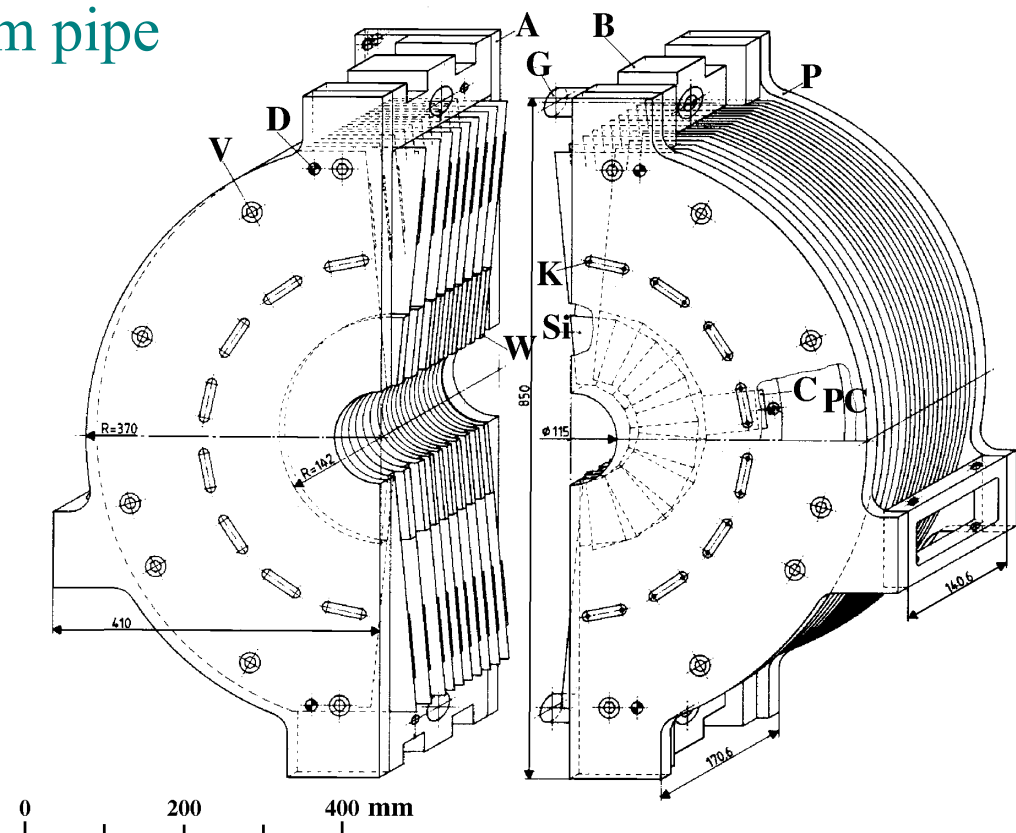
Total Depth  $22 X_0$

18 Tungsten layers

(14 cm)

Each detector layer divided into 16 overlapping wedges

Sensitive radius: 6.2 – 14.2 cm, corresponding to scattering angle of 25 – 58 mrad from the beam line



## Final Error on Luminosity

After all the effort on Radial reconstruction the dominant systematic error is related to Energy (mostly tail in the E response and nonlinearity)  
Quantitatively: (OPAL Collaboration, Eur.Phys.J. C14 (2000) 373)

|                  | Systematic<br>Error ( $\times 10^{-4}$ ) |
|------------------|------------------------------------------|
| Energy           | 1.8                                      |
| Inner Anchor     | 1.4                                      |
| Radial Metrology | 1.4                                      |

**Total Experimental Systematic Error :  $3.4 \times 10^{-4}$**

**Theoretical Error on Bhabha cross section:  $5.4 \times 10^{-4}$**

The Method used follows the above parametrization/factorization of the Bhabha cross-section

$$\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt} \left( \frac{\alpha(t)}{\alpha_0} \right)^2 (1 + \epsilon) (1 + \delta_\gamma) + \delta_Z$$
$$\frac{d\sigma^{(0)}}{dt} = \frac{4\pi\alpha_0^2}{t^2}$$

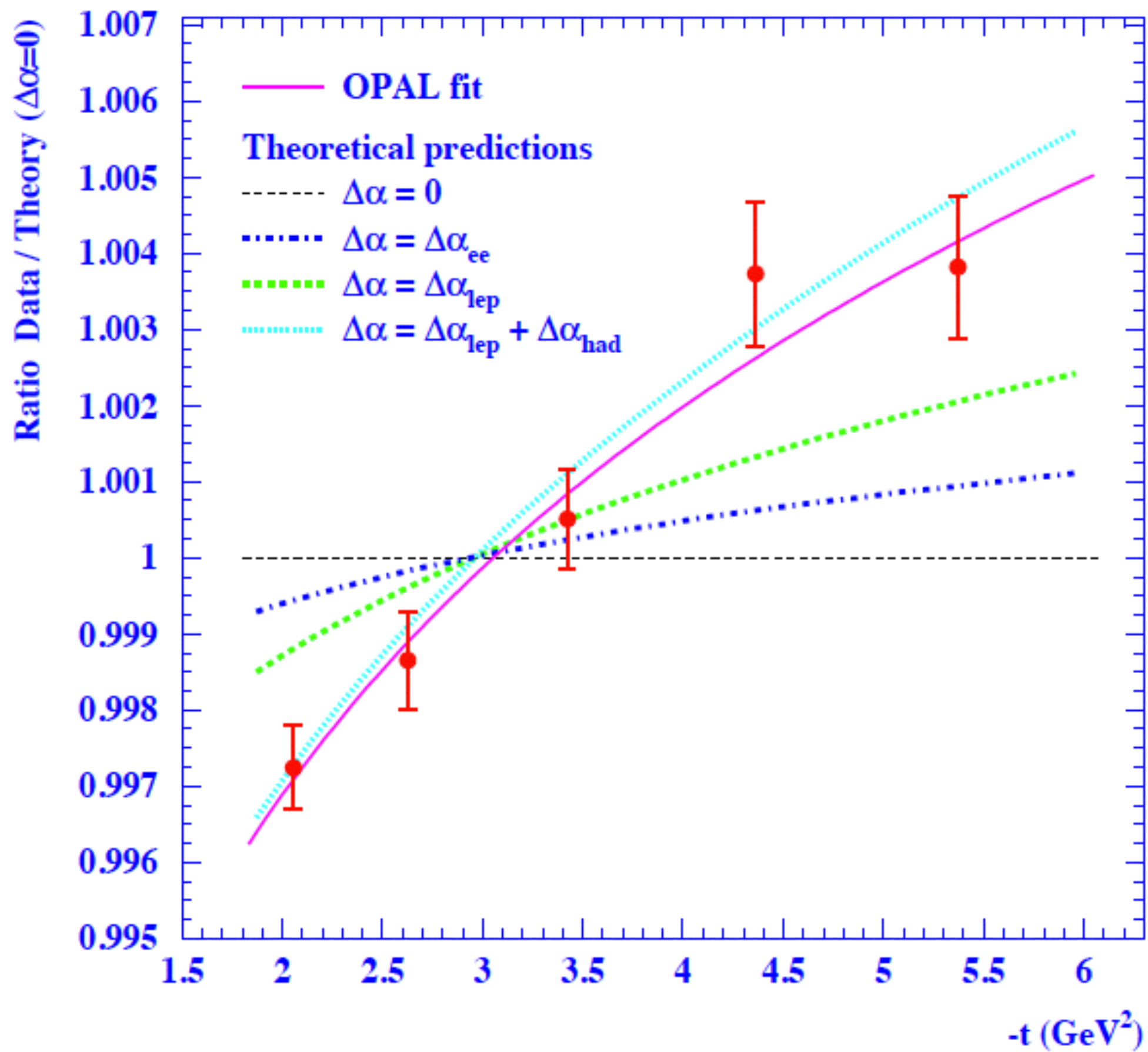
We determined the effective slope of the Bhabha momentum transfer distribution which is simply related to the average derivative of  $\Delta\alpha$  as a function of  $\ln t$  in the range  $2 \text{ GeV}^2 \leq -t \leq 6 \text{ GeV}^2$ . The observed  $t$ -spectrum is in good agreement with Standard Model predictions. We find:

$$\Delta\alpha(-6.07 \text{ GeV}^2) - \Delta\alpha(-1.81 \text{ GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5},$$

where the first error is statistical, the second is the experimental systematic and the third is the theoretical uncertainty.

This measurement is one of only a very few experimental tests of the running of  $\alpha(t)$  in the space-like region, where  $\Delta\alpha$  has a smooth behaviour. We obtain the strongest direct evidence for the running of the QED coupling ever achieved differentially in a single experiment, with a significance above  $5\sigma$ . Moreover we report clear experimental evidence for the hadronic contribution to the running in the space-like region, with a significance of  $3\sigma$ .

# OPAL



A new possibility  
via

$$\mu e \rightarrow \mu e$$

scattering

G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna,  
O. Nicrosini M. Passera, F. Piccinini, R. Tenchini, L.T. and G. Venanzoni,  
presented at the Physics Beyond Colliders Kickoff Meeting, September 2016, CERN  
“Measuring the leading hadronic contribution to the muon  $g-2$  via  $\mu$ - $e$  scattering”, arXiv:1609.08987 [hep-ex].

$$\mu e \rightarrow \mu e$$

- High intensity muon beam available in the CERN North Area  $E = 150 \text{ GeV}$
- pure t-channel process

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha(0)} \right|^2$$

$$s \simeq 0.16 \text{ GeV}^2 \quad -0.14 \leq t \leq 0 \text{ GeV}^2 \quad 0 \leq x \leq 0.93$$

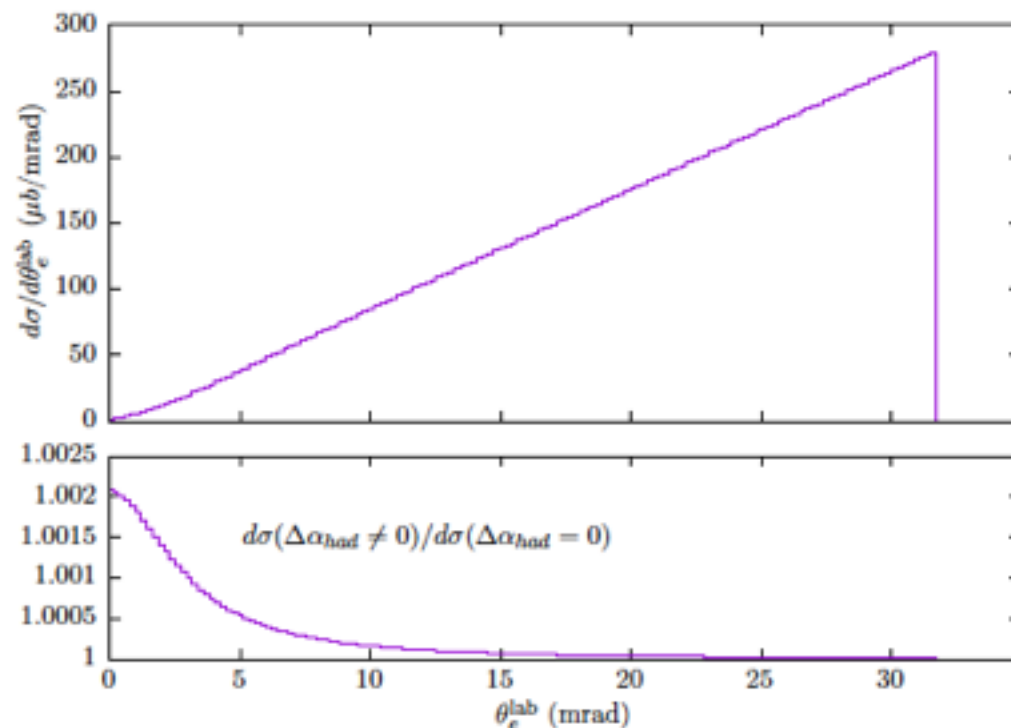
- the  $2 \rightarrow 2$  kinematics reads

$$t = 2m_e^2 - 2m_e E_e, \quad s = m_\mu^2 + m_e^2 + 2m_e E_\mu^i$$

$$E_e = m_e \frac{1 + r^2 c_e^2}{1 - r^2 c_e^2}, \quad \theta_e = \arccos \left( \frac{1}{r} \sqrt{\frac{E_e - m_e}{E_e + m_e}} \right)$$

$$r \equiv \frac{\sqrt{(E_\mu^i)^2 - m_\mu^2}}{E_\mu^i + m_e}, \quad c_e \equiv \cos \theta_e$$

- $0 < \theta_e < 31.85 \text{ mrad} \longleftrightarrow 139.8 > E_e > 1 \text{ GeV} \longleftrightarrow -0.143 < t < -10^{-3} \text{ GeV}^2$



→ differential cross-section at LO (including vacuum polarization) as a function of  $\theta_e$

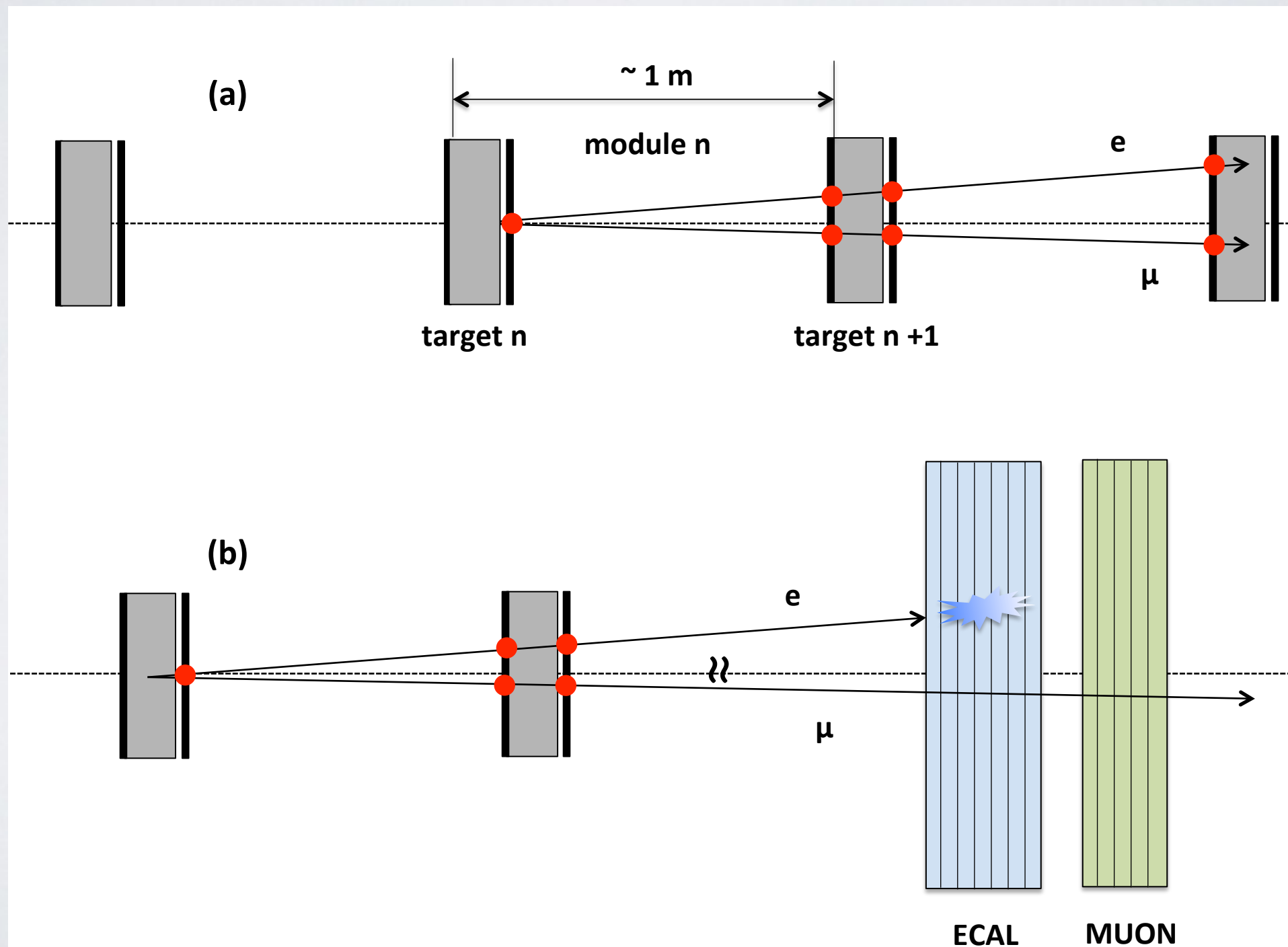
→ effect due to  $\Delta\alpha_{\text{had}}(t)$

→ for instance the region  $\theta_e > 20 \div 25 \text{ mrad}$  can be used as normalization

Same process can be used for signal and normalization

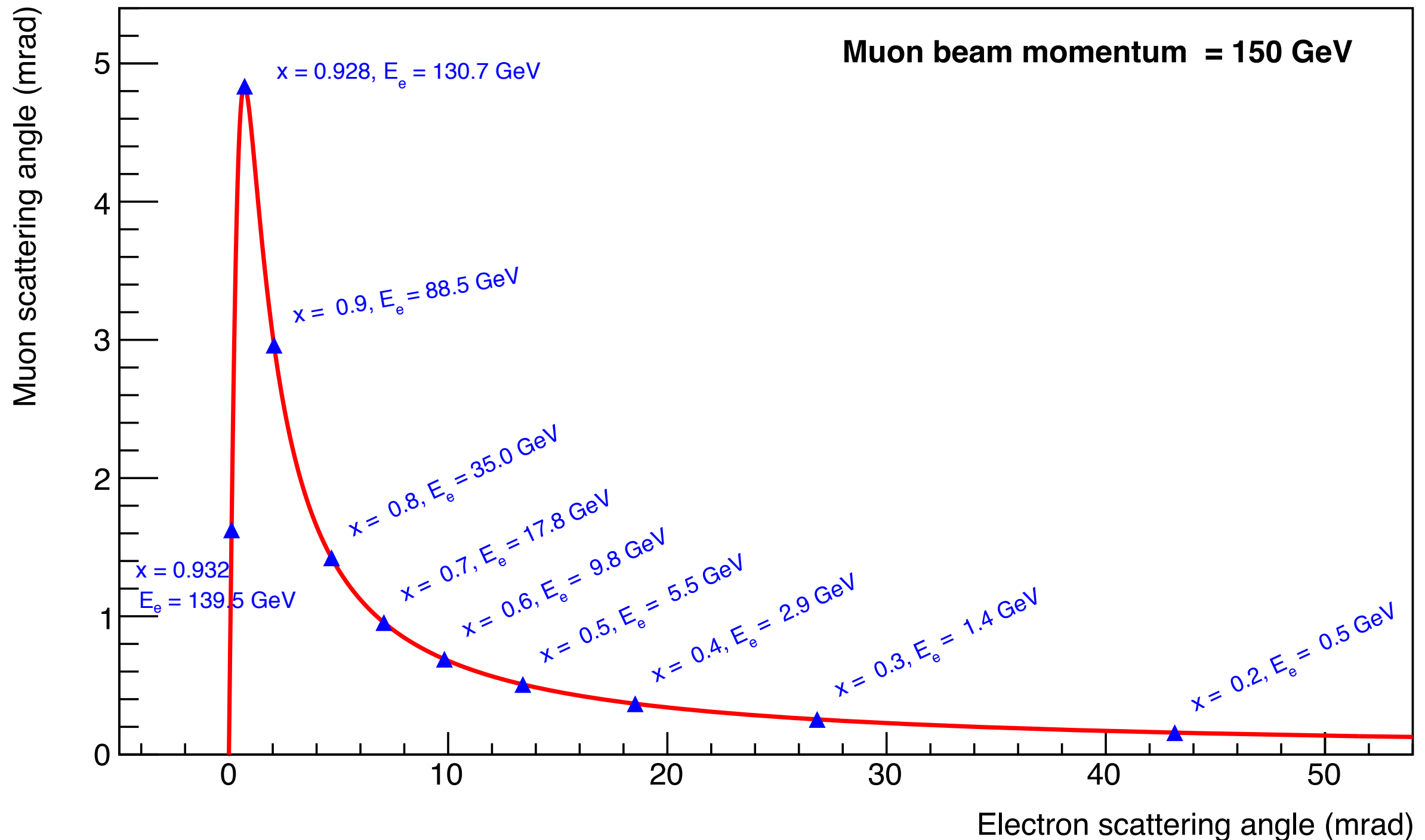
# The Detector

- i) Initial muons have to be tagged with their direction and momentum
- ii) 20 Be (C) layers interfaced with Si planes spaced by 1 m air gap modularly spaced
- iii) The use of a low  $Z$  material in order to reduce multiple scattering and background
- iv) A final EM calorimeter to discriminate e/mu at small angles ( 2-3 mrad )



Muon and electron scattering angles are correlated

This **very important** constraint may be used to select elastic events, reject background from radiative events and minimize systematics



# WORK IS IN PROGRESS

- To develop Monte Carlo codes to reduce theoretical systematics below the  $10^{*-5}$  bound ( as well as for the expt systematics )
- To improve the determination of QED radiative corrections ( to  $10^{*-4}$  accuracy )

Thiws new approach may become a path within an unexplored region of the field theoretical dynamics

It may lead to a possibly long series of phenomenological results

The (crossed) t-channel dynamics, as complementary and independent with respect to the s-channel one will permit an alternative new approach to the Standard Model precision physics

We propose a new approach to evaluate the leading-order hadronic contribution to the muon anomaly

The direct measurement of the hadronic contribution to anomalous muon magnetic moment will provide an independent determination, competitive with the time-like dispersive approach, and consolidate the theoretical prediction for the muon  $g-2$  in the Standard Model

It will allow a firmer interpretation of the measurements of the future muon  $g-2$  experiments at Fermilab and J-PARC.

I would like to thank  
Antonio Di Domenico  
for  
conceiving and organizing this workshop  
and for giving me the opportunity to participate to it

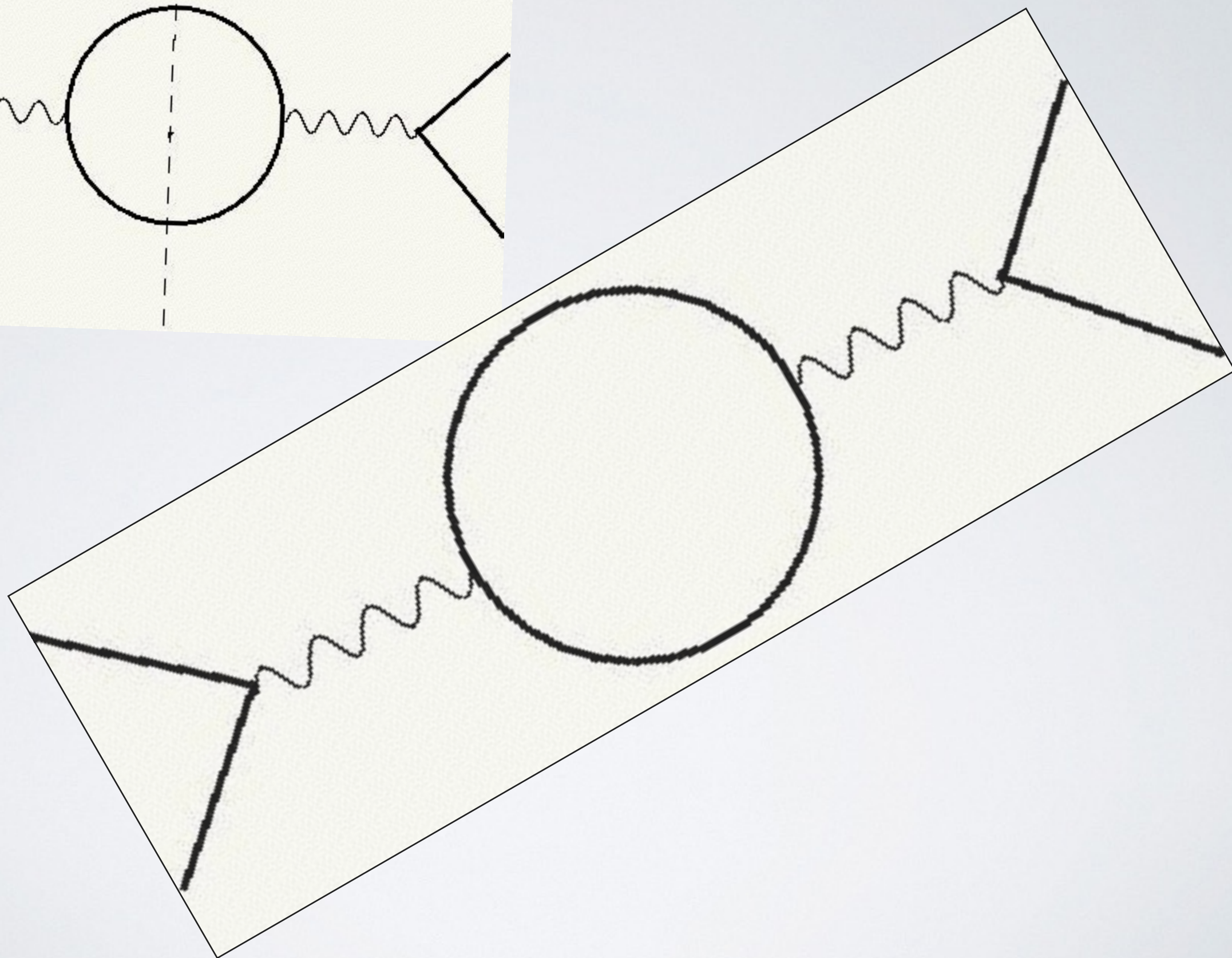
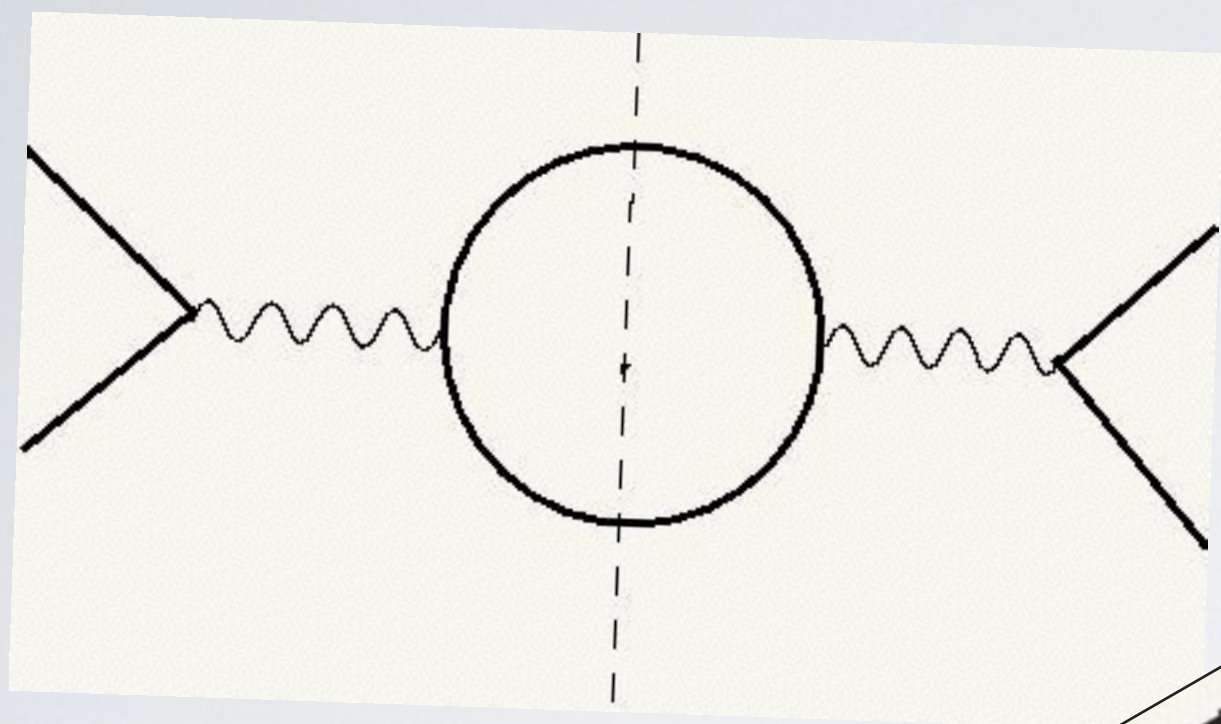
Thank you for your attention !







spare transparencies



$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)} ; \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s)$$