Planck-scale quantum mechanics with deformed relativistic symmetries

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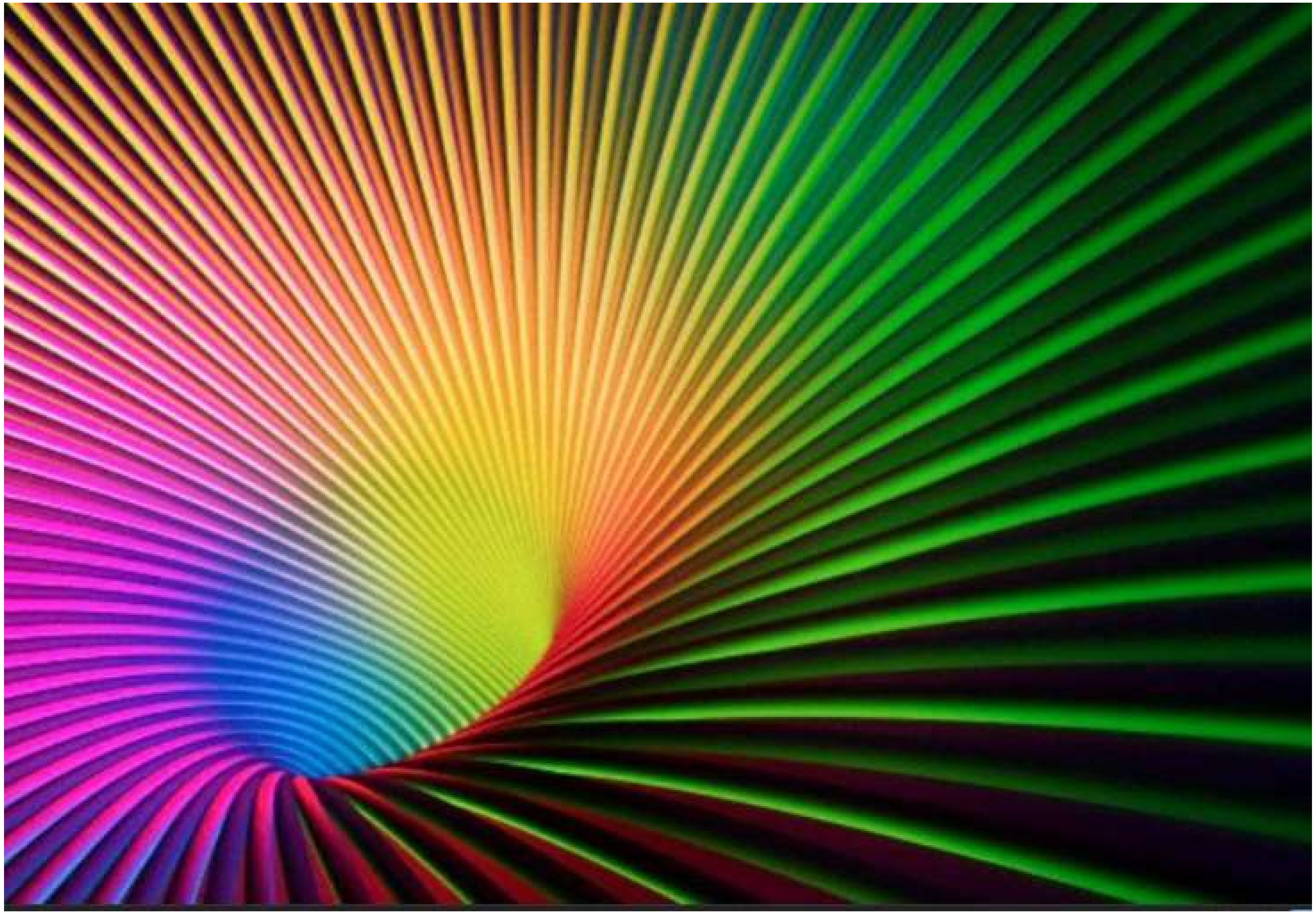


$$E_{QG} \sim E_{Planck} = 1.2 \bullet 10^{19}\,\mathrm{GeV} = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}}$$

i.e. 10^{-35}meters ("Planck length")

mainly comes from observing that at the Planck scale

$$\lambda_{compton} \sim \lambda_{schwartzschild}$$

Note that it is only <u>rough order-of-magnitude estimate at best</u>

in particular this estimate <u>assumes that G_{Newton} does not run at all</u>!!!!!!!!!
it most likely does run!!!
and we know the behaviour of gravitation only down to 10^{-6} meters!!!

Still, most likely quantum-gravity scale is very very high…

Nonetheless for more than a decade now there has been a worldwide effort showing that some plausible Planck-scale effects can be tested…

Long list of Qgphenomenology proposals made by several research groups around the world is in my "living review" of the field

GAC, Living Reviews in Relativity 16 (2013) 5

today I focus on the possibility of Planck-scale deformed relativistic symmetries, which has inspired many of these phenomenological proposals

Planck length as the minimum allowed value for wavelengths:

- suggested by several indirect arguments combining quantum mechanics and GR
- found in some detailed analyses of formalisms in use in the study of the QG problem

But the minimum wavelength is the Planck length for which observer?

GAC, ModPhysLettA (1994)
PhysLettB (1996)

Other results from the 1990s (mainly from spacetime noncommutativity and LoopQG) provided "theoretical evidence" of Planck-scale modifications of the on-shell relation, in turn inviting us to scrutinize the fate of relativistic symmetries at the Planck scale

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature(1998)
Alfaro+Tecotl+Urrutia,PhysRevLett(1999)
Gambini+Pullin, PhysRevD(1999)
Schaefer,PhysRevLett(1999)

a possibility worth exploring: "Planck-scale deformations of Lorentz symmetry"
[jargon: "DSR", for "doubly-special", or "deformed-special", relativity]

GAC, grqc0012051, IntJournModPhysD11,35
hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

change the laws of transformation between observers so that the new properties are observer-independent

* a law of minimum wavelength can be turned into a DSR law
* could be used also for properties other than minimum wavelength, such as deformed on-shellness, deformed uncertainty relations…

The notion of DSR-relativistic theories is best discussed in analogy with the transition from Galileian Relativity to Special Relativity

analogy with Galilean-SR transition

introduction to DSR case is easier starting from reconsidering the Galilean-SR transition (the SR-DSR transition would be closely analogous)

Galilean Relativity

on-shell/dispersion relation $E = \frac{p^2}{2m}$ (+m)

linear composition of momenta $p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$

linear composition of velocities $\vec{V} \oplus \vec{V}_0 = \vec{V} + \vec{V}_0$

from Galilean Relativity to Special Relativity

Maxwell theory was not pointing us toward the demise of relativity!
It was pointing to a "relativistic evolution"

The new law concerning the speed of light is not Galilean invariant but is invariant of a theory, special relativity, no less (and no more) relativistic than Galileo's

Relativistic invariance rescued at the "cost" of replacing Galileian boosts with special-relativistic boosts

of course (since c is invariant of the new theory) the <u>special-relativistic boosts act nonlinearly on velocities</u> (whereas Galilean boosts acted linearly on velocities)

and the <u>special-relativistic law of composition of velocities is nonlinear, noncommutative and nonassociative</u>

$$\mathbf{w} = \mathbf{v} \oplus \mathbf{u} \qquad \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \frac{1}{1 + \frac{v_1 u_1 + v_2 u_2 + v_3 u_3}{c^2}} \left\{ \left[1 + \frac{1}{c^2} \frac{\gamma_{\mathbf{v}}}{1+\gamma_{\mathbf{v}}} (v_1 u_1 + v_2 u_2 + v_3 u_3) \right] \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \frac{1}{\gamma_{\mathbf{v}}} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\}$$

much undervalued in the (horrible) textbooks we feed our students:

$$\frac{v+u}{1+(vu/c^2)}$$

from Special Relativity to DSR

If there was an observer-independent scale E_P (inverse of length scale ℓ) then, for example, one could have a modified on-shell relation as relativistic law

$$\boxed{m^2 = \Lambda(E,p;E_P) = E^2 - p^2 - \frac{E}{E_P}p^2 + O\left(\frac{E^4}{E_P{}^2}\right)}$$

For suitable choice of $\Lambda(E,p;E_P)$ one can easilyhave a maximum allowed value of momentum, i.e. minimum wavelength ($p_{max}=E_P$ for $\ell=-1/E_P$ in the formula here shown)

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2}e^{-\ell p_0}p_1^2$$

I shall later use in particular the fact that this onshellness takes the following form for massless particles

$$p_1 = \frac{1-e^{-\ell p_0}}{\ell}$$

it turns out that such laws could still be relativistic, part of a relativistic theory where not only c ("speed of massless particles in the infrared limit") but also E_P would be a nontrivial relativistic invariant

action of boosts on momenta must of course be deformed so that

$$\boxed{[N_k, \Lambda(E,p;E_P)] = 0}$$

then it turns out to be necessary to correspondingly deform the law composition of momenta

$$\boxed{p_\mu^{(1)} \oplus p_\mu^{(2)} \neq p_\mu^{(1)} + p_\mu^{(2)}}$$

(and even the simultaneity of coincident events may no longer be observer-independent)

Appreciating these technical and conceptual issues also allowed to shed light on previous results which were thought to be puzzling.
Let us see the case of the kappaMINKOWSKI noncommutative spacetime

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0$$

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlini,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

evidently not invariant under «classical translations»

$$[x'_0, x'_j] = [x_0 + a_0, x_j + a_j] = [x_0, x_j] = i\lambda x_j \neq i\lambda x'_j$$

but adding commutative numbers to the noncommutative coordinates of kappa-Minkowski is evidently not a sensible thing

Note that a more sensible starting point is to notice that translation transformations of a space are intimately related to the properties of the differential calculus…indeed in kappa-Minkowski it turns out that the properties of translation-transformation parameters ε_μ must be based on the (noncommutative!) differential calculus on kappa-Minkowski

$$[\varepsilon_0, x_\mu] = 0; [\varepsilon_j, x_l] = 0; [\varepsilon_j, x_0] = i\lambda\varepsilon_j$$

Sitarz, PhysLettB349(1995)42 Majid+Oeckl, math.QA/9811054

so that in particular $x_\mu+\varepsilon_\mu$ obeys the kappa-Minkowski commutation relations

Making a very long story short: these noncommutative properties of the translation-transformation parameters can be faithfully reflected on properties of translation generators, even by keeping a classical action of the generators on suitably ordered functions of the coordinates

Translation generators in kappa-Minkowski:

$$P_\mu \left(e^{ikx} e^{ik_0 t} \right) = k_\mu \left(e^{ikx} e^{ik_0 t} \right)$$

classical action

$$[x_j, t] = i\lambda x_j$$

$$[x_j, x_m] = 0$$

$$\Rightarrow \quad e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} = e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t}$$

!!

then "non-primitive coproduct"

$$P_\mu \left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} \right) = P_\mu \left(e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t} \right)$$

$$= \left(k_\mu + e^{-\lambda k_0} K_\mu \right) \left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} \right)$$

$$= \left[P_\mu \left(e^{ikx} e^{ik_0 t} \right) \right] \left(e^{iKx} e^{iK_0 t} \right) + \left[e^{-\lambda P_0} \left(e^{ikx} e^{ik_0 t} \right) \right] P_\mu \left(e^{iKx} e^{iK_0 t} \right)$$

!!

Generalization of Noether theorem applicable to this sort of Hopf-algebra symmetries of field theories in noncommutative spacetime has been achieved

PLB671(2009)298, PRD78(2008) 025005 ,MPLA22(2007)1779
(Agostini+Arzano+Gubitosi+Marciano+Martinetti+Mercati+GAC)

relativistic kinematics in kappa-Minkowski (based on nearly two decades of results)

GAC,arXiv:1111.5081,PhysRevD(2012)

on-shell/dispersion relation

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

[notice that this, for $\ell = -1/E_P$, sets maximum momentum E_P]

modified law of composition of momenta

$$(p \oplus_\ell p')_1 = p_1 + e^{\ell p_0} p'_1$$

$$(p \oplus_\ell p')_0 = p_0 + p'_0$$

$$\boxed{\ell \equiv \lambda \approx \frac{1}{E_P}}$$

modified boost action

$$[N, p_0] = p_1$$

$$[N, p_1] = \frac{e^{2\ell p_0} - 1}{2\ell} + \frac{\ell}{2} p_1^2$$

ensures observer-independence of on-shell relation

$$[N, \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2] = 0$$

in 3D quantum gravity

see, e.g., Freidel+Livine, PhysRevLett96,221301(2006)

consider a matter field ϕ coupled to gravity,

$$Z = \int Dg \int D\phi \, e^{iS[\phi,g]+iS_{GR}[g]}, \qquad (1)$$

where g is the space-time metric, $S_{GR}[g]$ the Einstein gravity action and $S[\phi, g]$ the action defining the dynamics of ϕ in the metric g.

integrate out the quantum gravity fluctuations and derive an *effective action* for ϕ taking into account the quantum gravity correction:

$$Z = \int D\phi e^{iS_{eff}[\phi]}.$$

the effective action obtained through this constructive procedure gives matter fields in a noncommutative spacetime (similar to, but not exactly given by, kappa-Minkowski) and with curved momentum space, as signalled in particular by the deformed on-shellness

(anti-deSitter momentum space) $$\cos(E) - e^{\ell E}\frac{\sin(E)}{E}P^2 = \cos(m)$$

a Planck-scale modification of the on-shell relation would produce time-of-arrival effects which at leading order are of the form ($n \in \{1,2\}$)

$$\Delta T = \left(\frac{E}{E_P} \right)^n T$$

and could be described in terms of an energy-dependent "physical velocity" of ultrarelativistic particles

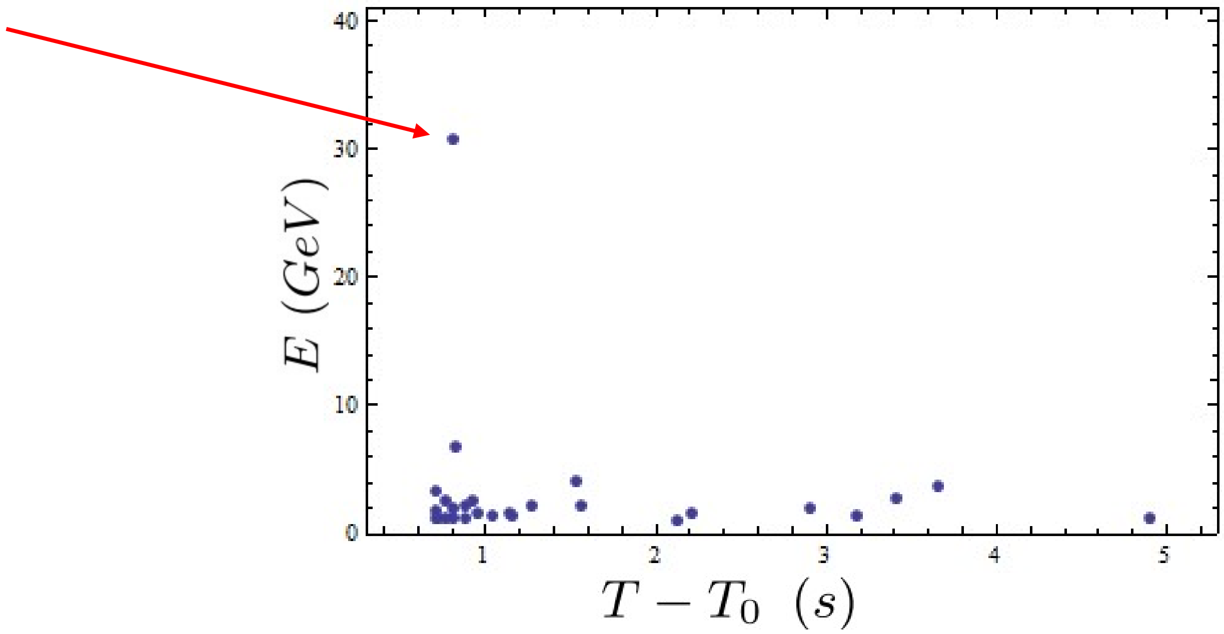
$$\mathrm{v} = c + s_{\pm} \left(\frac{E}{E_P} \right)^n c$$

<u>these are very small effects but they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!</u>

GRBs are ideally suited for testing this:
cosmological distances (established in 1997)
photons (and neutrinos) emitted nearly simultaneously
with rather high energies (GeV…..TeV…100 TeV…)

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
GAC, NaturePhysics10,254(2014)

example of GRB090510 (magnificent short burst) allowing to establish a limit at M_{planck} level on both signs of dispersion (soft and hard spectral lags)



a test with accuracy of about one part in 10^{20}!!!

interpretation of data still needs some theory work..
solid theory is for (curved momentum space and) flat spacetime

phenomenological opportunities are for propagation over cosmological distances, whose analysis requires curved spacetime

study of theories with both modified on-shellness and curved spacetime still in its infancy

GAC+Rosati, PhysRevD86,124035(2012)
KowalskiGlikman+Rosati,ModPhysLettA28,135101(2013)
Heckman+Verlinde,arXiv:1401.1810(2014)

Jacob and Piran [JCAP0801,031(2008)] used a compelling heuristic argument for producing a formula of energy-dependent time delay applicable to FRW spacetimes, which has been the only candidate so far tested

$$\Delta T = -s_{\pm} \frac{E}{M_{QG}} \frac{c}{H_0} \int_0^z d\zeta \frac{(1+\zeta)}{\sqrt{\Omega_\Lambda + (1+\zeta)^3 \Omega_m}}$$

where as usual H_0 is the Hubble parameter, Ω_Λ is the cosmological constant and Ω_m is the matter fraction.
However, it is now understood that Jacob-Piran formula implicitly makes restrictive assumptions about the nature of space-and-time translation transformations…next goal is to include nontrivial effects in the translation sector because explicit models suggest that the same effects affecting Lorentz sector also affect translation sector

Rosati + GAC +Marcianò +Matassa,
arXiv:1507.02056, PysRevD(2015)

*** we might not have seen yet the last “relativity evolution”**

*** (some) Planck-scale effects can be tested**