Decoherence and discrete symmetries in deformed relativistic kinematics

Michele Arzano

Dipartimento di Fisica
"Sapienza" University of Rome

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Fundamental decoherence in quantum gravity?

PHYSICAL REVIEW D VOLUME 14, NUMBER 10 15 NOVEMBER 1976

Breakdown of predictability in gravitational collapse*

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  are preserved by time evolution they (re)-discovered the **Lindblad equation**

$$\boxed{\dot{\rho} = -i[H,\rho] - \frac{1}{2}h_{\alpha\beta}\left(Q^\alpha Q^\beta \rho + \rho\, Q^\beta Q^\alpha - 2Q^\alpha \rho\, Q^\beta\right)}$$

$h_{\alpha\beta}$ is a hermitian matrix of constants and Q^α form a basis of hermitian matrices

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THIS TALK: show how **generalized quantum evolution** of Lindblad type emerges naturally when **four-momentum space is a non-abelian Lie group**

(MA: 1403.6457; Phys. Rev. D 90, 024016 (2014))

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Lie group-valued momenta are associated to **deformations of relativistic symmetries** and make their appearance when **one couples point particles to gravity** in $2+1$ dimensions

Point particles in 3d gravity

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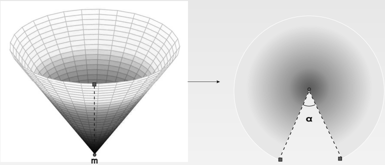
(3d Newton's constat G $\sim 1/M_{\mathrm{Planck}}$)



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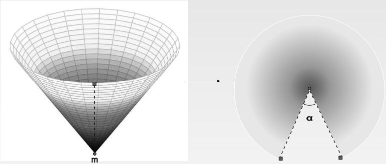

In such topological theory the particle's mass (**rest energy**) is described by a **rotation** $h_\alpha \in SL(2, \mathbb{R})$

$SL(2,\mathbb{R})$ momentum space: embedding coordinates

Matschull and Welling (Class. Quant. Grav. **15**, 2981 (1998)) showed that such "conical" particle's phase space is embedded in $\mathbb{R}^{2,1} \times SL(2,\mathbb{R})$

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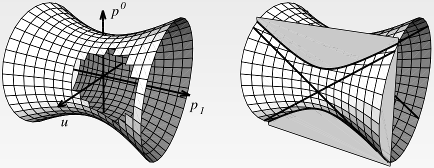

p^μ are *embedding coordinates on AdS space*

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i.e. just the familiar **adjoint action**... **Note:** Using the spectral theorem any operator can be written in terms of a combination of projectors $|k\rangle\langle k|$

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Key point: the action on operators will be deformed accordingly

Deformed translations and Lindblad evolution in three dimensions

For the deformed translation generators associated to $SL(2,\mathbb{R})$ momentum space:

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which leads to a **Lindlblad equation**

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with "decoherence" matrix given by

$$h = \frac{i}{\kappa}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

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κ-**momenta**: coordinates on **Lie group** $AN(3)$ obtained form the Iwasawa decomposition of $SO(4,1) \simeq SO(3,1)AN(3)$, sub-manifold of dS_4

$$-p_0^2 + p_1^2 + p_2^2 + p_3^2 + p_4^2 = \kappa^2\,; \qquad p_0 + p_4 > 0$$

with $\kappa \sim E_{Planck}$

These structures have been advocated as encoding the **kinematics of a "Minkowski-limit" of quantum gravity**...deformed relativistic kinematics at the **Planck scale (see Amelino-Camelia's talk)**

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In embedding coordinates we have *ordinary relativistic kinematics* at the **one-particle** level...all non-trivial structures confined to "co-algebra" sector

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A straightforward calculation of $\mathrm{ad}_{P_0}(\rho)$ leads to a *non-symmetric* **Lindblad equation**

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- the adjoint actions of N_i and P_0 satisfy

$$\mathrm{ad}_{\mathrm{ad}N_i(P_0)}(\cdot) = \mathrm{ad}_{N_i}(\mathrm{ad}_{P_0})(\cdot) - \mathrm{ad}_{P_0}(\mathrm{ad}_{N_i})(\cdot)$$

  in this sense the κ-Lindblad equation follows a **deformed notion of covariance**

Testing deformations via precision measurements of neutral kaons

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- **Natural question:** do the new structures introduced so far affect **discrete symmetries** ??
- **A first step**: use basic physical requirements and algebraic consistency to define the action of P, T and C on the generators of the κ-Poincaré group. (MA and J Kowalski-Glikman, Phys. Lett. B **760**, 69 (2016))

κ-deformation of discrete symmetries

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- **TIME REVERSAL**: require that in the limit $\kappa \to \infty$, $\mathbb{T}$ flips sign of M_i

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  - imposing that in the $\kappa \to \infty$ one recovers usual ordinary $\mathbb{C}$ we obtain

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