

Towards a dispersive determination of the pion transition form factor

Stefan Leupold

Frascati, October 2016



Collaborators

- Martin Hoferichter (Seattle)
- Jonny Jansson (Uppsala)
- Bastian Kubis (Bonn)
- Bai Long (Bonn)
- Franz Niecknig (Bonn)
- Sebastian Schneider (Bonn, now industry)

related previous work of the Bonn group:

F. Niecknig, B. Kubis and S. P. Schneider, Eur. Phys. J. C 72, 2014 (2012)

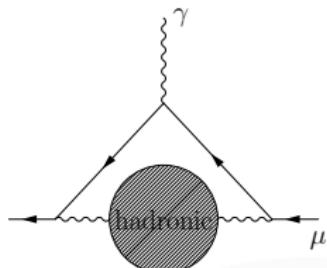
S. P. Schneider, B. Kubis and F. Niecknig, Phys. Rev. D 86, 054013 (2012)

M. Hoferichter, B. Kubis and D. Sakkas, Phys. Rev. D 86, 116009 (2012)

Data-driven approach to $g - 2$ of the muon

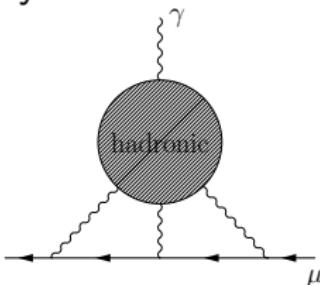
vacuum polarization (now dominant uncertainty)

- directly related to cross sect. $e^+e^- \rightarrow \text{hadrons}$
(by dispersion relation)
- ~~ measurable
- ~~ ongoing improvements by international efforts



light-by-light scattering

(soon dominant uncertainty)



- $\gamma^*\gamma^* \leftrightarrow \text{hadron(s)}$ not so easily accessible by experiment
- ~~ crank dispersive machinery further
- Colangelo/Hoferichter/Kubis/Procura/Stoffer, Phys.Lett. B738 (2014) 6
- ~~ defines extensive experimental and theoretical program

Unitarity and analyticity

- constraints from local quantum field theory:
partial-wave amplitudes for reactions/decays must be
 - unitary:

$$S S^\dagger = 1, \quad S = 1 + iT \quad \Rightarrow \quad 2 \operatorname{Im} T = T T^\dagger$$

→ note that this is a matrix equation:

$$\operatorname{Im} T_{A \rightarrow B} = \sum_X T_{A \rightarrow X} T_{X \rightarrow B}^\dagger$$

~~> in practice: use most relevant intermediate states X

- analytical (dispersion relations):

$$T(s) = T(0) + \frac{s}{\pi} \int_{-\infty}^{\infty} ds' \frac{\operatorname{Im} T(s')}{s'(s' - s - i\epsilon)}$$

~~> can be used to calculate whole amplitude from imaginary part

Dispersion theory in practice (schematic)

- concentrate on form factors $\langle B|j|A\rangle \sim F_{AB}$ (j is quark current)
- assume that lowest-mass state is **2-pion state**

$$\hookrightarrow F_{AB}(s) = F_{AB}(0) + \frac{s}{\pi} \int_0^\infty ds' \frac{T_{AB\pi\pi}^*(s') F_{\pi\pi}(s')}{s'(s'-s-i\epsilon)} + \dots$$

$4m_\pi^2$

- typically input $T_{AB\pi\pi}$, $F_{\pi\pi}$ is known for not too high energies
 - to further **suppress** unknown high-energy part and **not considered intermediate states** different from 2-pion state
 - oversubtract dispersion integral:

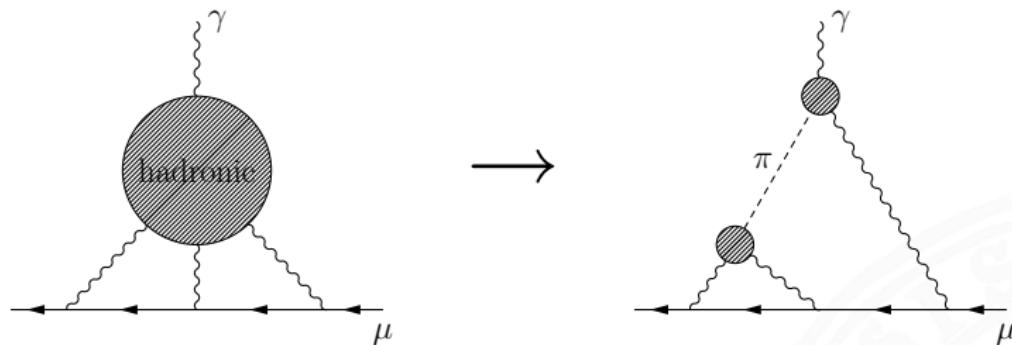
$$F_{AB}(s) = F_{AB}(0) + F'_{AB}(0)s + \frac{s^2}{\pi} \int_0^\infty ds' \frac{T_{AB\pi\pi}^*(s') F_{\pi\pi}(s')}{(s')^2(s'-s-i\epsilon)} + \dots$$

$4m_\pi^2$

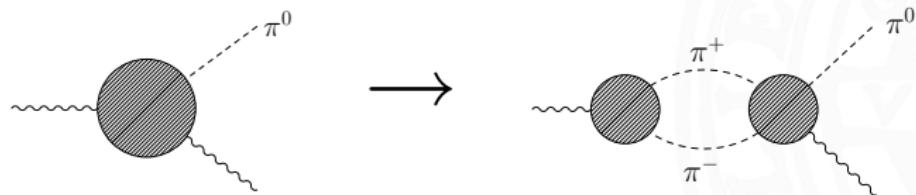
- get subtraction constants $F_{AB}(0)$, $F'_{AB}(0)$ from data, EFT, ...

Using lowest-mass states

hadronic light-by-light contribution

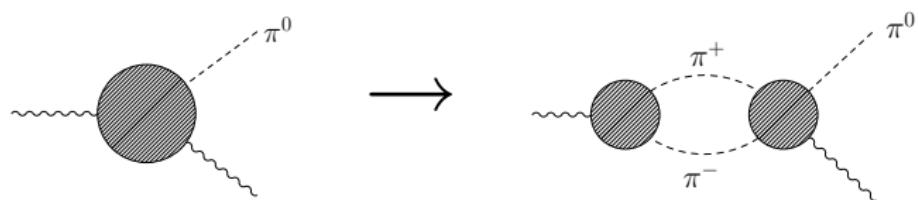


~~ need pion transition form factor

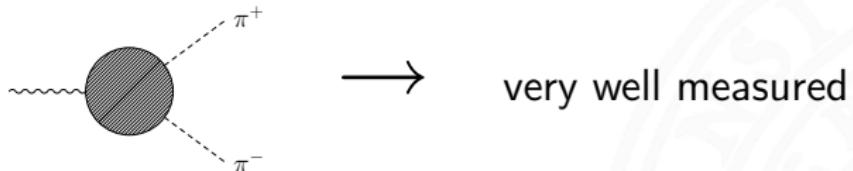


Dispersive reconstruction I

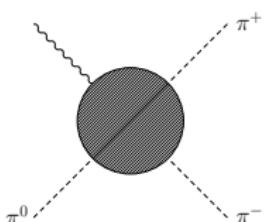
pion transition form factor



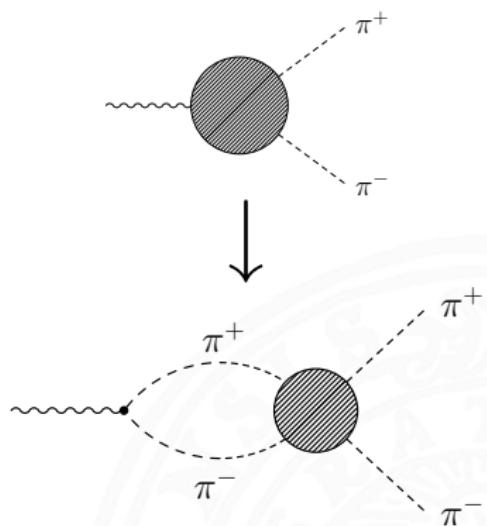
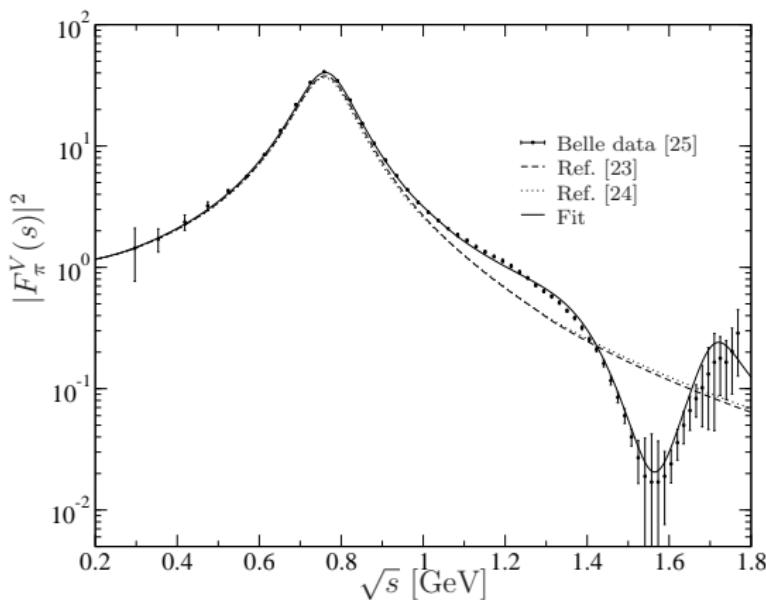
~~ need pion vector form factor



and amplitude $\gamma^* - 3\text{-pion}$



Pion vector form factor

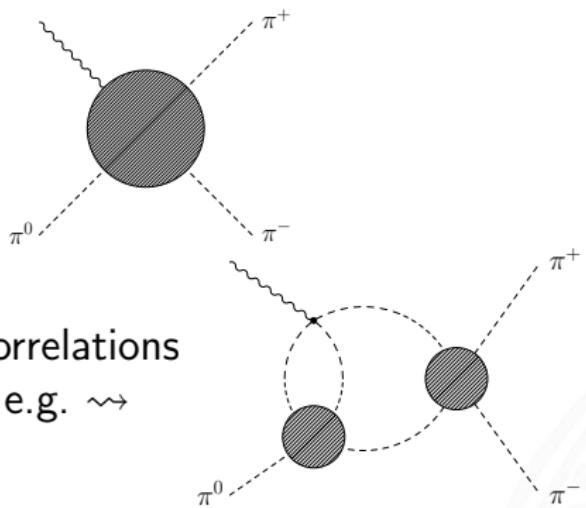


pion phase shift very well known; fits to pion vector form factor

Sebastian P. Schneider, Bastian Kubis, Franz Niecknig, Phys. Rev. D86:054013, 2012

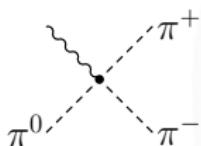
Dispersive reconstruction II

amplitude $\gamma^* - 3\text{-pion}$



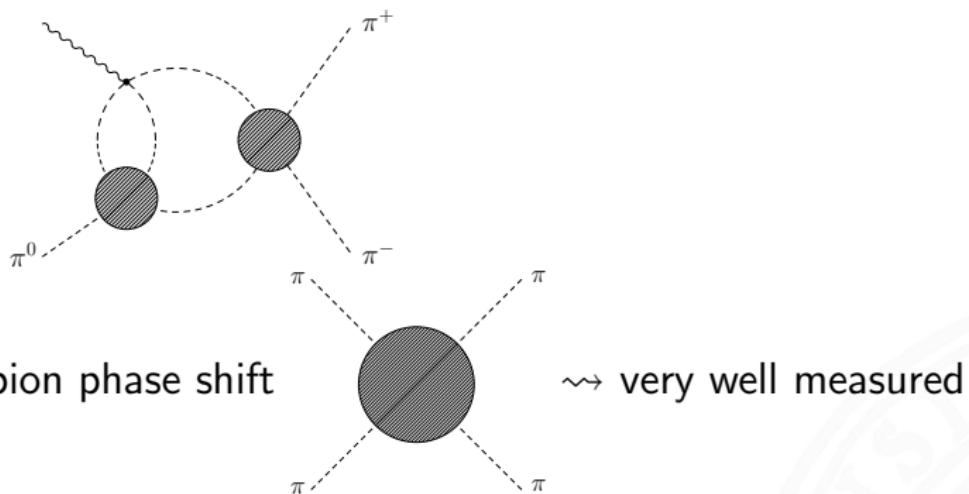
contains two-body correlations
(depend on s, t, u), e.g. \sim

and genuine three-body correlations
(depend on $m_{3\pi}^2 = m_{\gamma^*}^2$)

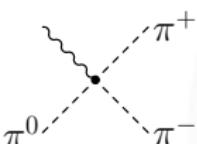


Required input

for



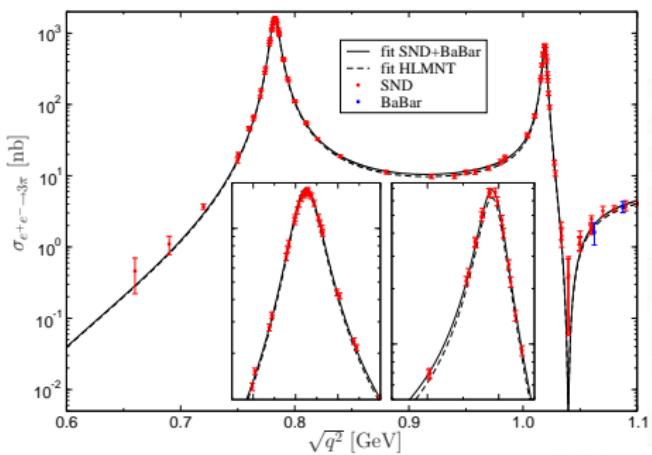
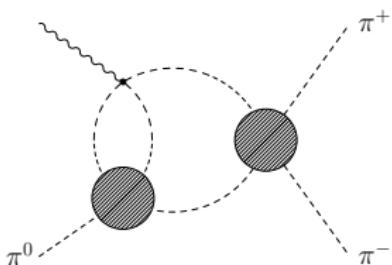
and genuine three-body correlations
(one-parameter function!)



\leadsto fit to cross section of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$

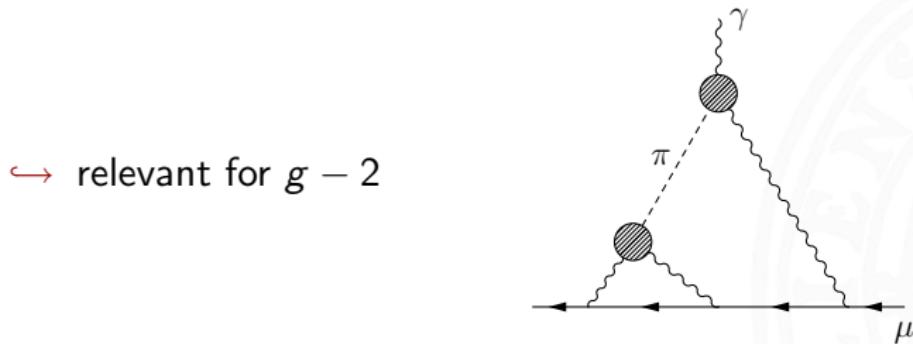
Fit to $e^+e^- \rightarrow \pi^+\pi^-\pi^0$

- dominated by narrow resonances ω, ϕ
- use Breit-Wigners plus background for genuine three-body correlations
- fully include cross-channel rescattering of pion pairs (two-body correlations)



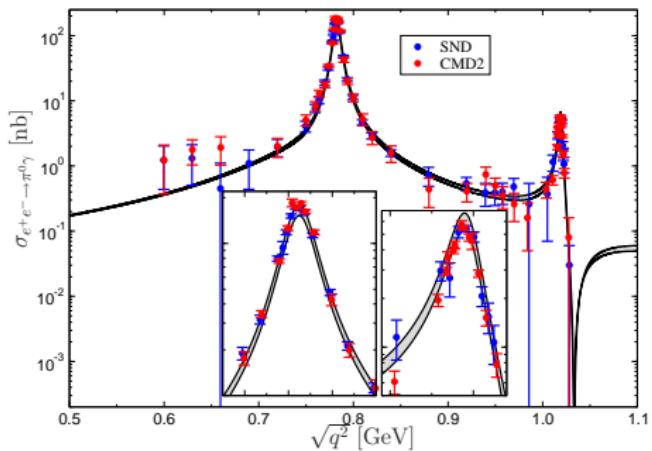
Results

- so far: single-virtual pion transition form factor
 - time-like: cross section $e^+e^- \rightarrow \pi^0\gamma$
→ compare to experimental data (**postdiction**)
 - space-like: reaction $\gamma^*\gamma \rightarrow \pi^0$
→ **prediction** for low energies
- final aim:** double-virtual pion transition form factor



Bai Long, Martin Hoferichter, Bastian Kubis, S.L.; work in progress

Time-like pion transition form factor



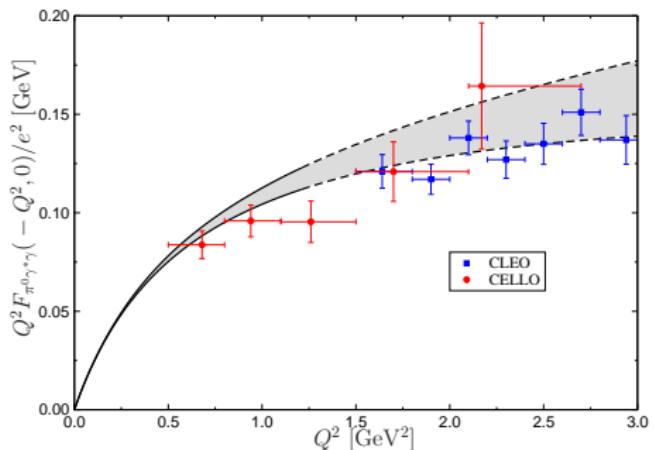
theory uncertainties from

- different data sets for $e^+ e^- \rightarrow 3\pi$
 - different pion phase shifts
 - other intermediate states than 2π neglected
- ↪ explored by different cutoff for range where 2π dominates

↝ excellent agreement

M. Hoferichter, B. Kubis, S.L., F. Niecknig, S. P. Schneider, Eur.Phys.J. C74 (2014) 3180

Space-like pion transition form factor



- this is a prediction, no data yet at low energies
 - expect new measurements from BESIII
 - final aim: double virtual transition form factor
- relevant for $g - 2$

M. Hoferichter, B. Kubis, S.L., F. Niecknig, S. P. Schneider, Eur.Phys.J. C74 (2014) 3180

Wish list

to improve our input

- resolve ambiguities in cross section $e^+e^- \rightarrow 3\pi$
- Dalitz plot $\omega \rightarrow 3\pi$ with same accuracy as $\phi \rightarrow 3\pi$
- cross section $\gamma\pi \rightarrow 2\pi$
 - determines subtraction constant and tests chiral-anomaly prediction
 - alternative: lattice QCD

Hadron Spectrum Collaboration, Phys.Rev. D93, 114508 (2016)

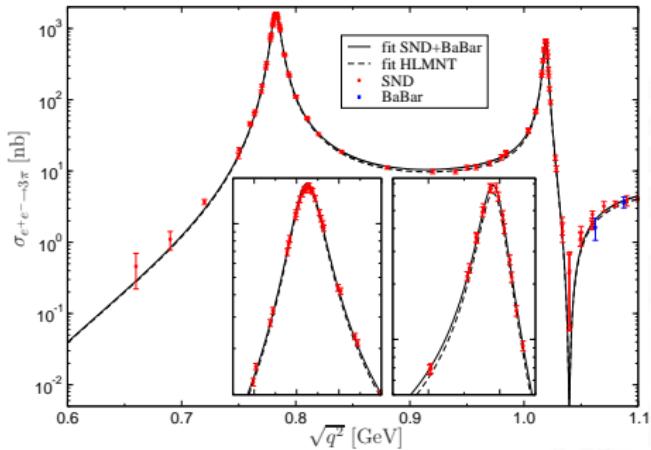
to check our approach

- transition form factor $\omega \rightarrow \pi^0 e^+ e^-$
- transition form factor $\phi \rightarrow \pi^0 e^+ e^-$ in ρ -peak region

Wish list I

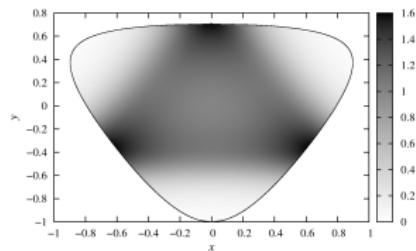
resolve ambiguities in cross section $e^+e^- \rightarrow \pi^+\pi^-\pi^0$

- largest uncertainty in our calculation of pion TFF
- for given s Kubis group can provide full angular distribution
- for acceptance corrections and extrapolation to full 4π angle

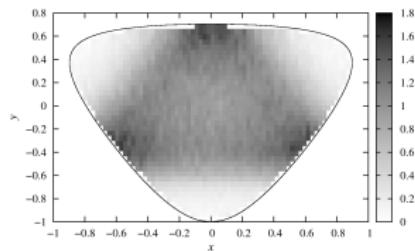


Wish list II

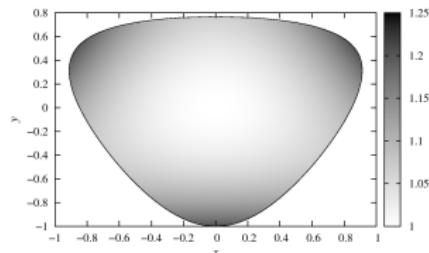
Dalitz plot $\omega \rightarrow 3\pi$ with same accuracy as $\phi \rightarrow 3\pi$
 \hookrightarrow allows for more subtractions in dispersive integrals



F. Niecknig, B. Kubis, S.P. Schneider,
 Eur.Phys.J.C72, 2014 (2012)

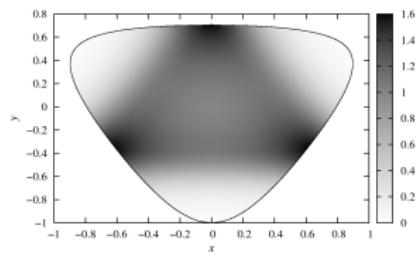


data: KLOE, Phys. Lett. B 561, 55 (2003)



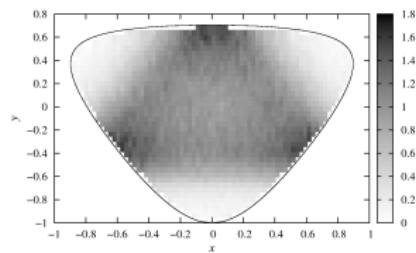
Wish list II

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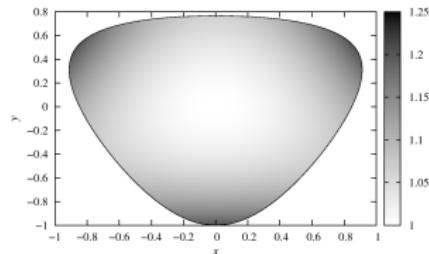


ϕ

F. Niecknig, B. Kubis, S.P. Schneider,
 Eur.Phys.J.C72, 2014 (2012)



data: KLOE, Phys. Lett. B 561, 55 (2003)



ω

first steps beyond pure phase space:

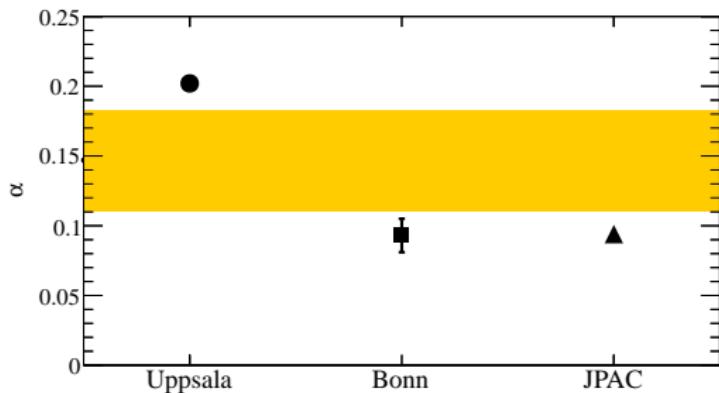
L. Heijkenskjöld, S. Sawant, PhD theses
 2016 and WASA@COSY, B. Kubis, S.L.,
 arXiv:1610.02187 [nucl-ex] \rightsquigarrow fig.

$\omega \rightarrow 3\pi$ — first steps beyond pure phase space

- for Dalitz distribution of $\omega \rightarrow 3\pi$:
use α to parametrize main deviation from (p-wave) phase space
- ↪ oversimplistic but qualitatively OK:
 $\alpha = 0$ no deviation, i.e. final pions would not interact
 $\alpha > 0$ tail of ρ meson

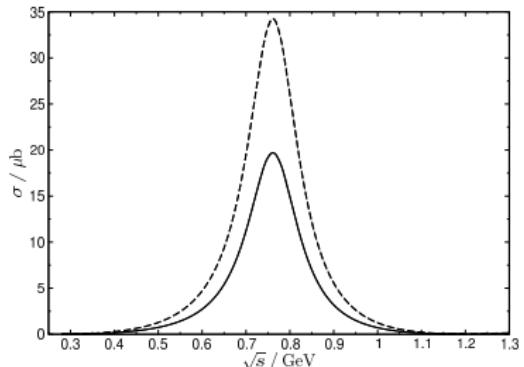
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 $\alpha = 0$ no deviation, i.e. final pions would not interact **disproven**
 $\alpha > 0$ tail of ρ meson **confirmed**



WASA@COSY, B. Kubis, S.L., arXiv:1610.02187 [nucl-ex]

Wish list III: cross section $\gamma + \pi \rightarrow 2\pi$



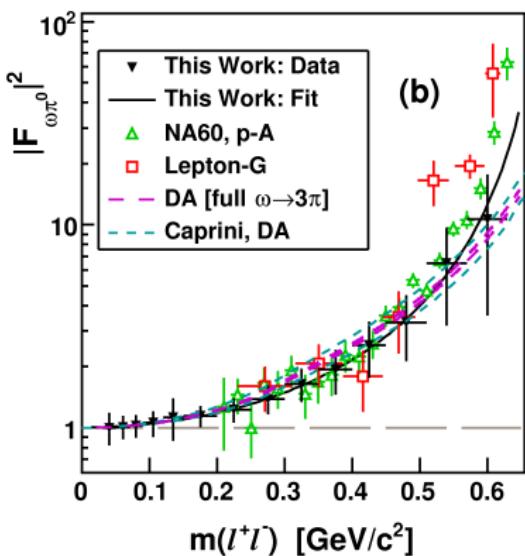
M. Hoferichter, B. Kubis, D. Sakkas,
Phys.Rev.D86, 116009 (2012)

- expect future data from COMPASS@CERN
(pion beam, Primakov effect)
- calculation includes chiral anomaly and pion-pion rescattering using dispersion theory
- **solid line:** prediction from anomaly
dashed line: size of anomaly scaled up by about 30%
- can use whole range to pin down anomaly = subtraction constant, not just threshold region

Wish list IV

- transition form factor $\omega \rightarrow \pi^0 e^+ e^-$

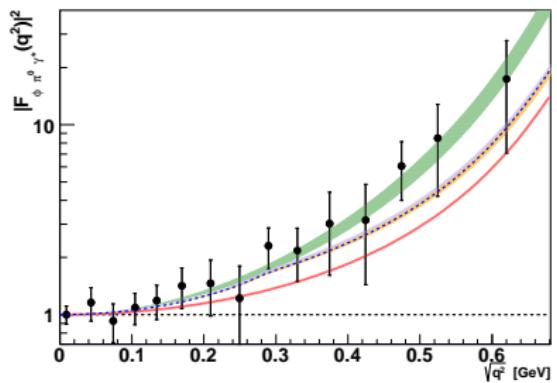
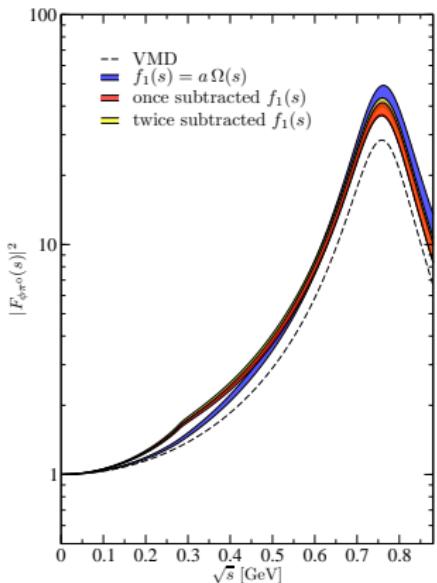
↪ resolve tension between NA60 ↔ theory, A2@MAMI



A2 Collaboration at MAMI, arXiv:1609.04503 [hep-ex]

Wish list V

transition form factor $\phi \rightarrow \pi^0 e^+ e^-$ in ρ -peak region

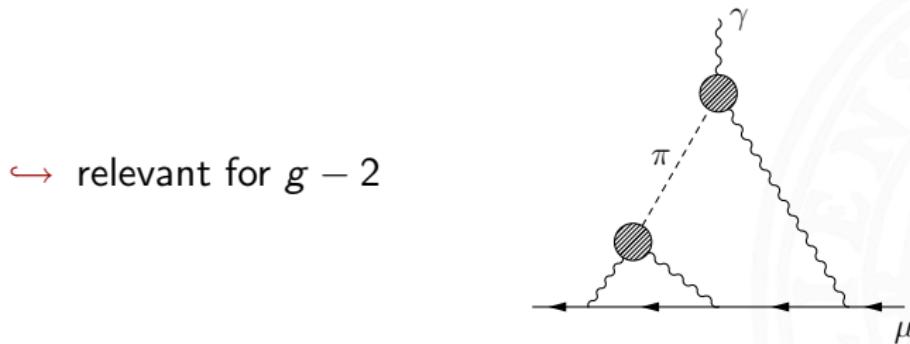


KLOE-2, Phys.Lett. B757, 362 (2016)

S.P. Schneider, B. Kubis, F. Niecknig,
Phys.Rev. D86, 054013 (2012)

Summary

- so far: single-virtual pion transition form factor
 - time-like: cross section $e^+e^- \rightarrow \pi^0\gamma$
→ compare to experimental data (**postdiction**)
 - space-like: reaction $\gamma^*\gamma \rightarrow \pi^0$
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Bai Long, Martin Hoferichter, Bastian Kubis, S.L.; work in progress

backup slides

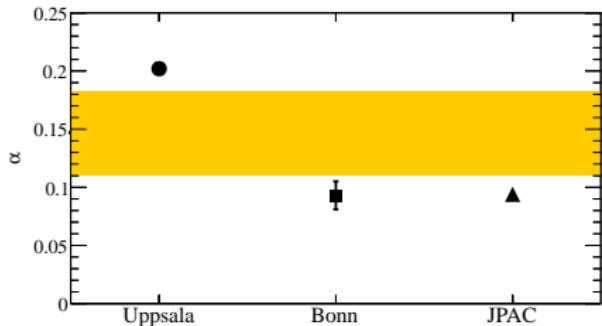
$\omega \rightarrow 3\pi$ — first steps beyond pure phase space

- Dalitz distribution of $\omega \rightarrow 3\pi$ is relatively flat

↪ to account for deviation from pure (p-wave) phase space
 ↪ use parametrization:

$$1 + 2\alpha Z + 2\beta Z^{3/2} \sin 3\phi + 2\gamma Z^2 + \mathcal{O}(Z^{5/2})$$

F. Niecknig, B. Kubis and S. P. Schneider, Eur. Phys. J. C 72, 2014 (2012)

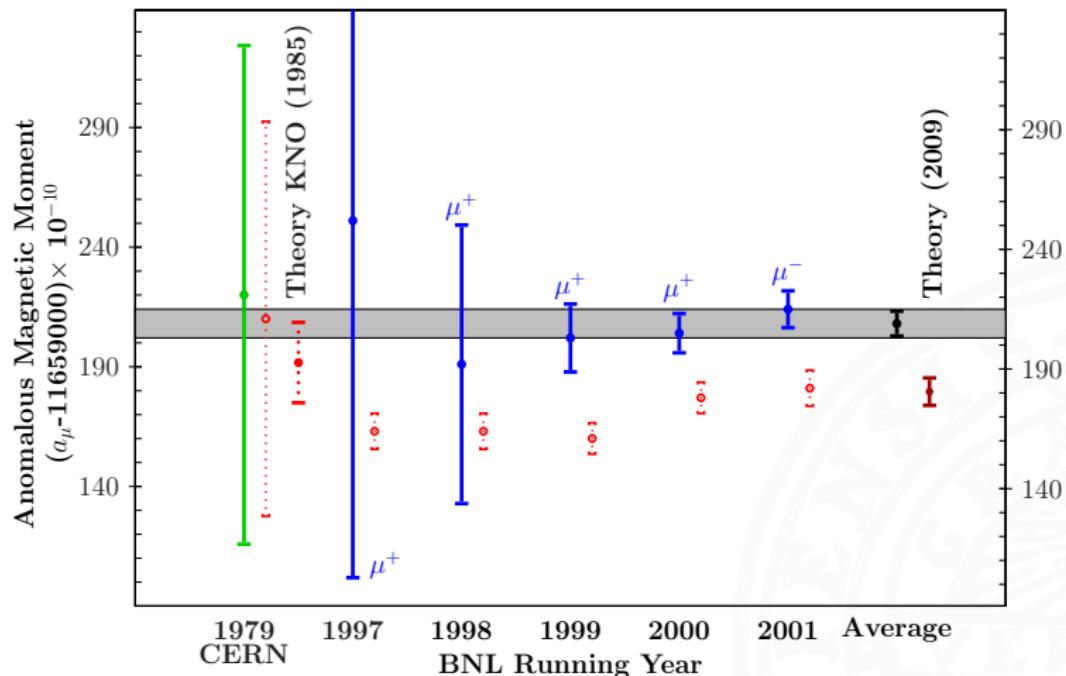


$$X =: \sqrt{Z} \cos \phi, \quad Y =: \sqrt{Z} \sin \phi$$

$$X := \frac{\sqrt{3}(T_+ - T_-)}{Q_\omega}, \quad Y := \frac{3T_0}{Q_\omega} - 1$$

$$Q_\omega := \text{sum of kinetic energies } T_i$$

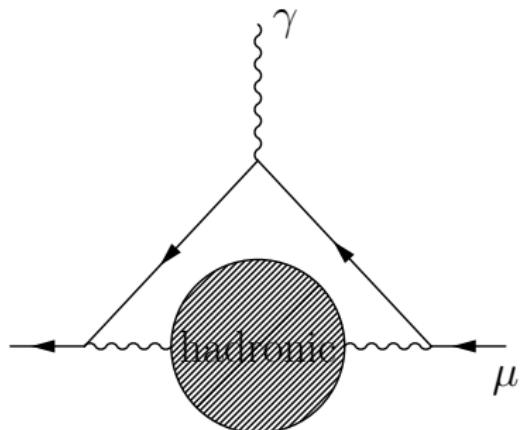
$g - 2$ of the muon — status: $\approx 3\sigma$ deviation



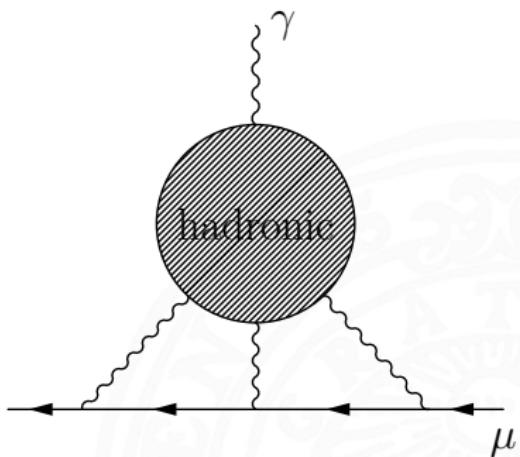
Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

$g - 2$ of the muon — theory

Largest uncertainty of standard model: hadronic contributions



vacuum polarization
 $\sim \alpha^2$



light-by-light scattering
 $\sim \alpha^3$

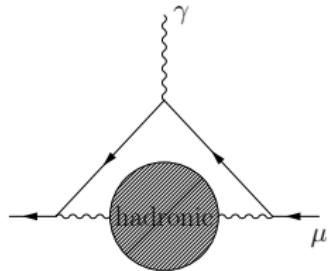
$g - 2$ of the muon — status

Standard model theory and experiment comparison [in units 10^{-11}].

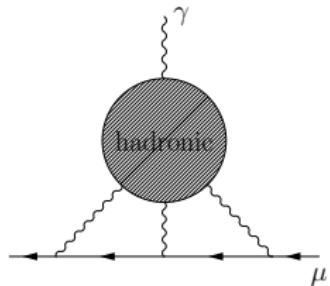
Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	-100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6
Experiment	116 592 080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3

Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

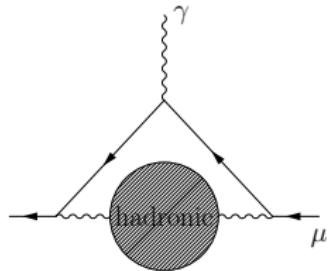
Hadronic contribution to $g - 2$ of the muon



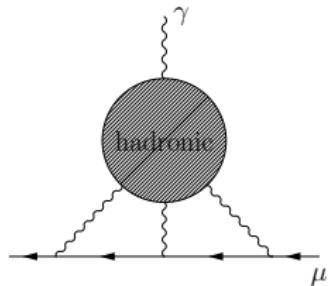
how to determine size of hadronic fluctuations?



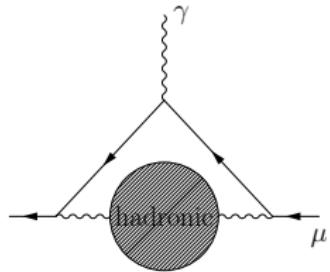
Hadronic contribution to $g - 2$ of the muon



how to determine size of hadronic fluctuations?
→ develop a **phenomenological hadronic model**
or quark model **P(?)**

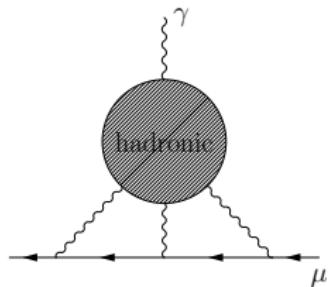


Hadronic contribution to $g - 2$ of the muon

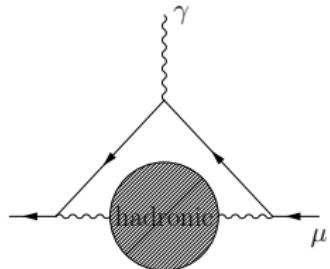


how to determine size of hadronic fluctuations?

- ↪ develop a **phenomenological hadronic model** or quark model **P(?)**
- ↪ this would yield a **P**-model prediction

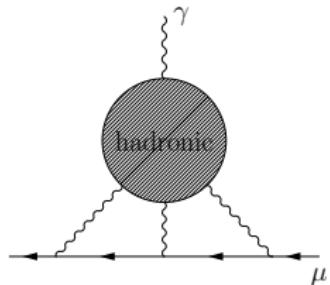


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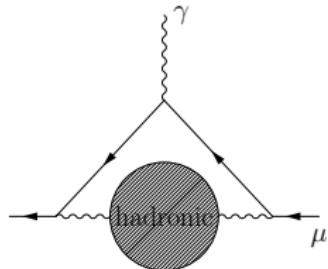


how to determine size of hadronic fluctuations?

- ↪ develop a **phenomenological** hadronic model or quark model **P(?)**
- ↪ this would yield a **P**-model prediction
- ↪ but we want a **standard-model** prediction

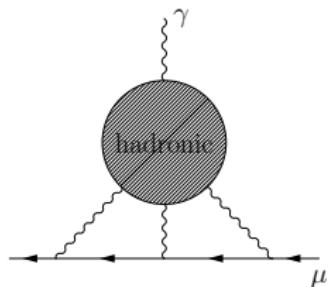


Hadronic contribution to $g - 2$ of the muon

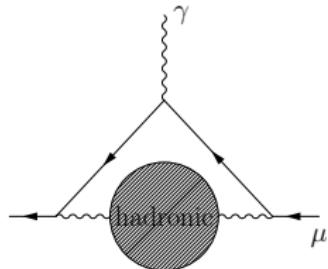


how to determine size of hadronic fluctuations?

- ↪ develop a **phenomenological** hadronic model or quark model **P(?)**
- ↪ this would yield a **P**-model prediction
- ↪ but we want a **standard-model** prediction and with a **reliable** uncertainty estimate!

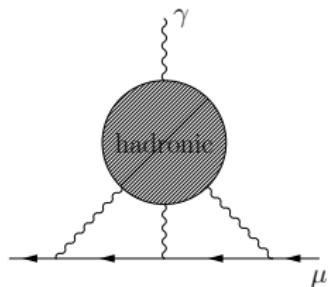


Hadronic contribution to $g - 2$ of the muon

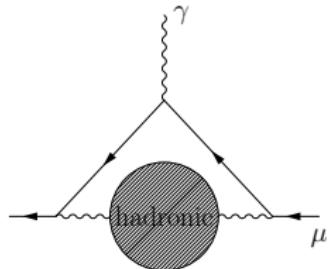


how to determine size of hadronic fluctuations?

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- ↪ this would yield a **P**-model prediction
- ↪ but we want a **standard-model** prediction and with a **reliable** uncertainty estimate!
- ↪ need a model independent approach
- ↪ lattice QCD, effective field theory (EFT) or “data”

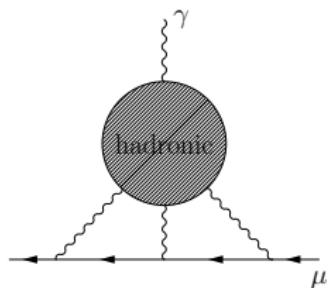


Hadronic contribution to $g - 2$ of the muon



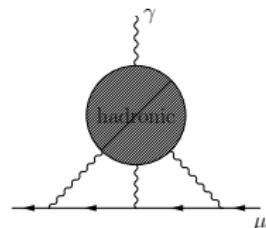
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- ↪ this would yield a **P**-model prediction
- ↪ but we want a **standard-model** prediction and with a **reliable** uncertainty estimate!
- ↪ need a model independent approach
- ↪ lattice QCD, effective field theory (EFT) or “data” (\leftarrow highest accuracy so far)



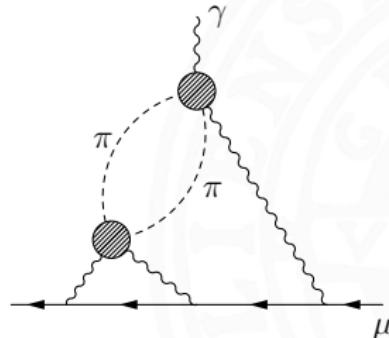
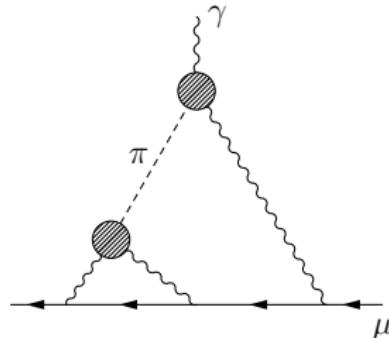
Hadronic light-by-light contribution

true for all hadronic contributions:



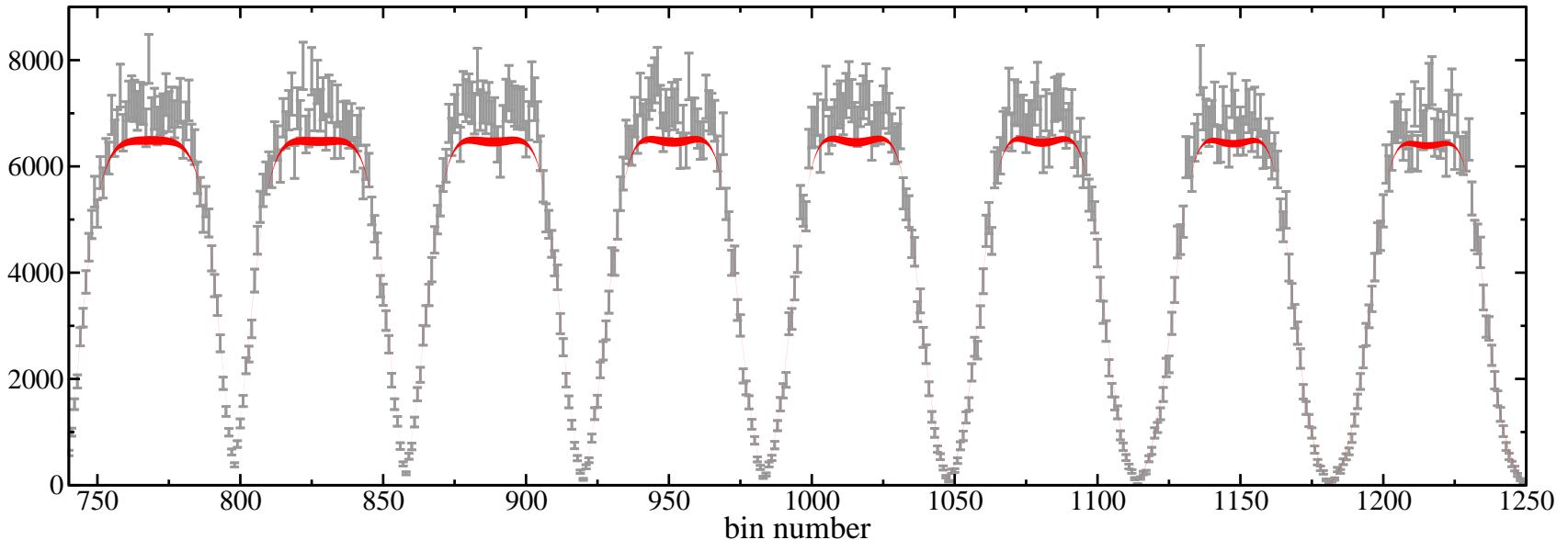
- the lighter the hadronic system, the more important
(though high-energy contributions not unimportant for light-by-light)

↪ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow \pi^0, \eta, \eta'$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow 2\pi, \dots$



Experimental comparison to $\phi \rightarrow 3\pi$

KLOE Dalitz plot: $2 \cdot 10^6$ events, 1834 bins Niecknig, BK, Schneider 2012



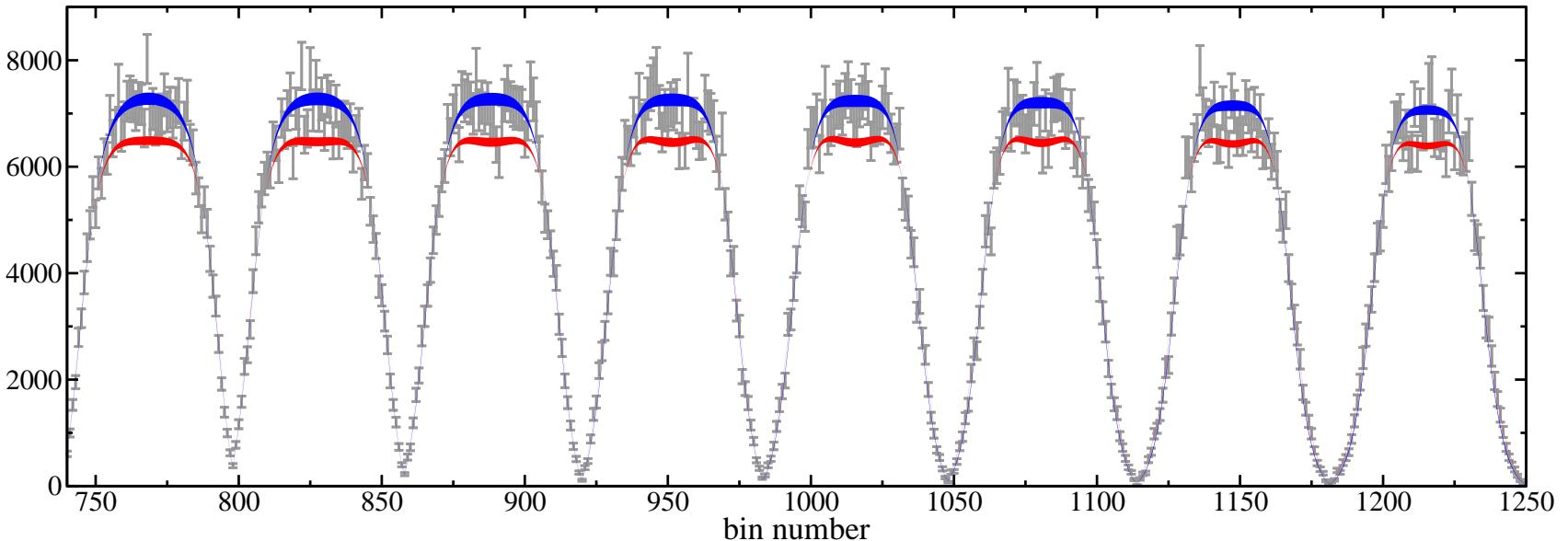
$$\hat{\mathcal{F}} = 0$$

$$\chi^2/\text{ndof} \quad 1.71 \dots 2.06$$

$$\mathcal{F}(s) = a \Omega(s) = a \exp \left[\frac{s}{\pi} \int_{4M_\pi^2}^\infty \frac{ds'}{s'} \frac{\delta_1^1(s')}{s' - s} \right]$$

Experimental comparison to $\phi \rightarrow 3\pi$

KLOE Dalitz plot: $2 \cdot 10^6$ events, 1834 bins Niecknig, BK, Schneider 2012



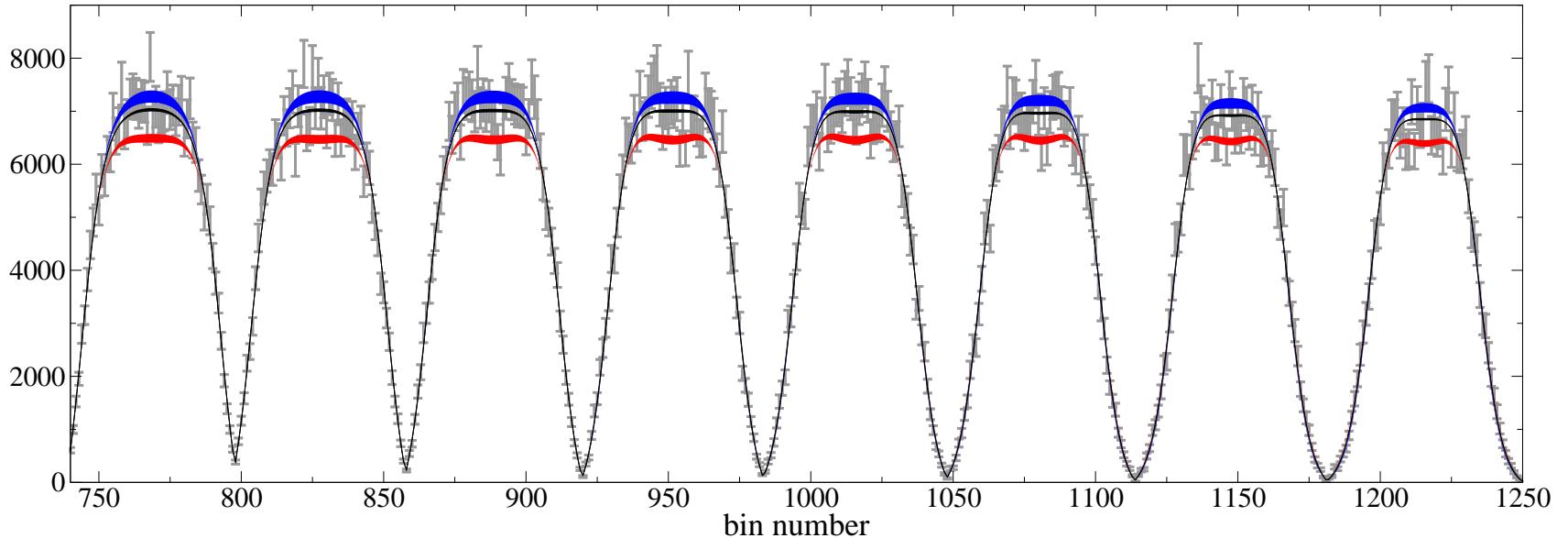
$\hat{\mathcal{F}} = 0$ once-subtracted

χ^2/ndof	1.71 ... 2.06	1.17 ... 1.50
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$$\mathcal{F}(s) = \textcolor{red}{a} \Omega(s) \left[1 + \frac{s}{\pi} \int_{4M_\pi^2}^\infty \frac{ds'}{s'} \frac{\hat{\mathcal{F}}(s') \sin \delta_1^1(s')}{|\Omega(s')|(s' - s)} \right]$$

Experimental comparison to $\phi \rightarrow 3\pi$

KLOE Dalitz plot: $2 \cdot 10^6$ events, 1834 bins Niecknig, BK, Schneider 2012



	$\hat{\mathcal{F}} = 0$	once-subtracted	twice-subtracted
χ^2/ndof	1.71 ... 2.06	1.17 ... 1.50	1.02 ... 1.03

$$\mathcal{F}(s) = \textcolor{red}{a} \Omega(s) \left[1 + \textcolor{blue}{b} s + \frac{s^2}{\pi} \int_{4M_\pi^2}^\infty \frac{ds'}{(s')^2} \frac{\hat{\mathcal{F}}(s') \sin \delta_1^1(s')}{|\Omega(s')|(s' - s)} \right]$$