Towards a dispersive determination of the η and η' transition form factors

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Outline

Introduction

- The anomalous magnetic moment of the muon:
  hadronic light-by-light scattering

Dispersion relations for meson transition form factors

- Ingredients for a data-driven analysis of $\eta, \eta' \to \gamma^*\gamma^{(*)}$

Summary / Outlook

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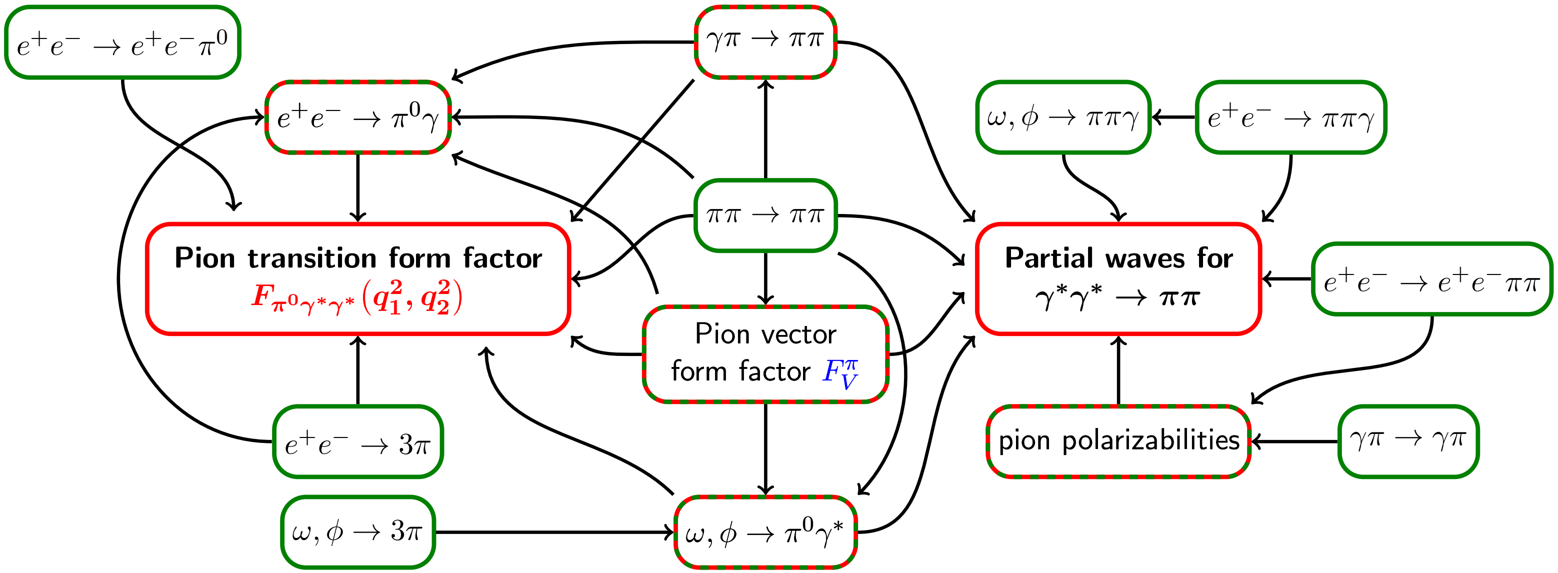
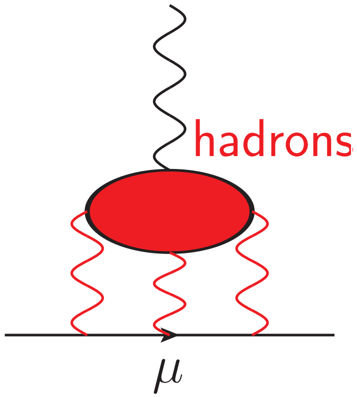


Colangelo, Hoferichter, BK, Procura, Stoffer 2014

Summary / Outlook

Hadronic light-by-light scattering

- hadronic light-by-light may soon dominate Standard Model uncertainty in $(g-2)_\mu$



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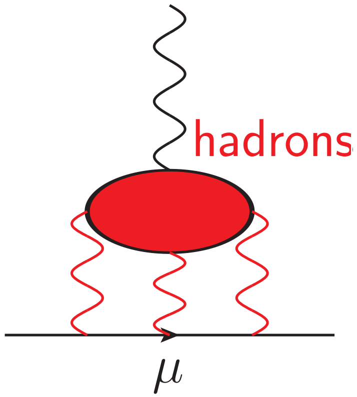
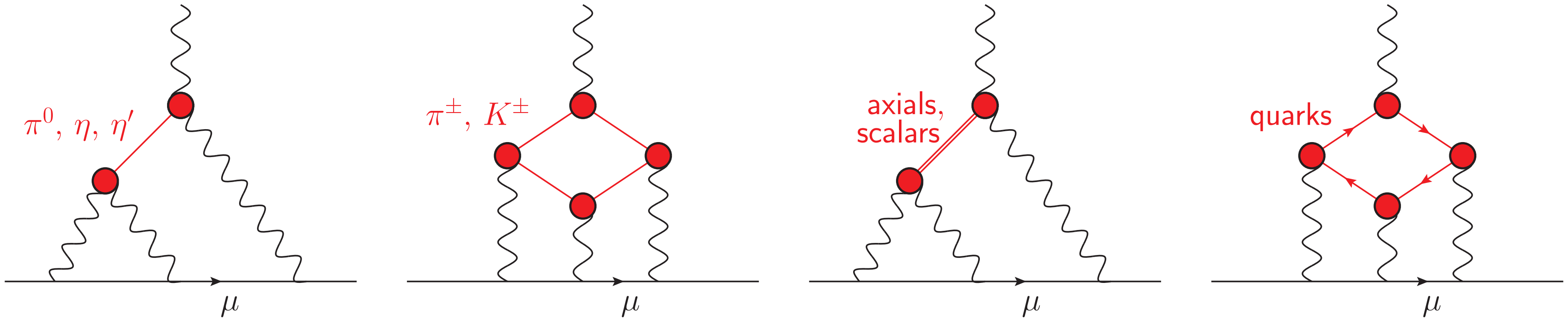


- different contributions estimated (in 10^{-11}):



99±16   –19±13   15±7   21±3

$\longrightarrow$ how to control hadronic modelling?   Jegerlehner, Nyffeler 2009

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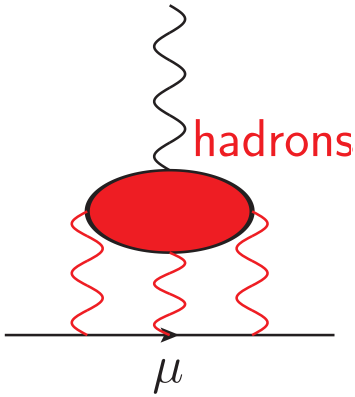

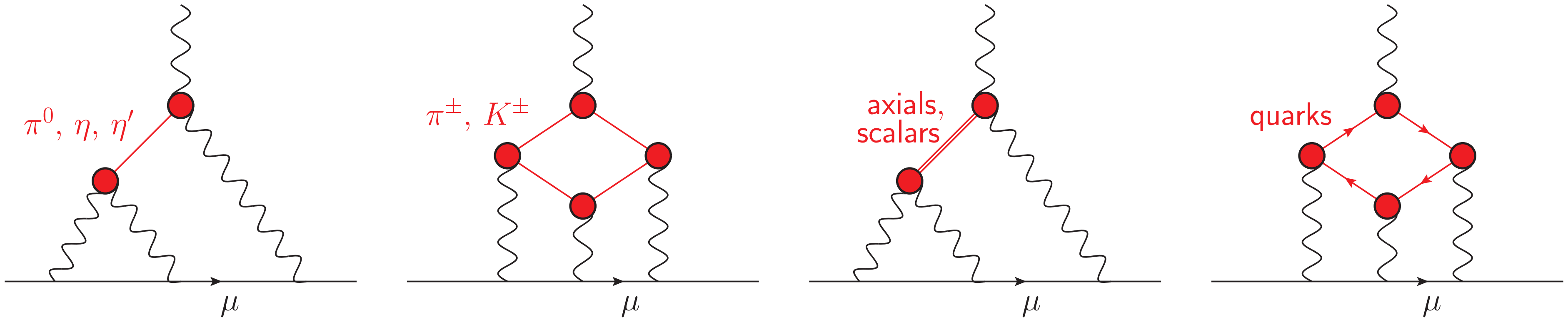

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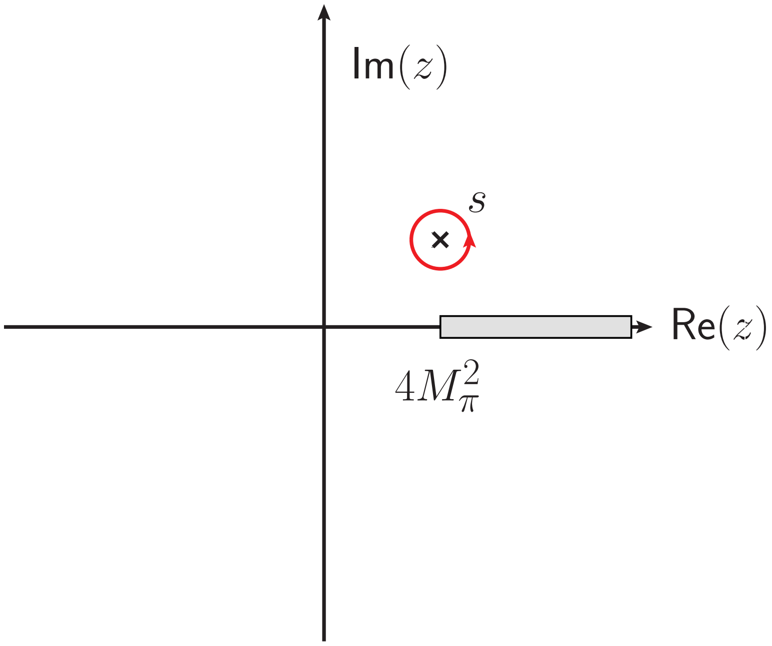
- dispersive point of view: analytic structure, cuts and poles
  $\longrightarrow$ (on-shell) form factors and scatt. amplitudes from experiment
  $\longrightarrow$ expansion in masses of intermediate states, partial waves

Dispersion relations for pedestrians



analyticity & Cauchy's theorem:

$$T(s) = \frac{1}{2\pi i} \oint_{\partial\Omega} \frac{T(z)dz}{z-s}$$

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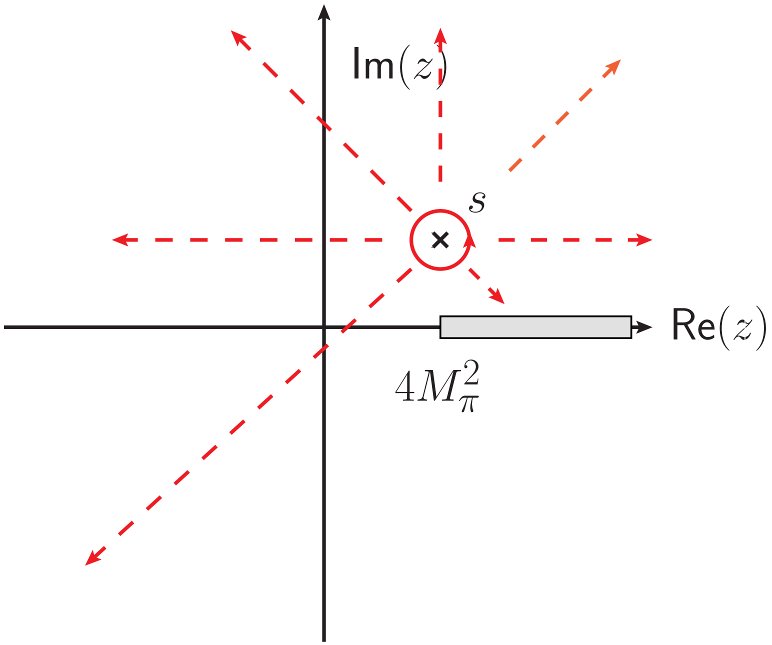

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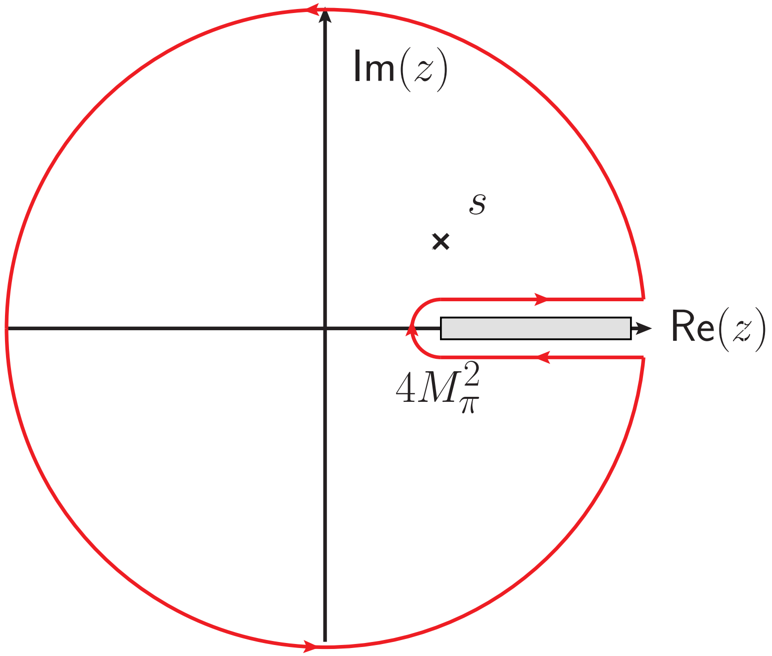

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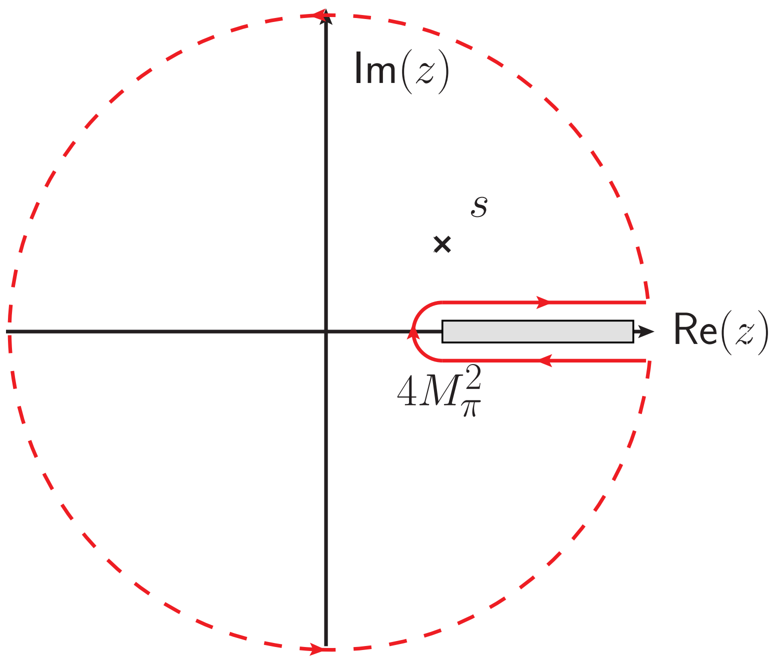

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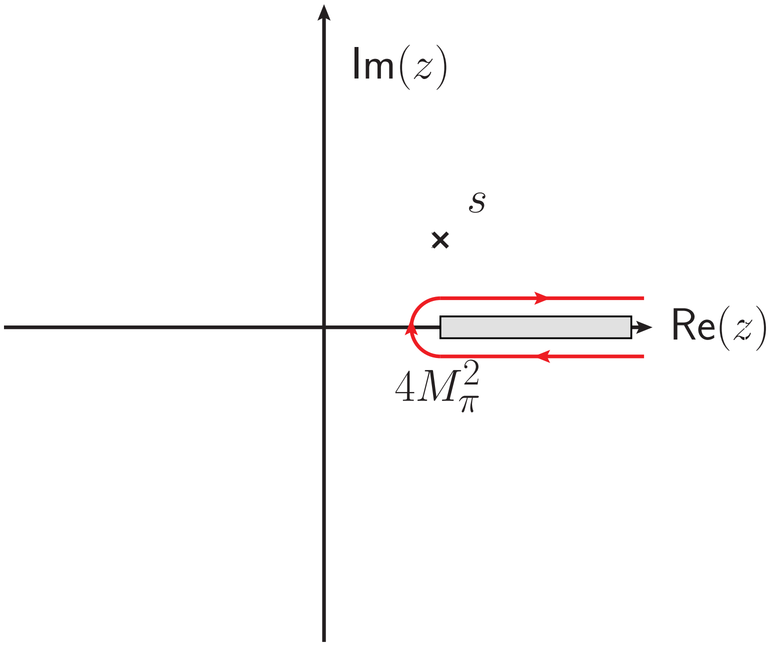

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$$T(s) = \frac{1}{2\pi i}\oint_{\partial\Omega}\frac{T(z)dz}{z-s}$$

$$\longrightarrow \frac{1}{2\pi i}\int_{4M_\pi^2}^{\infty}\frac{\operatorname{disc} T(z)dz}{z-s}$$

$$= \frac{1}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\operatorname{Im} T(z)dz}{z-s}$$

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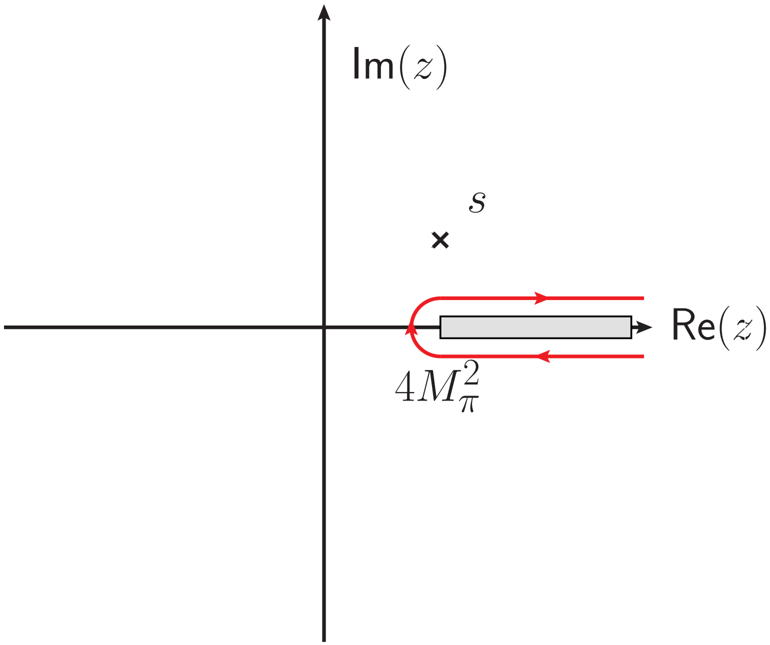

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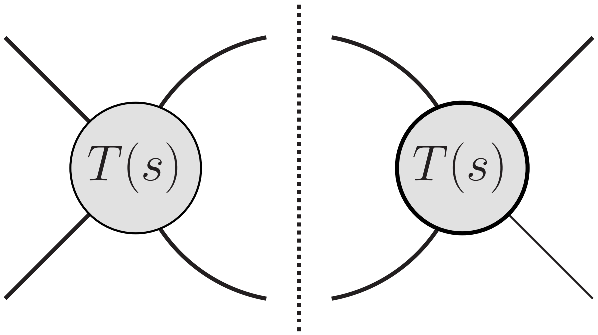
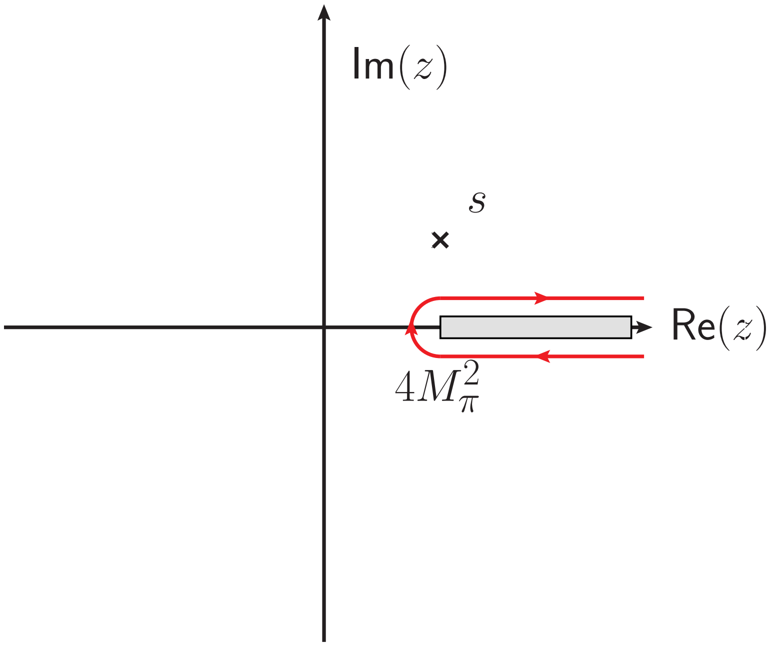


e.g. if $T(s)$ is a $\pi\pi$ partial wave $\longrightarrow$

$$\frac{\operatorname{disc}T(s)}{2i} = \operatorname{Im}T(s) = \frac{2q_\pi}{\sqrt{s}}\theta(s-4M_\pi^2)|T(s)|^2$$

Dispersion relations for pedestrians



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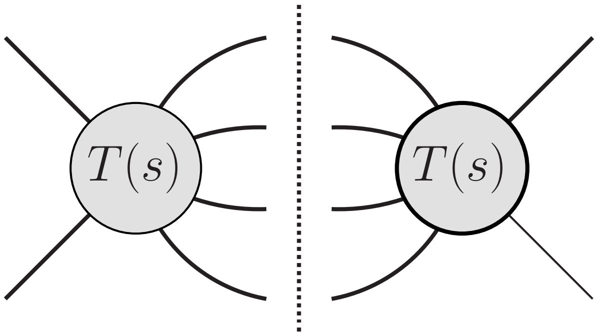

inelastic intermediate states ($K\bar{K}$, 4π) suppressed at low energies
$\longrightarrow$ will be neglected in the following

Dispersive analysis of $\pi^0/\eta \to \gamma^*\gamma^*$

- isospin decomposition:

see also following talk by S. Leupold

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{vs}(q_1^2, q_2^2) + F_{vs}(q_2^2, q_1^2)$$

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- analyse the leading hadronic intermediate states:

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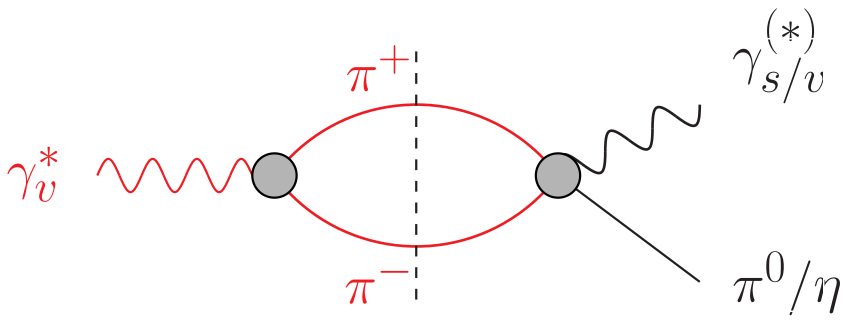

▷ isovector photon: 2 pions

$\propto$ pion vector form factor $\times$ $\gamma\pi \to \pi\pi$ / $\eta \to \pi\pi\gamma$

all determined in terms of pion–pion P-wave phase shift

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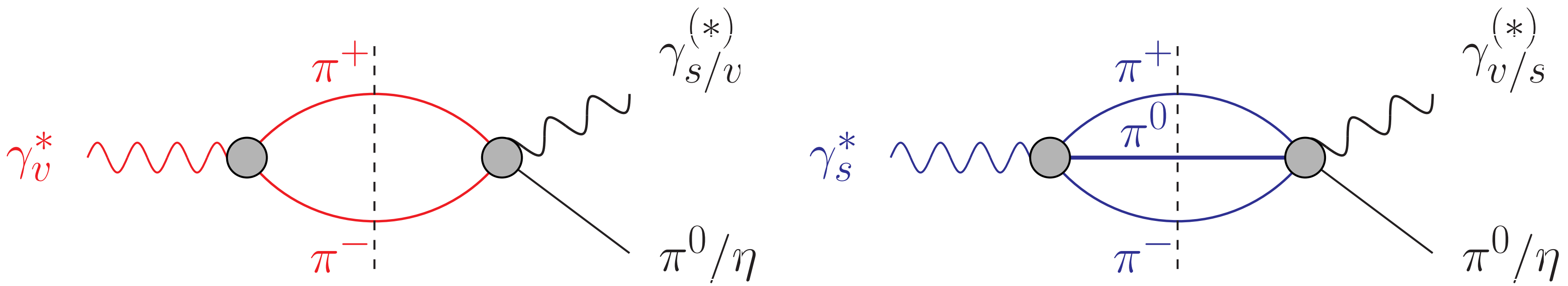

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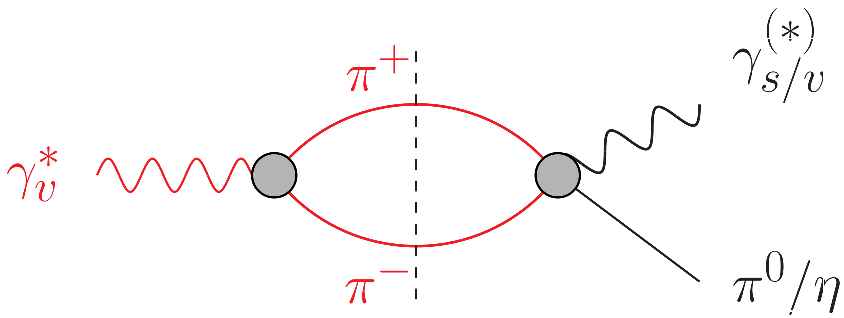

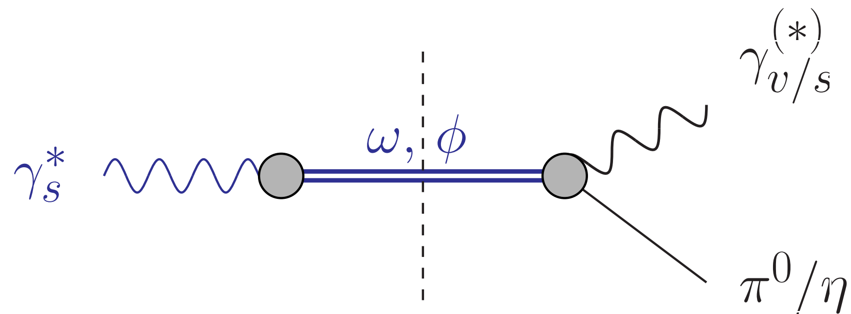

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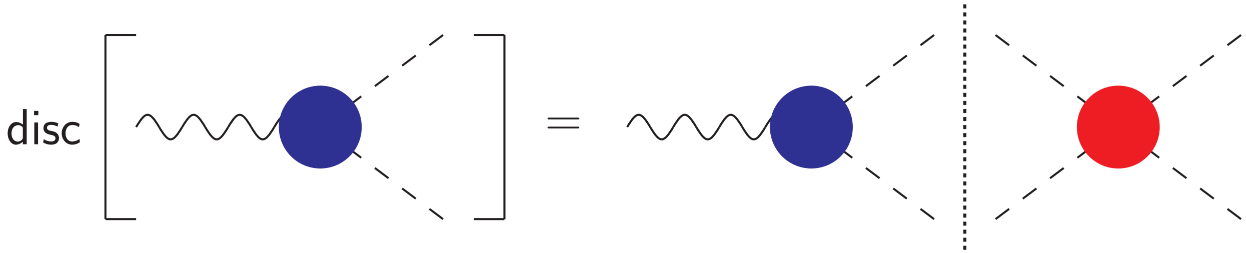
▷ isoscalar photon: 3 pions $\longrightarrow$ dominated by narrow ω, ϕ

$\leftrightarrow \omega/\phi$ transition form factors; very small for the η

Warm-up: pion form factor from dispersion relations

- just two hadrons: form factor, e.g. $e^+e^- \to \pi^+\pi^-$, $\tau^- \to \pi^-\pi^0\nu_\tau$



$$\text{Im}\, F(s) \;\propto\; F(s) \times \text{phase space} \times T^*_{\pi\pi}(s)$$

$\longrightarrow$ final-state theorem: phase of $F(s)$ is scattering phase $\delta(s)$

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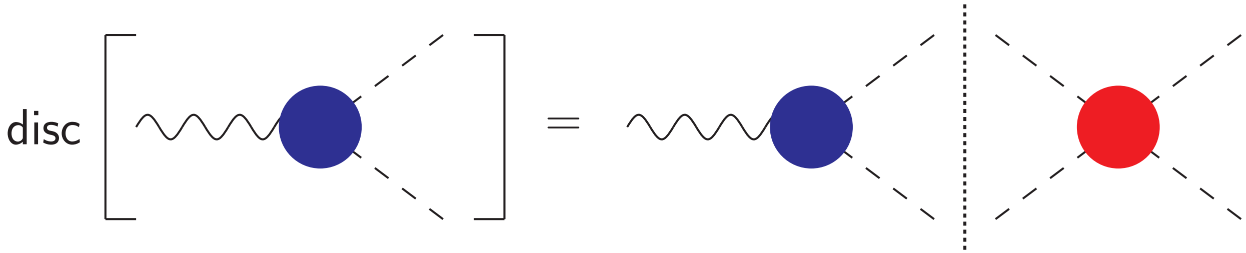

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- dispersion relations allow to reconstruct form factor from imaginary part $\longrightarrow$ elastic scattering phase $\delta(s)$:

$$F(s) = P(s)\Omega(s) \;, \quad \Omega(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)} \right\}$$

$P(s)$ polynomial, $\Omega(s)$ Omnès function

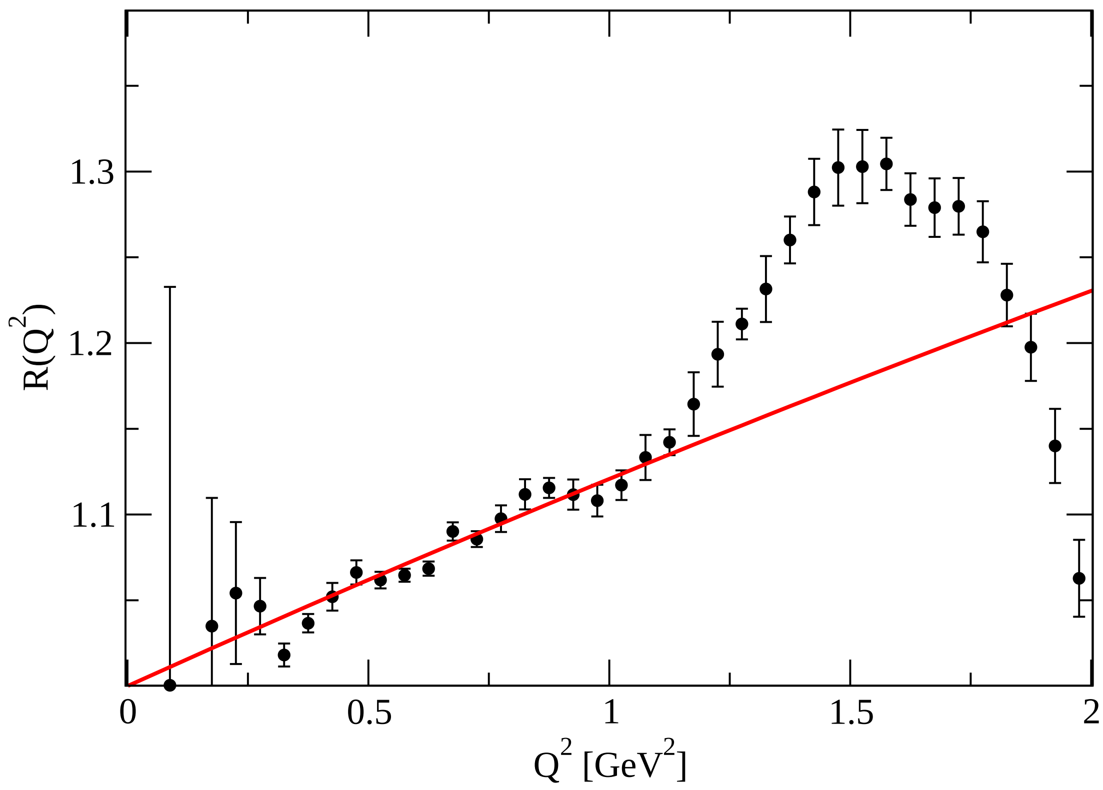
Omnès 1958

- today: high-accuracy $\pi\pi$ phase shifts available

Ananthanarayan et al. 2001, García-Martín et al. 2011

Pion vector form factor vs. Omnès representation

- divide $\tau^- \to \pi^-\pi^0\nu_\tau$ form factor by Omnès function:



Hanhart et al. 2013

$\longrightarrow$ linear below 1 GeV: $F_\pi^V(s) \approx (1 + 0.1\,\mathrm{GeV}^{-2}s)\Omega(s)$

$\longrightarrow$ above: inelastic resonances ρ', ρ''...

Final-state universality: $\eta,\ \eta' \to \pi^+\pi^-\gamma$

- $\eta^{(\prime)} \to \pi^+\pi^-\gamma$ driven by the chiral anomaly, $\pi^+\pi^-$ in P-wave $\longrightarrow$ final-state interactions the same as for vector form factor
- ansatz: $\mathcal{F}^{\eta^{(\prime)}}_{\pi\pi\gamma} = A \times P(t) \times \Omega(t)$, $P(t) = 1 + \alpha^{(\prime)} t$, $t = M^2_{\pi\pi}$

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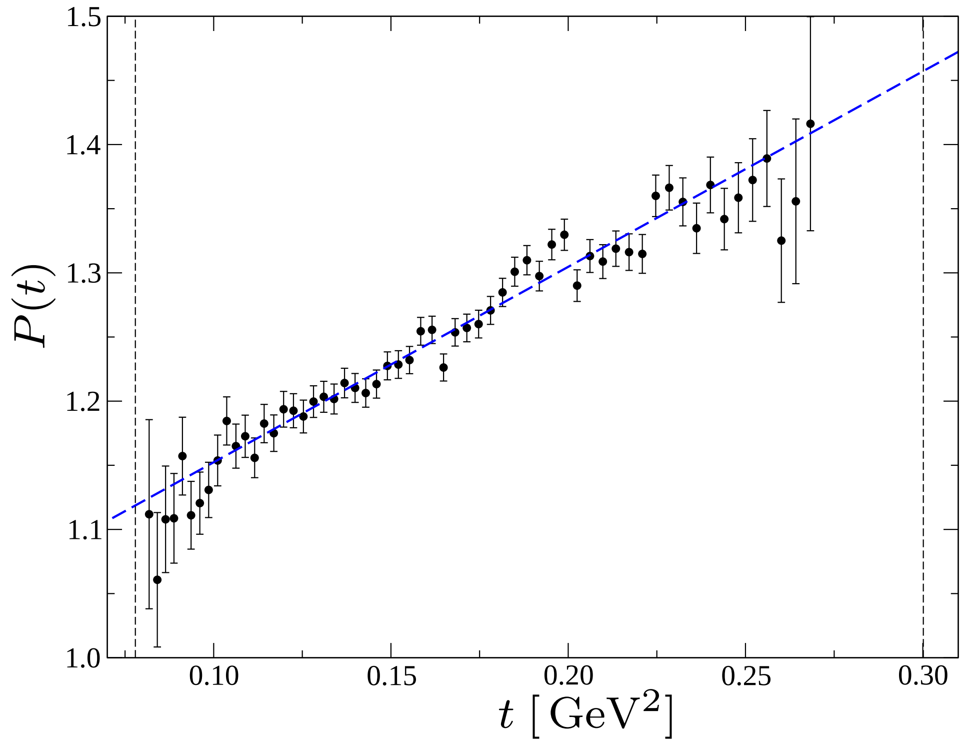
- divide data by pion form factor $\longrightarrow P(t)$ Stollenwerk et al. 2012

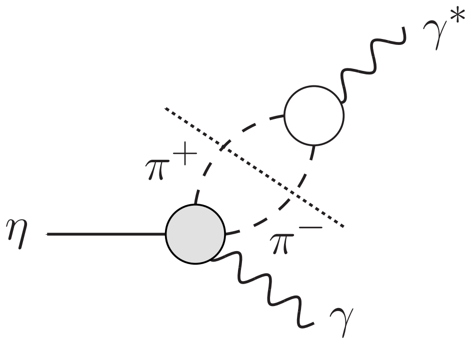


$\longrightarrow$ exp.: $\alpha_{\text{KLOE}} = (1.52 \pm 0.06)$ GeV^{-2} cf. KLOE 2013

Transition form factor $\eta \to \gamma^*\gamma$

Hanhart et al. 2013

$$\bar{F}_{\eta\gamma^*\gamma}(q^2,0) = 1 + \frac{\kappa_\eta q^2}{96\pi^2 F_\pi^2} \int_{4M_\pi^2}^{\infty} ds \sigma(s)^3 P(s) \frac{|F_\pi^V(s)|^2}{s-q^2}$$
$$+ \Delta F_{\eta\gamma^*\gamma}^{I=0}(q^2,0) \; [\longrightarrow \text{VMD}]$$



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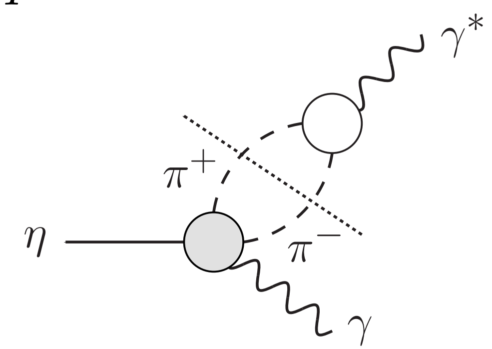
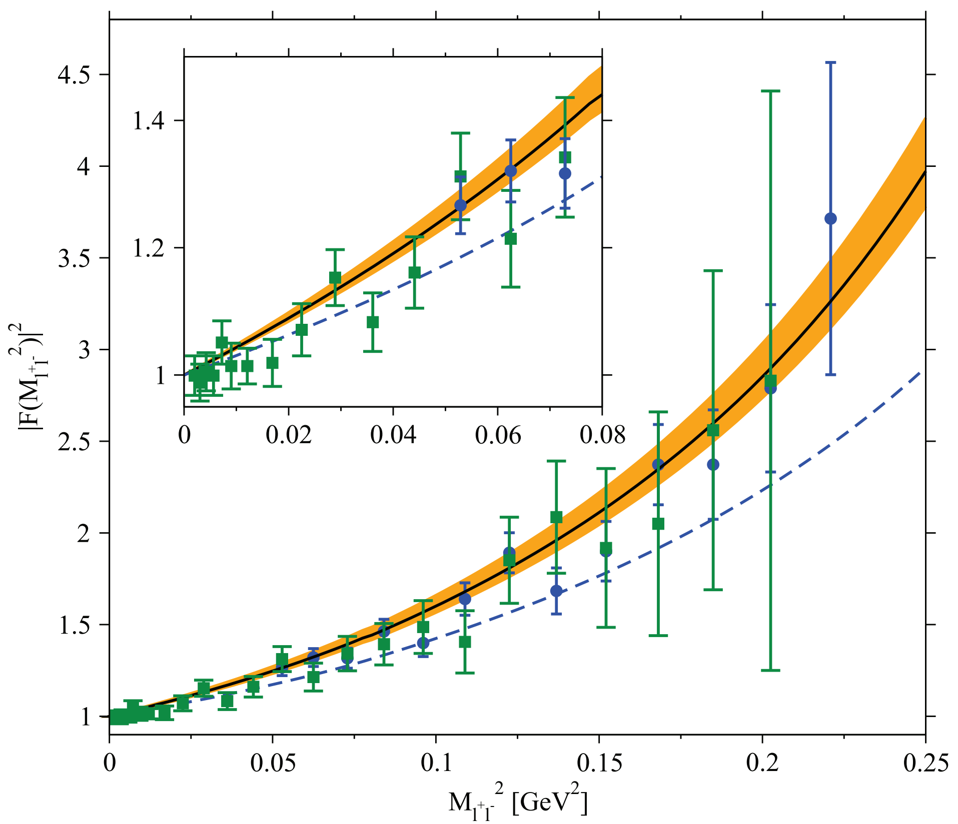




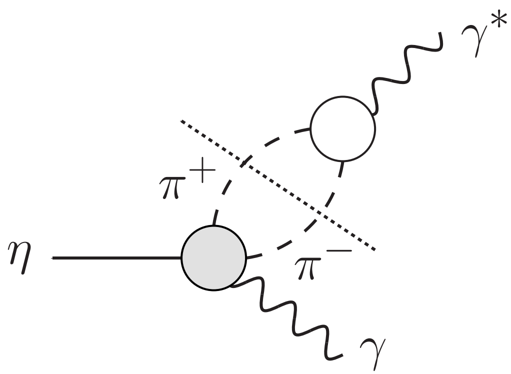
$\longrightarrow$ huge statistical advantage of using hadronic input for $\eta \to \pi^+\pi^-\gamma$ over direct measurement of $\eta \to e^+e^-\gamma$ (rate suppressed by α_{QED}^2)

figure courtesy of C. Hanhart

data: NA60 2011, A2 2014

Anomalous decay $\eta \to \pi^+\pi^-\gamma$

- $\alpha_{\text{KLOE}} = (1.52 \pm 0.06)$ GeV^{-2} large
  $\longrightarrow$ implausible to explain through ρ', ρ''...
- for large t, expect $P(t) \to$ const. rather
- $\eta \to \gamma^*\gamma$ transition form factor:
  $\longrightarrow$ dispersion integral covers
  larger energy range



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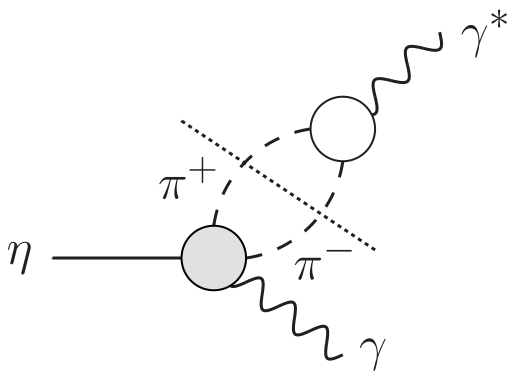


Intriguing observation:

- naive continuation of $\mathcal{F}^{\eta}_{\pi\pi\gamma} = A(1+\alpha t)\Omega(t)$ has zero
  at $t = -1/\alpha \approx -0.66\,\mathrm{GeV}^2$
  $\longrightarrow$ test this in crossed process $\gamma\pi^- \to \pi^-\eta$
  $\longrightarrow$ "left-hand cuts" in $\pi\eta$ system?

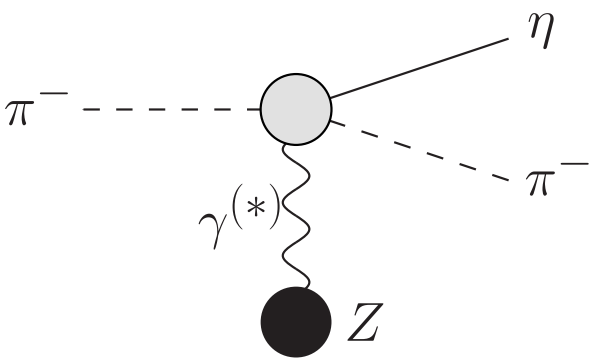
BK, Plenter 2015

Primakoff reaction $\gamma\pi \to \pi\eta$

- can be measured in Primakoff reaction COMPASS
- $\pi\eta$ S-wave forbidden

  P-wave exotic: $J^{PC} = 1^{-+}$

  D-wave $a_2(1320)$ first resonance



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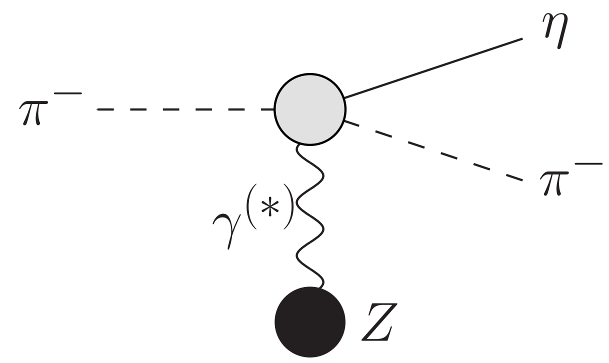

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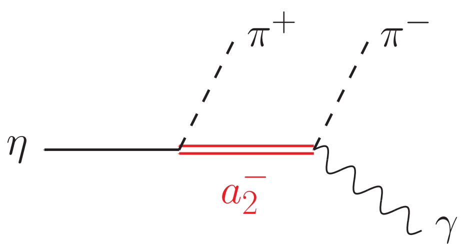
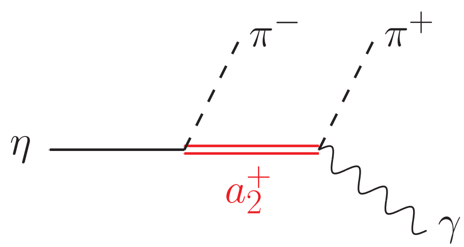
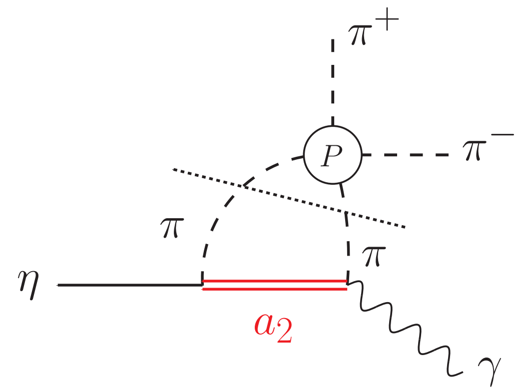
- include a_2 as left-hand cut in decay
  couplings fixed from $a_2 \to \pi\eta,\ \pi\gamma$





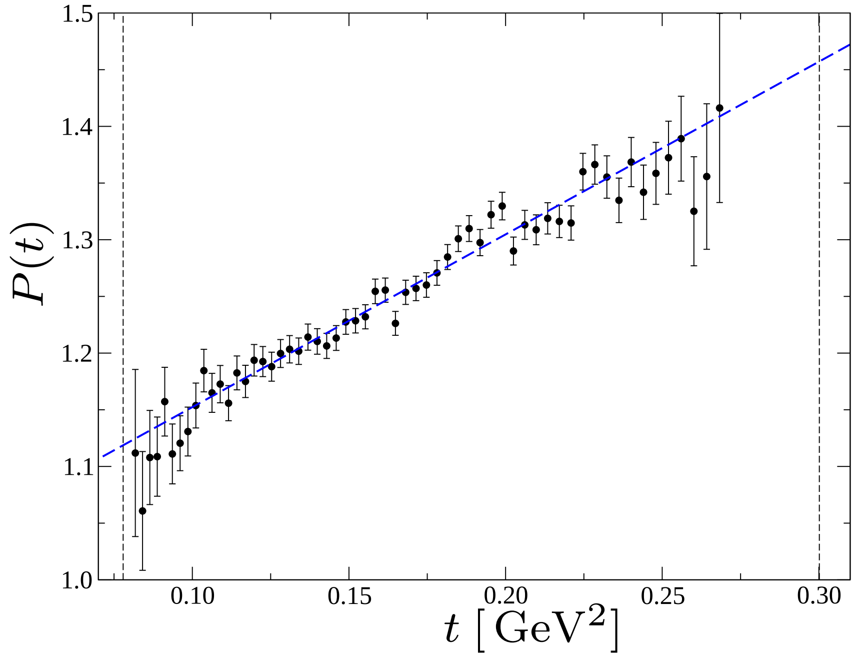


  ▷ compatible with decay data?

  ▷ predictions for $\gamma\pi \to \pi\eta$ cross sections and asymmetries [$\longrightarrow$ spares]

BK, Plenter 2015

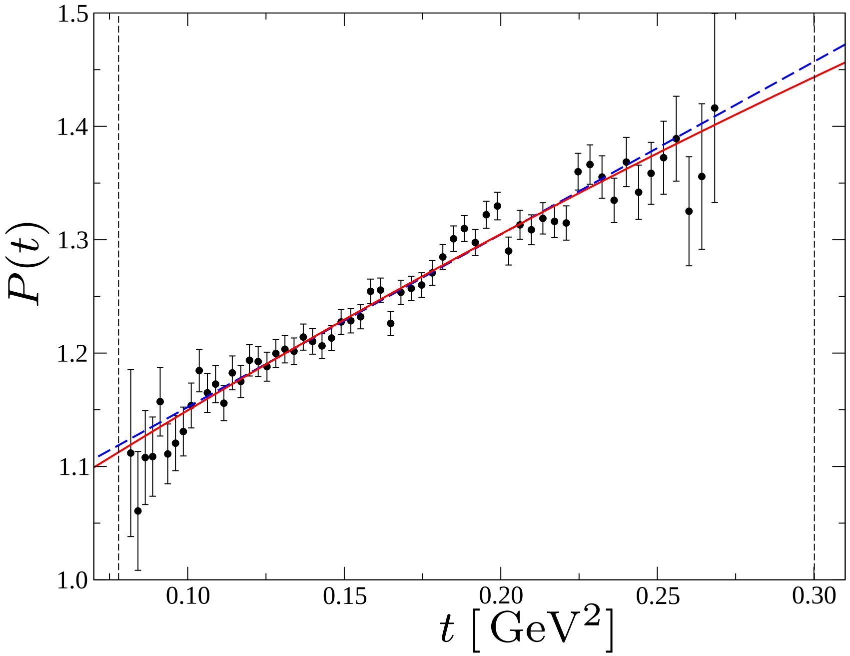
$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2



KLOE 2013

$\alpha = 1.52 \pm 0.06$, $\chi^2/$ndof $= 0.94$

$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2

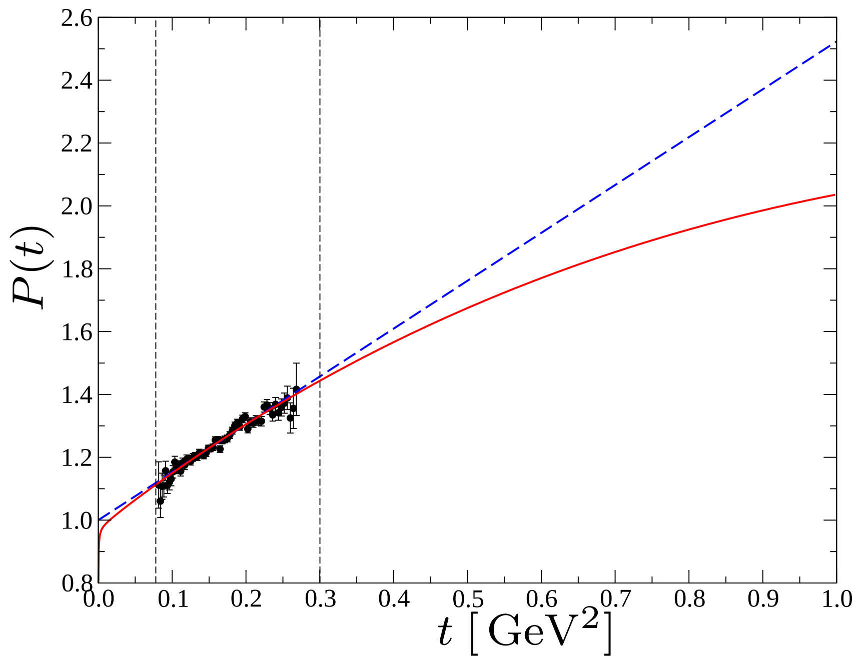


KLOE 2013

$\alpha = 1.52 \pm 0.06$, χ^2/ndof $= 0.94$

$\longrightarrow \alpha = 1.42 \pm 0.06$, χ^2/ndof $= 0.90$

$\eta,\, \eta' \to \pi^+\pi^-\gamma$ with a_2



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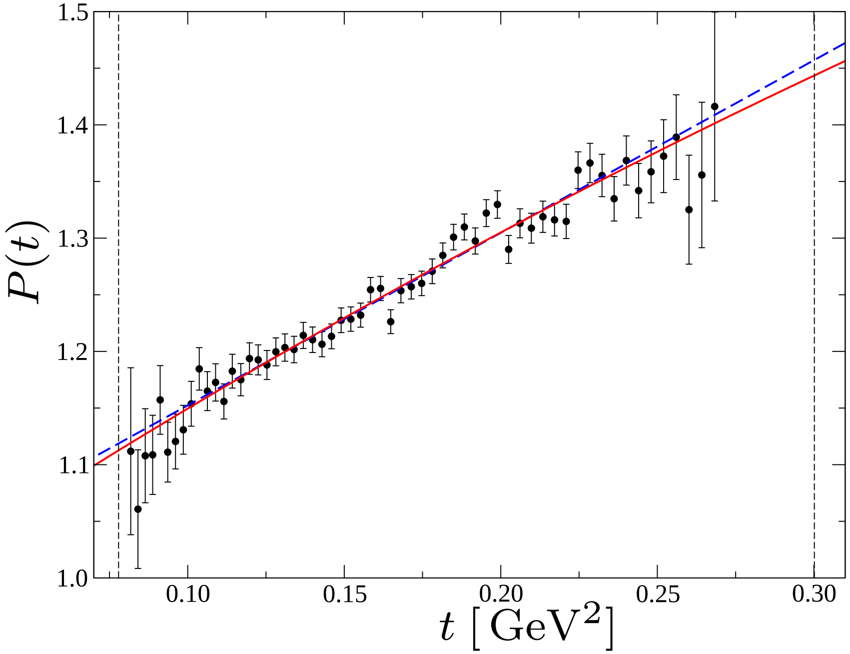
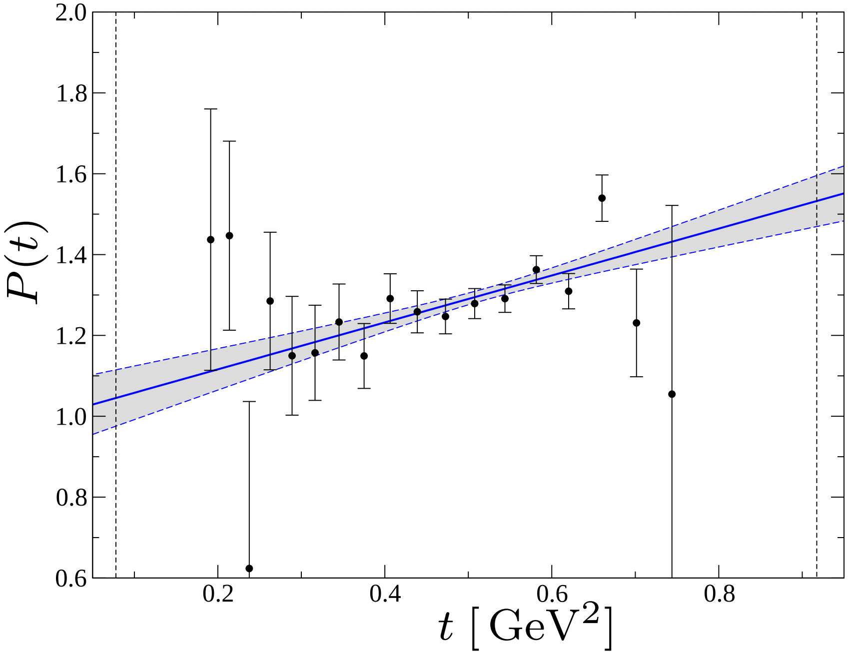
$\alpha = 1.52 \pm 0.06$, χ^2/ndof $= 0.94$

$\longrightarrow$ $\alpha = 1.42 \pm 0.06$, χ^2/ndof $= 0.90$

- equally good—why care? sum rule for $\eta \to \gamma^*\gamma$ transition form factor slope reduced by $7 - 8\%$ cf. Hanhart et al. 2013

$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2





KLOE 2013

$\alpha = 1.52 \pm 0.06$, χ^2/ndof $= 0.94$

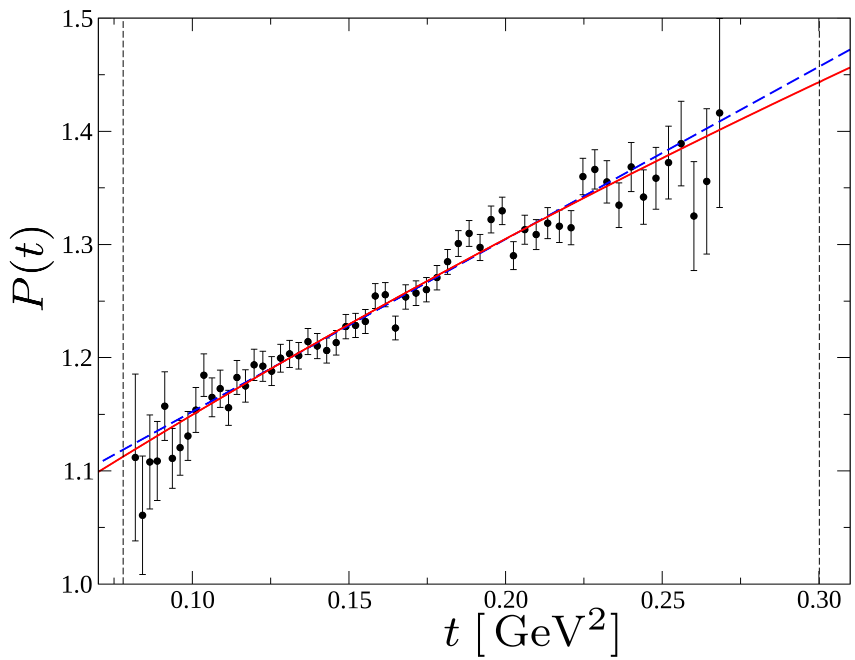
$\longrightarrow \alpha = 1.42 \pm 0.06$, χ^2/ndof $= 0.90$

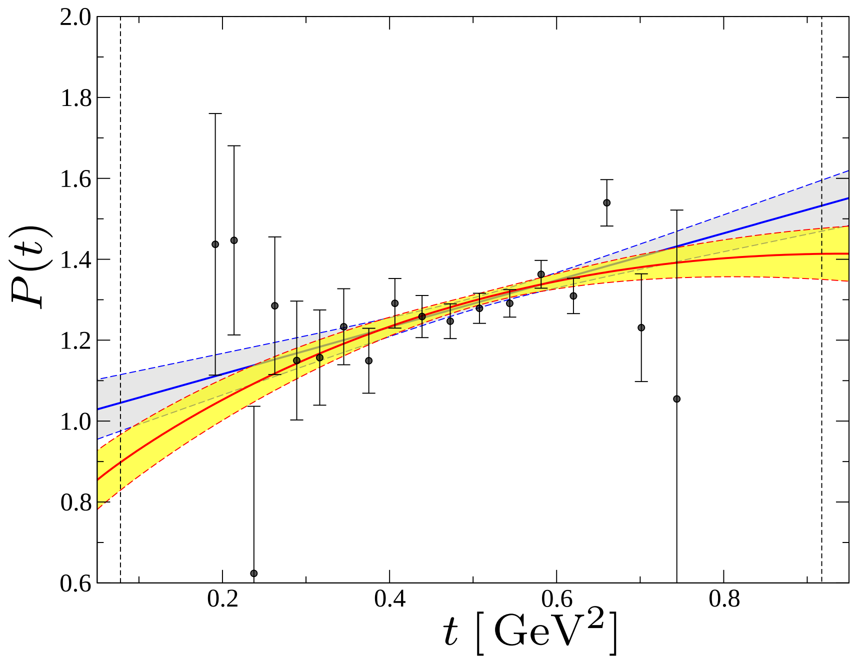
Crystal Barrel 1997

$\alpha' = 0.6 \pm 0.2$, χ^2/ndof $= 1.2$

- equally good—why care? sum rule for $\eta \to \gamma^*\gamma$ transition form factor slope reduced by $7-8\%$ cf. Hanhart et al. 2013

$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2





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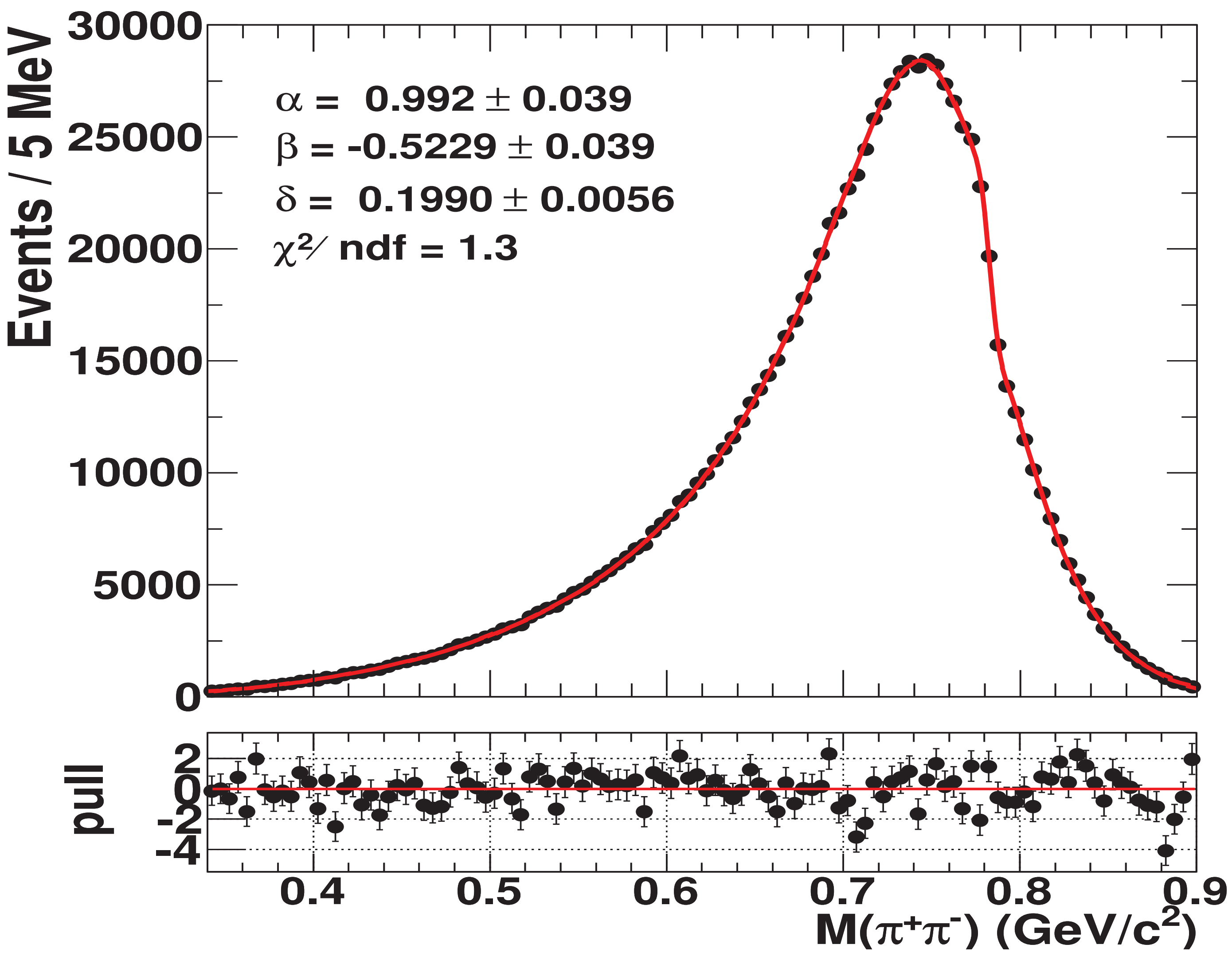
$\alpha' = 0.6 \pm 0.2$, χ^2/ndof $= 1.2$

$\longrightarrow \alpha' = 1.4 \pm 0.4$, χ^2/ndof $= 1.4$

- equally good—why care? sum rule for $\eta \to \gamma^*\gamma$ transition form factor slope reduced by $7-8\%$ cf. Hanhart et al. 2013
- $\alpha \approx \alpha'$ (large-N_c) better fulfilled including a_2 BK, Plenter 2015

New data on $\eta' \to \pi^+\pi^-\gamma$



BESIII preliminary, Fang 2015

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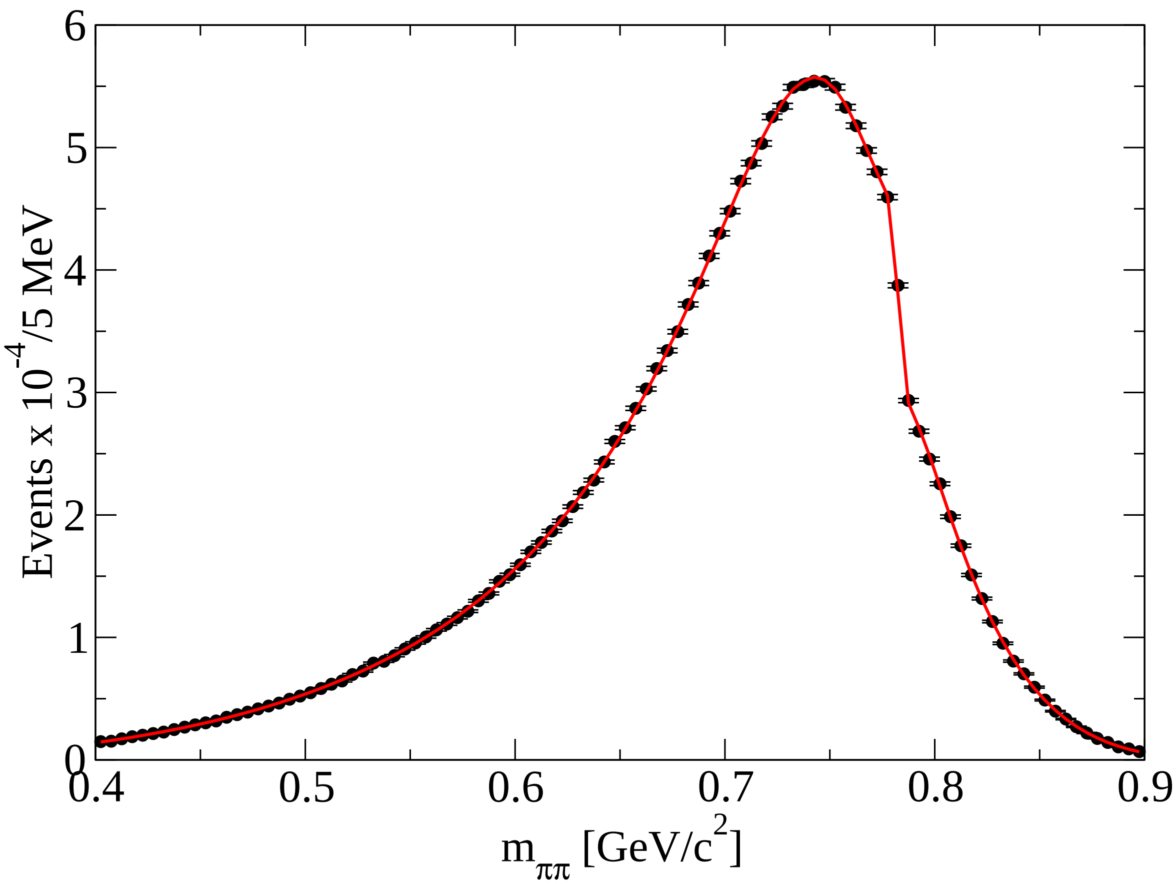


fit to pseudodata after BESIII preliminary

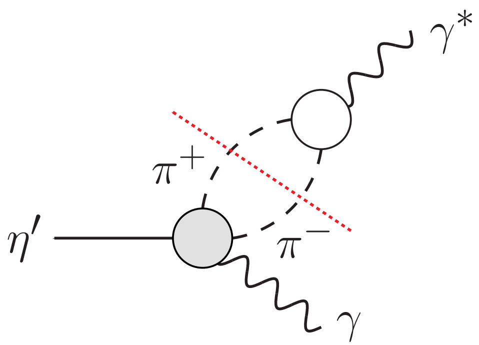
- fit form $$\left[A(1+\alpha t+\beta t^2)+\frac{\kappa}{m_\omega^2-t-im_\omega\Gamma_\omega}\right]\times\Omega(t)$$

  $\longrightarrow$ curvature $\propto \beta t^2$ essential (smaller than a_2 prediction)

  $\longrightarrow$ even ρ–ω mixing clearly visible

Prediction for η' transition form factor

- isovector: combine high-precision data on $\eta' \to \pi^+\pi^-\gamma$ and $e^+e^- \to \pi^+\pi^-$
- isoscalar: VMD, couplings fixed from $\eta' \to \omega\gamma$ and $\phi \to \eta'\gamma$



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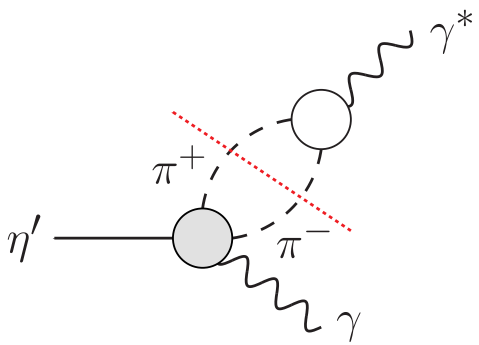
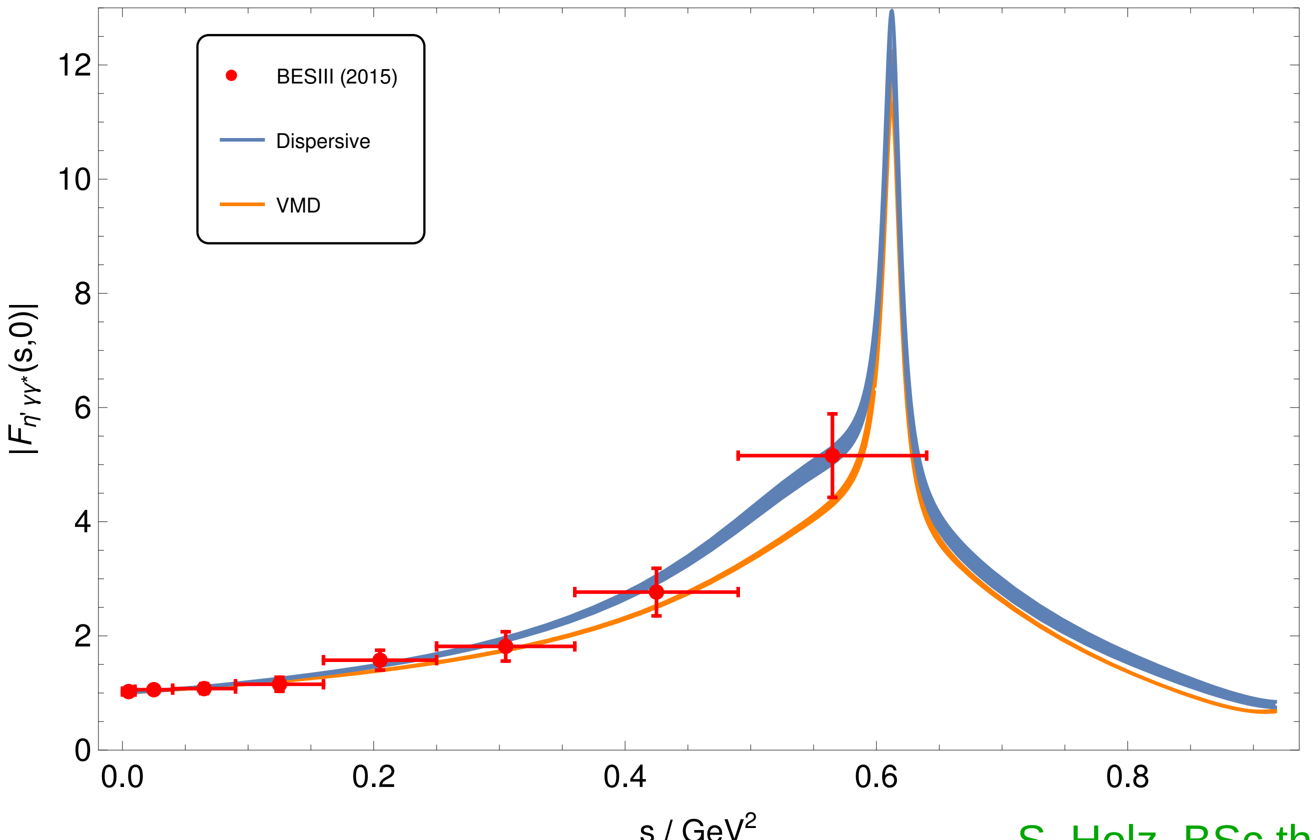
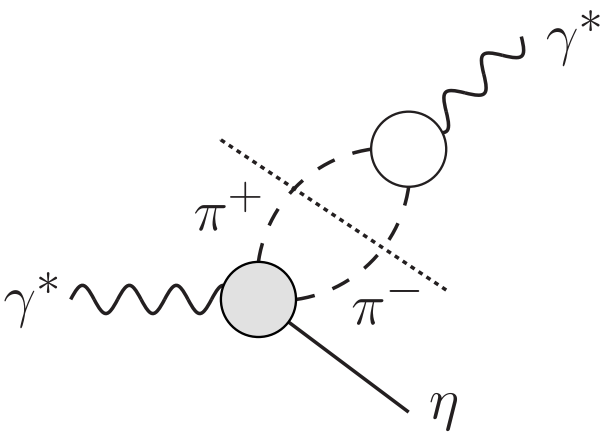




S. Holz, BSc thesis 2016

How to go *doubly* virtual? — $e^+e^- \to \eta\pi^+\pi^-$

- idea (again): beat α^2_{QED} suppression of $e^+e^- \to \eta e^+e^-$ by measuring $e^+e^- \to \eta\pi^+\pi^-$ instead



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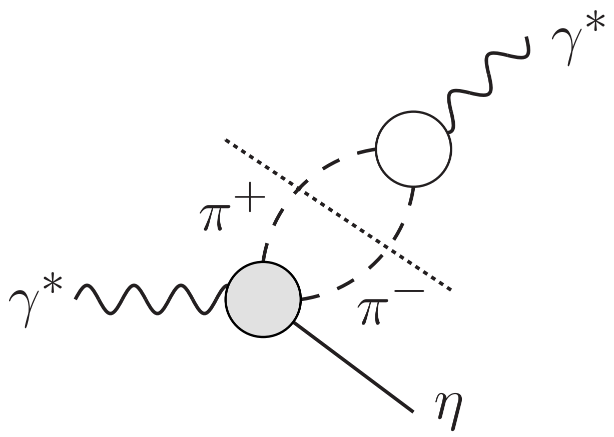


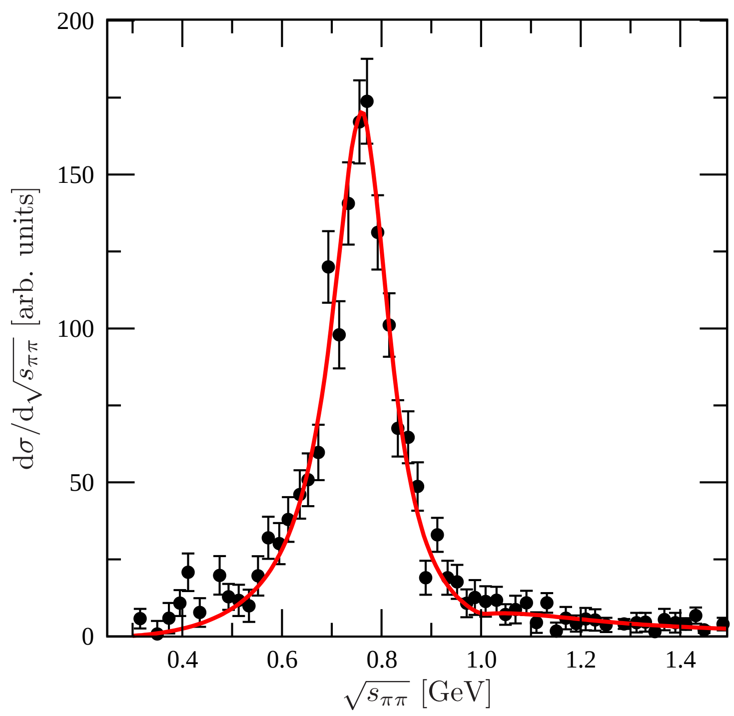
- test factorisation hypothesis in $e^+e^- \to \eta\pi^+\pi^-$:

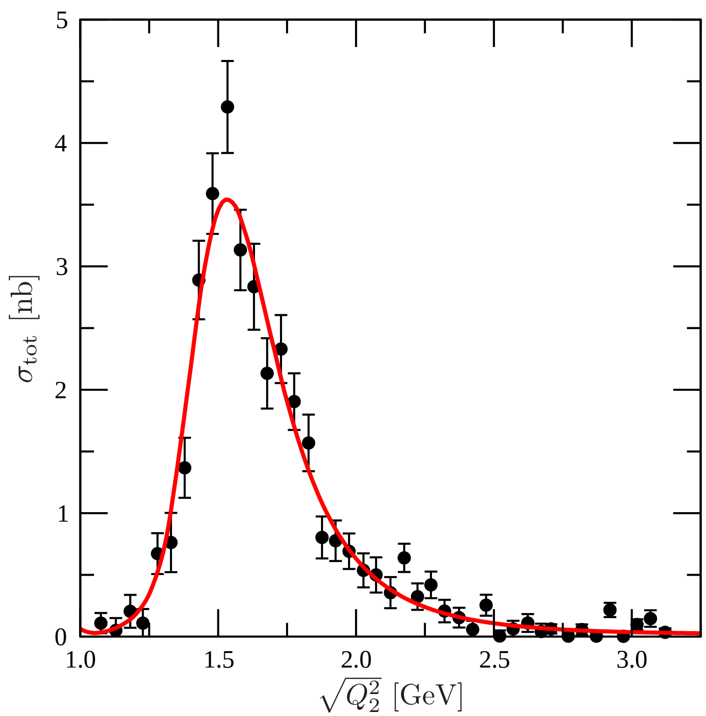
$$F_{\eta\pi\pi\gamma^*}(s_{\pi\pi}, Q_2^2) \stackrel{!?}{=} F_{\eta\pi\pi\gamma}(s_{\pi\pi}) \times F_{\eta\gamma\gamma^*}(Q_2^2)$$

  - ▷ allow same form for $F_{\eta\pi\pi\gamma}(s_{\pi\pi})$ as in $\eta \to \pi^+\pi^-\gamma$
  - ▷ fit subtractions to $\pi^+\pi^-$ distribution in $e^+e^- \to \eta\pi^+\pi^-$
    $\longrightarrow$ are they compatible to the ones in $\eta \to \pi^+\pi^-\gamma$?
  - ▷ parametrise $F_{\eta\gamma\gamma^*}(Q_2^2)$ by sum of Breit–Wigners (ρ, ρ')

Xiao et al. (preliminary)

How to go *doubly* virtual? — $e^+e^- \to \eta\pi^+\pi^-$





$$\frac{d\sigma}{d\sqrt{s_{\pi\pi}}}$$

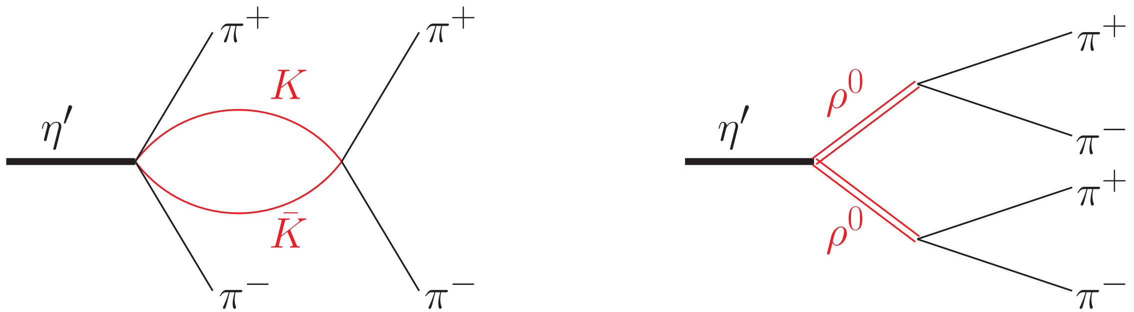
$\sigma_{\mathsf{tot}}(Q_2^2)$

Xiao et al. (preliminary); data: BaBar 2007

- $d\sigma/d\sqrt{s_{\pi\pi}}$ integrated over $1\,\text{GeV} \leq \sqrt{Q_2^2} \leq 4.5\,\text{GeV}$
- factorisation seems to work only if a_2 contribution retained
- more differential/binned data highly desirable!

How to go *doubly* virtual? — $\eta' \to \pi^+\pi^-\pi^+\pi^-$

- prediction of $\eta' \to 4\pi$ branching ratios based on ChPT + VMD:



$\longrightarrow$ $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (10 \pm 3) \times 10^{-5}$ Guo, BK, Wirzba 2012

exp: $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (8.5 \pm 0.7 \pm 0.6) \times 10^{-5}$ BESIII 2014

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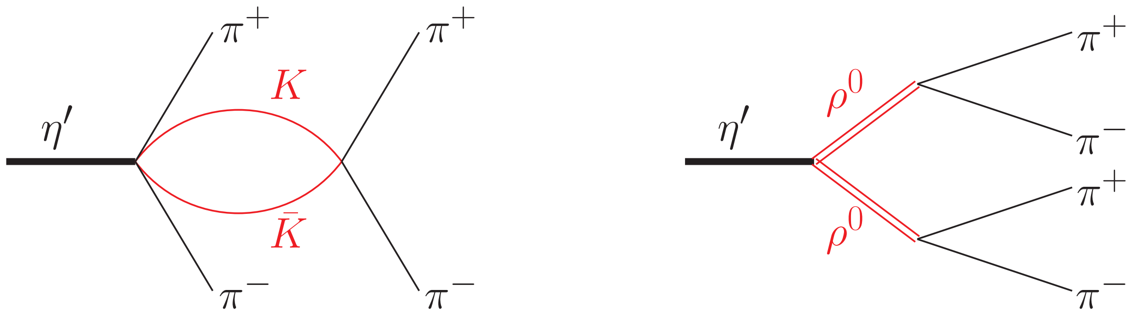

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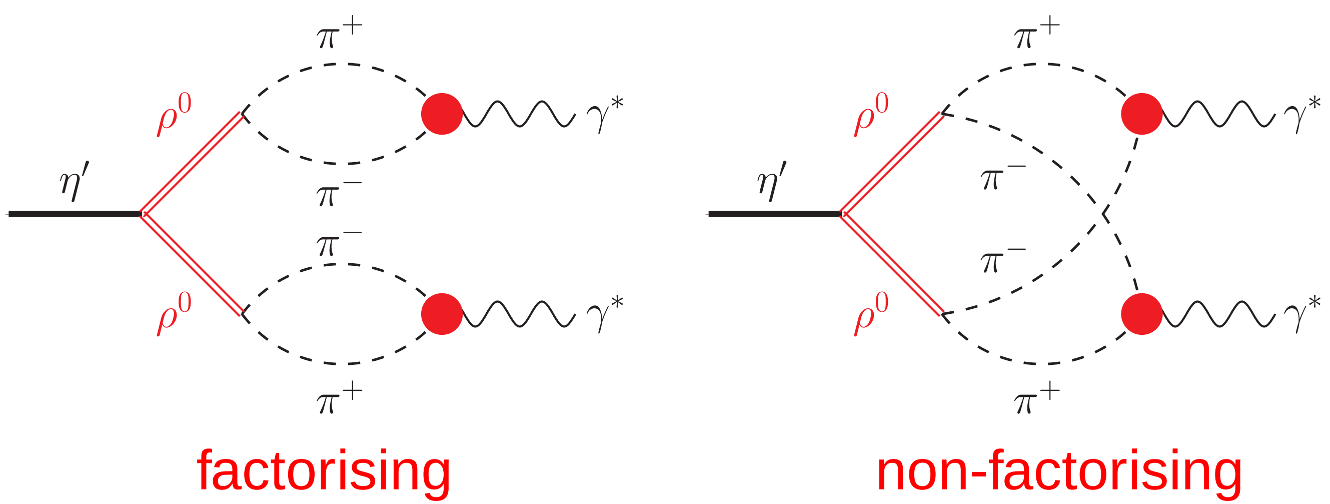
- start analysis of *doubly* virtual η' transition form factor from here?



$\longrightarrow$ more differential info on $\eta' \to \pi^+\pi^-\pi^+\pi^-$ highly desirable!

Summary / Outlook

Dispersive analyses of $\eta(')$ transition form factors:

- high-precision data on $\eta \to \pi^+\pi^-\gamma$ KLOE and $\eta' \to \pi^+\pi^-\gamma$ BESIII allow for high-precision dispersive predictions of $\eta(') \to \gamma\gamma^*$
- not discussed here: dispersive continuation of transition form factors to spacelike virtualities see S. Leupold for π^0

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Dispersive analyses of $\eta(')$ transition form factors:

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- not discussed here: dispersive continuation of transition form factors to spacelike virtualities see S. Leupold for π^0

Further useful experimental input (mainly for doubly virtual):

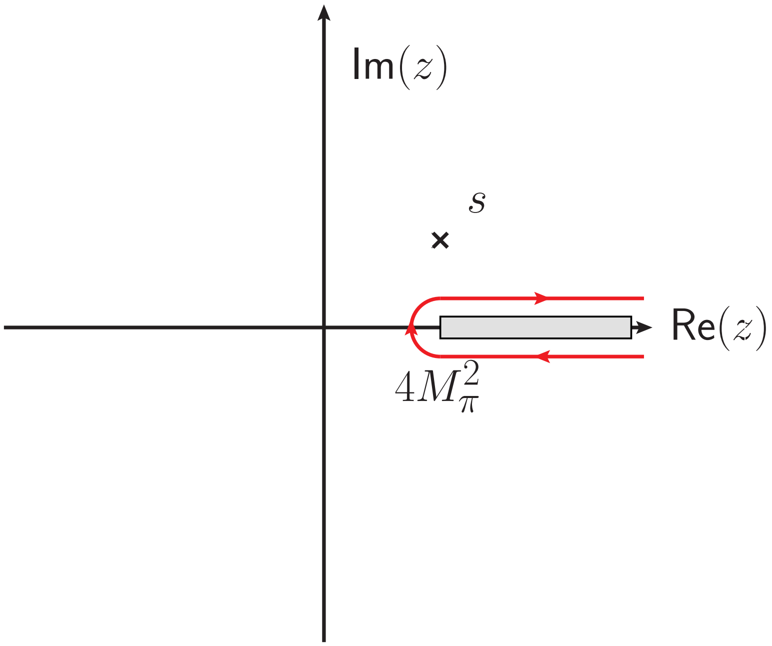
- Primakoff reaction $\gamma\pi \to \pi\eta$ COMPASS
- $e^+e^- \to \eta\pi^+\pi^-$ differential data C.-W. Xiao et al.
- given $\eta' \to \pi^+\pi^-\gamma$ — can you do $\eta' \to \pi^+\pi^-e^+e^-$ with precision?
- more detailed data on $\eta' \to \pi^+\pi^-\pi^+\pi^-$? work in progress

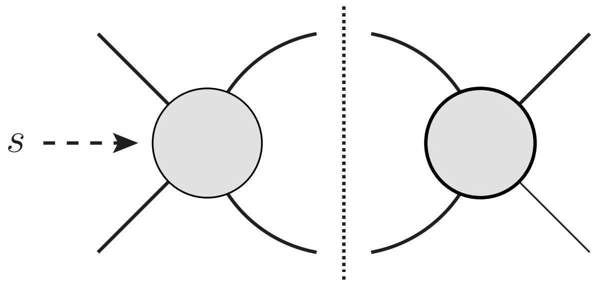
$\longrightarrow$ determine $(g-2)_\mu$ contributions with controlled uncertainty

Spares

What are left-hand cuts?

Example: pion–pion scattering





- right-hand cut due to unitarity: $s \geq 4M_\pi^2$

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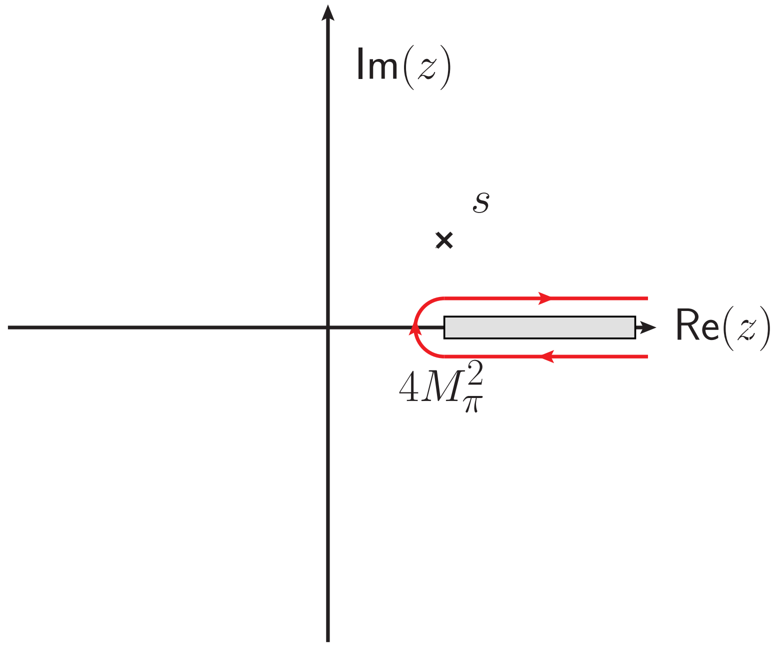

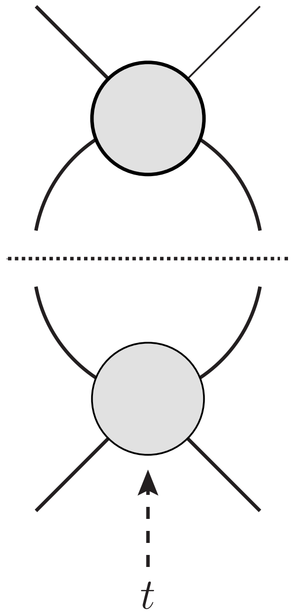

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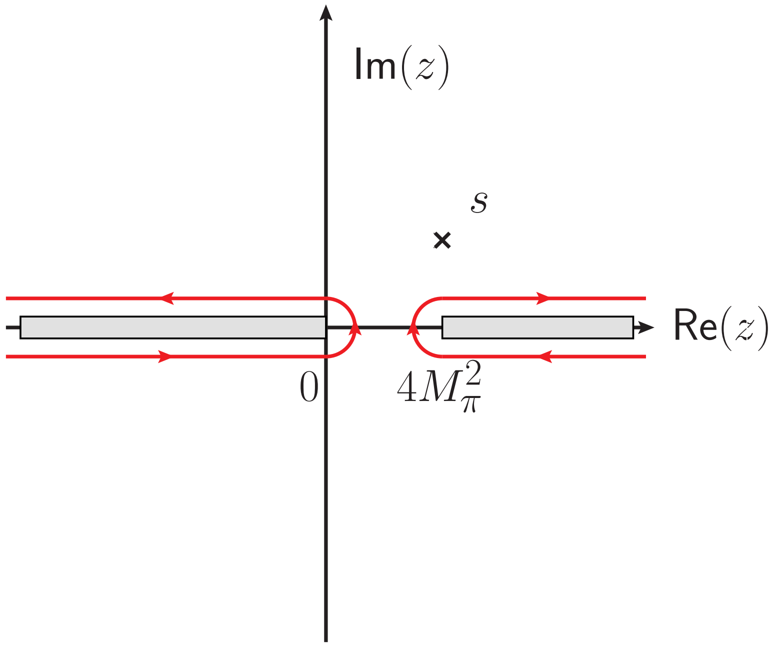
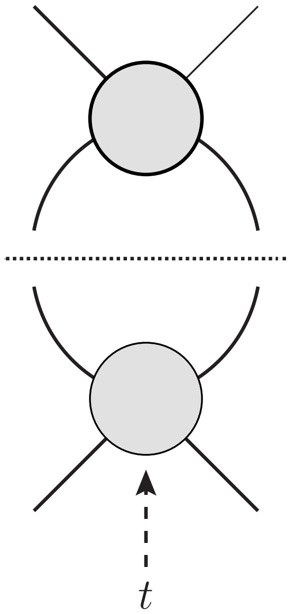


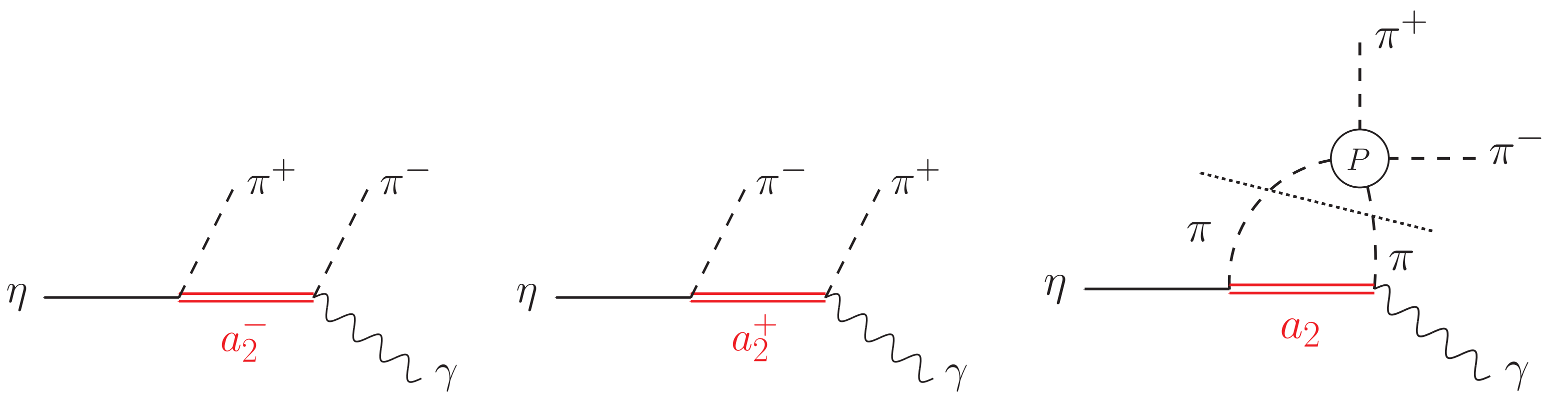


- right-hand cut due to unitarity: $s \geq 4M_\pi^2$
- crossing symmetry: cuts also for $t,\, u \geq 4M_\pi^2$
- partial-wave projection: $T(s,t) = 32\pi \sum_i T_i(s) P_i(\cos\theta)$

$$t(s,\cos\theta) = \tfrac{1-\cos\theta}{2}(4M_\pi^2 - s)$$

$\longrightarrow$ cut for $t \geq 4M_\pi^2$ becomes cut for $s \leq 0$ in partial wave

Formalism including left-hand cuts



- a_2 + rescattering essential to preserve Watson's theorem
- formally:

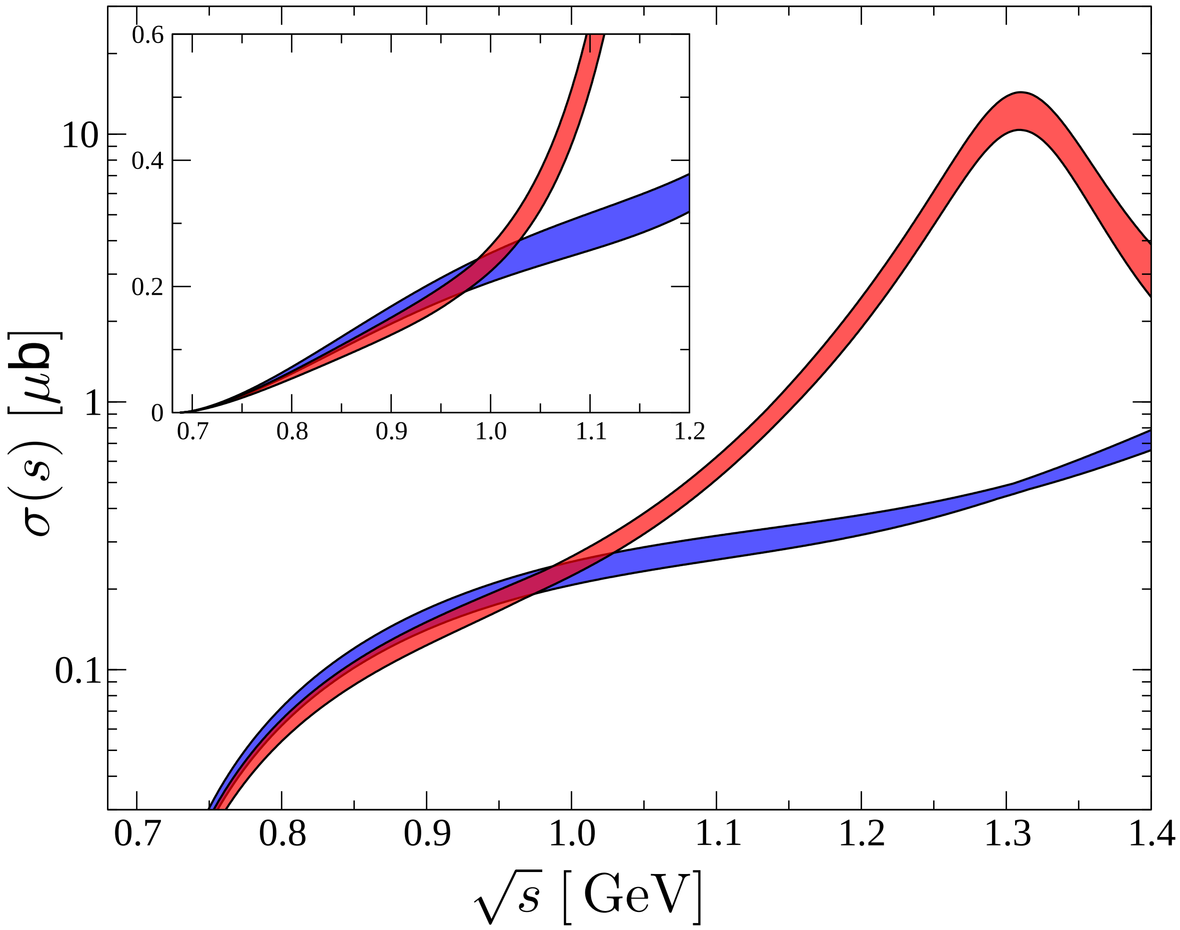
$$\mathcal{F}^{\eta}_{\pi\pi\gamma}(s,t,u) = \mathcal{F}(t) + \mathcal{G}_{a_2}(s,t,u) + \mathcal{G}_{a_2}(u,t,s)$$

$$\mathcal{F}(t) = \Omega(t)\left\{ A(1+\alpha t) + \frac{t^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{dx}{x^2} \frac{\sin\delta(x)\hat{\mathcal{G}}(x)}{|\Omega(x)|(x-t)} \right\}$$

$\hat{\mathcal{G}}$: t-channel P-wave projection of a_2 exchange graphs

- re-fit subtraction constants A, α

Total cross section $\gamma\pi \to \pi\eta$

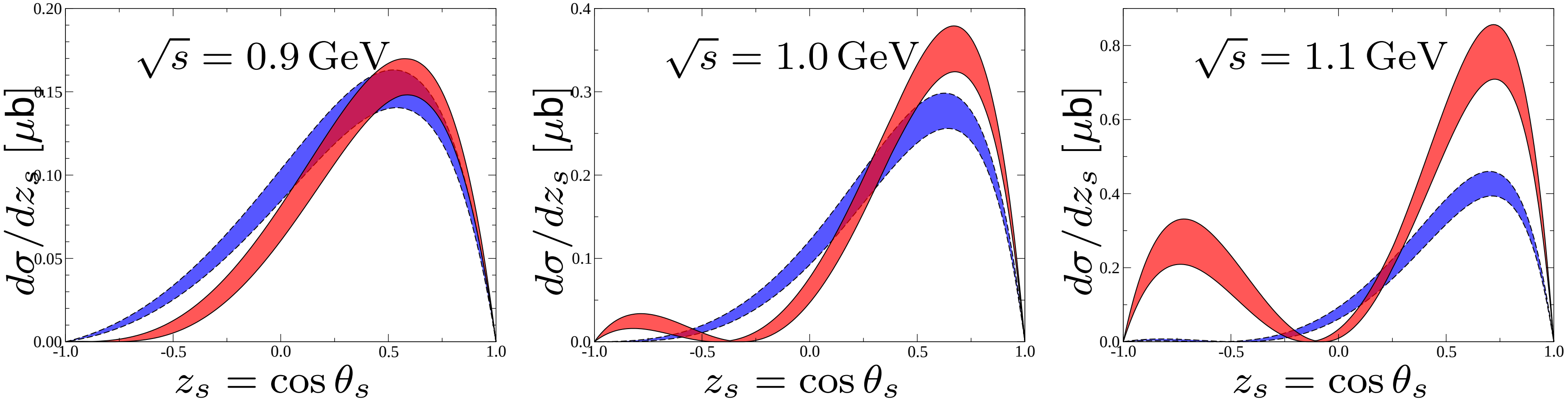


blue: t-channel dynamics / "ρ" only red: full amplitude

- t-channel dynamics dominate below $\sqrt{s} \approx 1\,\mathrm{GeV}$
- uncertainty bands: $\Gamma(\eta \to \pi^+\pi^-\gamma)$, α, a_2 couplings BK, Plenter 2015

Differential cross sections $\gamma\pi \to \pi\eta$

- amplitude zero visible in differential cross sections:



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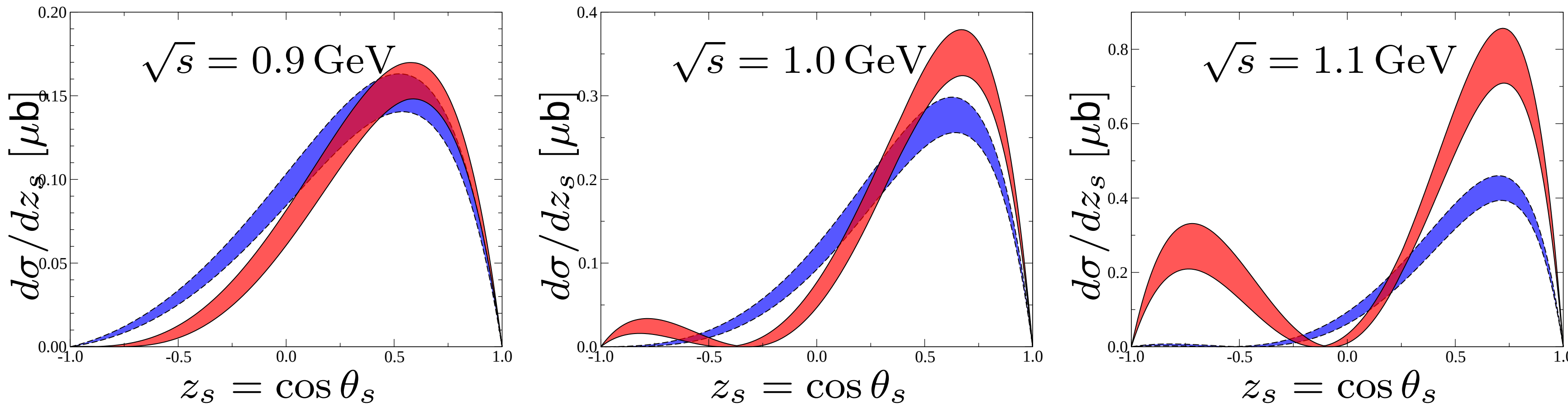
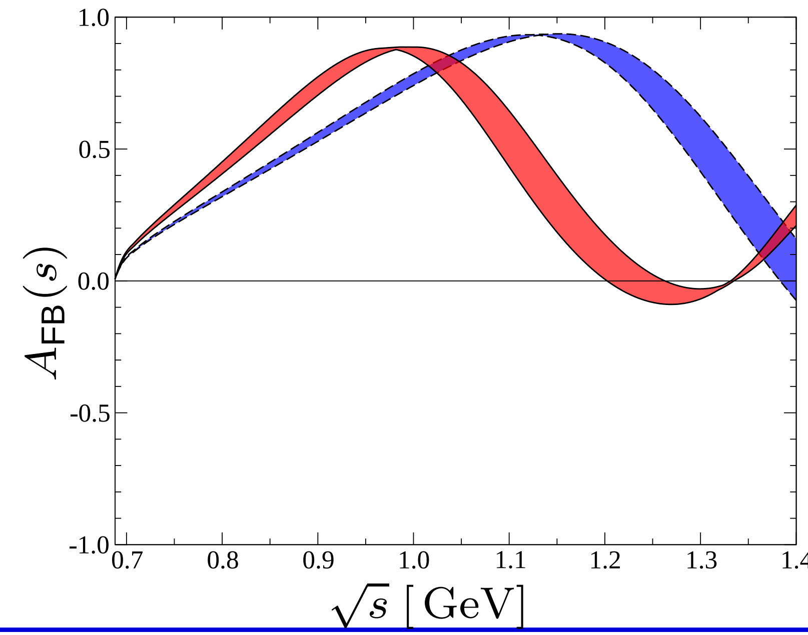


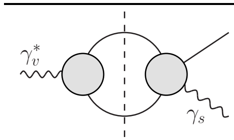
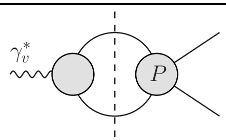
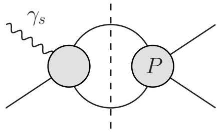
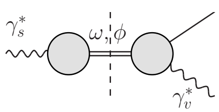
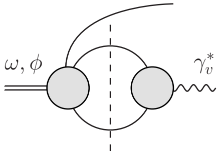
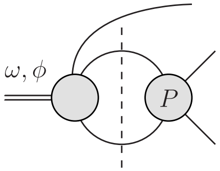
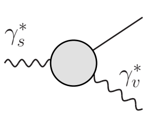
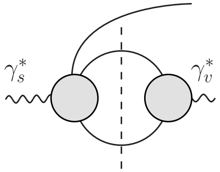
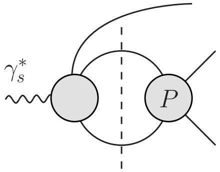
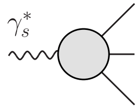
blue: t-channel dynamics / "ρ" only red: full amplitude

- strong P-D-wave interference
- can be expressed as forward-backward asymmetry

$$A_{\text{FB}} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma_{\text{total}}}$$



Summary: processes and unitarity relations for $\pi^0 \to \gamma^*\gamma^*$

process	unitarity relations	SC 1	SC 2
			$F_{\pi^0\gamma\gamma}$
		$F_{3\pi}$	$\sigma(\gamma\pi \to \pi\pi)$
			$\Gamma_{\pi^0\gamma}$
		$\Gamma_{3\pi}$	$\frac{d^2\Gamma}{ds\,dt}(\omega,\phi \to 3\pi)$
			$\sigma(e^+e^- \to \pi^0\gamma)$
		$\sigma(e^+e^- \to 3\pi)$	$\sigma(\gamma\pi \to \pi\pi)$ $\frac{d^2\Gamma}{ds\,dt}(\omega,\phi \to 3\pi)$
		$F_{3\pi}$	$\sigma(e^+e^- \to 3\pi)$

Colangelo, Hoferichter, BK, Procura, Stoffer 2014

$\gamma\pi \to \pi\pi$

$\omega \to 3\pi$, $\phi \to 3\pi$

$\gamma^* \to 3\pi$

common theme:
resum $\pi\pi$ rescattering