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Review on $e^+e^- \rightarrow \pi^+\pi^-\eta$

Experiment and theory

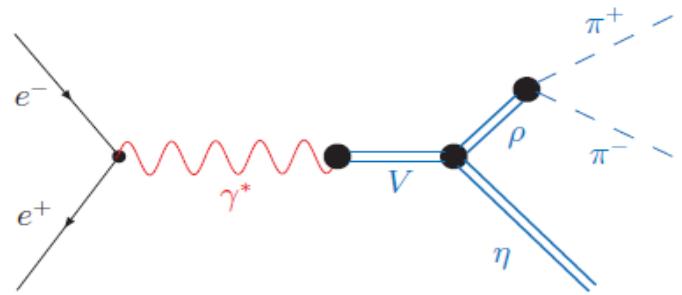
O. Shekhovtsova
INP Cracow / KIPT Kharkov

Frascati, 27.10.2016

Information from $e^+e^- \rightarrow \pi^+\pi^-\eta$ data

- measurements of ρ, ρ', ρ'' parameters
- Check of Conservation of Vector Current hypothesis (CVC)
- about 5% contribution to the total hadronic cross section for $\sqrt{s} = 1.5\text{GeV}$

Information from $e^+e^- \rightarrow \pi^+\pi^-\eta$ data



$V = \rho(770), \rho(1450), \rho(1750)$

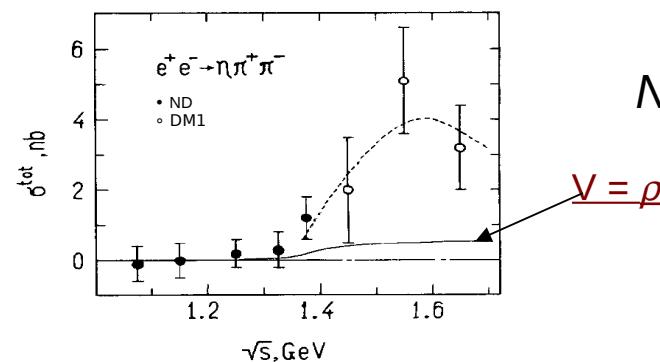
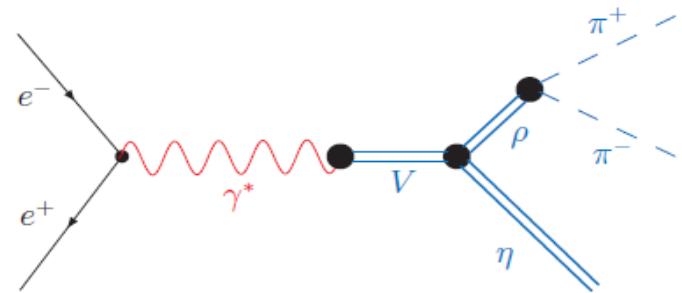
$$e^+e^- \rightarrow 4\pi : \quad e^+e^- \rightarrow \rho\rho; a_1\pi$$

Precise data on $e^+e^- \rightarrow \pi^+\pi^-\eta$ ρ' , ρ'' parameter measurements

Theoretical models

1. VMD with one resonance: $V = \rho$

N.N. Achasov and V.A. Karnakov, JETP Lett 39 (1984) 342
J.Layssac, F.Renard, Nuovo Cimento 6A (1971) 134



ND Collaboration, Phys. Lett. B 174 (1986) 115

$V = \rho$

Significant deviation for $2E > 1.35\text{GeV}$, an order below for $2E \sim 1.5\text{GeV}$

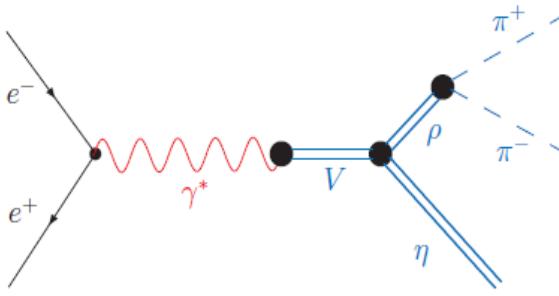
2. Inclusion of higher resonances

Old experiments DM1, DM2, DM2 two resonances $V = \rho(770), \rho(1600)$

Photoproduction data (Omega Photon Collaboration) $\gamma p \rightarrow \pi^+\pi^-\eta p$ shows $\rho(1300)$ Nucl. Phys. B242 (1984) 269

to reconcile the data $\rho(1600) \rightarrow \rho(1400) + \rho(1750)$ A. Donnachie and A. Clegg, Z. Phys. 34 (1987) 257

VMD with $V = \rho(770), \rho(1450), \rho(1750)$



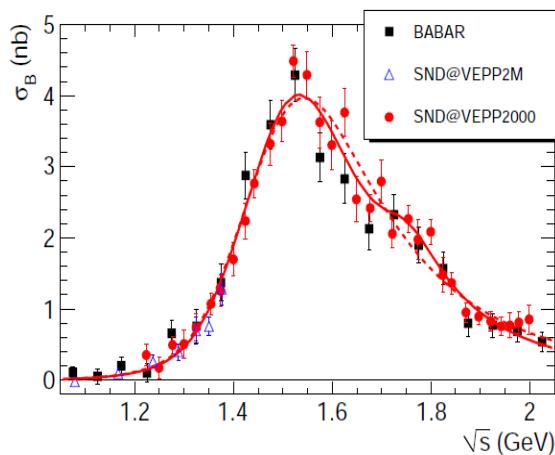
$$\frac{d\sigma}{dq^2}(s, q^2) = \frac{4\alpha^2}{3} \frac{1}{s\sqrt{s}} \frac{\sqrt{q^2}\Gamma_\rho(q^2)P_\eta^3(s, q^2)}{(q^2 - m_\rho^2)^2 + (\sqrt{q^2}\Gamma_\rho(q^2))^2} |F(s)|^2$$

$$P_\eta^2(s, q^2) = [(s - m_\eta^2 - q^2)^2 - 4m_\eta^2 q^2]/4s \quad \Gamma_\rho(q^2) = \Gamma_\rho(m_\rho^2) \frac{m_\rho^2}{q^2} \left(\frac{p_\pi^2(q^2)}{p_\pi^2(m_\rho^2)} \right)^{\frac{3}{2}}$$

with hadronic form factor $F(s)$

$$F(s) = \sum_V \frac{m_V^2}{g_{V\gamma}} \frac{g_{V\rho\eta}}{s - m_V^2 + i\sqrt{s}\Gamma_V(s)}, \quad V = \rho(770), \rho(1450), \rho(1700).$$

PDG mass and width values, as well as the $\rho(770)$ vertex constants common fit for SND + BaBar data



[V.Aulchenko et al \(SND\) Phys.Rev. D91 \(2015\) 052013](#)

— with $\rho(1700)$ $\chi^2/\nu = 37/31$

···· no $\rho(1700)$ $\chi^2/\nu = 42.6/32$

$$g_{V\rho\eta}/g_{V\gamma} = g_V e^{i\phi_V}$$

$$\begin{aligned} g_{\rho(1450)} &= 0.48^{+0.05}_{-0.06} \text{ GeV}^{-1} \\ g_{\rho(1700)} &= 0.02^{+0.03}_{-0.01} \text{ GeV}^{-1} \end{aligned}$$

No conclusion about necessity $\rho(1700)$ for the current statistics

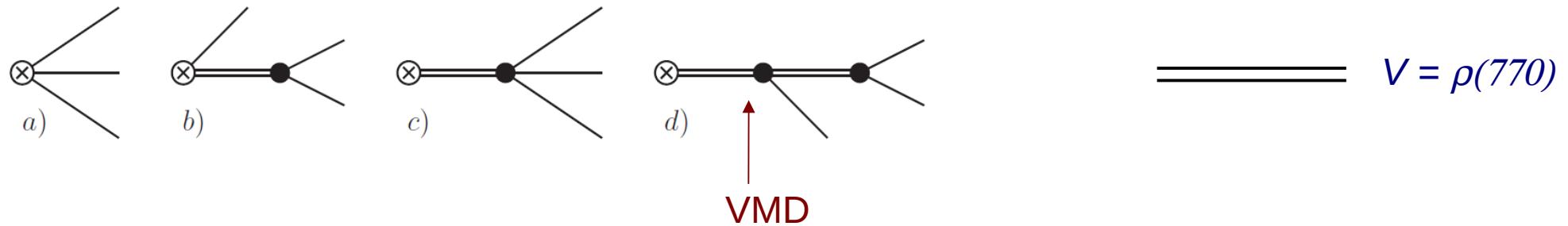
CMD-2 results for 5 pion production Phys.Lett. B489 (2000) 125

More precise data for 1.4-2 GeV

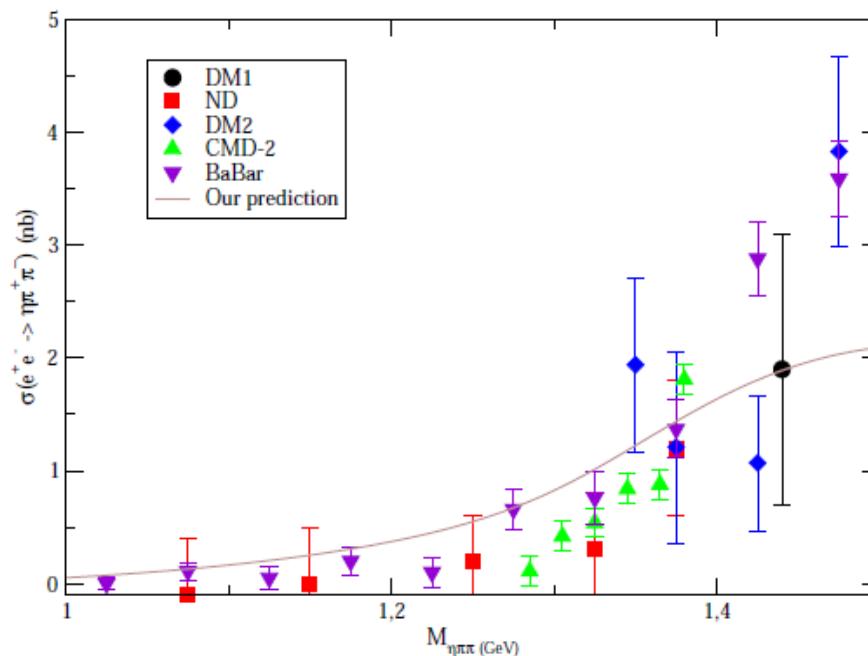
	$\rho(1450)$	$\rho(1700)$
$\sigma(m^2), \text{nb}$	2.53 ± 0.58	0.29 ± 0.24
$m, \text{ Mev}$	1421 ± 15	—
$\Gamma, \text{ MeV}$	211 ± 31	—
$\phi, \text{ rad}$	0	1.9 ± 0.5
$B_{e^+e^-} B_{fin} \cdot 10^{-7}$	3.5 ± 0.8	0.6 ± 0.5

3. Model on the base of RChL approach, *D.G. Dumm and P. Roig, Phys.Rev. D86 (2012) 076009*

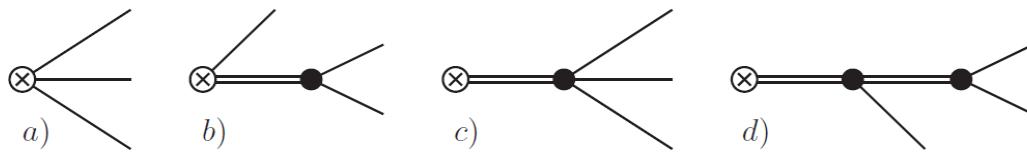
Described process $\tau^- \rightarrow \pi^0 \pi^- \eta^{(\prime)} \nu$, applying CVC estimated $e^+ e^- \rightarrow \pi^+ \pi^- \eta$



15 unknown Lagrangian constants, high energy restrictions on the hadronic form factors etc leave 2 unknown constants



4. Model on the base of RChL approach, [L.Y. Dai, J. Portoles, O.S., Phys.Rev. D88 \(2013\) 056001](#)



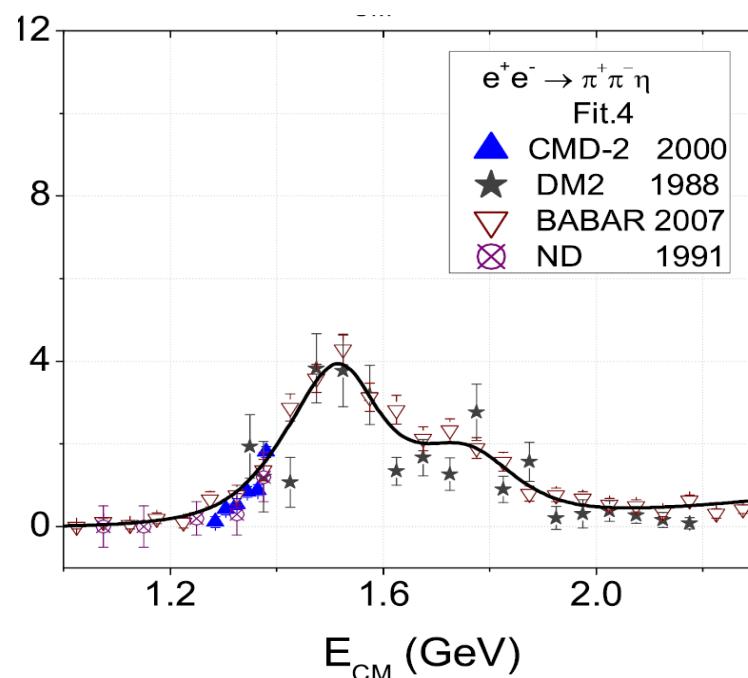
$$V = \rho(770), \rho(1450), \rho(1750)$$

$$\frac{1}{M_V^2 - x} \rightarrow \frac{1}{M_V^2 - x} + \frac{\beta'_\pi}{M_{V'}^2 - x} + \frac{\beta''_\pi}{M_{V''}^2 - x}$$

* Tiny effects: isospin symmetry breaking (mixing ρ - ω - ϕ)

* High energy restrictions only for 2π , 3π , $\eta\pi\pi$ form factors → about 20 constants (including the resonance masses)

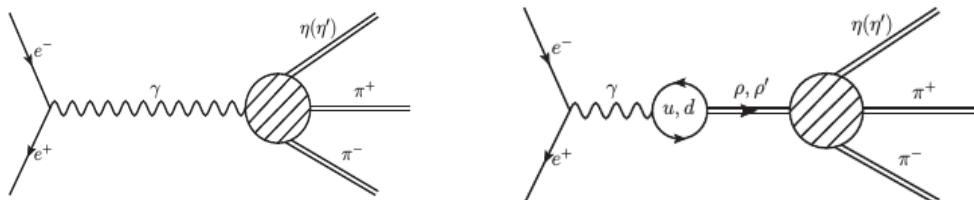
* Common fit to $e^+e^- \rightarrow \pi^+\pi^-\eta$, $e^+e^- \rightarrow \pi^+\pi^-\pi^0$



5. Extended Nambu-Jona-Lasino (NJL) model,

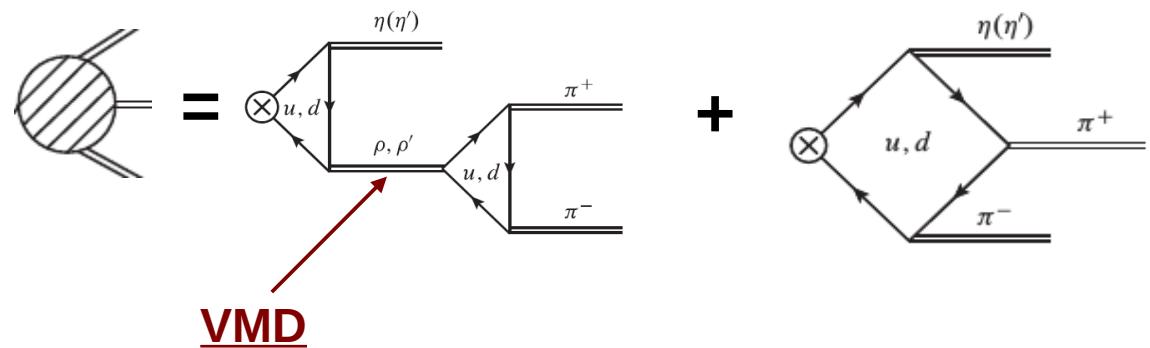
M. Volkov, A. Arbuzov, D.Kostunin, Phys.Rev. C89 (2014) 015202

Two type of Feynman diagrams to produce $\eta \pi\pi$ FS



The model Lagrangian contains quarks, mesons, photon fields

Hadronization mechanism is described by the quark loops (box and triangle)

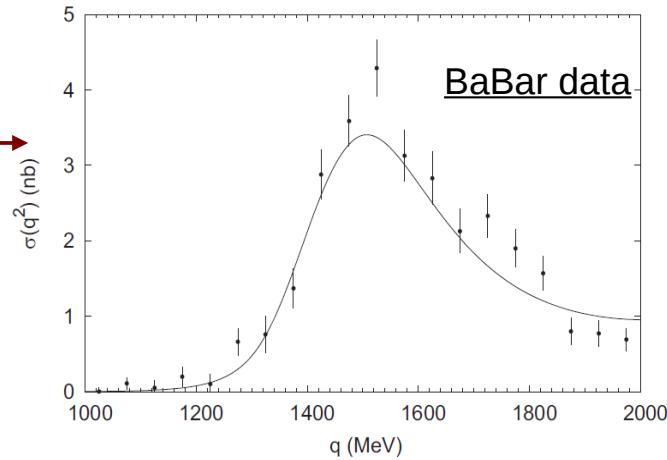


PDG for the resonance masses and widths

Qualitative agreement

Table II: Branching ratios for the processes
 $\tau \rightarrow \eta(\eta')2\pi\nu$

Process	Full amplitude	Only $\rho(770)$	PDG [28]
$\mathcal{B}(\tau \rightarrow \eta 2\pi\nu) \cdot 10^3$	1.46	1.01	1.39 ± 0.10
$\mathcal{B}(\tau \rightarrow \eta' 2\pi\nu) \cdot 10^5$	0.09	0.12	< 1.2

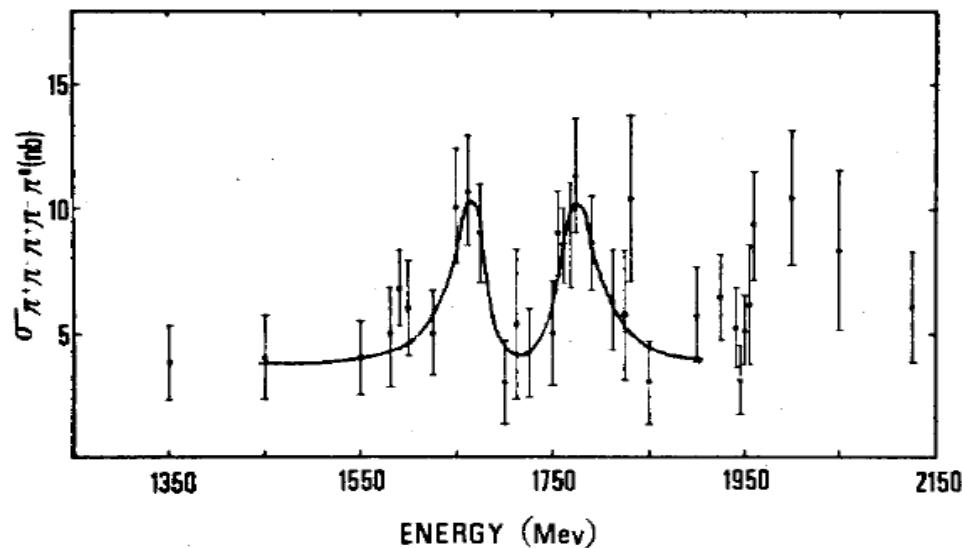


EXPERIMENT

η DECAY MODES

2γ	$(39.41 \pm 0.20) \%$
$\pi^+ \pi^- \pi^0$	$(22.92 \pm 0.28) \%$
$\pi^+ \pi^- \gamma$	$(4.22 \pm 0.08) \%$

$$e^+ e^- \rightarrow \pi^+ \pi^- \eta \rightarrow (\pi^+ \pi^- \pi^+ \pi^- \pi^0; \pi^+ \pi^- \gamma \gamma; \pi^+ \pi^- \pi^+ \pi^- \gamma)$$



1977 M3N experiment Orsay (DCI)
Nucl.Phys. B152 (1979) 215

First 5 pion production $\pi^+ \pi^- \pi^+ \pi^- \pi^0$
 $2E = (1.35; 2.125) \text{ GeV}, L = 310 \text{ nb}^{-1}$

Two BW amplitudes + linear
background

$$\chi^2/\text{d.o.f} = 10/15$$

$$m_1 = 1665 \pm 6 \text{ MeV } (\pm 10 \text{ MeV})$$

$$\Gamma_1 = 37 \pm 21 \text{ MeV}$$

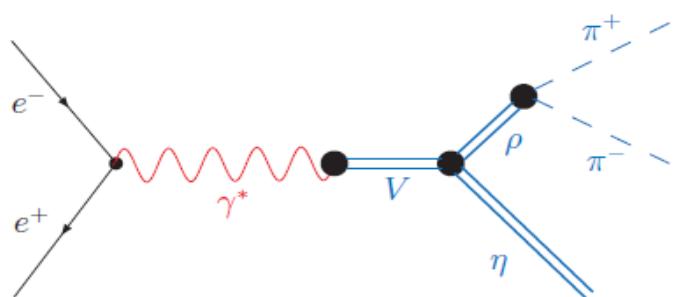
$$m_2 = 1772 \pm 6 \text{ MeV } (\pm 10 \text{ MeV})$$

$$\Gamma_2 = 49 \pm 25 \text{ MeV}$$

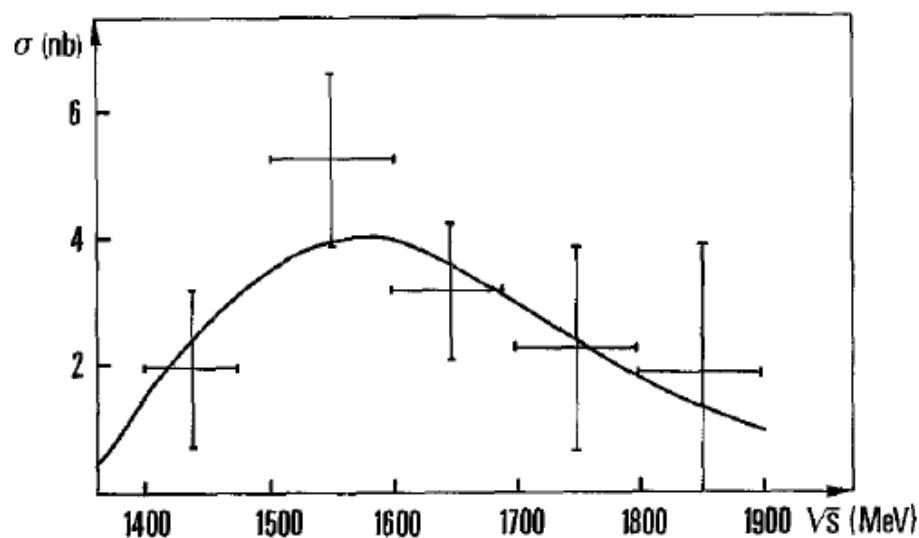
DM1 Collaboration Orsay (DCI)

DM1 $2E = (1.4; 2.18) \text{ GeV}$, $L = 1500 \text{ nb}^{-1}$

B. Delcourt et al, Phys. Lett. B 113 (1982) 93



VMD with two resonances ρ, ρ'
only ρ gives 0.5 nb



Events $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$

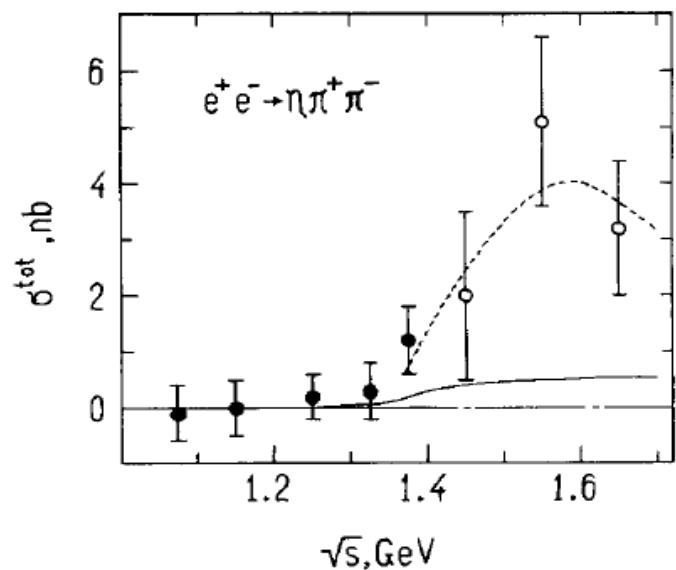
Fixed ρ' parameters: $M(\rho') = 1.57 \text{ GeV}$,
 $\Gamma(\rho') = 0.5 \text{ GeV}$

Fig. 3. $e^+e^- \rightarrow \rho\eta$ cross section. The full line is a fit with the $\rho'(M = 1.57 \text{ GeV}, \Gamma = 0.51 \text{ GeV})$.

Neutral Detector, VEPP-2M, Novosibirsk $2E = (1.05; 1.4)$ GeV

V. Druzhinin et al, Phys. Lett. B 174 (1986)

Events $e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$



VMD with two resonances ρ, ρ'

The value of cross section differs from zero only at 1.35-1.4 GeV

Fig. 3. Energy dependence of the total cross section of the reaction $e^+e^- \rightarrow \eta\pi^+\pi^-$. •: this work, ○: DM1 [10]. The solid line is the prediction of the vector dominance model with $\rho(770)$ only [1], the dashed line is the result of a fit taking into account $\rho'(1600)$ [10]

DM2 2E = (1.35; 2.4) GeV, L = 1814 nb⁻¹

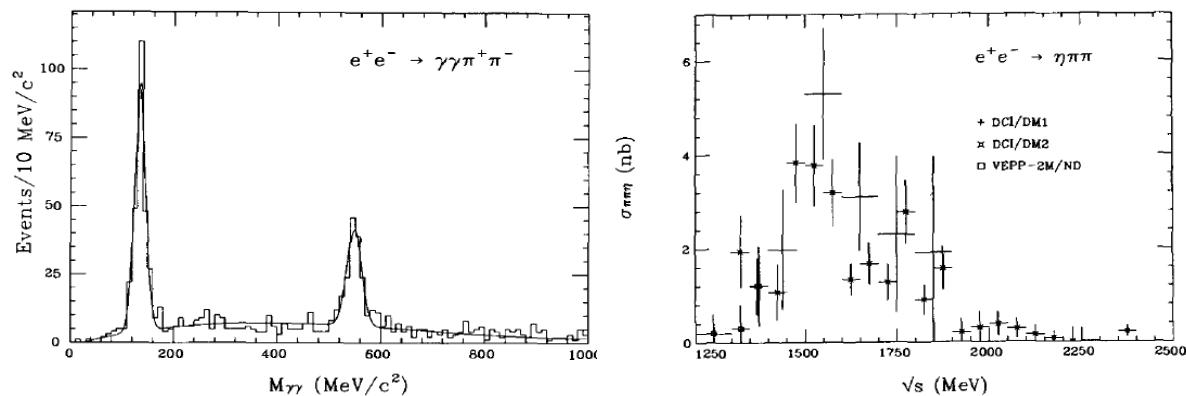
A. Antonelli et al, Phys. Lett. B 212 (1988) 133

Events

$e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$

$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$

$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\gamma$



VMD with two/ three resonances $\rho, \rho' / \rho''$

(nb)

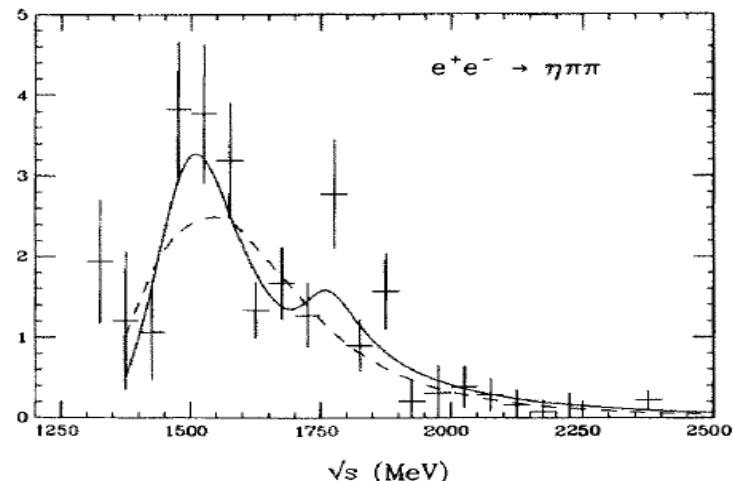


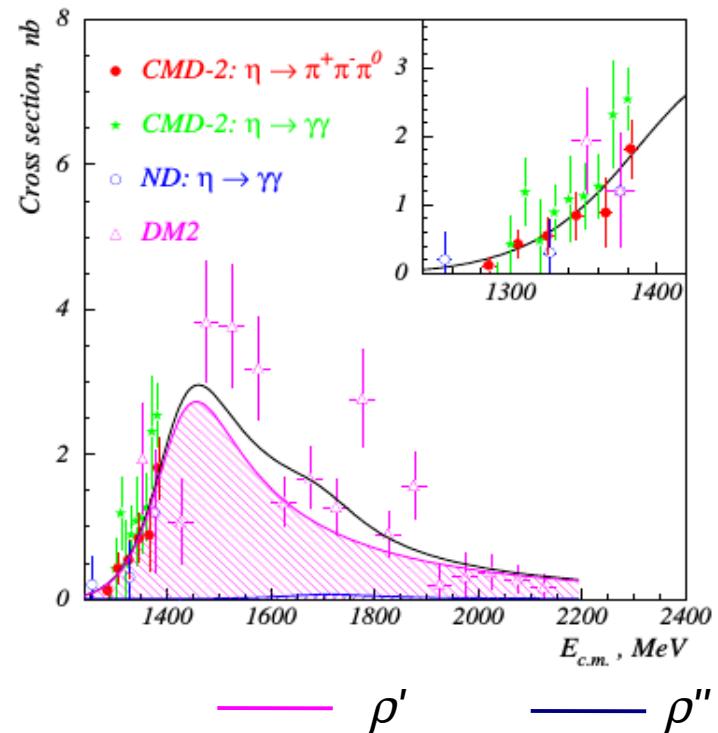
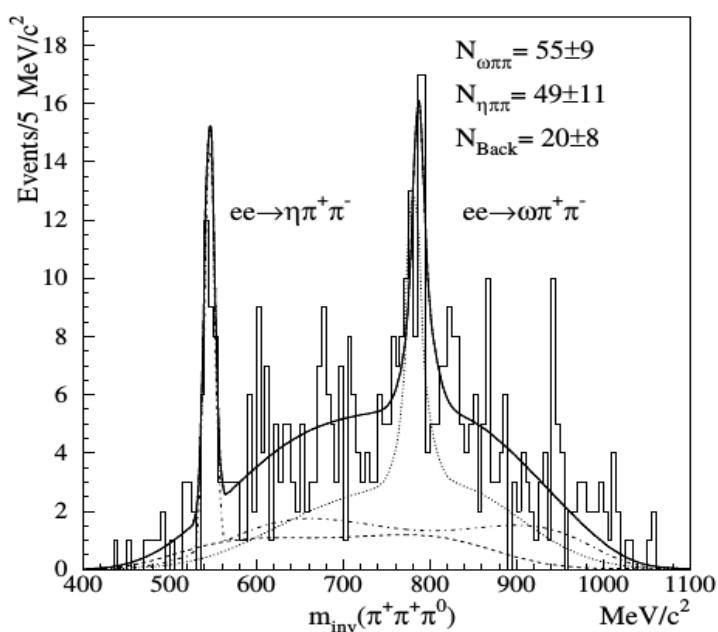
Fig. 6. Fit of the $\eta\pi^+\pi^-$ total cross section. The dashed line corresponds to the fit with just one resonance, the solid line to the fit described in the text.

M_1 (GeV/c ²)	Γ_1^{tot} (GeV)	$\text{BR} \cdot \Gamma_{ee}$ (eV)	M_2 (GeV/c ²)	Γ_2^{tot} (GeV)	$\text{BR} \cdot \Gamma_{ee}$ (eV)	CL %
1.57 [14]	0.51 [14]	176 ± 58	—	—	—	2
1.59 [16]	0.26 [16]	104 ± 62	—	—	—	0.1
1.47 ± 0.02	0.23 ± 0.03	91 ± 19	1.74 ± 0.02	0.15 ± 0.03	7 ± 3	35

CMD-2 detector, VEPP-2M, Novosibirsk $2E = (1.28; 1.38) \text{ GeV}$, $L = 3 \text{ pb}^{-1}$

R. Akhmetshin et al, Phys. Lett. B489 (2000) 125

Events $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$



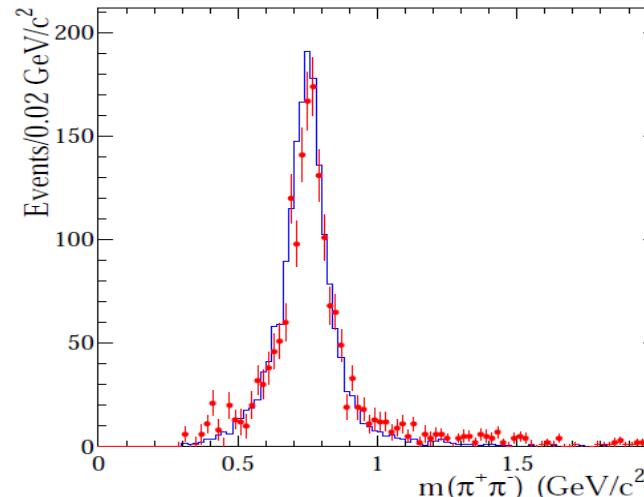
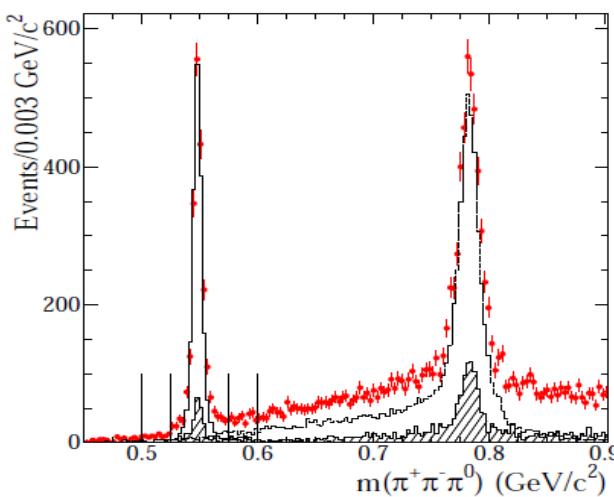
3 resonances

$\chi^2/\text{n.d.f} = 37/31$

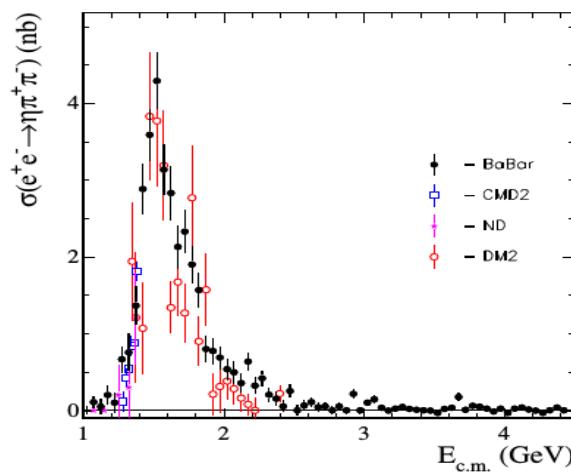
Only ρ'

$\chi^2/\text{n.d.f} = 43/35$

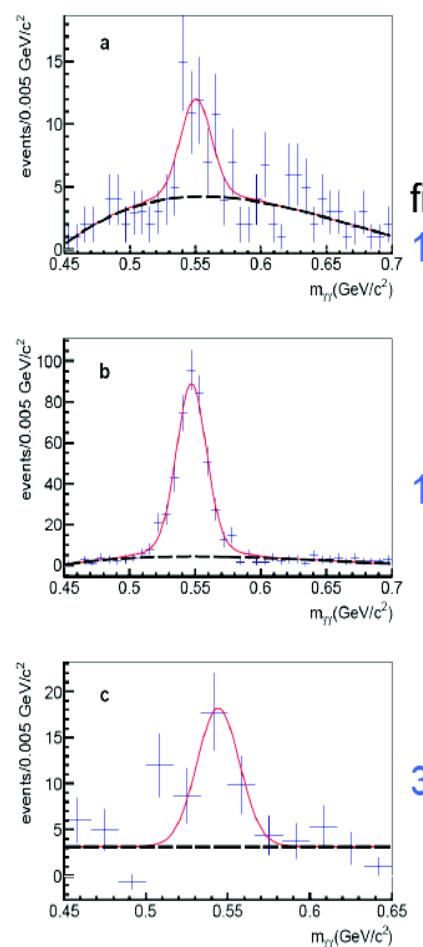
ISR method; events $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$



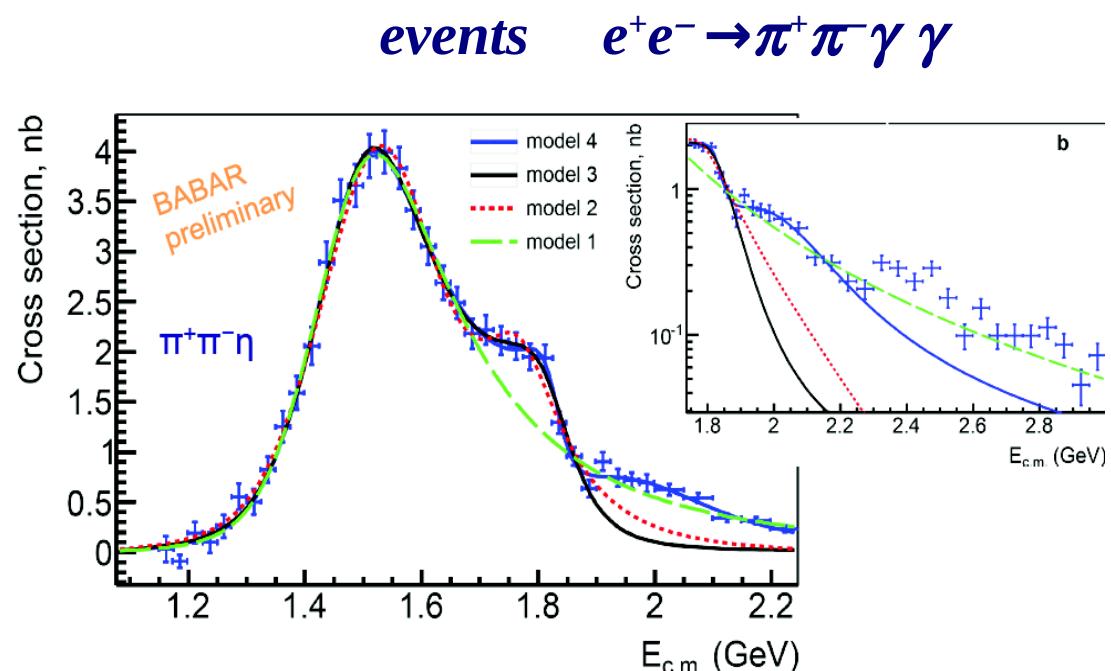
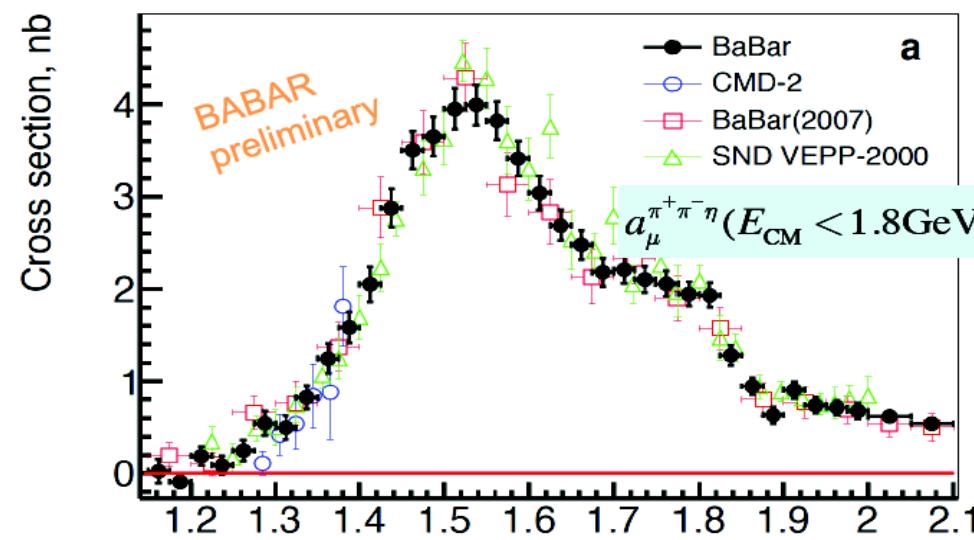
Data vs simulated $\eta\rho$ events



The cross section is in agreement with the direct experiment measurements; for the current statistics possible structure 1.6 and 1.8 GeV cannot be resolved



Cross section, nb



model 1: $\rho(770) - \rho(1450)$ fits $E_{CM} < 1.7 \text{ GeV}$
 model 2: $\rho(770) - \rho(1450) - \rho(1700)$ fits $< 1.9 \text{ GeV}$
 model 3: $\rho(770) - \rho(1450) + \rho(1700)$ fits $< 1.9 \text{ GeV}$
 model 4: $\rho(770) - \rho(1450) + \rho(1700) + \rho(2150)$ fits $< 2.2 \text{ GeV}$
 relative phases 0 (+) and 180° (-)

SND detector, VEPP-2000 (2011-2012 data), Novosibirsk 2E = (1.05; 2.00)
 GeV,L = 35 pb⁻¹, a scan step 25 MeV
V. Aulchenko et al, Phys.Rev. D91 (2015) 052013

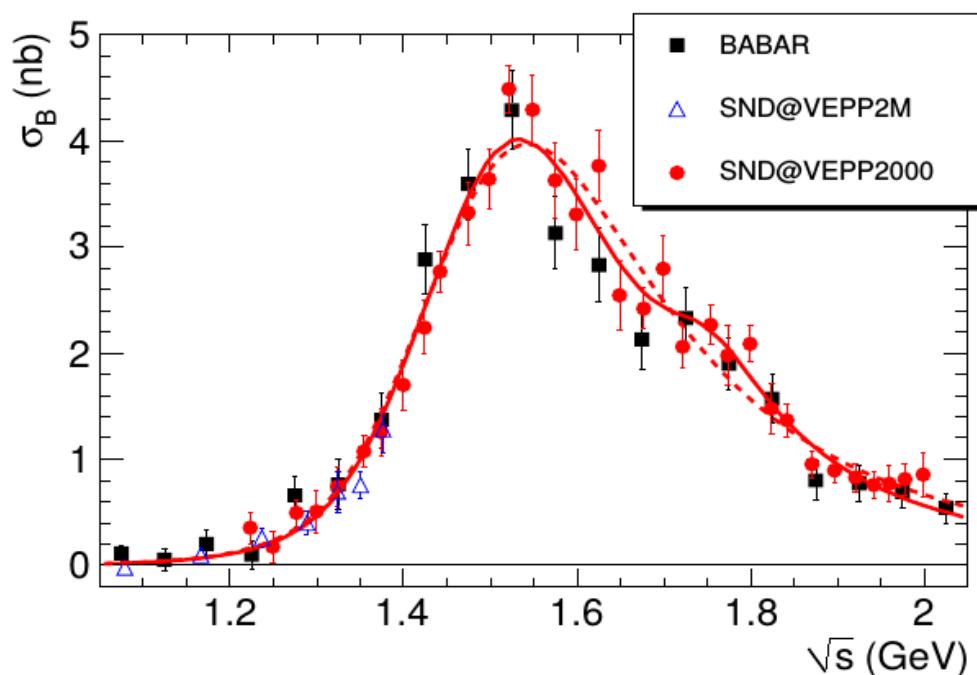
$e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$

Systematic uncertainty 8.3% for $\sqrt{s} < 1.45$ GeV, 5% for $\sqrt{s} = (1.45; 1.6)$,
 7.8% for $\sqrt{s} > 1.6$ GeV

Below 1.22 GeV no excess SIGNAL over BACKGROUND

VMD model with 3 resonances

$$F(s) = \sum_V \frac{m_V^2}{g_{V\gamma}} \frac{g_{V\rho\eta}}{s - m_V^2 + i\sqrt{s}\Gamma_V(s)}, \quad V = \rho(770), \rho(1450), \rho(1700)$$



RC estimated within VMD

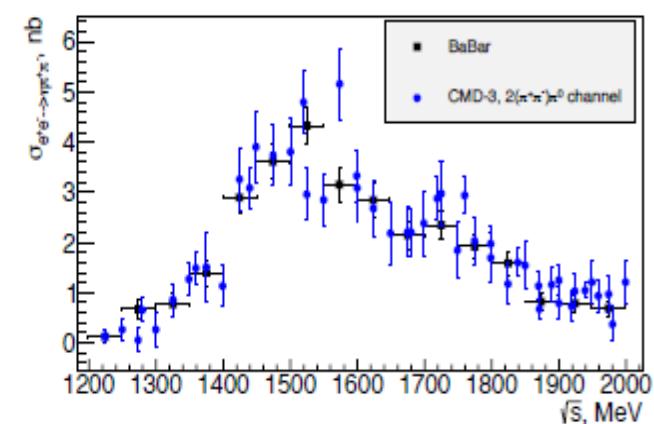
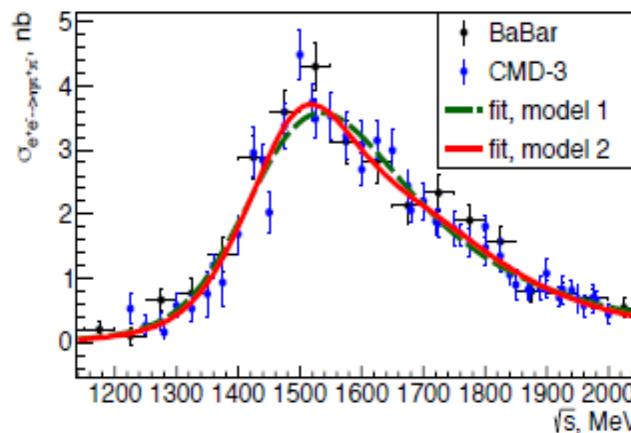
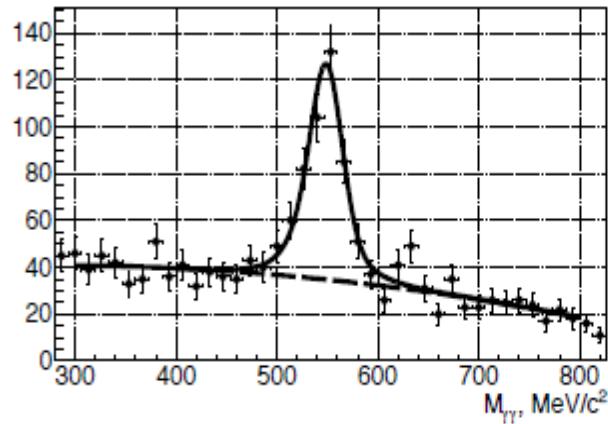
— with $\rho(1700)$ $\chi^2/\nu = 37/31$
 - - - - no $\rho(1700)$ $\chi^2/\nu = 42.6/32$

**No clear conclusion about necessity
 $\rho(1700)$ for the current statistics**

CMD-3 detector, VEPP-2000 (2011-2012 data), $2E = (1.2; 2.)$ GeV

E. Solodov et al, AIP Conf. Proc. 1735 (2016) 020005

2E = 1.5 GeV



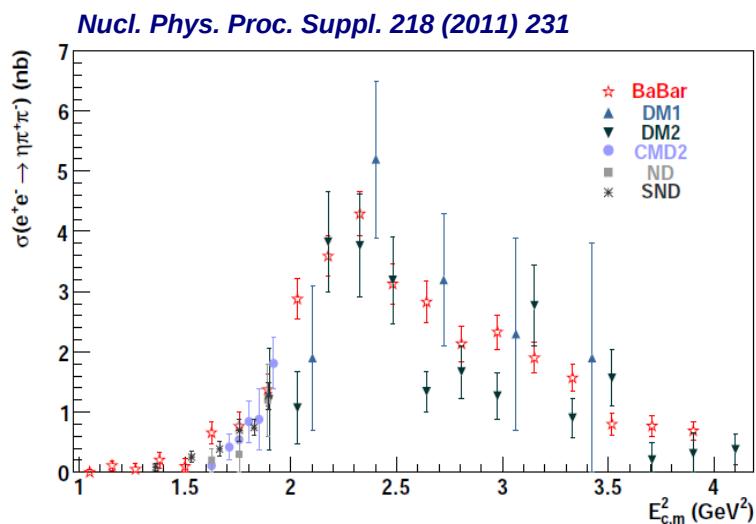
VMD with ρ' , ρ''

**The current statistics does not allow to make
a conclusion about the $\rho(1700)$ presence**

Table 1

Summary of $e^+e^- \rightarrow \eta\pi^+\pi^-$ data

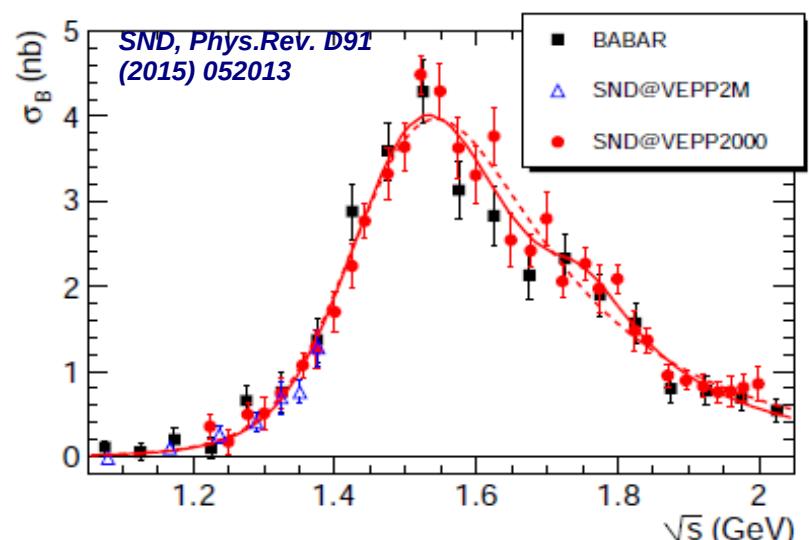
Group	\sqrt{s} , GeV	N _{points}	$\Delta_{\text{stat}}, \%$	$\Delta_{\text{syst}}, \%$
ND, 1986	1.25 - 1.40	3	50 - 100	10
CMD-2, 2000	1.25 - 1.40	6	30 - 60	15
SND, 2010	1.17 - 1.38	6	15 - 60	10.5
DM1, 1982	1.40 - 1.80	4	30 - 60	10
DM2, 1988	1.35 - 1.80	10	25 - 60	10
BaBar, 2007	1.00 - 1.80	16	10 - 60	8
SND, 2015	1.22-2.00	33	6-50	5-8.5
CMD-3				



Fair agreement.

$s < 1.4 \text{ GeV}^2$ BaBar higher

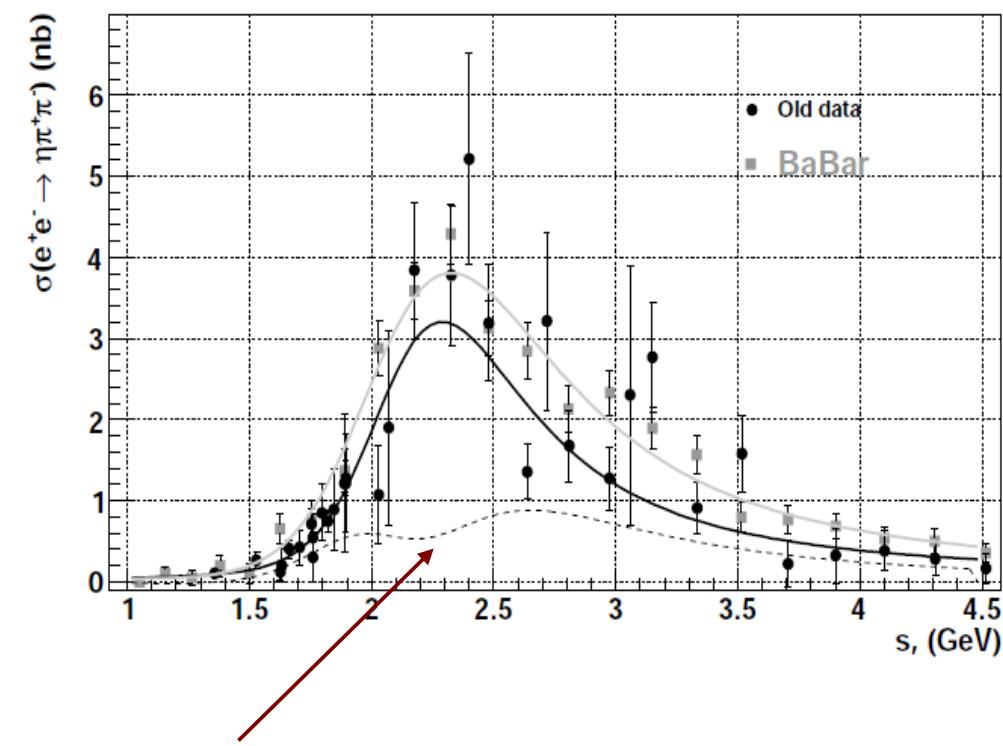
$s > 1.4 \text{ GeV}^2$ BaBar higher DM2, in agreement with DM1



The 3 experiment data are in agreement.
Detailed comparison?

Cross section comparison

V. Cherepanov, S. Eidelman,
Nucl. Phys. Proc. Suppl. 218 (2011) 231



Difference systematically higher zero

$$\sigma_{\eta\pi\pi}(s) = \frac{F_{\eta\pi\pi}(s)}{s} |A_{\rho'} + A_{\rho''} e^{i\delta_{\rho'\rho''}}|^2$$

$$A_V = \frac{m_V^2 \Gamma_V \sqrt{\sigma_V / F_{\eta\pi\pi}(m_V^2)}}{s - m_V^2 + i\sqrt{s} \Gamma_V(s)}, V = \rho', \rho''$$

A new fit to including SND 2015, BaBar-new,
CMD-3 data?

Check of CVC

V. Cherepanov, S. Eidelman, JETP Lett. 89 (2009) 429

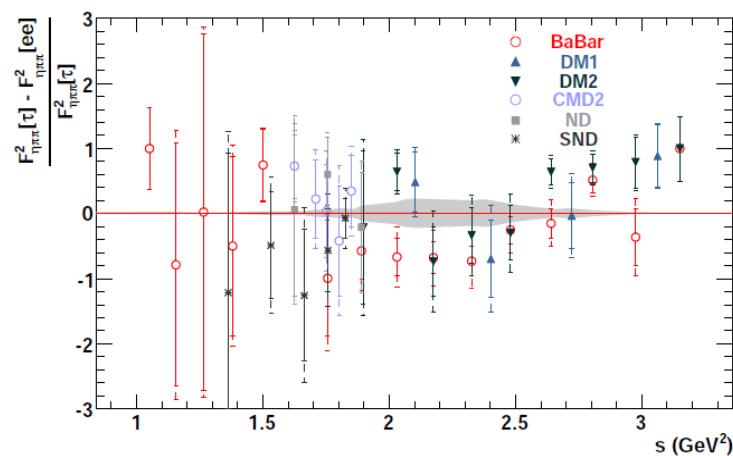
V. Cherepanov, S. Eidelman, Nucl.Phys.Proc.Suppl. 218 (2011) 231

Systematical study of CVC $2\pi, 4\pi \rightarrow \eta\pi\pi$

Hadronic decay of τ -lepton through the vector current ($G = +$)

$$J^{PG} = 1^{-+}, \tau \rightarrow 2n\pi\nu_\tau, \omega\pi\nu_\tau, \eta\pi\pi\nu_\tau, \dots$$

The differential width



$$\frac{d\Gamma}{dq^2} = \frac{G_F |V_{ud}|^2 S_{EW}}{32\pi^2 m_\tau^3} (m_\tau^2 - q^2)^2 (m_\tau^2 + 2q^2) v_1(q^2)$$

the spectral function

$$v_1(q^2) = q^2 \sigma_{e^+ e^-}^{I=1}(q^2) / 4\pi\alpha^2$$

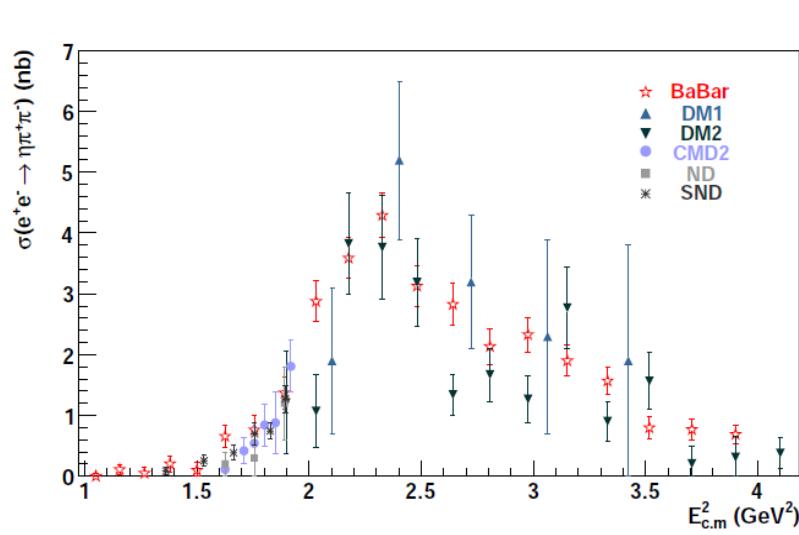
Spectral functions (form-factors) are in fair agreement

$$\mathcal{B}(\tau^- \rightarrow \eta\pi^-\pi^0\nu_\tau)$$

$$\frac{\mathcal{B}(\tau^- \rightarrow X^-\nu_\tau)}{\mathcal{B}(\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau)} = \frac{3|V_{ud}|^2 S_{EW}}{2\pi\alpha^2} \int_{4m_\pi^2}^{m_\tau^2} dq^2 \frac{q^2}{m_\tau^2} (1 - \frac{q^2}{m_\tau^2})^2 (1 + 2\frac{q^2}{m_\tau^2}) \sigma_{e^+e^-}^{I=1}(q^2)$$

$$S_{EW} = 1.0194$$

$$|V_{ud}|^2 = 0.9742$$



Experimental values of $\mathcal{B}(\tau^- \rightarrow \eta\pi^-\pi^0\nu_\tau)$

Group	$\mathcal{B}, \%$	Ref.
CLEO, 1992	$0.170 \pm 0.020 \pm 0.020$	[19]
ALEPH, 1997	$0.180 \pm 0.040 \pm 0.020$	[20]
Belle, 2009	$0.135 \pm 0.003 \pm 0.007$	[21]

$$\mathcal{B}(\tau \rightarrow e\bar{\nu}_e\nu_\tau) = (17.85 \pm 0.05)\%$$

Direct integration

1. $\sqrt{s} > 1.25$ GeV, old data $(0.130 \pm 0.015)\%$
2. $\sqrt{s} > 1.25$ GeV, BaBar $(0.165 \pm 0.015)\%$

Average1 $1.25 < \sqrt{s} < 1.77$: $(0.147 \pm 0.018)\%$

Total e+e- (average1 + Low energy BaBar): $(0.153 \pm 0.018)\%$

PDG 2010: $(0.139 \pm 0.008)\%$

Difference $\sim 0.9 \sigma$

SND, 2015, V. Aulchenko et al, Phys.Rev. D91 (2015) 052013 (1.22; 2.0)

$$\mathcal{B}(\tau^- \rightarrow \eta\pi^-\pi^0\nu_\tau) = (0.156 \pm 0.004 \pm 0.010)\%$$

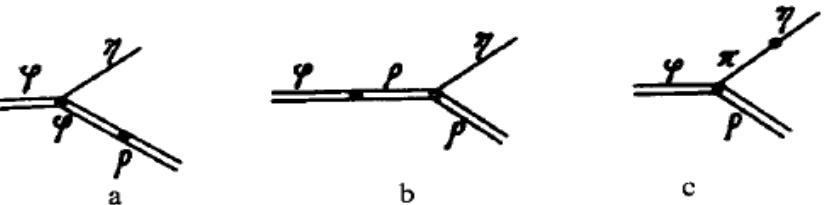
Ronald Waldi for BaBar Collaboration BaBar 2016 $\mathcal{B}(\tau \rightarrow v\pi^0\pi^-\eta) = (0.160 \pm 0.009)\%$

BaBar 2007+2016 $\mathcal{B}(\tau \rightarrow v\pi^0\pi^-\eta) = (0.162 \pm 0.008)\%$

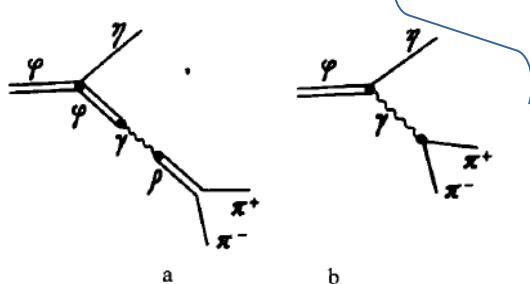
Rare decay $e^+e^- \rightarrow \phi \rightarrow \pi^+\pi^-\eta$

N.N. Achasov, V.A. Karnaughov, JETP Lett. 39 (1984) 342

OZI suppressed, G-parity violated



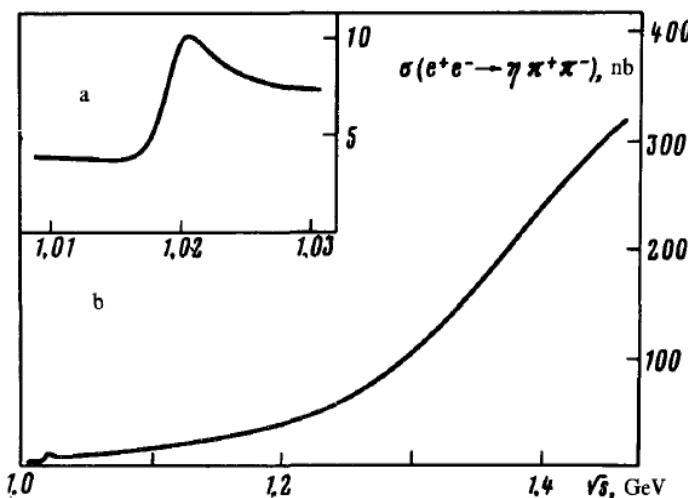
Main contribution



with photon a factor $m_\rho^2/q^2 \sim 4.4$

$$4m_\pi^2 < q^2 < (m_\phi - m_\eta)^2$$

With the ρ background amplitude



$$F(s) = \frac{m_\rho^2}{g_{\rho\gamma}} \frac{g_{\rho\rho\eta}}{s - m_\rho^2 + i\sqrt{s}\Gamma_\rho(s)} + \frac{m_\phi^2}{g_{\phi\gamma}} \frac{g_{\phi\rho\eta}}{s - m_\phi^2 + i\sqrt{s}\Gamma_\phi(s)}$$

$$\frac{d\sigma}{dq^2} = \frac{4\alpha^2}{3} \frac{1}{s\sqrt{s}} \frac{\sqrt{q^2} \Gamma_\rho(q^2) \rho_\eta^3(s, q^2)}{(q^2 - m_\rho^2)^2 + (\sqrt{q^2}\Gamma_\rho(q^2))^2} |F(s)|^2$$

$$\text{Br}(\phi \rightarrow \eta\pi^+\pi^-) = 0.35 \times 10^{-6}$$

L.Barkov et al (CMD), Journal of Nuclear Physics 47 (1988) 393

R. Akmetshin et al (CMD), Phys Lett B491 (2000) 81

$$\text{Br}(\phi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0) < 1.5 \times 10^{-4}$$

$$\text{Br}(\phi \rightarrow \eta\pi^+\pi^-) < 1.8 \times 10^{-5}$$

CONCLUSIONS

Experiments: direct measurements (Novosibirsk)

ISR, BaBar

two FS events:

* Fair agreement for 1-2 GeV: detailed comparison?

* The current statistics does not allow to make
a conclusion about the $\rho(1700)$ presence

Theory: VMD with 2 ρ' resonances; RChL approach; NJL model

ChPT prediction ??? (energy threshold-1GeV) → comparison with VMD and data

CVC test: $\mathcal{B}(\tau^- \rightarrow \eta\pi^-\pi^0\nu_\tau)$ calculated from e+e- data is $1-1.5\sigma$ higher PDG
isospin breaking corrections?

m_ϕ measurement: OZI suppressed and G-parity violated $\phi \rightarrow \eta \pi^+ \pi^-$

CMD-2 $Br(\phi \rightarrow \eta\pi^+\pi^-) < 1.8 \times 10^{-5}$ Theory $Br(\phi \rightarrow \eta\pi^+\pi^-) = 0.35 \times 10^{-6}$

Check of ChPT