$\eta \to 3\pi$
and
π^0 physics: status report

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Outline:

- $\eta \to 3\pi$
  - ChPT
  - dispersive approach
  - Adler zero
- π^0 physics
  - $\pi^0 \to \gamma\gamma$, $\pi^0 \to e^+e^-$, Dalitz, double Dalitz, future plans
- Summary

$\eta \to 3\pi$ in a nutshell

$\eta \to \pi\pi$ forbidden by CP

$\eta \to \pi\pi\pi$ allowed, however, G parity is violated, and it must proceed via isospin breaking effects (η is isosinglet and this $\pi\pi\pi$ system cannot have the isospin 0)

possible mechanisms:

- second order EM interactions

$$\mathcal{H}_{\mathsf{QED}}(x) = -\frac{1}{2}e^2 \int dy D^{\mu\nu}(x-y) T(j_\mu(x) j_\nu(y))$$

  apart from $m_{\pi^{+/0}}$ difference small: [Sutherland, Bell '68]; [Baur, Kambor, Wyler '95]; [Ditsche, Kubis, Meißner '09]

- mass difference between u and d quarks

$$\mathcal{H}_{\mathsf{QCD}}(x) = \frac{m_d - m_u}{2}(\bar{d}d - \bar{u}u)(x)$$

A major tool for studying $\eta \to 3\pi$: ChPT

$\eta \to 3\pi$ in ChPT

LO (current algebra result) [Cronin '67]

$$A(s,t,u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left(1 + \frac{3(s-s_0)}{m_\eta^2 - m_\pi^2}\right)$$

can be rewritten using

$$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \quad or \quad R = \frac{m_s - \hat{m}}{m_d - m_u}$$

We can pull out isospin breaking effects (up to first order in IB) defining

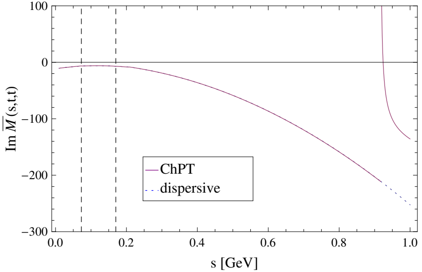
$$A(s,t,u) = \frac{\sqrt{3}}{4R}M(s,t,u), \qquad M(s,t,u) = \frac{1}{F_\pi^2}\left(\frac{4}{3}m_\pi^2 - s\right)$$

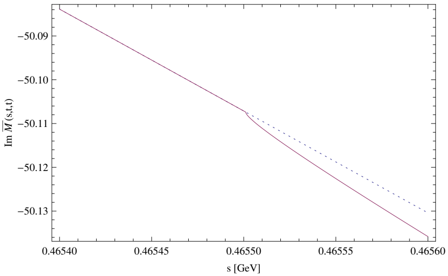
$\eta \to 3\pi$ in ChPT

NLO [Gasser, Leutwyler '85]: one loop calculation, large enhancement (as anticipated by [Roiesnel, Truong '81]) confirmed:

$$\Gamma^{LO}_{GL}(\eta \to \pi^0\pi^+\pi^-) \approx 66\,\text{eV} \to \Gamma^{NLO}_{GL}(\eta \to \pi^0\pi^+\pi^-) \approx 160\,\text{eV}$$

remark 1 one can use dispersive techniques to get(/verify) the absorptive part of p^4 amplitude with $\pi\pi$ intermediate states.
I.e. under KK (or more precisely $\eta\pi$) threshold one should get the same:





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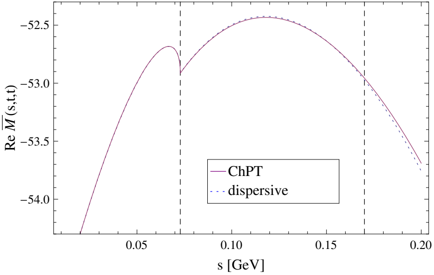
remark 1 one can use dispersive techniques to get(/verify) the absorptive part of p^4 amplitude with $\pi\pi$ intermediate states.

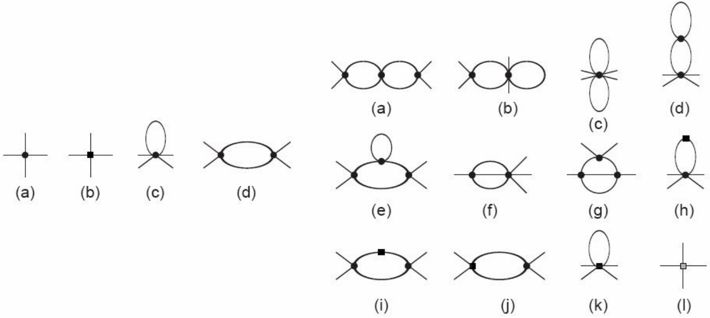
remark 2 consistent definition (GMO, etc.) is important

remark 3 real part in dispersive method via ChPT (masses, F, L_is)



$\eta \to 3\pi$ in ChPT; NNLO [Bijnens, Ghorbani '07]:

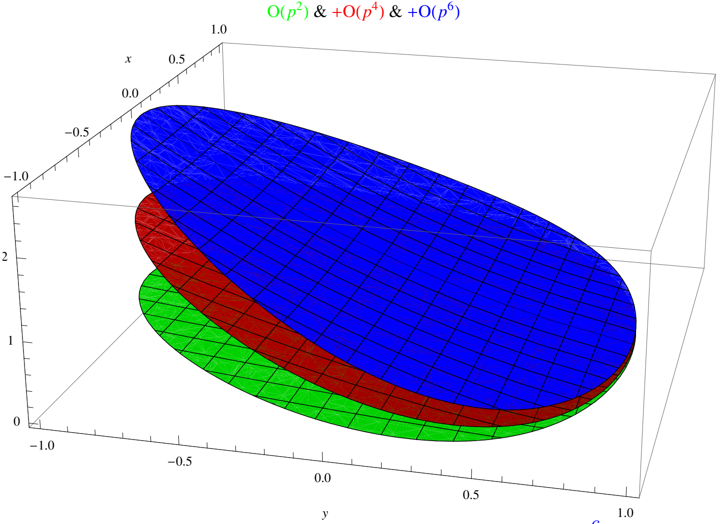
Diagrams



- Include mixing, renormalize, pull out factor $\frac{\sqrt{3}}{4R}$, ...
- Two independent calculations (comparison major amount of work)

$\eta \to 3\pi$ at two-loop order in ChPT: results

Dalitz plot of $|M(s,t,u)|^2$ [parameters: $x = \frac{\sqrt{3}}{2m_\eta(m_\eta - 3m_\pi)}(u-t)$, $y = \frac{3}{2m_\eta(m_\eta - 3m_\pi)}(s_0 - s)$]



normalization at center of Dalitz plot: $|M(s_0,s_0,s_0)|^{+p^6} = 1$

$\eta \to 3\pi$: dispersive analysis

For the general dispersive technique background see e.g. previous talks by Kubis and Leupold

When we started our project, there were two main attempts:

- [Anisovich, Leutwyler '96] $\longrightarrow$ more in the previous talk Passemar
- [Kambor, Wiesendanger, Wyler '95]
  - based on the extended Khuri-Treiman analysis
  - uses $O(p^4)$ ChPT
  - assumes the validity of ChPT

Dispersive approach for $\eta \to 3\pi$ is a reliable theoretical tool, based only on general assumptions: relativistic invariance, unitarity, analyticity and crossing symmetry together with chiral counting, which ensures that our amplitude is valid up to and including $O(p^6)$

Dispersive analysis: general idea

aim: construction of fully relativistic model independent representations of $\eta \to 3\pi$ valid up to and including two-loop corrections.
tools: the dispersive approach based only on very general principles, unitarity, analyticity and crossing symmetry, combined with chiral counting
already used for: $\pi\pi$ scattering [Stern, Sazdjian, Fuchs '93]

chiral counting: amplitude $\approx O(p^2)$, unitarity: amplitude is dominantly real (Im starts at $O(p^4)$). Analyticity (together with chiral limit behaviour): $M_{LO} = AM_\eta^2 + B(s - s_0)$, (where s_0 is the center of a Dalitz plot) n.b. $M_{LO}^{ChPT} = \frac{1}{F_\pi^2}\left(\frac{4}{3}m_\pi^2 - s\right)$ chiral counting: contribution of multi-pion states and D and higher waves suppressed
We will thus analytically continue $P\pi \to \pi\pi$ to the decay region together with decomposition into the partial waves

$$A(s,t,u) = 16\pi(f_0(s) + 3f_1(s)\cos\theta) + A_{\ell\geq 2}$$

Reconstruction theorem [Stern, Sazdjian, Fuchs '93; M.Z., Novotný '08]

Generally for process $AB \to CD$:

$$\boxed{\begin{aligned} S(s,t;u) =& P^3 + \Phi_0(s) + [s(t-u) + (m_A^2 - m_B^2)(m_C^2 - m_D^2)]\Phi_1(s) \\ &+ \text{crossed channels} + O(p^8), \end{aligned}}$$

P^3 - third order polynomial in s, t, u with same symmetries as $S(s,t;u)$, schematically (up to subtractions)

$$\boxed{\begin{aligned} \Phi_0(s) &\sim \int_\Sigma^\infty dx \frac{\mathrm{Im} f_0(x)}{x-s} + \ldots, \\ \Phi_1(s) &\sim \int_\Sigma^\infty dx \frac{\mathrm{Im} f_1(x)}{(x-s)\lambda_{AB}^{1/2}(x)\lambda_{CD}^{1/2}(x)}, \end{aligned}}$$

and similar for the $t-$ and $u-$ crossed channel

$\left[\lambda_{XY}(s) = (s-(m_X+m_Y)^2)(s-(m_X-m_Y)^2)\right]$

This can be used to fully reconstruct from data the whole amplitude (neglecting $O(p^8)$) in the whole low-energy region of the Mandelstam variables (including the unphysical domain).

Dispersive approach: unitarity relations

Unitarity relation

Assuming T-invariance and the real analyticity of the amplitude, the unitarity relation gives for the partial waves

$$\mathrm{Im} f_\ell^{i\to f}(s) = \sum_{(1,2)} \frac{1}{S} \frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{s} f_\ell^{i\to(k_1,k_2)}(s) \left[f_\ell^{f\to(k_1,k_2)}(s)\right]^* \theta(s-(m_1+m_2)^2)$$

$S = 1(2)$ for (un)distinguishable states k_1, k_2

- in the low-energy region the intermediate states other than those containing pairs of pseudoscalar mesons are suppressed up to $O(p^8)$
- intermediate states other than $\pi\pi$ induce singularities far from the central region of Dalitz plot of $P \to \pi\pi\pi$ processes $\Rightarrow$ can be expanded in series and included into the polynomial
- iterative process: correct analytic continuation important for the second iteration, e.g.

$$\tilde{\varphi}_l(s) \sim \int_{t_-(s)}^{t_+(s)} \mathrm{d}t\; P_l\left(\cos\theta(s,t)\right) \int\limits_{(m_a+m_b)^2}^{\infty} \frac{dx}{x^3} \frac{\mathrm{Im} f_0(x)}{x-t}$$

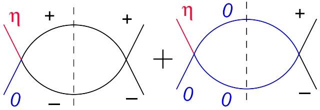
Dispersive approach: $\eta \to 3\pi$

Due to Bose symmetry, CP invariance P-wave of some (sub)processes can be zero

General form of $\eta \to \pi^+\pi^-\pi^0$, schematically up to $O(p^8)$

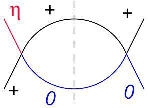
$$M(s,t,u) = P + W_0^{+-}(s) - \left[W_0^{+0}(t) + 3(u-s)W_1^{+0}(t) + \text{u-channel}\right]$$

$$W_0^{+-}(s) \sim \int \frac{dx}{x-s} \mathrm{Im} t_0^{+-}(x) \qquad \mathrm{Im} t_0^{+-}:$$



$$W_0^{+0}(s) \sim \int \frac{dx}{x-s} \mathrm{Im} t_0^{+0}(x)$$

$$W_1^{+0}(s) \sim \int \frac{dx}{x-s} \frac{\mathrm{Im} t_1^{+0}(x)}{\lambda^{1/2}\sigma} \qquad \mathrm{Im} t_i^{+0}:$$



note: for $\eta \to 3\pi^0$ there is no P-wave contribution.

$$\overline{M}(s,t,u) = \overline{P} + W^{00}(s) + W^{00}(t) + W^{00}(u)$$

Dispersive approach

All parameters hidden in P [note: at the same time we need to consider $\pi\pi \to \pi\pi$, that brings additional parameters in the corresponding polynomial; they can be fit or taken from somewhere else [Stern, Sazdjian, Fuchs '93]]

$$P = A_{+-} m_\eta^2 + B_{+-}(s - s_0) + C_{+-}(s - s_0)^2 + D_{+-}[(t - s_0)^2 + (u - s_0)^2] \\ + E_{+-}(s - s_0)^3 + F_{+-}[(t - s_0)^3 + (u - s_0)^3]$$

n.b. $M(s, t, u)$ is connected with $\overline{M}(s, t, u)$ via isospin relation

$$\overline{M}(s, t, u) = M(s, t, u) + M(t, u, s) + M(u, s, t)$$

and so are the parameters in $\overline{P}$ with $A_{+-}, \ldots$

Adler zero

$SU(2)$ theorem:
The amplitude for $\eta \to \pi^+(p_+)\pi^-(p_-)\pi^0(p_0)$ vanishes in $SU(2)$ chiral limit for $p_+ = 0$ or $p_- = 0$.
Consequence: expanding the amplitude

$$M(s,t,u) = \sum_{i,j\geq 0} c_{ij} s^i \left[(t-u)^2 - m_\eta^4\right]^j$$

$SU(2)$ theorem states:

$$c_{00} = O(m_\pi^2)$$

(and similarly around other points)
However, we expand in different place
(center of Dalitz plot).
Forcing the Adler zero can be dangerous

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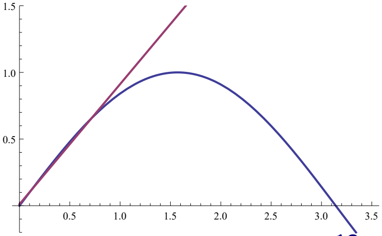

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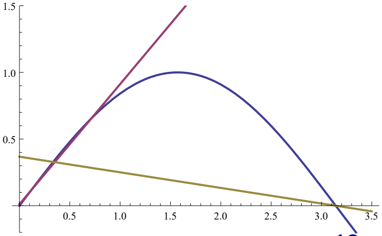

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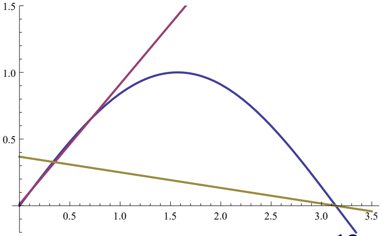


remark: error bars – important!

$\eta \to 3\pi$

Work in collaboration with M.Knecht, J.Novotny and M.Zdrahal
One crucial point: setting the normalization

- we use $O(p^6)$ ChPT calculation
- matching imaginary part (avoid the problems with LECs)
- for ChPT good convergence for $t = u$ when coming from NLO to NNLO
- using "plateau" argument (we expect a physical quantity as a function of matching point to be stable at some region)

$\eta \to 3\pi$

Work in collaboration with M.Knecht, J.Novotny and M.Zdrahal
Our work [published in 2011] based on KLOE 2008 analysis.

- final result $R = 37.7 \pm 2.2$
- seeming violation of Adler zero theorem (line $s = u$)
  - beyond $SU(2)$ limit, the absolute term proportional to m_π^2
  - we expand around the center of Dalitz plot, the studied Adler zero is relatively far (error bars)

New study based on the recent KLOE II analysis (2016) in progress [again in col. with M.Knecht, J.Novotny, M.Zdrahal]

- the highest $O(p^6)$ part of our polynomial

$$\sim E(s-s^c)^3 + F\left[(t-s^c)^3 + (u-s^c)^3\right]$$

  is polluted by the error from fitting to data.
- we would like to improve this issue

π^0 and related processes

status report on finished and ongoing projects

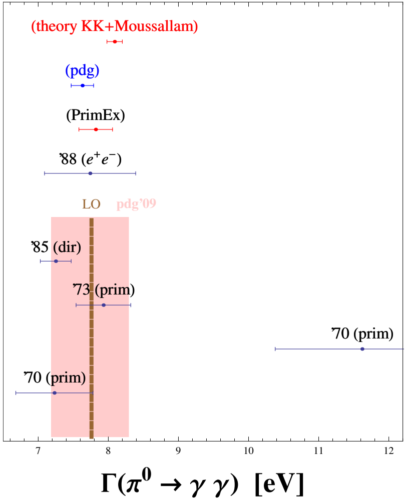
PDG 2016 on π^0

π^0 DECAY MODES

For decay limits to particles which are not established, see the appropriate Search sections (A^0 (axion) and Other Light Boson (X^0) Searches, etc.).

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	2γ		(98.823 ± 0.034) %	S=1.5
Γ_2	$e^+e^-\gamma$		(1.174 ± 0.035) %	S=1.5
Γ_3	γ positronium		$(1.82\pm0.29)\times10^{-9}$	
Γ_4	$e^+e^+e^-e^-$		$(3.34\pm0.16)\times10^{-5}$	
Γ_5	e^+e^-		$(6.46\pm0.33)\times10^{-8}$	
Γ_6	4γ		$<2\times10^{-8}$	CL=90%
Γ_7	$\nu\overline{\nu}$		[a] $<2.7\times10^{-7}$	CL=90%
Γ_8	$\nu_e\overline{\nu}_e$		$<1.7\times10^{-6}$	CL=90%
Γ_9	$\nu_\mu\overline{\nu}_\mu$		$<1.6\times10^{-6}$	CL=90%
Γ_{10}	$\nu_\tau\overline{\nu}_\tau$		$<2.1\times10^{-6}$	CL=90%
Γ_{11}	$\gamma\nu\overline{\nu}$		$<6\times10^{-4}$	CL=90%
Charge conjugation (*C*) or Lepton Family number (*LF*) violating modes				
Γ_{12}	3γ	*C*	$<3.1\times10^{-8}$	CL=90%
Γ_{13}	μ^+e^-	*LF*	$<3.8\times10^{-10}$	CL=90%

π^0 life time



π^0 mean life, PDG history:

1985 $(8.4 \pm 0.6) \times 10^{-17}$ s

...

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2012 $(8.52 \pm 0.18) \times 10^{-17}$ s $\leftarrow$ PrimEx col.

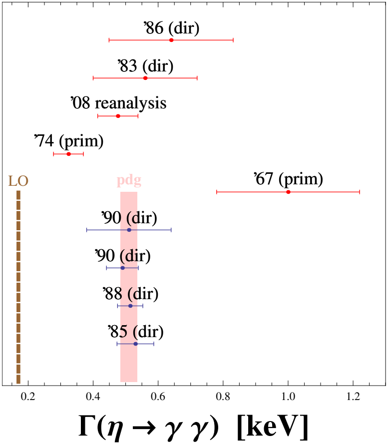
...

today $(8.52 \pm 0.18) \times 10^{-17}$ s

theory: [KK,Moussallam] $(8.04 \pm 0.11) \times 10^{-17}$ s

π^0 life time

compare with η decay:



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$\pi^0 \to \gamma\gamma$

- one of the most important processes for theory of particle physics
- π^0 lightest hadron $\Rightarrow$ dominant decay mode $\pi^0 \to \gamma\gamma$ (br=98.82%)
- non-existence of logarithmic correction to the current algebra result at NLO
- connection with the non-renormalization theorem ?
- new experimental activities
- theory – NNLO calculation: [KK,Moussallam'09] $\to$ see next

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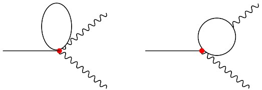
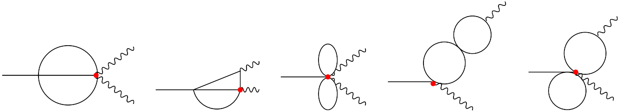
$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT: technical part

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector
- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]
- representation of chiral field: $U = \sigma + i\frac{\tau.\pi}{F}$, $\sigma = \sqrt{1-\vec{\pi}^2/F^2}$ (no $\gamma 4\pi$ vertex at LO)
- one-loop



- two-loop



- verification of Z-factor, F_π/F [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]
- double log checked by Weinberg consistency rel. [Colangelo '95]

$\pi \to \gamma\gamma$: Phenomenology

- $F_\pi = 92.22 \pm 0.07$ MeV from [Marciano, Sirlin '93] π_{l2} decay

  using quark mass ratio (from lattice), pseudo-scalar meson masses, R from $\eta \to 3\pi$ (ChPT: [Bijnens,Ghorbani '07])
- $\frac{m_d - m_u}{m_s} = (2.29 \pm 0.23)\, 10^{-2}$
- $B(m_d - m_u) = (0.32 \pm 0.03)\, M_{\pi^0}^2$
- $3L_7 + L_8^r(\mu) = (0.10 \pm 0.06)\, 10^{-3} \quad (\mu = M_\eta)$ (from pseudo-scalar meson masses formula [Gasser, Leutwyler '85])

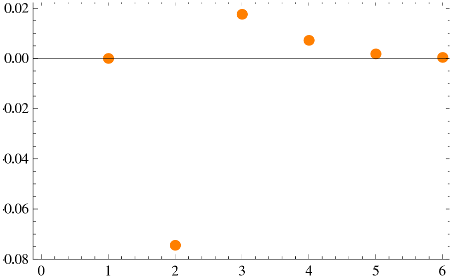
- $C_7^W = 0$ (more precisely $C_7^W \ll C_8^W$, motivated by simple resonance saturation)
- $C_8^W = (0.58 \pm 0.2) 10^{-3}$GeV^{-2} (from $\eta \to 2\gamma$)

result $\boxed{\Gamma_{\pi^0 \to 2\gamma} = (8.09 \pm 0.11)\text{eV}}$

[or $\tau_{\pi^0} = (8.04 \pm 0.11) \times 10^{-17}$ s]

$\pi^0 \to \gamma\gamma$: leading logs (for details see [Bijnens,KK,Lanz'12])

Leading logarithm contribution of individual orders in percent of the leading order:



Adler-Lee-Treiman-Zee-Terentev theorem on triangle and box anomaly

$$F^{3\pi}(0,0,0) = \frac{1}{eF_\pi^2} F_{\pi\gamma\gamma}(0,0)$$

is valid up to 2-loop order for LL beyond the soft-photon limit

$\pi^0 \to e^+e^-$

KTeV's measurement:

$$\frac{\Gamma(\pi^0 \to e^+e^-,\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}\,.$$

by extrapolating the Dalitz branching ratio to the full range of x

$$B(\pi^0 \to e^+e^-(\gamma),\, x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}\,.$$

Extrapolating the radiative tail using Bergström:

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}\,.$$

Theoretical prediction [Dorokhov, Ivanov '07, '10]

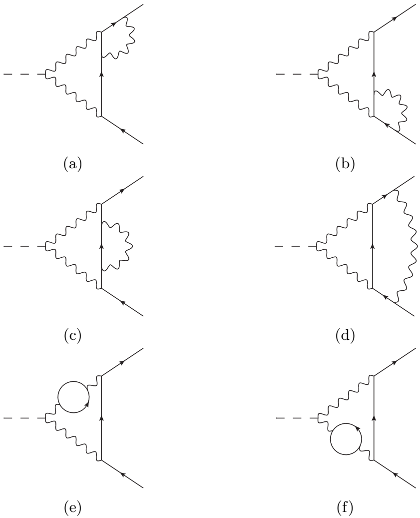
$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}\,. \qquad (1)$$

3.3 $\sigma \Rightarrow$ New physics?

In any case, radiative corrections play an important role in the analysis

$\pi^0 \to e^+e^-$

Radiative corrections $\to$ two-loop graphs



$\pi^0 \to e^+e^-$

- two-loop contributions, together with Bremsstrahlung (= Dalitz) [Dorokhov et al. '08], [Vasko,Novotny '11], [Husek,KK,Novotny'14]
- counter-term chiral Lagrangian for $\pi^0 l\bar{l}$ [Savage et al'92]
- modelled using the resonances [Knecht '99]

$$\chi^{(r)}_{\mathsf{LMD}}(M_\rho) = 2.2 \pm 0.9$$

- rem.: different models possible, see e.g. [Masjuan,Sanchez-Puertas '15], for $\chi = 2.76(23)$
- KTeV implies [Husek,KK,Novotny'14]

$$\chi^{(r)}_{\mathsf{KTeV}}(M_\rho) = 4.5 \pm 1.0$$

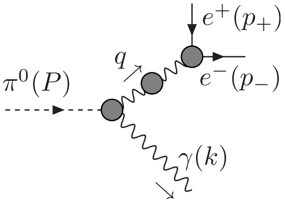
- original discrepancy down to 2 σ level
- note: weak contributions mediated via $\pi^0 \to Z^* \to e^+e^-$ three orders of magnitude smaller than EM

Dalitz decay

[K.K., Knecht, Novotný '06]

History

- First calculated by [Dalitz '51].
- Radiative corrections studied by [Joseph '60], [Lautrup, Smith'71], [Mikaelian, Smith'72]
- and during the 1980s by Tupper, Grose, Samuel, Lambin, Pestieau, Roberts...



$$x = m_{ee}^2/M_\pi^2, \quad y = \frac{E_+ - E_-}{E_\gamma}\Big|_{\pi^0\to 0}$$

NLO studied via $\delta(x,y)$ and $\delta(x)$:

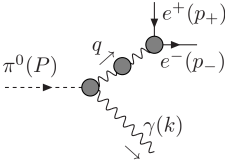
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}y} = \delta(x,y)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y}, \quad \frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \delta(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}.$$

with (point-like pion)

$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y} = \frac{\alpha^3}{(4\pi)^4}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\left[M_{\pi^0}^2 x(1+y^2) + 4m^2\right],$$

$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\alpha^3}{(4\pi)^4}\frac{8}{3}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\left(xM_{\pi^0}^2 + 2m^2\right).$$

Dalitz decay: slope parameter



$$\Gamma_\mu^{1\gamma R}(p_+,p_-,k) = \mathrm{i}e^2 \varepsilon_\mu{}^{\nu\alpha\beta} q_\alpha k_\beta \, \mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \, \mathrm{i}D^T_{\nu\rho}(q)(-\mathrm{i}e)\Lambda^\rho$$

$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2)$ is related to the doubly off-shell form factor $\mathcal{A}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

$$\int d^4x \, e^{il\cdot x} \langle 0|T(j^\mu(x) j^\nu(0)|\pi^0(P)\rangle = -\mathrm{i}\varepsilon^{\mu\nu\alpha\beta} l_\alpha P_\beta \, \mathcal{A}_{\pi^0\gamma^*\gamma^*}(l^2, (P-l)^2)$$

One can define a slope parameter a_π

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \;=\; \mathcal{F}_{\pi^0\gamma\gamma^*}(0) \left[1 \,+\, a_\pi \, \frac{q^2}{M_{\pi^0}^2} \,+\cdots\right],$$

$$\frac{\mathrm{d}\Gamma^{exp}}{\mathrm{d}x} - \delta_{QED}(x) \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} [1 + 2x \, a_\pi].$$

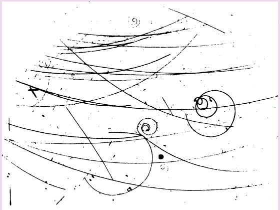
Dalitz decay: summary of [KK,Knecht,Novotny'06] & [Husek,KK,Novotny'15]

- Our works provide a detailed analysis of NLO radiative corrections to the Dalitz decay amplitude.
- The off-shell pion-photon transition form factor was included:
  this requires a treatment of non perturbative strong interaction effects
- The one-photon irreducible contributions, which had been usually neglected, were included.
  We have shown that, although these contributions are negligible as far as the corrections to the total decay rate are concerned, they are however sizeable in regions of the Dalitz plot which are relevant for the determination of the slope parameter a_π of the pion-photon transition form factor.
- Our prediction for the slope parameter $a_\pi = 0.029 \pm 0.005$ is in good agreement with the determinations obtained from the (model dependent) extrapolation of the CELLO and CLEO data.
  Unfortunately, the experimental error bars on the latest values of a_π extracted from the Dalitz decay are still too large
- used in NA48 analysis for the search of dark photon [1504.00607]

Double Dalitz decay

History

- determination of parity of pion via $\pi^0 \to \gamma\gamma$[Yang '50], experimentally difficult
- using internal conversion [Kroll, Wada '55]
- First measurement (and todays PDG number) [Samios et al. '62], hydrogen bubble chamber: 8×10^6 π^0-decays on approx 800 thousand pictures $\to$ 200 double-dalitzs (10t. dalitzs)



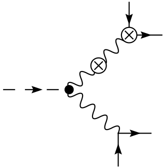
  - $B(\pi^0 \to e^+e^-e^+e^-) = (3.18 \pm 0.3) \times 10^{-5}$
  - π^0 is pseudoscalar (only 3.6σ significance)

- [Miyazaki and E. Takasugi '73] adding the effect of lepton exchange to Kroll-Wada
- new study: [Barker et al. '03] (some disagreement with previous)
- new measurement: KTeV '08
  - confirmation of negative π^0 parity
  - first searches for parity & CPT violation
  - $B(\pi^0 \to e^+e^-e^+e^-) = (3.46 \pm 0.19) \times 10^{-5}$

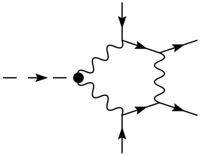
Double Dalitz decay

collaboration with T.Husek, M. Knecht, J. Novotný and P.Sanchez Puertas

It seems natural to convert the on-shell photon to the other Dalitz pair and obtain immediately Double Dalitz decay. This is true for LO:



However, for higher orders we have new topologies [Barker et al. '03]:



We are recalculating these results and try to put them together with our parameters introduced in the context of $\pi^0 \to e^+e^-\gamma$.

Further plans and projects

- $\pi^0 \to \mathit{Ps}\,\gamma$
  - similar to the Dalitz decay (threshold below e^+e^-)
  - non-relativistic bound state formalism Nemenov '72
  - radiative correction small Vysotskii '79
  - experiment Serpukhov '89 (=pdg)
- $\pi^0 \to 4\gamma$ and $\pi^0 \to 3\gamma$
  - interesting probe of the light-by-light scattering
  - theoretical estimate is 3 orders below the experimental limit
- $\pi^0 \to \nu\bar{\nu}$ and $\pi^0 \to \nu\bar{\nu}\gamma$
  - helicity suppression vs. weak radiative suppression
  - experimental limits far below reliable theoretical predictions
- η/K similar decays
  - already in preparation: $\eta \to \ell^+\ell^-\gamma$: in collaboration with T. Husek, S. Leupold and J. Novotny
  - work in progress: $\eta \to \gamma\gamma$: in col. with J.Bijnens, E. Passemar
- systematic study of odd intrinsic parity sector
  - e.g. transition formfactor $F_{\pi\gamma\gamma}$, continuation of [KK,Novotny'11], in collaboration with T.Kadavy and J.Novotny

π^0 and new physics

- one crucial ingredient for chiral dynamics: F_π
- F_π from π_{l2} based on SM; deviation from standard $V - A$ leads to an effective $\hat{F}_\pi$ [Bernard,Oertel,Passemar,Stern '08]

$$F_\pi^2 = \hat{F}_\pi^2(1+\epsilon), \quad \text{with} \qquad \epsilon \sim V_R^{ud}/V_L^{ud}$$

- connection between F_π and F_{π^0} tiny [KK,Moussallam '09]

$$\left.\frac{F_{\pi^+}}{F_{\pi^0}}\right|_{QCD} - 1 = \frac{B^2(m_d-m_u)^2}{F_\pi^4}\left[-16\,c_9^r(\mu) - \frac{l_7}{16\pi^2}\left(1+\log\frac{m_\pi^2}{\mu^2}\right)\right]$$
$$\simeq 0.7\times 10^{-4}\ .$$

- $\Rightarrow$ one can thus use $\pi^0 \to \gamma\gamma$ for determination of F_π:

$$F_\pi \approx F_{\pi^0} = 93.85 \pm 1.3(\text{exp.}) \pm 0.6(\text{theory})\,\text{MeV} = 93.85 \pm 1.4\,\text{MeV}$$

- n.b. $\hat{F}_\pi = 92.22(7) \Rightarrow \epsilon \approx 3-4\%$ 1σ significance for right-handed currents

Summary

$\eta \to 3\pi$: fashionable subject, theoretically studied e.g. by:

- ChPT (Gasser, Leutwyler '85; Bijnens, Ghorbani '07)
- EM corrections (Baur,Kambor,Wyler'96; Ditsche, Kubis, Meiner '08)
- NREFT (Gullstrom, Kupsc,Rusetsky '08; Schneider, B. Kubis, and C. Ditsche,'11),
- new dispersive analyses (KK,Knecht,Novotny,Zdrahal '11, Albaladejo, Moussallam '15, Guo et al '15+'16, Colangelo, Lanz, Leutwyler, Passemar '16)
- resummed ChPT (Kolesar,Novotny '16)

Short overview of π^0-related processes presented: $\pi^0 \to \gamma\gamma$, $\pi^0 \to e^+e^-$, $\pi^0 \to e^+e^-\gamma$ (Dalitz decay), $\pi^0 \to e^+e^-e^+e^-$ (double Dalitz) with some future plans.