Is it possible to detect a transient violation of the Pauli principle at the subfemtosecond scale ?

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Seminar organized within the project:

"Hunt for the "impossible atoms": the quest for a tiny violation of the Pauli Exclusion Principle. Implications for physics, cosmology and philosophy,"

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John Templeton Foundation

Outlook of the talk

Premise number 1: femtosecond pump-and-probe experiments

Premise number 2: Corinaldesi's paper

Diversion number 1: retarded interaction & the electromagnetic field

Diversion number 2: 'psychological' aspects

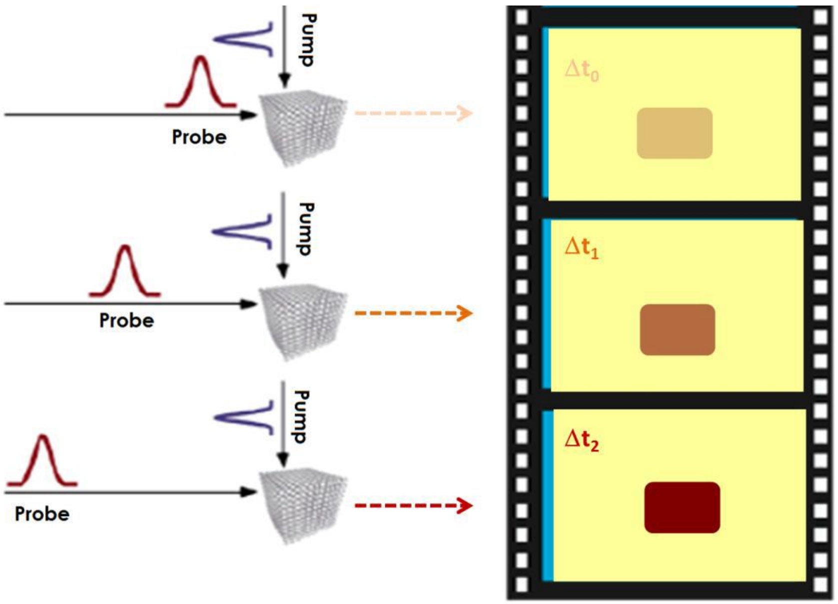
Analysis number 1: Dirac equation and the zitterbewegung

Analysis number 2: spin-statistics in the mesoscopic world

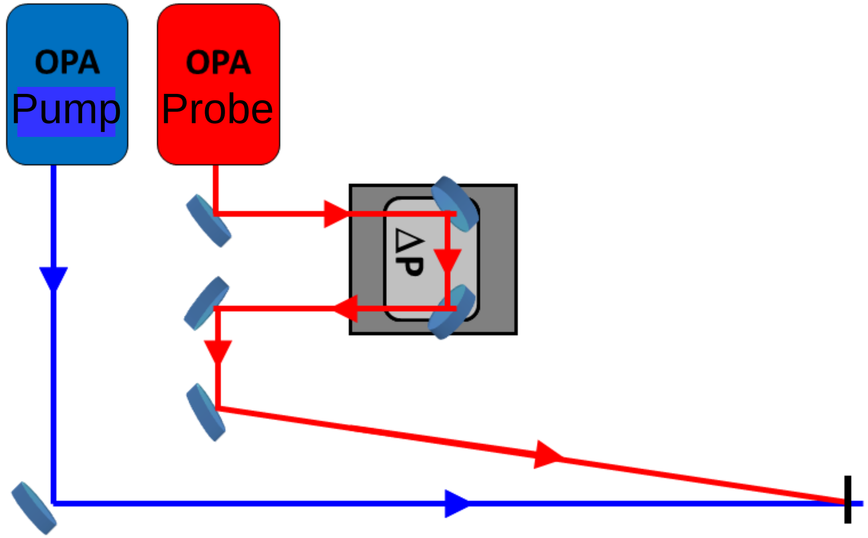
Conclusions: towards a non-orthodox view ?

Premise N. 1

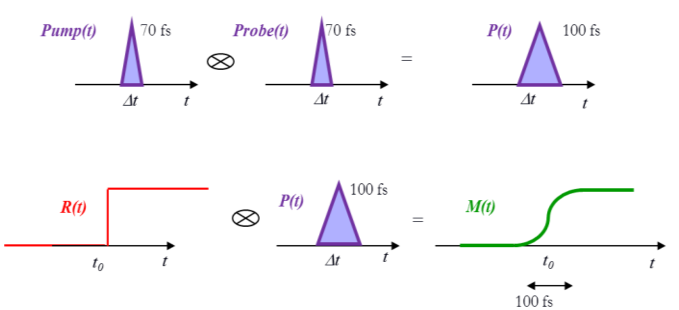
Nowadays pump-and-probe experiments are becoming extremely important in condensed-matter and atomic physics.



Schematic representation of the time-resolved "film" of pump-and-probe dynamics. At different time delay the probe monitors different states by the change physical properties.

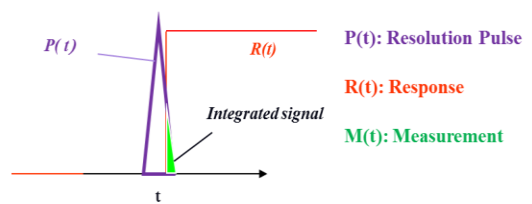


30 cm → 1 ns ; 30 μm → 100 fs



From A. Marino, Ph.D. Thesis, 2015

$$P(t) = \sqrt{\Delta t^2_{pump} + \Delta t^2_{probe}}$$



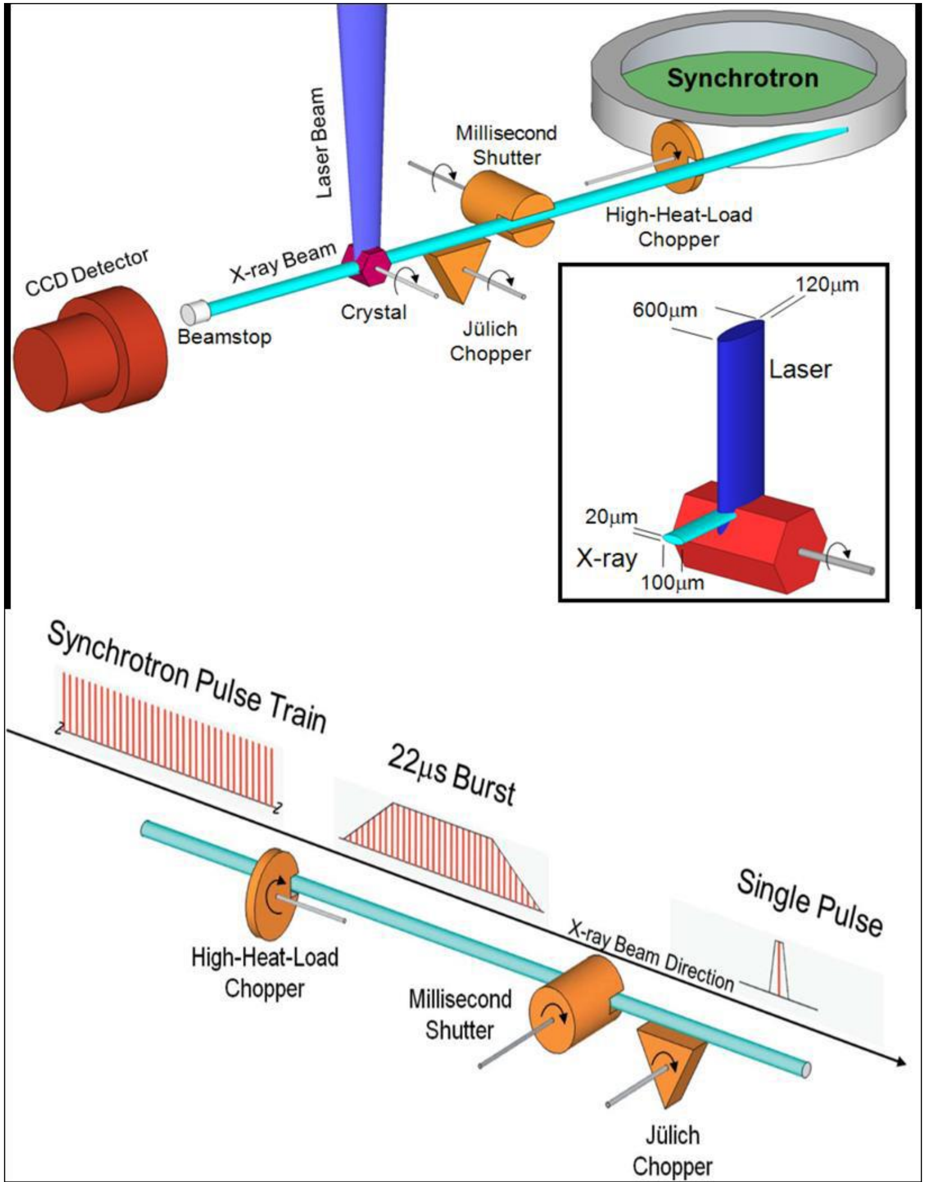
Premise N. 1

Optical regime

Solid state group laboratory femtosecond pump-probe equipment, Institute de Physique de Rennes, University of Rennes 1, France.

Setup of the BioCARS beamline at APS Synchrotron. A mechanical chopper system is used to isolate single X-ray pulses from the storage ring. The laser beam is oriented orthogonal to the X-ray beam and intersects the crystal at the center of the goniometer rotation. The chopper/shutter includes a high-heat-load chopper, which produces a 22 ms burst of X-rays and the Julich chopper capable of isolating a single 50 ps X-ray pulse at a rate of 1 kHz.
From T. Graber et al. *J. Synchrotron Rad.* 18, 658–670 (**2011**)

X-ray regime



Premise N. 1

PRL 113, 227402 (2014) PHYSICAL REVIEW LETTERS week ending 28 NOVEMBER 2014

Sequential Activation of Molecular Breathing and Bending during Spin-Crossover Photoswitching Revealed by Femtosecond Optical and X-Ray Absorption Spectroscopy

Marco Cammarata,[1] Roman Bertoni,[1] Maciej Lorenc,[1] Hervé Cailleau,[1] Sergio Di Matteo,[1] Cindy Mauriac,[2] Samir F. Matar,[2] Henrik Lemke,[3] Matthieu Chollet,[3] Sylvain Ravy,[4] Claire Laulhé,[4] Jean-François Létard,[2] and Eric Collet[1,*]

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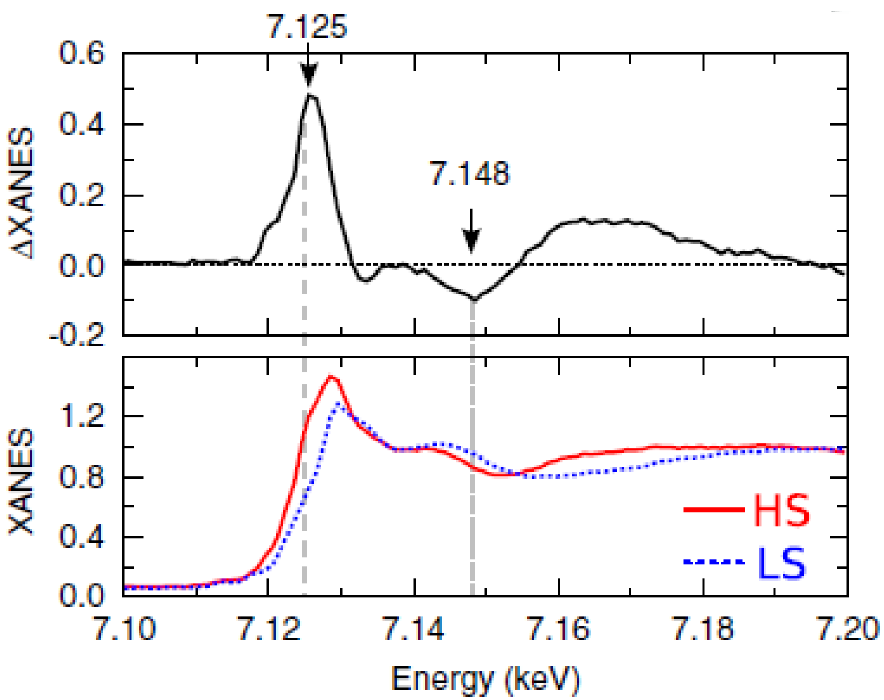

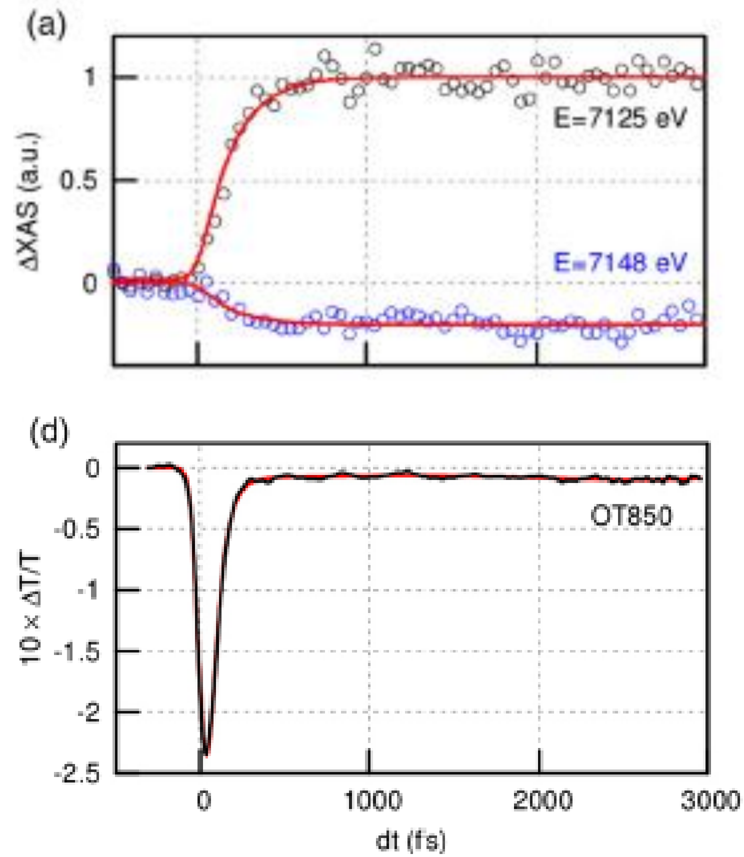

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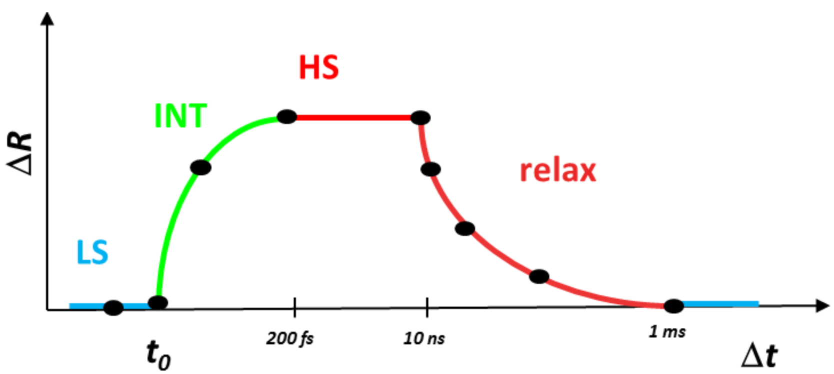
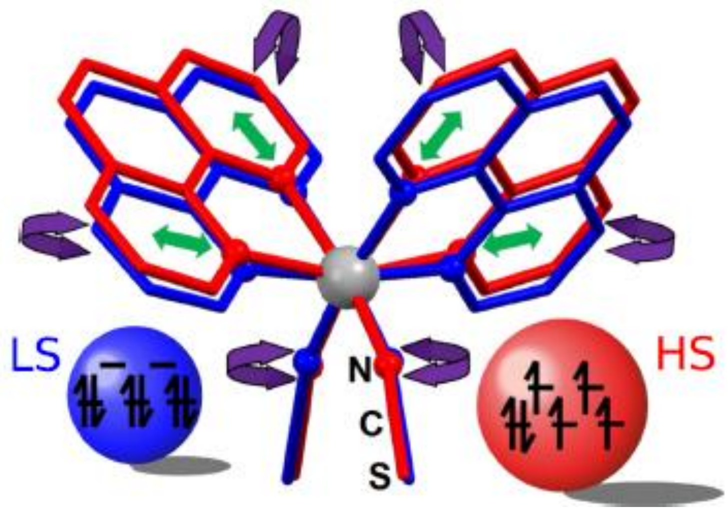
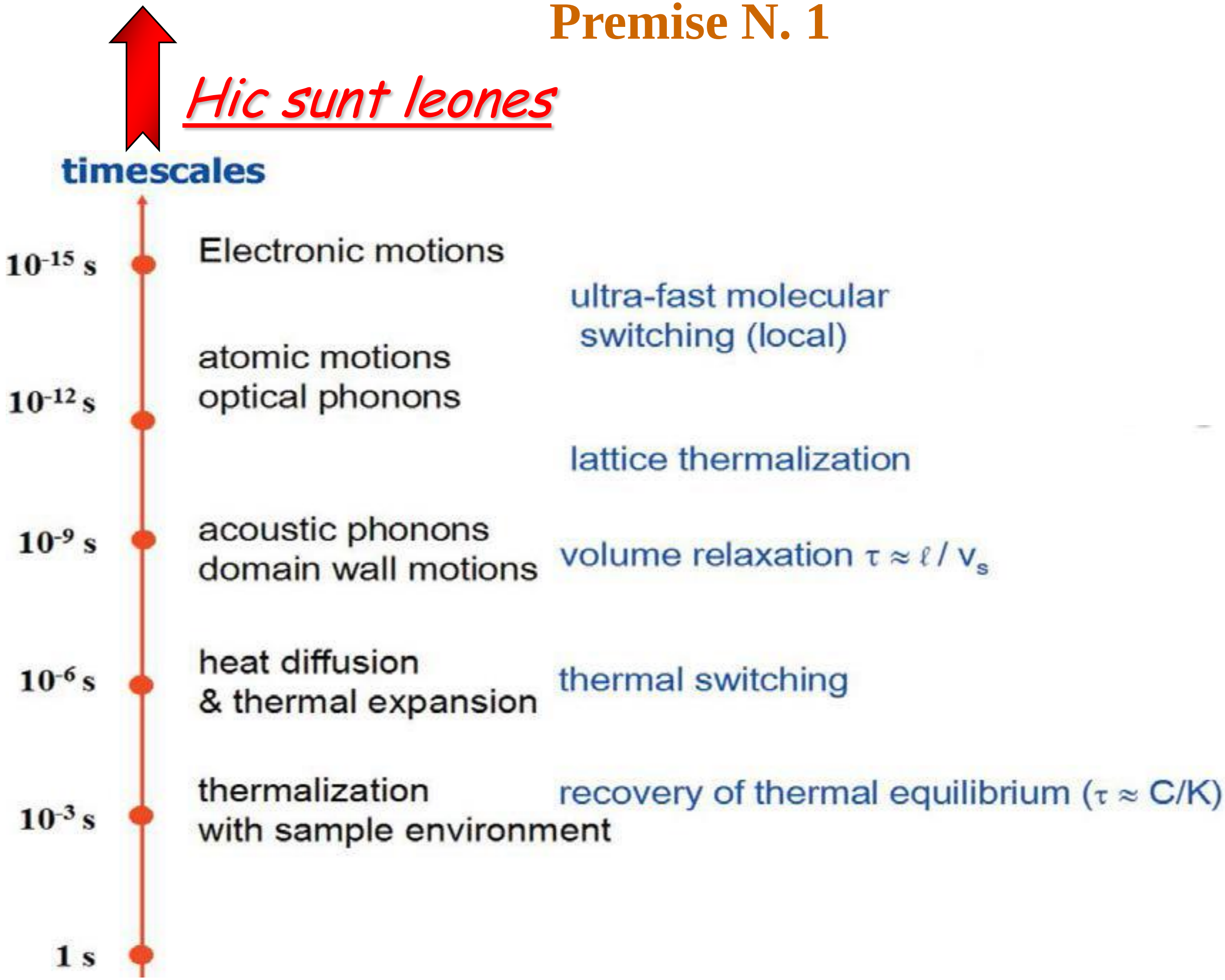


Temporal evolution of the spin conversion.



Premise N. 1



Hic sunt leones
timescales
10^{-15} s
Electronic motions
ultra-fast molecular switching (local)
10^{-12} s
atomic motions
optical phonons
lattice thermalization
10^{-9} s
acoustic phonons
domain wall motions
volume relaxation $\tau \approx \ell / v_s$
10^{-6} s
heat diffusion
& thermal expansion
thermal switching
10^{-3} s
thermalization
with sample environment
recovery of thermal equilibrium ($\tau \approx C/K$)
1 s

Is it possible to detect a transient violation of the Pauli principle at the subfemtosecond scale ?

Premise N. 2

Corinaldesi's idea that Pauli principle can be violated in short time transients

SUPPLEMENTO AL NUOVO CIMENTO N. 3, 1967
VOLUME V

Model of a Dynamical Theory of the Pauli Principle.

E. CORINALDESI

Department of Physics, Boston University - Boston, Mass.

(ricevuto il 4 Marzo 1967)

This note does not question the fact that nature seems to order systems of « identical » bosons and fermions in a special way which we describe by means of symmetric and antisymmetric wave functions. Our only aim is to show that this ordering may be conceived as a dynamical process of which only the final stage is normally observed.

Premise N. 2

Corinaldesi's idea that Pauli principle can be violated in short time transients

Consider the 2-particle Lagrangian of the conventional non-relativistic theory:

$$\mathscr{L} = -\frac{\hbar^2}{2m}\left(\nabla_1\psi^\dagger\cdot\nabla_1\psi + \nabla_2\psi^\dagger\cdot\nabla_2\psi\right) - V\psi^\dagger\psi + \ldots + \frac{\hbar}{2i}\left(\frac{\partial\psi^\dagger}{\partial t}\psi - \psi^\dagger\frac{\partial\psi}{\partial t}\right)$$

And add to it the following non-linear term (written here for fermions):

$$\mathcal{L}_{non-lin} = \tfrac{1}{2}\left(\Psi^+(1,2)+\Psi^+(2,1)\right)\left(\Psi(1,2)+\Psi(2,1)\right)\left(i\ln\xi\right)^3$$

where: $\xi = \dfrac{\left(\Psi^+(1,2)+\Psi^+(2,1)\right)\left(\Psi(1,2)-\Psi(2,1)\right)}{\left(\Psi^+(1,2)-\Psi^+(2,1)\right)\left(\Psi(1,2)+\Psi(2,1)\right)}$ is a phase !

Notice that the non-linear term is zero for both non-overlapping fermions (ξ=1, so lnξ =0), and for symmetrized wave-functions, because $\Psi(1,2) = -\Psi(2,1)$ (!!!)

Premise N. 2

Corinaldesi's idea that Pauli principle can be violated in short time transients

Define: $$N^{(sym)} = \tfrac{1}{2}\int \left(\Psi^{+}(1,2) - \Psi^{+}(2,1)\right)\left(\Psi(1,2) - \Psi(2,1)\right) d^3x_1 d^3x_2$$

$$N^{(no-sym)} = \tfrac{1}{2}\int \left(\Psi^{+}(1,2) + \Psi^{+}(2,1)\right)\left(\Psi(1,2) + \Psi(2,1)\right) d^3x_1 d^3x_2$$

In this framework, the equation of motion leads to the interesting properties:

1) When the two wave-packets do not overlap, then: $N^{(sym)} = N^{(no-sym)} = 1$

2) When the two wave-packets start overlapping, then: $\begin{cases} \dfrac{dN^{(sym)}}{dt} \geq 0 \\ \dfrac{dN^{(no-sym)}}{dt} \leq 0 \end{cases}$ **up to:** $\begin{cases} N^{(sym)} = 2 \\ N^{(no-sym)} = 0 \end{cases}$

with the property (conservation of probability) : $$\frac{dN^{(sym)}}{dt} + \frac{dN^{(no-sym)}}{dt} = 0$$

Premise N. 2

Corinaldesi's idea is that Pauli principle can be violated in short time transients

Conclusions of Corinaldesi's paper:

The new Schrödinger equation can be expected to yield physical predictions differing from those derived from the conventional theory, when times are involved which are shorter than a characteristic « symmetrization time ».

For charged fermions this would amount to a reformulation of electromagnetic interactions in which the electromagnetic field would play the role of a symmetrizing agent (!)

This, of course, leaves three questions open:

1) How could the electromagnetic field act this way ?
2) What would be a typical value for the «symmetrization time» ?
3) How could it be possible to measure it ?

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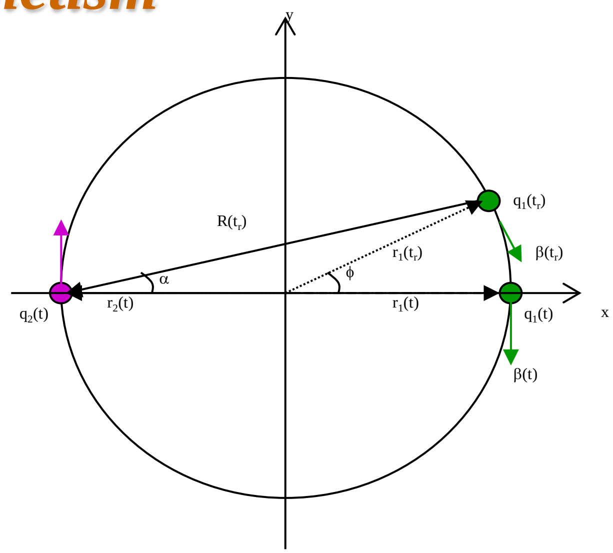
Reminder of classical electromagnetism



Expression of the Lienard-Wieckert retarded electric field at q_2:

$$\vec{E}(R,t) = \frac{q_1(1-\beta^2)(\hat{R}-\vec{\beta}(t_r))}{R^2(1-\hat{R}\cdot\vec{\beta}(t_r))^3} + \frac{q_1\hat{R}\times[(\hat{R}-\vec{\beta}(t_r))\times\dot{\vec{\beta}}(t_r)]}{cR(1-\hat{R}\cdot\vec{\beta}(t_r))^3}$$

The rate of work done by q_1 on q_2 to order β^4 is: $W_2 = \dfrac{\mu_0 q_1 q_2 a^2}{6\pi c}$

Oscillating dipoles: Lienard/Wiechert emitting power $p_{emit} = \dfrac{\mu_0 q_1^2}{6\pi c}\dfrac{a^2 - \frac{(\vec{v}\times\vec{a})^2}{c^2}}{1-\frac{v^2}{c^2}}$

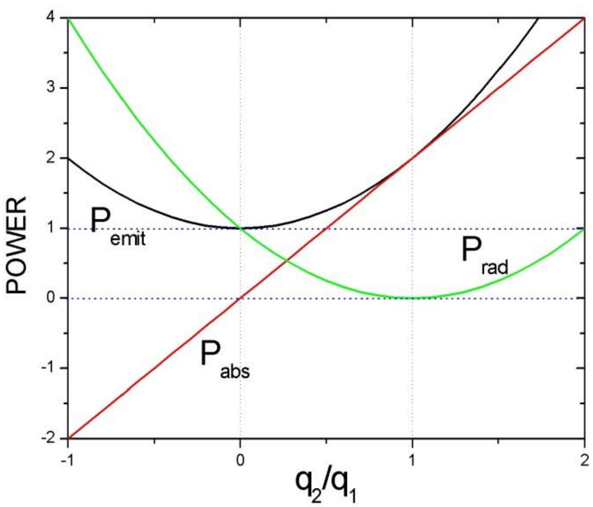
$$\Rightarrow p_{abs} = W_1 + W_2 \cong 2\frac{\mu_0 q_1 q_2}{6\pi c}a^2$$

$$\Rightarrow p_{rad} = p_{emit} - p_{ab}$$

$$\Rightarrow \textit{If } \vec{v}\perp\vec{a} \Rightarrow p_{emit} = \frac{\mu_0 q_1^2}{6\pi c}a^2$$

$\Rightarrow$ Total radiated power of the system proportional to the square of the dipole moment:

$$\boxed{p_{rad} = \frac{\mu_0(q_1-q_2)^2}{6\pi c}a^2}$$



Ping-pong motion in hydrogen atom

[Jayme De Luca, Phys. Rev. E 73, 026221 (2006)]

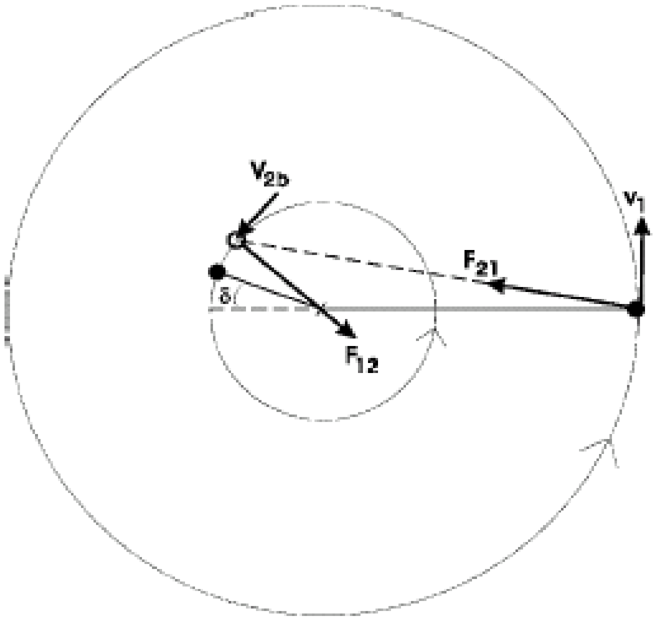
The infinite proton-mass limit is a singular condition that cannot be treated perturbatively (it does not allow retardation effects)

Action for the electron: $$\int \frac{1-\vec{v}_1 \cdot \vec{v}_{2r}}{r_{12r}(1-\vec{n}_{12r} \cdot \vec{v}_{2r})} dt_1$$

$\vec{v}_1$ **= electron velocity**

$\vec{v}_{2r}$ **= retarded proton velocity ;** $\vec{n}_{12r} = \dfrac{\vec{r}_{12r}}{r_{12r}}$

r_{12r} **= electron-proton distance at the retarded time**



Results of Lyapunov stability analysis:

1) Resonant orbits are quantized naturally because of delay
2) Angular momenta are ~ integer multiples of a constant

$\Rightarrow$ ***Ping-pong phenomenon is a non-trivial feature absent in ODE***

Functional differential equations

General characteristics of FDE:

1) Solutions are quantized due to retardation (no scale invariance)

Instead of an algebraic associated equation, you end up with a transcendental (trigonometric) associated equation → quantized solutions

2) Need for a whole set of past data in the interval [0,t_r]

For example: $\dot{x}(t) = x(t - \frac{\pi}{2})$

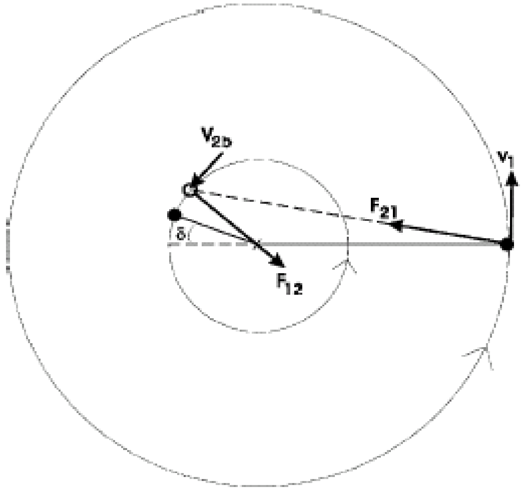
$\Rightarrow$ $x(t) = a\cos t + b\sin t$ …for any a and b ! .

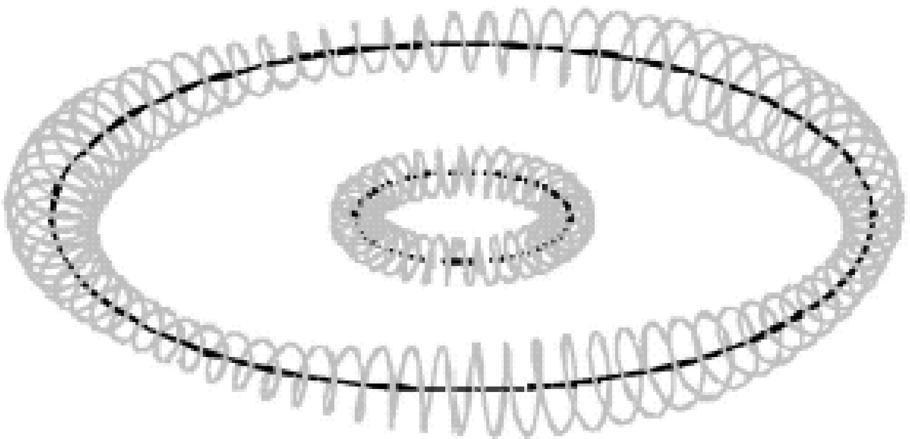
Ping-pong motion in hydrogen atom (I)

[Jayme De Luca, Phys. Rev. E 73, 026221 (2006)]

The infinite proton-mass limit is a singular condition that cannot be treated perturbatively (it does not allow retardation effects)

Angular momentum is not conserved !
(purely under the action of internal forces)





Beatings of modes leads to a no-radiation Poynting condition !

A parenthesis: some ‘psychological’ considerations

Here we analyze some ‘truths’ that are not usually viewed as such...

...for reasons usually dependent on the way quantum mechanics is taught to us

Psychological aspects (I)

Wave-like behaviour should not be identified with Ψ !

1 Newton eq.

(in Hamilton-Jacobi form for a statistical set)

$$\boxed{i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}=-\frac{\hbar^2\nabla^2}{2m}\Psi(\vec{r},t)+V(\vec{r},t)\Psi(\vec{r},t)+\frac{\hbar^2\nabla^2\left|\Psi(\vec{r},t)\right|}{2m\left|\Psi(\vec{r},t)\right|}\Psi(\vec{r},t)}$$

$$\boxed{i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}=-\frac{\hbar^2\nabla^2}{2m}\Psi(\vec{r},t)+V(\vec{r},t)\Psi(\vec{r},t)}$$

2 Schrödinger eq. with

$$\boxed{\left|\Psi(\vec{r},t)\right|^2=1,\quad\forall\ (\vec{r},t)}$$

Psychological aspects (II)

Schrödinger's equation can be written non-linearly:

$$\boxed{i\hbar\frac{\partial\Psi(r,t)}{\partial t} = -\frac{\hbar^2\nabla^2}{2m}\Psi(r,t) + V(r,t)\Psi(r,t)}$$

If we put: $\Psi(\vec{x},t) = R(\vec{x},t)e^{iS(\vec{x},t)/\hbar}$ and separate Re and Im:

$$\Rightarrow \begin{cases} -\dfrac{\partial S}{\partial t} = \dfrac{(\nabla S)^2}{2m} + V(r) + Q \equiv E & \text{(Hamilton-Jacobi equation)} \\ \vec{\nabla}\cdot\vec{J} - \dfrac{\partial \rho}{\partial t} = 0 & \text{(continuity equation)} \quad \begin{cases} \rho = \Psi^*\Psi = R(\vec{x},t)^2 \\ \vec{j} = \frac{i\hbar}{2m}(\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi) \end{cases} \end{cases}$$

where: $Q = -\dfrac{\hbar^2}{2m}\dfrac{\nabla^2 R}{R}$ is called quantum potential

Psychological aspects (III)

Ψ has nothing to do with our (3+1)D space (manybody !)

$$i\hbar\frac{\partial\Psi(\vec{r}_1,\vec{r}_2,t)}{\partial t}=-\left(\frac{\hbar^2\nabla_1^2}{2m_1}+\frac{\hbar^2\nabla_2^2}{2m_2}\right)\Psi(\vec{r}_1,\vec{r}_2,t)+V(\vec{r}_1,\vec{r}_2,t)\Psi(\vec{r}_1,\vec{r}_2,t)$$

For 2 particles, it ‘lives’ in (6+1)D space (manybody !)

…and in the relativistic case we would even have two proper times…

Psychological aspects (IV)

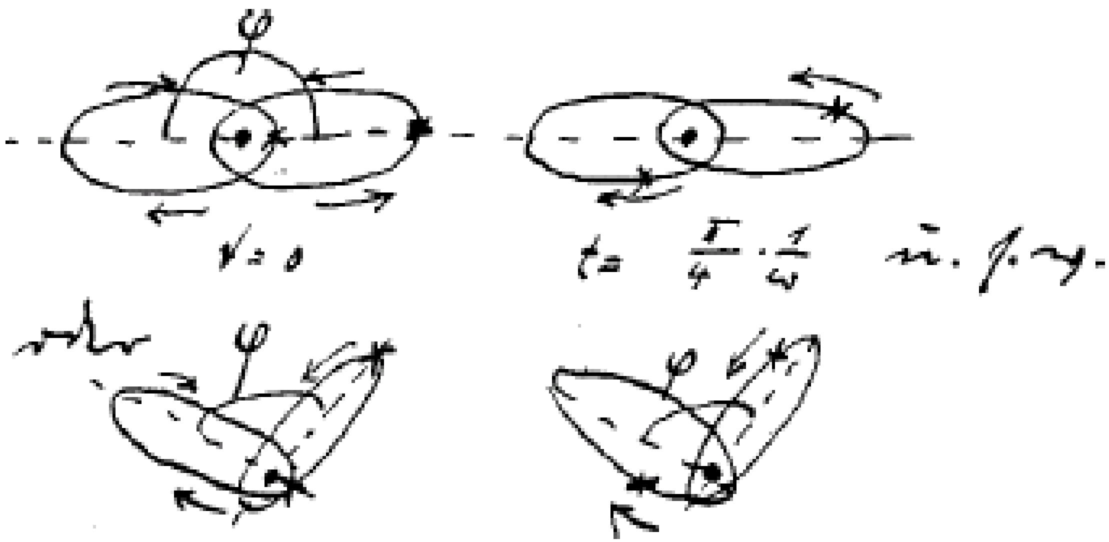


FIG. 3. Sketch of the periodic electron-pair motion proposed by Heisenberg and Sommerfeld as a candidate for a classical ground-state configuration of helium. The figure is copied from a letter of Heisenberg to Sommerfeld (Heisenberg, 1922). It was never published by Heisenberg.

We move to a hydrodynamic analogy to QM: wave-particle symbiosis

Movie 1

Movie 2

The electron spin in real space.

If I can't picture it, I can't understand it

(A. Einstein)

…what is proved by impossibility proofs…

…is lack of imagination…

(J. Bell)

Is it true ?

"These symbols (operators q and p), as indicated by the use of imaginary numbers, are not susceptible of pictorial representations…" (N.Bohr, Dialectica **34**, 312 (1948))

"Spin is an essential quantum-mechanical property,…a classically not describable two-valuedness" *and* *"The concrete picture of rotation must be replaced by mathematical characteristics of the representation of rotations in 3-dimensional space…"* (W. Pauli)
in M. Jammer, "The conceptual development of quantum mechanics, pp. 152 and 153

A step back to Hamilton's findings in 1850

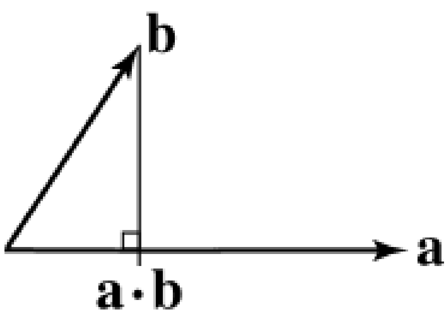
Pauli matrices in geometric algebra (GA)

Geometric Product **ab** of two vectors **a** and **b** implies two other products with familiar geometric interpretations:

$$\boxed{\mathbf{ab} = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \wedge \mathbf{b}}$$

Inner Product: $\mathbf{a} \cdot \mathbf{b} = \tfrac{1}{2}(\mathbf{ab} + \mathbf{ba}) = \mathbf{b} \cdot \mathbf{a}$

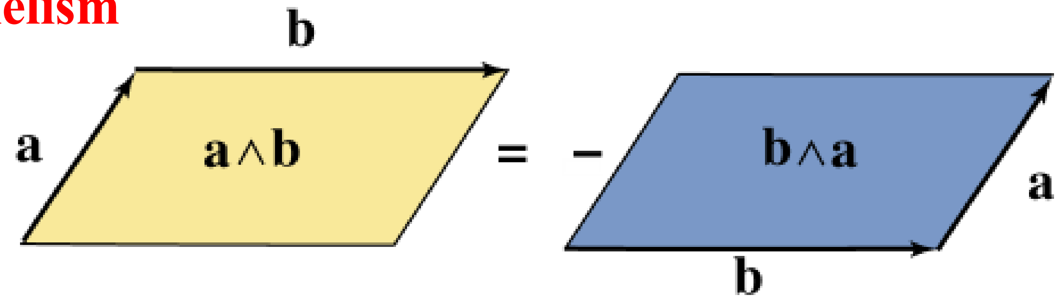
Anticommutator = 0 means orthogonality



Outer Product: $\mathbf{a} \wedge \mathbf{b} = \tfrac{1}{2}(\mathbf{ab} - \mathbf{ba}) = -\mathbf{b} \wedge \mathbf{a}$

Commutator = 0 means parallelism

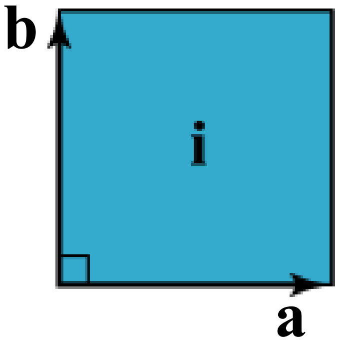
Bivector represents an oriented area



Geometric interpretation of 'i'

$$\mathbf{a}\cdot\mathbf{b}=0 \quad \Rightarrow \quad \mathbf{i}=\mathbf{a}\wedge\mathbf{b}=\mathbf{ab}=-\mathbf{ba}$$

$$\mathbf{a}^2=\mathbf{b}^2=1 \quad \Rightarrow \quad \boxed{\mathbf{i}^2=-1}$$



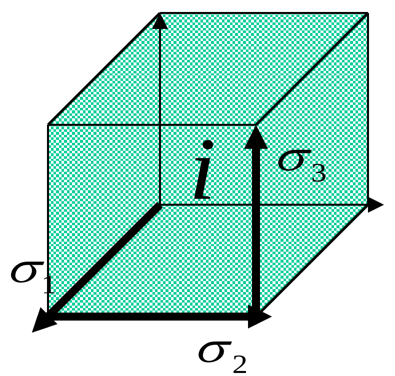
So
- **i** ≈ *oriented unit area* for a plane

Proof: $(\mathbf{ab})^2=(\mathbf{ab})(\mathbf{ab})=-(\mathbf{ba})(\mathbf{ab})=-\mathbf{a}^2\mathbf{b}^2=-1$

The Pauli algebra is recovered geometrically:

$$\sigma_a\sigma_b=\delta_{ab}+\mathbf{i}\varepsilon_{abc}\sigma_c$$

$$\Rightarrow \begin{cases} \sigma_a\sigma_b+\sigma_b\sigma_a=0 & \textbf{(orthogonal)} \\ \sigma_a\sigma_b-\sigma_b\sigma_a=0 & \textbf{(parallel)} \end{cases}$$



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Dirac equation & ZBW

Dirac equation: $$\left(\alpha_0 mc^2 + \sum_{j=1}^{3} \alpha_j p_j\, c\right)\psi(\mathbf{x},t) = i\hbar\frac{\partial\psi}{\partial t}(\mathbf{x},t)$$

with $\alpha_\mu\alpha_\nu+\alpha_\nu\alpha_\mu=2\delta_{\mu\nu}$

Zitterbewegung: $$\vec{v} \equiv \frac{d\vec{x}}{dt} = \frac{i}{\hbar}[H,\vec{x}] = c\vec{\alpha}$$

$$i\hbar\frac{d\vec{\alpha}}{dt} = -2c\vec{p} + 2H\vec{\alpha}$$ $\Rightarrow$ **free particle (p & H constant):** $$\frac{d\vec{x}}{dt} = c^2H^{-1}\vec{p} + c\vec{\alpha}_0 e^{-2iHt/\hbar}$$

$\Rightarrow$ **from which we get:** $$\vec{x}(t) = \vec{x}_0 + c^2H^{-1}\vec{p}t + \tfrac{1}{2}i\hbar c\vec{\alpha}_0 H^{-1}e^{-2iHt/\hbar}$$
$$= \vec{x}_A(t) + \vec{\xi}(t)$$

with $$\omega_{ZBW} = \frac{H}{\hbar} \geq \frac{2mc^2}{\hbar}$$

Moreover: $$\vec{p} \to \vec{p} - e\vec{A}(\vec{x},t)$$

From Dirac to Schrodinger equation

Non-relativistic limit of Gordon decomposition:

$$m\rho\,\vec{\mathrm{v}} = m\rho\,\vec{u} + \vec{\nabla}\times(\rho\,\vec{S}) \quad \Rightarrow \quad \vec{\mathrm{v}} = \vec{u} + \vec{w}$$

usual definition of momentum: $\vec{p} = i\hbar[\Psi^*(\vec{\nabla}\Psi) - (\vec{\nabla}\Psi^*)\Psi]$

However:
$$\begin{cases} \vec{\nabla}\cdot m\rho\,\vec{\mathrm{v}} = \vec{\nabla}\cdot m\rho\,\vec{\mathrm{u}} \qquad \text{and} \\ \left\langle \vec{r}\times m\rho\,\vec{\mathrm{v}} \right\rangle = \left\langle \vec{r}\times m\rho\,\vec{u} \right\rangle + 2\left\langle \vec{S} \right\rangle \equiv \left\langle \vec{L} \right\rangle + 2\left\langle \vec{S} \right\rangle \end{cases}$$

$\Rightarrow$ v = charge velocity ; u = velocity of the center of mass

From Dirac to Schrodinger equation

Kinetic energy of u, v and w: $$\frac{m\mathrm{v}^2}{2} = \frac{m\mathrm{u}^2}{2} + \frac{m\mathrm{w}^2}{2}$$

$\Rightarrow$ if the spin is independent of position: $\vec{S}(\vec{r}) = \vec{S}$

the contribution of the α–ZBW motion $$\vec{w} = \frac{\vec{\nabla}\rho \times \vec{S}}{m\rho}$$

in the Hamiltonian is: $$\frac{1}{2}mw^2 = \frac{S^2(\nabla\rho)^2}{2m\rho^2} = \frac{\hbar^2(\nabla\rho)^2}{8m\rho^2} \equiv Q$$

This shows that, in the Schrödinger equation, $\hbar$ stands for twice the spin !

Non-relativistic hydrogen atom

Given the Hamilton-Jacobi equation: $$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V(r) + Q \equiv E$$

where: $Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ and: $\Psi(\vec{x},t) = R(\vec{x},t)e^{iS(\vec{x},t)/\hbar}$

the spin kinetic energy term Q is responsible for H-atom eigenvalues:

$$\boxed{Q(100) = E_{1s} - V(r)} \quad \boxed{Q(200) = E_{2s} - V(r)} \quad \boxed{Q(210) = E_{2s} - V(r)}$$

$$\boxed{Q(21\pm 1) = E_{2s} - V(r) - \frac{\hbar^2}{2mr^2 \sin^2 \vartheta} = E_{2s} - V(r) - \frac{(\nabla S)^2}{2m}}$$

the spin velocity field **w**(**r**) stabilizes only "true" orbitals

Dirac equation, ZBW & Schrodinger equation

Foldy-Wouthuysen transformation:

$$e^{iS} = \frac{\beta[\beta mc^2 + c\vec{\alpha}\cdot\vec{p} + \beta W]}{\sqrt{2W(W+mc^2)}}$$

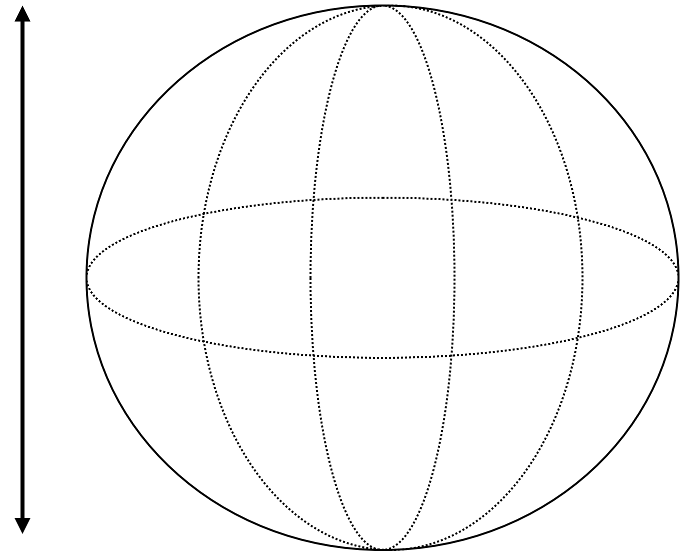
$$\boxed{H_{FW} = \beta\sqrt{m^2c^4 + p^2c^2}} \equiv \beta\, W$$

$$\boxed{\vec{J}_{FW} = \beta(\vec{L} + 2\vec{S})} \;;\; |S| = \frac{\hbar}{2}$$

$$\boxed{\vec{\mathrm{v}}_{FW}} = \frac{d}{dt}\vec{\mathrm{x}}_{FW} = \boxed{\beta\frac{pc^2}{W}} \;;\; \boxed{\Psi(\vec{r}) = \int K(\vec{r},\vec{r}\,')\Psi(\vec{r}\,')d\vec{r}\,'}$$

$$\vec{\mathrm{x}}_{FW} = e^{iS}xe^{-iS} = x - \frac{i\beta\vec{\alpha}c\hbar}{2W} + c^2\hbar\frac{i\beta(\vec{\alpha}\cdot\vec{p})\vec{p} - p(\vec{\sigma}\times\vec{p})}{2W(W+mc^2)p}$$

$\frac{\hbar}{mc}$



The nucleus K(r,r’) of the order of the Compton wavelength

Time-like behaviour of Dirac electron

Separation of positive and negative energies:

$$\Psi^{(4)} = (\psi;\varphi)$$

$$\begin{cases} i\hbar\partial_t\psi = (V + mc^2)\psi + c(\vec{\sigma}\cdot\vec{\pi})\varphi \\ i\hbar\partial_t\varphi = (V - mc^2)\varphi + c(\vec{\sigma}\cdot\vec{\pi})\psi \end{cases}$$

$$\vec{\pi} \equiv \vec{p} - e\vec{A}(\vec{x},t)$$

Exact formal separation of ψ and φ:

$$i\hbar\partial_t\psi = (V + mc^2)\psi +$$

$$+\frac{c^2}{i\hbar}\left(\vec{\sigma}\cdot\vec{\pi}(\vec{x},t)\right)\int_{-\infty}^{t} e^{-i\frac{(V-mc^2)(t-\tau)}{\hbar}}\left(\vec{\sigma}\cdot\vec{\pi}(\vec{x},\tau)\right)\psi(\tau)d\tau$$

Dirac-relativistic hydrogen atom (I)

$$\Psi_{1s}(\vec{r},t)=\begin{pmatrix} g_{1s}(\vec{r}) \\ f_{1s}(\vec{r}) \end{pmatrix} e^{-iWt/\hbar}$$ with: $W=mc^2\sqrt{1-(Z\alpha)^2}\cong mc^2(1-\tfrac{1}{2}(Z\alpha)^2)$

$$g_{1s}=\sqrt{\frac{8Z^3}{a_0^3}}\sqrt{\frac{W+mc^2}{2mc^2\Gamma(3)}}e^{-Zr/a_0}$$ and $$f_{1s}=\sqrt{\frac{8Z^3}{a_0^3}}\sqrt{\frac{W-mc^2}{2mc^2\Gamma(3)}}e^{-Zr/a_0}$$

$\Rightarrow$ $\langle \vec{p}_{cl}\rangle_{1s}=\langle p_{cl}^2\rangle_{1s}=0$ and $\vec{p}_{nc}=\dfrac{\hbar}{2\rho(\vec{r})}\left(\vec{\nabla}\times(\overline{\Psi}\vec{\sigma}\Psi)-\tfrac{1}{c}\partial_t(\overline{\Psi}i\vec{\alpha}\Psi)\right)$

with: $\overline{\Psi}\sigma_z\Psi=g_{1s}^2+f_{1s}^2$ and $\overline{\Psi}i\alpha_y\Psi=2g_{1s}f_{1s}$

$\Rightarrow$ We average over ZBW and get the same result as for Schrodinger equation… what if we did not average ?

Dirac-relativistic hydrogen atom (II)

Two oscillatory motions determined by W: $\Psi_{1s}(\vec{r},t) = \begin{pmatrix} g_{1s}(\vec{r}) \\ f_{1s}(\vec{r}) \end{pmatrix} e^{-iWt/\hbar}$

Composition of two frequencies:

$$\left(W \cong 2mc^2\left(1-\tfrac{1}{2}(Z\alpha)^2\right)\right)$$

$$\begin{cases} \hbar\omega_{free} = 2mc^2 \\ \hbar\omega_{1s} = Z^2\alpha^2 mc^2 \end{cases}$$

The 2 energies sum up as if the two motions were orthogonal

A possible composition:

Toroidal pattern

Dirac equation and the zitterbewegung (summary)

1) The motion of the electron is determined by the composition of two momenta: $\vec{p} = \vec{p}_{cl} + \vec{p}_{nc}$

2) p_{cl} is the motion of the center of mass and p_{nc} is the motion of a massless charge (moving at c):

3) both Schrodinger and Dirac equations (if properly interpreted) agree with this description: their expectation values correspond to averages on the ZBW frequency

4) Interestingly, the relativistic time-dilation and length-contraction are determined by the c.o.m. velocity, u.

5) The toroidal motion is responsible of the spin (and might be related to high-frequency parity-violation effects)

We were left with three open questions:

1) How could the electromagnetic field act this way ?
2) What would be a typical value for the «symmetrization time» ?
3) How could it be possible to measure it ?

1) How could the electromagnetic field act this way ?
   Retardation + ZBW

2) What would be a typical value for the «symmetrization time» ?
   If ZBW picture is true, extremely short : ~ 10^{-19} s (at a frequency of ~ 10^{20} Hz)

3) How could it be possible to measure it ?
   Subfemtosecond pump-and-probe… presently unreachable

4) A new question: how does the two-electron system behave ?
   ZBW picture only clear for one electron…

Back to PEP: two-electron atoms

How to extend this ZBW picture to the case of 2 electrons ?

$$\Rightarrow \begin{cases} P_{nc}^{(1)} = P^{(1)} - \hbar \nabla_{x_1} S(x_1, x_2) \\ P_{nc}^{(2)} = P^{(2)} - \hbar \nabla_{x_2} S(x_1, x_2) \end{cases}$$

$\Rightarrow$ ***Non-classical momentum of particle 1 depends on the position of particle 2 and vice-versa***

AJP, 1945 →

Action and Reaction Between Moving Charges

LEIGH PAGE AND NORMAN I. ADAMS, JR.
Yale University, New Haven, Connecticut

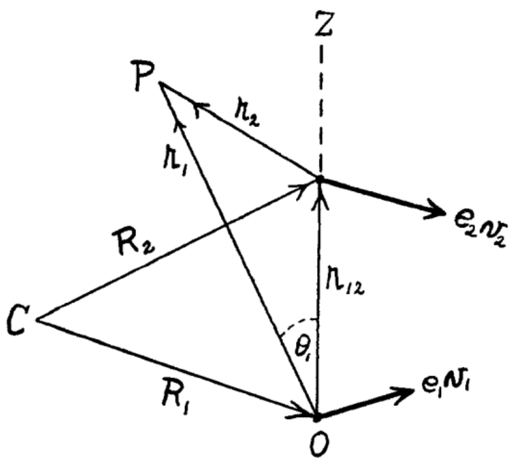


FIG. 1. Two moving charges.

$$\mathbf{G}_a = \frac{e_1 e_2}{2c^2}\left\{ \mathbf{R}_2 \times \left(\frac{\mathbf{v}_1}{r} + \frac{\mathbf{v}_1 \cdot \mathbf{r}_{12}\mathbf{r}_{12}}{r^3} \right) + \mathbf{R}_1 \times \left(\frac{\mathbf{v}_2}{r} + \frac{\mathbf{v}_2 \cdot \mathbf{r}_{12}\mathbf{r}_{12}}{r^3} \right) \right\}$$

Comparing with Eq. (8) we see that the portion of the linear momentum involving the velocity $\mathbf{v}_1$ of the first particle is to be considered as located at the second particle, and *vice versa*.

Outlook of the talk

Premise number 1: femtosecond pump-and-probe experiments

Premise number 2: Corinaldesi's paper

Diversion number 1: retarded interaction & the electromagnetic field

Diversion number 2: 'psychological' aspects

Analysis number 1: Dirac equation and the zitterbewegung

Analysis number 2: spin-statistics in the mesoscopic world

Conclusions: towards a non-orthodox view ?

Statistical physics & the Pauli principle (I)

Distinguishable vs. indistinguishable particles.

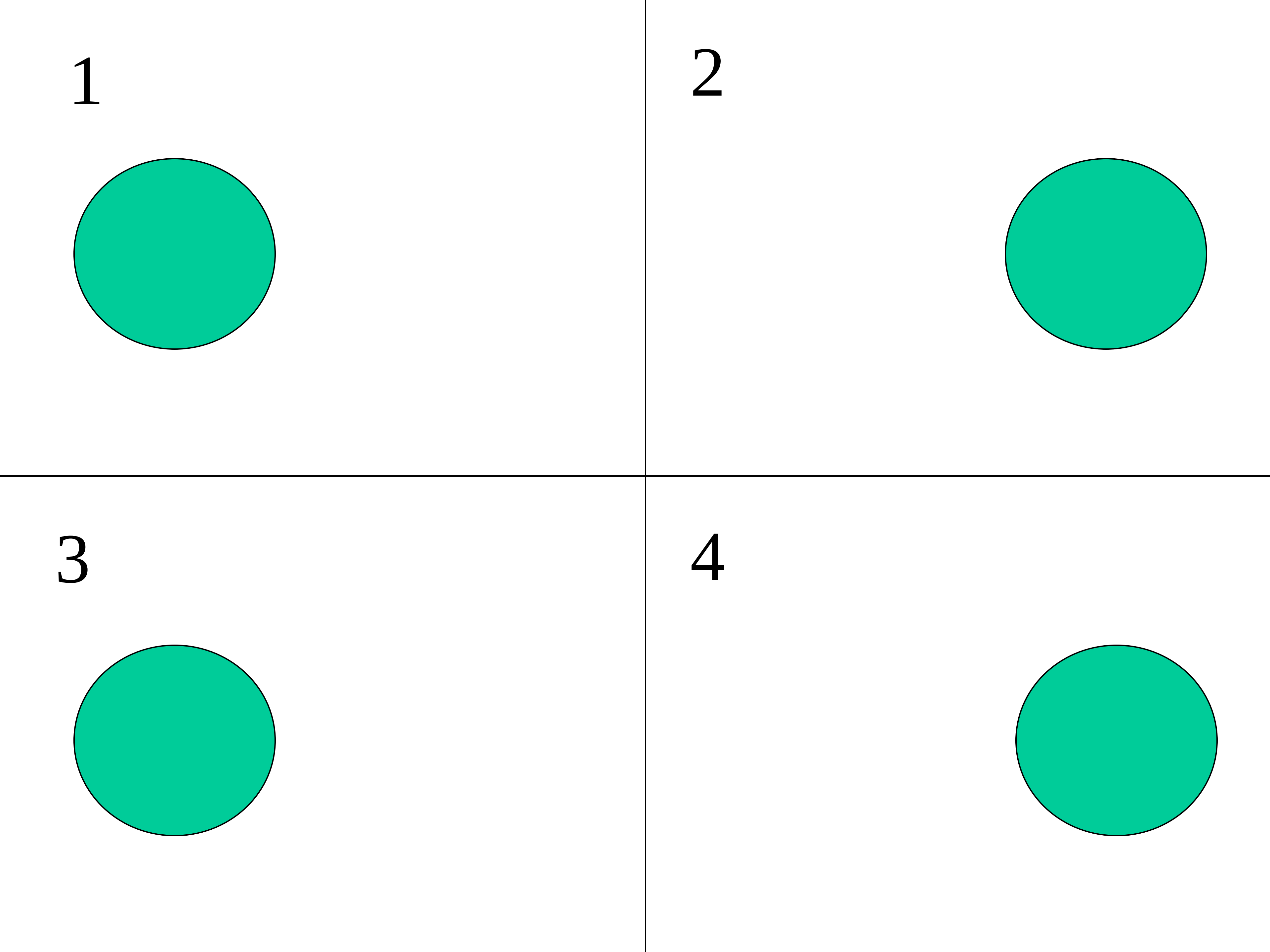
$$\Delta S_{\exp} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} \Leftarrow \text{ Entropy } \Rightarrow S_{stat} = k_B \sum_n p_n \ln p_n$$

$\Delta S_{exp}=0$ for mixing of like gases $\Rightarrow$ no count of permutations in S_{stat}, ie, principle of indistinguishability of identical particles

Classical particles = distinguishable (unique world line)
Quantum particles = indistinguishable (no unique world line)

Yet, not always true (well-separated wave-packets). Also, ergodic theory imposes to visit the whole configuration space and $S=k_B \ln\Omega$ would give the wrong result (!!!)

Moreover… experimental entropy is a state function , ie, it is independent of the history of the system !!!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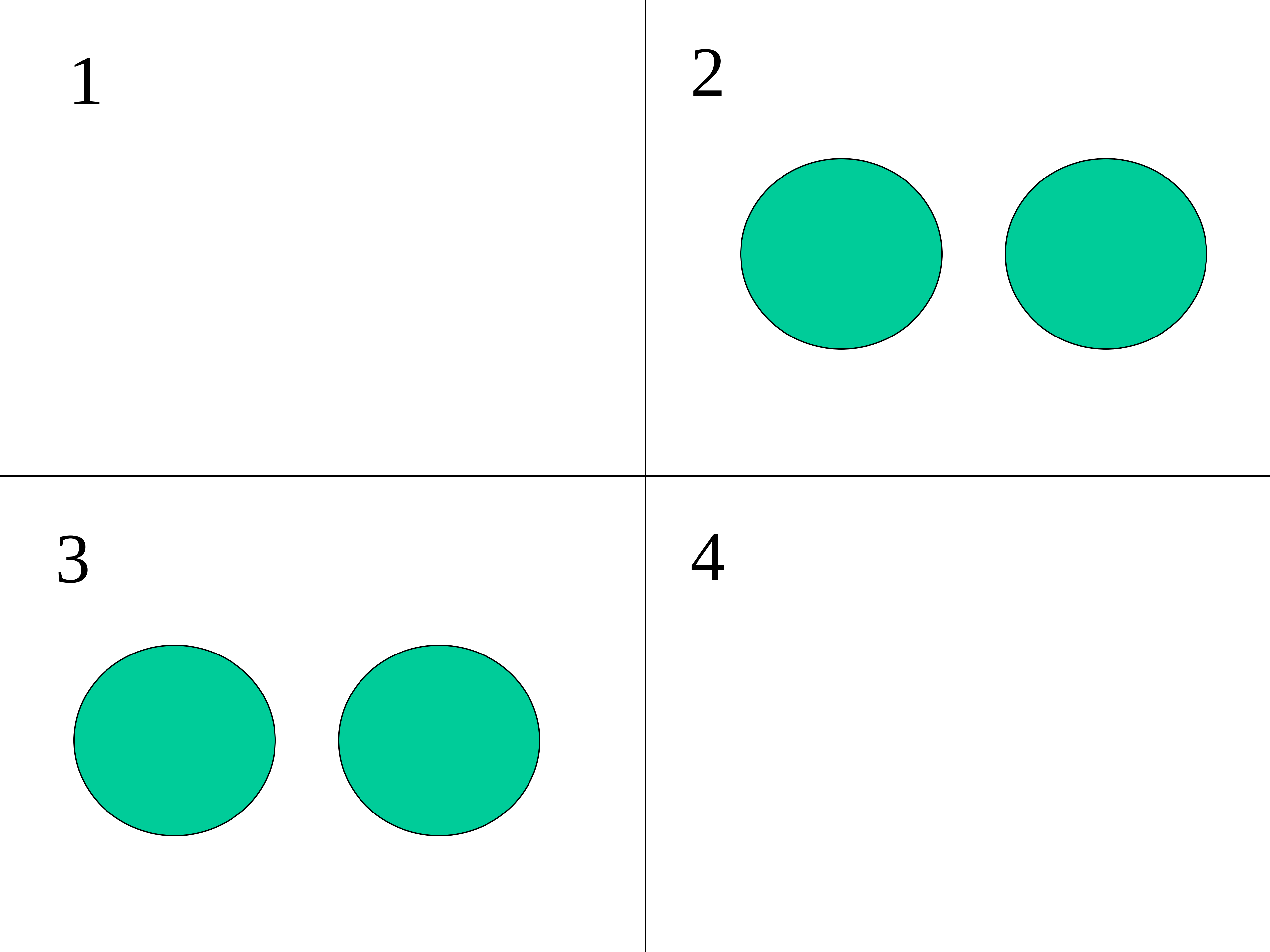
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Statistical physics & the Pauli principle (II)

Only way out is to adopt an information-statistical approach to entropy (à la Jaynes)

Indistinguishability is an expression of the information that can be obtained by mixing and filtering like-particles and has nothing whatever to say about the motion of the particles.

Antisymmetrization (eg) of wave functions can be:

1) a restatement of the principle of indistinguishability
2) a formulation of the Pauli principle

However, if Pauli principle (and exchange interactions) can be conceived as a dynamical constraint on the motion of the particles determined by ZBW-like electromagnetic interactions, then it has nothing to do with indistinguishability!

What is then the relation of Pauli principle with statistics ?

Indistinguishability is not necessarily related to quantum particles, as demonstrated by the study of colloidal particles in suspension in milk (Swendsen, J. Stat. Phys. **107**, 1143, 2002 & Am. J. Phys. **74**, 187 2005). Colloids are 'macroscopic' particles (therefore, classical) but must be treated as indistinguishable in order to have the correct statistics.

If Pauli principle can be really described by some subfemtosecond dynamics, through ZBW and retarded electrodynamics, then also the spin-statistics relations should be revisited. Spin would be determined by this dynamics, whereas quantum statistics (eg, Fermi-Dirac) would be a consequence of correlation effects (like for drops in phase or antiphase).

But remember that, for the moment, all this is just theoretical speculations ...

Conclusion – the beauty of time-lapse