What is time?

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I KNOW WHAT IS TIME,
BUT IF SOMEBODY ASK ME,
I D'ONT KNOW WHAT TO SAY

AGOSTINO
FROM IPPONA
AD 345 – 430

Time in Physics

- In relativity, time is the **fourth component** of position tetra-vector.
- In Q.M., the first three components become **observable operators** $[x, p_x] = i\hbar, \ldots.$
- Time remains a **parameter**.
- All attempts to quantize time have been failed (and I will NOT do it).
- $\Delta x \cdot \Delta p_x \geq \hbar/2$ (Jordan Lemma).
- $\Delta t \cdot \Delta E \sim h$ remains a **principle** with imprecise meaning for Δt.
- NO TIME ARROW $(t \to -t)$ OK?

What is wrong?

- In all fundamental Eqs. of physics, time evolution is given at each **instant**.
- Nothing happens in an instant !
- In Q.M., the specification of initial state requires MEASURMENT of a **complete set** of observables which **CANNOT BE DONE in an instant**.
- The instant is a **mathematical artifact** in the differential calculus with **no physical counterpart**.
- The instant concept MUST be removed from Physics.

Short Review

- **Please remember:** $$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H, \rho], \qquad (1)$$

  where ρ is the density operator, H is a system Hamiltonian.

- If $\rho = |\Psi\rangle\langle\Psi|$ (pure state), $\mathrm{Tr}\,(\rho^2)=1$, Eq. (1) is equivalent to

$$\frac{\partial |\Psi\rangle_t}{\partial t} = \frac{1}{i\hbar} H|\Psi\rangle_t, \qquad (2)$$

- If ρ is a statistical mixture, Eq. (2) is meaningless.

- Eq. (1) → Unitary time evolution. **NO TIME ARROW.**

Finite-difference Equation

- Substitute Eq. (1) with:

$$\frac{\rho(t)-\rho(t-\tau)}{\tau}=\frac{1}{i\hbar}[H,\rho(t)], \qquad (3)$$

where τ is a **characteristic CRONON of the system**.

- NON UNITARY time evolution ⇒ **TIME ARROW.**
- $\mathrm{Tr}(\rho)$=1 ; $\mathrm{Tr}(\rho^2)$ NOT preserved.
- Pure state **can evolve to a statistical mixture.**
- ρ(t) can be specified only for $\boldsymbol{t}=\boldsymbol{k\tau}$ (k=1,2,…).
- Time evolution by **finite steps τ WITHOUT quantizing t**.

[1] R. Bonifacio, P. Caldirola, Lett. Nuovo Cimento **33**, 97 (1982).
[2] R. Bonifacio, S. Olivares, P. Tombesi, D. Vitali, Phys Rev. A **61**, 053802 (2000).
Ref. [1] (IGNORED) & Ref. [2] (HERETICAL) .

Energy representation $H|n\rangle = E_n|n\rangle$

$$\frac{\rho_{n,m}(t)-\rho_{n,m}(t-\tau)}{\tau} = -i\omega_{n,m}\rho_{n,m}(t)$$

$\omega_{n,m} = (E_n - E_m)/\hbar$ Bohr frequencies.

- Trivial solution:

$$\rho_{n,m}(t) = \left(1 + i\omega_{n,m}\tau\right)^{-\frac{t}{\tau}}\rho_{n,m}(0).$$

$$\rho_{n,m}(k) = \left(1 + i\omega_{n,m}\tau\right)^{-k}\rho_{n,m}(1); \quad (t = k\tau)$$

- In the limit $\tau \to 0$,

$$\rho_{n,m}(t) = e^{-i\omega_{n,m}t}\rho_{n,m}(0)$$

- The limit $\tau \to 0$ CAN NOT be taken (see later).
- $\rho_{n,n}(t) = P_n = const.$

Irreversible state reduction to diagonal form

$$\rho_{n,m}(t) = \left(1 + i\omega_{n,m}\tau\right)^{-\frac{t}{\tau}} \rho_{n,m}(0)$$
$$= e^{-\gamma_{n,m}t} e^{-i\nu_{n,m}t} \rho_{n,m}(0)$$

$$\gamma_{n,m} = \frac{1}{2\tau}\ln\left(1 + \omega_{n,m}^2\tau^2\right); \;\; \nu_{n,m} = \frac{1}{\tau} arctg\left(\omega_{n,m}\tau\right)$$

- $\tau \rightarrow 0$, $\gamma_{n,m} \rightarrow 0$, $\nu_{n,m} = \omega_{n,m}$

- $\tau \neq 0$, $n \neq m$, $\gamma_{n,m} > 0 \Rightarrow \rho_{n,m} \rightarrow 0$

$$\rho_{n,m}(t) \Rightarrow \sum_n P_n \, |n\rangle\langle n|$$ **Equilibrium**

Time Arrow

What is "cronon" τ ?

- My proposal: $\tau = \frac{\hbar}{2\Delta E}; \Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2}$
- The limit $\tau \rightarrow 0$ **CANNOT** be taken.
- Precise statement of **time-energy** uncertainty relation with **NO time quantization**.
- τ is **NOT** a universal time , but it is a **characteristic time of the system**. It depends on H and the initial state.
- Did τ already appear in the literature ? Is there any **expermintal evidence** for my statements ?

YES.

The Caldirola cronon

- Consider a free electron in a minimum uncertanity wavepacket

$$\tau = \frac{\hbar}{2\Delta E} \approx \frac{\Delta x}{v} \qquad v = \frac{\langle p \rangle}{m}$$

- From the finite-difference Equation:

$$m\frac{v(t) - v(t-\tau)}{\tau} = F_{ext} = 0$$

- Expanding $v(t-\tau)$ up to second order in τ

$$\dot{v}(t) - \frac{\tau}{2}\ddot{v}(t) = 0 \Rightarrow \quad v = 0; \quad \boldsymbol{v(t) = v(0)e^{\frac{t}{\tau}}} \text{ !!}$$

- Abraham-Lorentz equation of motion **with self-acclerating solution** (Jackson)
- If $\Delta x = r_e \approx \frac{e^2}{mc^2}; \quad v \sim c \Rightarrow \quad \boldsymbol{\tau = \frac{r_e}{c} = \frac{e^2}{mc^3}}$ **(Caldirola Cronon)**
- Self-accelerating solution is an **artifact** of $v(t-\tau)$ expansion. Exact solution is $\boldsymbol{v(t) = v(\mathrm{t} - \tau) = \mathrm{const.}}$

[1] P. Caldirola, Lett. Nuovo Cimento 45, 549 (1978). [2] P. Caldirola, and R. Bonifacio, ibiden 33, 197 (1994).

Expermintal Evidence [1]

- Cavity Q.E.D: observation of **vacuum induced Rabi oscillations** between $|e\rangle$ and $|g\rangle$ in R_b atoms prepared in $|e\rangle$ injected in high-Q cavity **with selected velocity** $\boldsymbol{v}$ as a function of $t = L/v$ (just changing velocity v).
- Expected results [1]: $$P_g = \sin^2(\Omega_R t) = \frac{1}{2}[1 - \cos(2\Omega_R t)],$$
- Observed results **fitted by** [2]:

$$P_g = \frac{1}{2}[1 - e^{-\gamma t}\cos(2\Omega_R t)].$$

- **Damped oscillations** $P_g \Rightarrow 0.5$ with $1/\gamma \sim 40\ \mu s$. Ref. [1] **gives unrealistic explanation for the observed damping.**
- Our theory leads to fitting formula with $1/\gamma$ in agreement with the observed value [2]. This paper has been considered **heretic**.

[1] M. Brune et al., Phy. Rev. Lett., **76**, 1800 (1996).
[2] R. Bonifacio, S. Olivares, P. Tombesi, D. Vitali, Phys Rev. A **61**, 053802 (2000).

Expermintal Result

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Quantum Rabi Oscillation: A Direct Test of Field Quantization in a Cavity

M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J.M. Raimond, and S. Haroche
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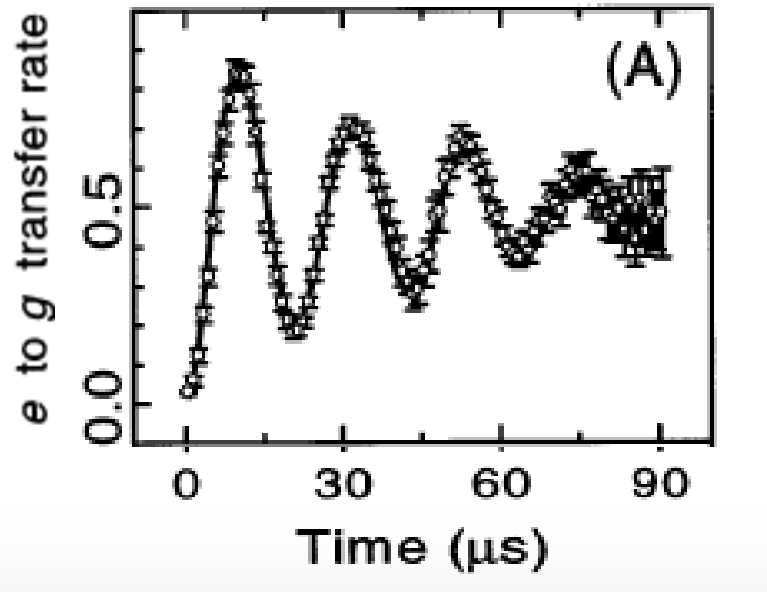

Conclusions

- Anomalies in describing time as a continuous parameter.
- Criticism of the INSTANT concept in physics.
- Introduction of a finite-difference equation giving a time evolution for finite time-steps $\tau=\hbar/2\,\Delta E \Rightarrow$ TIME ARROW.
- **Precise statement of time-energy uncertainty relation with no time quantization**.
- The **Caldirola cronon** obtained as a particular case **cancelling the Abraham-Lorentz self-acclerating free electron**.
- Interpretation of the **expermintal results** in cavity Q.E.D.
- If some young researcher did like my talk , **please FORGET IT, NOT TO BE CONSIDERED HERETIC.**

ACKNOWLEDGMENT

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  * If anything is wrong, it is his fault.

Thank You