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One More Technique for Burst Source Position Identification: X-Wigner Spectra

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Summary - We present preliminary results of numerical simulations implementing a simple technique for estimating the delays in the arrival times of gravitational wave transients on a network of interferometric detectors, based on the cross-Wigner spectra between the data gathered from all detector-pairs in the network.
Tthe salient performance as a function of signal strength, source location and noise power specral density are briefly discussed, the nice glitch-rejection capabilities of the proposed methods are illustrated, and its computational burden is evaluated.

Rationale - The cross Wigner-Ville (XWV) spectrum [1] between two signals is:

$$W_{ij}(t,f)=\int_{-\infty}^{\infty} x_i\left(t-\frac{\theta}{2}\right)\tilde{x}_j\left(t+\frac{\theta}{2}\right)\exp[\iota 2\pi f\theta]\,d\theta$$

If the two signals $x_i(t)$ and $x_j(t)$ are sine-Gaussian waveforms with carrier frequencies f_i and peak times τ_i, $i=1,2$, their XWV spectrum X_{ij} is peaked at $f=(f_1+f_2)/2$ and $t=(\tau_1+\tau_2)/2$. The above *localization* property holds essentially true for *all* signals representing oscillatory transients with *uni-modal* envelopes.

Each of the three XWV spectra constructed from the data gathered by three interferometers observing a single (unimodal) gravitational wave burst (GWB) will thus exhibit a (magnitude) peak at:

$$(t_{ij}, f_{ij}),\ \text{with}\ t_{ij}=(\tau_i+\tau_j)/2\ \text{and}\ f_{ij}=(f_i+f_j)/2=f_0 \qquad i,j=1,2,3$$

allowing, in principle, to estimate the two (independent) GWB wavefront propagation delays between e.g. interferometer #3 and interferometers #1 and #2 as

$$\tau_{31}=(\tau_3-\tau_1)=2\,(t_{23}-t_{12}),\ \text{and}\ \tau_{32}=(\tau_3-\tau_2)=2\,(t_{13}-t_{12})$$

whereby (except for the well known ambiguity affecting the three interferometers case) the source direction can be uniquely inferred [2].

In order to reject spurious transients (glitches), given a candidate *hot* time-frequency pixel e.g. in the (discrete) X_{12} spectrum at (t_{12}, f_0), one should *consistently* look for the corresponding *hot* pixels in the X_{13} and X_{23} spectra, located at *the same frequency*, and *within* the time intervals

$$2\,|t_{13}-t_{12}|\le|\mathbf{r}_{32}|/c \ \text{and}\ 2\,|t_{23}-t_{12}|\le|\mathbf{r}_{31}|/c, \qquad \text{(a)}$$

expressing the maximal wavefront propagation delays (corresponding to line-of-sight propagation).

The following pseudo-code will produce a *histogram* of *candidate* propagation delays (and hence, of candidate directions of arrival) in the (τ_{31}, τ_{32}) plane

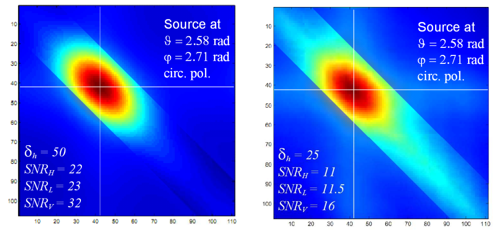
```
for all pixels (t12,f12) in X12
  for all pixels (t13,f13) in X13 such that f13=f12 and 2|t13-t12| ≤ |r32|/c
    for all pixels (t23,f23) in X23 such that f23=f12 and 2|t23-t12| ≤ |r13|/c
            accumulate  R=|X12(t12,f12) X13(t13,f13) X23(t23,f23)|
         At {τ31=2(t23-t12), τ32=2(t13-t12)}
    end for
  end for
end for
```

Candidate peaks should be obviously sought in the *subset* of the (τ_{31}, τ_{32}) plane where the further maximal wavefront propagation delay condition

$$|\tau_{31}-\tau_{32}|\le|\mathbf{r}_{12}|/c \qquad \text{(b)}$$

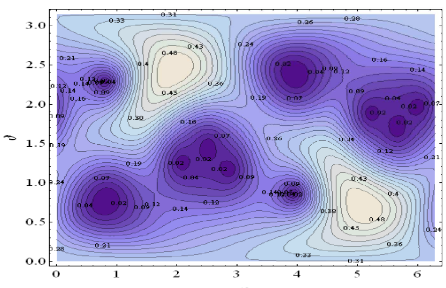
has been enforced (corresponding the diagonal strip in the figures below).

Performances - In the presence of "simple" (white, bandlimited) Gaussian noise in the data, the peak of the histogram in the time delay plane is found to be statistically significant, provided the product $\rho(\vartheta,\varphi)\delta_h$ is sufficiently large (~ 20), where $\rho(\vartheta,\varphi)$ is the (squared) product of the pertinent antenna pattern functions, and $\delta h=(2h^2_{rss}/N)^{1/2}$ is the GWB *intrinsic* SNR, N being the one-sided noise PSD (assumed here for simplicity as white and equal in all detectors,).



Level plot of XWV based SG-burst arrival-delays estimator. The x and y axis are τ_{31} and τ_{32}, with extremal values $\pm|\mathbf{r}_{32}|/c$ and $\pm|\mathbf{r}_{31}|/c$, respectively. Axes units are in time bins.

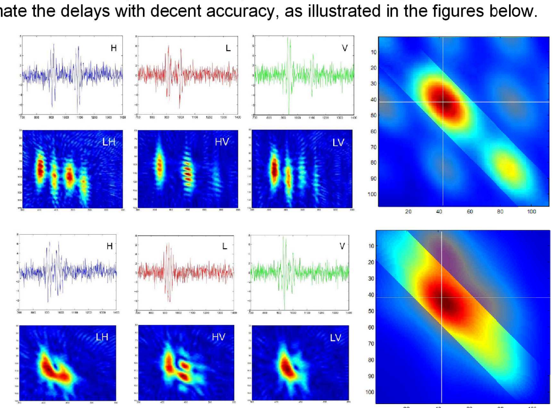
The angular converage for the LIGO-Virgo triad is decent, as seen from the $\rho(\vartheta,\varphi)$ factor shown below.



Contour levels of the $\rho(\vartheta,\varphi)$ factor for the LIGO-Virgo observatory. Circular polarization. The North pole is $\vartheta=0$. The prime meridian is $\varphi=0$.

Extensive MonteCarlo simulations are presently under way toward full statistical characterization of the histogram peak level and position. Preliminary evidence suggests that the delays *estimated* from the position of the peak in the histogram are *unbiased*, and their variance reaches the limit set by time-discretization for large (> 25) values of the product $\rho(\vartheta,\varphi)\delta_h$.

Robustness - By construction, the histogram algorithm is basically *immune* from spurious glitches whose timings do not fulfil the delay constraints (a) and (b). There is obviously no way (except based on consistency with the instruments' directional responses) of discriminating a GWB from a triplet of glitches for which (a) and (b) hold true. However, even when this is not the case, when one or more glitches coexist, and possibly overlap with bursts, the algorithm still manages to estimate the delays with decent accuracy, as illustrated in the figures below.



Time series (top left), X-Wigner spectra level plots (bottom left) and XWV based SG-burst arrival-delays estimator (right). SG burst with one glitch in each detector. Top panel: time resolved glitches. Bottom panel: time overlapping glitches.

Conclusions – The X-Wigner transform could be a quick-and-dirty method for estimating the direction of arrival of gravitational wave transients in a network of interferometric detectors The computational burden of the proposed algorithm is extremely cheap, being dominated by the $O(N^2 \log N)$ floating point operations needed to compute the XWVs, N being of the order of 10^3 (e.g., 100*ms* of data at *16 KHz*)

References - *[1]* B. V. Kumar and C . W. Carroll, "Performance of Wigner distribution function based detection methods," Opt. Eng., 23 (732) 1984; *[2]* J. Markowitz, M. Zanolin, L.Cadonati, E. Katsavounidis, "Gravitational Wave Burst Source Direction Estimation Using Time and Amplitude Information," Phys. Rev. D78 (2008) 122003;