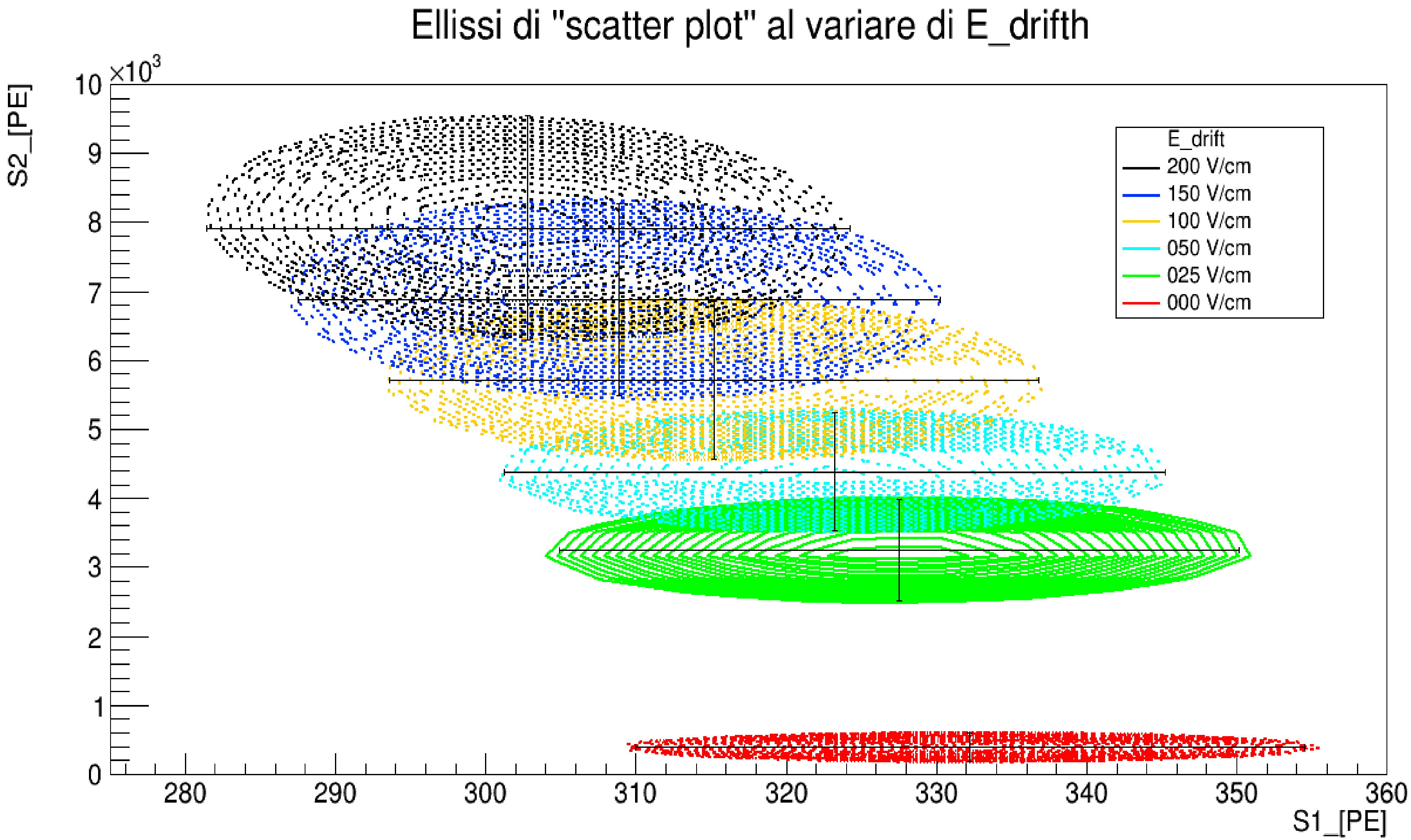
Prima bozza dell’analisi dei dati della calibrazione 2016 in DS50 con Kr

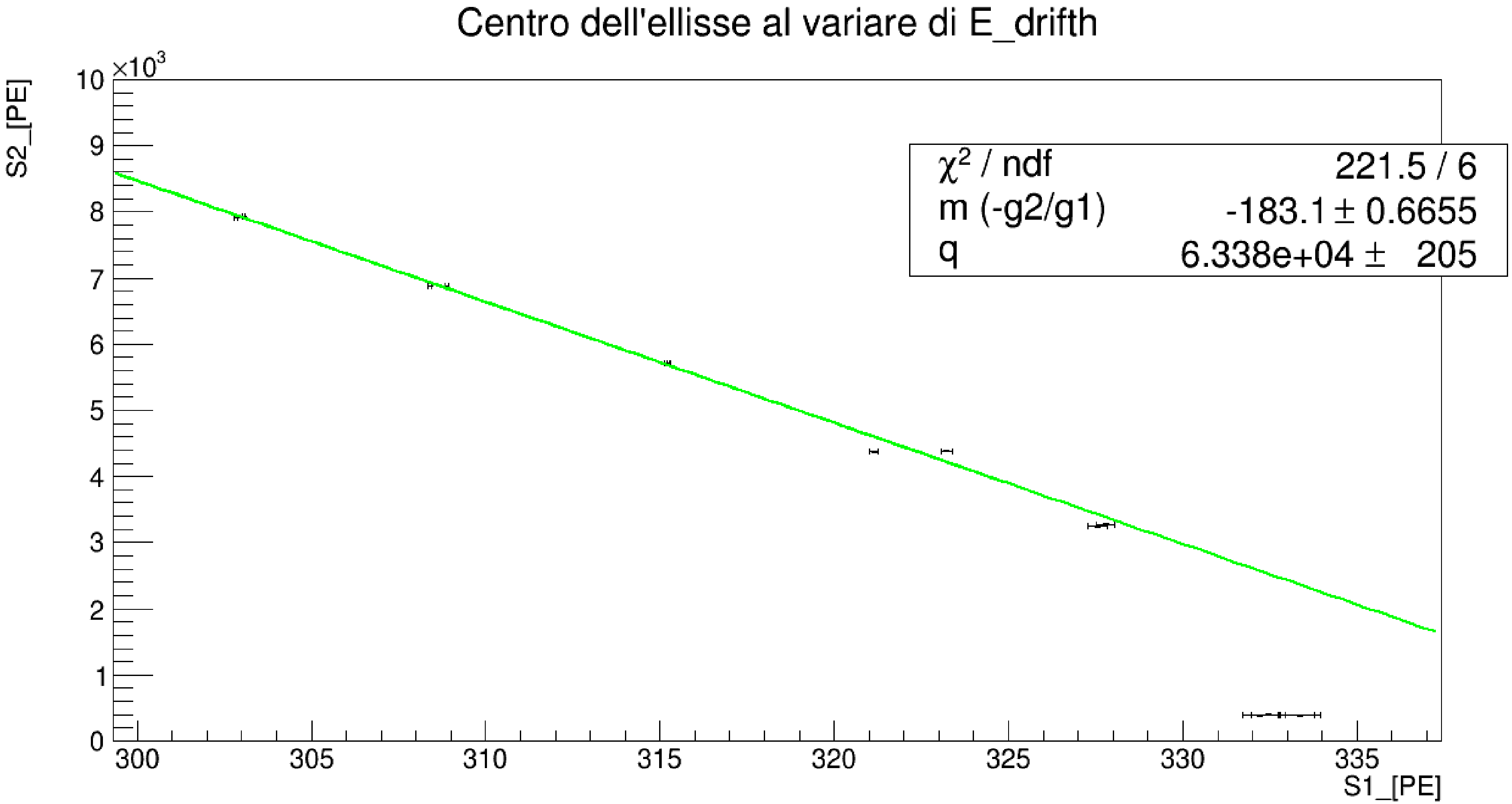
Work in progress

Effetto di drifth del centro ellisse

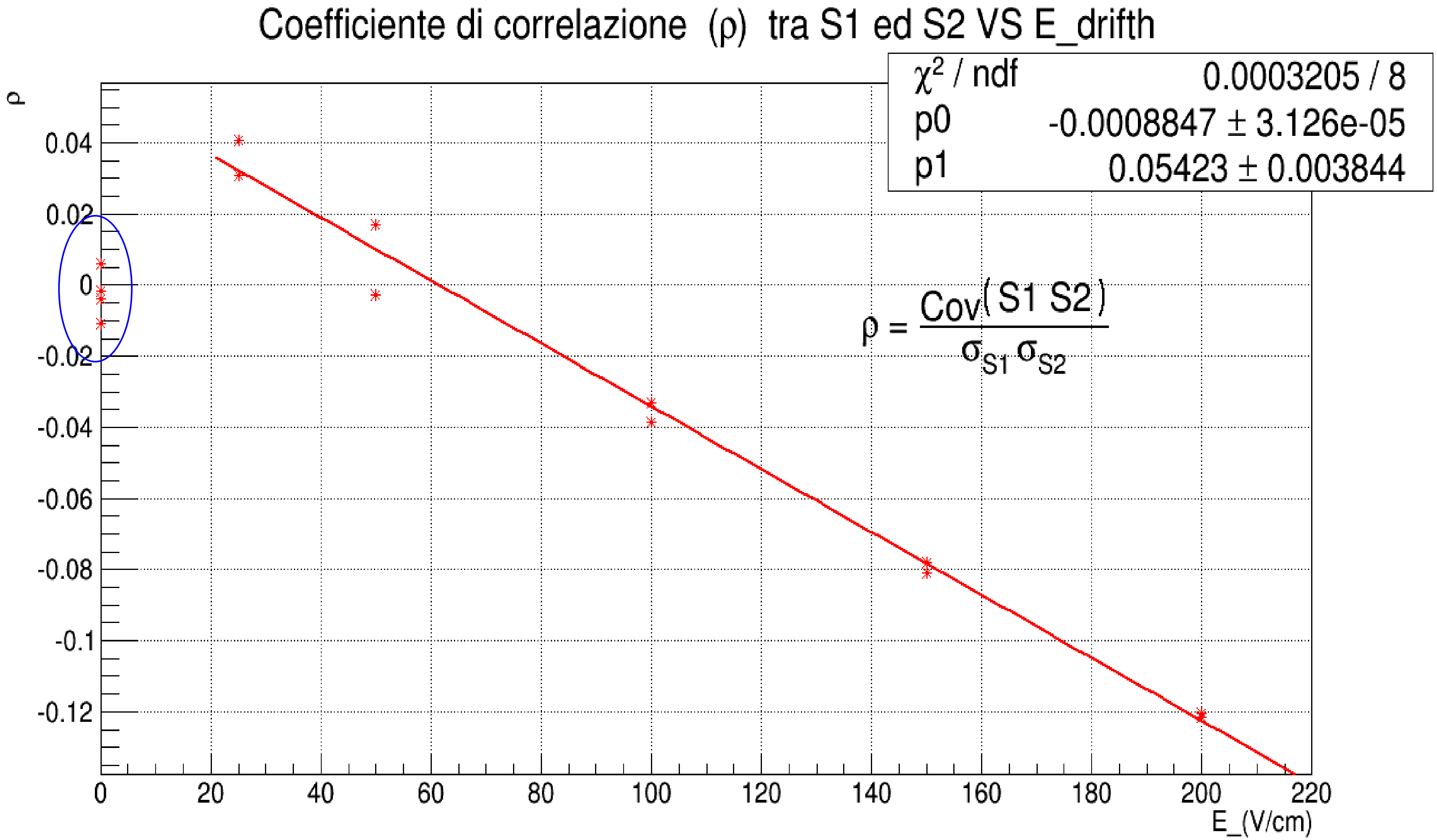


Fit del drifth

(escluso E = 0 V/cm)



Nota: Rho non dipende dalla scala delle variabili



Rotazione? ..qualche appunto teorico.

(Da "Cowan – Statistical data analysis")

In two dimensions, for example, the covariance matrix for the variables $\mathbf{x} = (x_1, x_2)$ can be expressed as

$$V = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}. \quad (1.67)$$

The eigenvalue equation $(V - I\lambda)\mathbf{r} = 0$ (where I is the 2×2 unit matrix) is solved by requiring that the determinant of the matrix of coefficients be equal to zero,

$$\det(V - I\lambda) = 0. \quad (1.68)$$

The two eigenvalues $\lambda_\pm$ are found to be

$$\lambda_\pm = \frac{1}{2}\left[\sigma_1^2 + \sigma_2^2 \pm \sqrt{(\sigma_1^2 + \sigma_2^2)^2 - 4(1-\rho^2)\sigma_1^2\sigma_2^2}\right]. \quad (1.69)$$

The two orthonormal eigenvectors $\mathbf{r}_\pm$ can be parametrized by an angle θ,

$$\mathbf{r}_+ = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \qquad \mathbf{r}_- = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}. \quad (1.70)$$

Substituting the eigenvalues (1.69) back into the eigenvalue equation determines the angle θ,

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}\right). \quad (1.71)$$

The rows of the desired transformation matrix are thus given by the two eigenvectors,

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \quad (1.72)$$

This corresponds to a rotation of the vector (x_1, x_2) by an angle θ. An example is shown in Fig. 1.11 where the original two variables have $\sigma_1 = 1.5$, $\sigma_2 = 1.0$, and a correlation coefficient of $\rho = 0.7$.

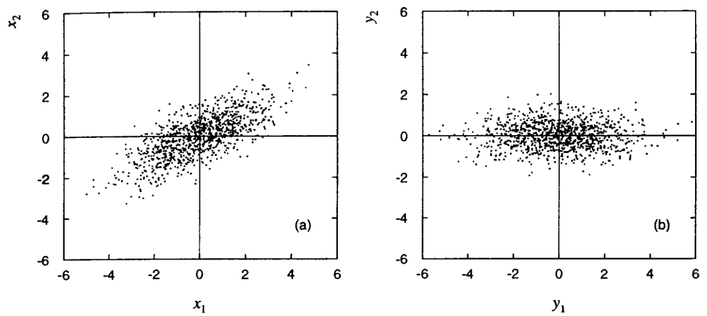
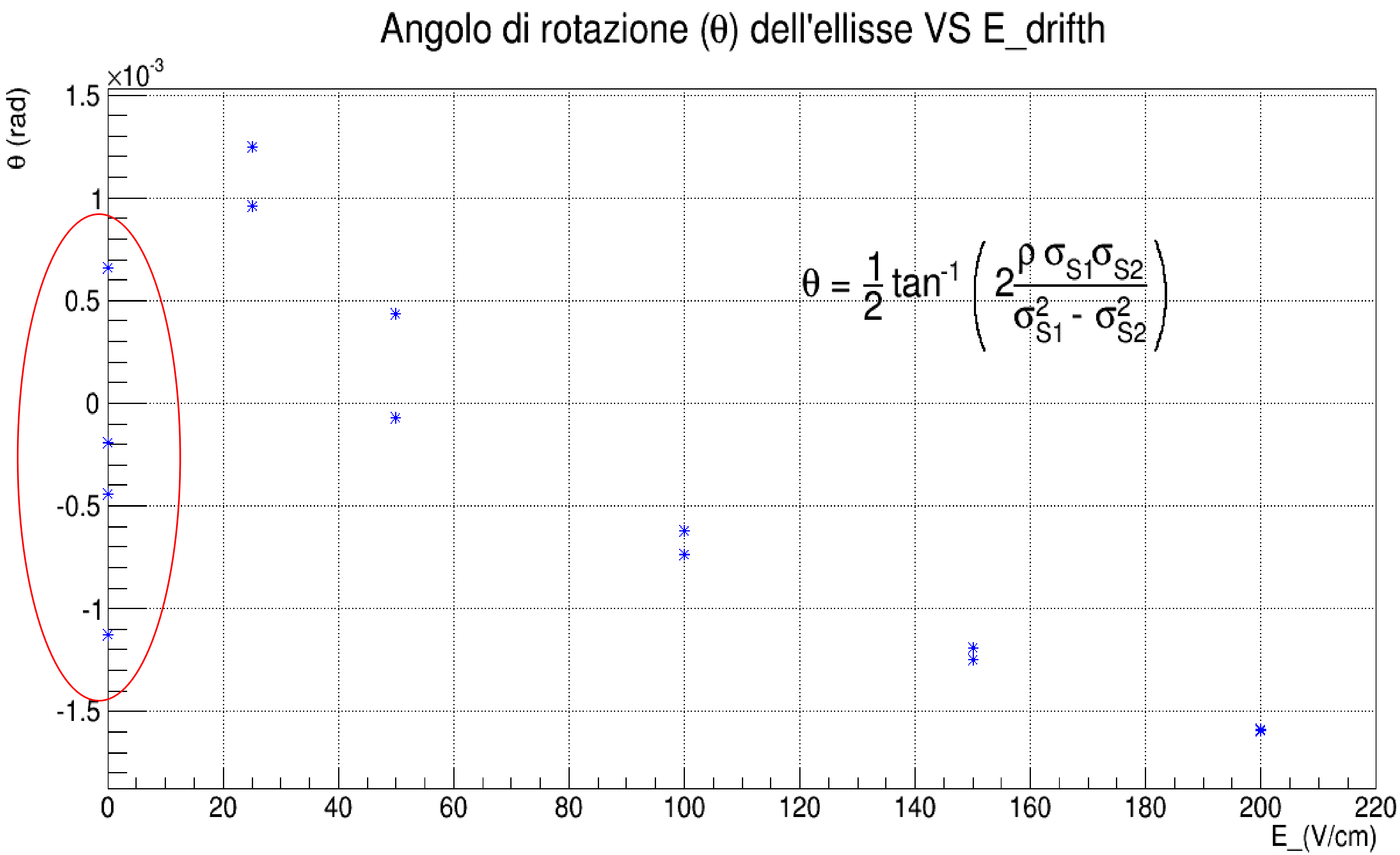


Fig. 1.11 Scatter plot of (a) two correlated random variables (x_1, x_2) and (b) the transformed variables (y_1, y_2) for which the covariance matrix is diagonal.

Attenzione: l’angolo dipende dalla scala delle variabili!



Se “normalizziamo” l’angolo rispetto alla media dei valori delle variabili... (Ma quanto è lecito?)