

# Gluon TMDs (very low- $x$ )

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3D Parton Distributions: path to the LHC

Frascati, November 30, 2016

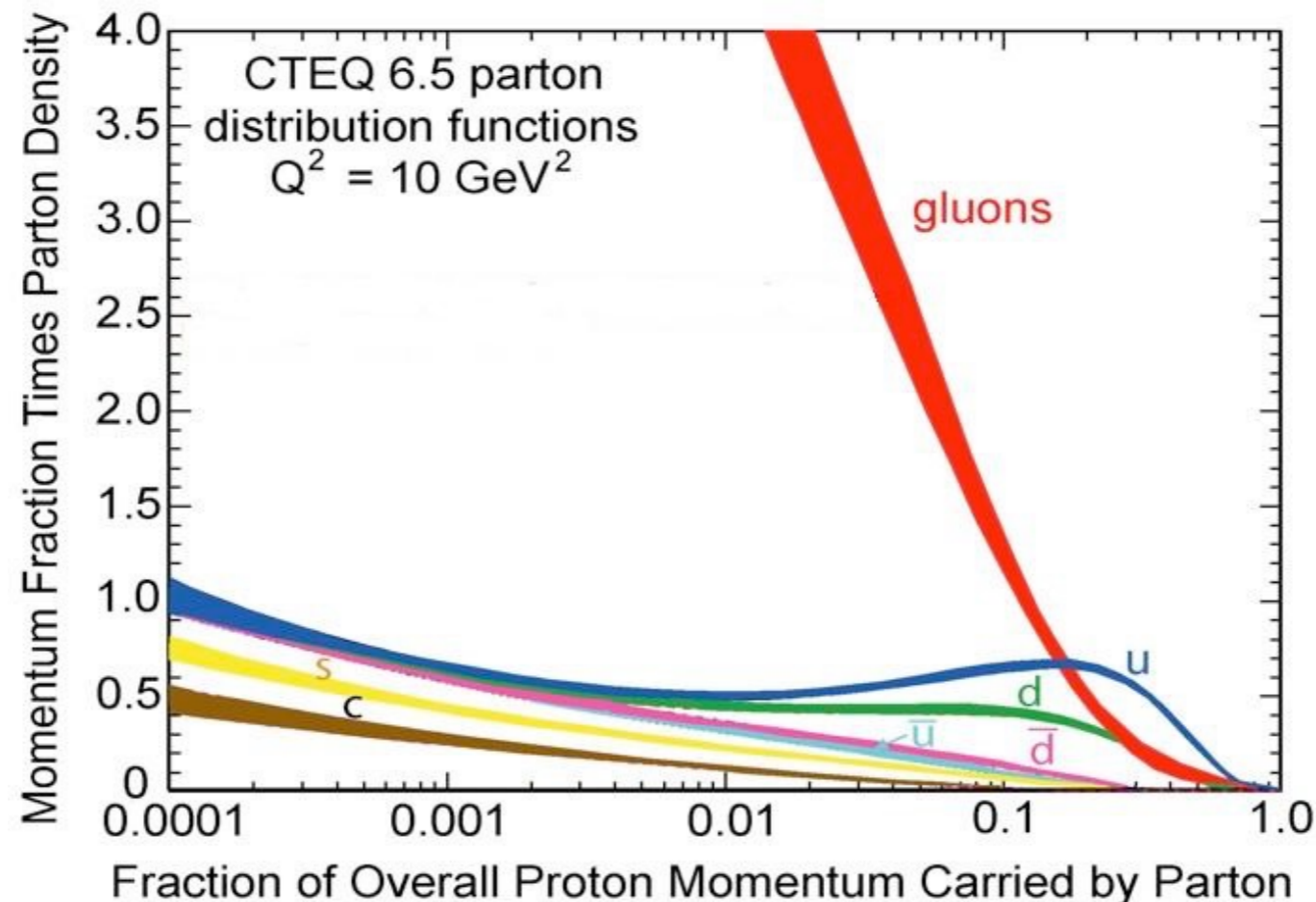


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# Partons at small $x$

Gluons dominate at high center of mass energy  $s$ , where the gluons carry a small fraction of the proton momentum:  $x \approx Q^2/s \ll 1$



At small  $x$  it becomes natural to consider the transverse momentum dependence

TMD = *transverse momentum dependent parton distribution*

Because of the additional  $k_T$  dependence there are more TMDs than collinear pdfs

# Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x, k_T) \equiv \text{F.T.} \langle P | \text{Tr}_c \left[ F^{+\nu}(0) \mathcal{U}_{[0,\xi]} F^{+\mu}(\xi) \mathcal{U}'_{[\xi,0]} \right] | P \rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{p}_T^2) + \left( \frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{p}_T^2) \right\}$$

unpolarized gluon TMD

linearly polarized  
gluon TMD

Gluons inside *unpolarized* protons can be polarized!

For transversely polarized protons:

gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, \mathbf{p}_T^2) + \dots \right\}$$



# Process dependence

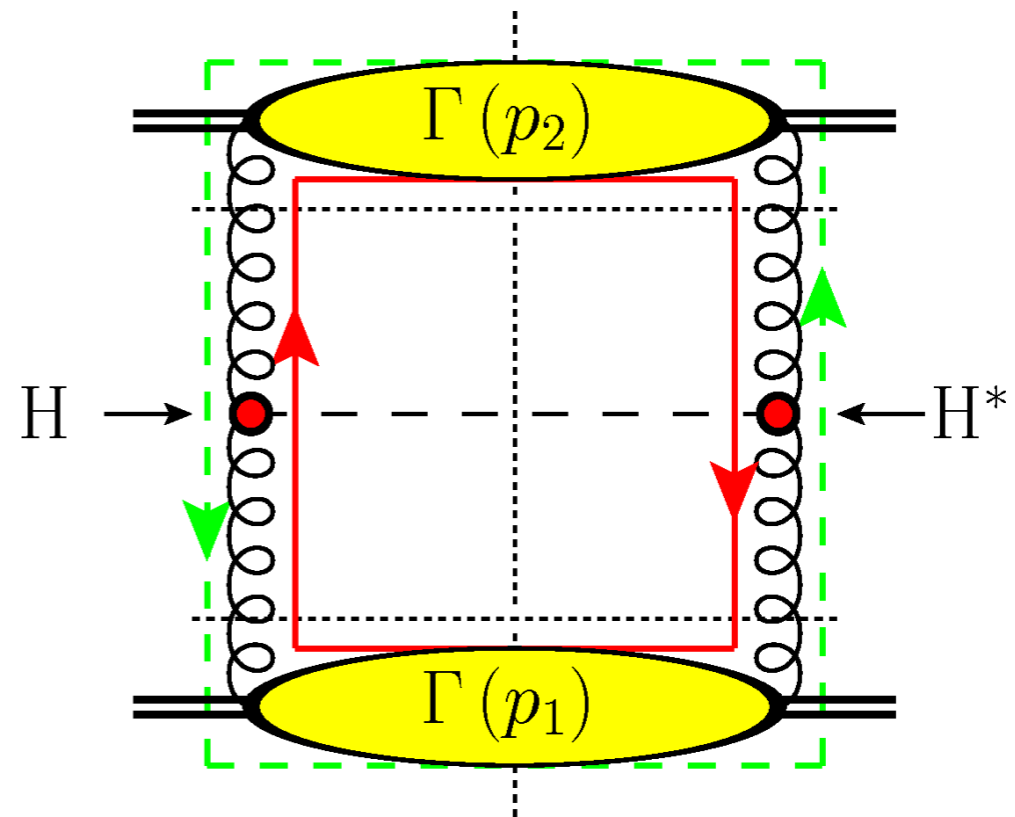
# Factorization and color flow

Theoretical description of high-energy scattering cross sections is based on **factorization** in perturbative partonic hard scattering factors (H) and nonperturbative hadronic correlators ( $\Phi, \Gamma, \Delta$ ), i.e. parton distributions

Higgs production:  $pp \rightarrow HX$

Color treatment is simple at high energies: separate traces, not dependent on kinematics

But in the actual process there are no colored final states and there are many soft gluons exchanged to balance the color



This cartoon version of the color flow works fine in most cases, when collinear factorization applies

In TMD factorization the color flow in a process leads to distinct correlators

## Process dependence of gluon TMDs

$$\Gamma_g^{\mu\nu}[\mathcal{U}, \mathcal{U}'](x, k_T) \equiv \text{F.T.} \langle P | \text{Tr}_c \left[ F^{+\nu}(0) \mathcal{U}_{[0, \xi]} F^{+\mu}(\xi) \mathcal{U}'_{[\xi, 0]} \right] | P \rangle$$

$$\mathcal{U}_c[0, \xi] = \mathcal{P} \exp \left( -ig \int_{\mathcal{C}[0, \xi]} ds_\mu A^\mu(s) \right) \quad \xi = [0^+, \xi^-, \xi_T]$$

The gauge links are process dependent, affecting even the unpolarized gluon TMDs as was first realized in a small-x context

Dominguez, Marquet, Xiao, Yuan, 2011

Kharzeev, Kovchegov & Tuchin (2003): "A tale of two gluon distributions"

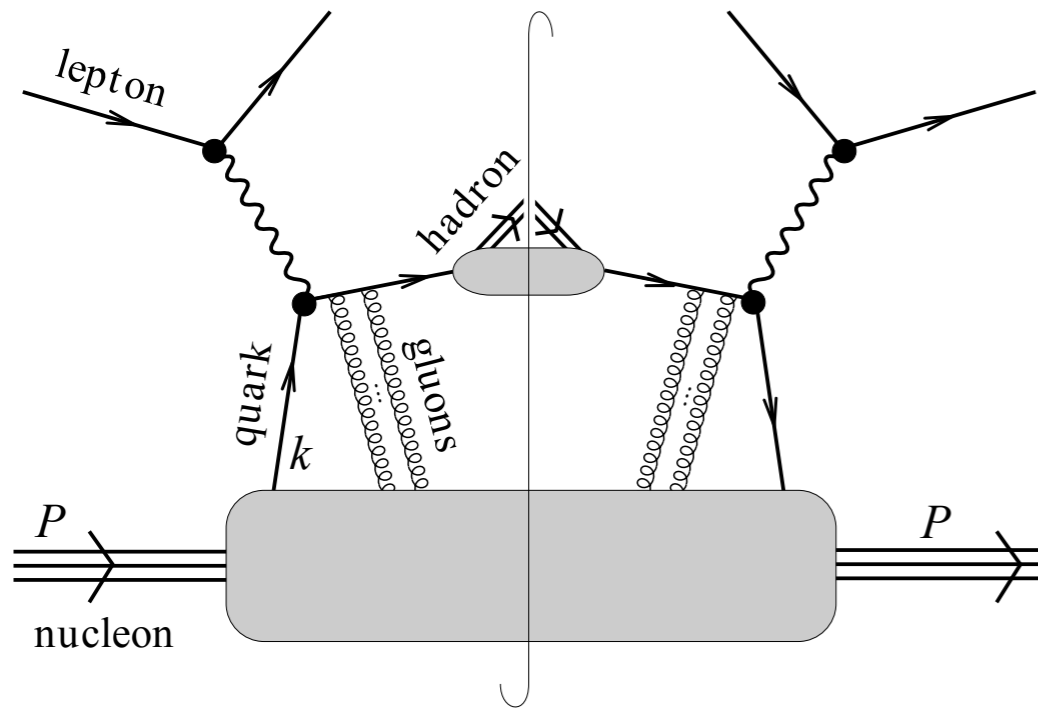
They noted there are 2 distinct but equally valid definitions for the small-x gluon distribution: the Weizsäcker-Williams (WW) and the dipole (DP) distribution

KKT: "cannot offer any simple physical explanation of this paradox"

The explanation turns out to be in the process dependence of the gluon distribution, in other words, its sensitivity to the initial and/or final state interactions (ISI/FSI) in a process, without them WW and DP would be the same



# Initial and final state interactions



summation of all gluon rescatterings leads to path-ordered exponentials in correlators

$$\mathcal{L}_c[0, \xi] = \mathcal{P} \exp \left( -ig \int_{c[0, \xi]} ds_\mu A^\mu(s) \right)$$

$$\Phi \propto \langle P | \bar{\psi}(0) \mathcal{L}_c[0, \xi] \psi(\xi) | P \rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

Resulting Wilson lines depend on whether the color is incoming or outgoing

[Collins & Soper, 1983; D.B. & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, X. Ji & F.Yuan, 2003; D.B., Mulders & Pijlman, 2003]

This does not automatically imply that the ISI and/or FSI affect observables, but it turns out that they do in certain cases, for example, Sivers effect asymmetries

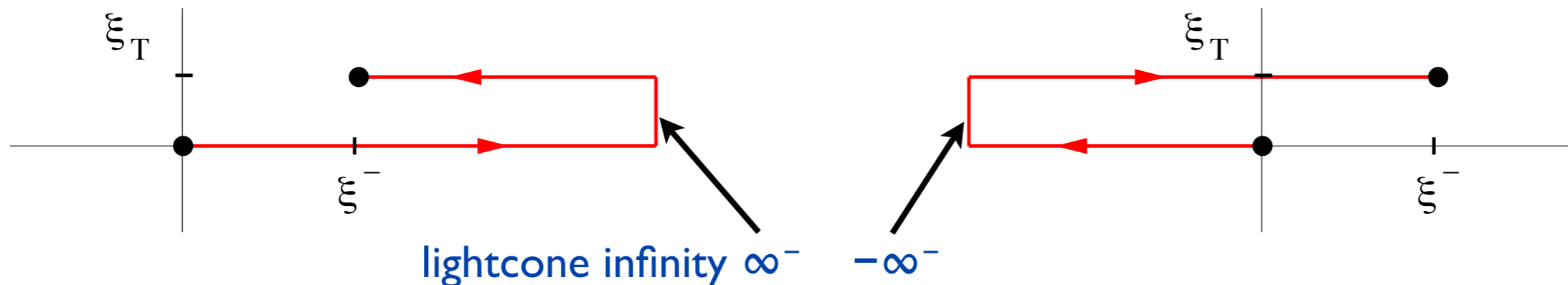
[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

# Process dependence of Sivers TMDs

SIDIS

DY

FSI lead to a future pointing Wilson line (+ link), whereas ISI to past pointing (- link)



One can use parity and time reversal invariance to relate these

$$f_{1T}^{\perp q[\text{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp q[\text{DY}]}(x, k_T^2) \quad [\text{Collins '02}]$$

A similar sign change relation for gluon Sivers functions holds, but due to the appearance of two gauge links, there are more possibilities



# Sign change relation for gluon Sivers TMD

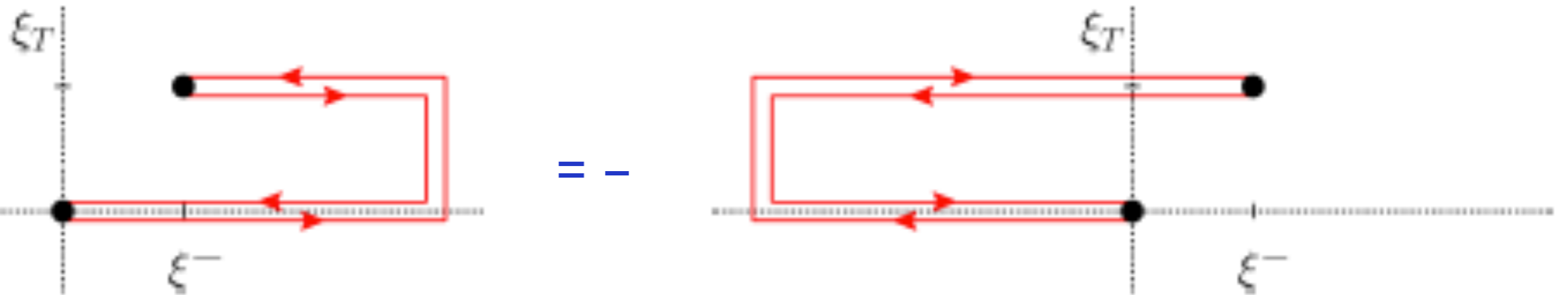
$$e p^\uparrow \rightarrow e' Q \bar{Q} X \quad \gamma^* g \rightarrow Q \bar{Q} \text{ probes } [+, +]$$

$$p^\uparrow p \rightarrow \gamma \gamma X$$

Qiu, Schlegel, Vogelsang, 2011

In the kinematic regime where pair rapidity is central, one effectively selects the subprocess:

$$g g \rightarrow \gamma \gamma \text{ probes } [-, -]$$



$$f_{1T}^\perp g [e p^\uparrow \rightarrow e' Q \bar{Q} X] (x, p_T^2) = -f_{1T}^\perp g [p^\uparrow p \rightarrow \gamma \gamma X] (x, p_T^2)$$

D.B., Mulders, Pisano, Zhou, 2016

Important role for EIC

# f and d type gluon Sivers TMD

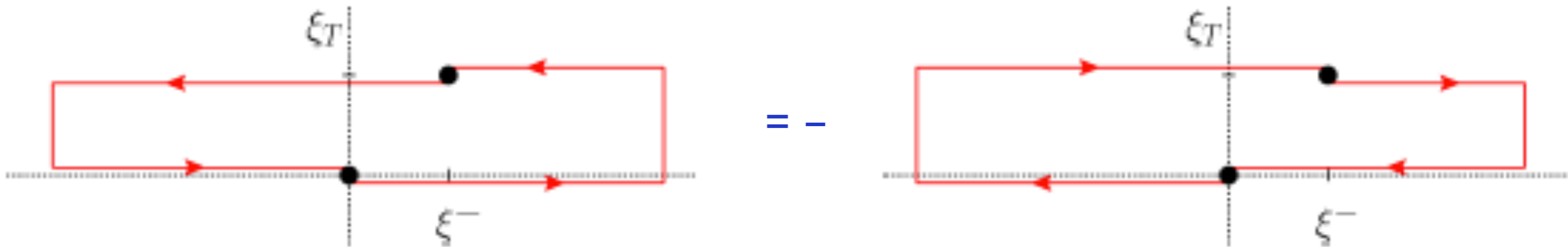
$$e p^\uparrow \rightarrow e' Q \bar{Q} X$$

$$\gamma^* g \rightarrow Q \bar{Q} \text{ probes } [+,+]$$

$$p^\uparrow p \rightarrow \gamma \text{ jet } X$$

In the kinematic regime where gluons in the polarized proton dominate, one effectively selects the subprocess:

$$g q \rightarrow \gamma q \text{ probes } [+,-]$$



These processes probe 2 distinct, independent gluon Sivers functions

Related to antisymmetric ( $f^{abc}$ ) and symmetric ( $d^{abc}$ ) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Conclusion: gluon Sivers TMD studies at EIC and at RHIC or AFTER@LHC can be related or complementary, depending on the processes considered



# Unpolarized gluon TMDs at small $x$

# WW vs DP

For most processes of interest there are 2 relevant unpolarized gluon distributions

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x, k_{\perp}) = 2 \int \frac{d\xi^{-} d\xi_{\perp}}{(2\pi)^3 P^+} e^{ixP^+ \xi^{-} - ik_{\perp} \cdot \xi_{\perp}} \langle P | \text{Tr} [F^{+i}(\xi^{-}, \xi_{\perp}) \mathcal{U}^{[+]\dagger} F^{+i}(0) \mathcal{U}^{[+]}] | P \rangle \quad [+, +]$$

$$xG^{(2)}(x, k_{\perp}) = 2 \int \frac{d\xi^{-} d\xi_{\perp}}{(2\pi)^3 P^+} e^{ixP^+ \xi^{-} - ik_{\perp} \cdot \xi_{\perp}} \langle P | \text{Tr} [F^{+i}(\xi^{-}, \xi_{\perp}) \mathcal{U}^{[-]\dagger} F^{+i}(0) \mathcal{U}^{[+]}] | P \rangle \quad [+, -]$$

For unpolarized gluons  $[+, +] = [-, -]$  and  $[+, -] = [-, +]$

At small  $x$  the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x, k_{\perp}) = -\frac{2}{\alpha_S} \int \frac{d^2 v}{(2\pi)^2} \frac{d^2 v'}{(2\pi)^2} e^{-ik_{\perp} \cdot (v - v')} \langle \text{Tr} [\partial_i U(v)] U^{\dagger}(v') [\partial_i U(v')] U^{\dagger}(v) \rangle_{x_g} \quad \text{WW}$$

$$xG^{(2)}(x, q_{\perp}) = \frac{q_{\perp}^2 N_c}{2\pi^2 \alpha_s} S_{\perp} \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-iq_{\perp} \cdot r_{\perp}} \frac{1}{N_c} \langle \text{Tr} U(0) U^{\dagger}(r_{\perp}) \rangle_{x_g} \quad \text{DP}$$

Different processes probe one or the other or a mixture, so this can be tested



# MV model

In the MV model one may not notice the origin for the difference between WW and DP, because the two TMDs become related:

$$xG_g^{(2)}(x, q_\perp) \stackrel{\text{MV}}{\propto} q_\perp^2 \nabla_{q_\perp}^2 xG_g^{(1)}(x, q_\perp)$$

Processes involving  $G^{(1)}$  (WW)  $[+,+]$  in the MV model can be expressed in terms of  $G^{(2)} \sim C(k_\perp)$ , e.g.

$$\gamma A \rightarrow Q\bar{Q} X$$

Gelis, Peshier, 2002

$$\frac{d\sigma_T}{dy dk_\perp} = \pi R^2 \frac{2N_c(Z\alpha)^2}{3\pi^3} \ln\left(\frac{\gamma}{2mR}\right) k_\perp C(k_\perp) \times \left\{ 1 + \frac{4(k_\perp^2 - m^2)}{k_\perp \sqrt{k_\perp^2 + 4m^2}} \operatorname{arcth} \frac{k_\perp}{\sqrt{k_\perp^2 + 4m^2}} \right\}$$

$$C(k_\perp) = \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \langle U(0)U^\dagger(x_\perp) \rangle$$

Heavy quark pair production in DIS probes the WW distribution, like  $pp \rightarrow \text{Higgs} X$

For general x expressions, see Pisano, D.B., Brodsky, Buffing, Mulders, 2013

# WW vs DP

Selection of processes that probe the WW or DP unpolarized gluon TMD:

	DIS	DY	SIDIS	$pA \rightarrow \gamma \text{jet } X$	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$	$pp \rightarrow \eta_{c,b} X$ $pp \rightarrow H X$	$pp \rightarrow J/\psi \gamma X$ $pp \rightarrow \Upsilon \gamma X$
$f_1^g^{[+,+]}$ (WW)	×	×	×	×	✓	✓	✓
$f_1^g^{[+,-]}$ (DP)	✓	✓	✓	✓	×	×	×

Dijet production in pA probes a combination of 6 distinct unpolarized gluon TMDs  
In the large  $N_c$  limit it probes a combination of DP and WW functions

Akcakaya, Schäfer, Zhou, 2013; Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren, 2015

Dijet production in pA generally suffers from factorization breaking contributions

Collins, Qiu, 2007; Rogers, Mulders, 2010

Single color singlet (CS)  $J/\psi$  or  $\Upsilon$  production from two gluons is not allowed by the Landau-Yang theorem, while color octet (CO) production involves a more complicated link structure. C-even (pseudo-)scalar quarkonium production is easier

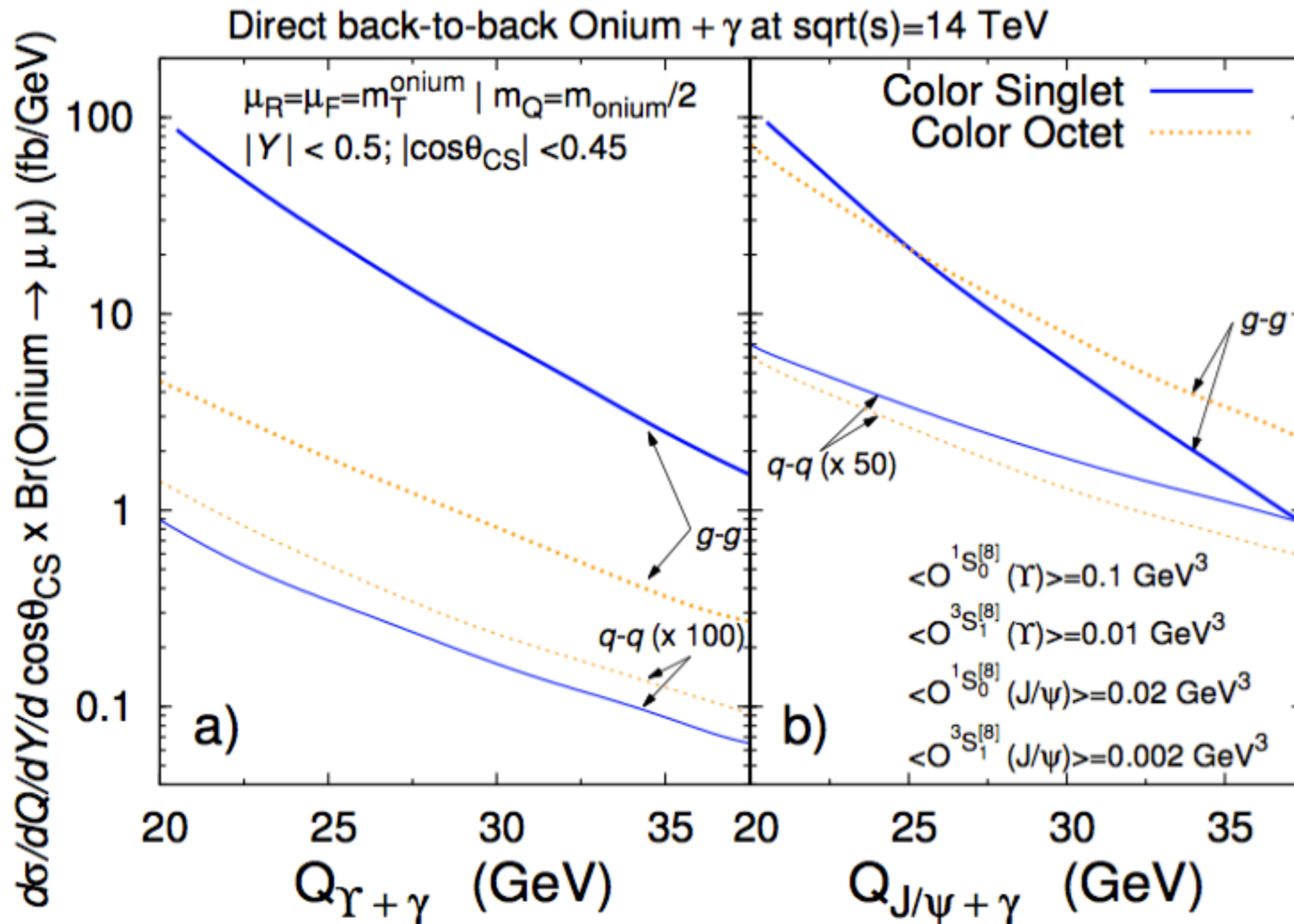
D.B., Pisano, 2012



# CS vs CO

In  $\Upsilon + \gamma$  production the color singlet contribution dominates and in  $J/\psi + \gamma$  production for a specific range of invariant mass of the pair

den Dunnen, Lansberg, Pisano, Schlegel, 2014



Linearly polarized gluons in  
unpolarized hadrons  
at small  $x$



# Gluon polarization inside unpolarized protons

Linearly polarized gluons can exist in unpolarized hadrons

[Mulders, Rodrigues, 2001]

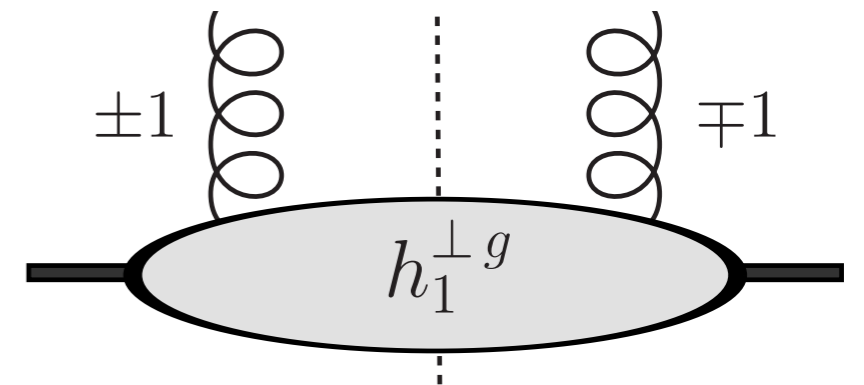
It requires nonzero transverse momentum: TMD

For  $h_1^{\perp g} > 0$  gluons prefer to be polarized along  $\mathbf{k}_T$ , with a  $\cos 2\phi$  distribution of linear polarization around it, where  $\phi = \angle(\mathbf{k}_T, \boldsymbol{\varepsilon}_T)$

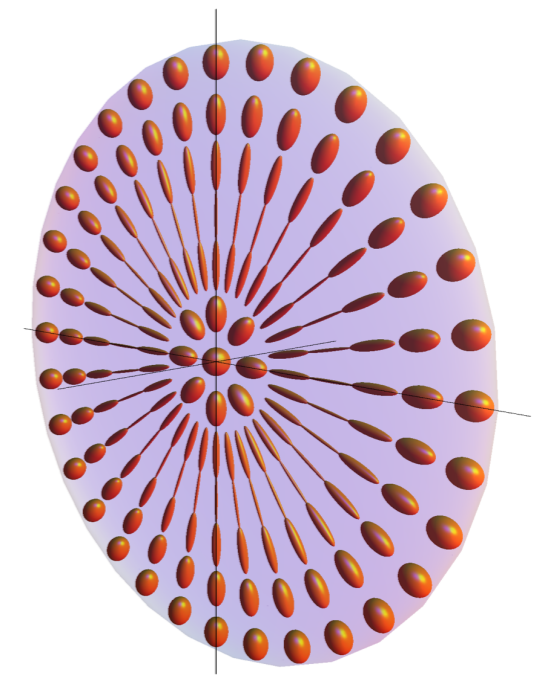
This TMD is  $\mathbf{k}_T$ -even, chiral-even and T-even:

$$\Gamma_U^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{p}_T^2) + \left( \frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{p}_T^2) \right\}$$

For linearly polarized gluons also  $[+,+] = [-,-]$  and  $[+,-] = [-,+]$



an interference between  $\pm 1$  helicity gluon states





# Linear gluon polarization at small x

$h_1^{\perp g}$  is more difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or  $\gamma$ +jet production in pp or pA collisions

Selection of processes that probe the WW or DP linearly polarized gluon TMD:

	$pp \rightarrow \gamma \gamma X$	$pA \rightarrow \gamma^* \text{jet } X$	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$	$pp \rightarrow \eta_{c,b} X$ $pp \rightarrow H X$	$pp \rightarrow J/\psi \gamma X$ $pp \rightarrow \Upsilon \gamma X$
$h_1^{\perp g [+,+]}$ (WW)	✓	×	✓	✓	✓
$h_1^{\perp g [+, -]}$ (DP)	×	✓	×	×	×

Higgs and  $0^{\pm\pm}$  quarkonium production allows to measure the linear gluon polarization using the angular independent  $p_T$  distribution

All other suggestions use angular modulations

EIC and RHIC/LHC can probe same  $h_1^{\perp g}$

# Linear gluon polarization in $pp \rightarrow HX$

$h_1^{\perp g}$  affects Higgs production at the LHC

Boer, Den Dunnen, Pisano, Schlegel, Vogelsang, PRL 2012

The LHC is actually a *polarized* gluon collider

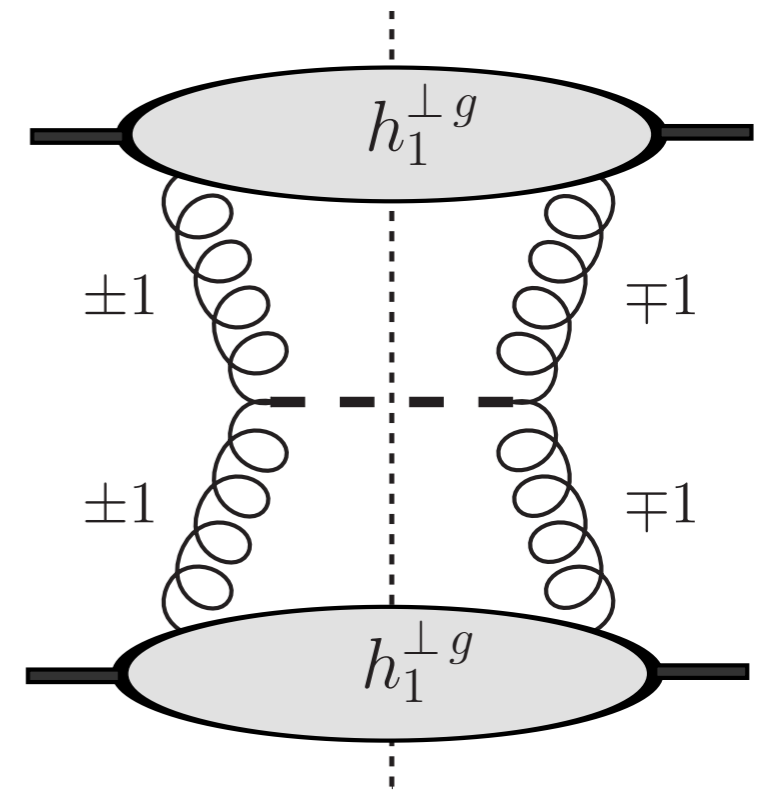
It remains to be seen whether this can be exploited

The relative effect of linearly polarized gluons:

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$$

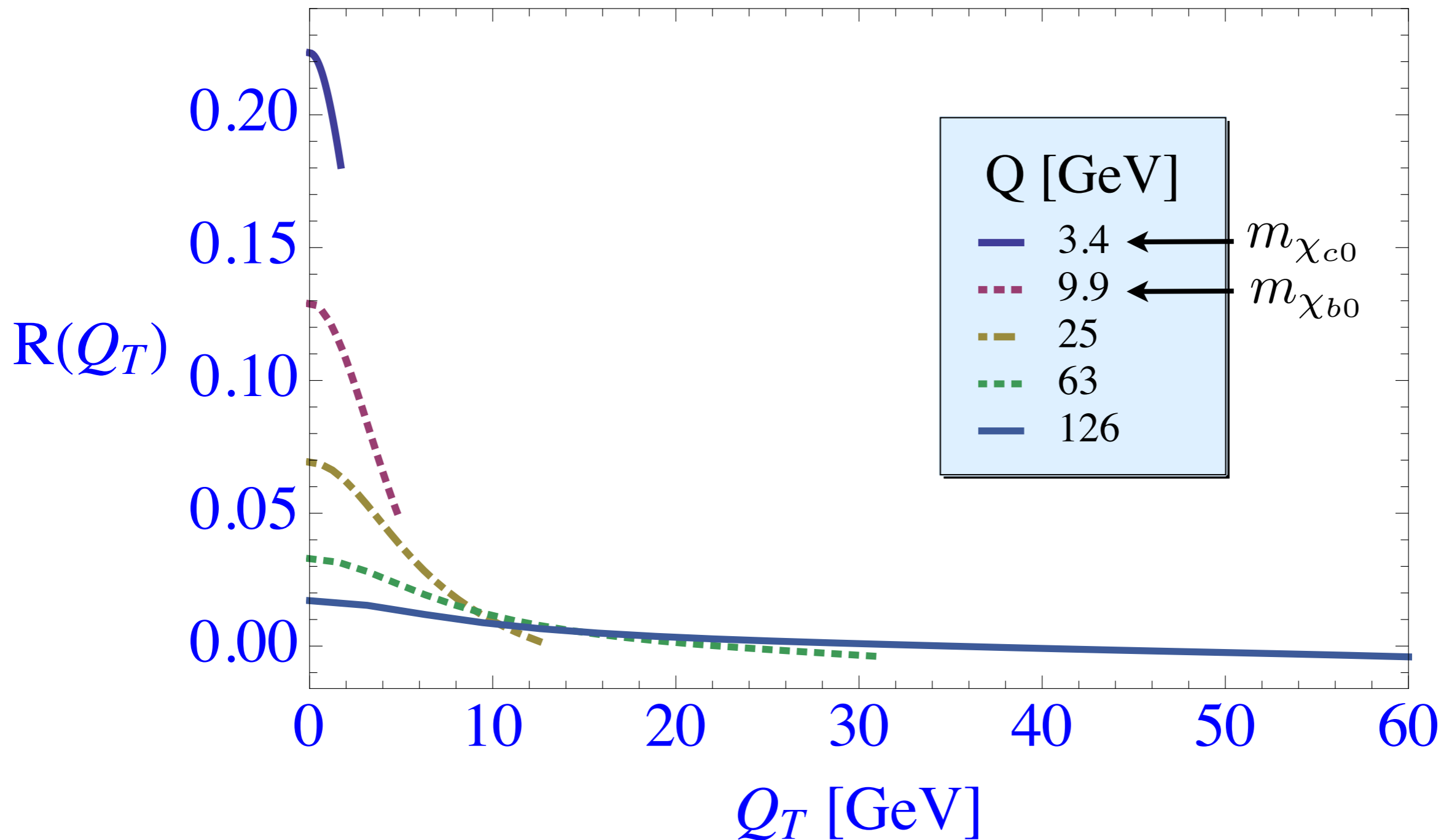
TMD evolution suppresses this ratio

D.B. & den Dunnen, 2014



$$w_H = \frac{(\mathbf{p}_T \cdot \mathbf{k}_T)^2 - \frac{1}{2} \mathbf{p}_T^2 \mathbf{k}_T^2}{2M^4}$$

# Color singlet scalar production



Quantum corrections imply that the effect of linear gluon polarization decreases with the mass of the scalar produced as:  $Q^{-0.85}$

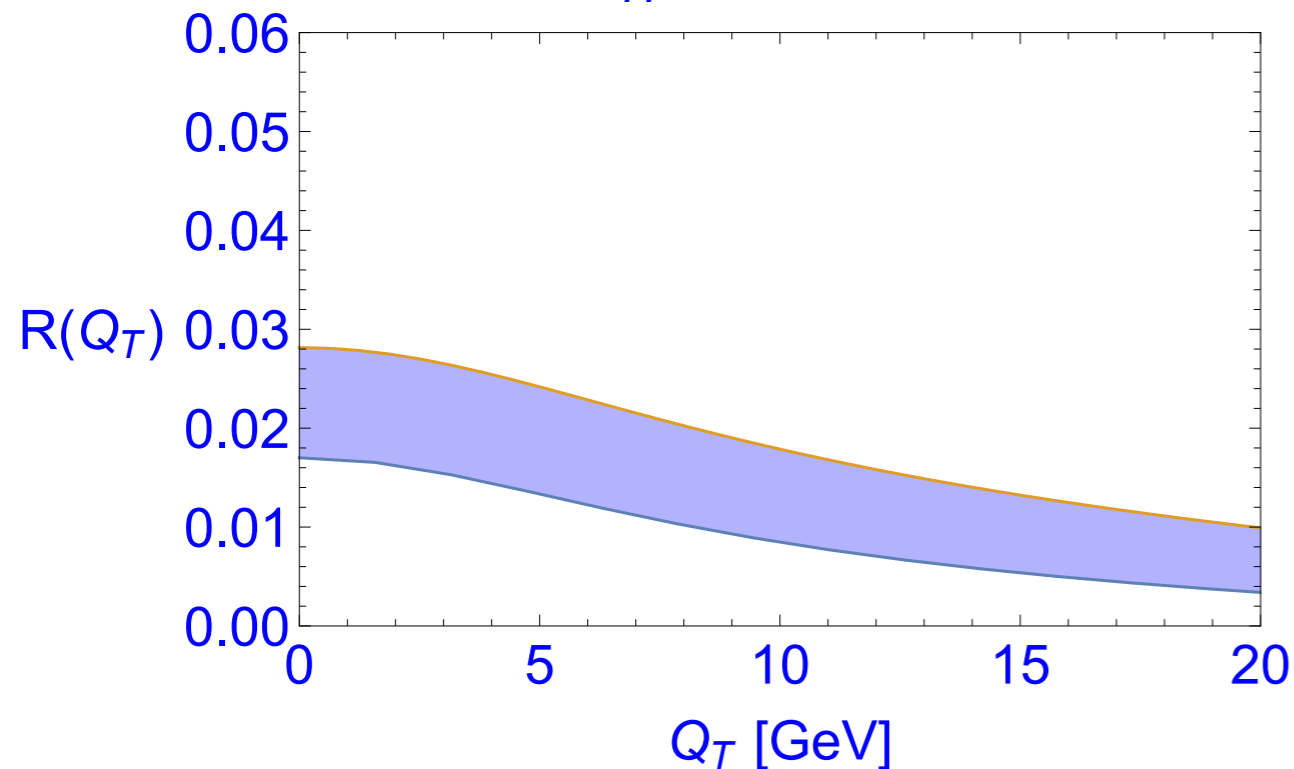
D.B. & den Dunnen, 2014

**Conclusion: in Higgs production linear gluon polarization contributes at few % level**



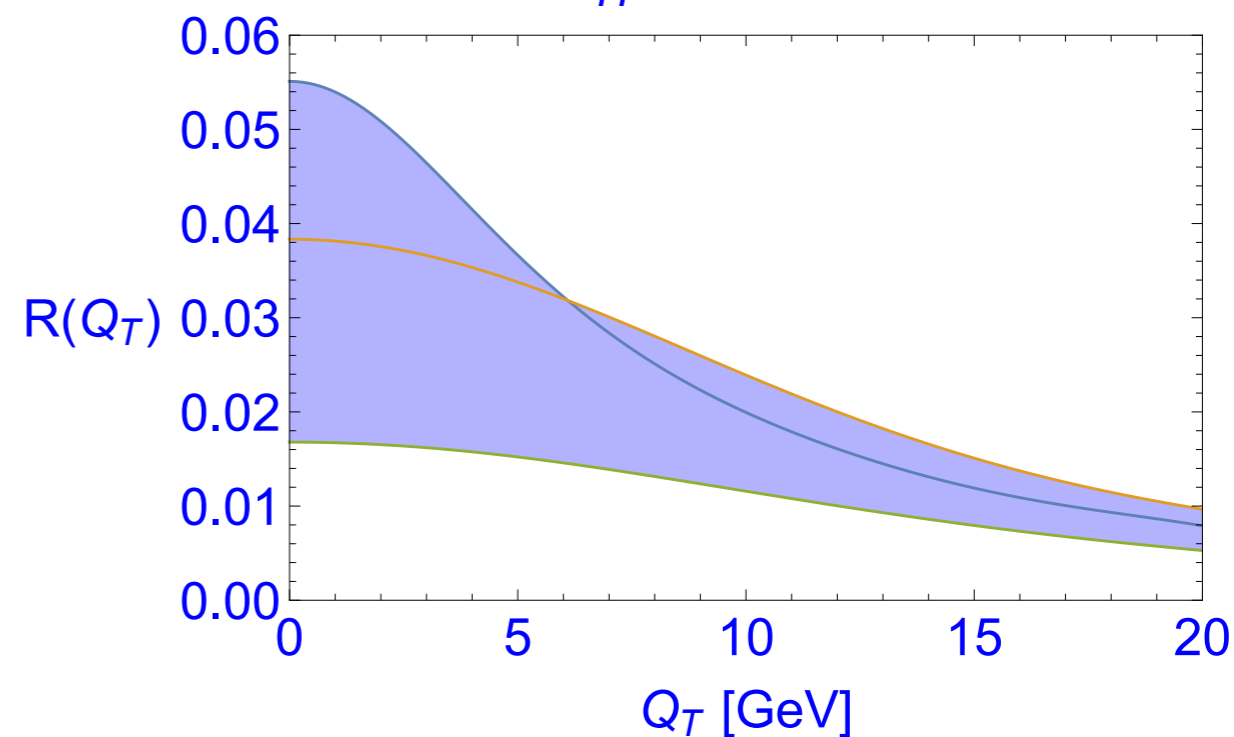
# Range of predictions

$m_H = 126 \text{ GeV}$



Boer & den Dunnen, 2014

$m_H = 126 \text{ GeV}$



Echevarria, Kasemets, Mulders, Pisano, 2015

Left: variation of the nonperturbative input and of the large  $Q_T$  behavior

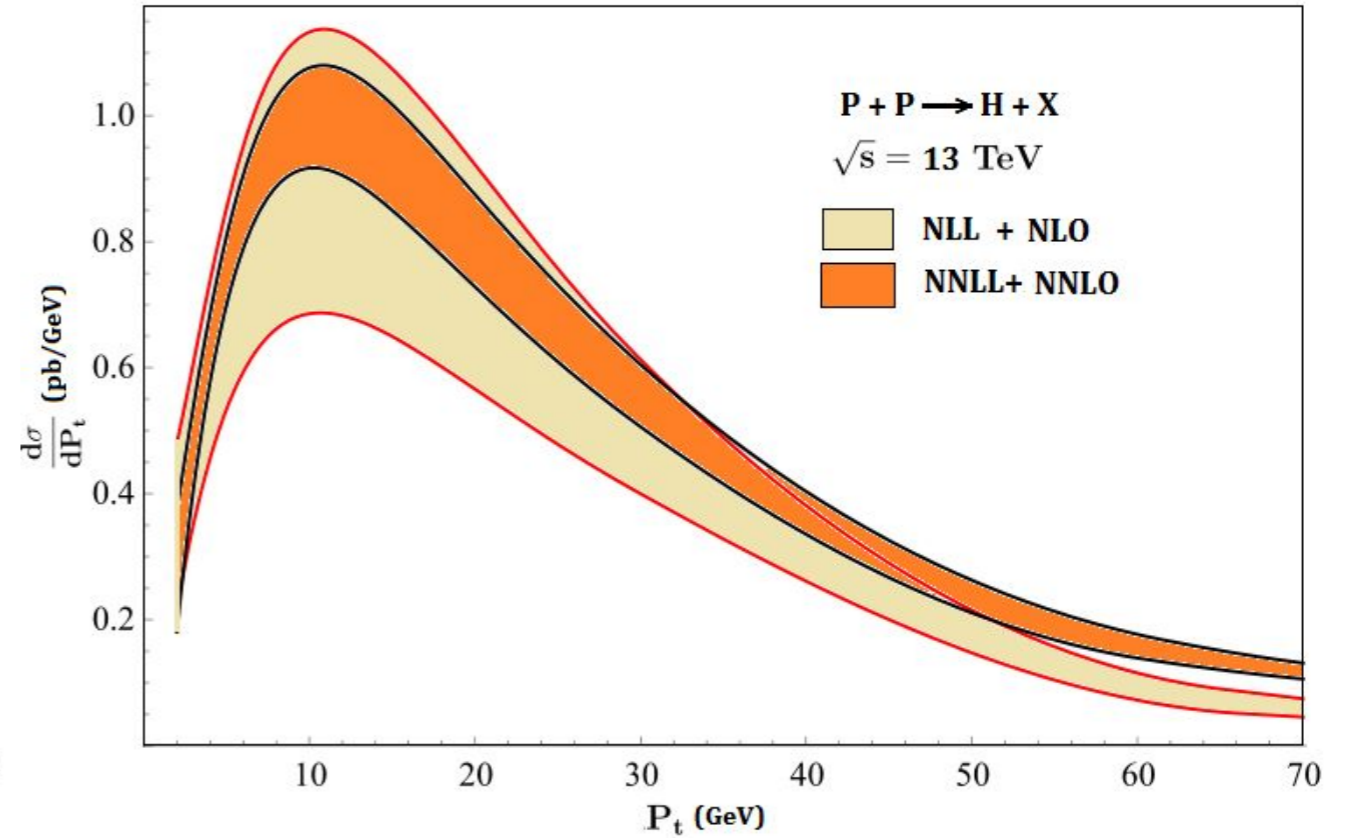
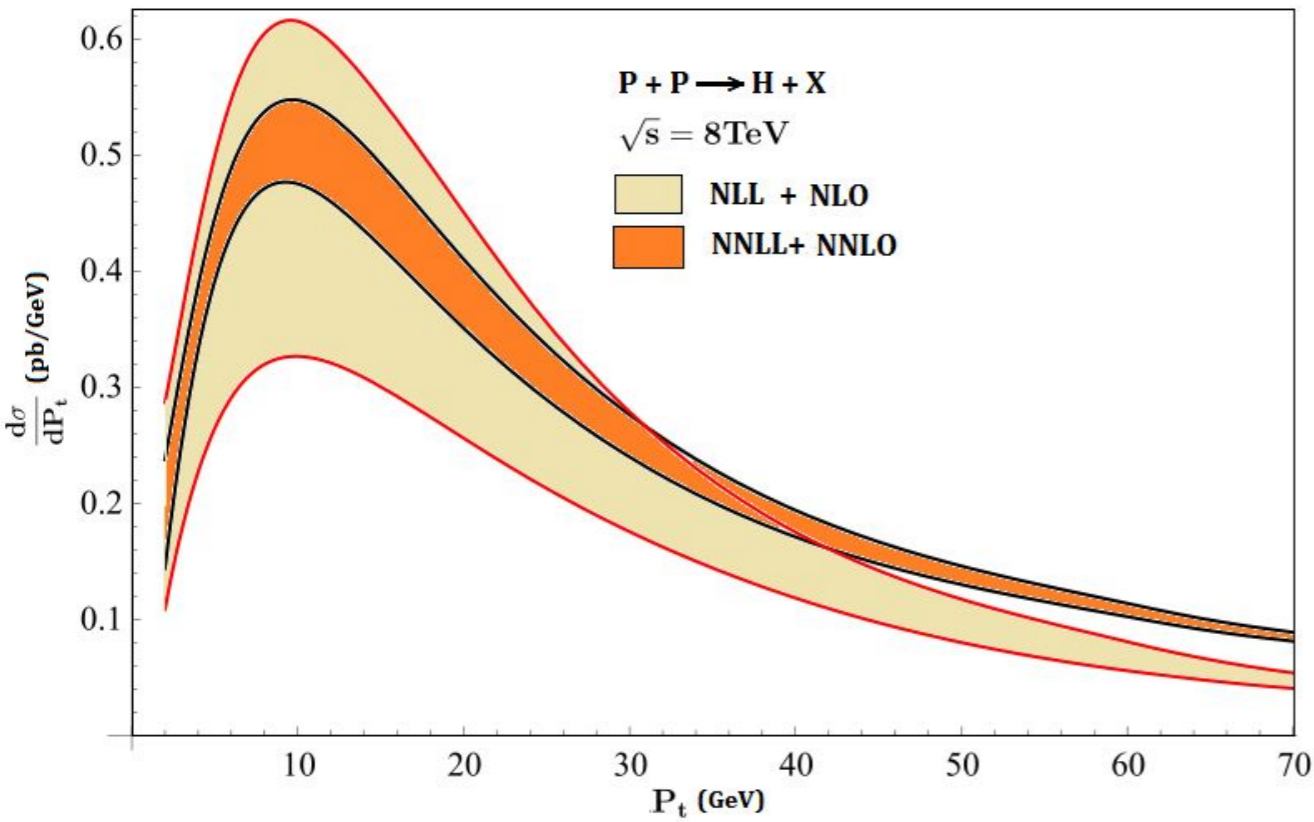
Right: variation of the nonperturbative input and the renormalization scale

## Conclusions:

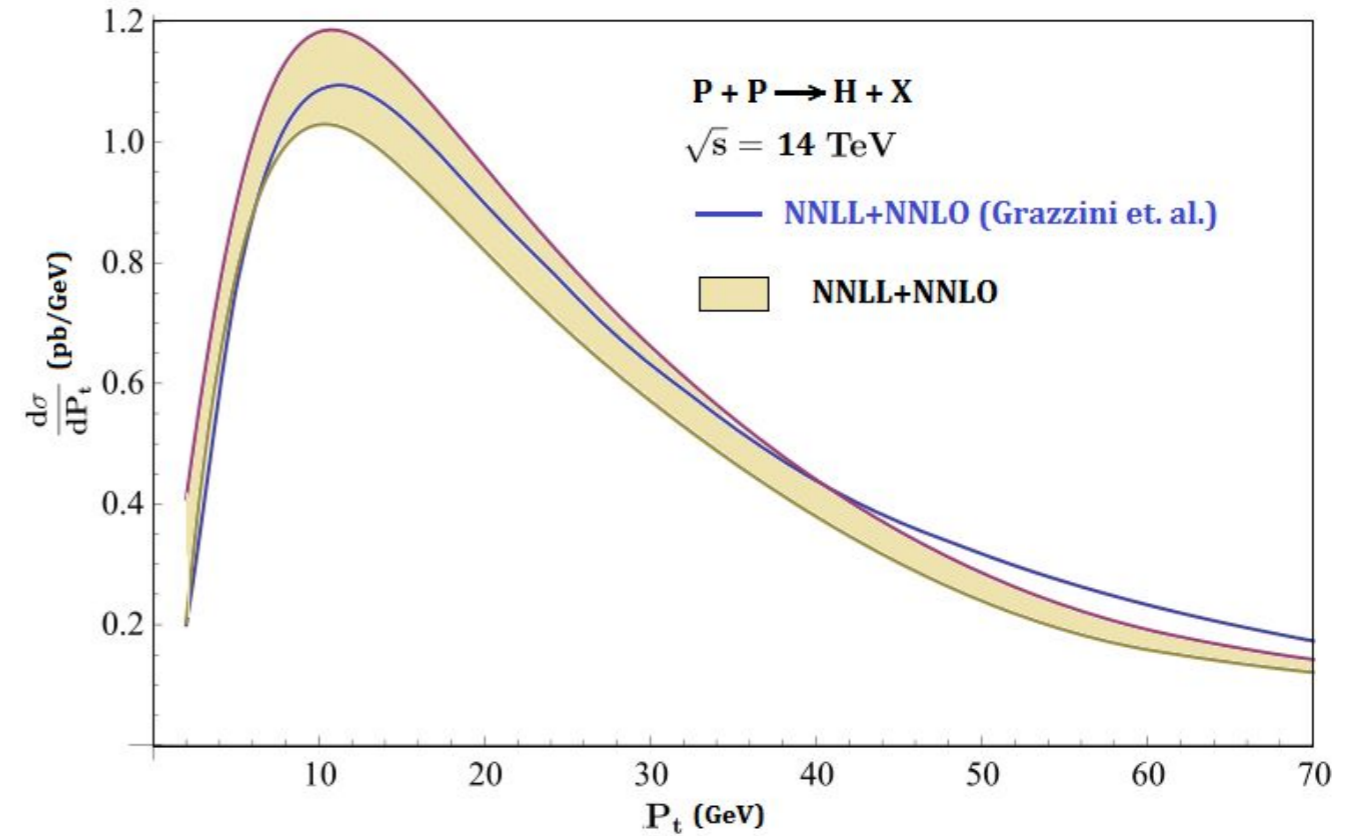
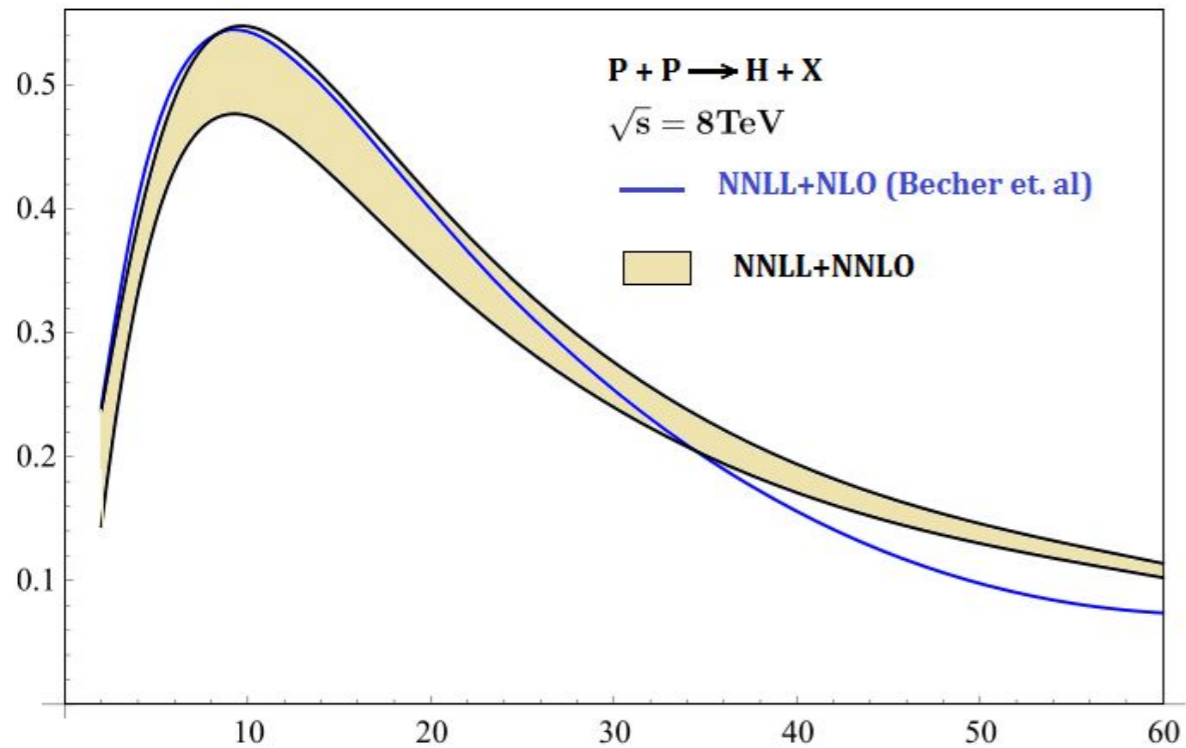
- effect of linear gluon polarization in Higgs production on the order of 2-5%
- extraction of  $h_1^{\perp g}$  from Higgs production may be too challenging

Effects larger at smaller  $Q$  ( $0^{\pm+}$  quarkonia) and at small  $x$  (plots are for  $x \sim 0.016$ )

# Perturbative state-of-the-art

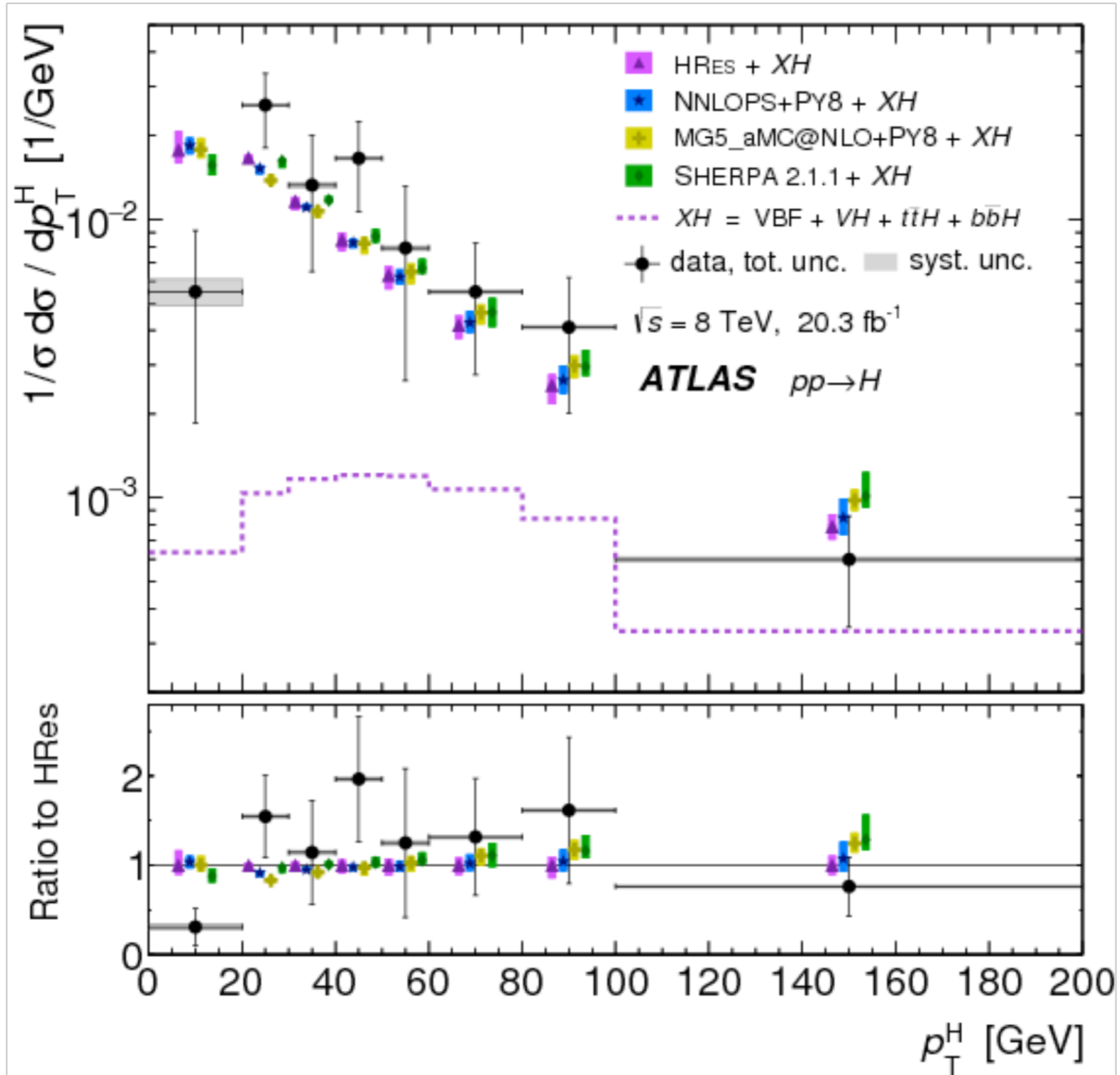


Neill, Rothstein, Vaidya, arXiv:1503.00005



NNLL+NNLO has 10-20% uncertainty, plus an unknown nonperturbative contribution

# Current data



Current  $p_T$  resolution of Higgs too low at low  $p_T$ , will eventually be around 5 GeV



# Quarkonium production

C-even (pseudo-)scalar quarkonium production promising for studying  $h_1^{\perp g}$

Using the CS model and LO NRQCD we obtain:

$$\frac{d\sigma(\eta_Q)}{dy d^2\mathbf{q}_T} = \frac{2}{9} \frac{\pi^3 \alpha_s^2}{M^3 s} \langle 0 | \mathcal{O}_1^{\eta_Q} (^1S_0) | 0 \rangle \mathcal{C} [f_1^g f_1^g] [1 - R(\mathbf{q}_T^2)]$$

$$\frac{d\sigma(\chi_{Q0})}{dy d^2\mathbf{q}_T} = \frac{8}{3} \frac{\pi^3 \alpha_s^2}{M^5 s} \langle 0 | \mathcal{O}_1^{\chi_{Q0}} (^3P_0) | 0 \rangle \mathcal{C} [f_1^g f_1^g] [1 + R(\mathbf{q}_T^2)]$$

$$\frac{d\sigma(\chi_{Q2})}{dy d^2\mathbf{q}_T} = \frac{32}{9} \frac{\pi^3 \alpha_s^2}{M^5 s} \langle 0 | \mathcal{O}_1^{\chi_{Q2}} (^3P_2) | 0 \rangle \mathcal{C} [f_1^g f_1^g]$$

D.B., Pisano, PRD 86 (2012) 094007

These are color singlet model expressions, which at least may be justified for C=+ bottomonium states

Bodwin, Braaten, Lepage, 1995; Hägler, Kirschner, Schäfer, Teryaev, 2001; Maltoni, Polosa, 2004; Bodwin, Braaten, Lee, 2005; ...

# Bottomonium production

To extract  $R(Q_T)$  one can consider 3 bottomonia and ratios of ratios:

$$\frac{\sigma(\chi_{b0})}{\sigma(\eta_b)} \frac{d\sigma(\eta_b)/d^2\mathbf{q}_T}{d\sigma(\chi_{b0})/d^2\mathbf{q}_T} \approx \frac{1 + R(\mathbf{q}_T^2)}{1 - R(\mathbf{q}_T^2)}$$

$$\frac{\sigma(\chi_{b0})}{\sigma(\chi_{b2})} \frac{d\sigma(\chi_{b2})/d^2\mathbf{q}_T}{d\sigma(\chi_{b0})/d^2\mathbf{q}_T} \approx 1 + R(\mathbf{q}_T^2)$$

Uncertainties about the hadronic wave function (approximately) cancel

Very small scale differences:  $m_{\eta_b} \approx m_{\chi_{b0}} \approx m_{\chi_{b2}}$

Therefore, hardly any TMD evolution effects

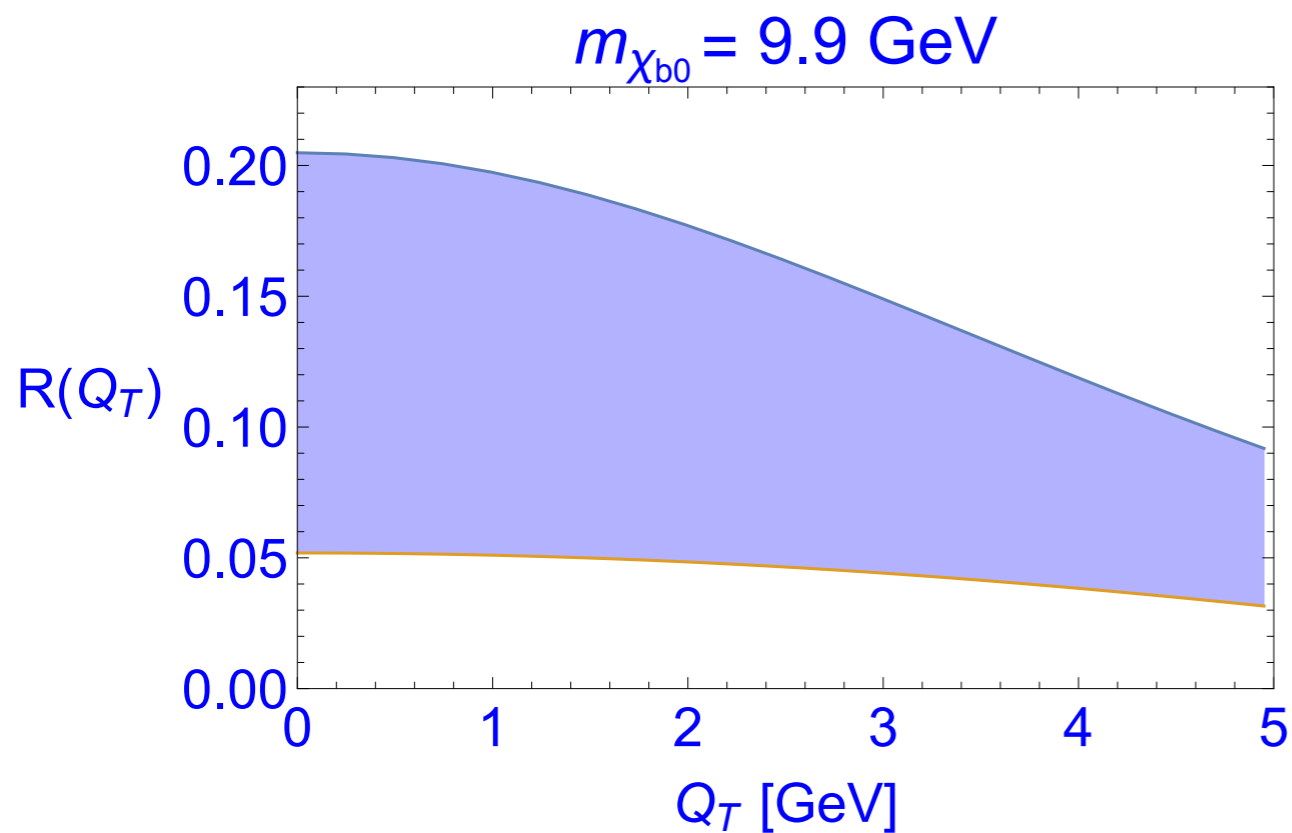
TMD factorization for the p-wave states  $\chi_{bj}$  has been called into question, but can be tested with these ratios as well

J.P. Ma, Wang, Zhao, 2014

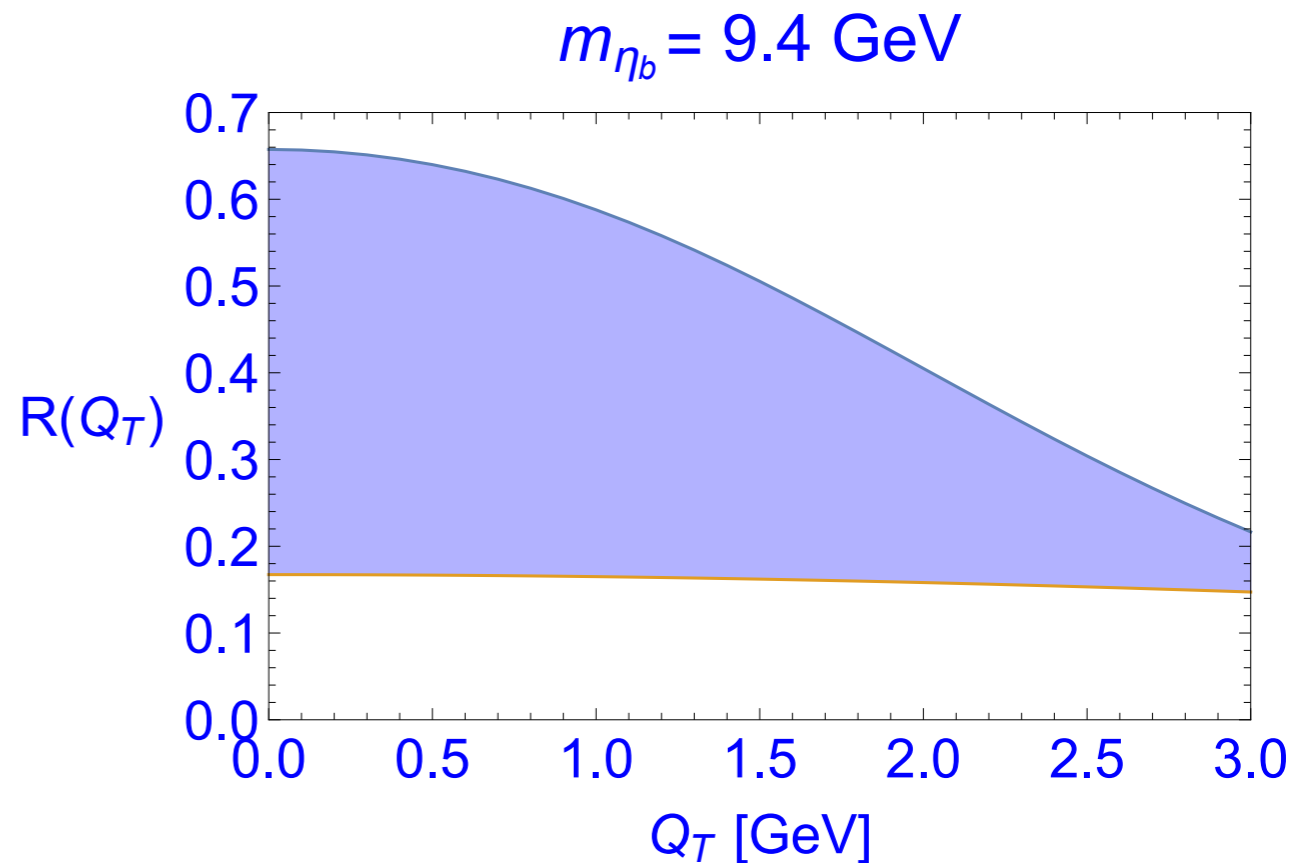
Of course, not easy experimentally, but sizeable effects are expected

# Bottomonium production

The range of predictions for C-even bottomonium production:



Boer & den Dunnen, 2014



Echevarria, Kasemets, Mulders, Pisano, 2015

**Conclusion: very large theoretical uncertainties in quarkonium production (more sensitive to unknown nonperturbative part than Higgs production), but larger effects**



# Linear gluon polarization at small x

There is no theoretical reason why  $h_1^{\perp g}$  effects should be small, especially at small x

Evolution:  $h_1^{\perp g}$  has the same  $1/x$  growth as  $f_1$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left( \frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

The small-x limit of the DP correlator in the TMD formalism:

$$\Gamma^{[+,-]ij}(x, \mathbf{k}_T) \xrightarrow{x \rightarrow 0} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\square]}(\mathbf{k}_T) \quad U^{[\square]} = U_{[0,y]}^{[+]} U_{[y,0]}^{[-]}$$

$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[ -g_T^{ij} f_1(x, \mathbf{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^{\perp}(x, \mathbf{k}_T^2) \right] \xrightarrow{x \rightarrow 0} \frac{k_T^i k_T^j}{2M^2} e(\mathbf{k}_T^2)$$

$$\lim_{x \rightarrow 0} x f_1(x, \mathbf{k}_T^2) = \frac{\mathbf{k}_T^2}{2M^2} \lim_{x \rightarrow 0} x h_1^{\perp}(x, \mathbf{k}_T^2) = \frac{\mathbf{k}_T^2}{2M^2} e(\mathbf{k}_T^2)$$

The DP  $h_1^{\perp g}$  becomes maximal when  $x \rightarrow 0$

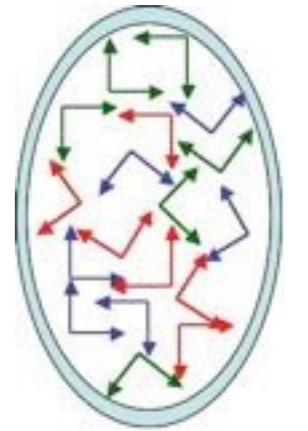
# Polarization of the CGC

CGC framework calculations show the CGC gluons are in fact linearly polarized

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_{\perp} \ll Q_s, \quad h_{1,WW}^{\perp g} = 2f_{1,WW}^{\perp g} \quad \text{for } k_{\perp} \gg Q_s$$

$$xh_{1,DP}^{\perp g}(x, k_{\perp}) = 2xf_{1,DP}^g(x, k_{\perp})$$

Metz, Zhou '11



The WW  $h_1^{\perp g}$  is (moderately) suppressed for small transverse momenta:

$$\frac{h_{1,WW}^{\perp g}}{f_{1,WW}} \propto \frac{1}{\ln Q_s^2/k_{\perp}^2}$$

The CGC can be 100% polarized, but its observable effects depend on the process

The “ $k_T$ -factorization” approach (CCFM) yields maximum polarization too:

$$\Gamma_g^{\mu\nu}(x, \mathbf{p}_T)_{\text{max pol}} = \frac{p_T^\mu p_T^\nu}{p_T^2} x f_1^g$$

Catani, Ciafaloni, Hautmann, 1991

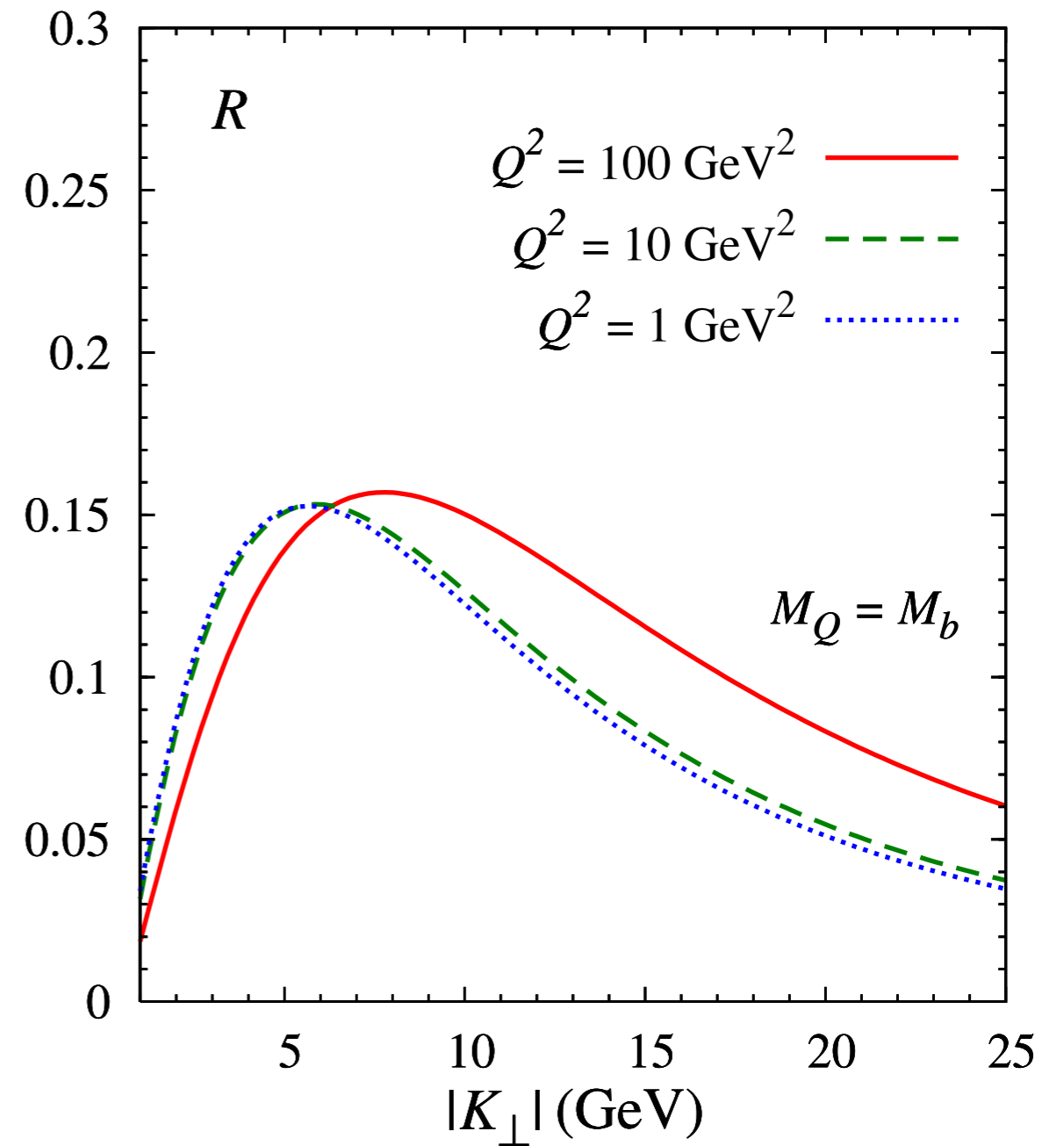
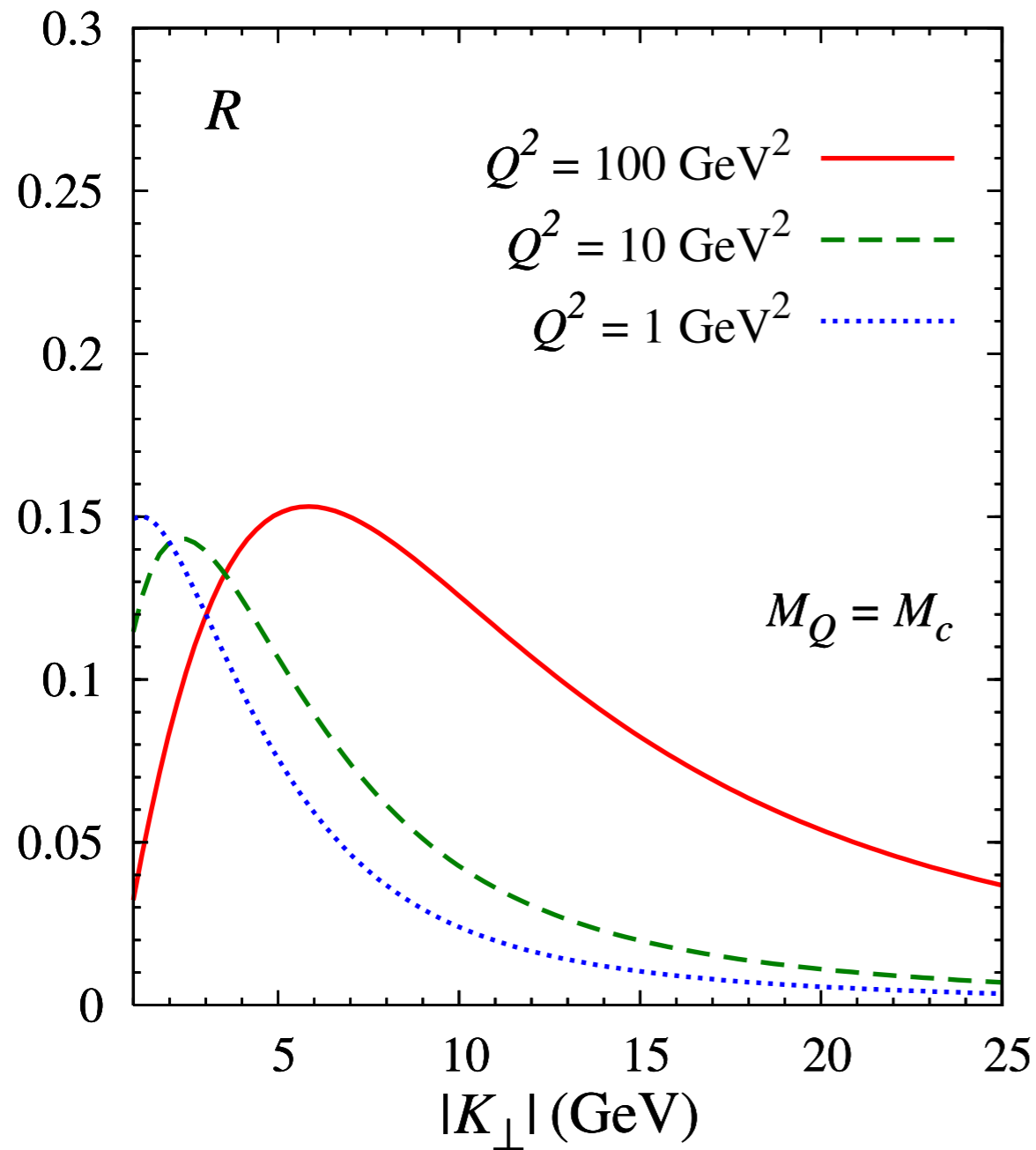




# Maximum asymmetries in heavy quark production

$$ep \rightarrow e' Q \bar{Q} X$$

$$R = \text{bound on } |\langle \cos 2(\phi_T - \phi_\perp) \rangle|$$

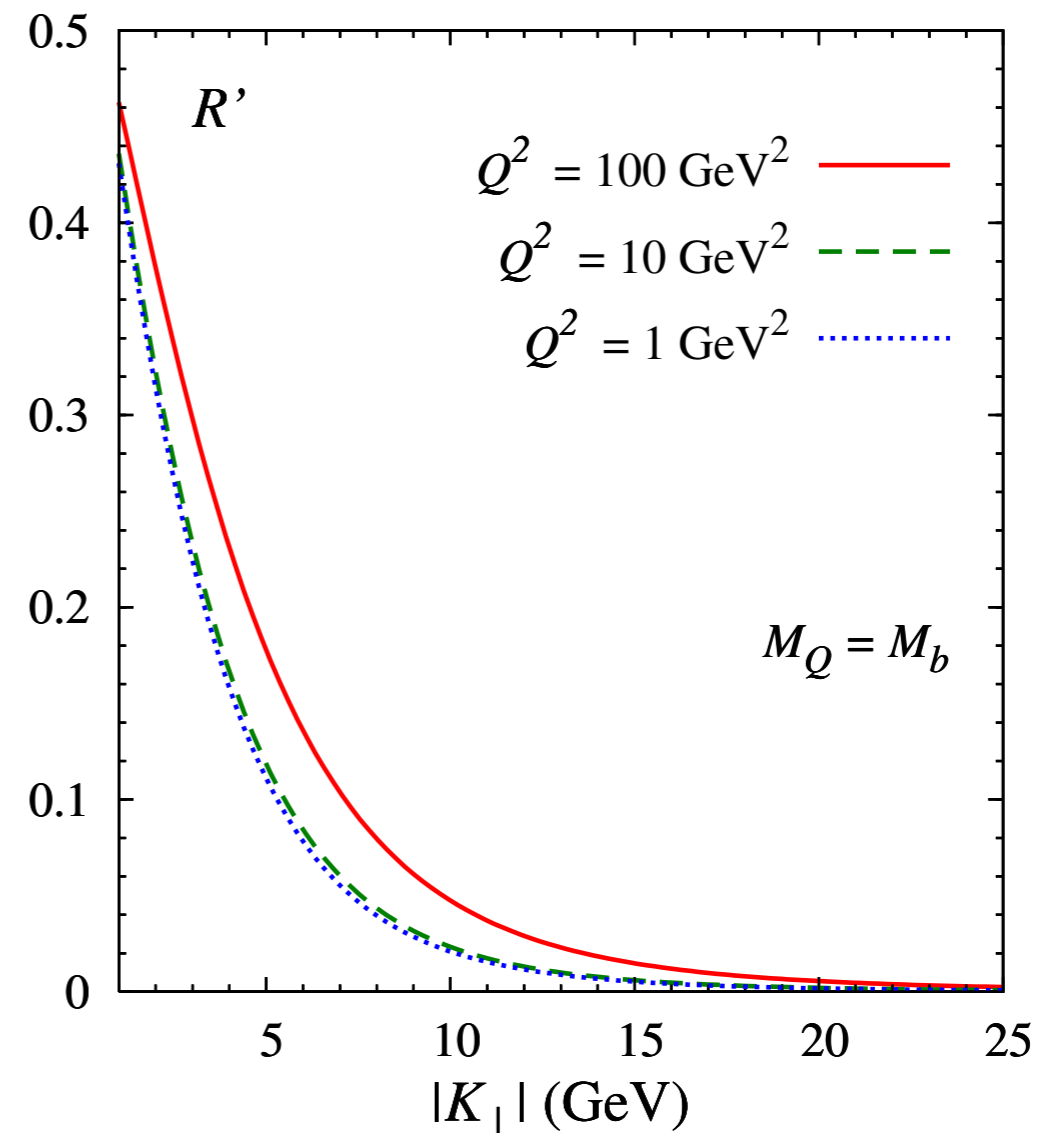
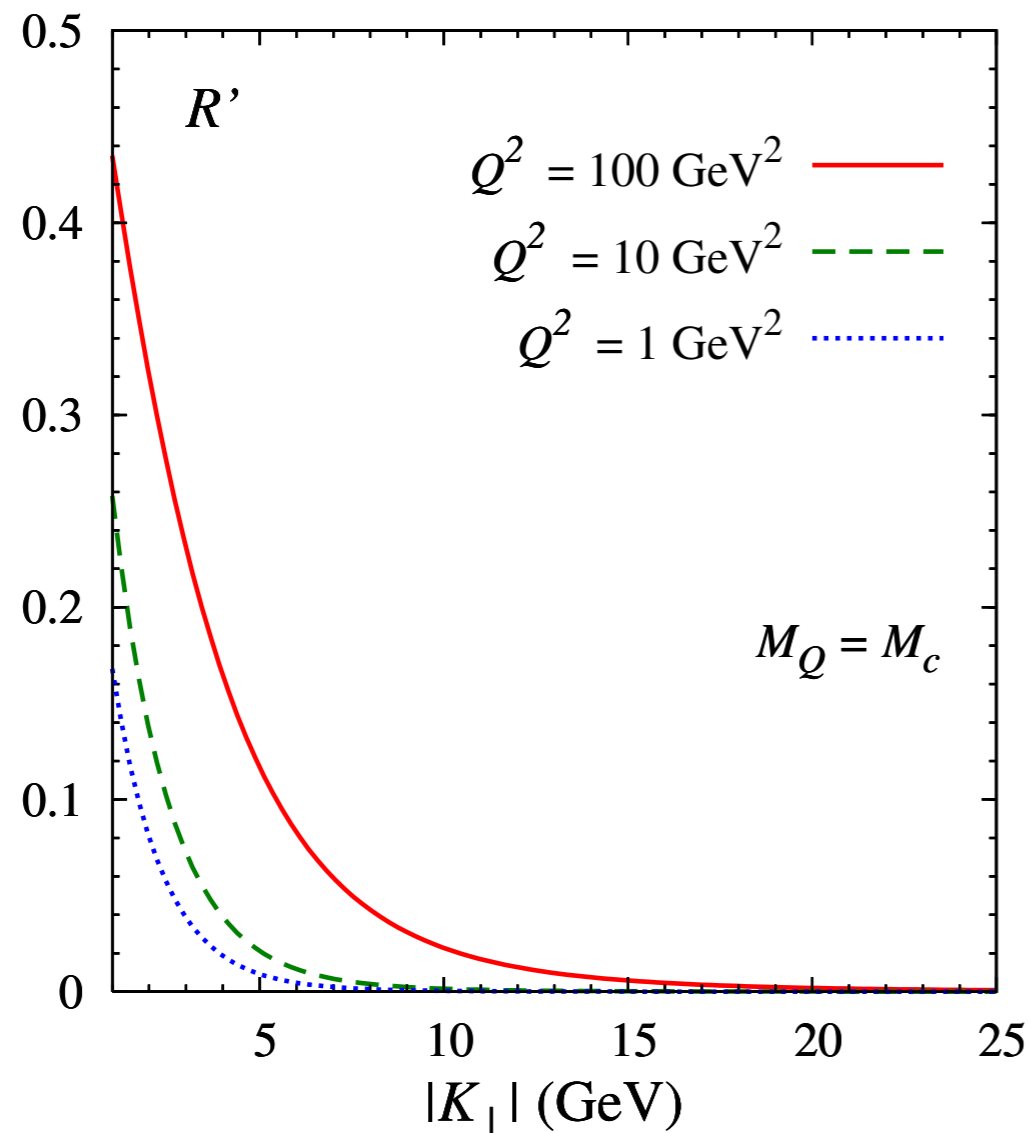


$$y = 0.01$$

# Maximum asymmetries in heavy quark production

There are also angular asymmetries w.r.t. the lepton scattering plane, which are mostly relevant at smaller  $|K_{\perp}|$

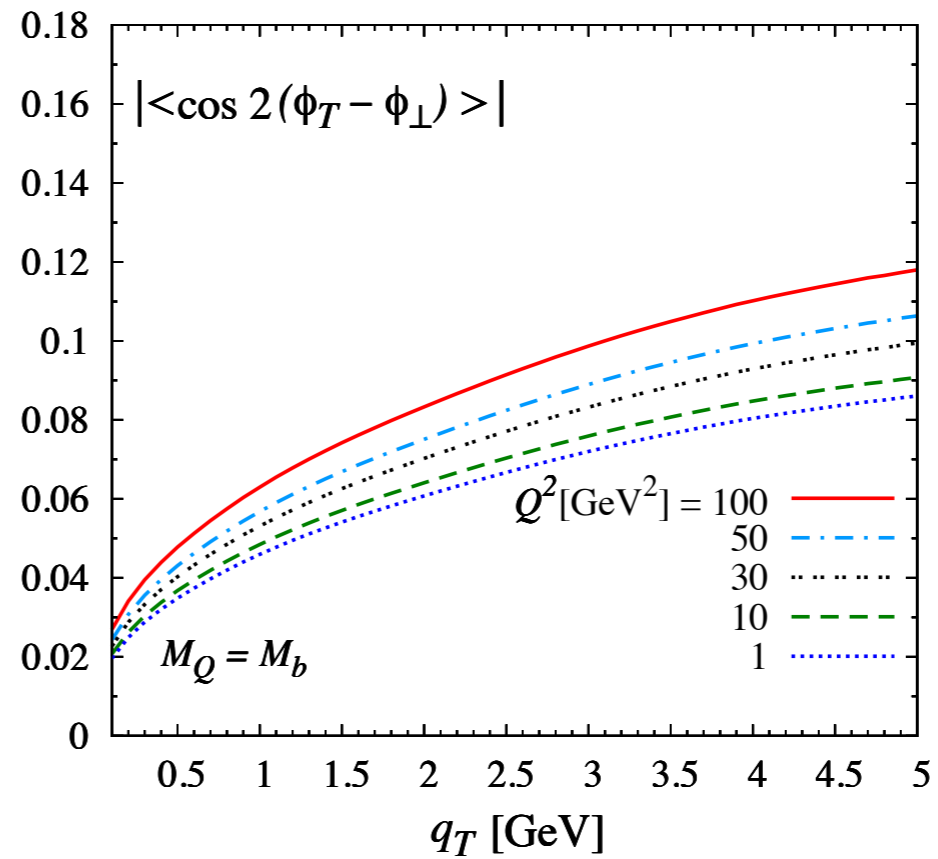
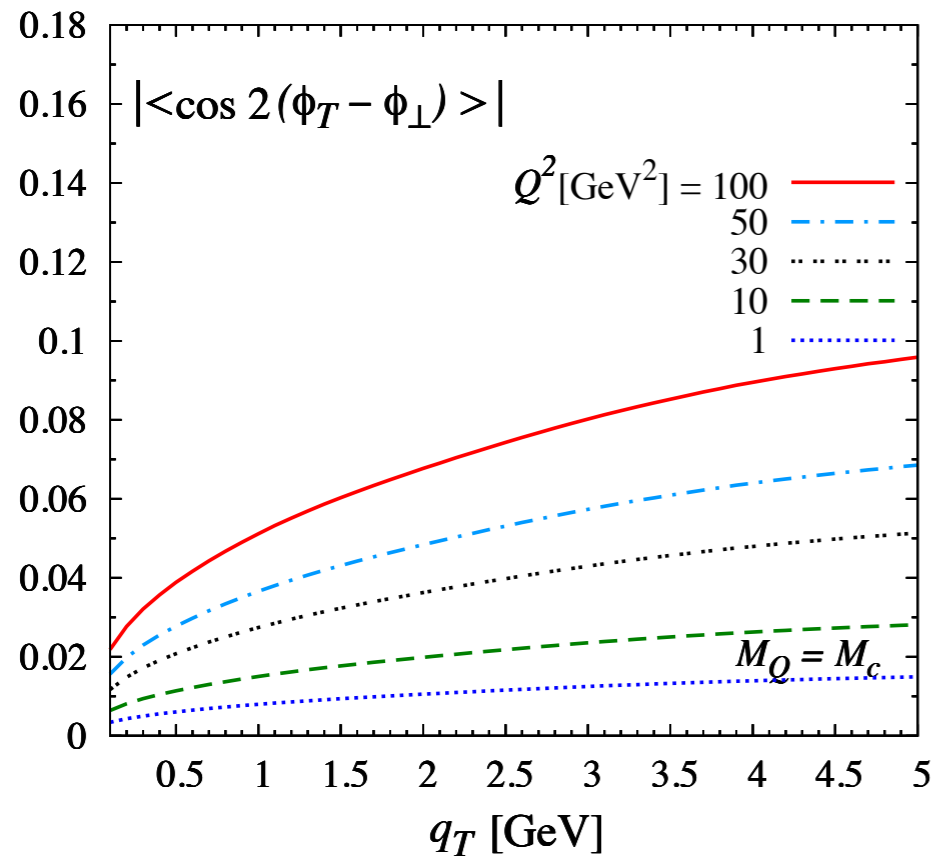
$$ep \rightarrow e' Q \bar{Q} X \quad R' = \text{bound on } |\langle \cos 2(\phi_{\ell} - \phi_T) \rangle|$$



$$y = 0.01$$

[Pisano, D.B., Brodsky, Buffing & Mulders, 2013]

# Heavy quark pair production at EIC

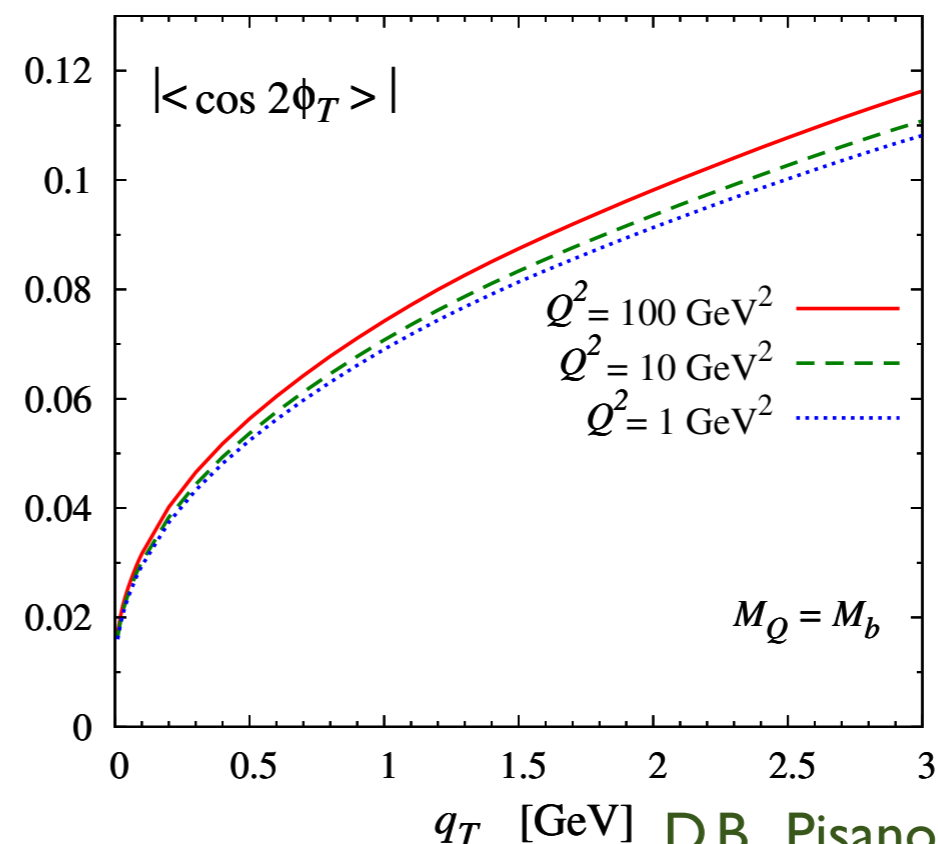
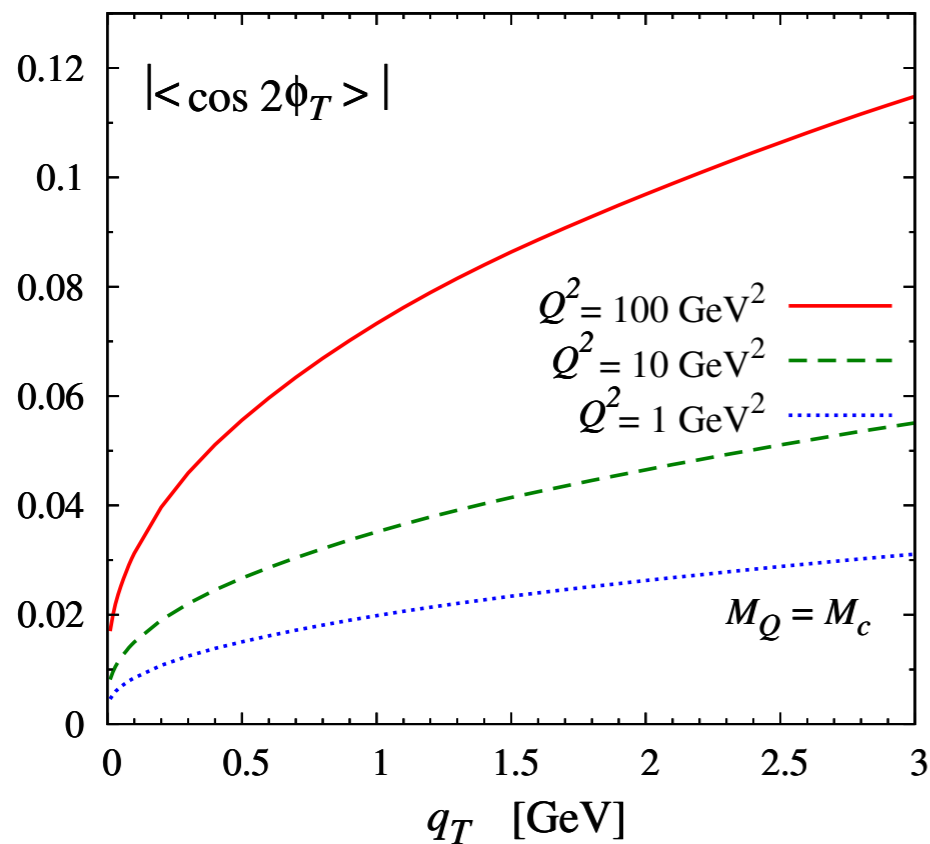


MV model

$$|\mathbf{K}_\perp| = 10 \text{ GeV}$$

$$z = 0.5$$

$$y = 0.3$$



$$|\mathbf{K}_\perp| = 6 \text{ GeV}$$

$$z = 0.5$$

$$y = 0.1$$

# Dijet production at EIC

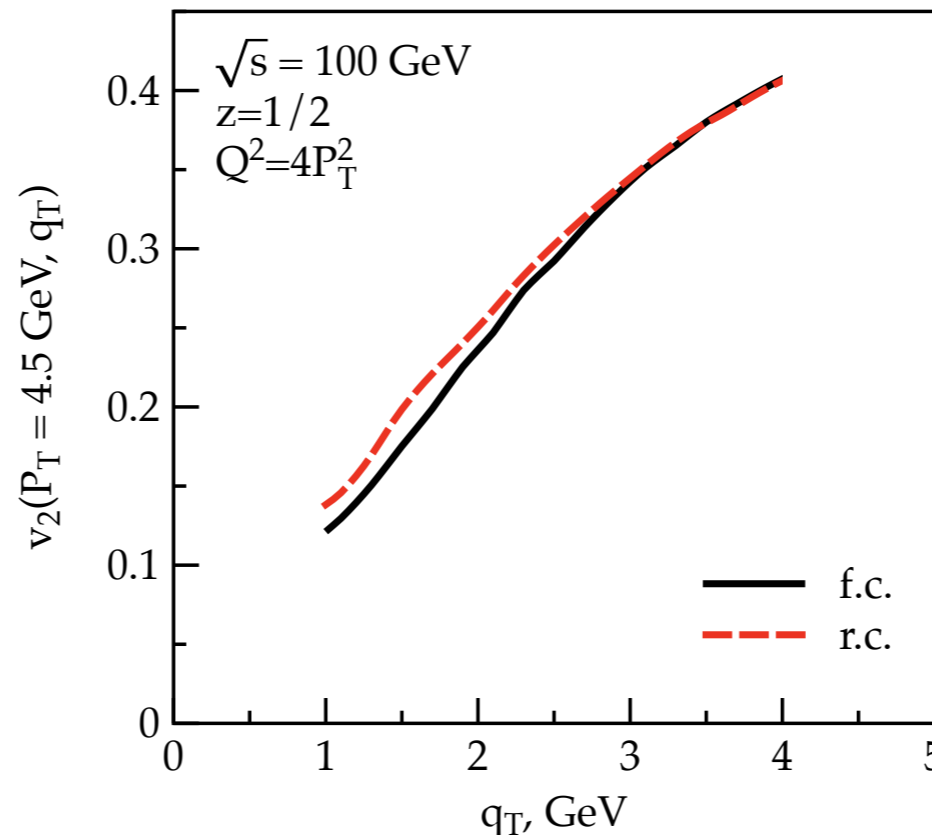
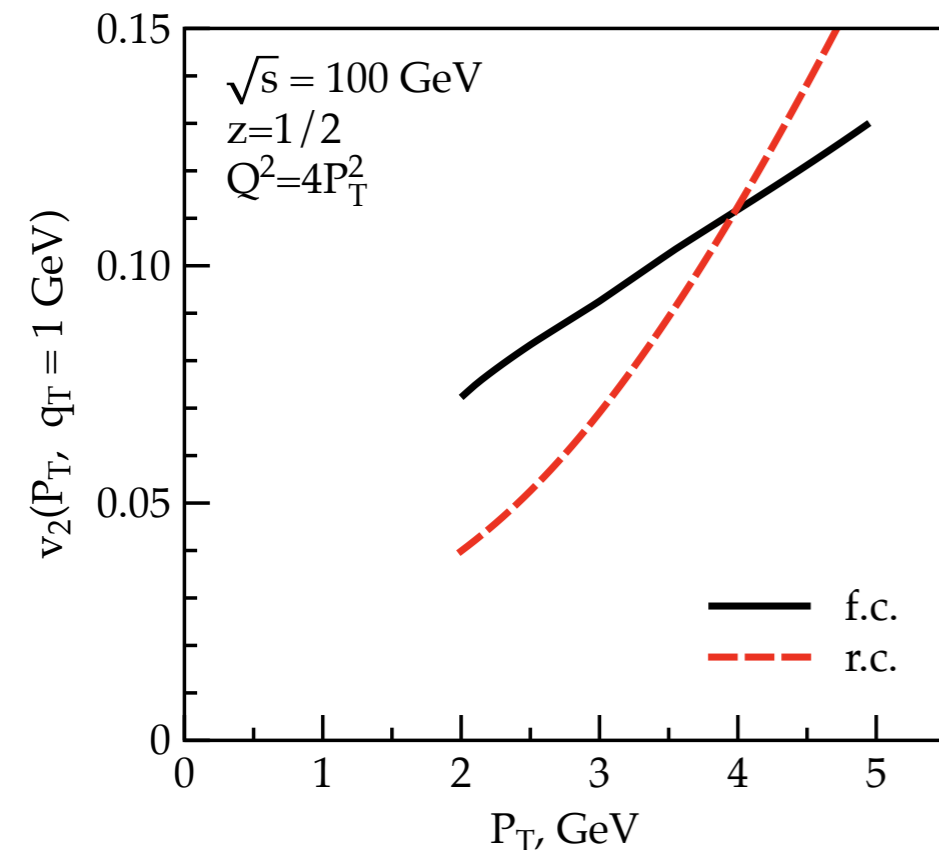
**WW  $h_1^{\perp g}$  accessible in dijet production in eA collisions at a high-energy EIC**  
 [Metz, Zhou 2011; Pisano, D.B., Brodsky, Buffing, Mulders, 2013; D.B., Pisano, Mulders, Zhou, 2016]

The WW  $h_1^{\perp g}$  is (moderately) suppressed for small transverse momenta:

$$\frac{h_{1WW}^{\perp g}}{f_{1WW}} \propto \frac{1}{\ln Q_s^2/k_{\perp}^2}$$

Metz, Zhou '11

Polarization shows itself through a  $\cos 2\phi$  distribution



Large effects are found  
 Dumitru, Lappi, Skokov, 2015



# Gluon Sivers effect at small $x$

# Small gluon Sivers effect?

Arguments suggesting gluon Sivers is small:

- Burkardt sum rule already (approximately) satisfied by up and down quarks

$$\sum_{a=q,g} \int f_{1T}^{\perp(1)a}(x) dx = 0$$

- small Sivers asymmetry on deuteron target as found by COMPASS  
[Brodsky & Gardner, 2006]
- $1/N_c$  suppressed at not too small  $x$  ( $x \sim 1/N_c$ ), of order of the flavor singlet  $u+d$   
[Efremov, Goeke, Menzel, Metz, Schweitzer, 2005]
- small  $A_N$  at midrapidity (small gluon Sivers function in the GPM)  
[Anselmino, D'Alesio, Melis & Murgia, 2006; D'Alesio, Murgia, Pisano, 2015]

Note however that  $A_N$  in pion production is not a TMD factorizing process  
COMPASS high- $p_T$  hadron pairs and other constraints are about fairly large  $x$

Gluon Sivers function is constrained to be  $\lesssim 30\%$  of nonsinglet quark Sivers function  
This is of natural size and will lead to smaller asymmetries, but not necessarily tiny

# Gluon Sivers effect at small x

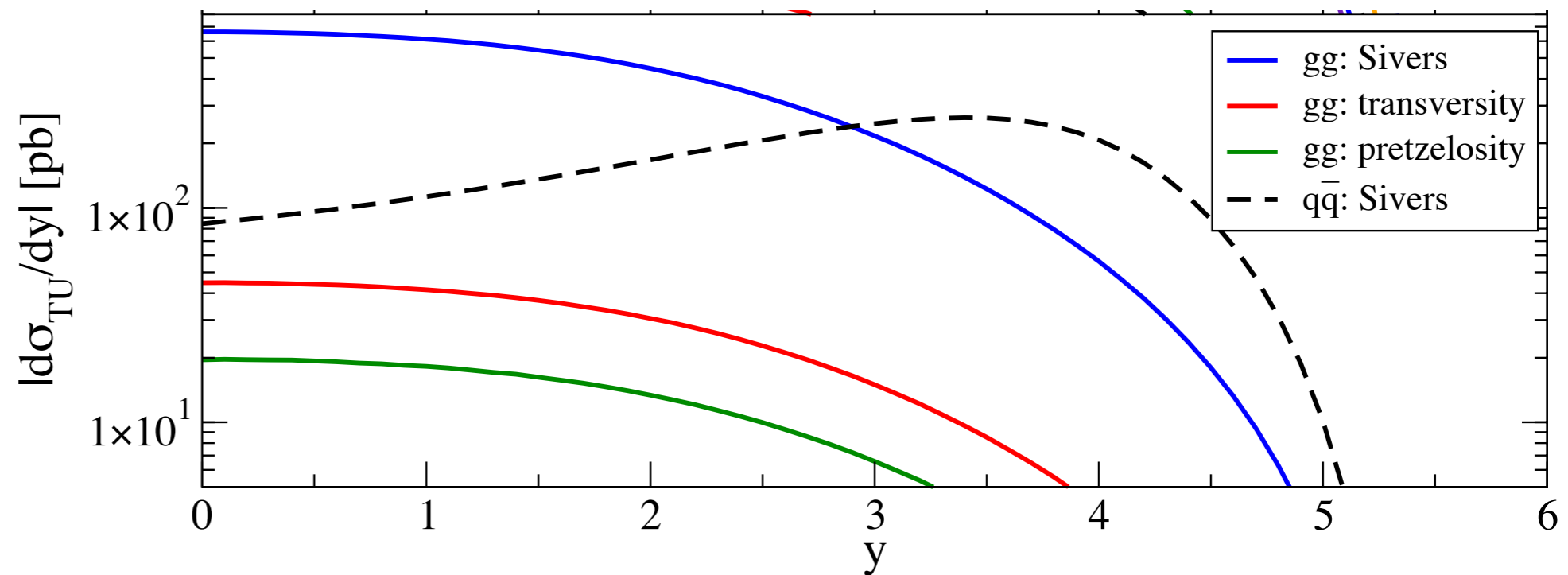
Selection of processes that probe the WW (f type) or DP (d type) Sivers gluon TMD:

	DY	SIDIS	$p^\uparrow A \rightarrow h X$	$p^\uparrow A \rightarrow \gamma^{(*)} \text{jet } X$	$p^\uparrow p \rightarrow \gamma\gamma X$ $p^\uparrow p \rightarrow J/\psi \gamma X$ $p^\uparrow p \rightarrow J/\psi J/\psi X$	$e p^\uparrow \rightarrow e' Q \bar{Q} X$ $e p^\uparrow \rightarrow e' j_1 j_2 X$
$f_{1T}^\perp g^{[+,+]}$ (WW)	×	×	×	×	✓	✓
$f_{1T}^\perp g^{[+,-]}$ (DP)	✓	✓	✓	✓	×	×

↑  
backward hadron production

[Qiu, Schlegel, Vogelsang, 2011]

$p^\uparrow p \rightarrow \gamma\gamma X$



$\sqrt{s}=500 \text{ GeV}, p_{T^\gamma} \geq 1 \text{ GeV}$ , integrated over  $4 < Q^2 < 30 \text{ GeV}^2, 0 \leq q_T \leq 1 \text{ GeV}$   
 At photon pair rapidity  $y < 3$  gluon Sivers dominates and  $\max(d\sigma_{TU}/d\sigma_{UU}) \sim 30\text{-}50\%$



# Gluon Sivers effect at small $x$

At small  $x$  the large  $k_T$  tail of the WW Sivers function is suppressed by a factor of  $x$  compared to the unpolarized gluon function

The DP-type Sivers function is not suppressed and can be probed in pA collisions

$$\Gamma^{[+,-]ij}(x, \mathbf{k}_T) \xrightarrow{x \rightarrow 0} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\square]}(\mathbf{k}_T)$$

The DP-type Sivers function at small  $x$  turns out to be the *spin-dependent odderon*

$$\Gamma_{(d)}^{(T-\text{odd})} \equiv \left( \Gamma^{[+,-]} - \Gamma^{[-,+]} \right) \propto \text{F.T.} \langle P, S_T | \text{Tr} \left[ U^{[\square]}(0_T, y_T) - U^{[\square]\dagger}(0_T, y_T) \right] | P, S_T \rangle$$

D.B., Echevarria, Mulders, Zhou, 2016

a single Wilson loop matrix element

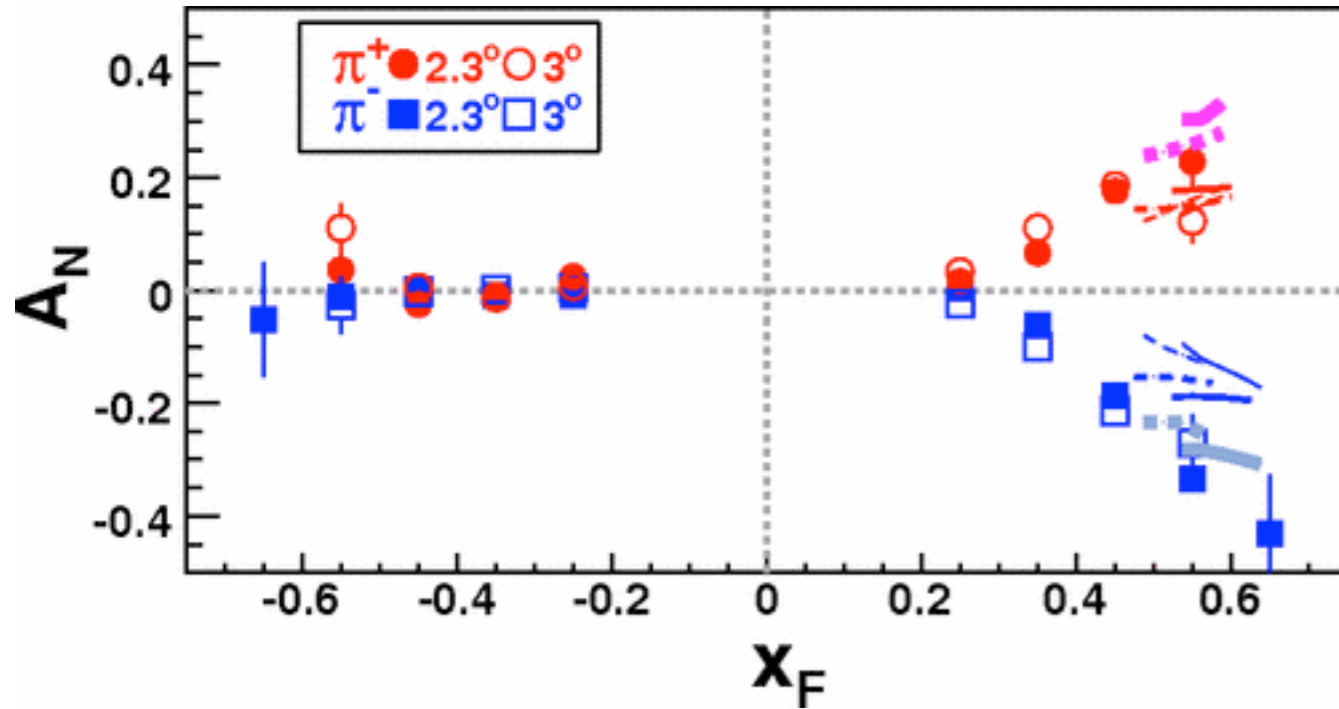
$$U^{[\square]} = U_{[0,y]}^{[+]} U_{[y,0]}^{[-]}$$

It is the only relevant contribution in  $A_N$  at negative  $x_F$ , as opposed to the many contributions at positive  $x_F$

The imaginary part of the Wilson loop determines the gluonic single spin asymmetry



$$p^\uparrow p \rightarrow h^\pm X \text{ at } x_F < 0$$

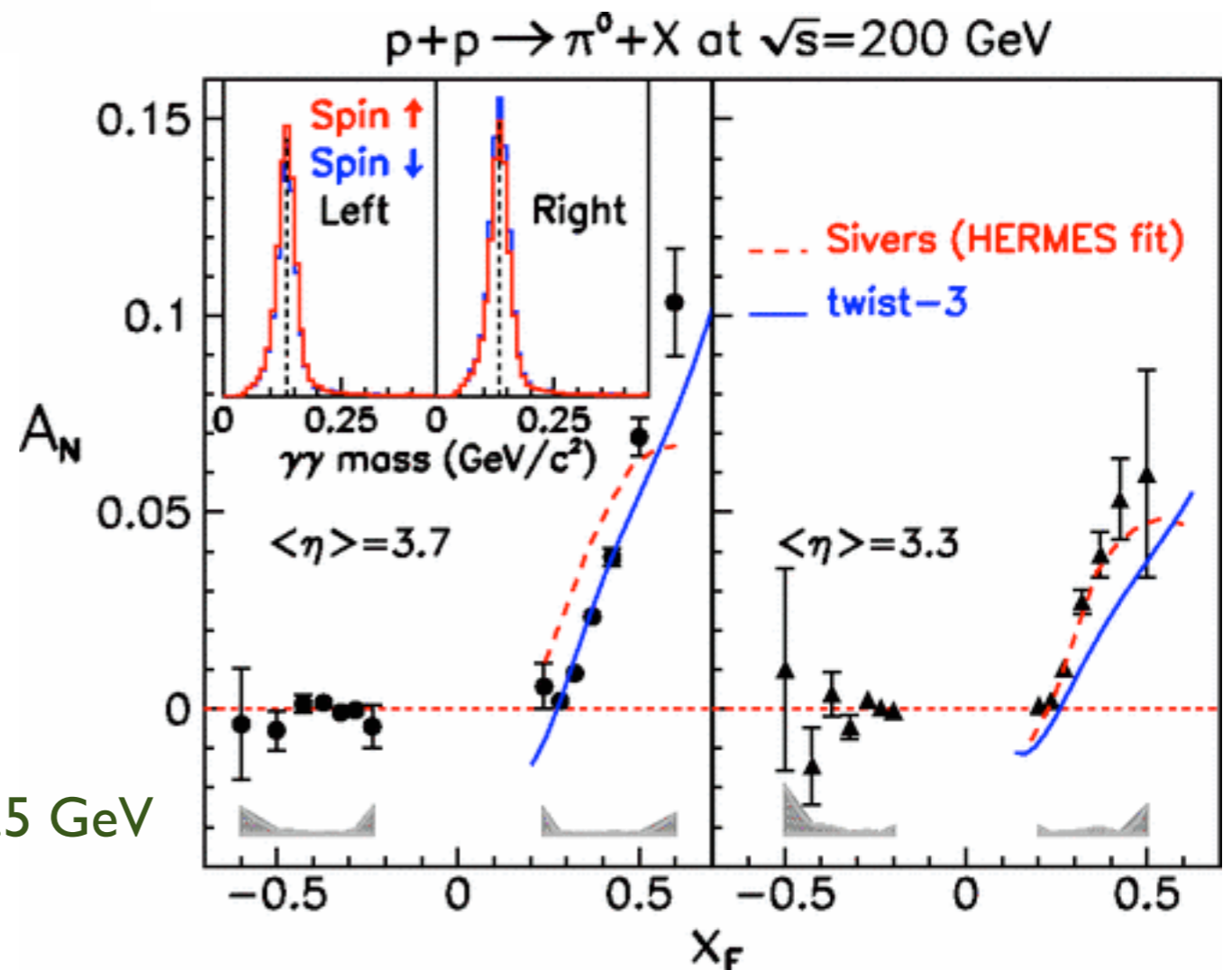


BRAHMS, 2008  $\sqrt{s} = 62.4$  GeV  
 low  $p_T$ , up to roughly 1.2 GeV  
 where gg channel dominates

spin-dependent odderon is C-odd,  
 whereas gg in the CS state is C-even

expect smaller asymmetries  
 in neutral pion and jet production

STAR, 2008  
 $\sqrt{s} = 200$  GeV  
 $p_T$  between 1 and 3.5 GeV



# Conclusions



# Conclusions

- All TMDs are process dependent, with observable and testable effects
- At small  $x$  the unpolarized WW and DP gluon TMDs both matter and there are sufficient processes in ep and pp collisions to test the expectations
- Same applies to the linear polarization of gluons inside unpolarized hadrons:  
In pp collisions percent level effects, except in quarkonium production  
In ep collisions it could be much larger (10% or more) & its sign can be determined
- The CGC can be maximally polarized, although not all processes will be (fully) sensitive to it
- Two distinct gluon Sivers TMDs can be measured in  $p^\uparrow p$  and  $p^\uparrow A$  collisions (RHIC & AFTER@LHC), the WW-type allows for a sign-change test w.r.t.  $ep^\uparrow$  (EIC)
- As  $x \rightarrow 0$  only the DP gluon Sivers TMD remains, which then corresponds to the spin-dependent odderon, a T-odd and C-odd single Wilson loop matrix element that determines  $A_N$  at negative  $x_F$

Back-up slides

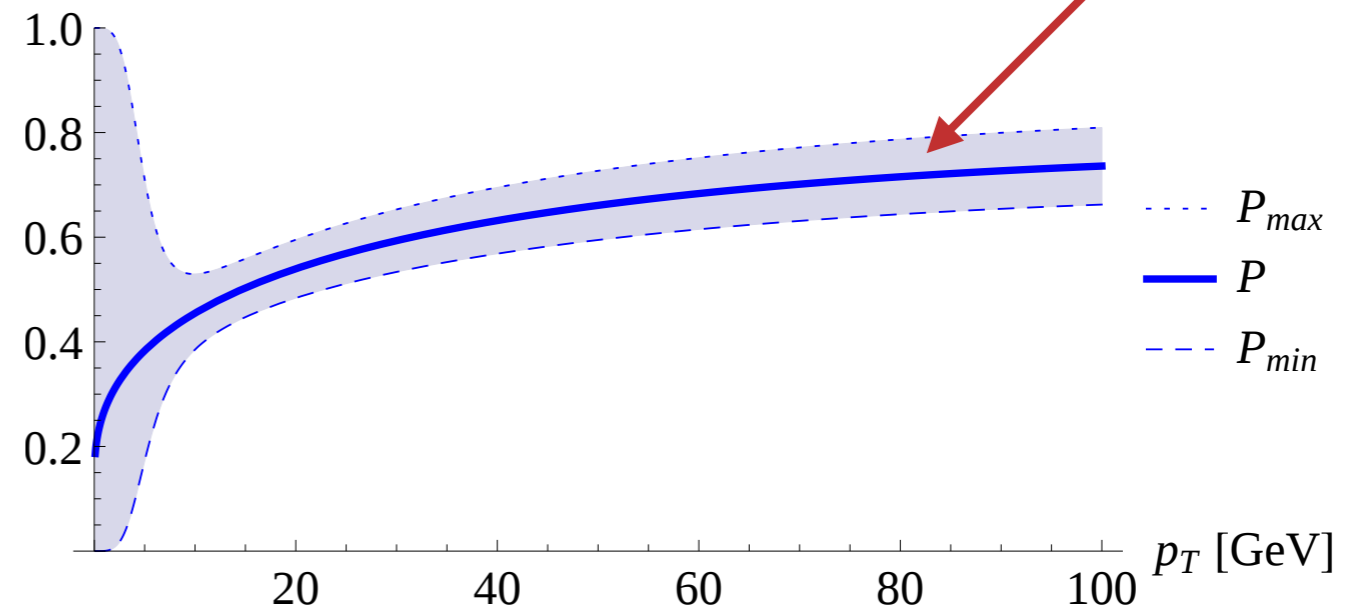


# Size of the effect

$$\frac{\alpha_s P' \otimes f_1}{\alpha_s P \otimes f_1}$$

Amount of linear gluon polarization:

D.B., Den Dunnen, Pisano, Schlegel '13



Ratio of large- $k_T$  tails of  $h_1^\perp$  and  $f_1$  is large, does *not* mean large effects at large  $Q_T$  (observables involve *integrals* over all partonic  $k_T$ )

What matters is the small- $b$  behavior of the Fourier transformed TMD:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

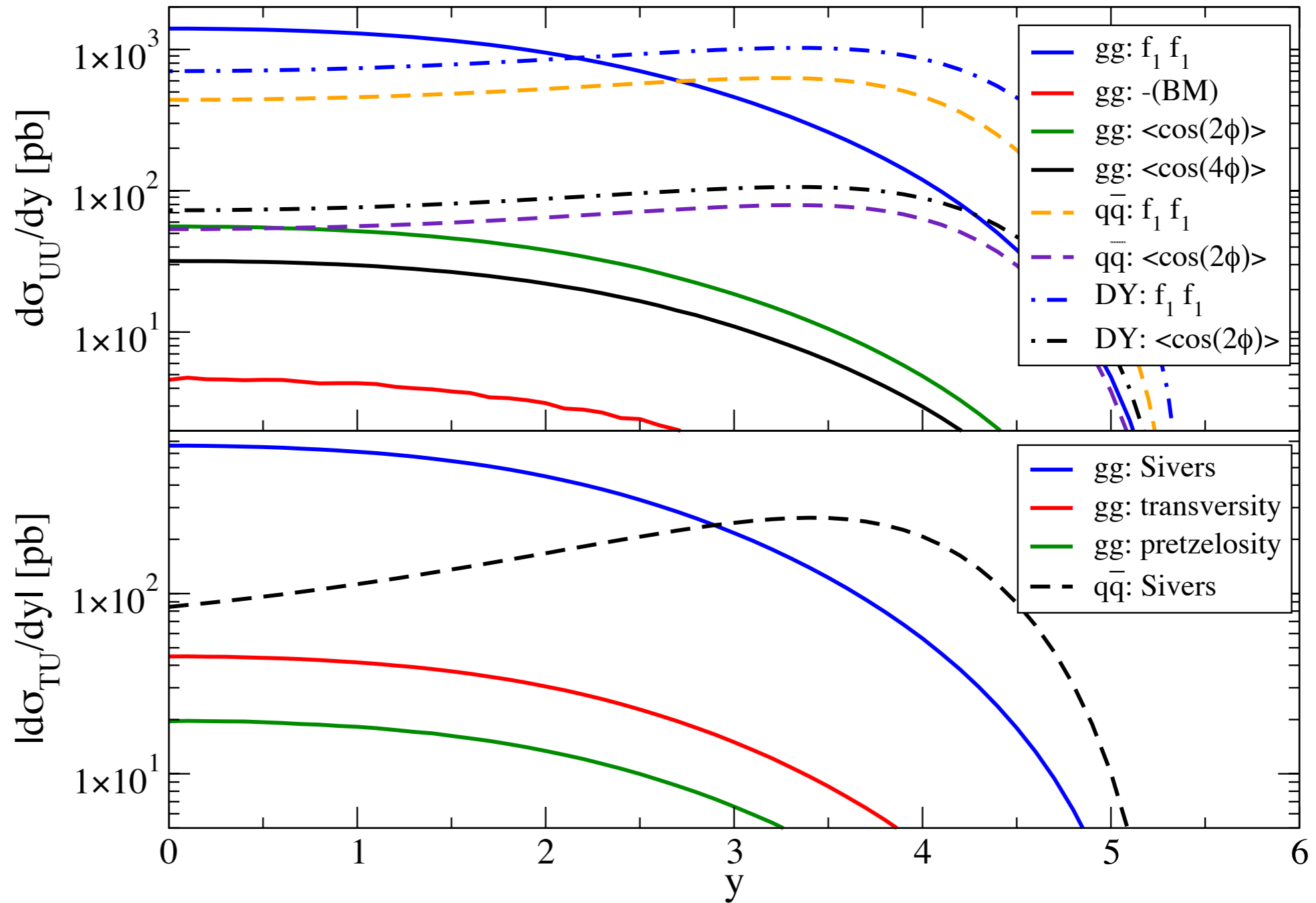
$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left( \frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

[Nadolsky, Balazs, Berger, C.-P.Yuan, 2007; Catani, Grazzini, 2010; P. Sun, B.-W. Xiao, F.Yuan, 2011]

The linear polarization starts at order  $\alpha_s$ , leading to a **suppression w.r.t.  $f_1$**

# Photon pair production

$pp \rightarrow \gamma\gamma X$



$\sqrt{s}=500$  GeV,  $p_{T^\gamma} \geq 1$  GeV, integrated over  $4 < Q^2 < 30$  GeV<sup>2</sup>,  $0 \leq q_T \leq 1$  GeV  
 At photon pair rapidity  $y < 3$  gluon Sivers dominates and  $\max(d\sigma_{TU}/d\sigma_{UU}) \sim 30\text{-}50\%$

# Photon-jet production

$$M_N^{\gamma j}(\eta_\gamma, \eta_j, x_\perp) = \frac{\int d\phi_j d\phi_\gamma \frac{2|\mathbf{K}_{\gamma\perp}|}{M} \sin(\delta\phi) \cos(\phi_\gamma) \frac{d\sigma}{d\phi_j d\phi_\gamma}}{\int d\phi_j d\phi_\gamma \frac{d\sigma}{d\phi_j d\phi_\gamma}}$$

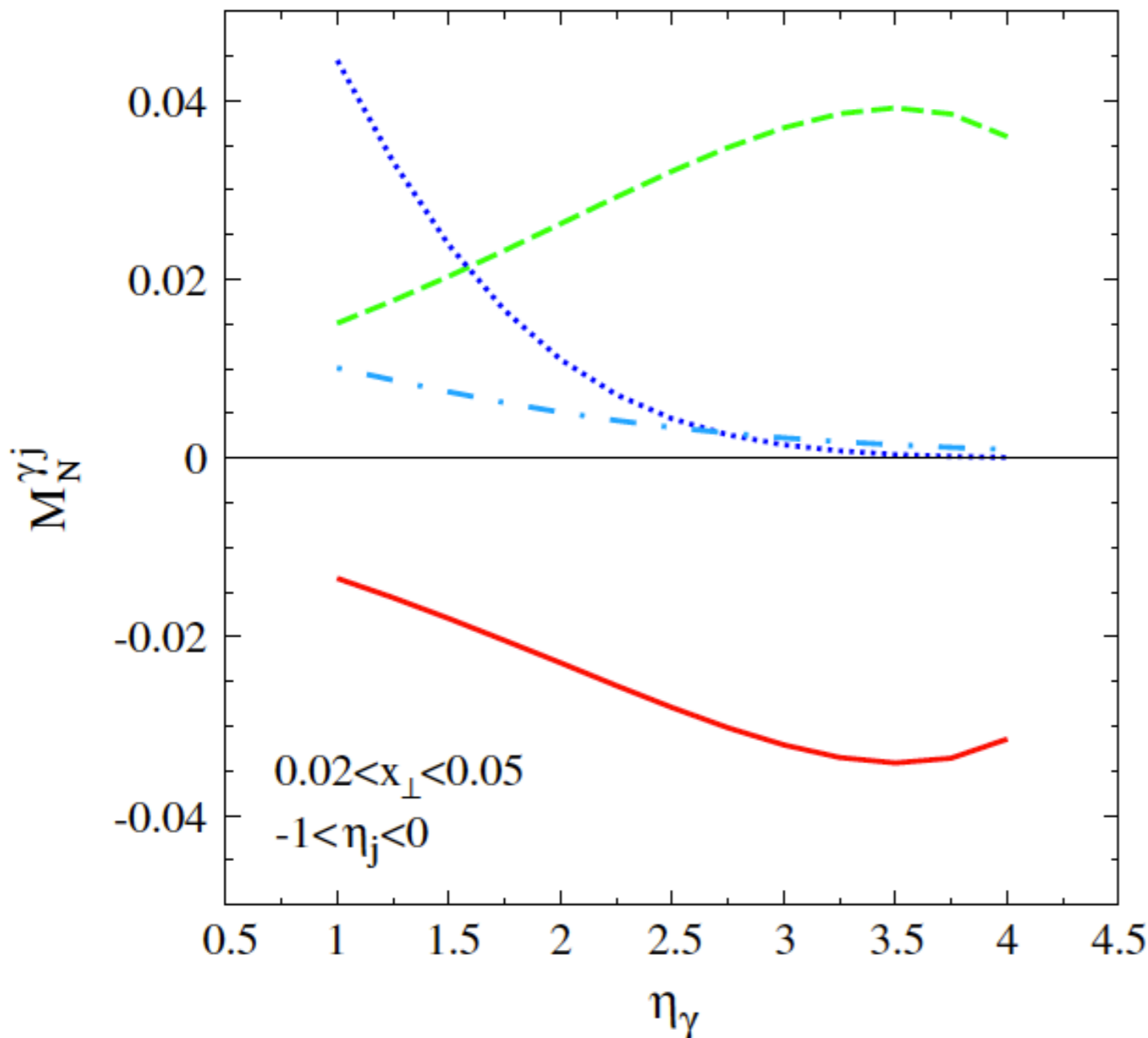
Prediction for the azimuthal moment at  $\sqrt{s}=200$  GeV,  $p_{T^\gamma} \geq 1$  GeV, integrated over  $-1 \leq \eta_j \leq 0$ ,  $0.02 \leq x_\perp \leq 0.05$

Dashed line: GPM

Solid line: using gluonic-pole cross sections

Dotted line: maximum contribution from the gluon Sivers function (absolute value)

Dot-dashed line: maximum contribution from the Boer-Mulders function (abs. value)



# WW vs DP

At small  $x$  the unpolarized WW and DP gluon TMDs both matter and there are sufficient processes in ep and pp collisions to test the expectations

How different can the two unpolarized gluon distributions be?

The first transverse moment must coincide

$$\int d\mathbf{k}_T f_1^{g[+,+]}(x, \mathbf{k}_T^2) = \int d\mathbf{k}_T f_1^{g[+,-]}(x, \mathbf{k}_T^2)$$

Also the large  $k_T$  tail of the functions must coincide

Therefore, the two functions can have rather different shapes and magnitudes