Nucleon Structure from Lattice QCD

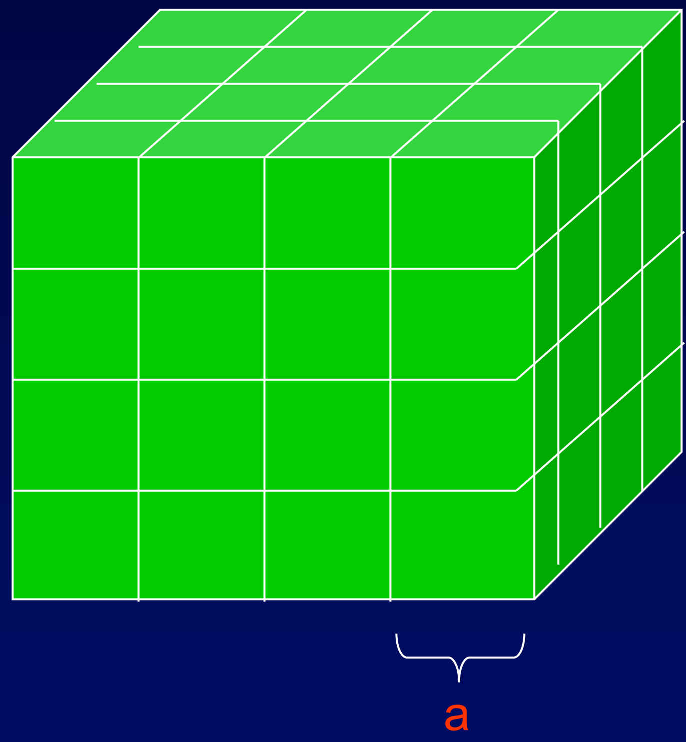
- Pion-nucleon and strange sigma terms
- Nucleon mass decompostion
- Strange quark magnetic moment
- Quark and glue momenta and angular momenta
- Glue spin

χ QCD Collaboration

Frascati, Dec. 1, 2016

Lattice QCD

Why Lattice?



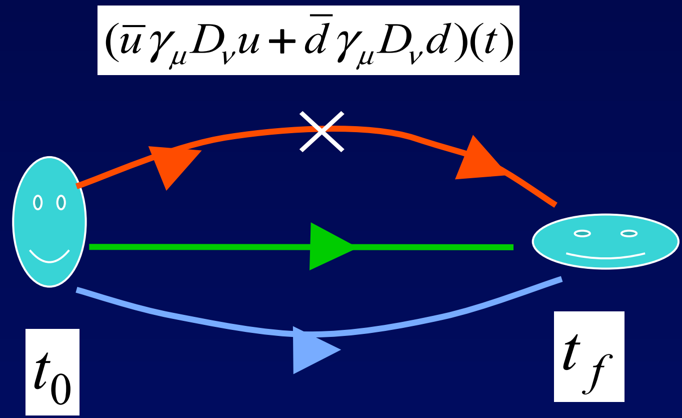
- Regularization
  - Lattice spacing a
  - Hard cutoff, $p \leq \pi/a$
  - Scale introduced (dimensional transmutation)
- Renormalization
  - Perturbative
  - Non-perturbative
- Numerical Simulation
  - Euclidean quantum field theory ⟹ classical statistical mechanics
  - Monte Carlo simulation (importance sampling)

$$e^{-S_G} \det M \geq 0$$

- Systematic errors
  - 2+1 flavor dynamical fermions
  - Continuum limit ($a \to 0$)
  - Infinite volume limit ($L \to \infty$)
  - Physical quark mass (pion and kaon masses)
  - Chiral symmetry at finite lattice spacing (overlap, domain-wall fermions)

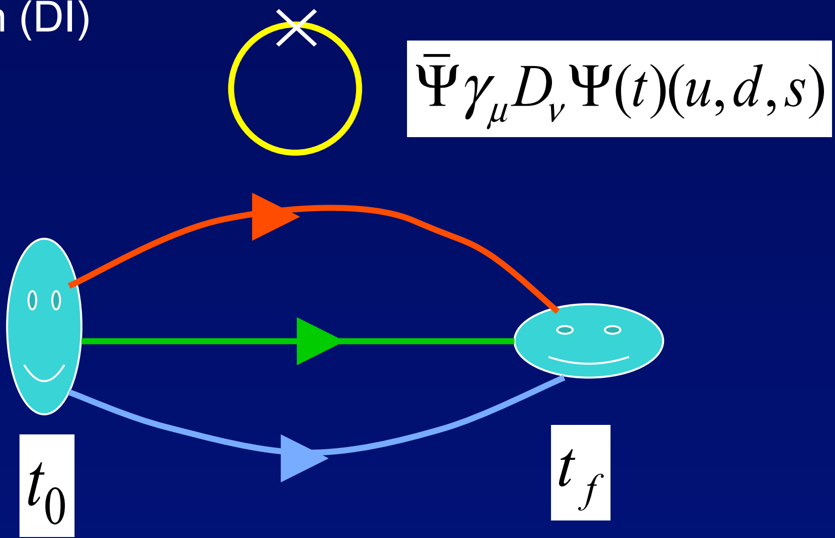
Hadron Structure with Quarks and Glue

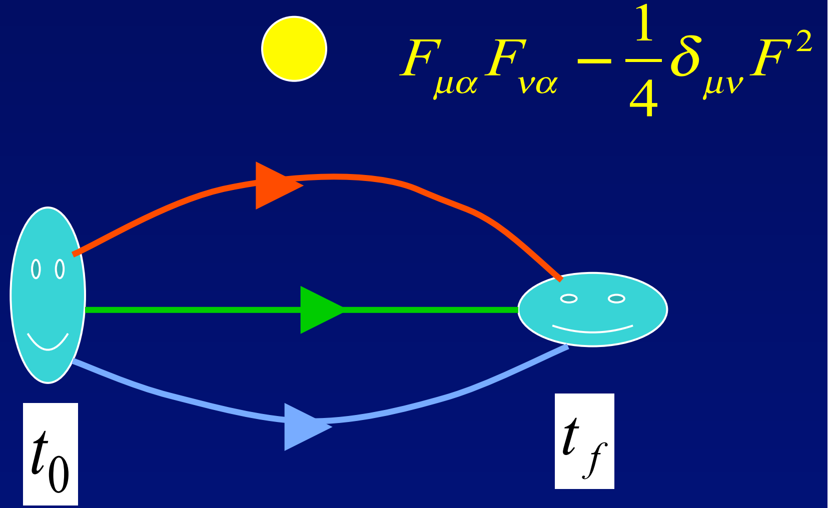
- Quark and Glue Momentum and Angular Momentum in the Nucleon



Connected insertion (CI)

Disconnected insertion (DI)





2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.6 fm
m_π ~ 170 MeV

32^3 x 64, a =0.143 fm

↓ ($O(a^2)$ extrapolation)

La ~ 2.65 fm
m_π ~ 330 MeV

24^3 x 64, a =0.111 fm

→

La ~ 5.48 fm
m_π ~ 139 MeV

48^3 x 96, a =0.114 fm

La ~ 2.65 fm
m_π ~ 295 MeV

32^3 x 64, a =0.0828 fm

→

La ~ 5.35 fm
m_π ~ 139 MeV

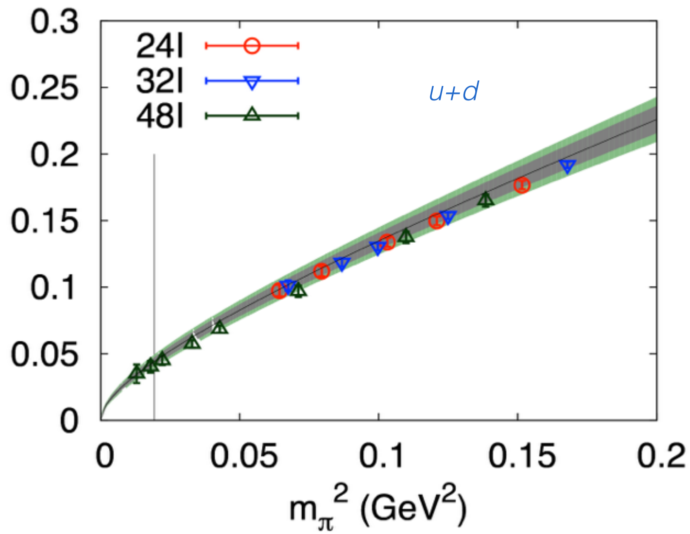
64^3 x 128, a =0.0837 fm

Pion-Nucleon and Strange Sigma terms

Spontaneous and explicit chiral symmetry breaking
Pion-nucleon scattering
Quark mass content in nucleon
Dark matter search via Higgs coupling

$$\sigma = m \langle N | \bar{\psi}\psi | N \rangle$$

The πNσ term



$$\sigma_{\pi N}(m_\pi, a, L) = C_0^\pi m_\pi + C_1^\pi m_\pi^2 + C_2^\pi a^2 + C_3^\pi \left(\frac{m_\pi^2}{L} - m_\pi^3\right) e^{-m_\pi L},$$

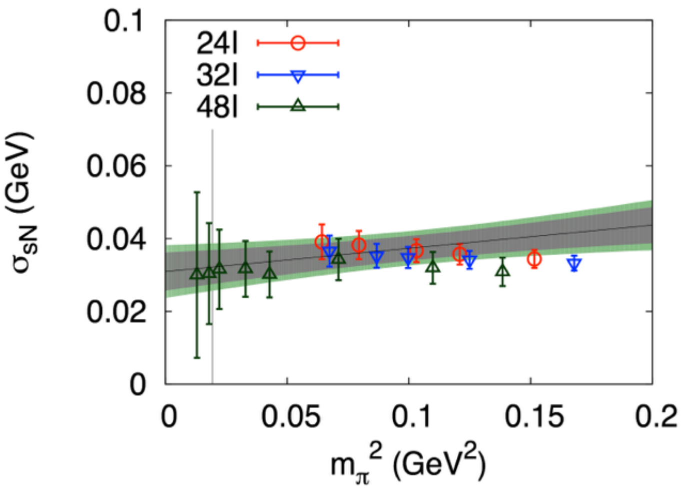
Y. Yar

- Th pc wi la
- Th ap
- Th m si
- Th st ba th
- Th

σ_π

The strange σ term

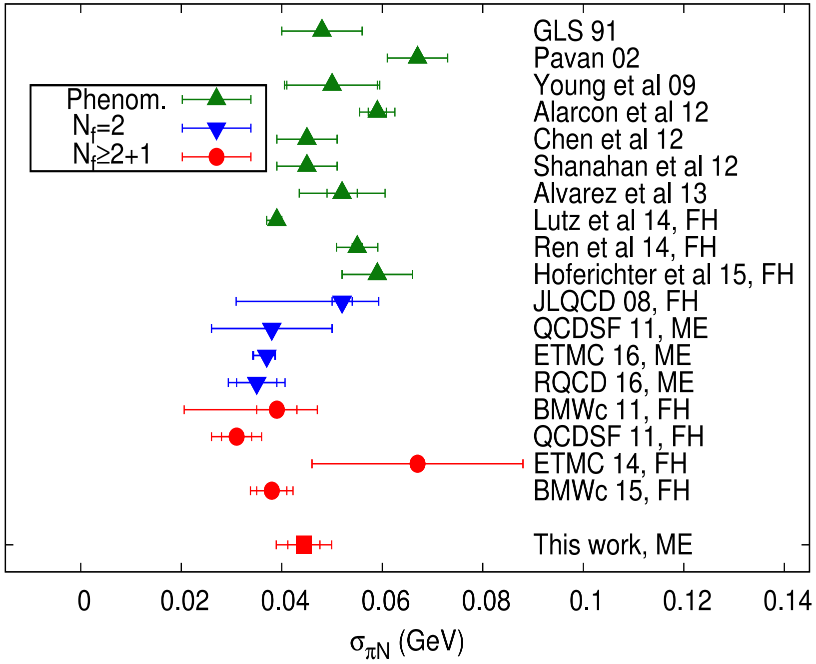
Y. Yang, et al. [χQCD], 1511.09089

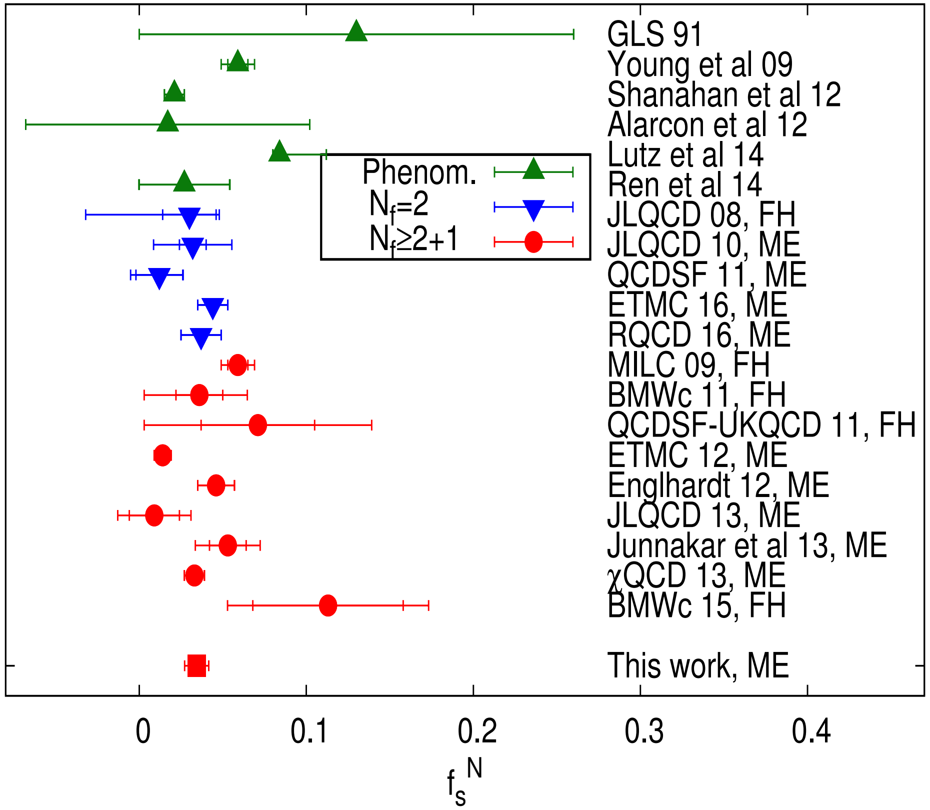


$$\sigma_{sN}(m_\pi, a, L) = C_0^s + C_1^s m_\pi^2 + C_2^s a^2 + C_3^s e^{-m_\pi L}.$$

- Due the finite lattice spacing, volume and partially quenching effects, biases exist between the data points and curve.
- It can be cured by the global fit.
- The final prediction is,

$$\sigma_{sN} = 32.3(4.7)(4.8)\ \text{MeV}.$$





Yibo Yang (χ QCD) et al.,

arXiv:1511.15089

$$\sigma_{\pi N} = 45.9(7.4)(2.8)\ \text{MeV}$$

$$\sigma_{sN} = 40.2(11.7)(3.5)\ \text{MeV}$$

$$H_m(u,d,s)/m_N = 8(1)\%$$

Quark and Glue Components of Hadron Mass

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

Canonical Conformal Anomaly

M. Chanowitz and J. Ellis, PRD 7, 2490 (1973)

$$\langle P | T_{\mu\nu} | P \rangle = P_\mu P_\nu / M$$

- Trace anomaly

$$T_{\mu\mu} = -(1+\gamma_m)\bar{\psi}\psi + \frac{\beta(g)}{2g}G^2$$

Xiangdong Ji, PRL 74, 1071 (1995); PRD 52, 271 (1995)

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_a\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle;$$

$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle;$$

where

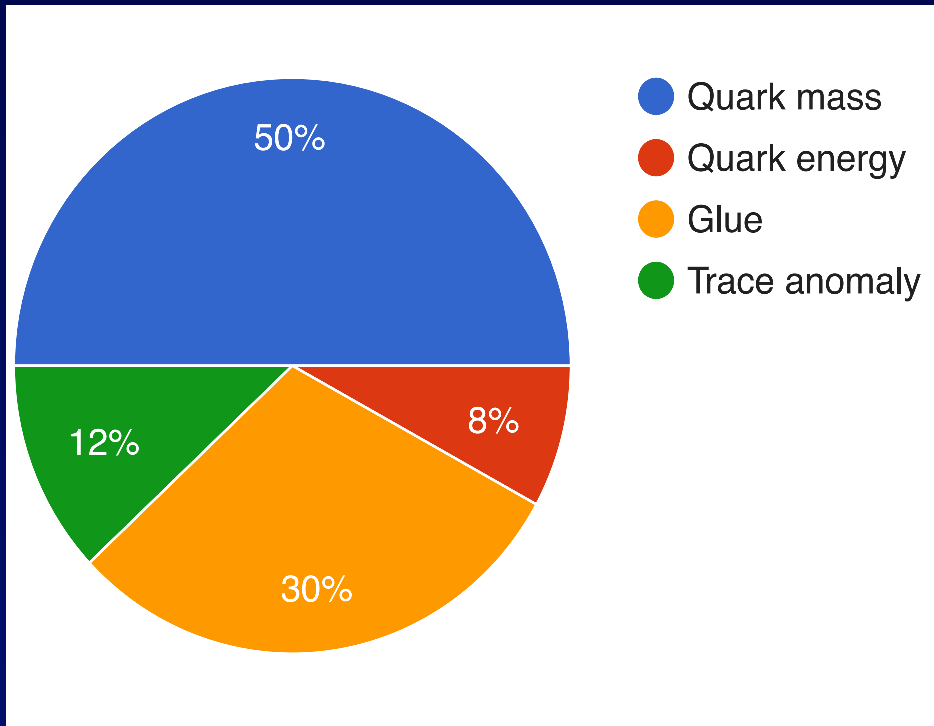
$$H_q = \sum_{u,d,s\ldots}\int d^3x - \bar{\psi}(\gamma_4 D_4)\psi;\;\; H_E = \sum_{u,d,s\ldots}\int d^3x\; \bar{\psi}(\vec{\gamma}\cdot\vec{D})\psi;\;\; H_m = \sum_{u,d,s\ldots} m_f \int d^3x\; \bar{\psi}\psi;$$

$$H_g = \int d^3x\,(B^2 - E^2);\;\; H_a = \int d^3x\; \frac{-\beta(g)}{2g}(B^2 + E^2)$$

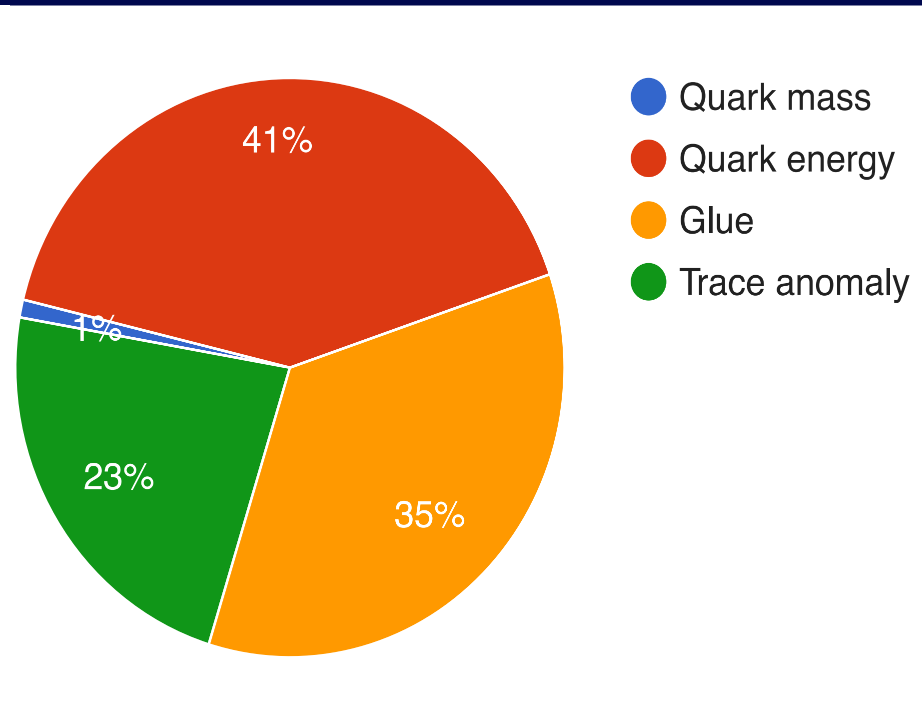
Decomposition of Meson Masses

Y. Yang et al, 1405.4440 (PRD)

π



ρ

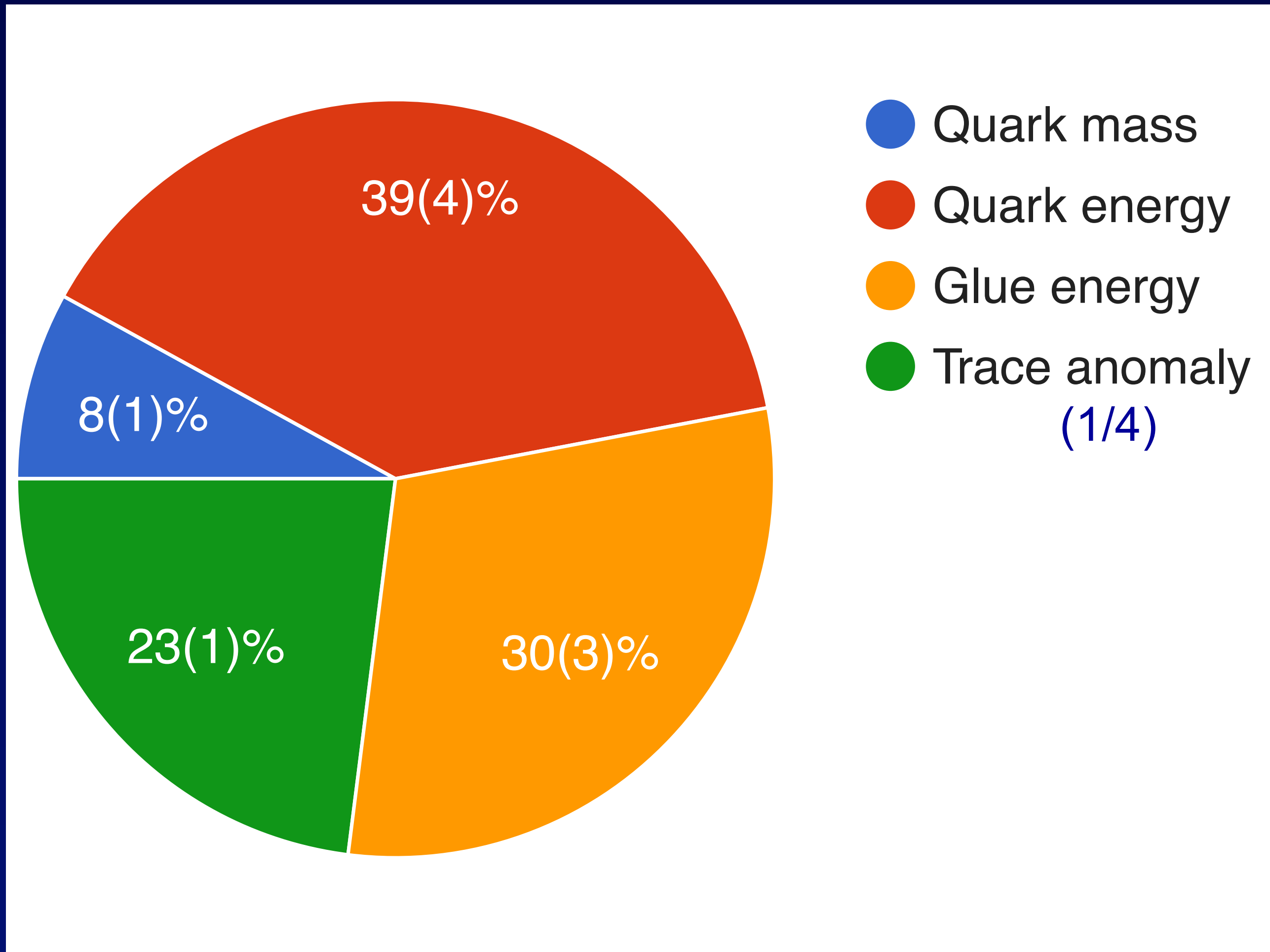


Feynman-Hellman Theorem

$$m_\pi^2 \sim m_q, \quad \mathrm{H}_m(\pi) = m_q \frac{dm_\pi}{dm_q} \sim \frac{1}{2} m_\pi$$

- Quark mass contribution negligible for ρ.
- H_g and H_a are fairly constant for light quarks → origin of constituent quark mass (?)
- For the Φ, the quark mass term is ~ 200 eV.

Nucleon Mass Components

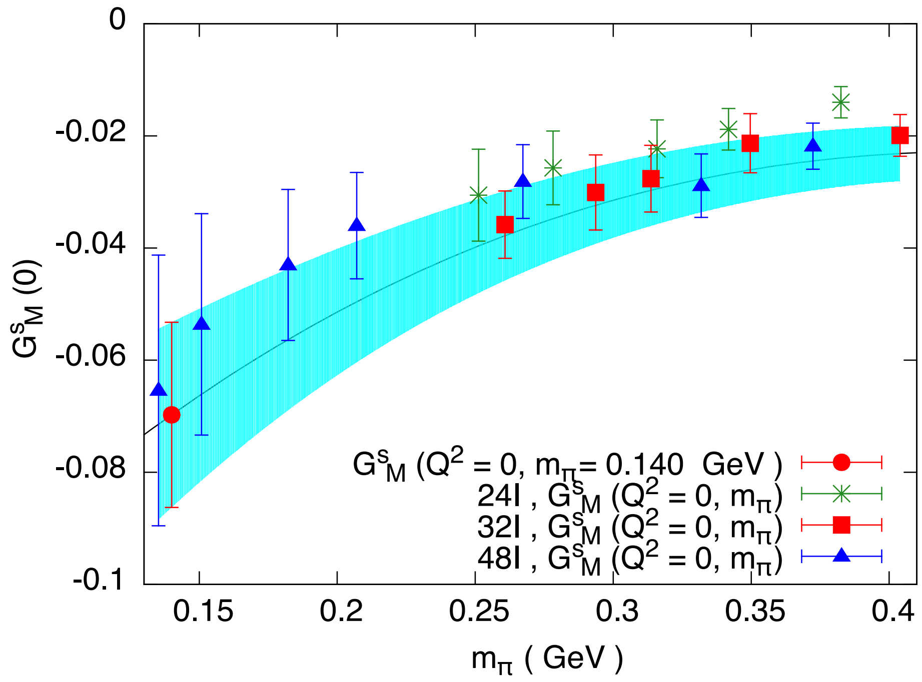


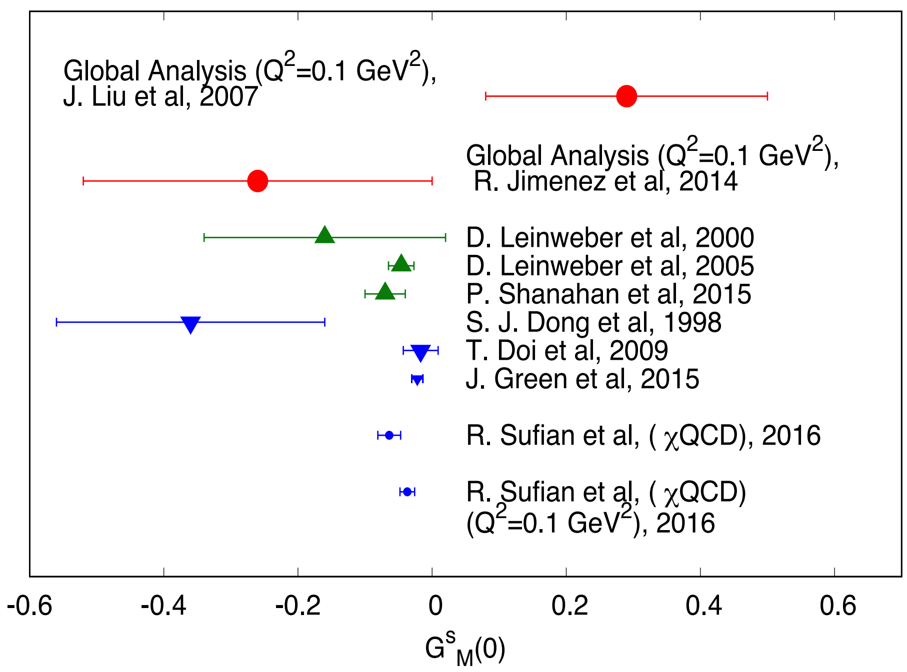
Y. Yang

Strange quark magnetic moment

Parity-violating ep scattering with radiative correction

R. Sufian et al, 1606.07075 (PRL)



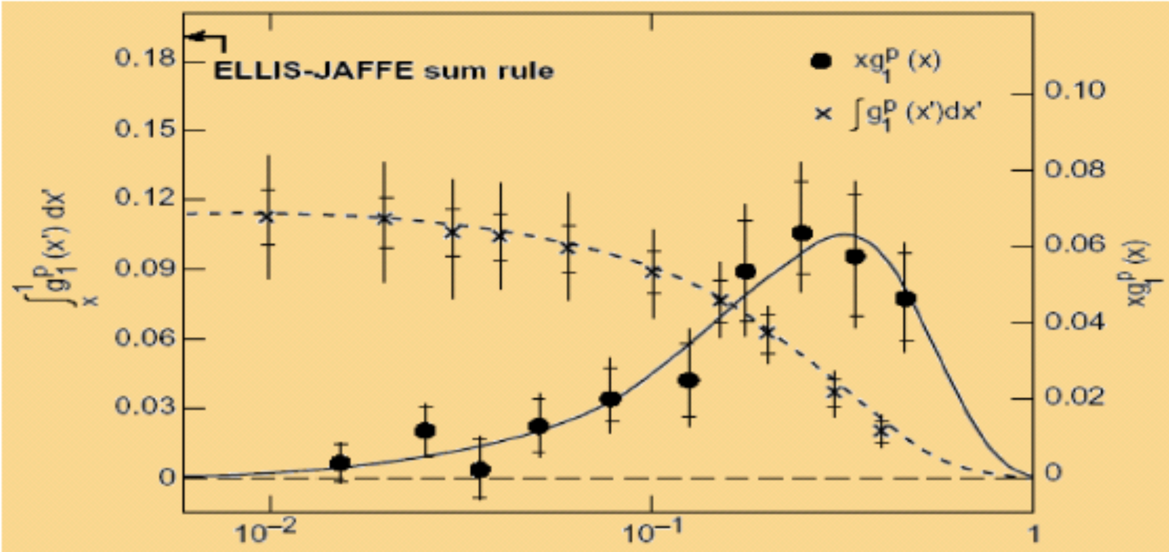


$G_M^S(0) = -\,0.073(17)(8)\ \mu_N$ R. Sufian

Where does the spin of the proton come from?

Twenty ^7 years since the "spin crisis"

❑ **EMC experiment in 1988/1989 – "the plot":**

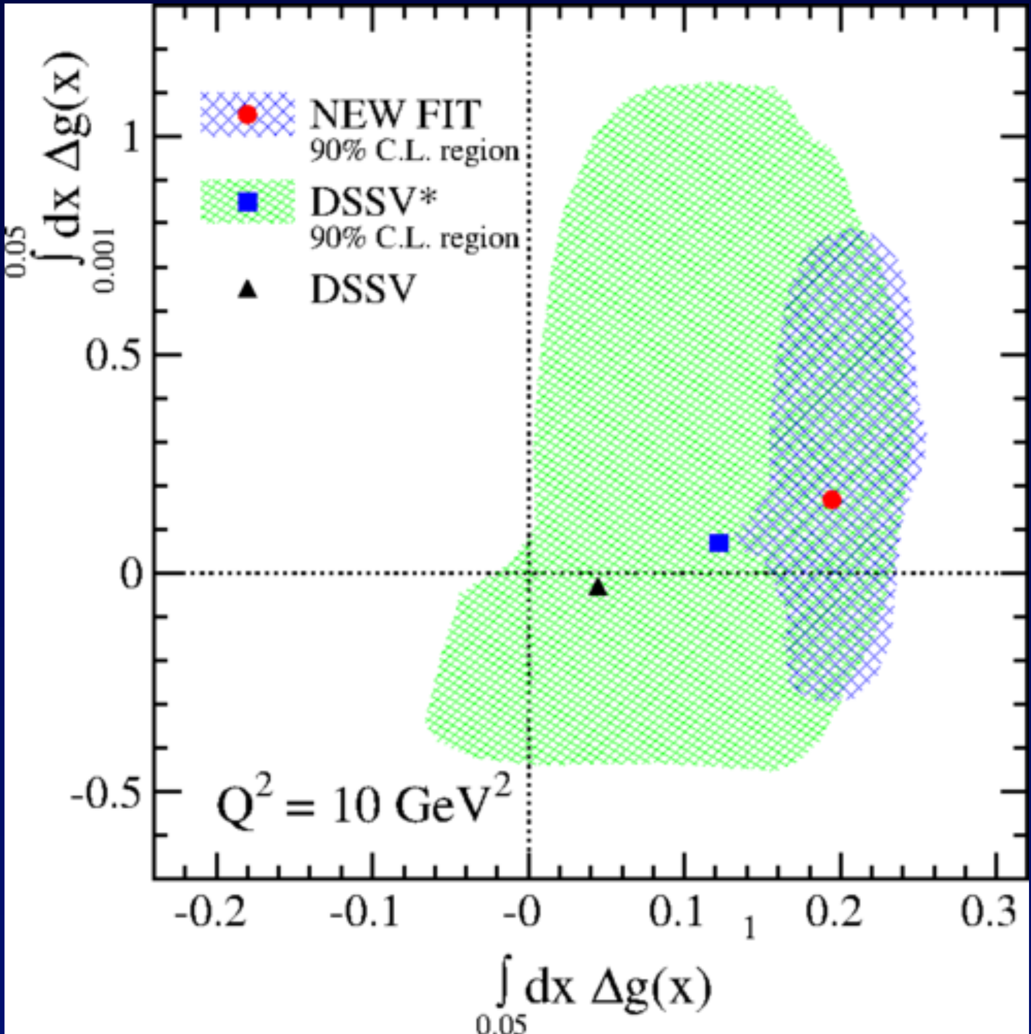


$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\|}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{\|}\rangle$$

❑ **"Spin crisis" or puzzle:** $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

Glue Helicity ΔG



Experimental results from
STAR [1404.5134]
PHENIX [1402.6296]
COMPASS [1001.4654]

ΔG ~ 0.2 with large error

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang,
PRL 113, 012001 (2014)

Spin Sum Rules

- Jaffe and Manohar sum rule (1990)

$$J = \frac{\Sigma}{2} + L_q + S_G + L_G$$

$$\vec{J}_{Tot} = \int d^3x \ \psi^\dagger \frac{1}{2} \Sigma \psi + \int d^3x \ \vec{x} \times \psi^\dagger \vec{\nabla} \psi + \int d^3x \ \vec{E}^a \times \vec{A}^a$$

$$+ \int d^3x \ \vec{x} \times E^{aj} (\vec{x} \times \nabla) A^{aj}$$

  - Canonical EM tensor on light-cone with light-cone gauge
  - Not directly accessible on the lattice

- Ji sum rule (1997)

$$J = \frac{\Sigma}{2} + L_q + J_G$$

$$\vec{J}_{Tot} = \int d^3x \ \psi^\dagger \frac{1}{2} \Sigma \psi + \int d^3x \ \vec{x} \times \psi^\dagger \vec{D} \psi + \int d^3x \ \vec{x} \times \ (\vec{E}^a \times \vec{B}^a)$$

  - Symmetric EM tensor (Belinfante) → gauge invariant and frame independent.

Quark Spin from Anomalous Ward Identify

- Calculation of the point axial-vector in the DI is not sufficient.
- AWI needs to be satisfied. $\partial_\mu A_\mu^0 = i2mP - \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
- Unrenormalized AWI for overlap fermion for point current

$$\kappa_A \partial_\mu A_\mu^0 = i2mP - iN_f 2q(x)$$

Renormaliztion and mixing:

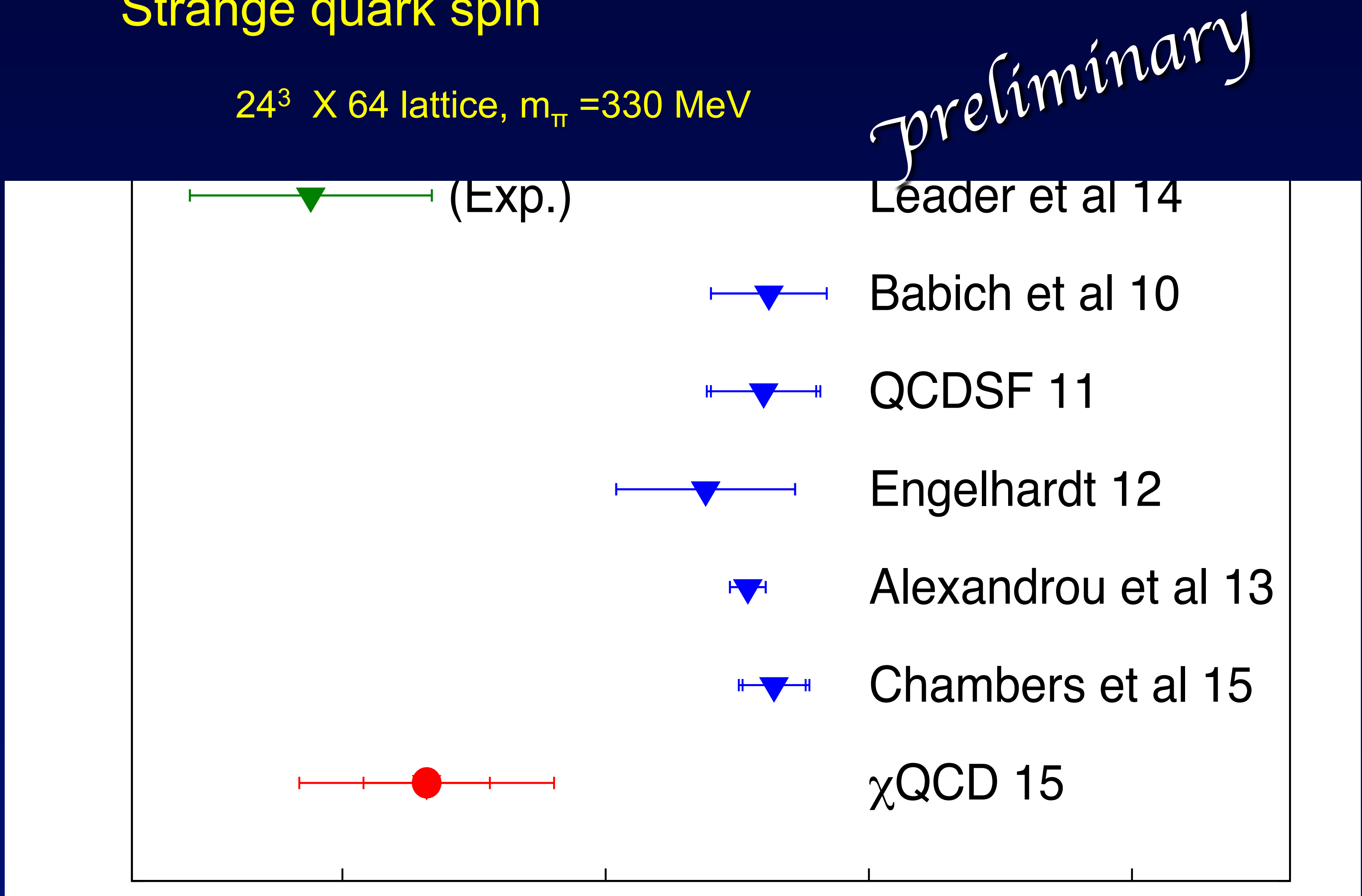
$$Z_A \kappa_A \partial_\mu A_\mu^0 = i2Z_m m Z_P P - iN_f 2(Z_q q(x) + \lambda \partial_\mu A_\mu^0)$$

  - Overlap fermion --> mP is RGI ($Z_m Z_P$=1)
  - Overlap operator for $q(x) = -1/2\, \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ has no multiplicative renormalization.
  - Espriu and Tarrach (1982) $Z_A(2-\mathrm{loop}) = 1 - \left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{8} C_2(R) N_f \frac{1}{\varepsilon},$

$$\lambda = -\left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{16} C_2(R) \frac{1}{\varepsilon}$$

Strange quark spin

24^3 X 64 lattice, m_π =330 MeV



κ_A(isovector) = 1.10, κ_A(singlet) ~ 1.4 - 1.7

Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \rightarrow \vec{\mathrm{J}}_g = \int d^3x \left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\left\langle p,s \,|\, T_{\mu\nu} \,|\, p's' \right\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu \bar{p}_\nu - T_2(q^2)\bar{p}_\mu \sigma_{\nu\alpha} q_\alpha / 2m$$

$$\text{-i}\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu} m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g} \;\left[\mathrm{OPE}\right] \rightarrow \langle x \rangle_{q/g}(\mu, \overline{\mathrm{MS}}), \quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu, \overline{\mathrm{MS}})$$

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2}$$

$$\Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

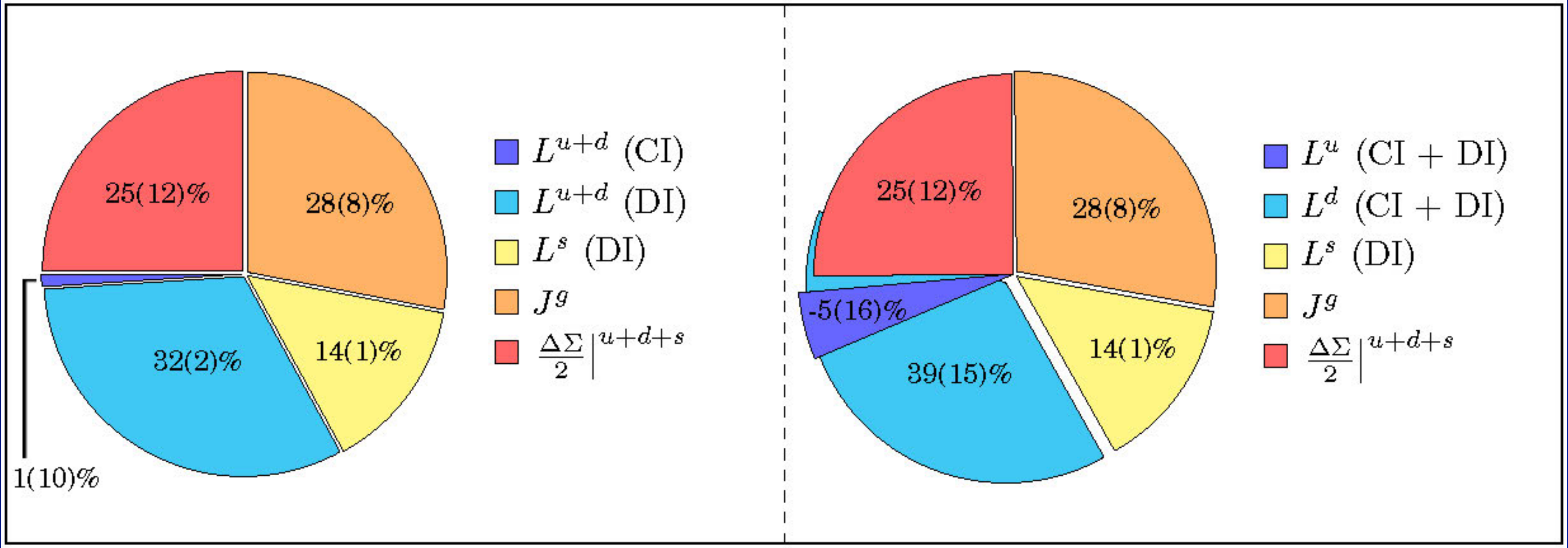
Mixing

$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

M. Glatzmaier, KFL
arXiv:1403.7211

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum (M. Deka *et al*, 1312.4816, PRD)

pizza cinque stagioni



$\Delta q \approx 0.25;$

$2\,L_q \approx 0.47\ (0.01(\mathrm{CI}){+}0.46(\mathrm{DI}));$

$2\,\mathrm{J}_g \approx 0.28$

These are quenched results so far.

Summary of Quenched Lattice Calculations

- Complete calculation of momentum fractions of quarks (both valence and sea) and glue have been carried out for a quenched lattice:
  - Glue momentum fraction is ~ 33%.
  - g_A^0 ~ 0.25 in agreement with expt.
  - Glue angular momentum is ~ 28%.
  - Quark orbital angular momentum is large for the sea quarks (~ 47%).
- These are quenched results so far.

Orbital Angular Momentum

skyrmion

Trinacria, Erice

Glue Spin and Helicity ΔG

- Jaffe and Manohar -- spin sum rule on light cone

  $S_g = \int d^3x\, \vec{E} \times \vec{A}$ in light-cone gauge $(A^+ = 0)$ and IMF frame.

  - Not gauge invariant
  - Light cone not accessible on the Euclidean lattice.

- Manohar – gauge invariant light-cone distribution

  $$\Delta g(x)\, S^+ = \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS \,|\, F_a^{+\alpha}(\xi^-) L^{ab}(\xi^-, 0) \tilde{F}_{\alpha,b}^{+}(0) \,|\, PS \rangle$$

  - After integration of x, the glue helicity operator is

  $$\mathrm{H}_g(0) = \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+} \left(\vec{\nabla} A^{+,b} \right) L^{ba}(\xi^-, 0) \right)$$

  - Non-local and on light cone

Glue Spin and Helicity ΔG

- X.S. Chen, T. Goldman, F. Wang; Wakamatsu; Hatta, etc.

Gauge invariant decomposition

$$\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G$$

$$S_g = \int d^3x \, \mathrm{Tr}\,(\vec{E} \times \vec{A}_{phys}), \quad A^\mu = A^\mu_{phys} + A^\mu_{pure}, \quad F^{\mu\nu}_{pure} = 0;$$

$$A^\mu_{phys} \to g^\dagger A^\mu_{phys} g, \quad A^\mu_{pure} \to g^\dagger A^\mu_{pure} g - \frac{i}{g} g^\dagger \partial^\mu g$$

$$D^i A^i_{phys} = \partial^i A^i_{phys} - ig\,[A^i, A^i_{phys}] = 0$$

  – Gauge invariant but frame dependent

- X. Ji, J.H. Zhang, Y. Zhao; Y. Hatta, X. Ji, Y. Zhao

Infinite momentum frame

$$\vec{E}^a(0) \times \vec{A}^a_{phys} \xrightarrow[p_z \to \infty]{} \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+} \left(\vec{\nabla} A^{+,b} \right) L^{ba}(\xi^-, 0) \right)$$

Glue Spin and Helicity ΔG

- Large momentum limit

$$S_g = \frac{\langle PS | \int d^3x \ \mathrm{Tr}\ (\vec{E} \times \vec{A}_{phys})_z \ | PS \rangle}{2E_P} \xrightarrow[P_Z \to \infty]{} \Delta G$$

  - Calculate S_g at finite P_z
  - Match to MS-bar scheme at 2 GeV
  - Large momentum effective theory to match to IMF
  - Similar proof for the quark and glue orbital angular momenta which are related to form factors in generalized TMD (GTMD)

  (Y. Zhao, KFL, and Y. Yang, arXiv:1506.08832 (PRD))

- Solution of A_{phys} -- related to A in Coulomb gauge

$$U^\mu(x) = g_c(x) U_c^\mu(x) g_c^{-1}(x + a\hat{\mu}),$$

$$U_{pure}^\mu(x) \equiv g_c(x) g_c^{-1}(x + a\hat{\mu}),$$

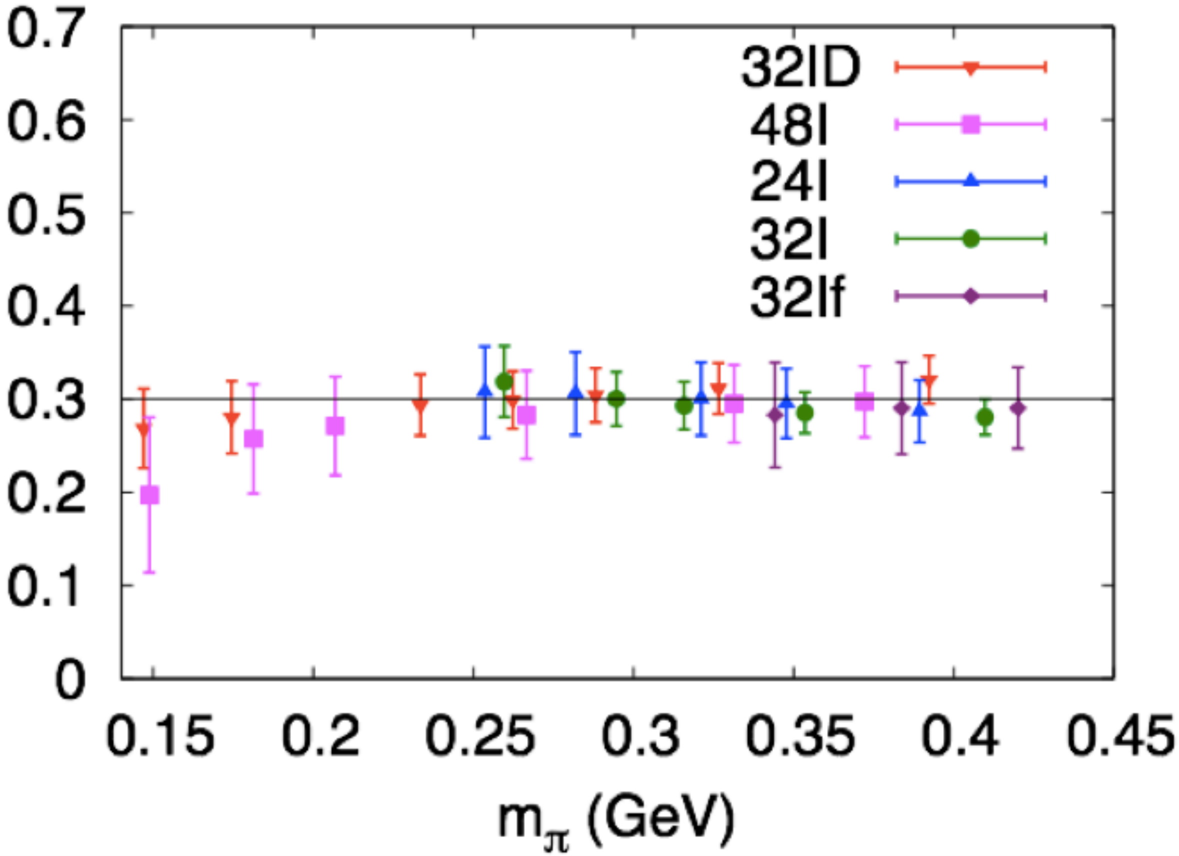
$$A_{phys}^\mu(x) \equiv \frac{i}{ag_0}\left(U^\mu(x) - U_{pure}^\mu(x)\right) = g_c(x) A_c(x) g_c^{-1}(x) + O(a).$$

$$Tr(\vec{E} \times \vec{A}_{phys}) = Tr(\vec{E} \times g_c \vec{A}_c g_c^{-1}) = Tr(\vec{E}_c \times \vec{A}_c)$$

The dependence

Y. Yang, R. S. Sufian, et al, χQCD Collaboration, arXiv 1609.05937.

of m_π, a, and V

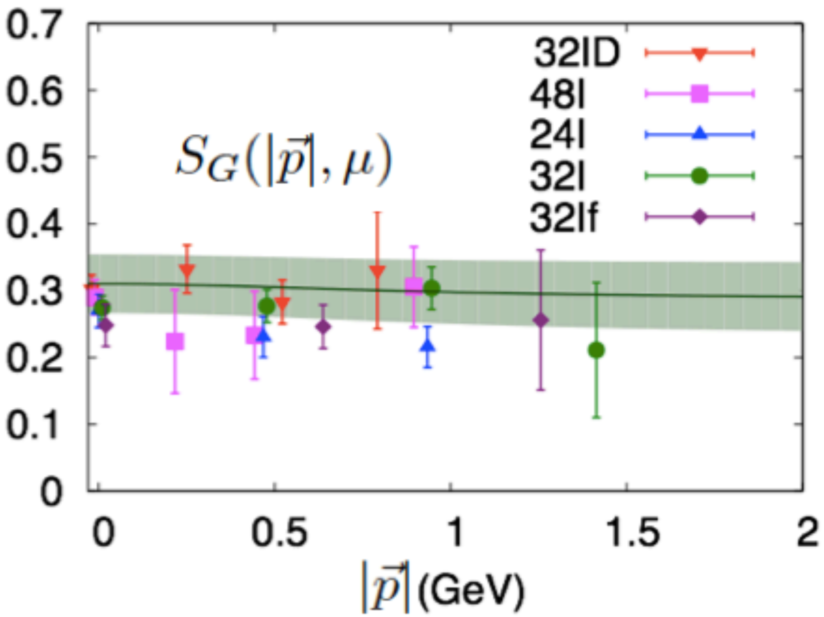


μ^2=10 GeV^2

In the rest frame, *the pion mass (both valence and sea), lattice spacing and volume dependences are mild.*

From glue spin to helicity

with *Large-momentum effective field theory*



X. Ji, J.-H. Zhang, and Y. Zhao, Phys. Lett. B743, 180 (2015)

$$S_G(|\vec{p}|,\mu) = \left[1 + \frac{g^2 C_A}{16\pi^2}\left(\frac{7}{3}\log\frac{(\vec{p})^2}{\mu^2} - 10.2098\right)\right]\Delta G(\mu) + \frac{g^2 C_F}{16\pi^2}\left(\frac{4}{3}\log\frac{(\vec{p})^2}{\mu^2} - 5.2627\right)\Delta\Sigma(\mu) + O(g^4) + O(\frac{1}{(\vec{p})^2}) \,.$$

- **The large finite pieces indicates a convergence problem**
- **Large frame dependence need re-summation.**

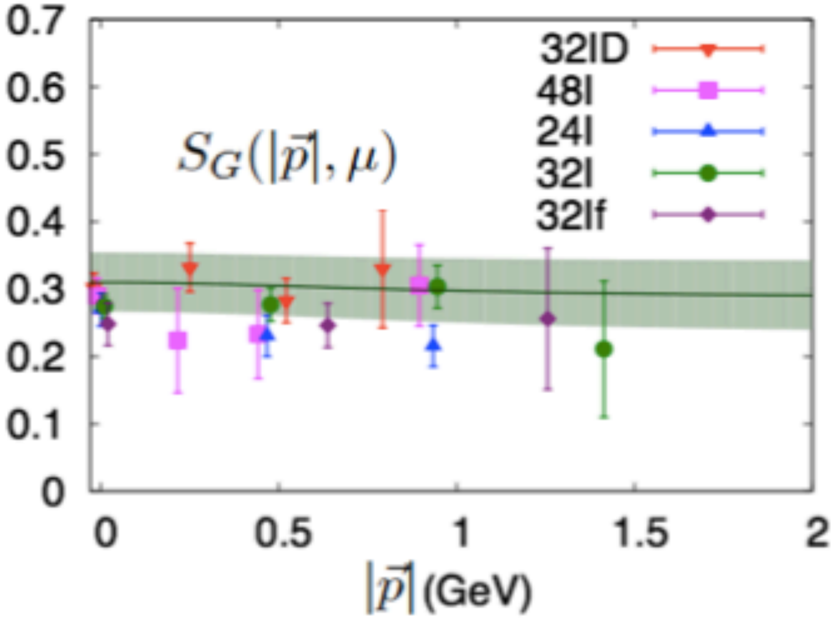
With $|\vec{p}| = 1.5$ GeV and $\mu^2 = 10$ GeV^2,

the factor before Δ_G is 0.22.

Glue spin

Y. Yang, R. S. Sufian, et al, χQCD Collaboration, arXiv 1609.05937.

The final result



We neglect the matching and use the following empirical form to fit our data,

$$S_G(|\vec{p}|) = S_G(\infty) + \frac{C_1}{M^2 + |\vec{p}|^2} + C_2(m^2_{\pi,vv} - m^2_{\pi,phys})$$
$$+C_3(m^2_{\pi,ss} - m^2_{\pi,phys}) + C_4 a^2$$

$m_{\pi,phys}$=0.139 GeV $\quad M = 0.939$ GeV

The glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2:

S_G=0.287(55)(16)

Present experiment

$\Delta G(Q^2=10$ GeV$^2) \sim 0.2$, de Florian et al., 2014

Summary and Challenges

- Lattice calculations of the physical 2+1 flavor dynamical fermions at the physical pion point and with extrapolations to continuum and infinite volume limits are becoming available even with chiral fermions.
- Decomposition of proton spin and hadron masses into quark and glue components on the lattice is feasible. Large momentum frame for the proton to calculate glue helicity remains a challenge.
- Together with evolution, factorization, perturbative QCD, lattice QCD results with small enough statistical and systematic errors can compare directly with experiments and have an impact in advancing our understanding of the underline physics of the hadron structure (form factors, PDF, neutron electric dipole moment, muon g-2, etc).