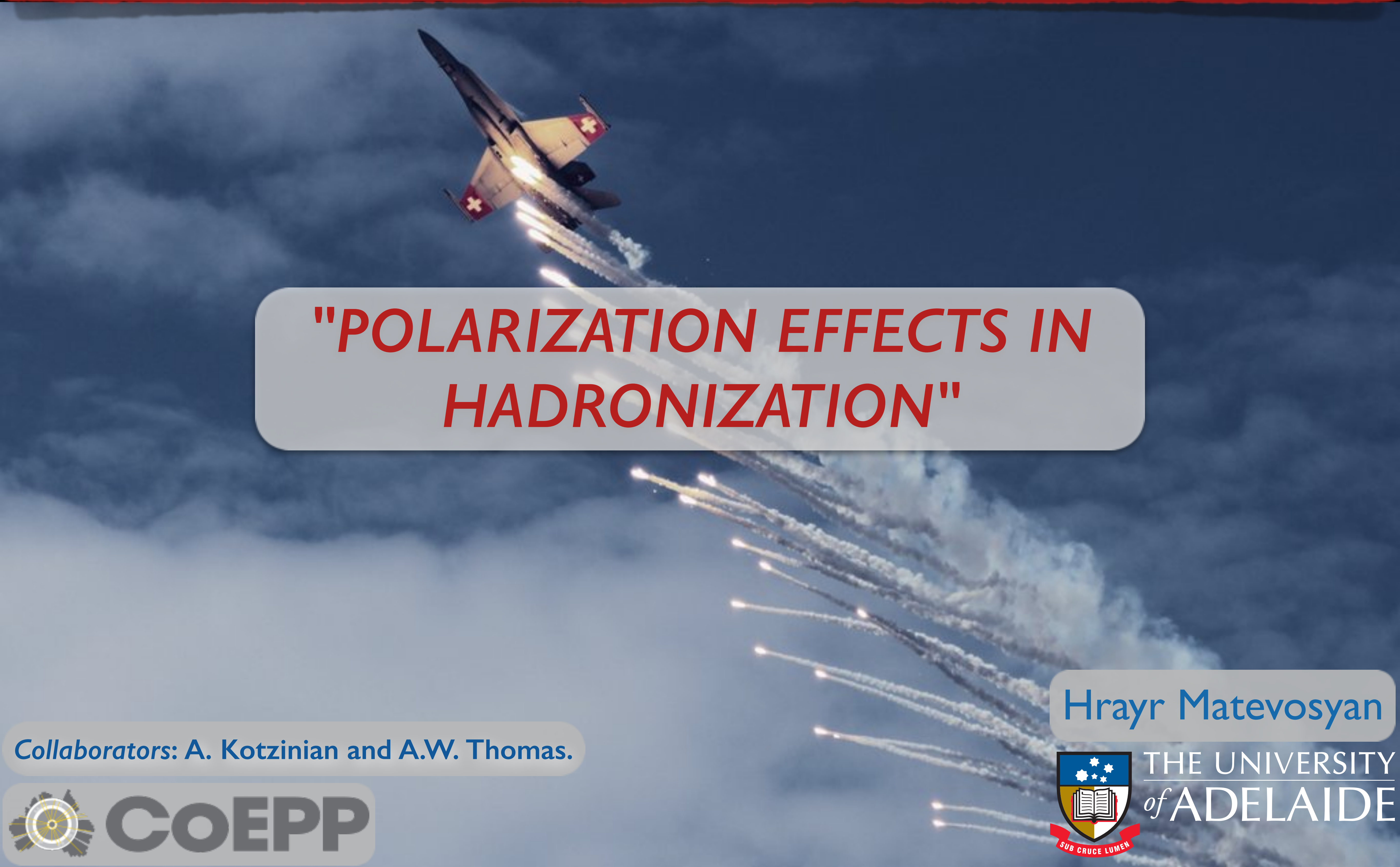
“3D Parton Distributions: path to the LHC”
INFN Frascati: 29/11 - 2/12/2016.



"POLARIZATION EFFECTS IN HADRONIZATION"

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Collaborators: A. Kotzinian and A.W. Thomas.

CoEPP

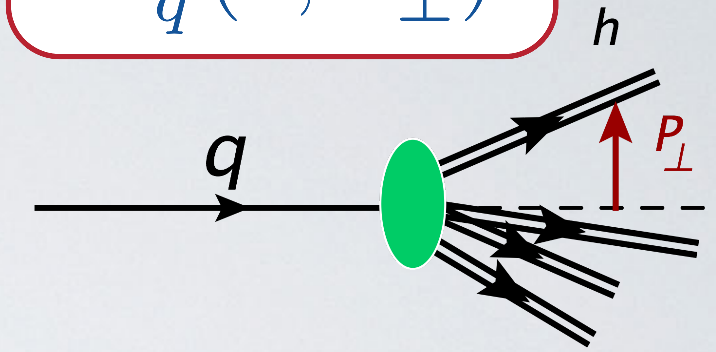
Outlook

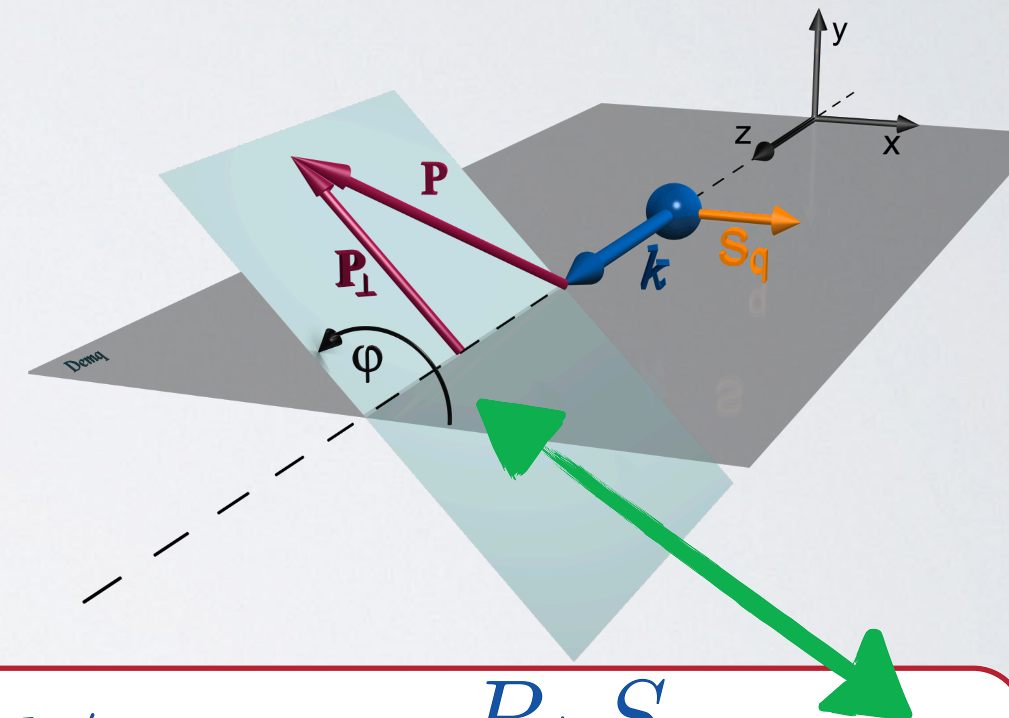
- *Introduction and Motivation.*
- *Short Overview of models for polarized fragmentation functions.*
- *Quark-jet model.*
- *Recent Results from Monte Carlo Simulations: both single and dihadron FFs. Can we learn about hadronization mechanisms from polarized FFs?*
- *Conclusions.*

TMD FFs and Collins Fragmentation Function

- **Unpolarized TMD FF:** number density for quark ***q*** to produce unpolarized hadron ***h*** carrying LC fraction **z** and TM $P_\perp$.

$$D_q^h(z, P_\perp^2)$$





- **Collins Effect:** Azimuthal Modulation of Transversely Polarized Quark' FF. Fragmenting quark's transverse spin couples with produced hadron's TM!

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

Unpolarized   Collins

- ***Collin FF*** is **Chiral-ODD:** Should to be coupled with another chiral-odd PDF/FF in observables.

TMD FFs for Spin-0 and Spin-1/2 Hadrons

❖ ***The transverse momentum (TM) of the hadron can couple with both its own spin and the spin of the quark!***

$$F^{q\to\pi}(z, \boldsymbol{p}_\perp; \boldsymbol{s})$$

π/q	U	L	T
U	D_1		$H_1^\perp$

$$F^{q\to h^\uparrow}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S})$$

h/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

✦ **TMD Polarized Fragmentation Functions at LO.**

- *Only two for unpolarised final state hadrons.*
- *8 for spin 1/2 final state (including quark). Similar to TMD PDFs.*

Field-Theoretical Definitions

- ***The quark-quark correlator.***

$$\Delta^{[\Gamma]}(z,\vec{p}_T) \equiv \frac{1}{4}\int \frac{dp^+}{(2\pi)^4} Tr[\Delta\Gamma]|_{p^-=zk^-}$$

$$= \frac{1}{4z}\sum_X \int \frac{d\xi^+ d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(p^-\xi^+/z-\vec{\xi}_T\cdot\vec{p}_T)} \langle 0|\psi(\xi^+,0,\vec{\xi}_T)|p,S_h,X\rangle\langle p,S_h,X|\bar{\psi}(0)\Gamma|0\rangle$$

- ***The definitions of FFs from the quark correlator***

$$\Delta^{[\gamma^+]} = D(z,p_\perp^2) - \frac{1}{M}\epsilon^{ij} k_{Ti} S_{Tj} D_T^\perp(z,p_\perp^2)$$

$$\Delta^{[\gamma^+\gamma_5]} = S_L G_L(z,p_\perp^2) + \frac{\boldsymbol{k}_T\cdot S_T}{M} G_T(z,p_\perp^2)$$

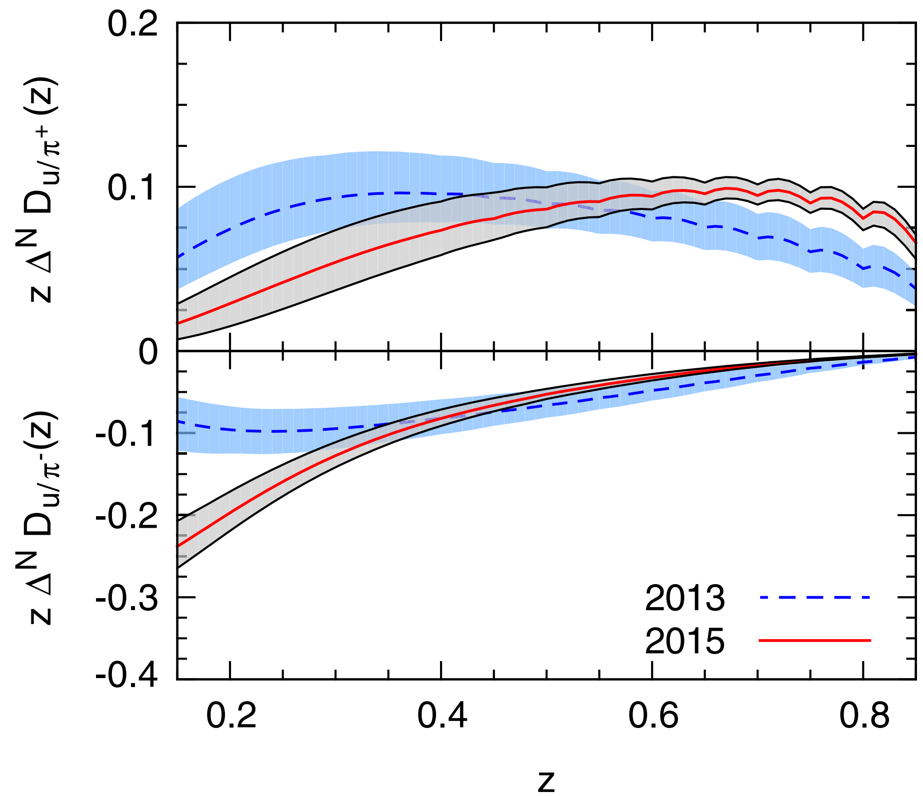
$$\Delta^{[i\sigma^{i+}\gamma_5]} = S_T^i H_T(z,p_\perp^2) + \frac{S_L}{M} k_T^i H_L^\perp(z,p_\perp^2) + \frac{k_T^i(\boldsymbol{k}_T\cdot S_T)}{M^2} H_T^\perp(z,p_\perp^2) - \frac{\epsilon^{ij}k_{Tj}}{M} H^\perp(z,p_\perp^2)$$

Current Challenges

1) Phenomenological Extractions of TMD FFs.

- Still Large Uncertainties.
- Simplistic Approximations.
- Limited kinematic region.

Anselmino et al: PRD 92, 114023 (2015).



2) Full Event Generators:

- ***No*** Mainstream MC generator ***includes spin*** in Full Hadronization: *PYTHIA, HERWIG, SHERPA…*
- MC generators are needed to support mapping of the 3D structure of nucleon at *JLab12, BELLE II, EIC.*

Modelling Hadronization with Spin: The Objectives.

1) Phenomenological Extractions of TMD FFs.

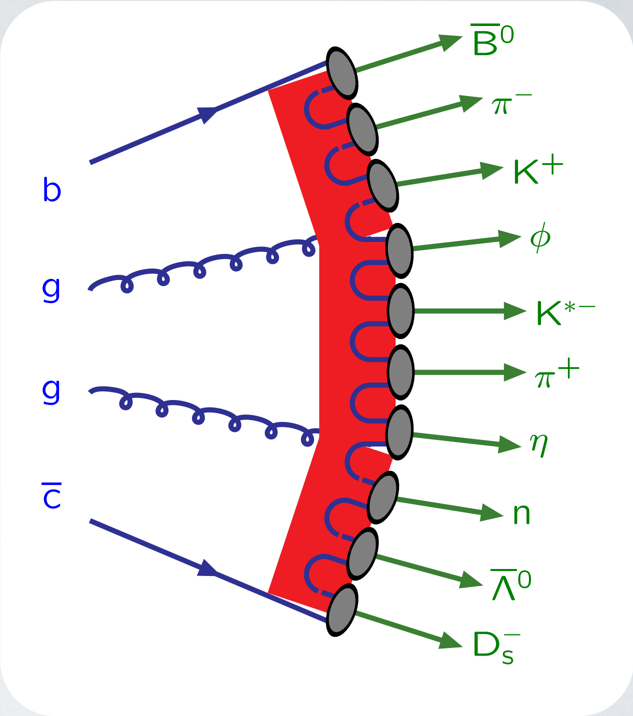
- ***Quantitative*** **extract. of fav. and unfav. polarised TMD FF. Provide guidance for empirical fits to data.**
- **Both single and dihadron FFs in the same framework!**

2) Interpretation in Full Event Generators:

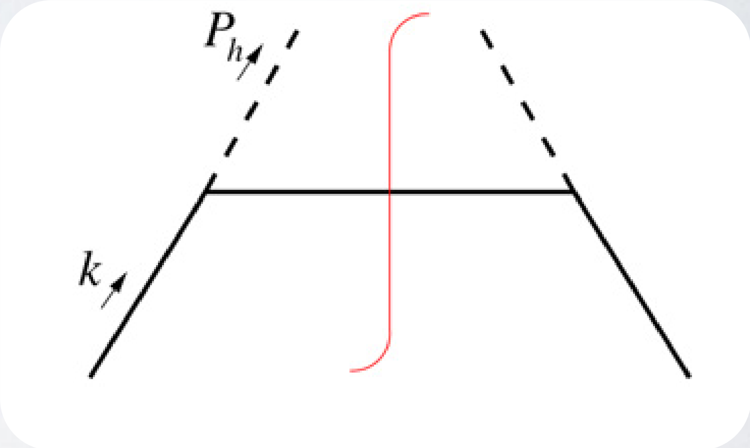
- **Probabilistic Mechanism for Full Hadronization.**
- **Iterative picture for MC framework: spin transfer!**
- ***Should not break any of the unpolarised observables! (PYTHIA fits to existing data, etc.)***

(SOME of the) MODELS FOR FRAGMENTATION

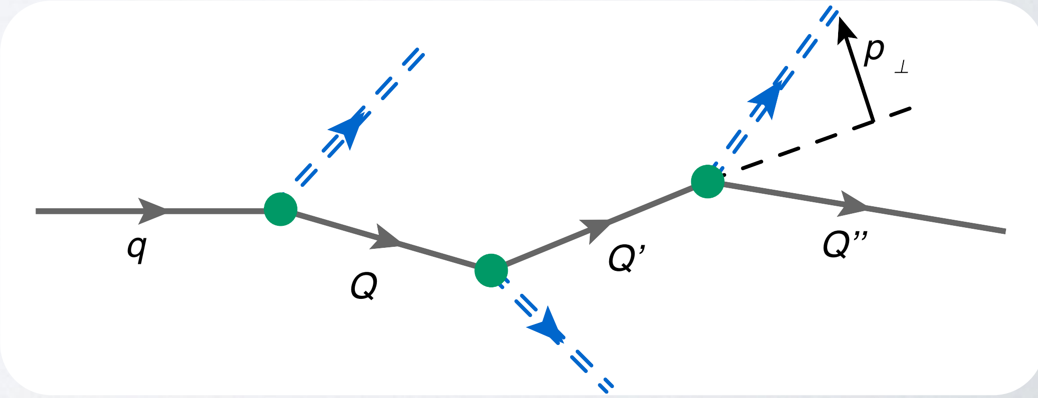
- *Lund String Model*
  - *Very Successful* implementation in *JETSET, PYTHIA*.
  - Highly Tunable.
  - Spin Effects - see *X. Artru's talk*.

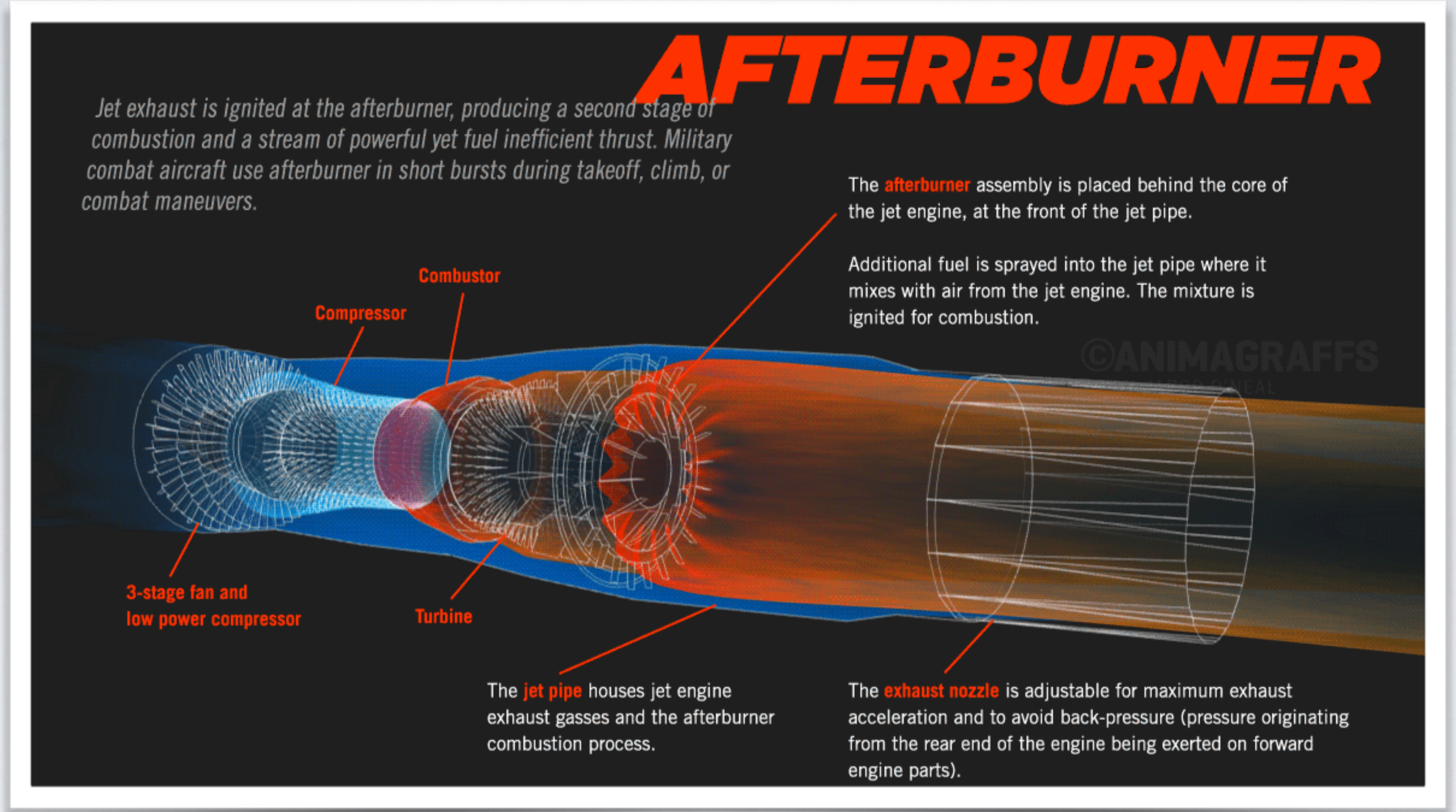


- *Spectator Model*
  - Quark model calculations with empirical form factors.
  - No unfavored fragmentations.
  - Need to tune parameters for small z dependence.



- *NJL-jet Model*
  - *Multi-hadron* emission framework with effective quark model input.
  - *Monte-Carlo framework* allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.



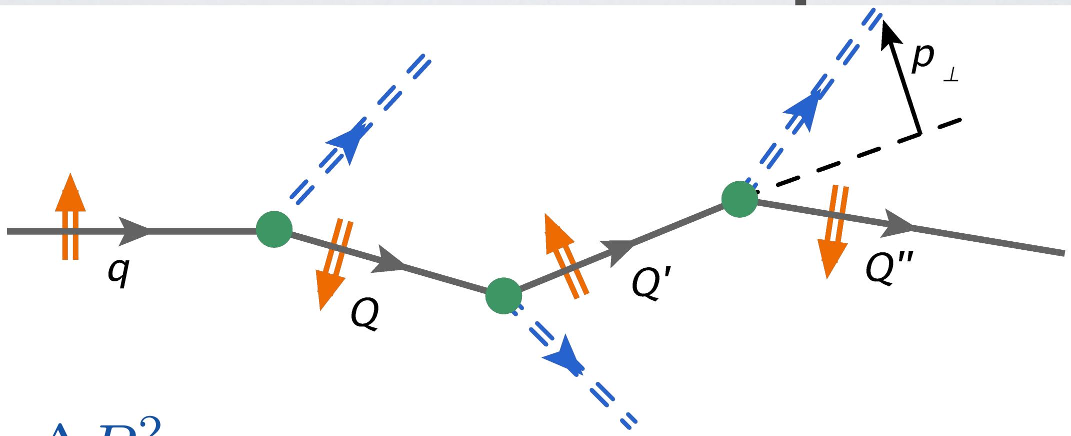


POLARISATION IN QUARK-JET FRAMEWORK

COLLINS FRAGMENTATION FUNCTION IN QUARK-JET

H.M.,Bentz, Thomas, PRD.86:034025, (2012). H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- **Extend Quark-jet Model to include Spin.**

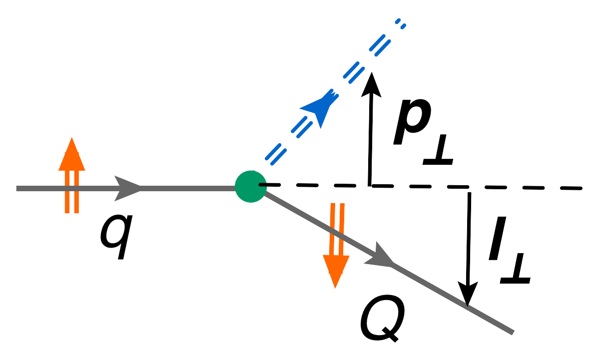


$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

- **Input Elementary Collins Function: *Model or Parametrization***

- ***Calc. Spin of the remnant quark:*** $\mathbf{S}'$

  Previously: constant values for spin flip probability: $\mathcal{P}_{SF}$



✦ Use fit form to extract unpol. and Collins FFs from $D_{h/q^\uparrow}$.

$$F(c_0, c_1) \equiv c_0 - c_1 \sin(\varphi_C)$$

$$D_{h/q^\uparrow}(z, p_\perp^2, \varphi) = D^{h/q}(z, p_\perp^2) - H^{\perp h/q}(z, p_\perp^2) \frac{p_\perp s_T}{z m_h} \sin(\varphi_C)$$

COLLINS FRAGMENTATION FUNCTION IN QUARK-JET

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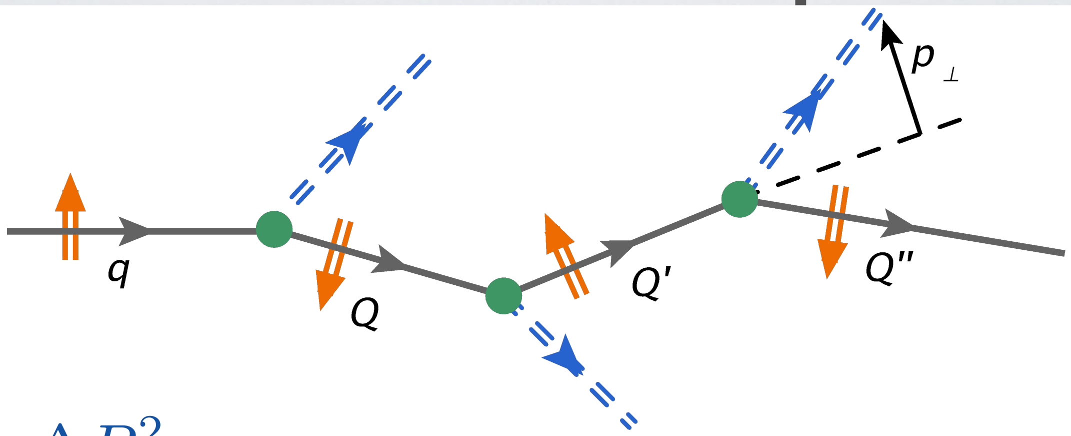

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

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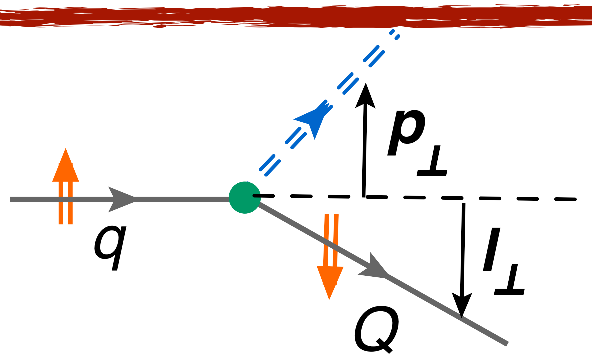

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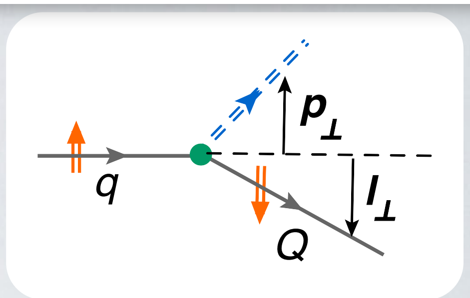
COLLINS EFFECT - NJL-jet MKII

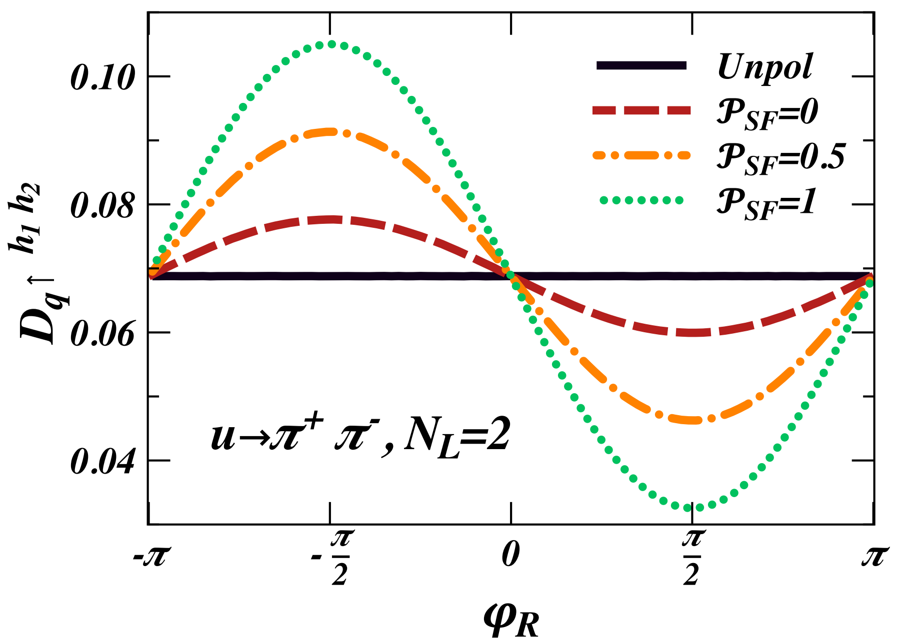
MKII Model Assumptions:

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

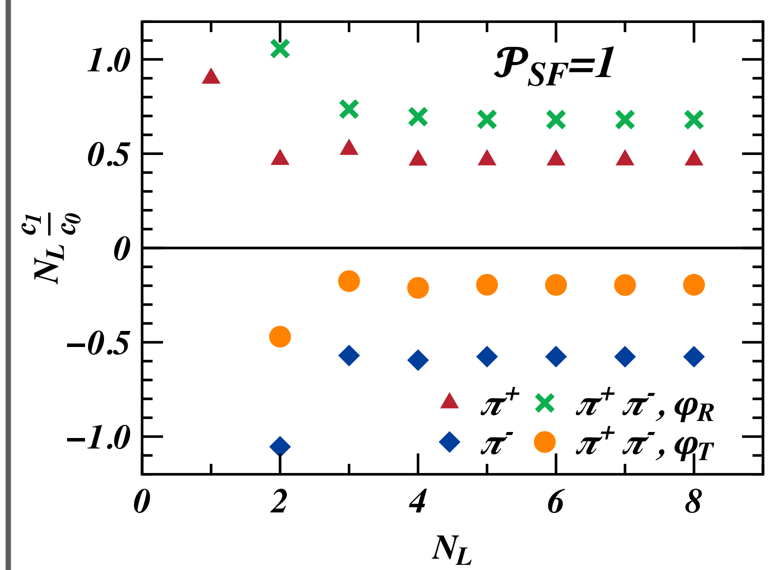
1. Allow for Collins Effect only in a SINGLE emission vertex (N_L^{-1} scaling of the resulting Collins function).
2. Use constant values for spin flip probability: $\mathcal{P}_{SF}$.
3. Extreme ansatz for the elem. Collins function: $d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9 \sin\varphi)$

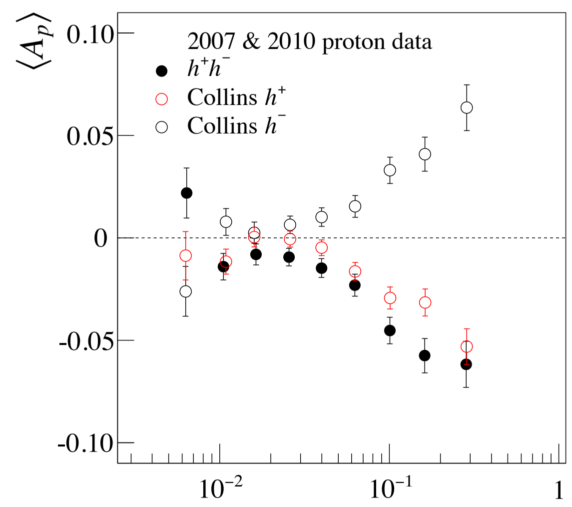


✓ ***First-ever** model calc. for two-hadron modulations induced by Collins effect!*



✓ ***NJL-jet** model results are **consistent** with **COMPASS** data on interplay between one- and two- hadron SSAs.*





COMPASS: PLB736, 124-131 (2014).

✘ ***A self-consistent** model is needed that naturally avoids complications with higher-order modulations: **the need for N_L^{-1} scaling, etc.***

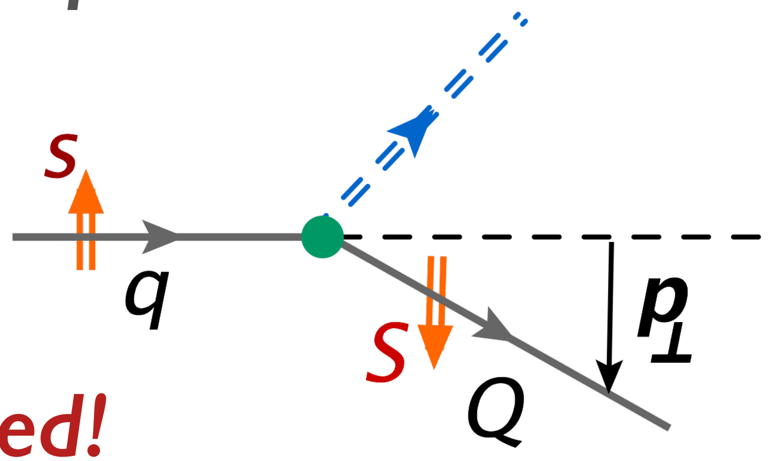
Spin Transfer in quark-jet Framework.

✦NJL-jet MKIII:

Bentz et al, Phys.Rev. D94 034004 (2016).

- *The probability for the process* $q \to Q$, *initial spin* $\mathbf{s}$ *to* $\mathbf{S}$

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) = \alpha_\mathbf{s} + \boldsymbol{\beta}_\mathbf{s} \cdot \mathbf{S}$$

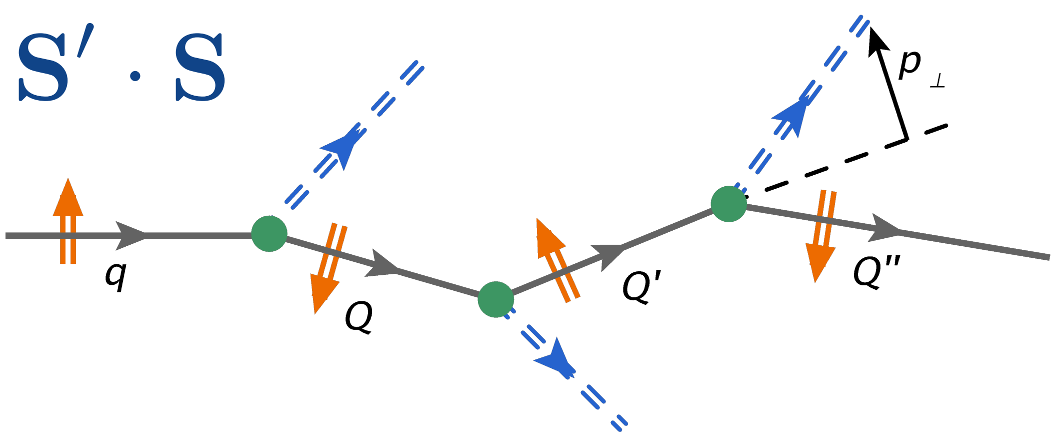


- ***Intermediate** quarks in quark-jet are **unobserved!***

Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii: **QUANTUM ELECTRODYNAMICS (1982).**

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'} \rho^{\mathbf{S}}] \sim 1 + \mathbf{S}' \cdot \mathbf{S}$$

$$\mathbf{S}' = \frac{\boldsymbol{\beta}_\mathbf{s}}{\alpha_\mathbf{s}}$$



- *Remnant quark's* $\mathbf{S}'$ ***uniquely determined by*** $z, \mathbf{p}_\perp$ ***and*** $\mathbf{s}$ ***!***

- *Process probability is **the same** as transition to **unpolarized state.***

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{0}) = \alpha_s$$

Example: Pion production.

✦ We can express the spin of the remnant quark $\mathrm{S}' = \dfrac{\beta_s}{\alpha_s}$ in terms of *quark-to-quark* *TMD FFs*.

$$\alpha_q \equiv D(z, \boldsymbol{p}_\perp^2) + (\boldsymbol{p}_\perp \times \boldsymbol{s}_T) \cdot \hat{\boldsymbol{z}} \frac{1}{z\mathcal{M}} \; H^\perp(z, \boldsymbol{p}_\perp^2)$$

$$\beta_{q\|} \equiv s_L \; G_L(z, \boldsymbol{p}_\perp^2) - (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z\mathcal{M}} H_L^\perp(z, \boldsymbol{p}_\perp^2)$$

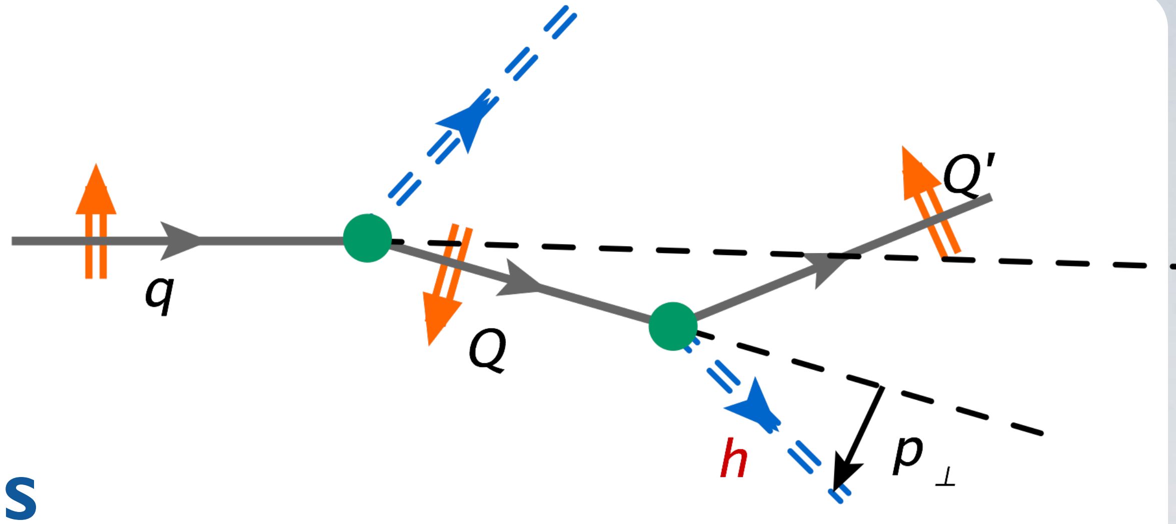
$$\boldsymbol{\beta}_{q\perp} \equiv \boldsymbol{p}'_\perp \frac{1}{z\,\mathcal{M}} D_T^\perp(z, \boldsymbol{p}_\perp^2) - \boldsymbol{p}_\perp \frac{1}{z\mathcal{M}} s_L G_T(z, \boldsymbol{p}_\perp^2)$$
$$+ \boldsymbol{s}_T \; H_T(z, \boldsymbol{p}_\perp^2) + \boldsymbol{p}_\perp (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z^2\mathcal{M}^2} \; H_T^\perp(z, \boldsymbol{p}_\perp^2)$$

$$F^{q\to Q}(z, \boldsymbol{p}_\perp; \boldsymbol{s}, \boldsymbol{S})$$

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

Example: Pion prod. up to Rank 2

✦ **Only consider pion produced in the first two emission steps!**



✦ **Then the polarised number density is**

1st rank | 2nd rank

$$F^{(2)q\to\pi} = f^{q\to\pi} + f^{q\to Q} \otimes f^{Q\to\pi}$$

✦ **"Elementary" number densities:** *only favoured types are non-zero.*

$$f^{q\to\pi} = d^{q\to\pi} - \frac{p_\perp}{zM_h} s_T h_1^{\perp q\to\pi}$$

$$f^{u\to\pi^-} = 0$$

✦ *It is shown <u>analytically</u> that only Collins modulations appear!*

$$F^{(2)q\to\pi}(z, p_\perp^2, \varphi_C) = F_0^{(2)}(z, p_\perp^2) - \sin(\varphi_C) F_1^{(2)}(z, p_\perp^2)$$

Example: Pion prod. up to Rank 2

✦ *It is shown analytically that only Collins modulations appear!*

$$F^{(2)q\to\pi}(z,p_\perp^2,\varphi_C) = F_0^{(2)}(z,p_\perp^2) - \sin(\varphi_C)F_1^{(2)}(z,p_\perp^2)$$

✦ *Up to **unspecified** coefficients, using.*

Unpolarised term:

From TM-induced Spin of intermediate quark

$$F_0^{(2)q\to\pi} = d^{q\to\pi} + (d^{q\to Q}\otimes d^{Q\to\pi} + d_T^{\perp q\to Q}\otimes h^{\perp Q\to\pi})$$

Collins term:

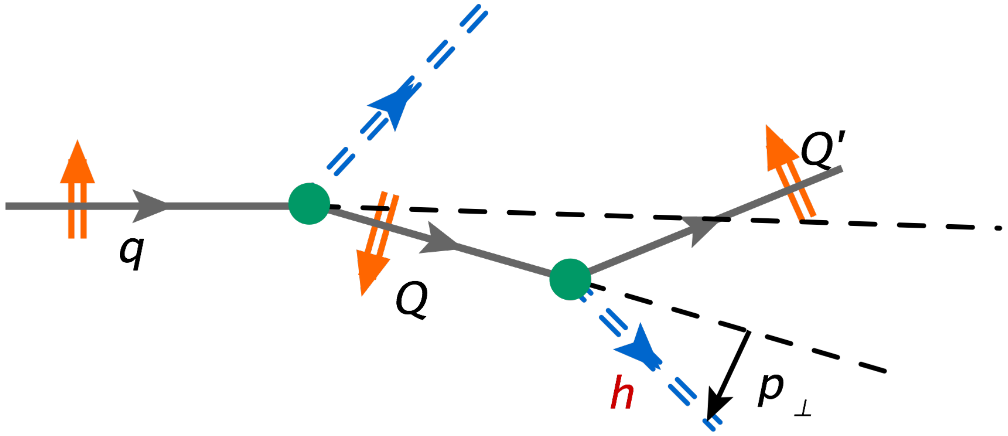
"Recoil" TM contribution

$$F_1^{(2)q\to\pi} \sim h^{\perp q\to\pi} + [h^{\perp q\to Q}\otimes d^{Q\to\pi} + (h_T^{q\to Q} + h_T^{\perp q\to Q})\otimes h^{\perp Q\to\pi}]$$

Transferred Spin of intermediate quark

✦ *Reminder*

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$



Integral Equations

✦ **In the limit of infinite produced hadrons, we can derive integral equations for the FFs within quark-jet framework.**

✦ **Unpolarized FF**

$$D^{(q\to\pi)}(z,\mathbf{p}_\perp^2) = \hat{d}^{(q\to\pi)}(z,\mathbf{p}_\perp^2) + 2\int \mathcal{D}^2\eta \int \mathcal{D}^4 p_\perp \delta(z-\eta_1\eta_2)\delta^{(2)}(\mathbf{p}_\perp - \mathbf{p}_{2\perp} - \eta_2\mathbf{p}_{1\perp})$$
$$\times \left[\hat{d}^{(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)D^{(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2) + \frac{1}{M m_\pi z}(\mathbf{p}_{1\perp}\cdot\mathbf{p}_{2\perp})\hat{d}_T^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)H^{\perp(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)\right]$$

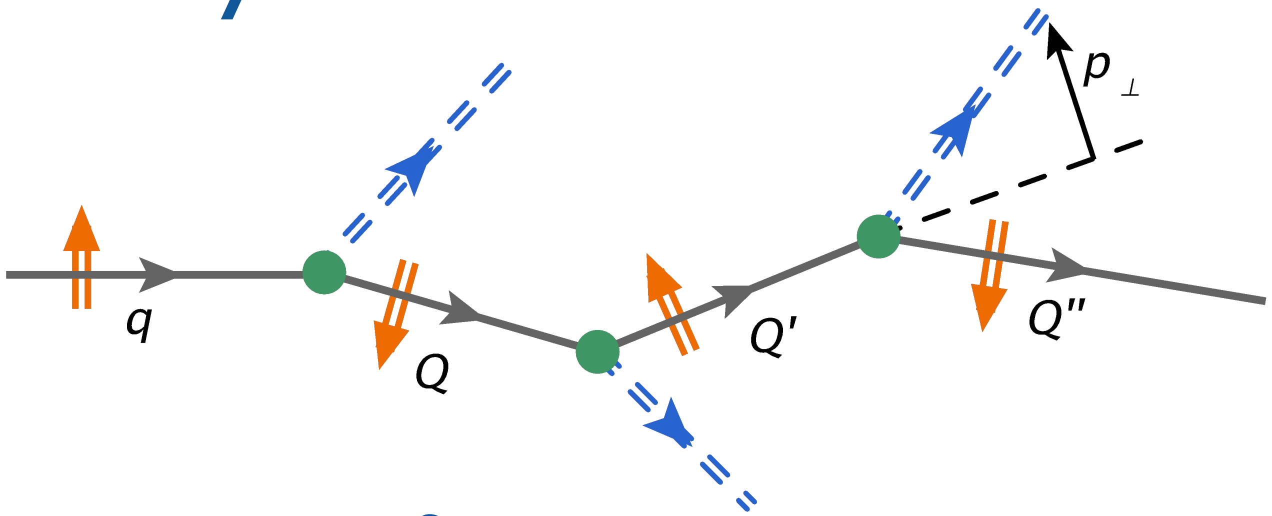
✦ **Collins FF**

$$(\mathbf{p}_\perp\times\mathbf{s}_T)^3 H^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2) = (\mathbf{p}_\perp\times\mathbf{s}_T)^3\hat{h}^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2) + 2\int \mathcal{D}^2\eta \int \mathcal{D}^4 p_\perp \delta(z-\eta_1\eta_2)\delta^{(2)}(\mathbf{p}_\perp - \mathbf{p}_{2\perp} - \eta_2\mathbf{p}_{1\perp})$$
$$\times \left[\frac{m_\pi}{M}\eta_2(\mathbf{p}_{1\perp}\times\mathbf{s}_T)^3\hat{h}^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)D^{(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)\right.$$
$$+ (\eta_1(\mathbf{p}_{2\perp}\times\mathbf{s}_T)^3\hat{h}_T^{(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2) - \frac{1}{M^2\eta_1}(\mathbf{s}_T\cdot\mathbf{p}_{1\perp})$$
$$\left.\times(\mathbf{p}_{1\perp}\times\mathbf{p}_{2\perp})^3\hat{h}_T^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2))H^{\perp(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)\right].$$

MC Simulation of Full Hadronization

HM et al, arXiv:1610.05624

✦ We can consider many hadron emissions.



✦ We can sample the $h, z, p_\perp^2, \varphi_h$ using

$$f^{q\to h}(z, p_\perp^2, \varphi_h; \mathbf{S}_T)$$

✦ Determine the momenta in the initial frame and calculate

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

✦ *Calculate the remnant quark's spin:* $\mathbf{S}' = \dfrac{\boldsymbol{\beta}_\mathbf{s}}{\alpha_\mathbf{s}}$

✦ We only need the "elementary" splittings.

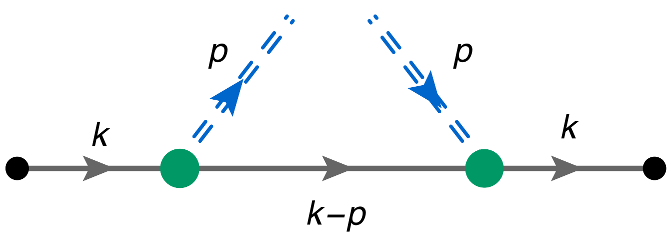
$f^{q\to h}$ $f^{q\to Q}$

Model Calculations of $q \to Q$ Splittings

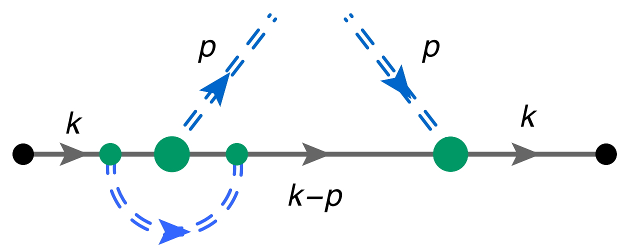
E.G. - Meissner et al, PLB 690, 296 (2010).

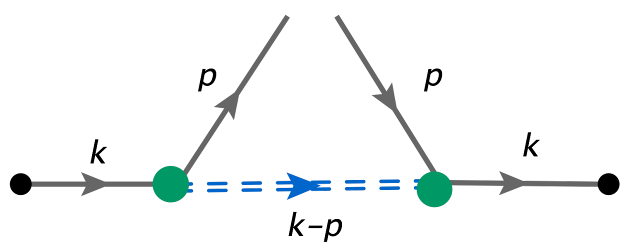
✦We can use the same "spectator" type calculations as for pion.

T-even ***T-odd***

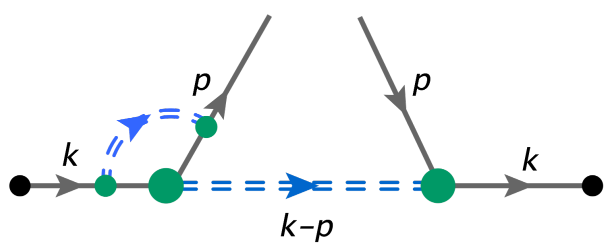


$$q \to h$$





$$q \to Q$$



✦ Positivity Constraints on TMD FFs:

Bacchetta et al, P.R.L. 85, 712 (2000).

$$(H_L^{\perp[1]})^2 + (D_T^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2}D^2$$

$$(G_T^{[1]})^2 + (H^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2}D^2$$

✦ T-odd parts from previous models ***violate positivity***!

$$(\hat{G}_T^{[1]})^2 = (\hat{H}_L^{\perp[1]})^2 = \frac{p_\perp^2}{4z^2M^2}(\hat{D}+\hat{G}_L)(\hat{D}-\hat{G}_L) \leq \frac{p_\perp^2}{4z^2M^2}\hat{D}^2$$

$$\hat{H}^\perp(z, p_\perp^2) = 0, \quad \hat{D}_T^\perp(z, p_\perp^2) = 0.$$

Model Calculations of $q \to Q$ Splittings

✦ Simple Model that is positive-definite:

$$\hat{d}(z, p_\perp^2) = 1.1\, \hat{d}_{tree}(z, p_\perp^2),$$

✦ Use Collins-ansatz for T-odd

J. C. Collins, NPB 396, 161 (1993)

$$\frac{p_\perp}{zM} \frac{\hat{h}^{\perp(q\to h)}(z, p_\perp^2)}{\hat{d}^{(q\to h)}(z, p_\perp^2)} = 0.4 \frac{2\, p_\perp M_Q}{p_\perp^2 + M_Q^2}$$

$$d_T^\perp = -h^\perp$$

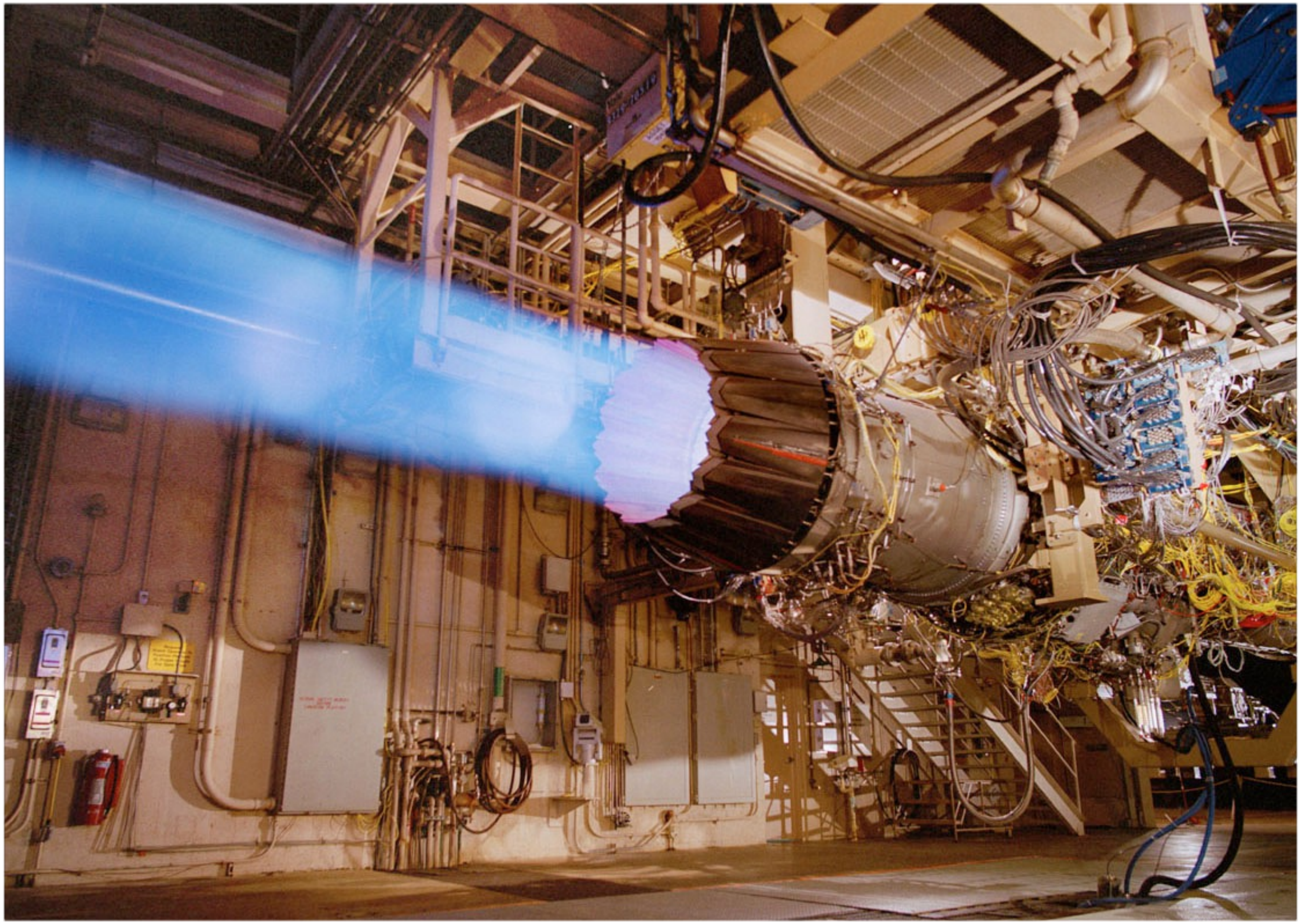
✦ Ensures the inequalities

$$(H_L^{\perp[1]})^2 + (D_T^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2} D^2$$

$$(G_T^{[1]})^2 + (H^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2} D^2$$

✶ Also: ***Evolution - mimicking ansatz***

$$\hat{d}'(z, p_\perp^2) = (1-z)^4 \hat{d}(z, p_\perp^2)$$

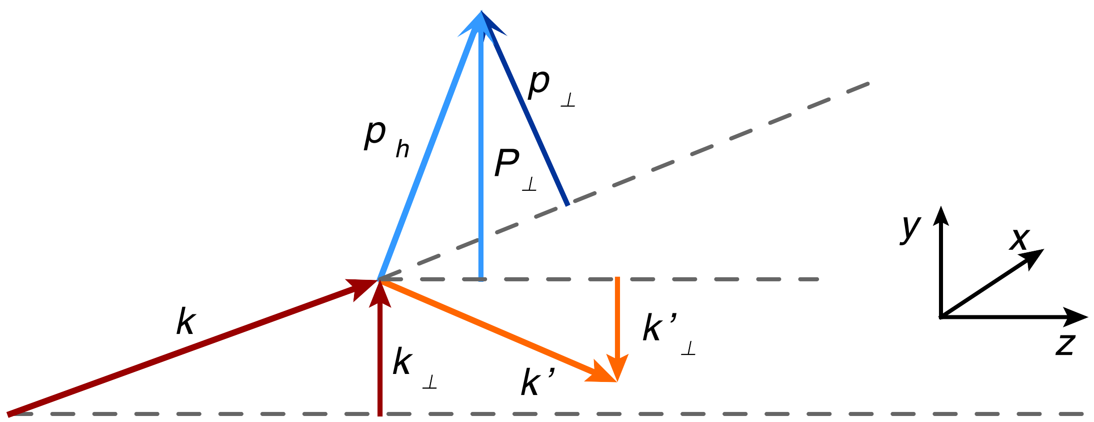


VALIDATION TESTS

Recoil TM Contribution: Rank 2 Hadron

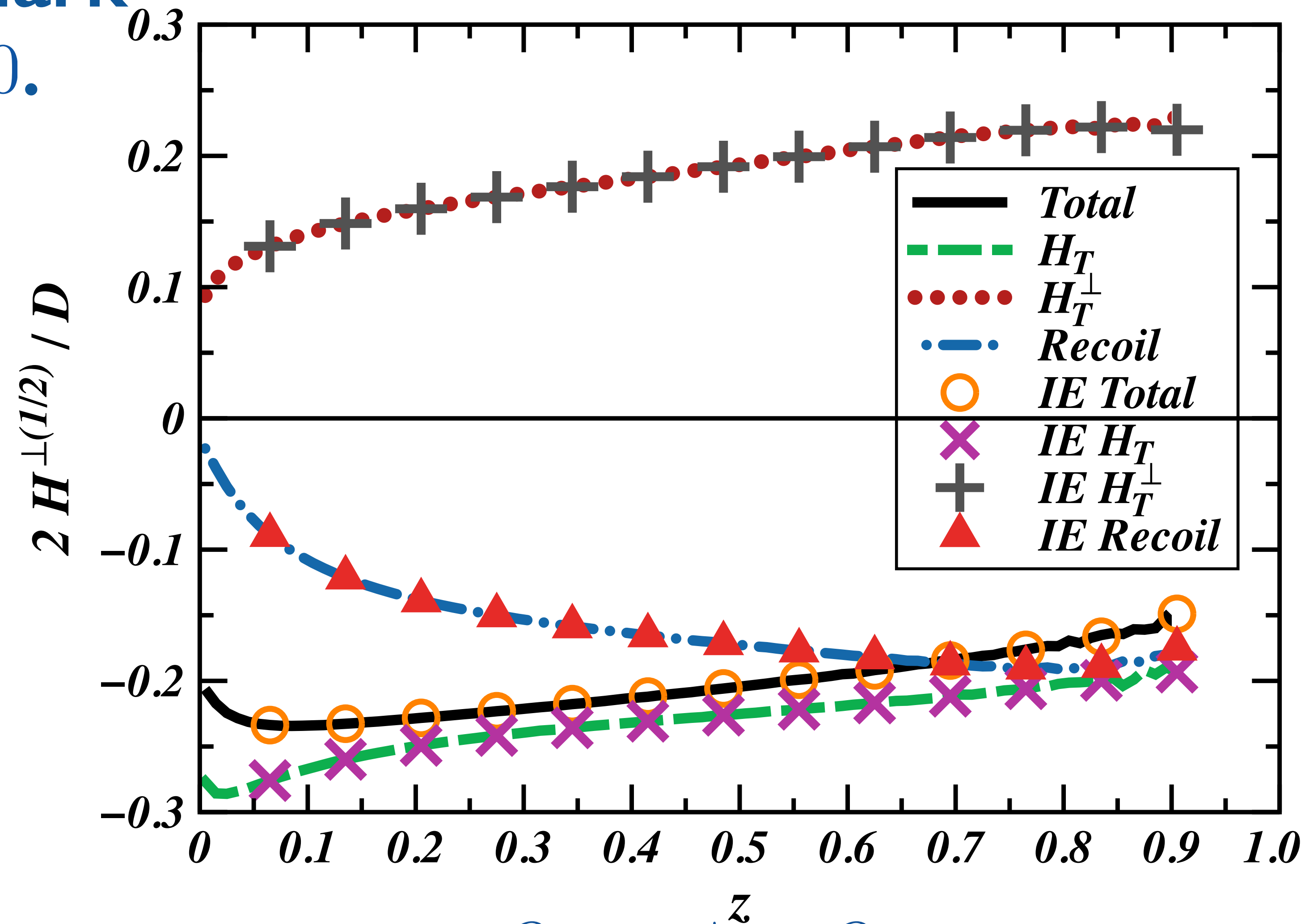
✦ **Full vs "Recoil TM" contributions:**

- **Simulate by *depolarizing* quark after the first emission** $\mathrm{S}' = 0$.



$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



$$F_1^{(2)q\to\pi} \sim h^{\perp q\to\pi} + [h^{\perp q\to Q} \otimes d^{Q\to\pi} + (h_T^{q\to Q} + h_T^{\perp q\to Q}) \otimes h^{\perp Q\to\pi}]$$

"Recoil" TM contribution

Transferred Spin of intermediate quark

✓ **Recoil TM contribution has distinct *z* dependence!**

Higher Order Modulations

✦ The FFs should be *linear functions* of S ! This means linear dependence on sine of Collins angle φ_C .

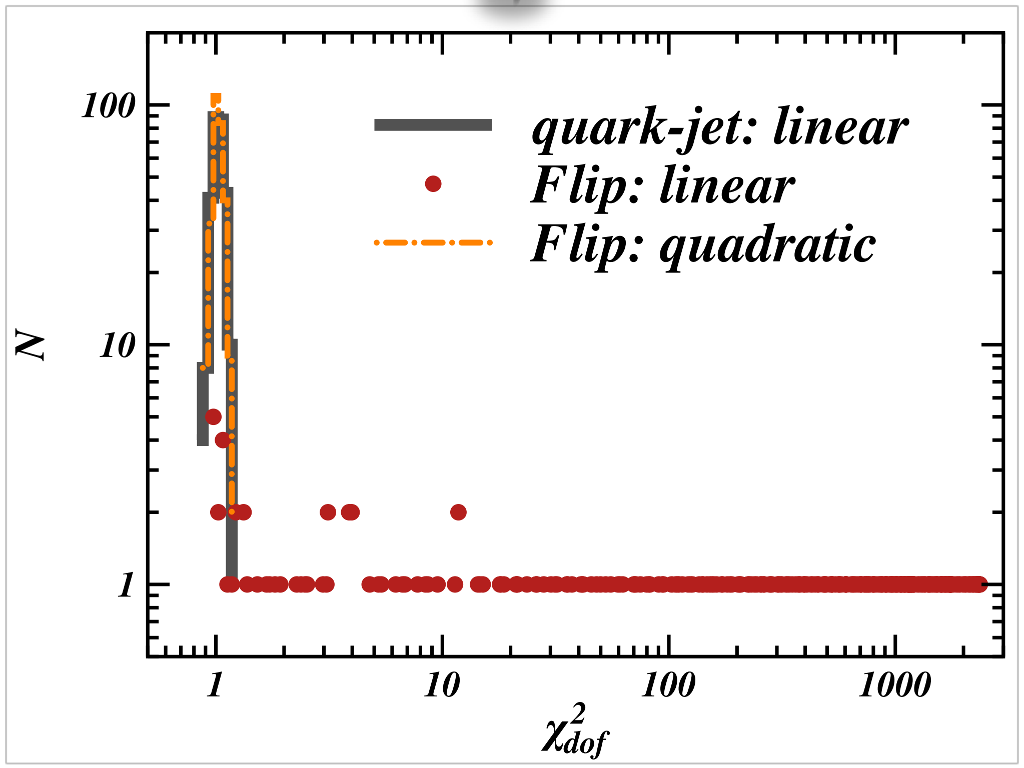
$$F(c_0, c_1) \equiv c_0 - c_1 \sin(\varphi_C)$$

- Also test a simple anstaz: spin *Flip*

$$\mathcal{P}_{SF} = 1 \qquad S'_T = -S_T$$

- *High precision* tests: **10[12] events** for 2 hadron emissions!

- Fit polarized FF for each z: ~ 300 fits.



✓ Linearity on the transverse spin is confirmed at high precision !

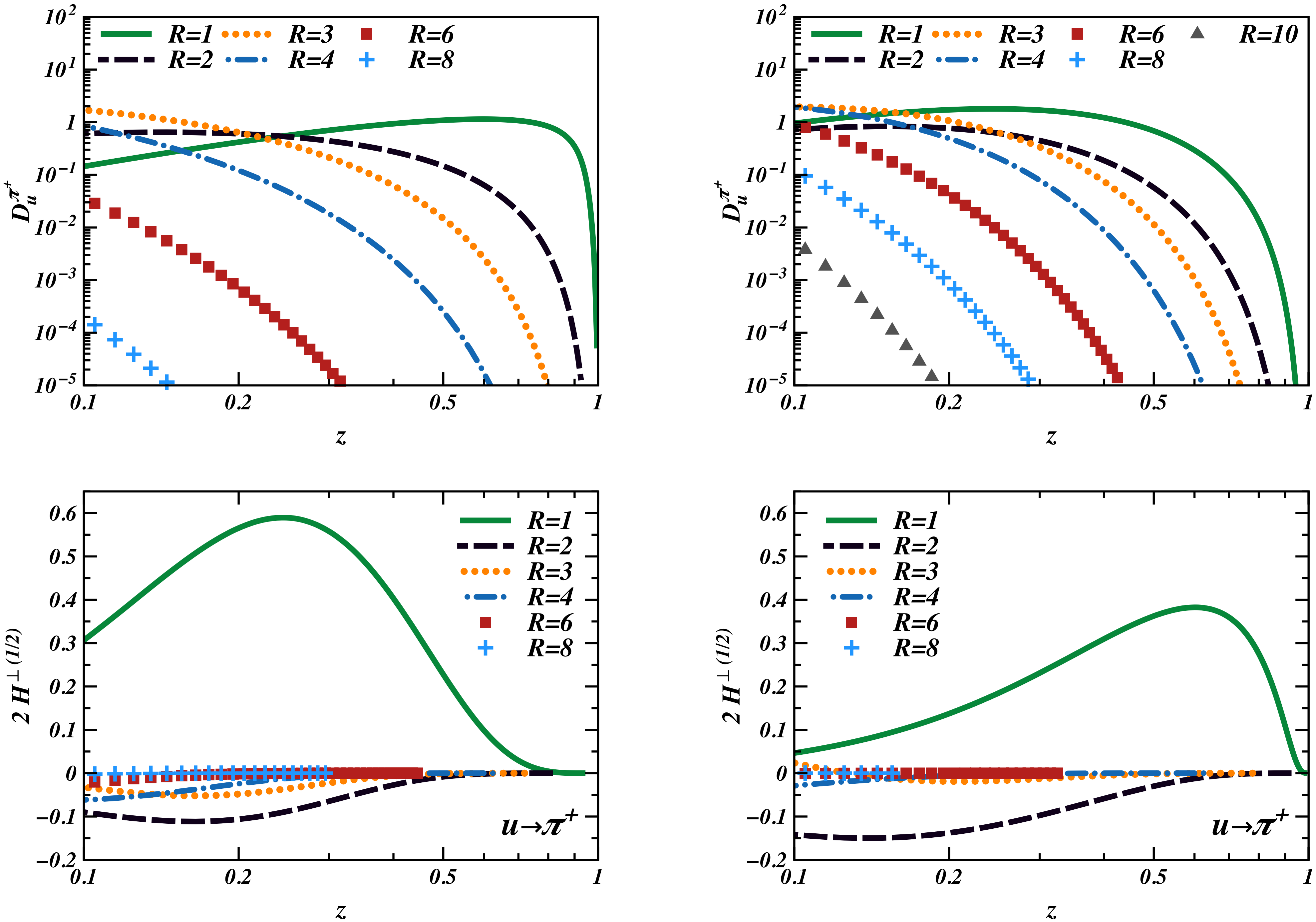
✗ Simplistic spin flip ansatz results in unphysical results !

RESULTS

COLLINS EFFECT IN QUARK-JET MODEL

Saturations of FFs with h Rank

- FFs vs Rank of produced hadron.
  - *NJL* Model
  - Evolution-mimicking *Ansatz.*

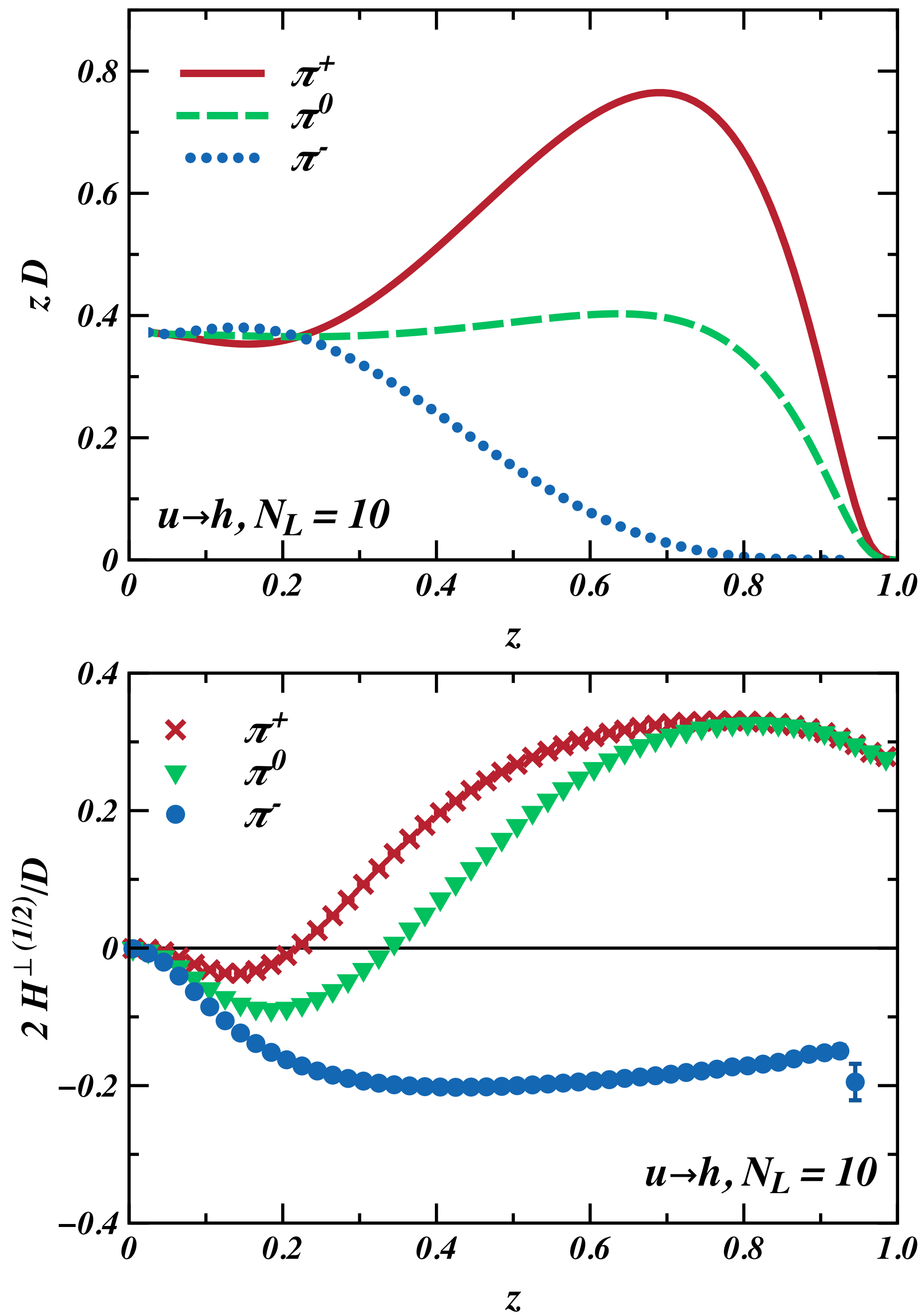


✓ Hadrons of *Rank > 4* are negligible for FFs at *z > 0.1*

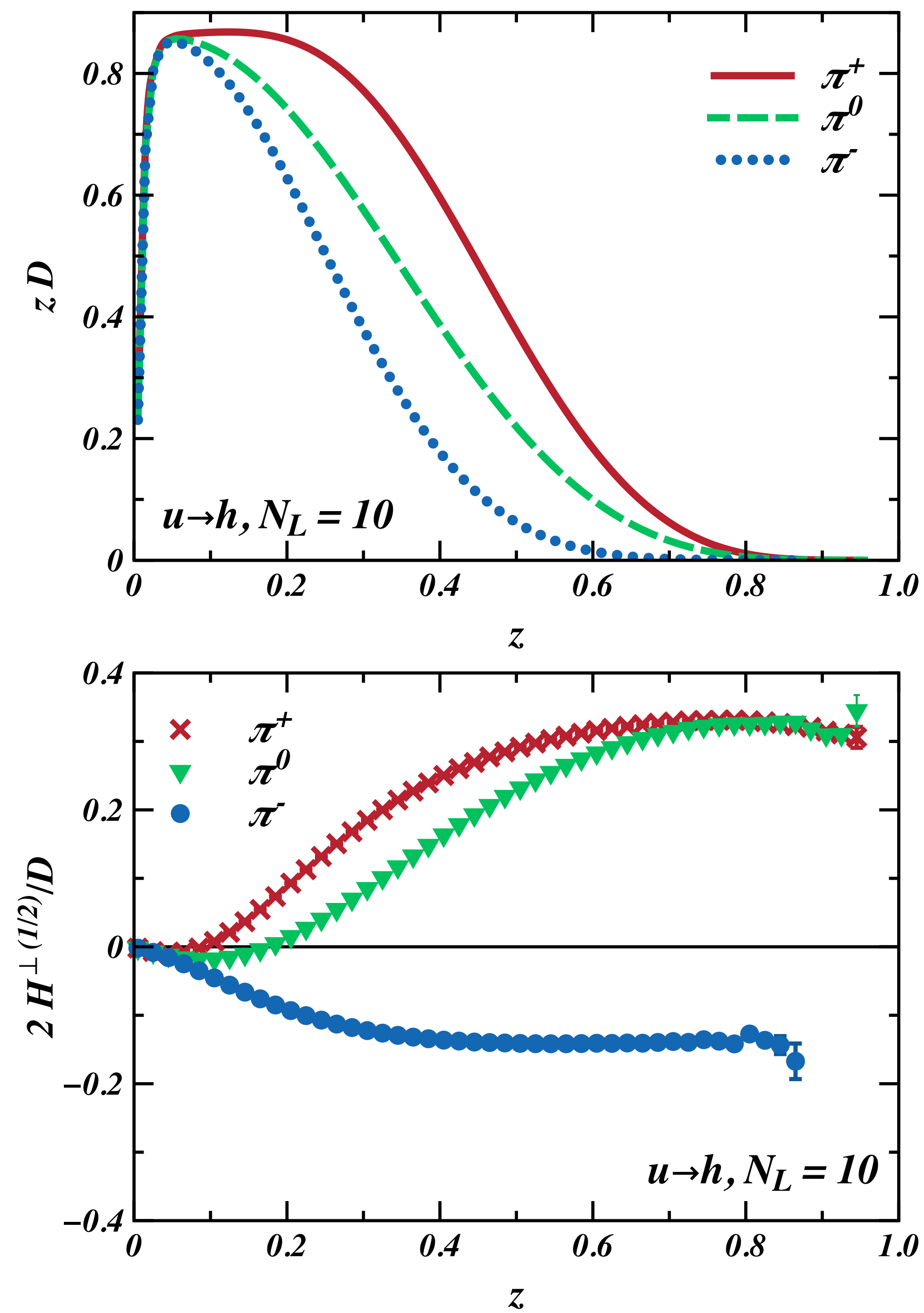
MC Simulation in Toy Model

HM et al, arXiv:1610.05624

- ***NJL* Model**



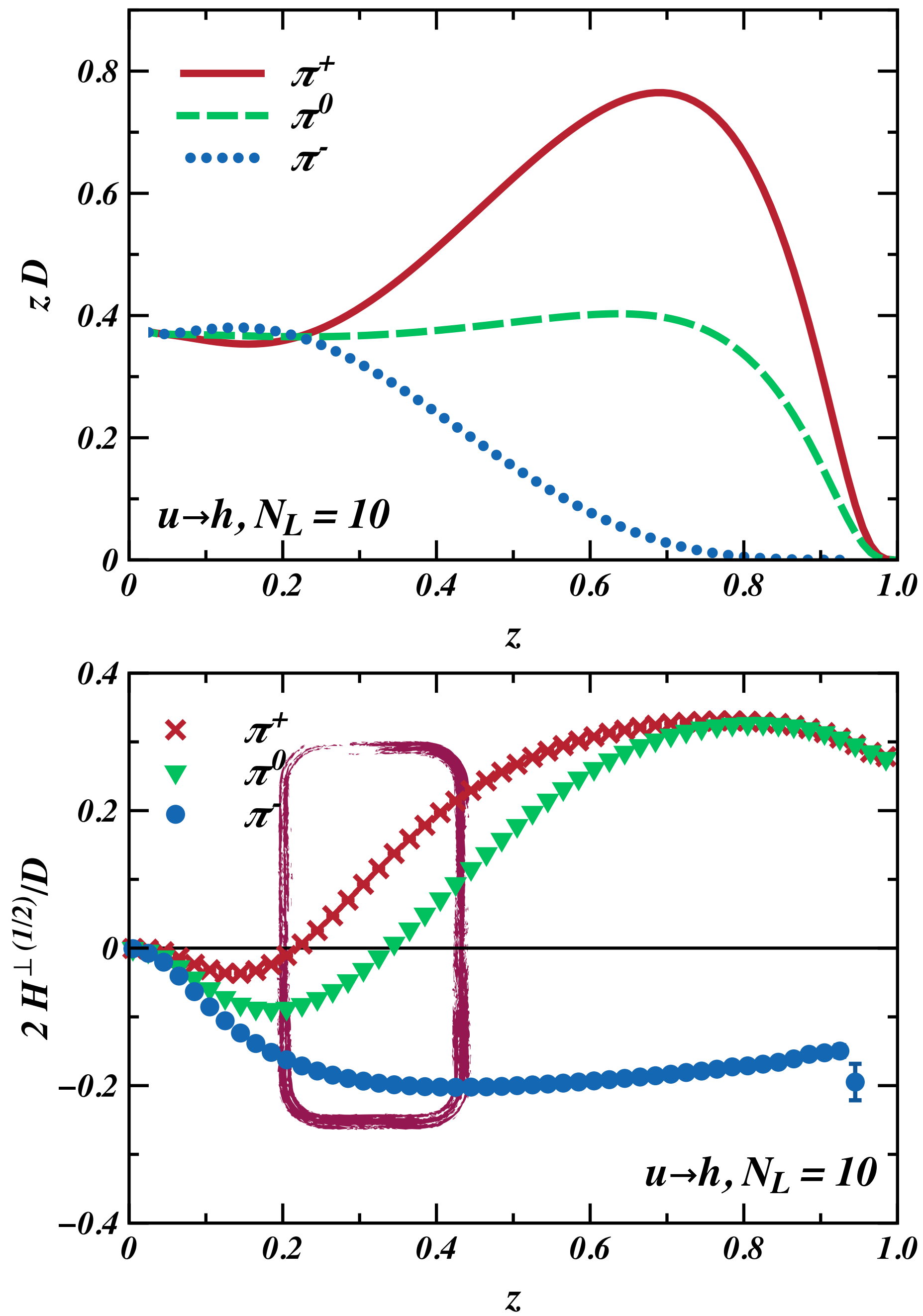
- **Evolution-mimicking *Ansatz.***



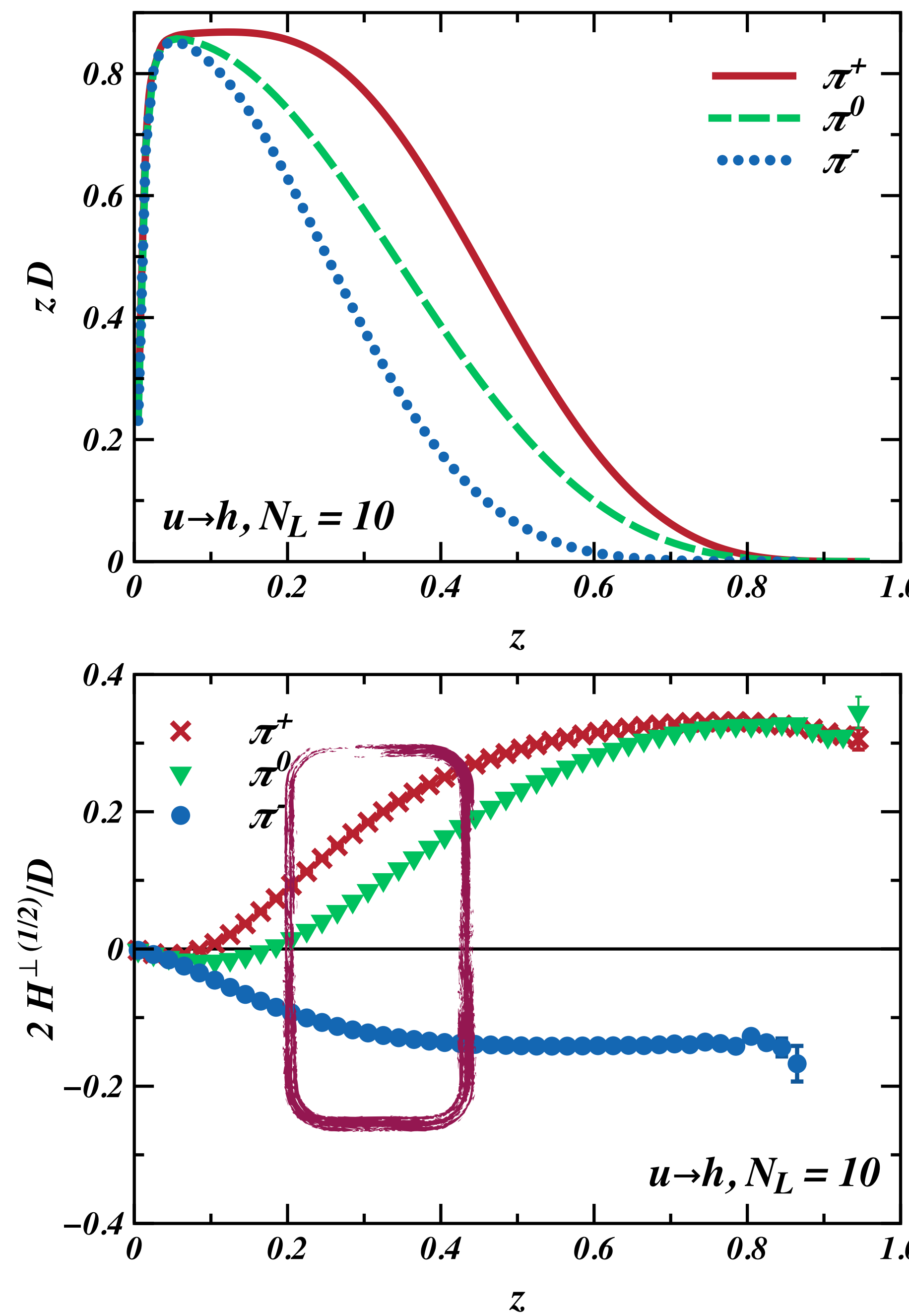
MC Simulation in Toy Model

HM et al, arXiv:1610.05624

‣ *NJL* Model



‣ Evolution-mimicking *Ansatz.*



✦ ***Opposite sign and similar size*** in mid-z range for charged pions. (Similar to empirical extractions).

✦ ***Dependence on model inputs:*** can be tuned to data.

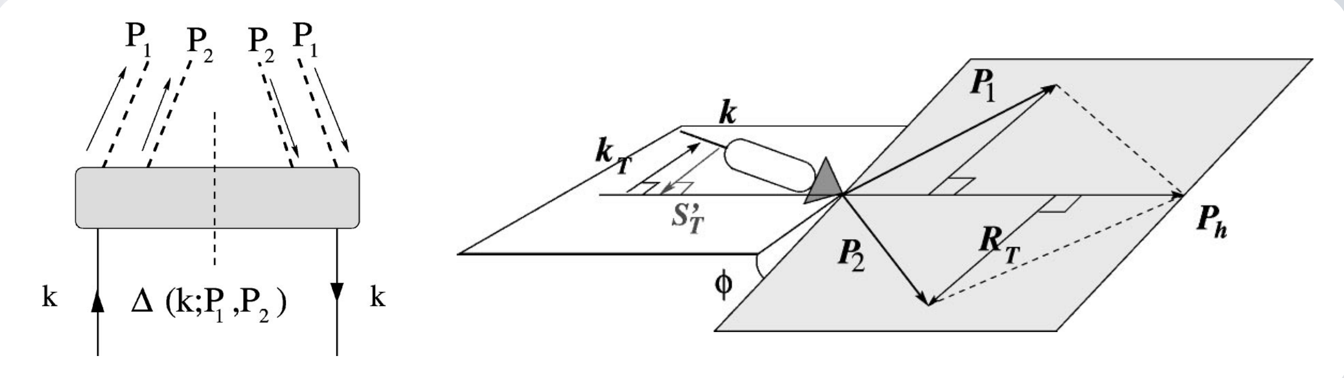
TWO HADRON CORRELATIONS:
DIHADRON FRAGMENTATION FUNCTIONS

TWO-HADRON FRAGMENTATION

A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

- **Total and Relative TM of hadron pair.**

$$\mathbf{P}_T = \mathbf{P}_{h_1}^{\perp} + \mathbf{P}_{h_2}^{\perp}$$
$$\mathbf{R} = (\mathbf{P}_{h_1}^{\perp} - \mathbf{P}_{h_2}^{\perp})/2$$



- Correlation of the *transverse polarisation* of quark and one of the momenta:

$$D_{q^{\uparrow}}^{h_1 h_2}(\varphi_R) = D_q^{h_1 h_2} + s_T \sin(\varphi_R - \varphi_S)\mathcal{F}[H^{\sphericalangle}, H^{\perp}]$$
$$D_{q^{\uparrow}}^{h_1 h_2}(\varphi_T) = D_q^{h_1 h_2} + s_T \sin(\varphi_T - \varphi_S)\mathcal{F}'[H^{\sphericalangle}, H^{\perp}]$$

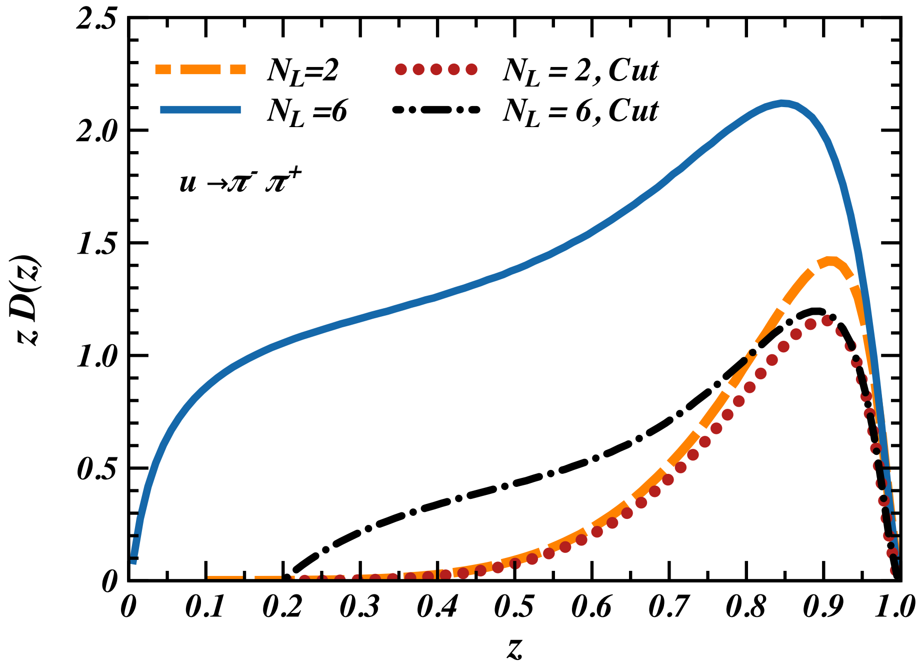
- Correlation of the *longitudinal polarisation* of quark and *both* momenta:

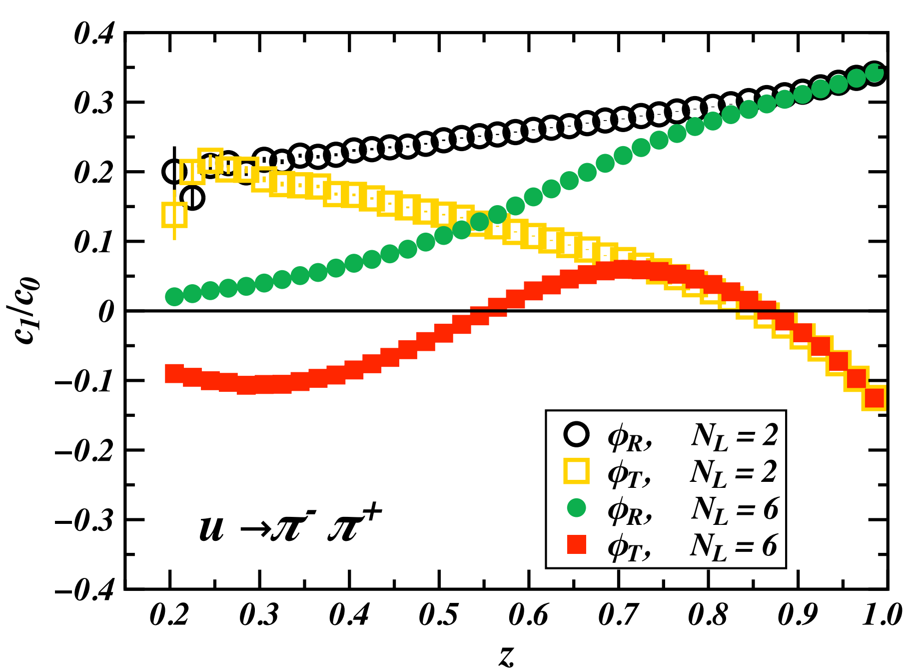
$$D_{q^{\rightarrow}}^{h_1 h_2}(\varphi_{R-T}) = D_q^{h_1 h_2}[cos(\varphi_{R-T})] + s_L \sin(\varphi_{R-T})\mathcal{G}[cos(\varphi_{R-T})]$$
$$\varphi_{R-T} \equiv \varphi_R - \varphi_T$$

Transverse Spin

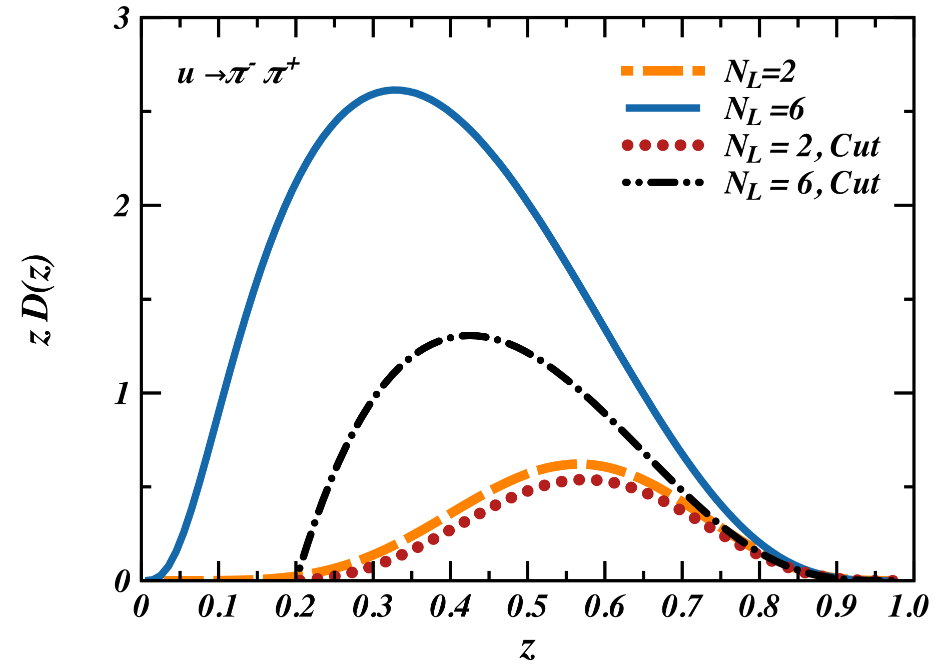
✦ **Results for unpolarized DiFF and analysing power, impose cut** $z_{1,2} \geq 0.1$

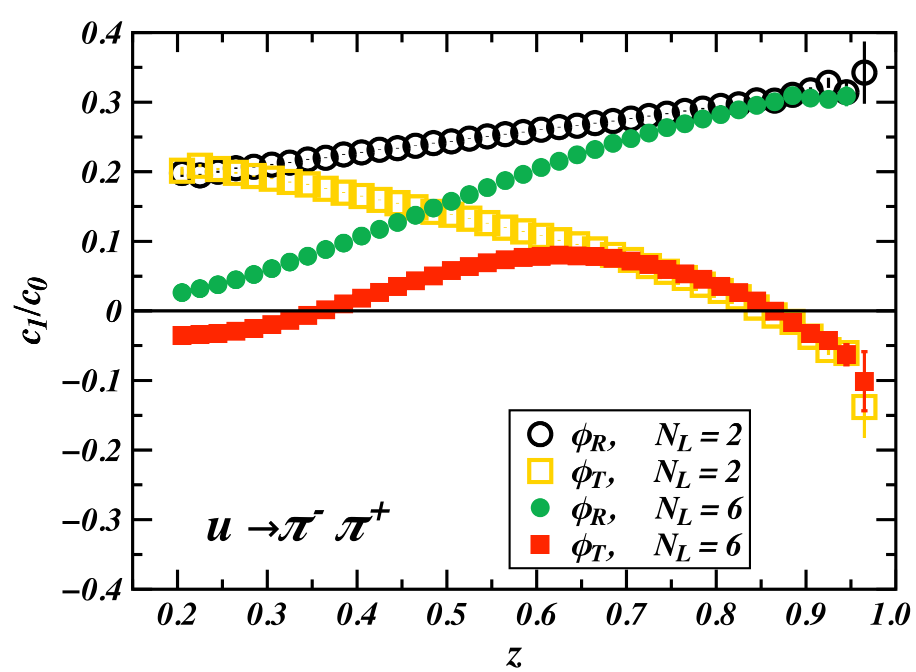
‣ *NJL* Model





‣ Evolution-mimicking *Ansatz.*



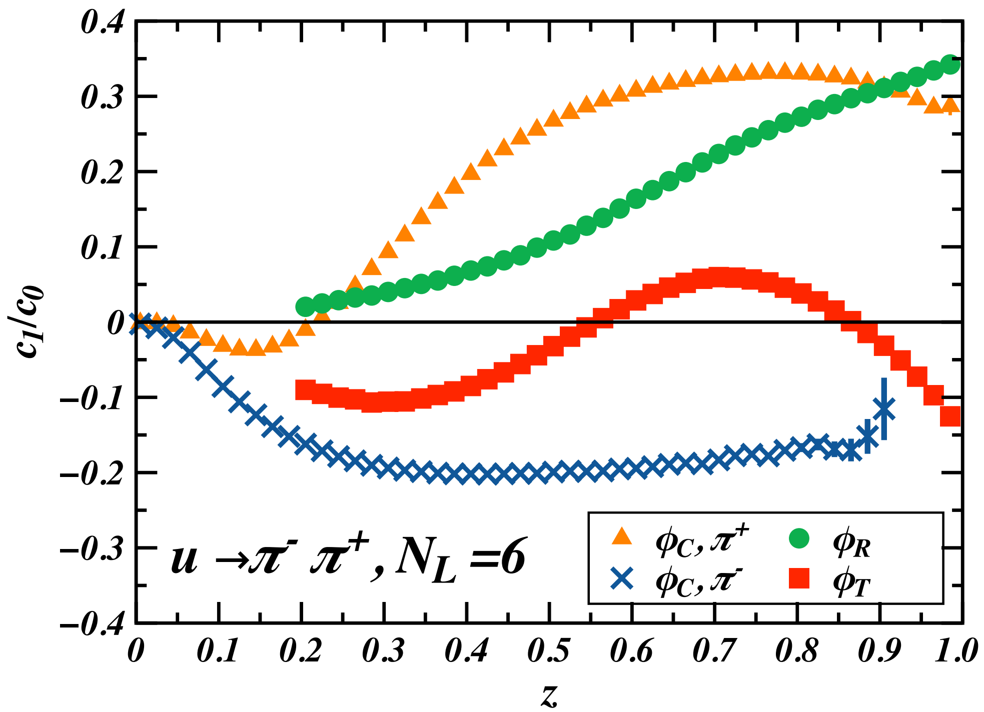


✦ ***Destructive interference with increasing N_L!***

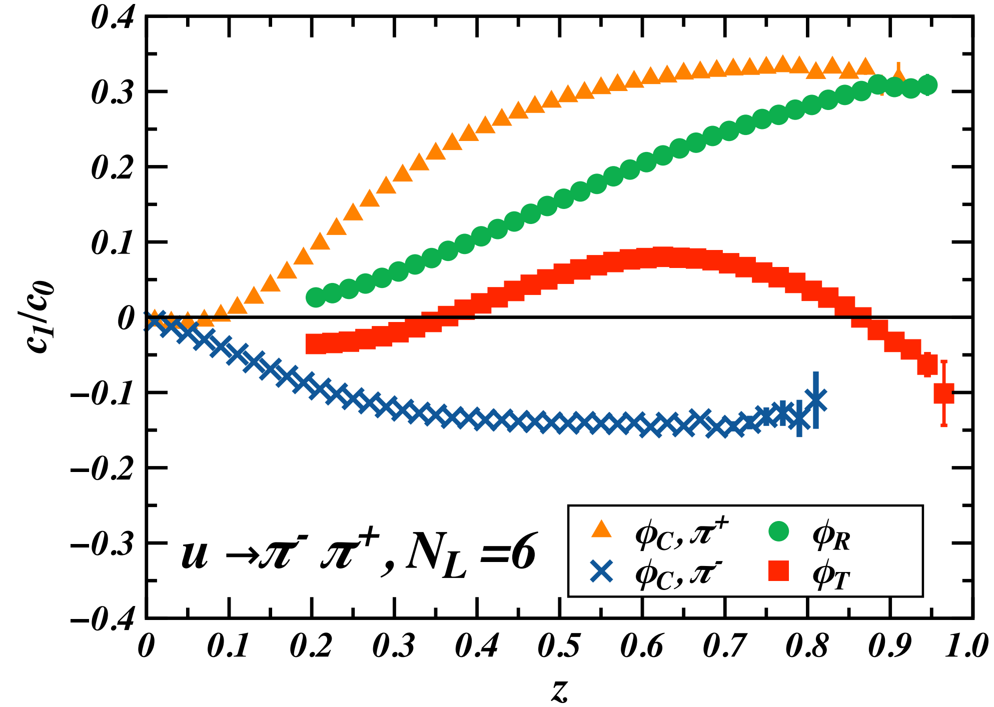
Collins and IFF

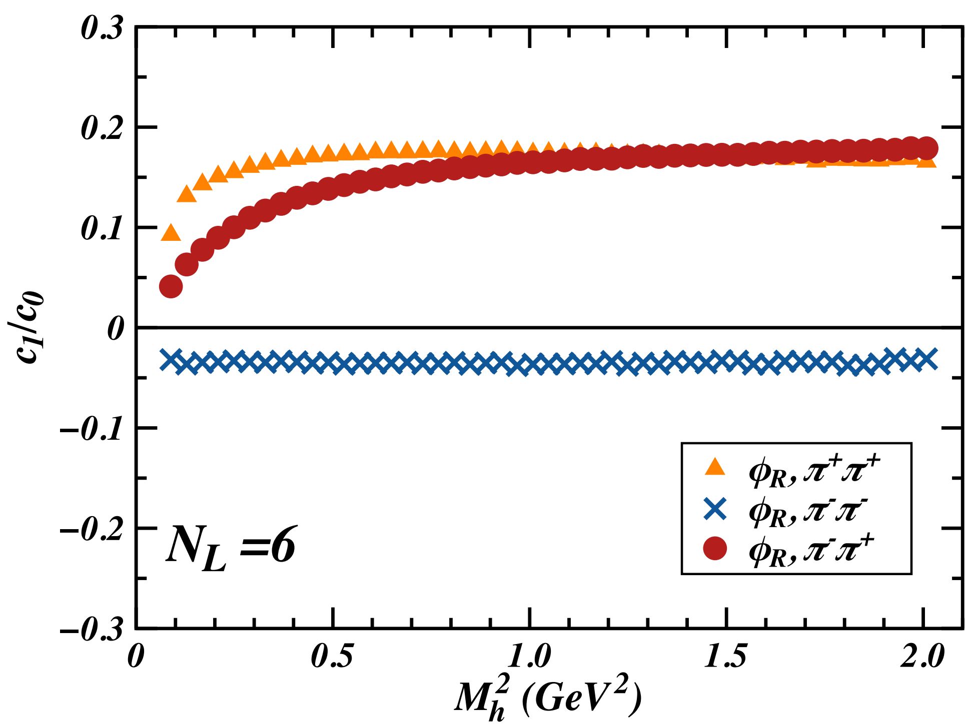
✦ Comparing the analysing powers for Collins effect and IFFs.

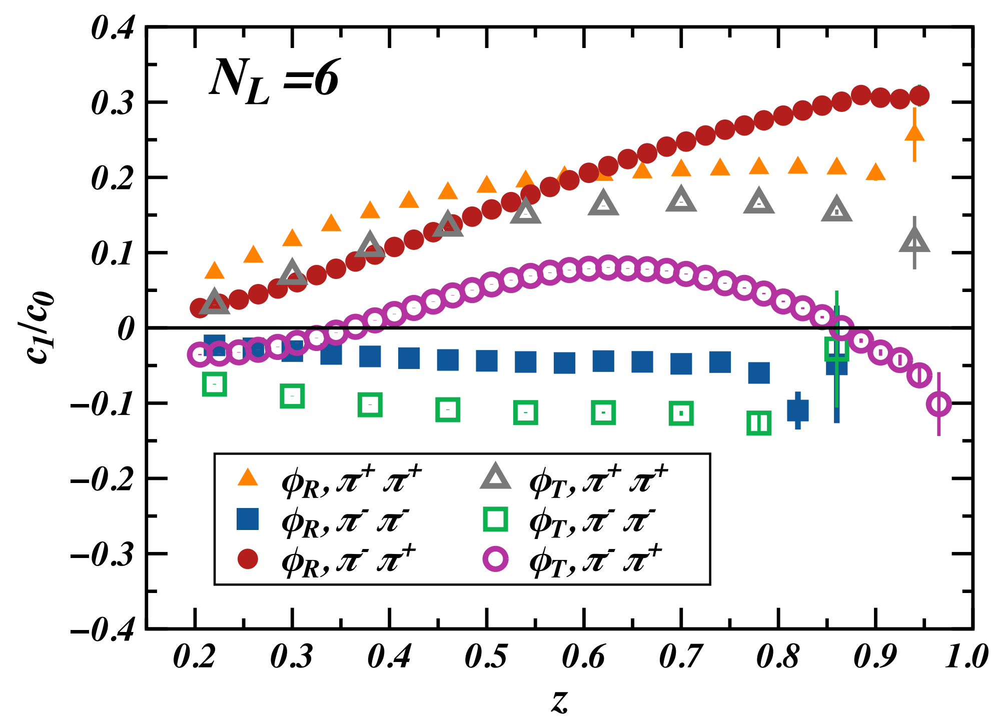
▸ *NJL* Model



▸ Evolution-mimicking *Ansatz.*



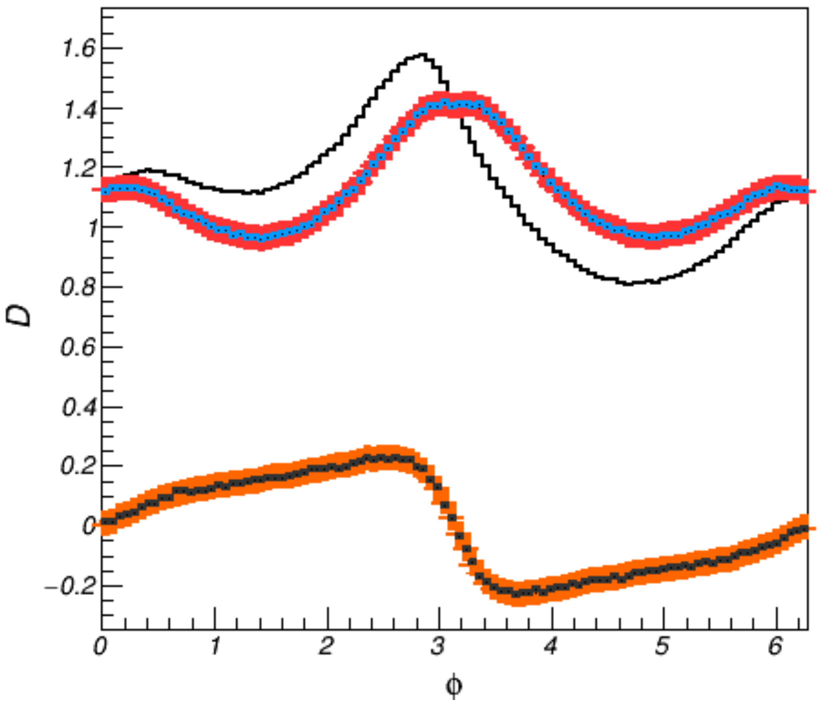




Longitudinal Polarisation in DiHadron FFs

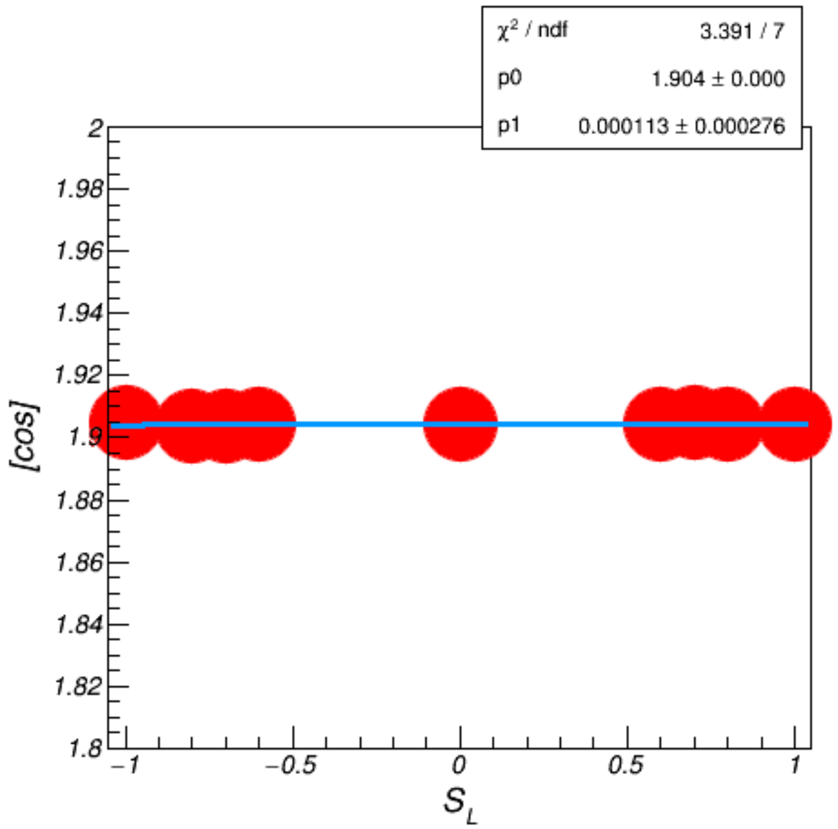
Longitudinal Spin

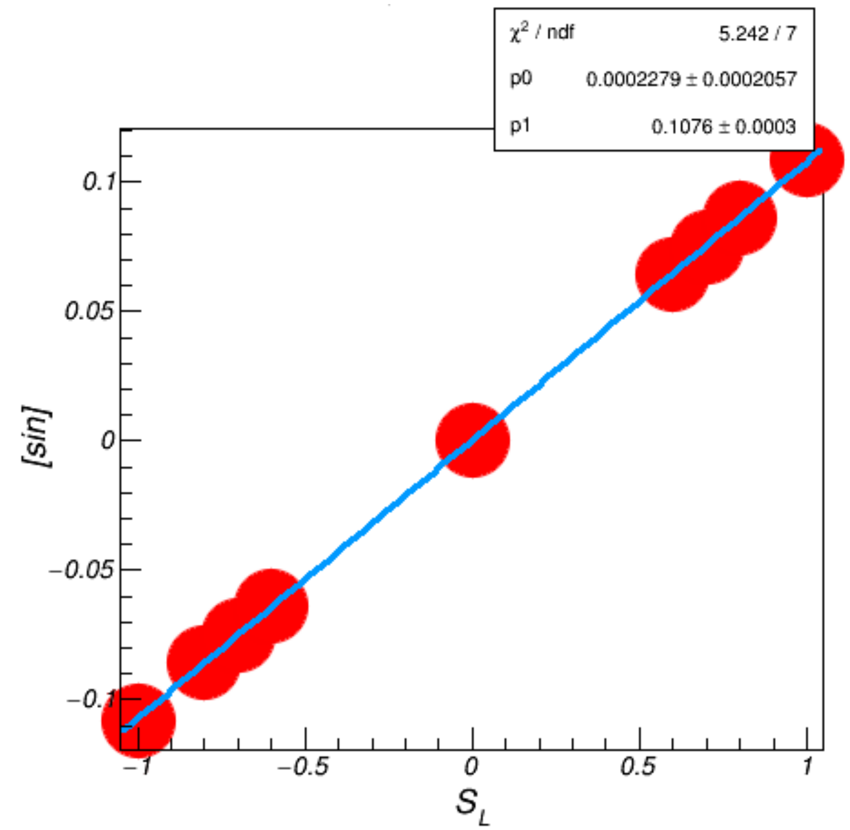
✦ **FF for longitudinally polarized quark:** $(\mathbf{R} \times \mathbf{T}) \cdot \mathbf{s}_L$



$$D_{q^{\rightarrow}}^{h_1 h_2}(\varphi_{R-T}) = D_q^{h_1 h_2}[cos(\varphi_{R-T})] + s_L \sin(\varphi_{R-T})\mathcal{G}[cos(\varphi_{R-T})]$$
$$\varphi_{R-T} \equiv \varphi_R - \varphi_T$$

✦ **Proof of linear dependence on s$_L$: 9 values of** $(s_L, \mathbf{s}_T)$ **for** $N_L = 6$ **.**

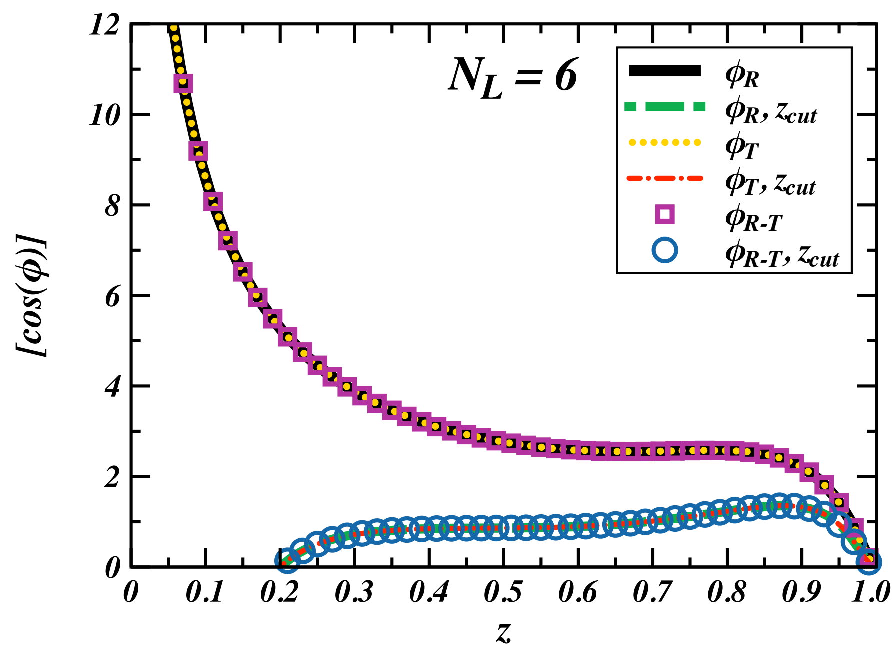


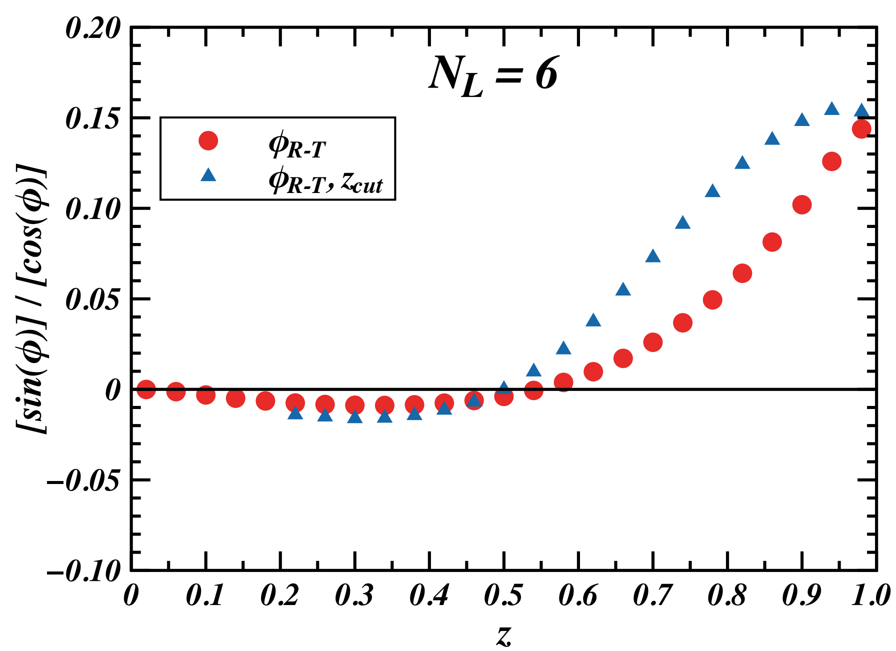


Cross-check for unpolarized DiFF

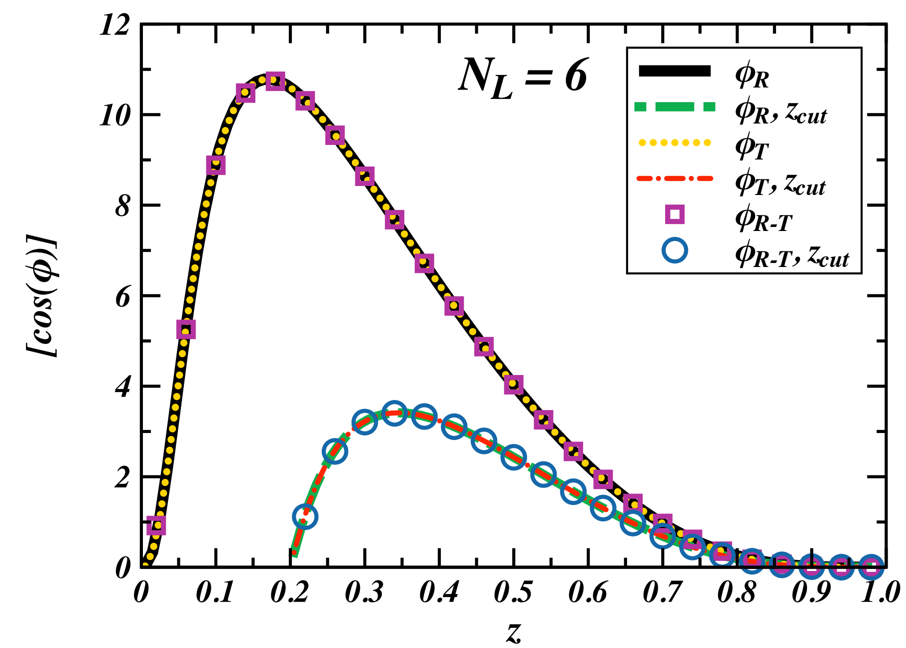
✦ **Results for unpolarized DiFF and analysing power, impose cut $z_{1,2} \geq 0.1$**

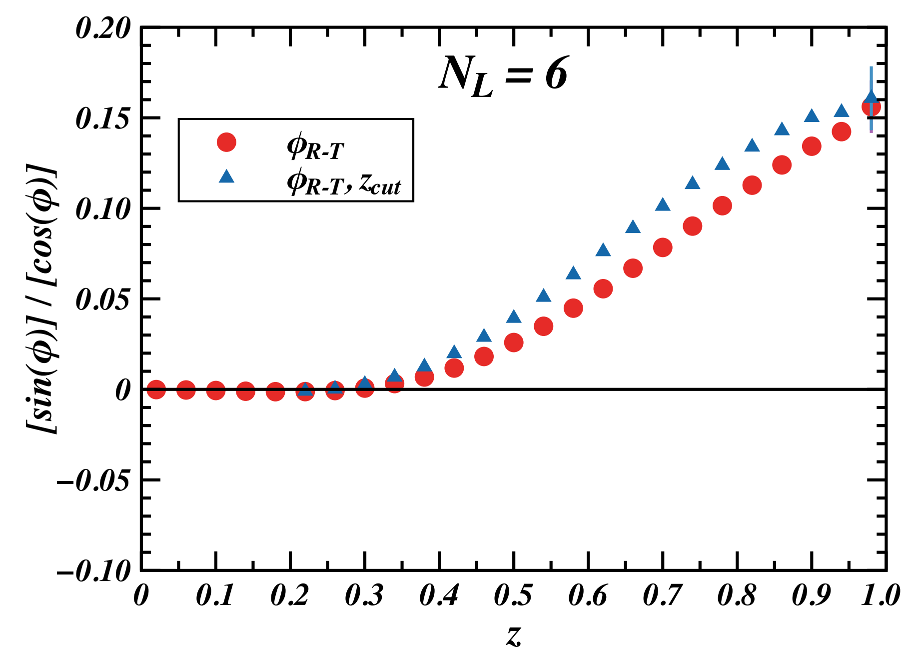
‣ ***NJL* Model**





‣ **Evolution-mimicking *Ansatz.***



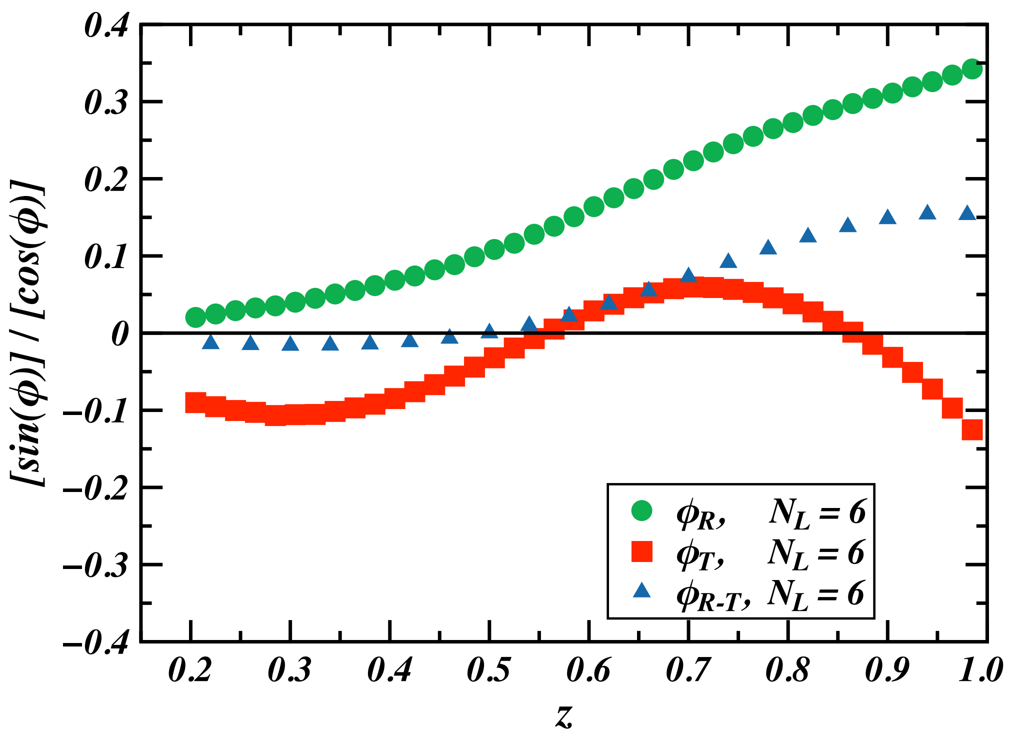


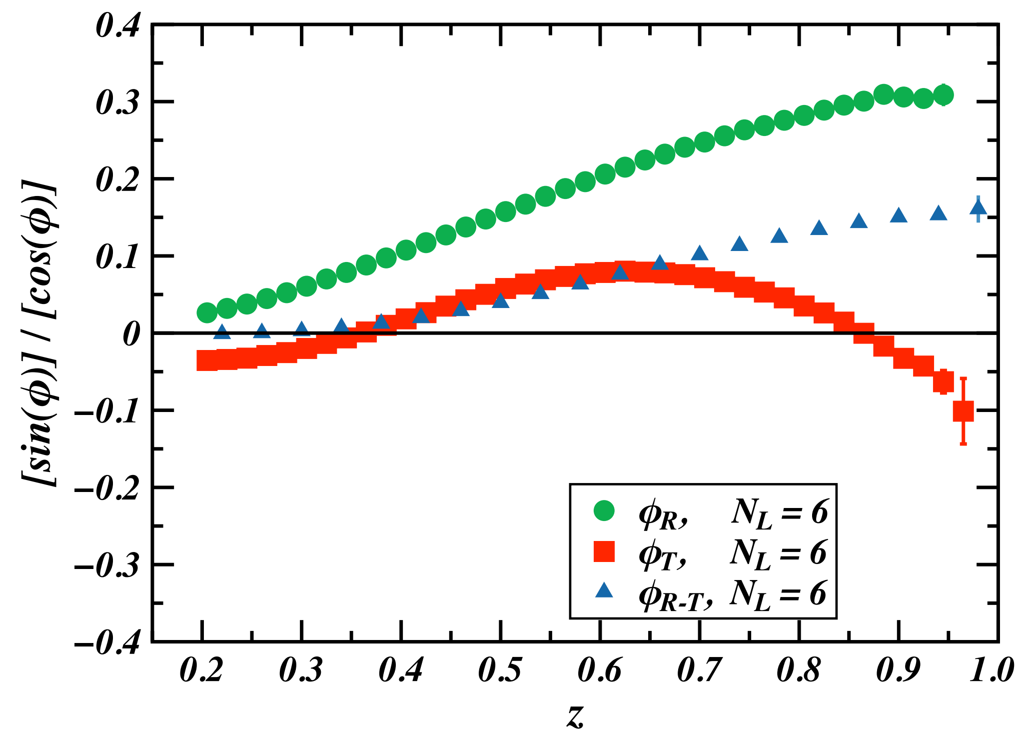
✦ $z_{1,2} \geq 0.1$ ***cut enhances the analysing power at high-z for larger N_L!***

Analysing Power for Longitudinal Spin

✦ **Comparing the analysing power for Collins effect and IFFs.**

- *NJL* **Model**
- **Evolution-mimicking** *Ansatz.*





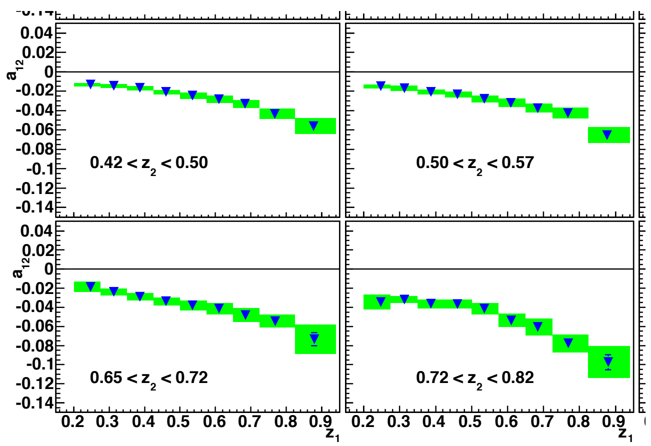
✦ ***Might explain BELLE results.***

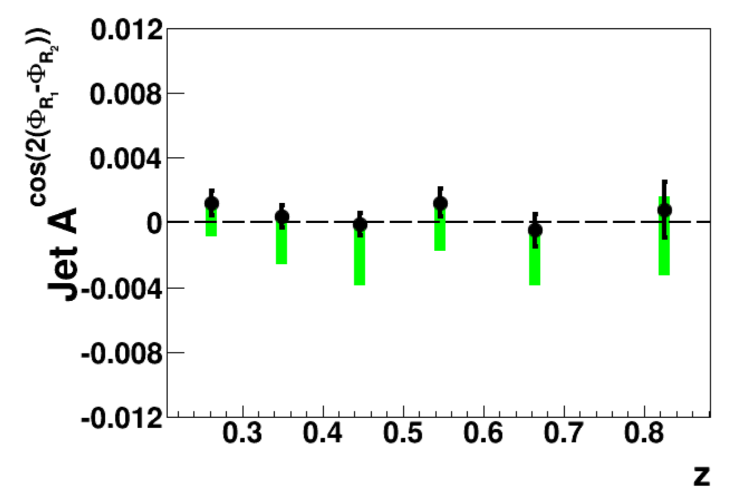
Phys.Rev.Lett. 107 (2011) 072004

PoS DIS2015 (2015) 216

$$\sim H_q^{\sphericalangle}(z_1, m_1^2) H_{\bar{q}}^{\sphericalangle}(z_2, m_2^2)$$

$$\sim G_q^{\perp}(z_1, m_1^2) G_{\bar{q}}^{\perp}(z_2, m_2^2)$$





FUTURE PLANS

THE EFFECT OF VECTOR MESONS (VM)

- A naive assumption: VMs should have modest contribution due to relatively small production probability $P(\pi^+)/P(\rho^+) \approx 1.7$
- ***But***: Combinatorial factors enhance VM contribution significantly!
- Let's consider only two hadron emission

Direct: $$u \to d + \pi^+ \to u + \pi^- + \pi^+$$

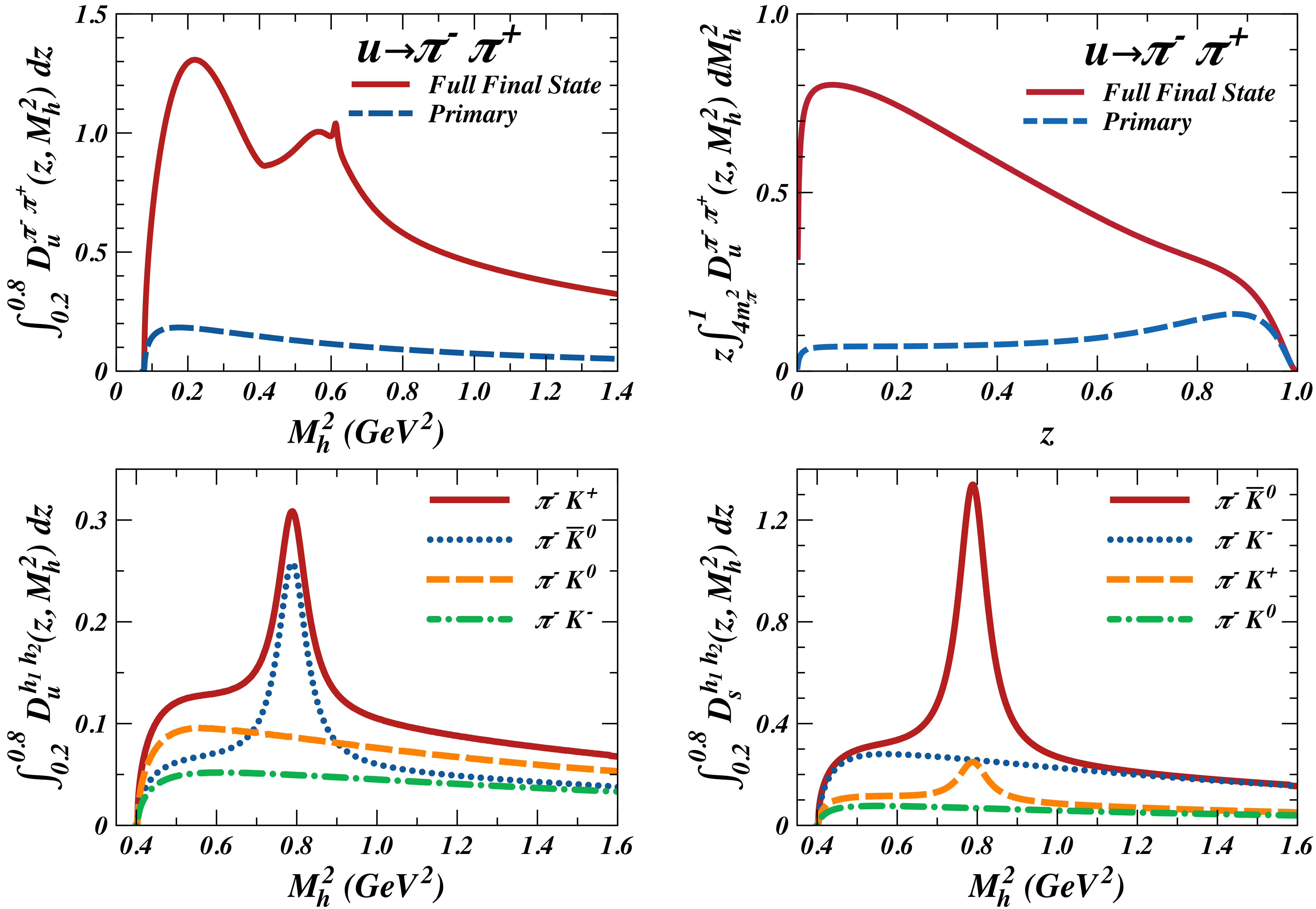
VM: $$u \to d + \pi^+ \to u + \rho^- + \pi^+$$
$$\rho^- \to \pi^- \pi^0$$

...

$$u \to u + \rho^0 \to u + \rho^0 + \rho^0$$
$$\rho^0 \to \pi^+ \pi^-, \quad \rho^0 \to \pi^+ \pi^-$$

$$P_{Dir}(\pi^+\pi^-)/P_{VM}(\pi^+\pi^-) \approx \frac{1}{4}$$

Effect of Vector Mesons on Unpol. DiFFs



Conclusions

- ❖ (Polarised) TMD FFs provide a wealth of information about the *spin-spin* and *spin-momentum* correlations in hadronisation.
- ❖ Hadronization Models are needed to calculate polarised FFs and study various correlations (Collins and IFF, etc).
- ❖ Polarised hadronisation in *MC generators: support for future experiments* to map the 3D structure of nucleon (*COMPASS, JLab12, BELLE II, EIC*).
- ❖ The *NJL-jet* model provides a robust and extendable framework for *microscopic* description of hadronization using MC: *TMD, Collins, DiHadron*.
- ❖ *All 3* Di-Hadron spin correlations from single-hadron effects in quark-jet!
- ❖ The extension of the underlying *quark-jet* mechanism to include polarisation can be incorporated into *mainstream MC frameworks*.
- ❖ Inclusion of *vector mesons* in polarized hadronization is the next step to *accurately describe di-hadron effects*.

Thanks!

BACKUP SLIDES

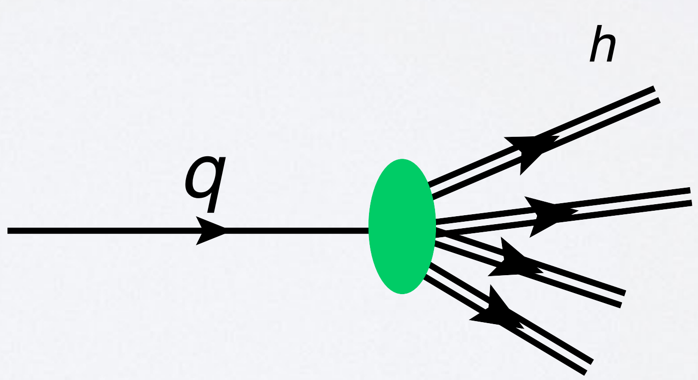
Fragmentation Functions

- The non-perturbative, universal functions encoding parton hadronization are the: ***Fragmentation Functions (FF)***.

$$\frac{1}{\sigma}\frac{d}{dz}\sigma(e^- e^+ \to hX) = \sum_i \mathcal{C}_i(z, Q^2) \otimes D_i^h(z, Q^2)$$

- **Unpolarized FF** is the **number density** for parton *i* to produce hadron *h* with LC momentum fraction **z**.

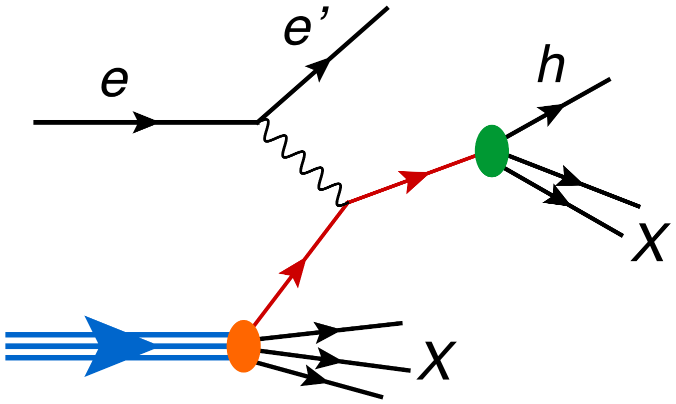
$$D_i^h(z, Q^2)$$



- **z** is the light-cone mom. fraction of the parton carried by the hadron

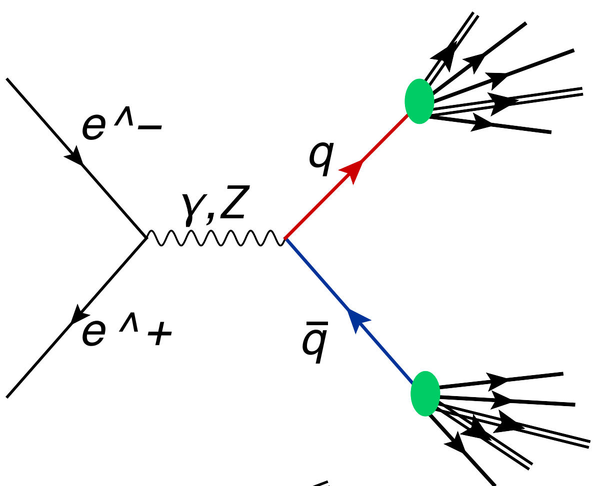
$$z = \frac{p^-}{k^-} \approx z_h = \frac{2E_h}{Q} \qquad a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$$

FACTORIZATION AND *UNIVERSALITY*



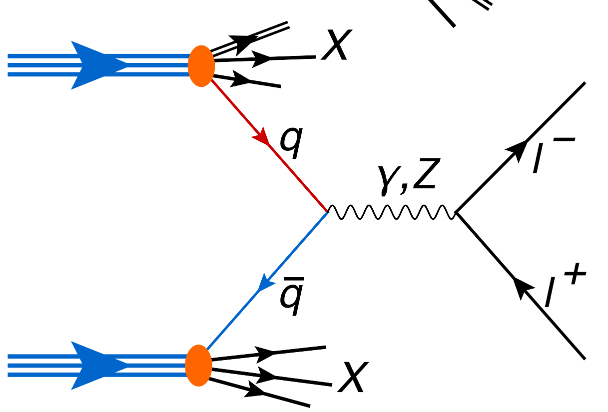
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h$$



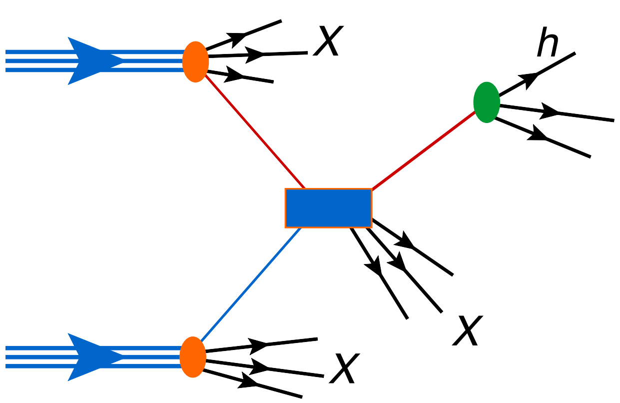
- e^+e^-

$$\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



- DRELL-YAN (DY)

$$\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}$$



- Hadron Production

$$\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h$$

3D Nucleon Structure with TMD PDFS

❖ ***TMDs: Momentum Space*** ❖ ***GPDs: Impact Parameter***

❖ ***The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon!***

❖ *Leading Order TMD PDFs*

✦ Survive after TM integration!

N/q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	h_1 $h_{1T}^{\perp}$

Boer-Mulders

Kotzinian-Mulders

Pretzelocity

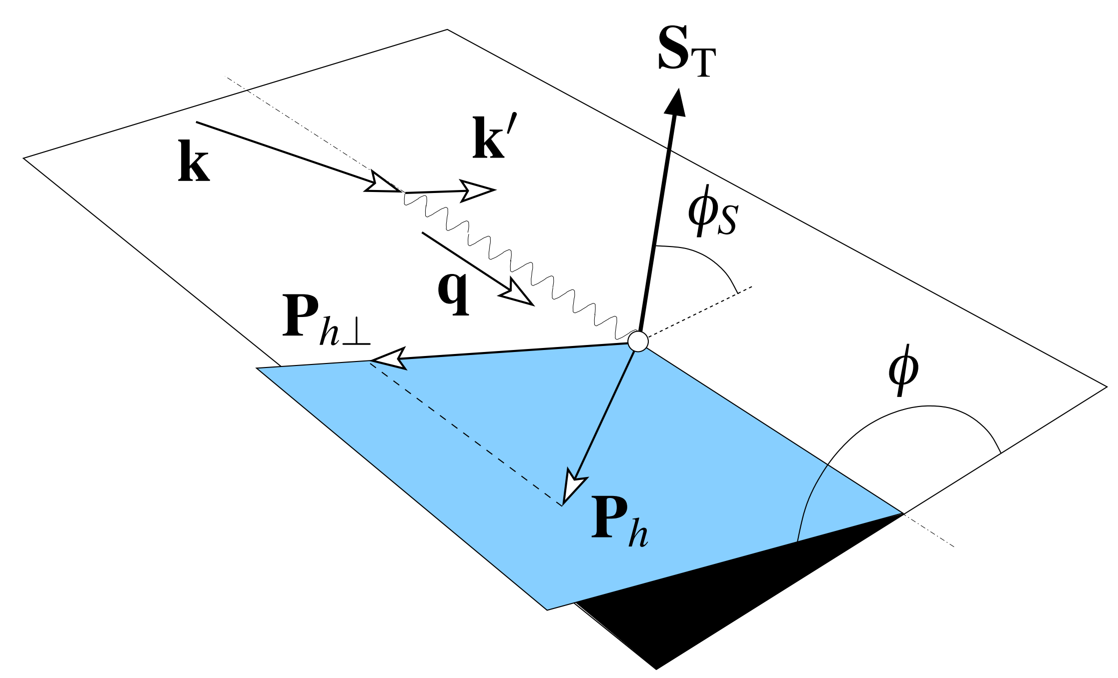
Sivers

Worm Gear

TMDs from SIDIS $e\,P \to e'\,h\,X$

A. Bacchetta et al., JHEP08 023 (2008).

- ***For polarized SIDIS cross-section there are 18 terms in leading twist expansion:***



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \ldots$$

$$+ |\boldsymbol{S}_\perp| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + .. \right]$$

Collins term

- Access the structure functions via *specific* modulations.
- LO Matching to *convolutions* of PDFs and FFs: $P_T^2 \ll Q^2$

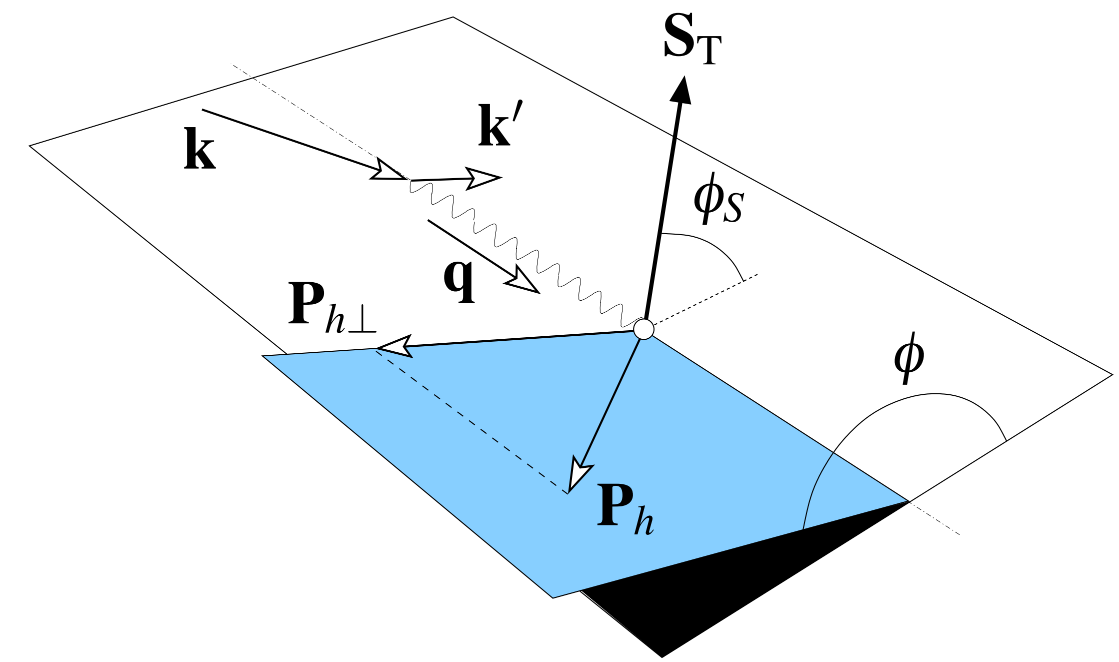
$$F_{UU,T} \sim \mathcal{C}[f_1\ D_1] \qquad F_{UT}^{\sin(\phi_h+\phi_S)} \sim \mathcal{C}[h_1\ H_1^\perp]$$

- ***NEED Collins Fragmentation Function to access Transversity PDF from SIDIS!*** [BELLE (II) , BaBar]

TMDs from SIDIS $e\,P \to e'\,h\,X$

A. Bacchetta et al., JHEP08 023 (2008).

- *For polarized SIDIS cross-section there are* ***18 terms*** *in leading twist expansion:*

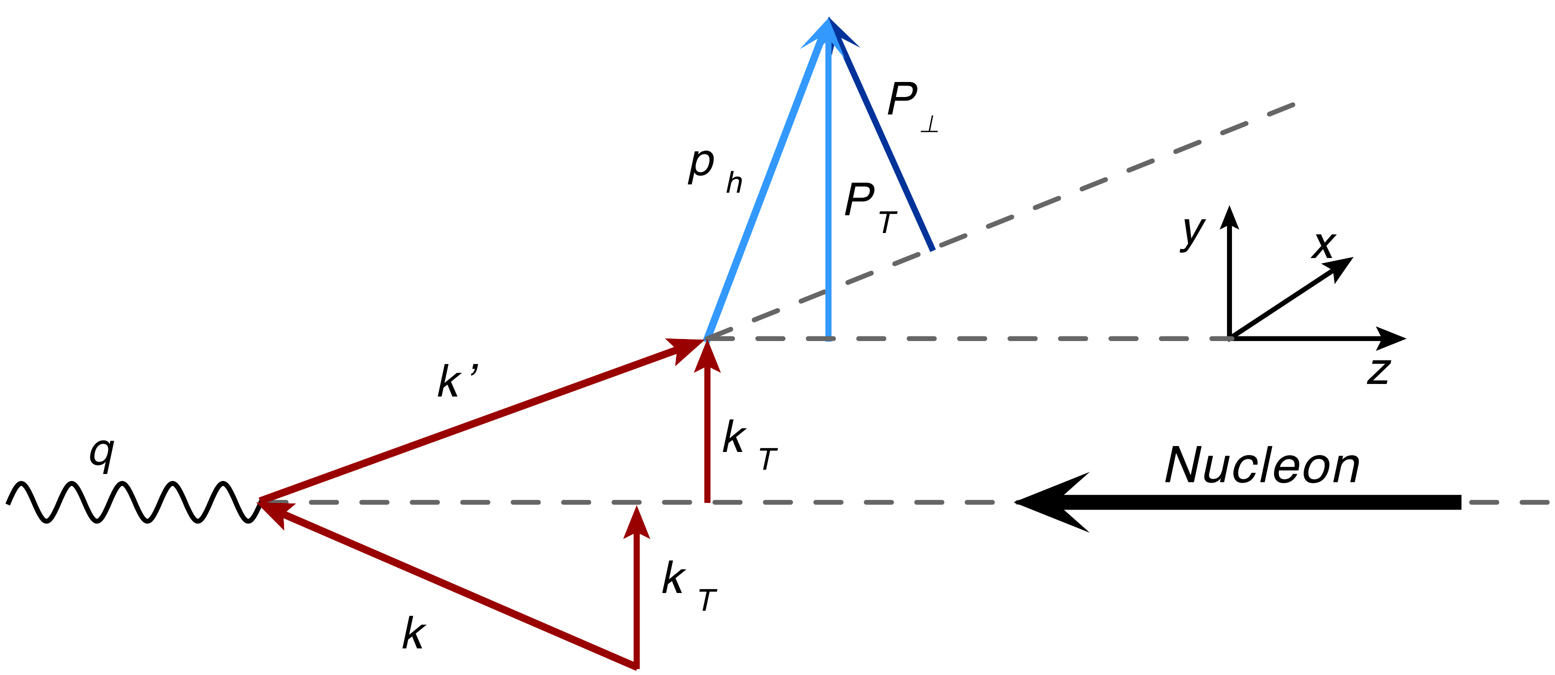


$$\mathcal{C}[f\;g\;...] \equiv \sum_q e_q^2 \int d^2\vec{k}_T\; d^2\vec{P}_\perp\; f\;g\;...\;\delta^2(\vec{P}_T - \vec{P}_\perp - z\vec{k}_T)$$

$$\int d^2\mathbf{k}_\perp f(x, k_\perp^2) = f(x)$$

$$\int d^2\mathbf{P}_\perp D(z, P_\perp^2) = D(z)$$

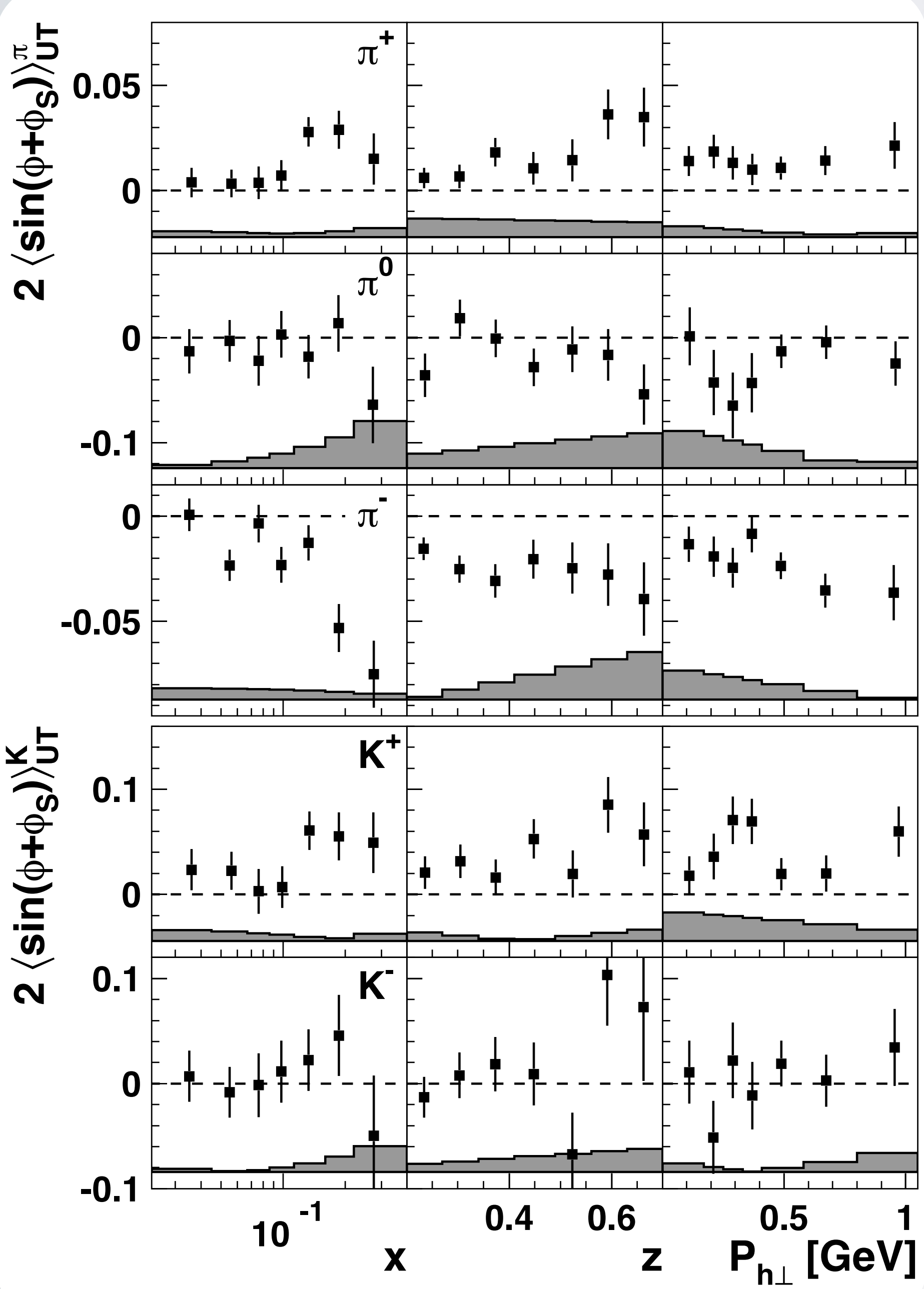
$$\vec{P}_T = \vec{P}_\perp + z\vec{k}_T$$



EMPIRICAL EXTRACTIONS OF TRANSVERSITY

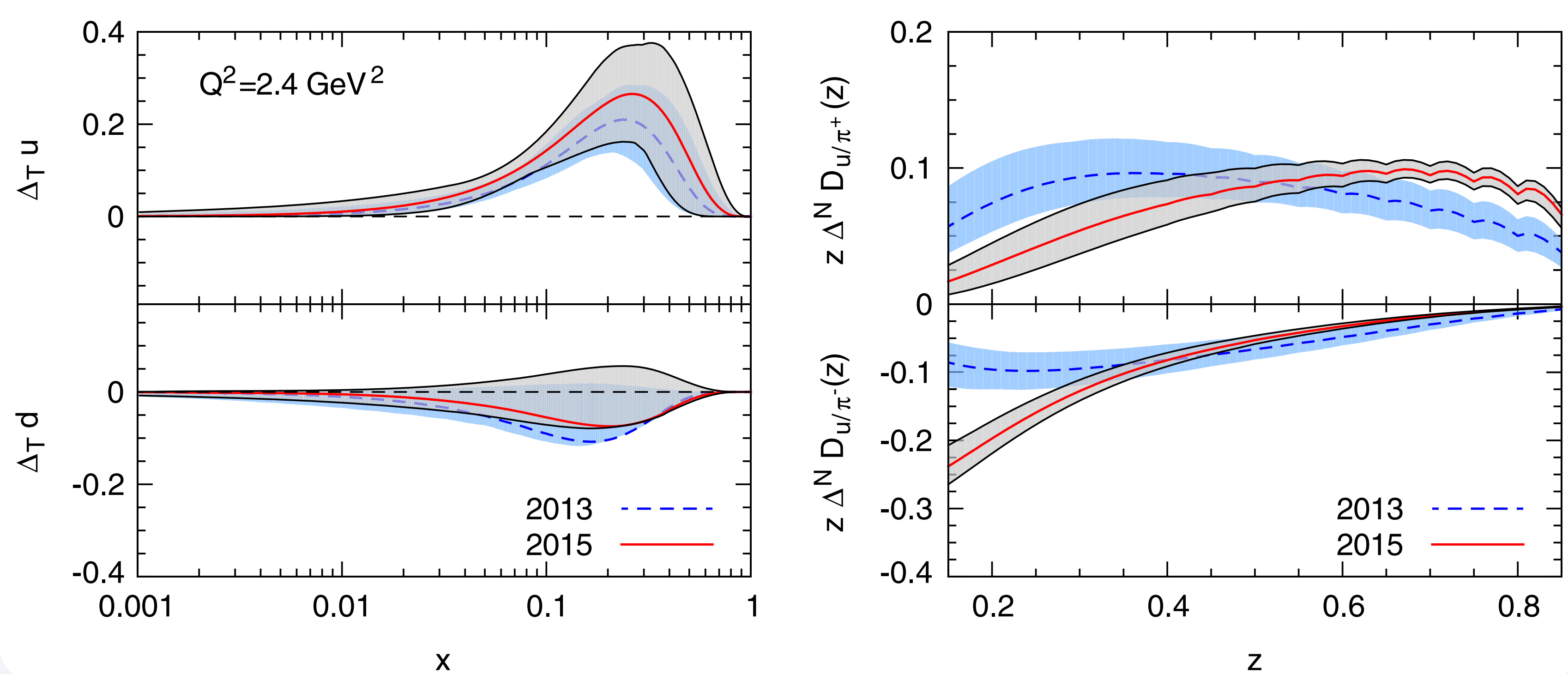
- ***SIDIS at HERMES***
  PLB693 (2010) 11-16.

$$\langle \sin(\phi + \phi_S) \rangle_{UT}^{h} \sim \frac{\mathcal{C}[h_1^q \; H_{1q}^{\perp h/q}]}{\mathcal{C}[f_1^q \; D_1^{h/q}]}$$



- Opposite sign for the charged **pions.**
- Large positive signal for K^+.
- Consistent with **0** for π^0 and K^-.

❖ ***Fits to HERMES, COMPASS and BELLE/BaBar:*** PRD 92, 114023 (2015).



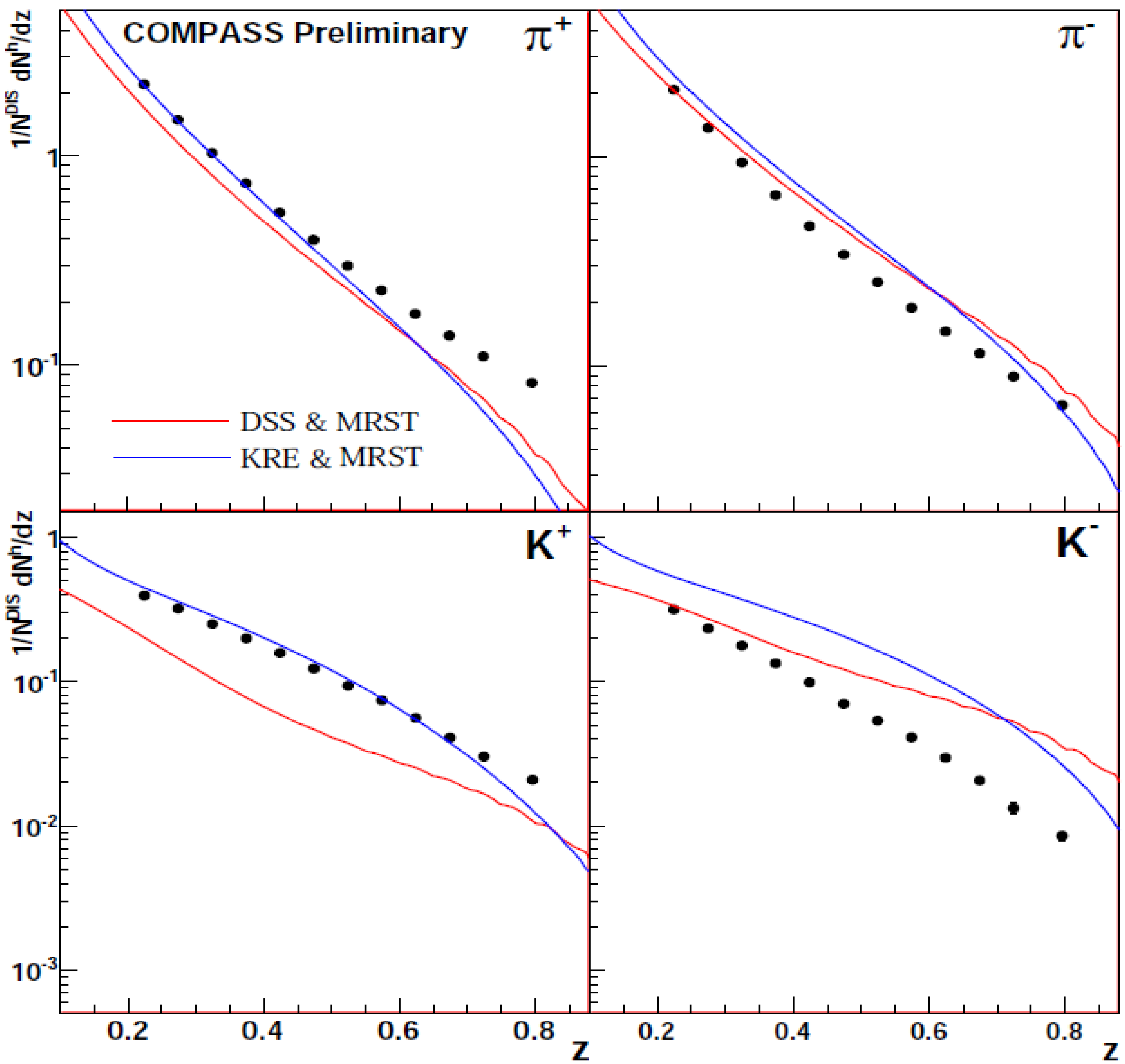
- **Still Large Uncertainties!**
- **Simplistic Approximations !**

Unfavored FFs NOT well known!

Hadron Multiplicities

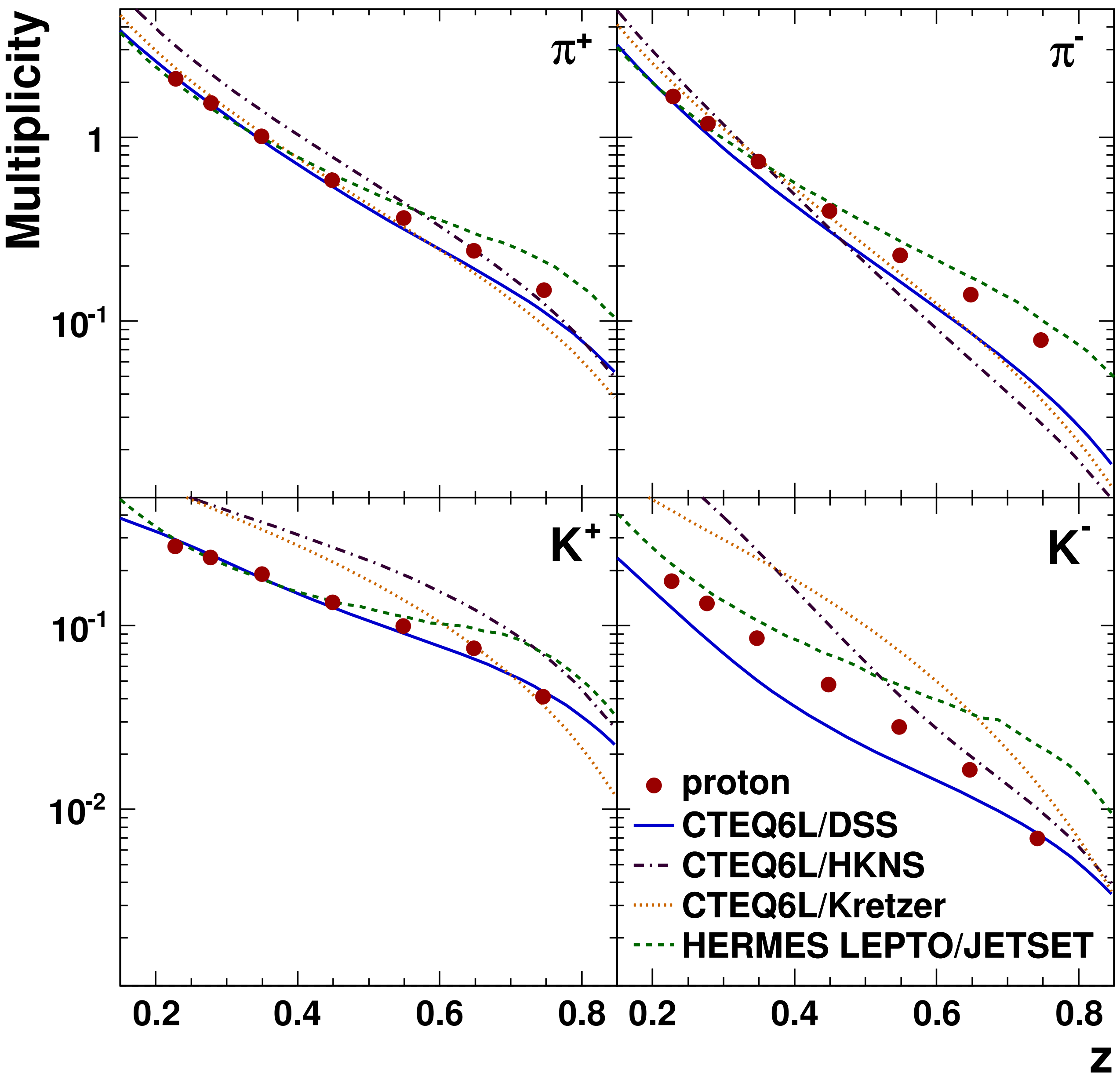
- Preliminary from COMPASS

Talk by C.Franco at CIPANP 2012.



- Also results from HERMES

Phys. Rev. D 87, 074029 (2013)

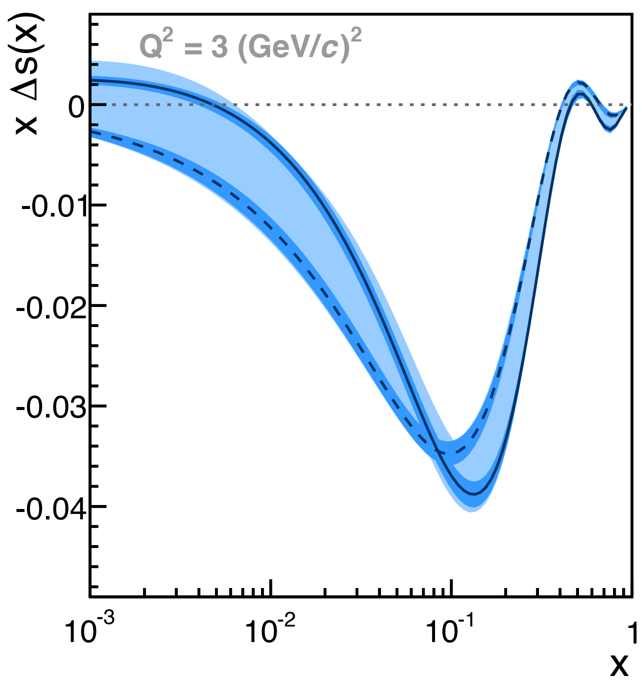


Impact of FF uncertainties on extracted PDFs

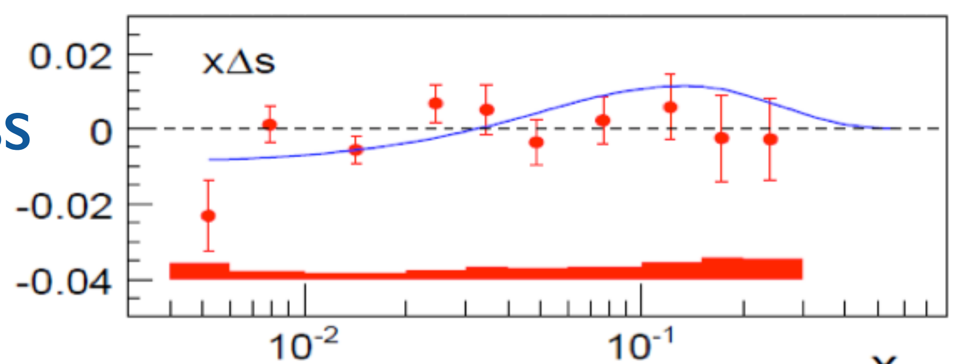
‣ Δs puzzle: DIS vs SIDIS.

Platchkov: Talk in Chile, 2016.

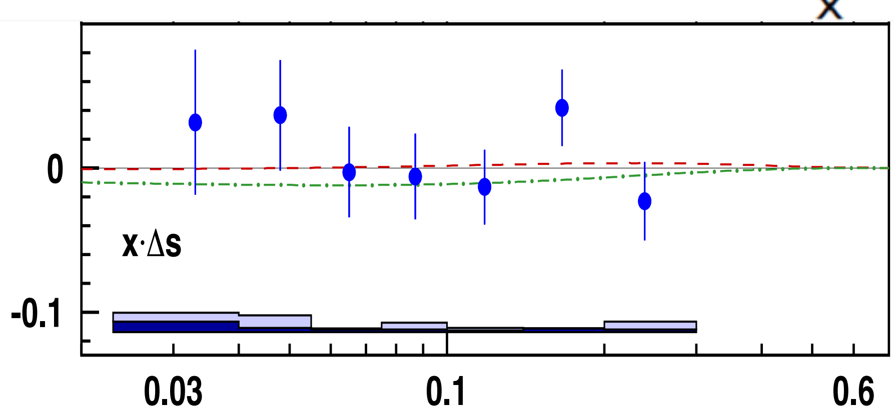
DIS COMPASS



SIDIS COMPASS



SIDIS HERMES

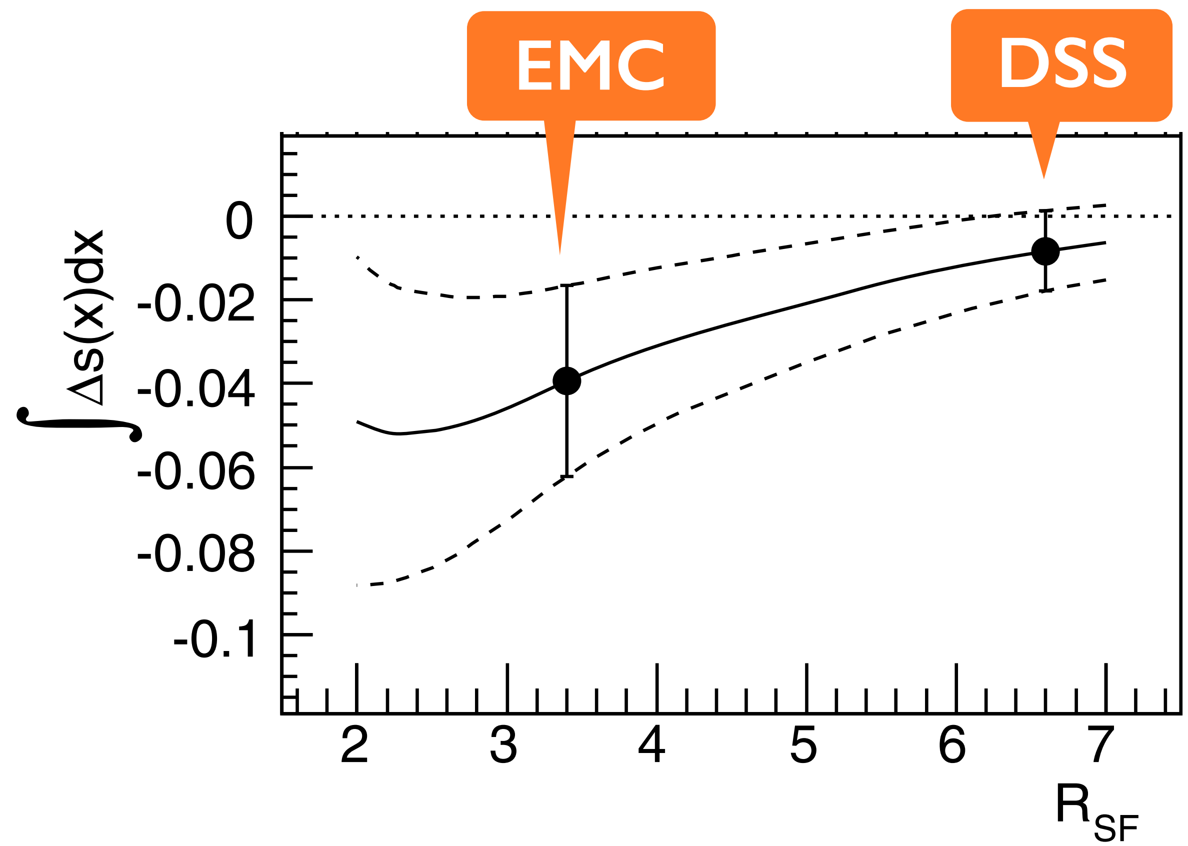


‣ Impact on extracted Δs

COMPASS: PLB 693 (2010) 227–235.

$$A_1^h(x,z) = \frac{\sum_q e_q^2(\Delta q(x) D_q^h(z) + \Delta\bar{q}(x) D_{\bar{q}}^h(z))}{\sum_q e_q^2(q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z))}.$$

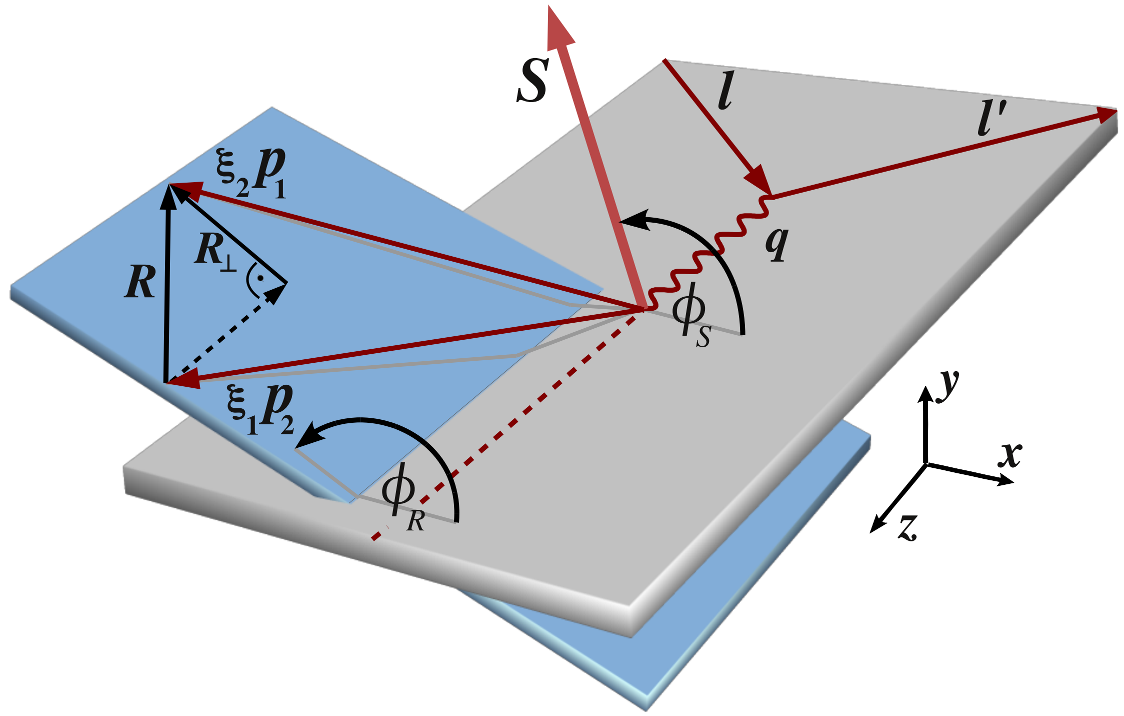
$$R_{UF} = \frac{\int D_d^{K^+}(z)\,dz}{\int D_u^{K^+}(z)\,dz}, \qquad R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z)\,dz}{\int D_u^{K^+}(z)\,dz}.$$



RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

✦ SIDIS with transversely polarized target.



✦ Collins single spin asymmetry:

$$A_{Coll} = \frac{\sum_q e_q^2\, h_1^q \otimes H_1^{\perp h/q}}{\sum_q e_q^2\, f_1^q \otimes D_1^{h/q}}$$

✦ Two hadron single spin asymmetry:

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

✦ Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2\boldsymbol{P}_1 - z_1\boldsymbol{P}_2}{z_1 + z_2}$$

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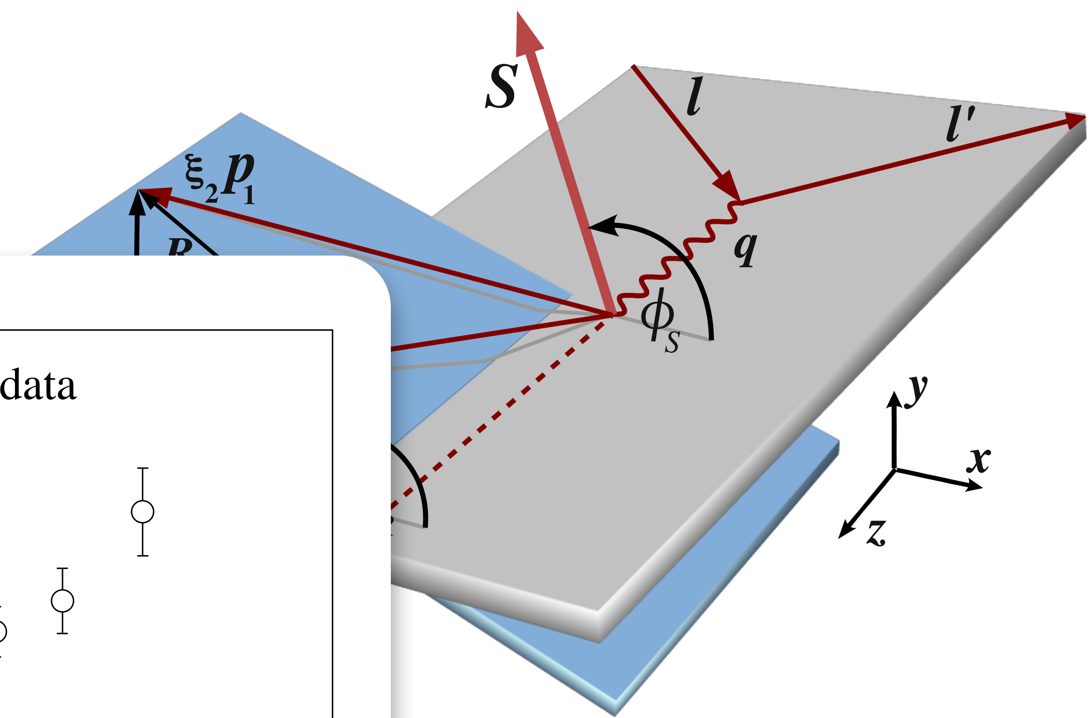
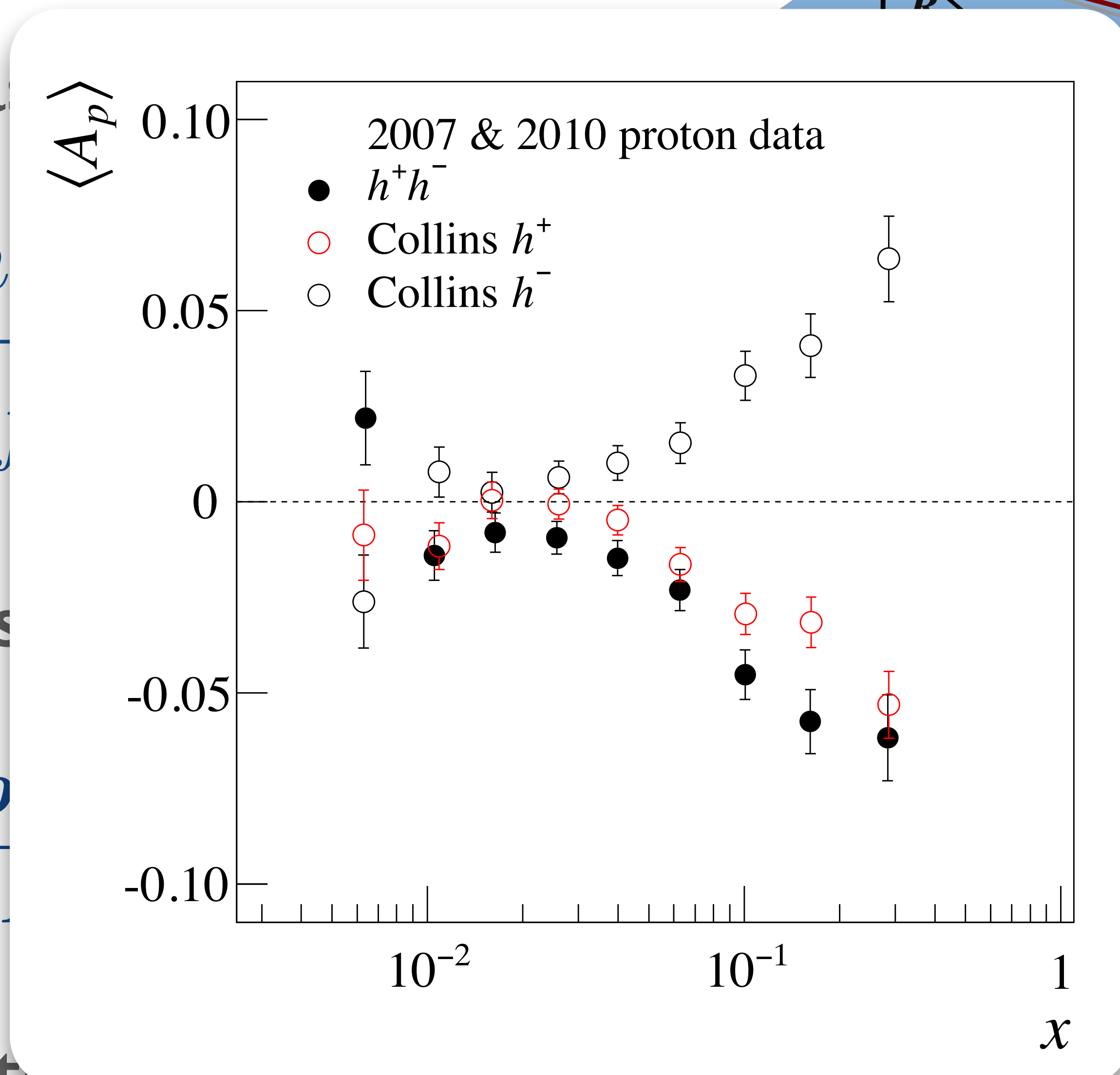


✦ Collins single spin a

$$A_{Coll} = \frac{\sum_q e_q^2\ h}{\sum_q e_q^2\ j}$$



✦ Two hadron single s

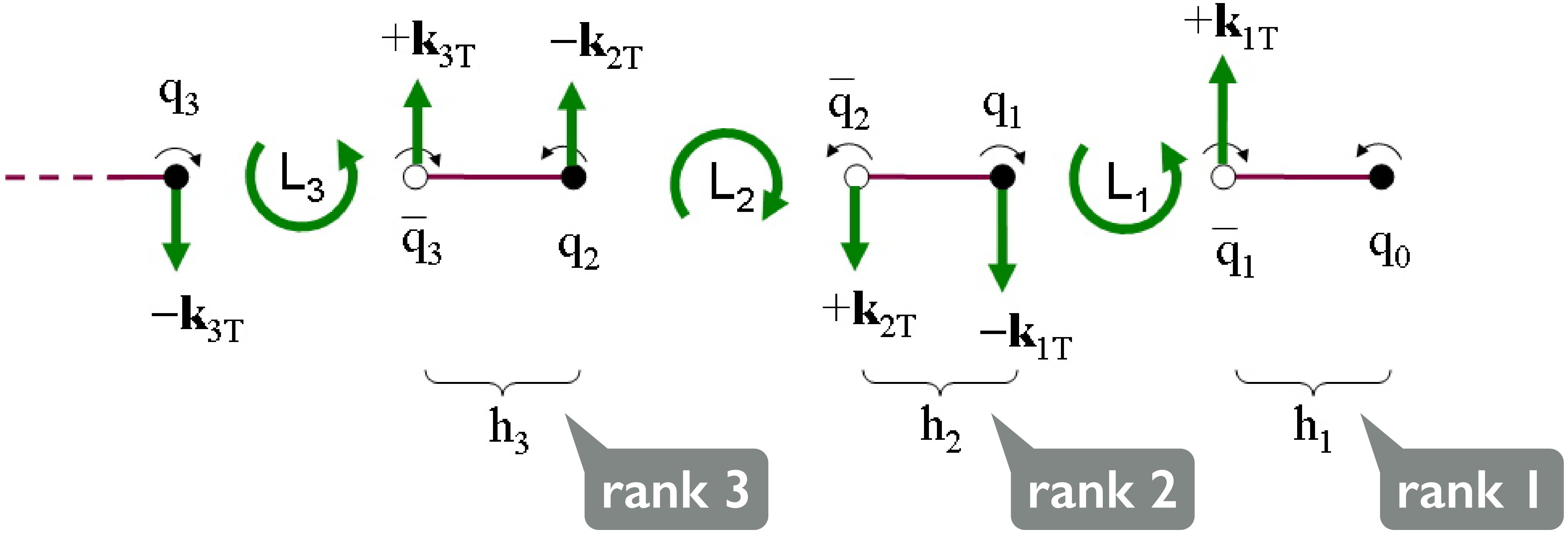
$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}\ \ldots M^2_{h^+h^-},\ \cos\theta)}{2\ \ldots M^2_{h^+h^-},\ \cos\theta)}$$

✦ Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2\boldsymbol{P}_1 - z_1\boldsymbol{P}_2}{z_1 + z_2}$$

String Model: Artru Mechanism

- $q\bar{q}$ **created in** $^3P_0$ **state.**
- **Local compensation of TM.**



- *Qualitatively implies opposite signs for favoured and unfavored. (Omitting complications from favoured production at rank 2, etc .)*

- *Simple and intuitive quantum-mechanical picture.*

SPECTATOR MODELS

E.G. - Bacchetta et al, PLB 659:234, 2008

✦ ***Use Field-theoretical definition of FFs from a Correlator.***

$$\Delta(z,k_T) = \frac{1}{2z}\int dk^+\,\Delta(k,P_h) = \frac{1}{2z}\sum_X\int\frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}e^{ik\cdot\xi}\,\langle 0|\mathcal{U}^{n_+}_{(+\infty,\xi)}\psi(\xi)|h,X\rangle\langle h,X|\bar{\psi}(0)\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\big|_{\xi^-=0}$$

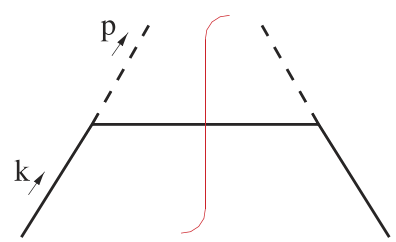
$$D_1(z,z^2\vec{k}_T^2) = \mathrm{Tr}[\Delta(z,\vec{k}_T)\gamma^-].$$

$$\frac{\epsilon_T^{ij}k_{Tj}}{M_h}H_1^\perp(z,k_T^2) = \frac{1}{2}\mathrm{Tr}[\Delta(z,k_T)i\sigma^{i-}\gamma_5]$$

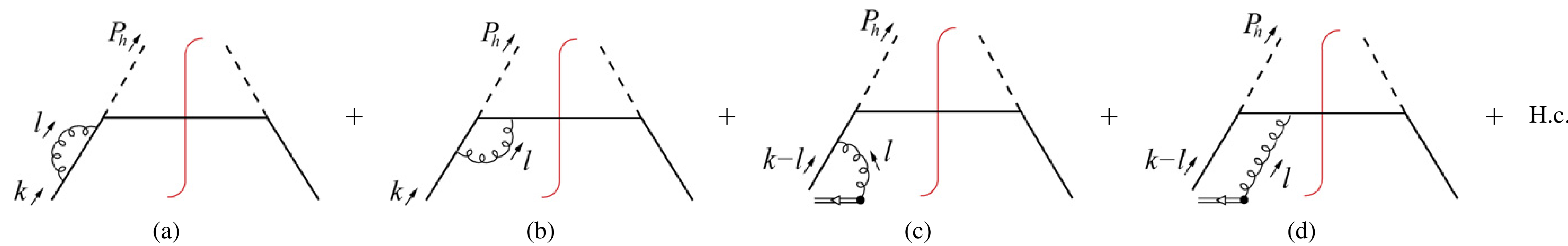
✦ ***Approximate the remnant X as a "spectator" (quark).***

✦ ***Calculate the FFs at leading-order in favourite quark model.***

$D_1(z,p_\perp^2)$



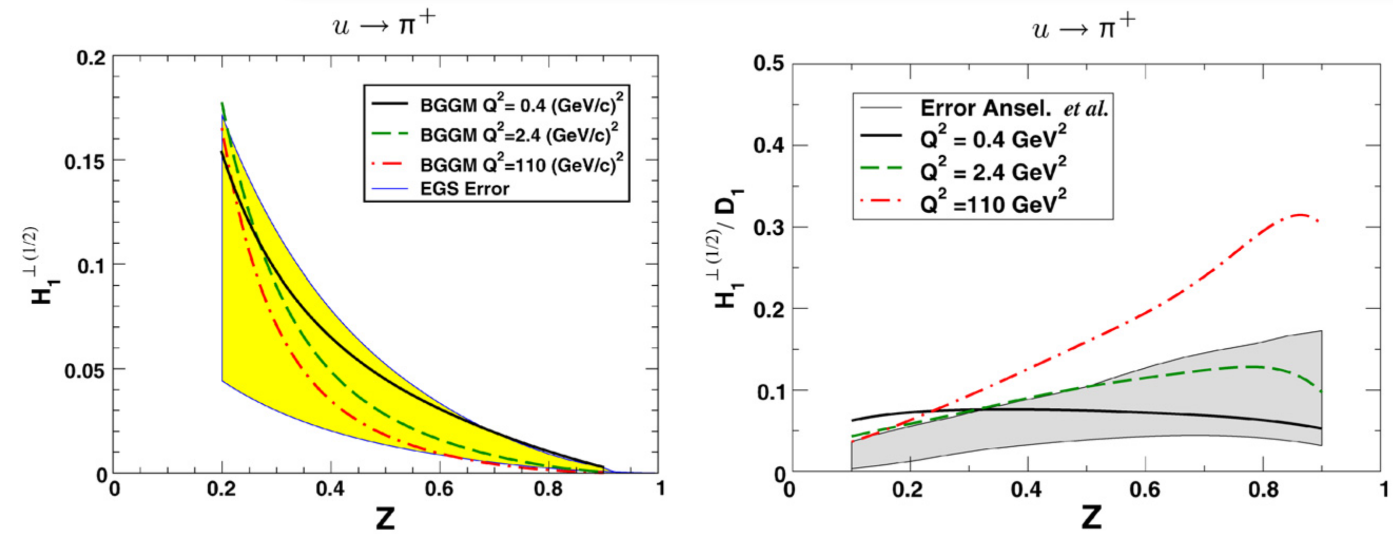
$H_1^\perp(z,p_\perp^2)$



SPECTATOR MODELS

E.G. - Bacchetta et al, PLB 659:234, 2008

✦ *Calculated Collins FF.*

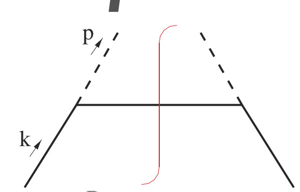


Issues with ALL the model calculations *to date*:

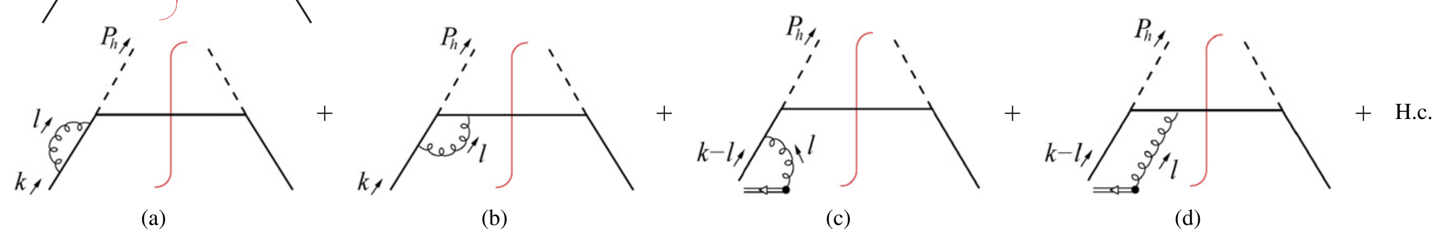
✦ *Mismatch in orders of calculations* : *VIOLATION OF POSITIVITY*

Bacchetta et al, PRL 85, 712 (2000) .

$D_1(z, p_\perp^2)$

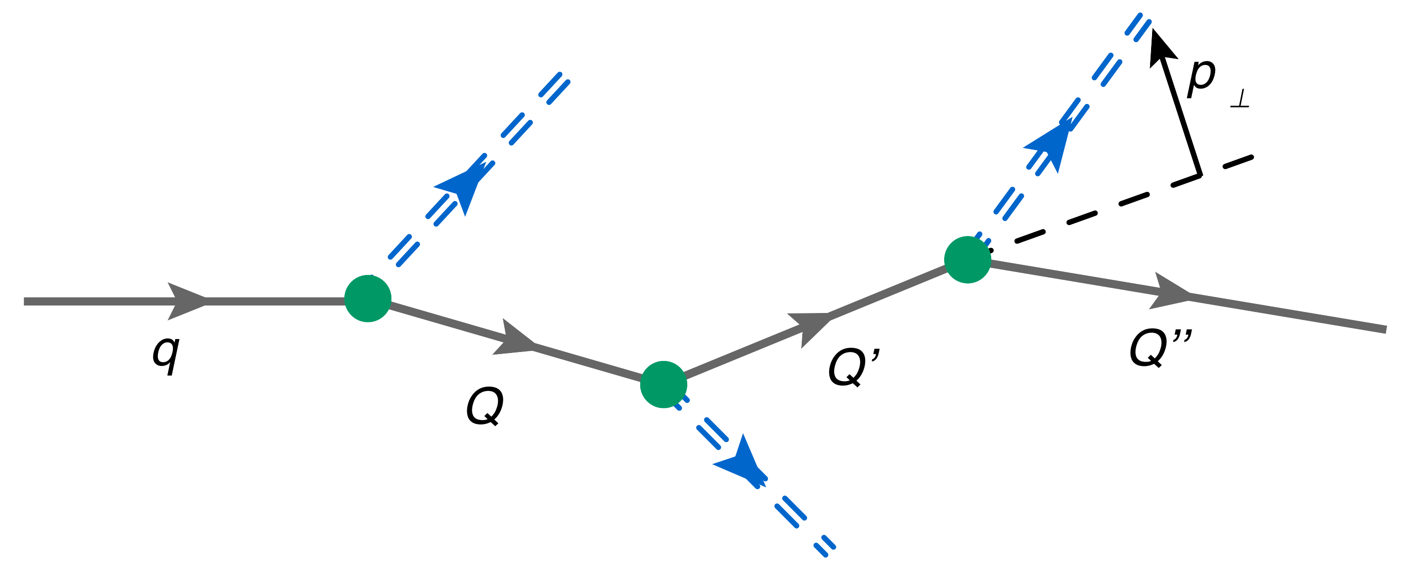


$H_1^\perp(z, p_\perp^2)$



✦ *Missing **multi-hadron** emission effect:*

- *No direct access to unfavored FFs.*
- *Description of small-z region.*



TRANSVERSE MOMENTUM DEPENDENCE

SLIDE STOLEN FROM P. SKANDS

Andersson - Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 7 (1997) 1-471

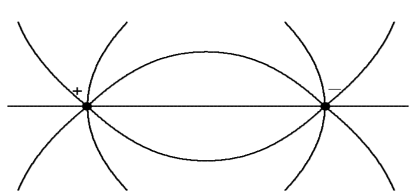
The Ultimate Limit: Wavelengths > 10^{-15} m

Quark-Antiquark Potential

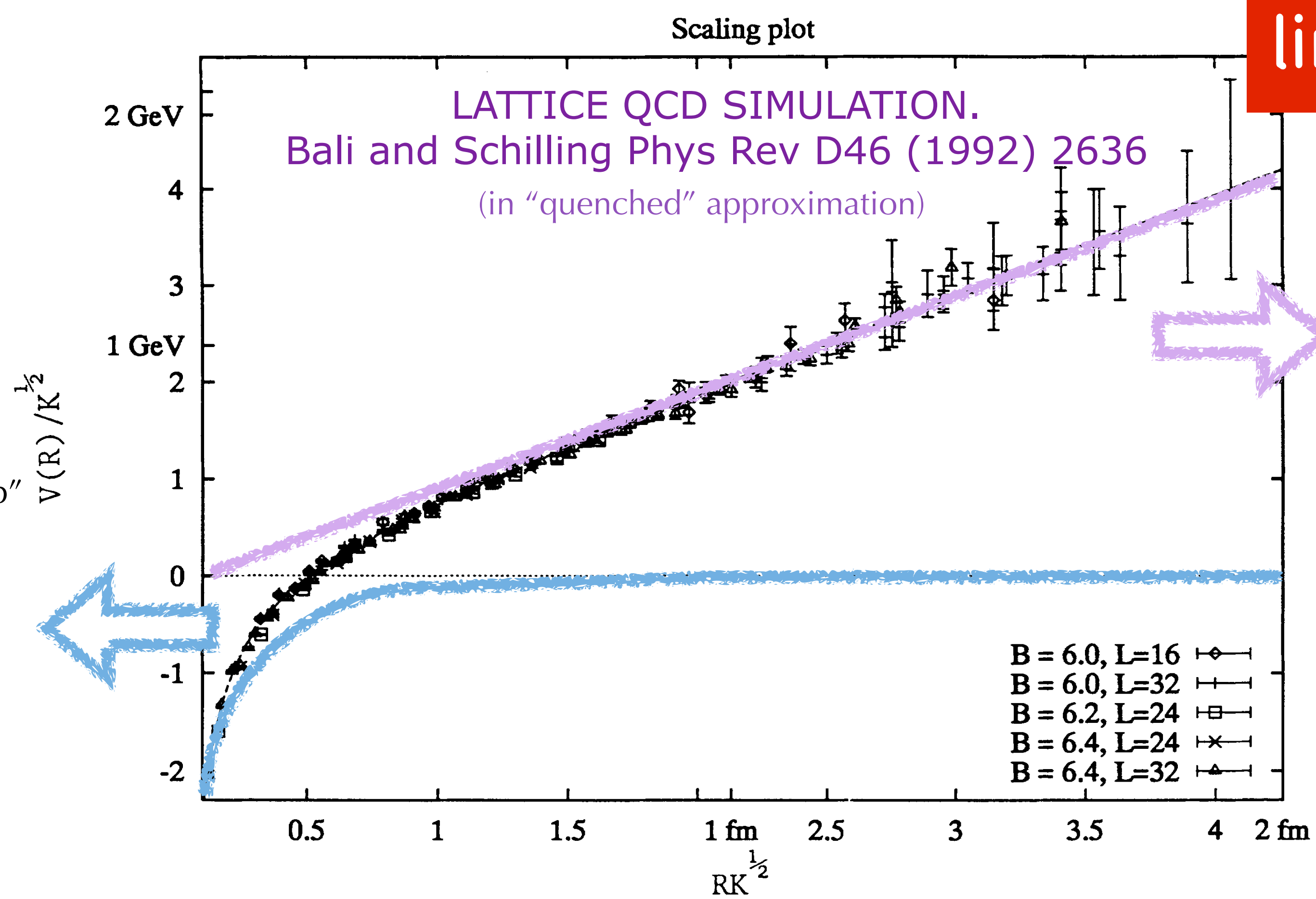
As function of separation distance

What physical system has a linear potential?

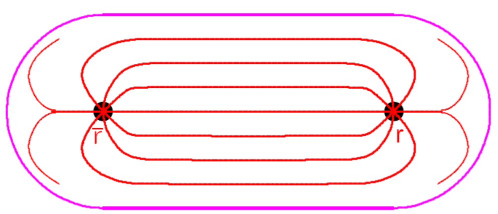
Short Distances ~ "Coulomb"



"Free" Partons



Long Distances ~ Linear Potential



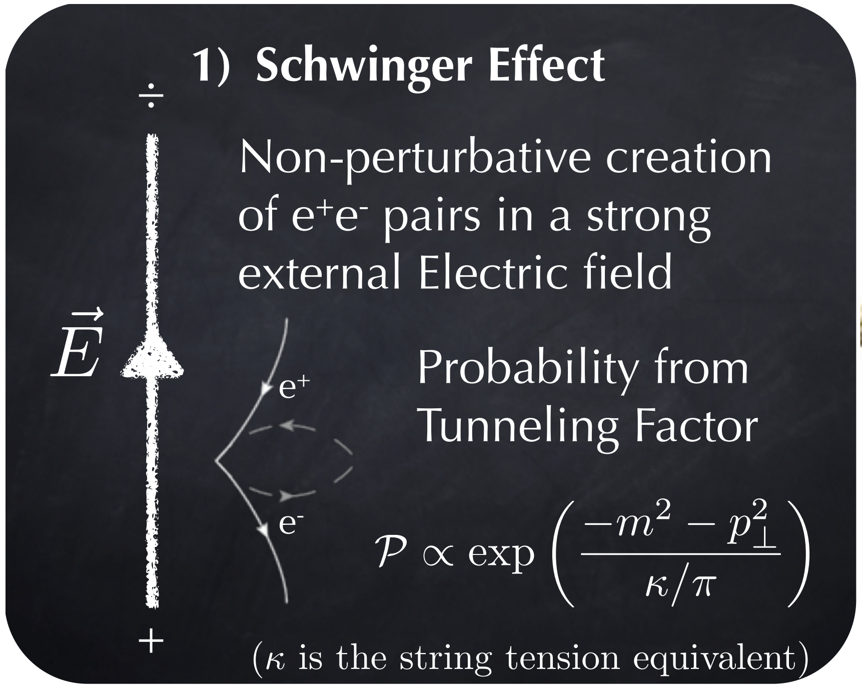
"Confined" Partons
(a.k.a. Hadrons)

$$F(r) \approx \text{const} = \kappa \approx 1\ \text{GeV/fm} \iff V(r) \approx \kappa r$$

~ Force required to lift a 16-ton truck

LUND SYMMETRIC FF

- String breaks: quark-antiquark pair creation via tunnelling in strong "chromoelectric" field.
  - ✦ Does NOT depend on the type of produced hadron!

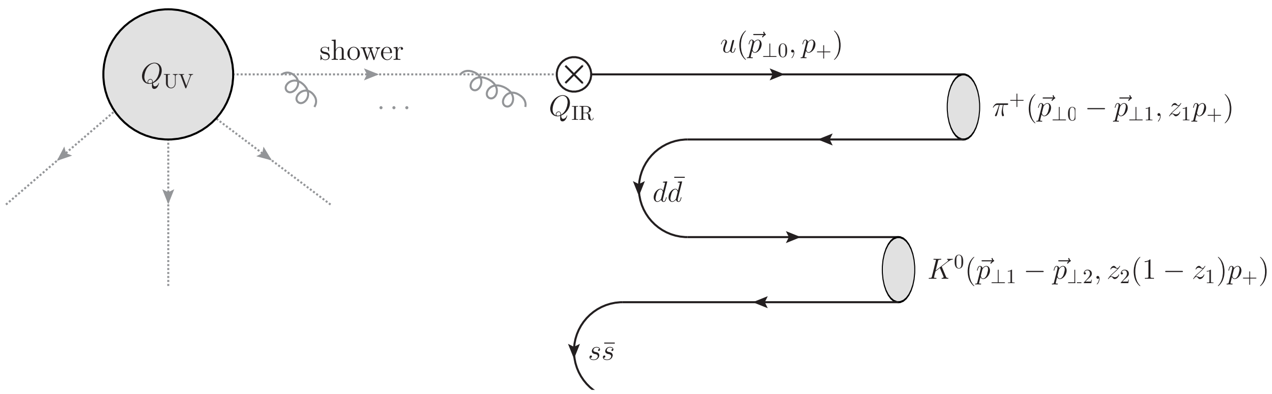


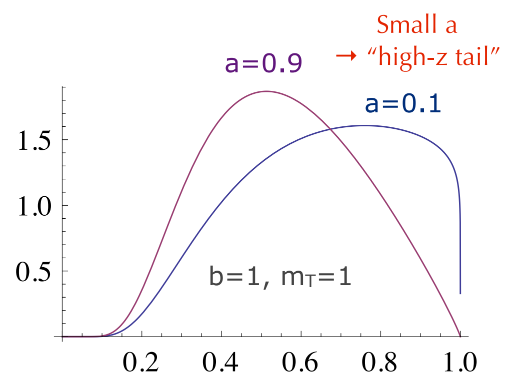
- Causality: independent breaking of the string:

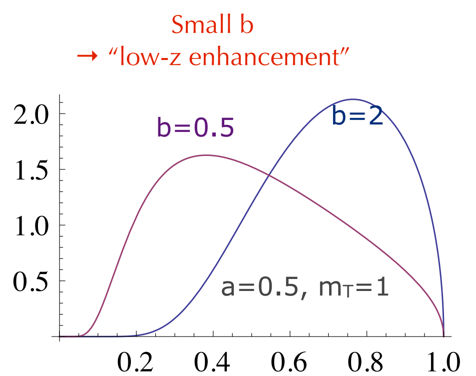
❖ Constrained form of FF

❖ May produce h in <u>*any*</u> order.

$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{b\,(m_h^2 + p_{\perp h}^2)}{z}\right)$$







Note: In principle, a can be flavour-dependent. In practice, we only distinguish between baryons and mesons

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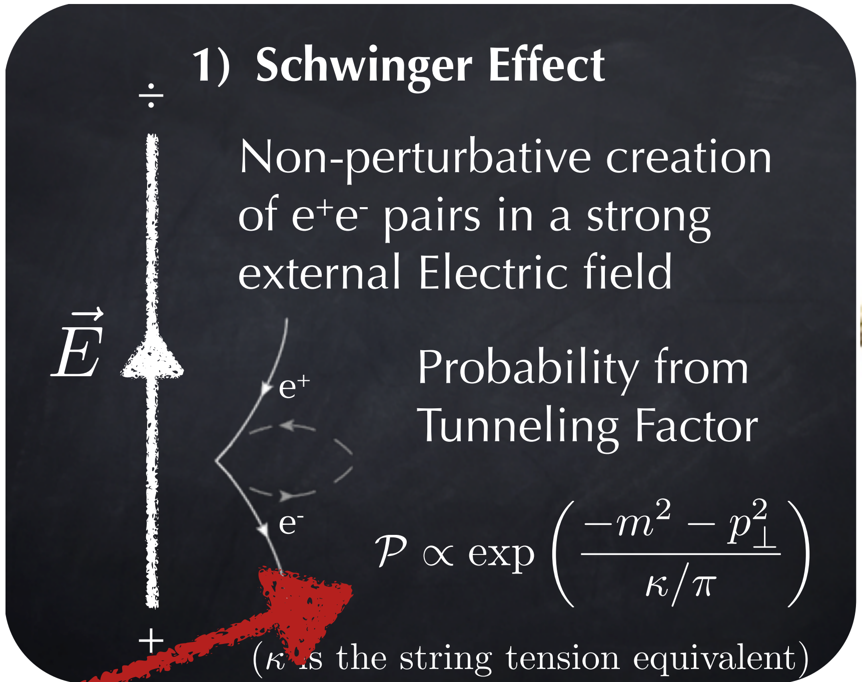

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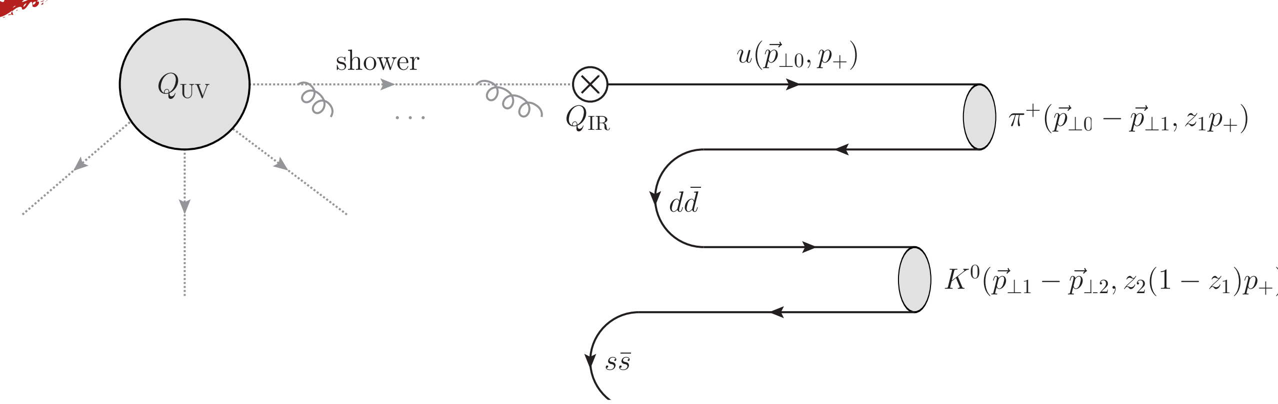
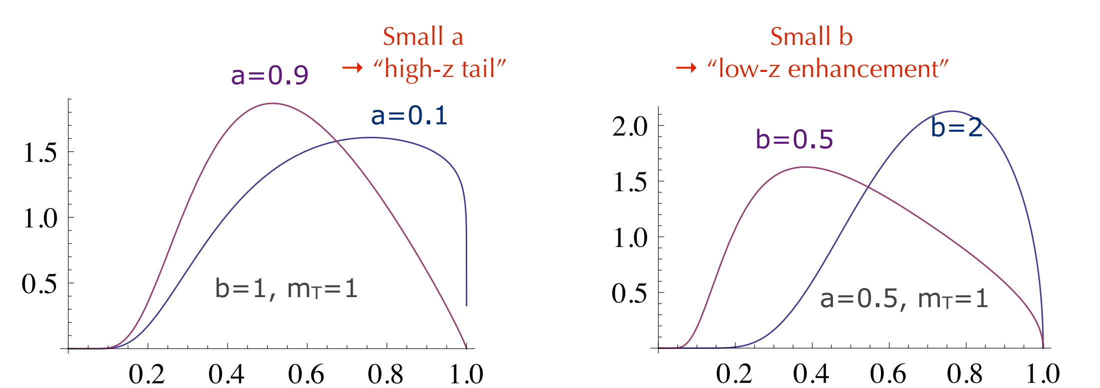


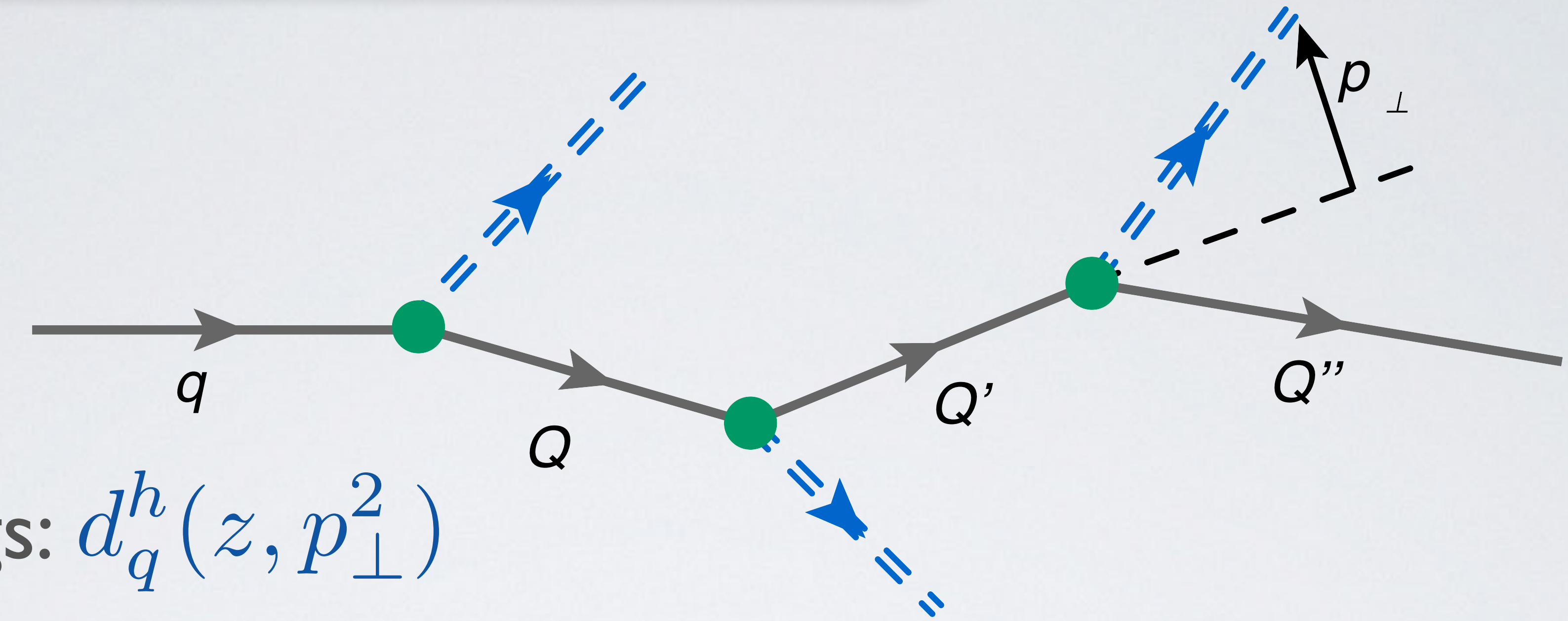


Note: In principle, a can be flavour-dependent. In practice, we only distinguish between baryons and mesons

The hadron z *depends* on combined TM of antiquark and a quark from previous string break!

TM FFS IN QUARK-JET

H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

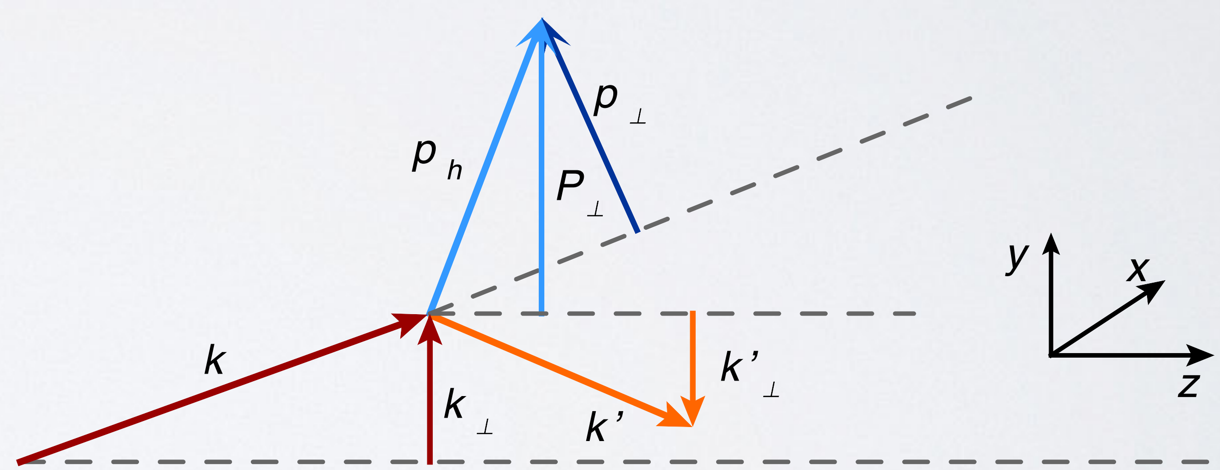


- TMD splittings: $d_q^h(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



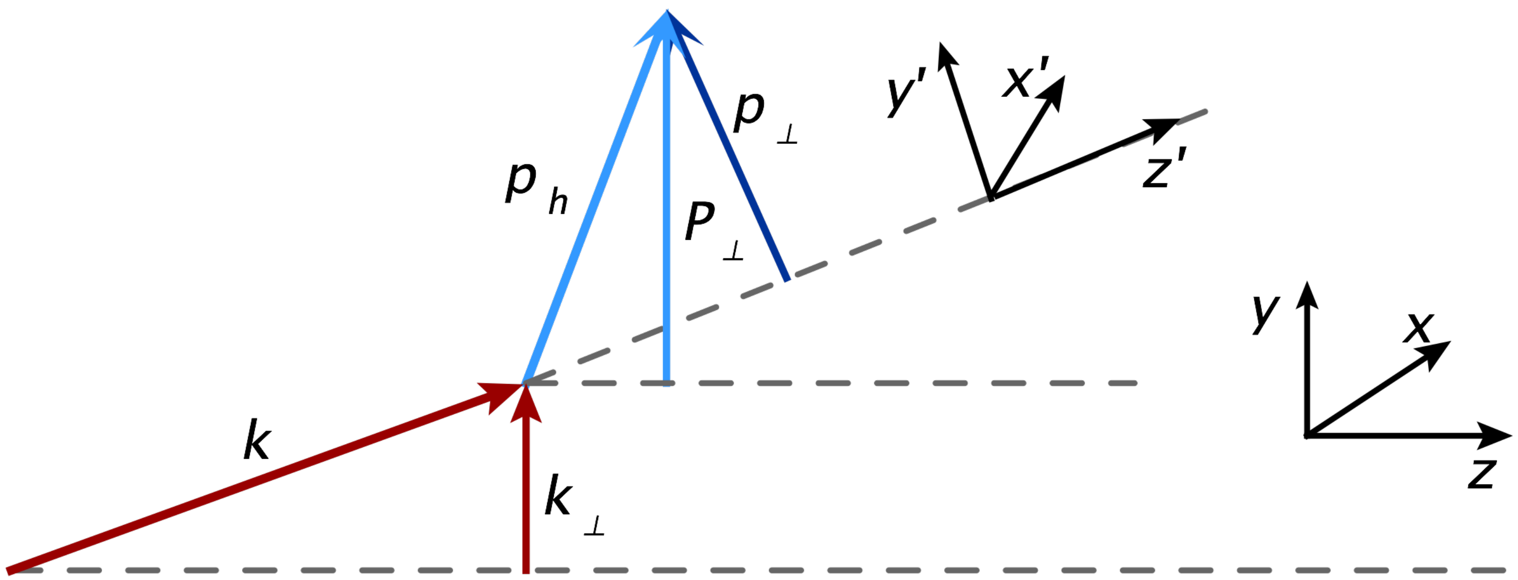
- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

Lorentz Transforms of TM

Diehl: NPB 596, 33 (2001)(2015)

- **Boosts from *0* TM frame that preserve "-" *component.***



$$\begin{pmatrix} 1 & \frac{\boldsymbol{k}_\perp^2}{2(k^-)^2} & \frac{k_1}{k^-} & \frac{k_2}{k^-} \\ 0 & 1 & 0 & 0 \\ 0 & \frac{k_1}{k^-} & 1 & 0 \\ 0 & \frac{k_2}{k^-} & 0 & 1 \end{pmatrix}$$

	q	h
$\mathcal{L}'$	$(k'^+, k'^-, \boldsymbol{k}'_\perp = 0)$	$(p^+, p^-, \boldsymbol{p}_\perp)$
$\mathcal{L}$	$(k^+, k^- = k'^-, \boldsymbol{k}_\perp)$	$(P^+, P^- = p^-, \boldsymbol{P}_\perp = \boldsymbol{p}_\perp + z\boldsymbol{k}_\perp)$

$$z \equiv \frac{p^-}{k^-} = \frac{p'^-}{k'^-}$$

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

- **In case of two (or more) hadrons: same story!**

$$\boldsymbol{P}_{1\perp} = \boldsymbol{p}_{1\perp} + z_1\boldsymbol{k}_\perp \qquad \boldsymbol{P}_{2\perp} = \boldsymbol{p}_{2\perp} + z_2\boldsymbol{k}_\perp$$

ELEMENTARY TMD SPLITTINGS

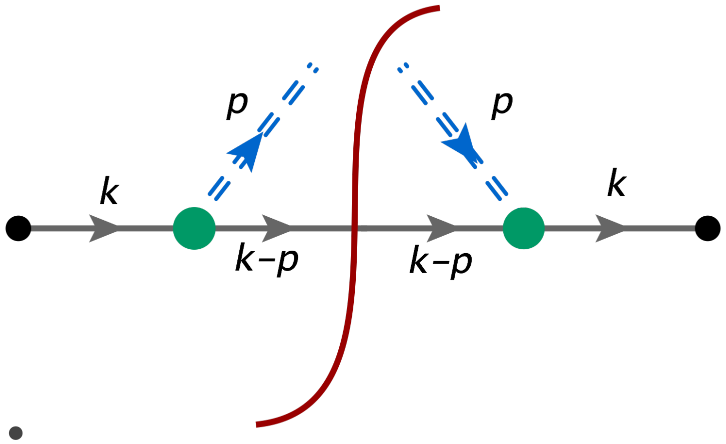
H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

- **Quark-quark correlator:**

$$\Delta_{ij}(z,p_\perp) = \frac{1}{2N_c\ z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\ e^{ip\cdot\xi} \times \langle 0|\mathcal{U}_{(\infty,\xi)}\ \psi_i(\xi)|h,X\rangle_{\text{out out}}\langle h,X|\bar{\psi}_j(0)\,\mathcal{U}_{(0,\infty)}|0\rangle\Big|_{\xi^-=0}$$

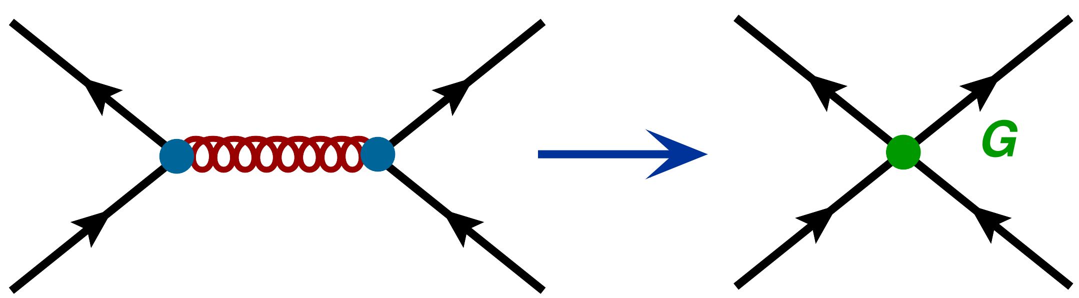
- **One-quark truncation of the wavefunction:** $q \to Qh$

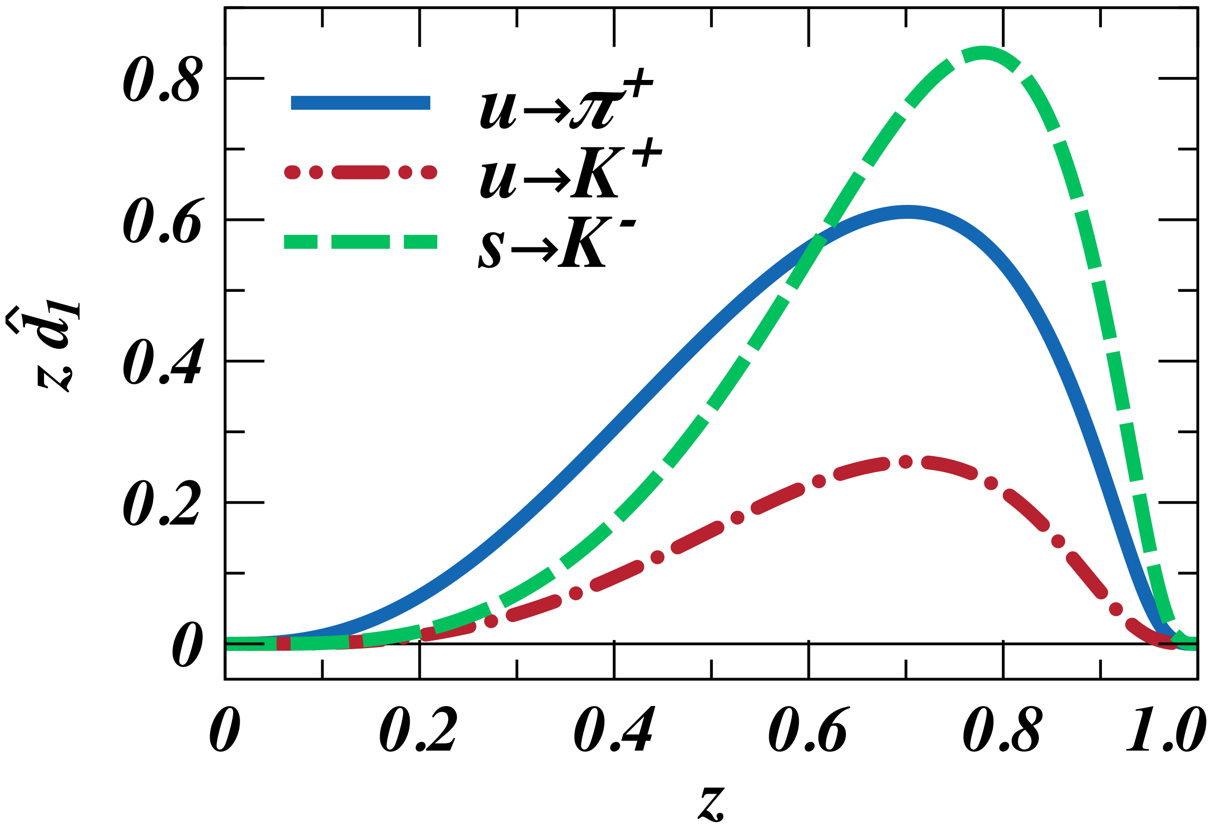
$$d_q^h(z,p_\perp^2) = \frac{1}{2}\text{Tr}[\Delta_0(z,p_\perp^2)\gamma^+]$$



- **NJL Effective quark model calculations:**

$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$

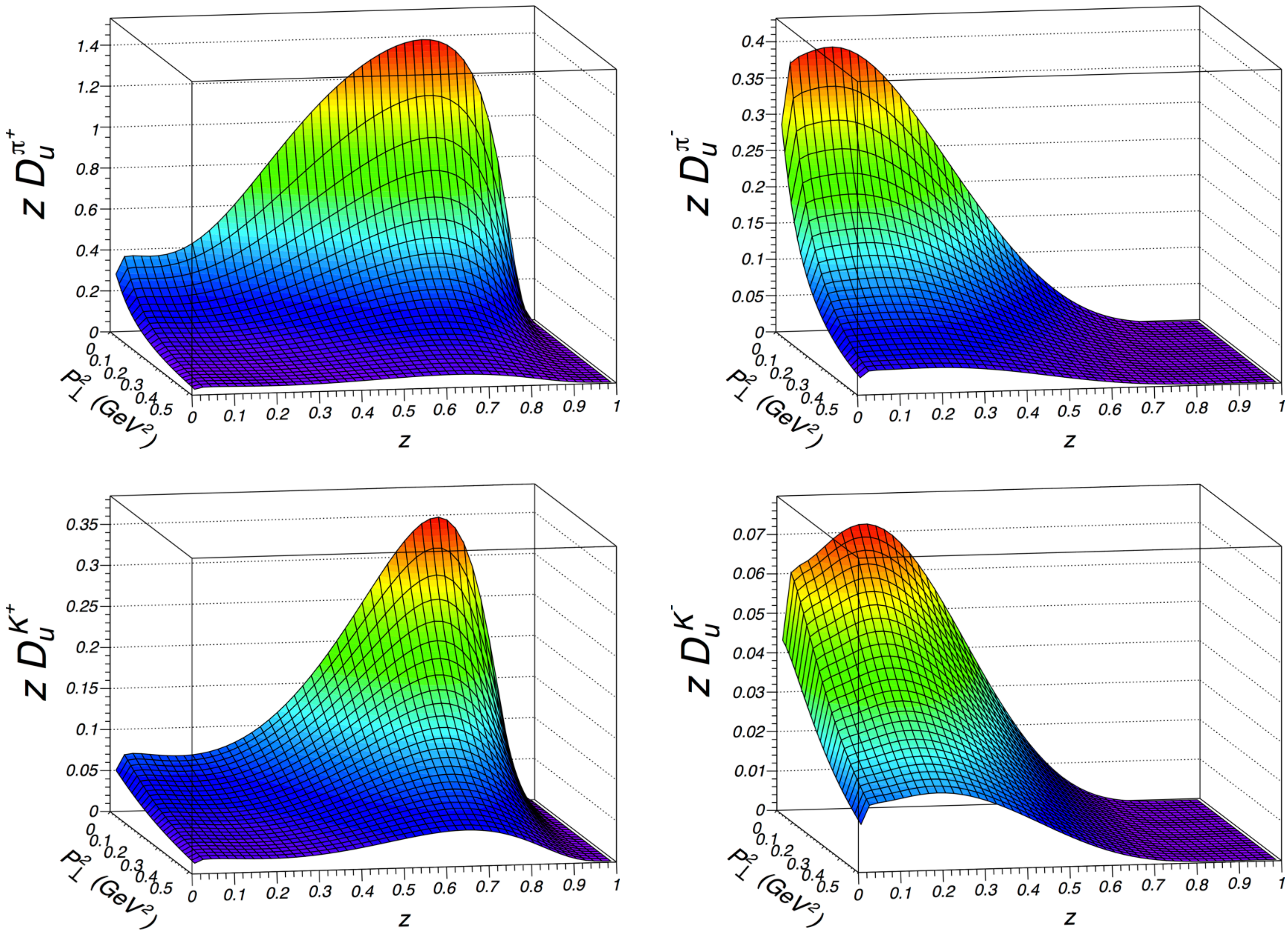




TMD FRAGMENTATION FUNCTIONS

FAVORED

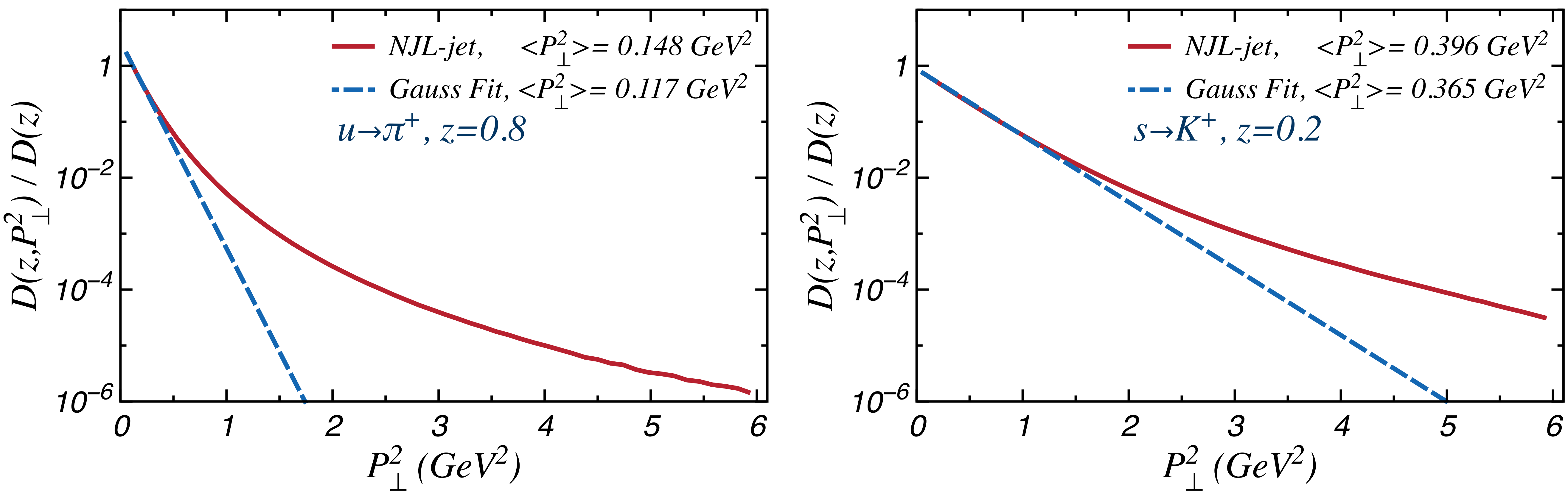
• UNFAVORED



π

K

COMPARISON WITH GAUSSIAN ANSATZ



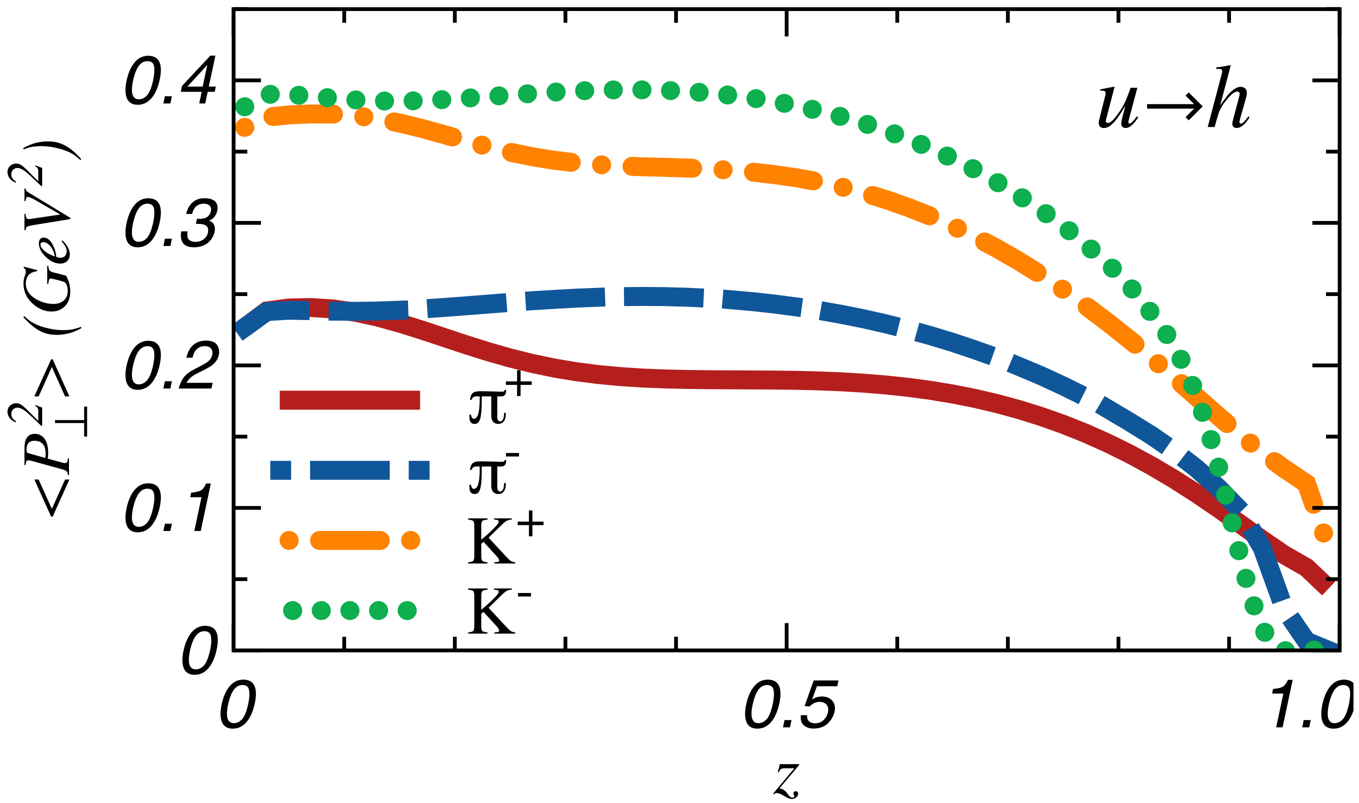
- Average TM: $\langle P_\perp^2 \rangle \equiv \dfrac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$
- Gaussian ansatz assumes: $D(z, P_\perp^2) = D(z) \dfrac{e^{-P_\perp^2/\langle P_\perp^2 \rangle}}{\pi \langle P_\perp^2 \rangle}$

AVERAGE Transverse Momenta vs z

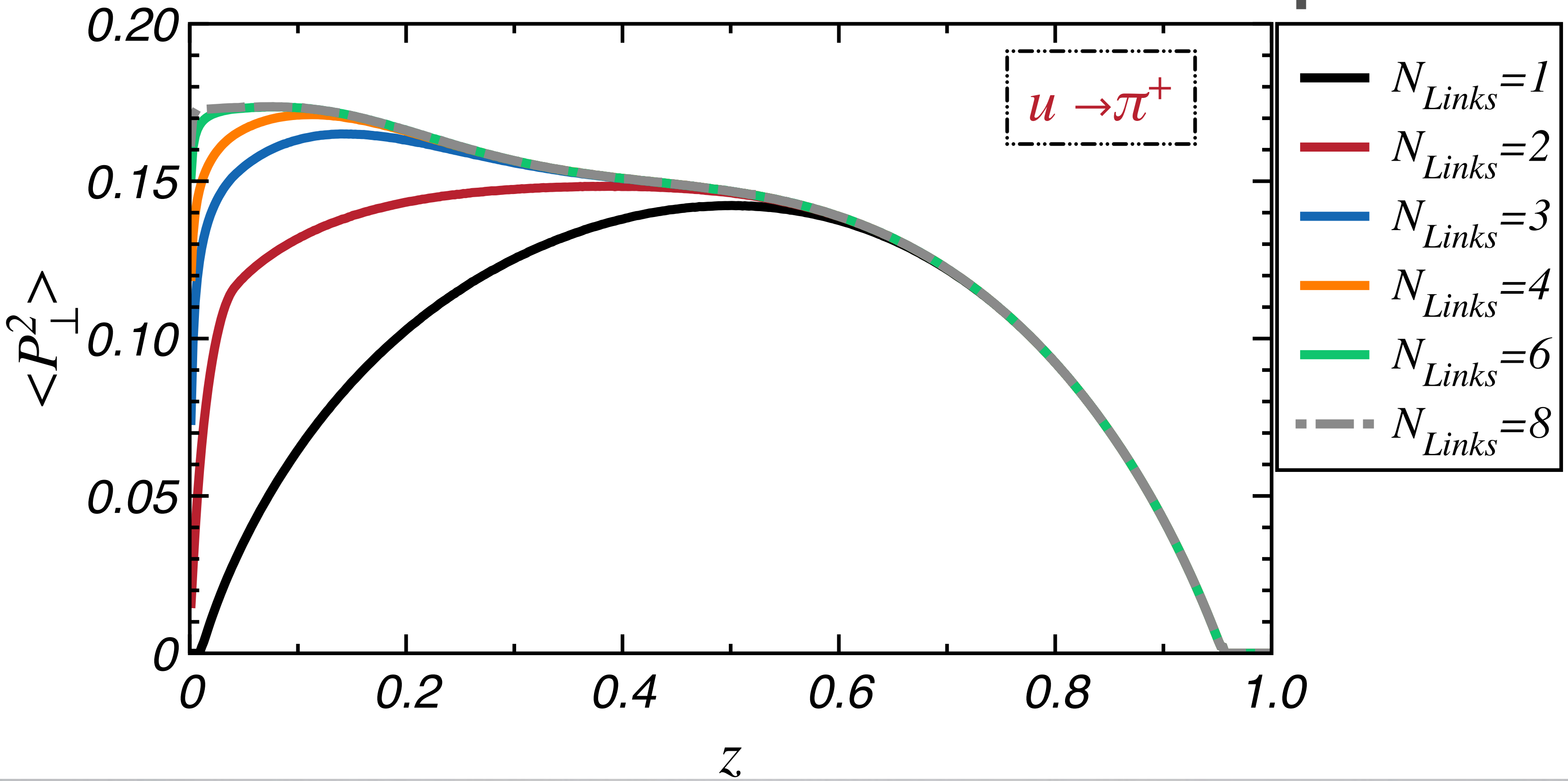
FRAGMENTATION

$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦Indications from HERMES data: A. Signori, et al: JHEP 1311, 194 (2013)



✓Multiple hadron emissions: ***broaden*** the TM dependence at ***low z***!



Different Hadronization Mechanisms.

LUND Model

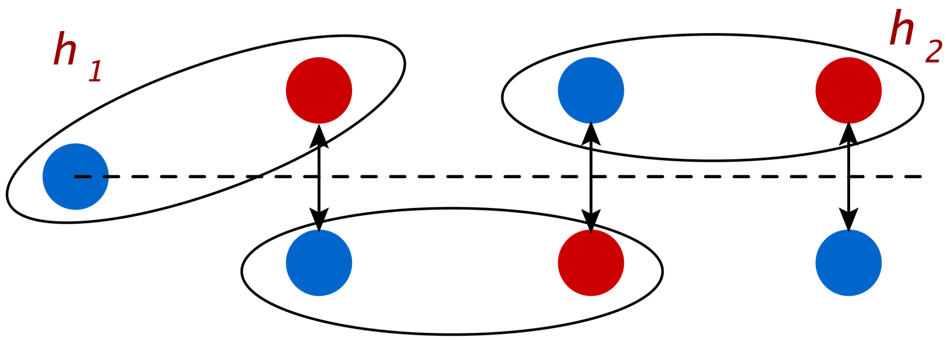
✦ Fragmentation of $q\bar{q}$ pair: break-up of the string.

✦ Independent breaking of the string.

✦ Quark TM indep. of hadron type.

$$u \to u + s\bar{s}, \quad s \to s + s\bar{s}$$

✦ No correlation in TM: **h$_1$** and **h$_2$**.



Quark-Jet

✦ Fragmentation of q, similar to QFT definition of FFs.

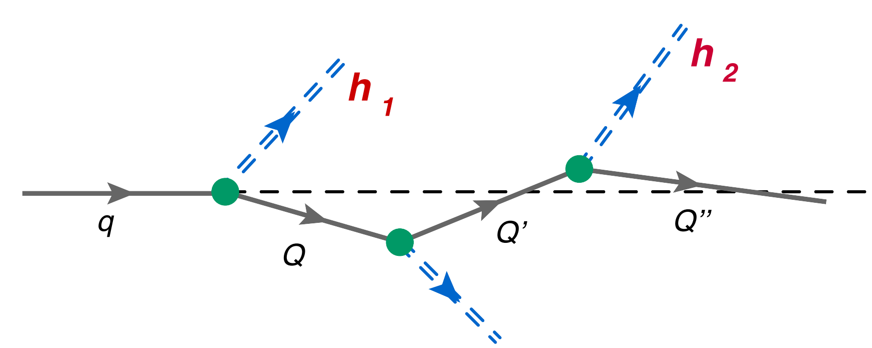
✦ Time-ordered hadron emissions.

✦ $q \to Qh$ depends on **h** (spin, mass).

$$u \to K^+ + s, \quad s \to \phi + s$$

$$u \to K^{*+} + s$$

✦ Recoil TM of **h$_1$** affects **h$_2$**



❖ Can we find a signature in polarized FFs? Perhaps Dihadron FFs?

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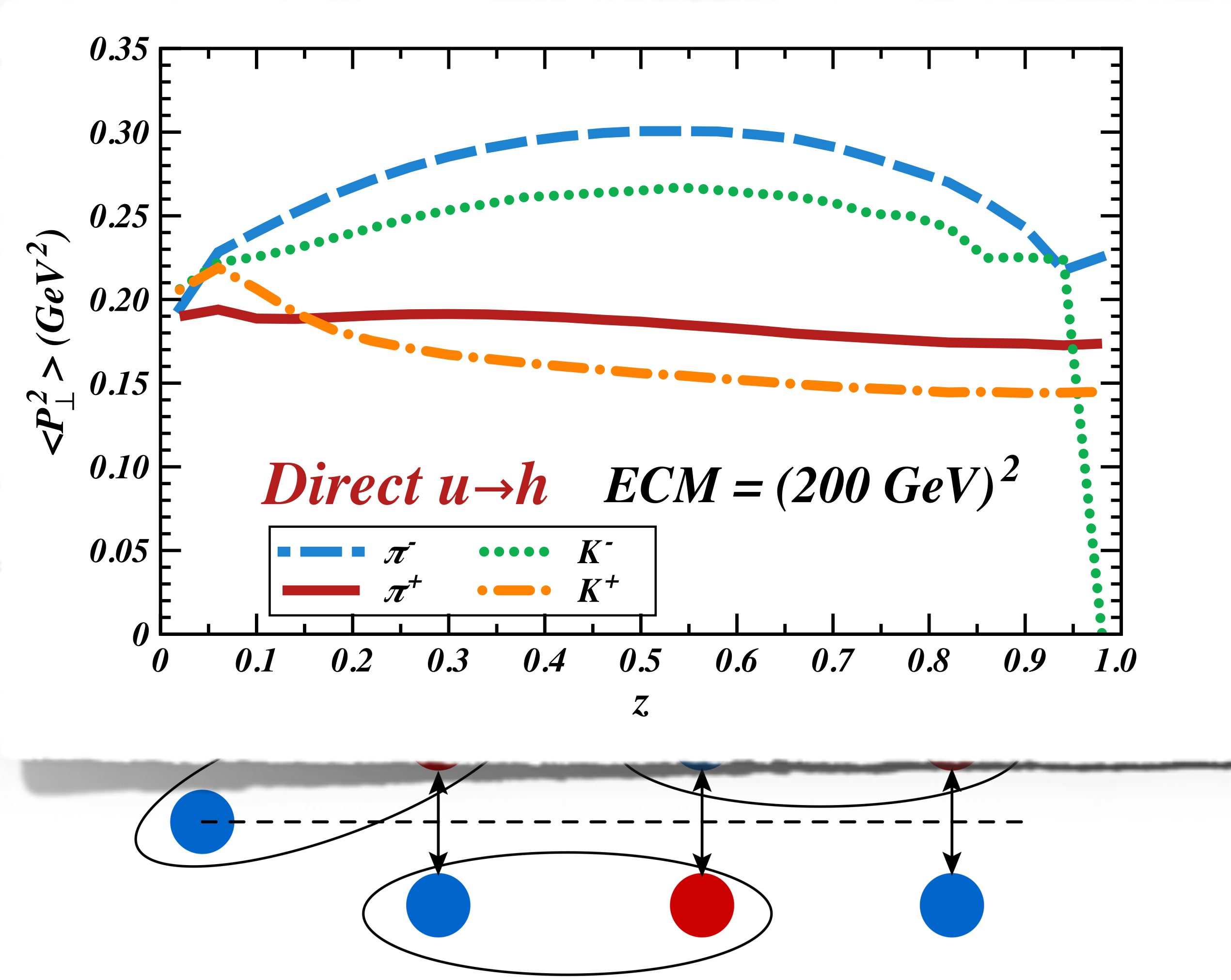

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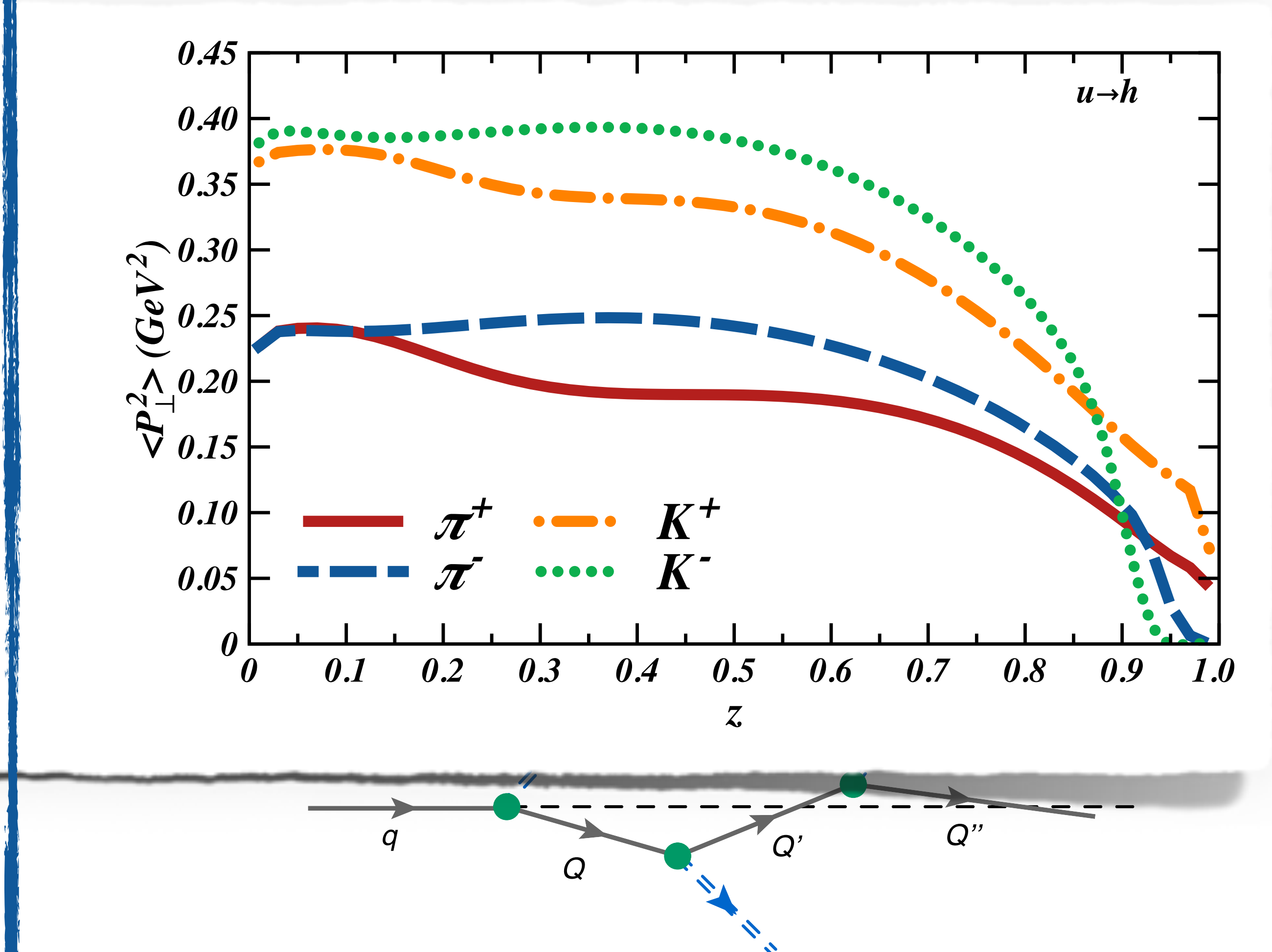
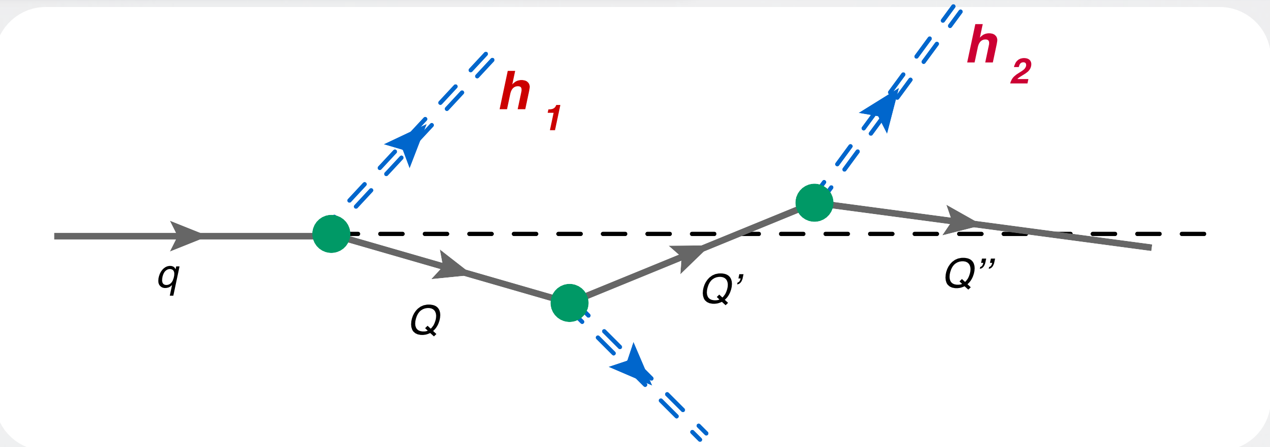


❖ Can we find a signature in polarized FFs? Perhaps Dihadron FFs?

UNPOLARIZED DIHADRON FRAGMENTATIONS

H.M. Thomas, Bentz, PRD.88:094022, 2013.



- The probability density for observing two hadrons:

$$P_1 = (z_1 k^-, P_1^+, \boldsymbol{P}_{1,\perp}),\ \ P_1^2 = M_{h1}^2$$

$$P_2 = (z_2 k^-, P_2^+, \boldsymbol{P}_{2,\perp}),\ \ P_2^2 = M_{h2}^2$$

- The corresponding number density:

$$D_q^{h_1 h_2}(z, M_h^2)\ \Delta z\ \Delta M_h^2 = \langle N_q^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2 + \Delta M_h^2) \rangle$$

$$z = z_1 + z_2 \qquad M_h^2 = (P_1 + P_2)^2$$

- Kinematic Constraint.

$$z_1 z_2 M_h^2 - (z_1 + z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) \geq 0$$

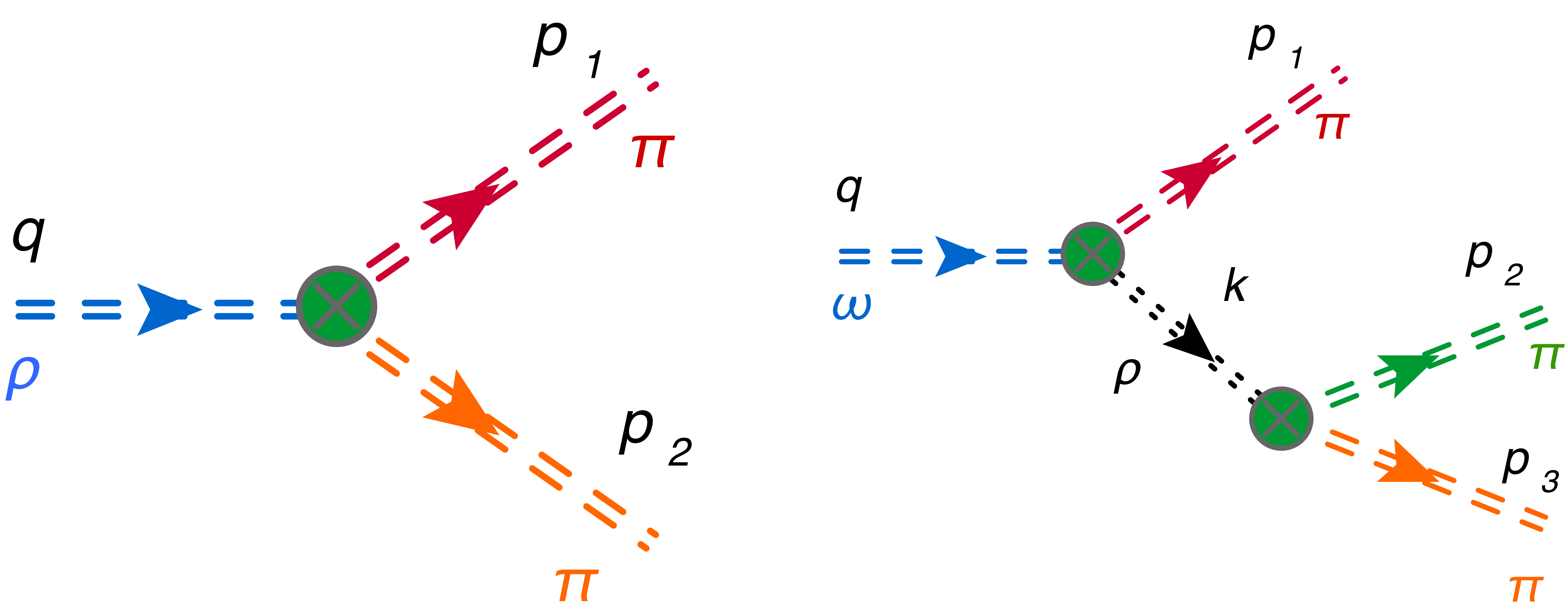
- In MC simulations record all the pairs in every decay chain.

2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .

Achasov et al. (SND), PRD 68, 052006, (2003).

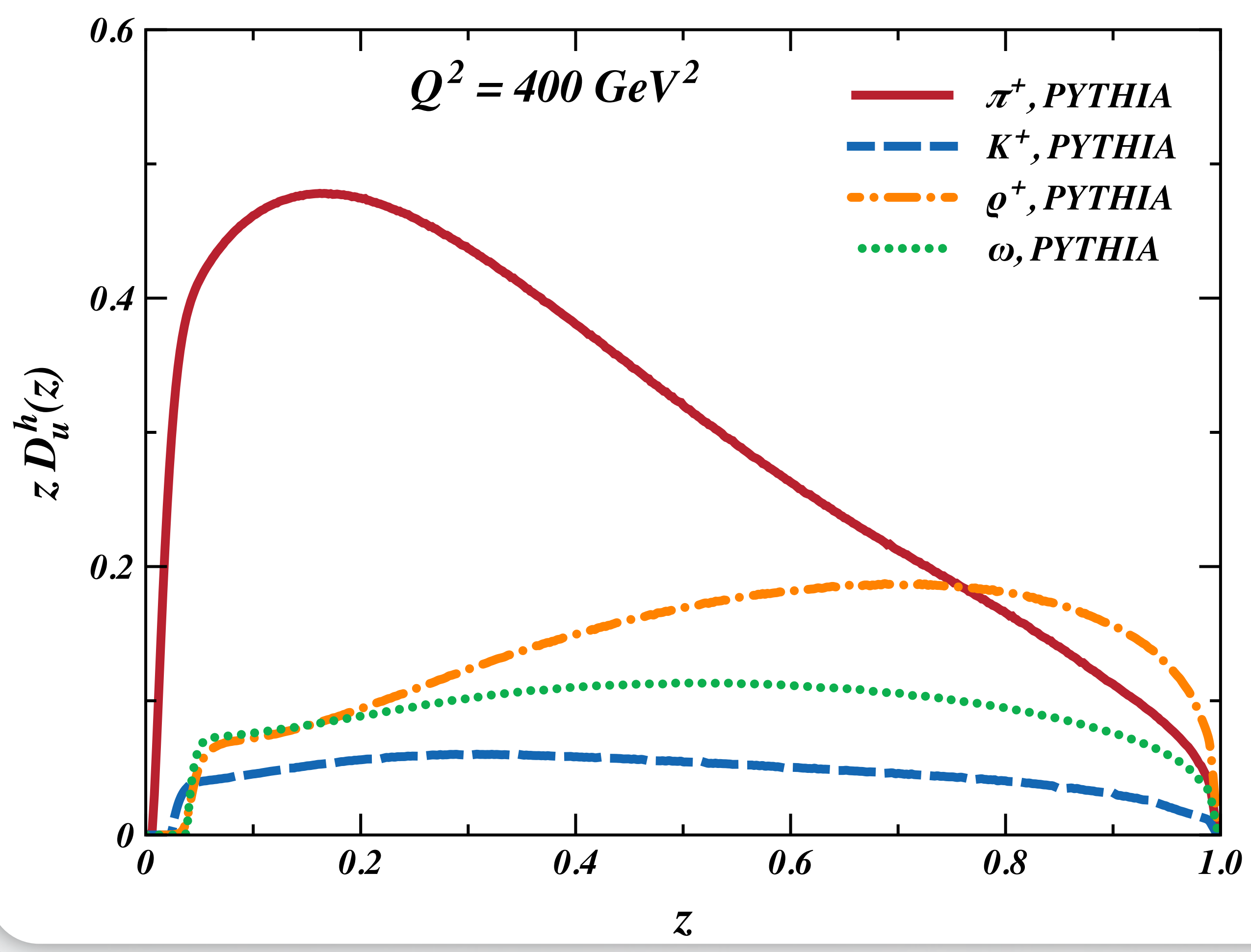


PYTHIA SIMULATIONS

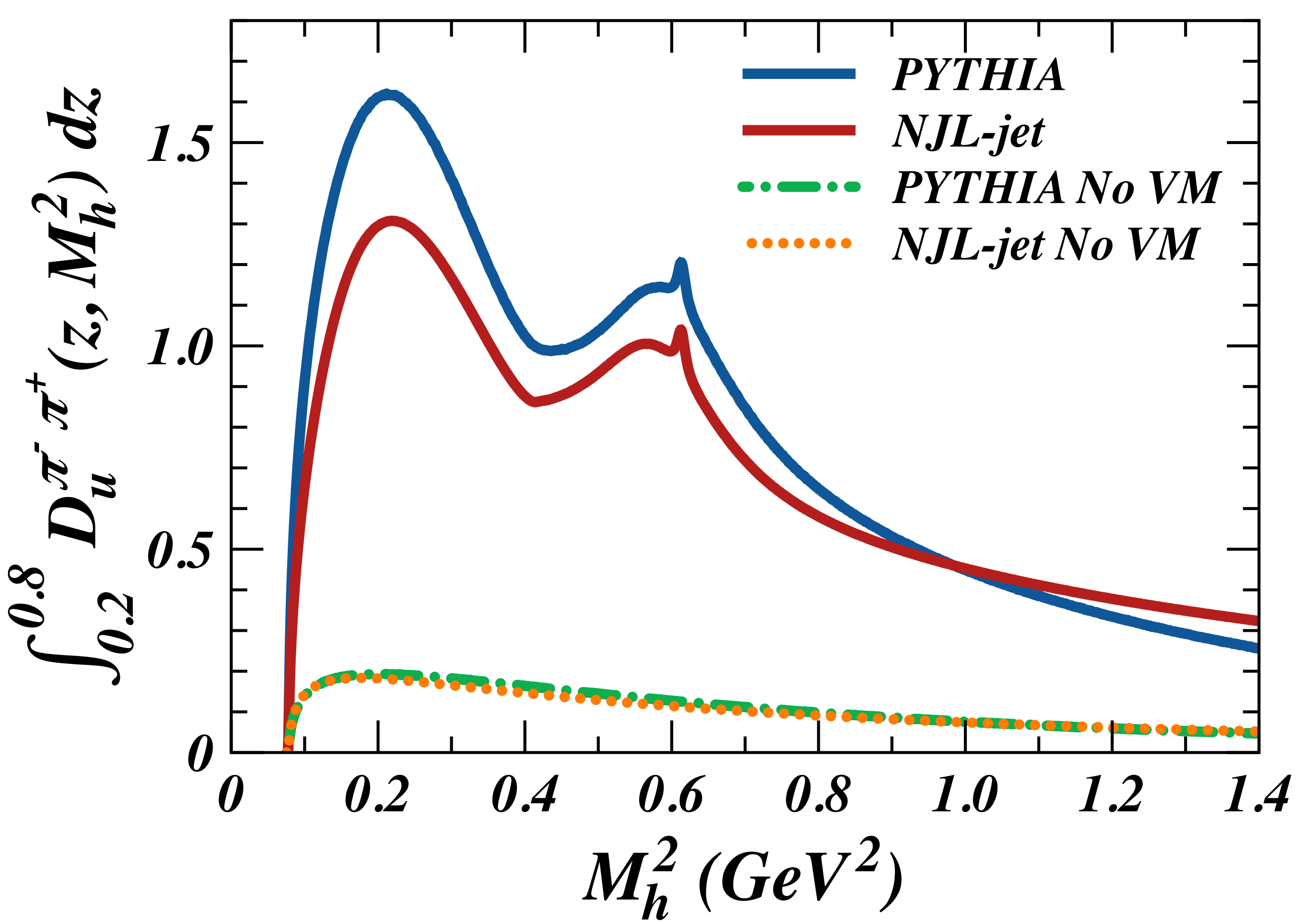
- Setup hard process with back to back $q\ \bar{q}$ along **z** axis.
- ***Only Hadronize.*** Allow the same resonance decays as NJL-jet.
- Assign hadrons with positive p_z to q fragmentation.

$$E_q = 10\ \mathrm{GeV}$$

Single Hadron



Dihadron



Positivity and Polarisation of Quark

Bacchetta et al, PRL 85, 712 (2000) .

✦ The probability density is Positive Definite: constraints on FFs.

✦ Leading-order T-Even functions FULLY Saturate these bounds!

✦ For non-vanishing $H^{\perp}$ and $D_T^{\perp}$, need to calculate T-Even FFs at next order!

✦ Average value of remnant quark's spin.

$$\langle \boldsymbol{S}_T \rangle_Q = \boldsymbol{s}_T \frac{\int dz \left[h_T^{(q\to Q)}(z) + \frac{1}{2z^2 M_Q^2} \; h_T^{\perp[1](q\to Q)}(z) \right]}{\int dz \; d^{(q\to Q)}(z)}$$

✦ In spectator model, at leading order: $h_T(z) = -d(z)$

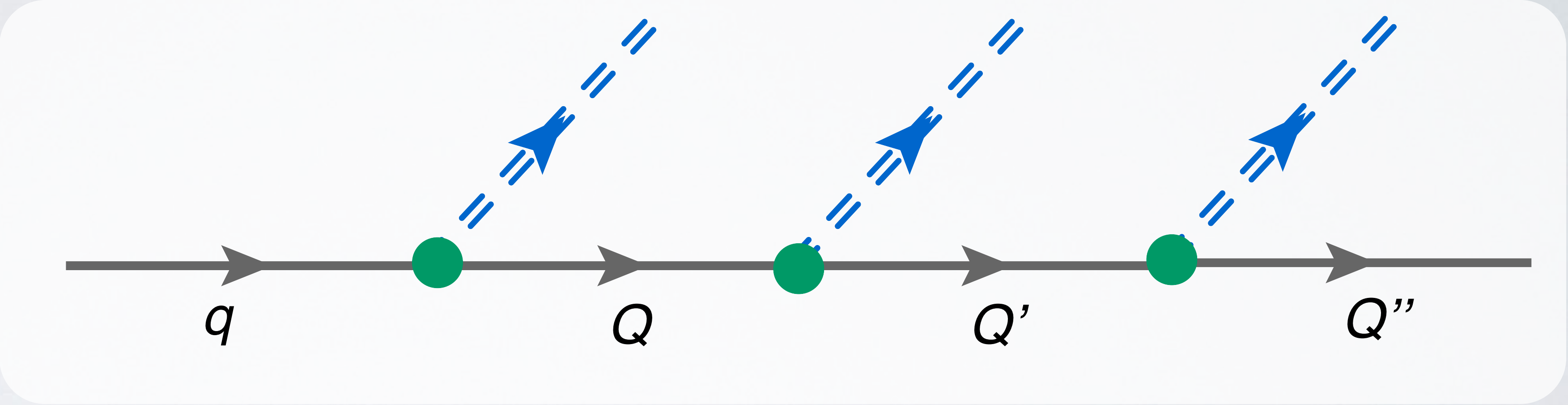
✦ Non-zero $h_T^{\perp}$ means $\langle \boldsymbol{S}_T \rangle_Q \neq -\boldsymbol{s}_T$ (full flip of the spin)!

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



$$D_q^h(z) = \hat{d}_q^h(z) + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h(\frac{z}{y})\frac{1}{y}$$

$$\hat{d}_q^h(z) = \hat{d}_q^{Q'}(1-z)|_{h=\bar{Q}'q}$$

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
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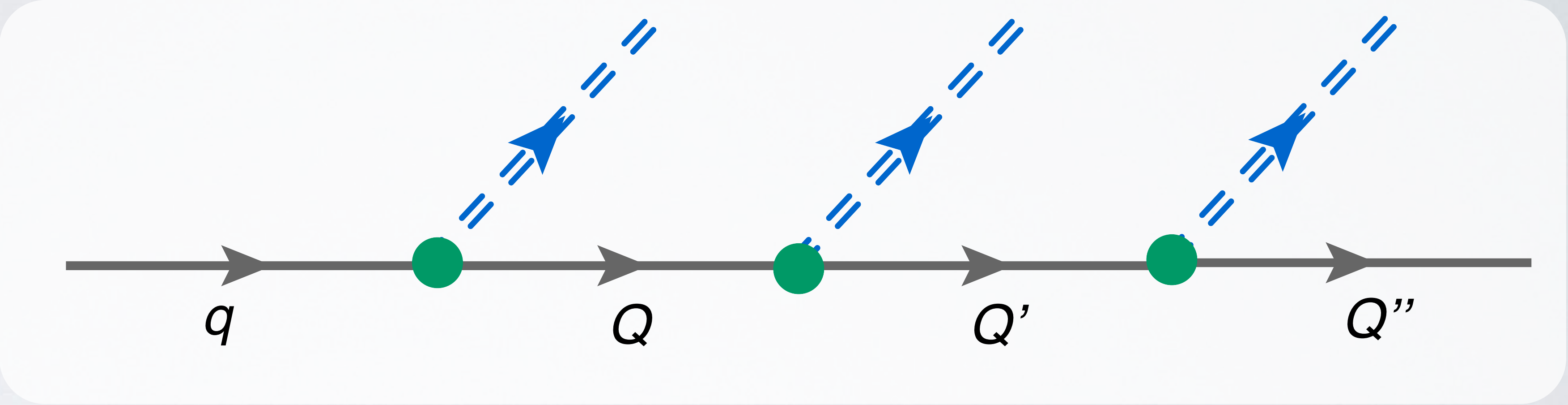
- ∞ hadron emissions



Probability of finding hadron h with mom. frac. [z, z+dz] in a jet of quark q

The probability scales with mom. fraction

$$D_q^h(z)dz = \hat{d}_q^h(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h\left(\frac{z}{y}\right)\frac{dz}{y}$$

Prob. of emitting at step 1

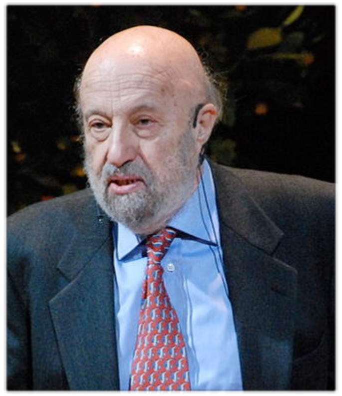
Prob. of mom. [y, y+dy] is transferred to jet at step 1.

NAMBU--JONA-LASINIO MODEL

Yoichiro Nambu and Giovanni Jona-Lasinio:

"Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I"

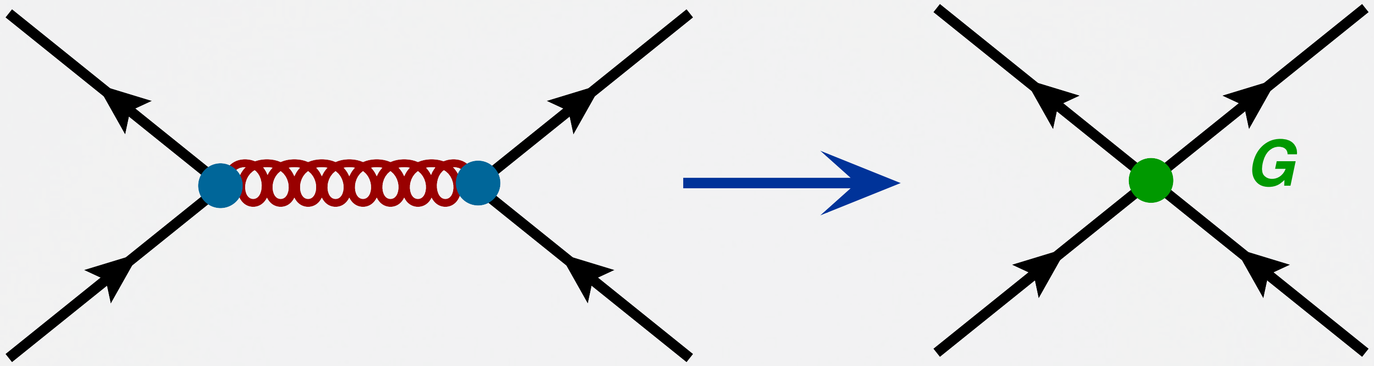
Phys.Rev. 122, 345 (1961)



Effective Quark model of QCD

- Effective Quark Lagrangian

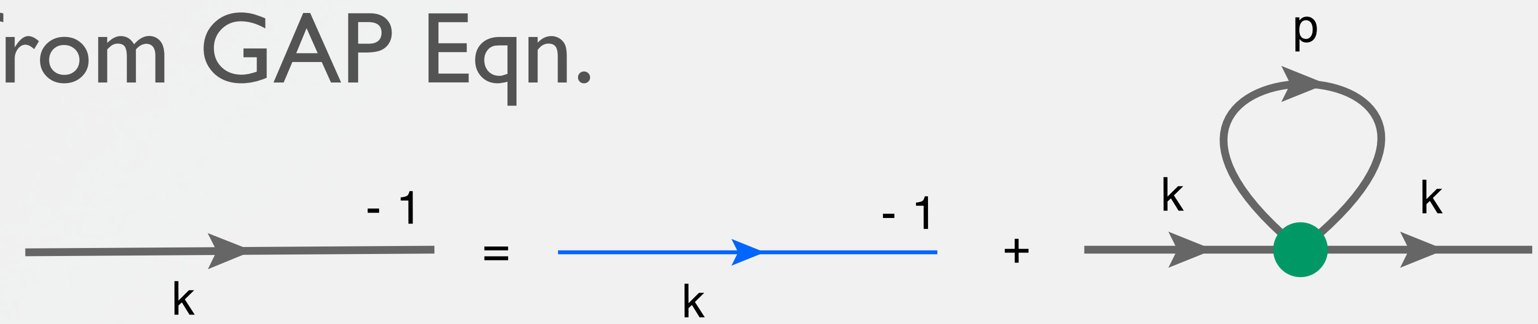
$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$

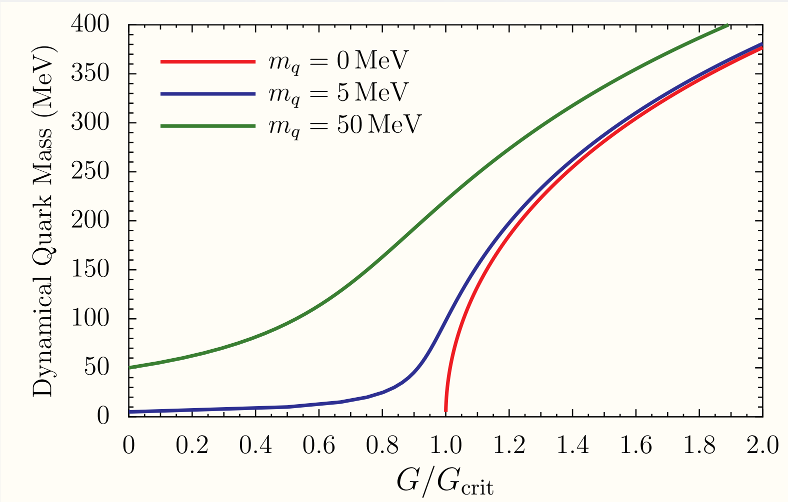


- Low energy chiral effective theory of QCD.
- Covariant, has the same flavor symmetries as QCD.

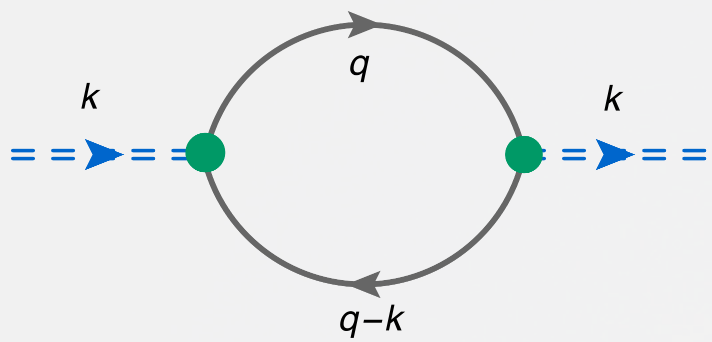
NAMBU--JONA-LASINIO MODEL

•Dynamically Generated Quark Mass from GAP Eqn.

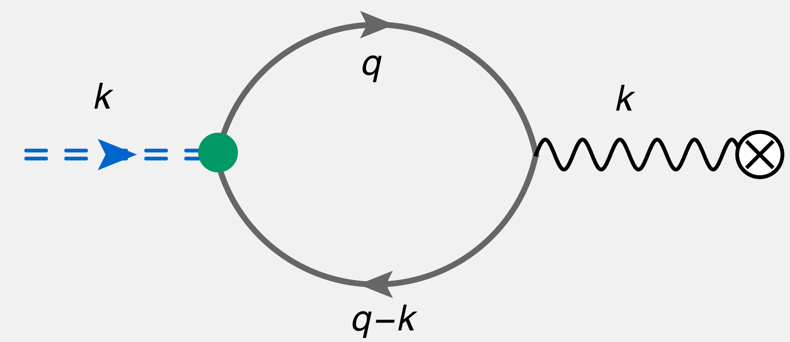




•Pion mass and quark-pion coupling from t-matrix pole.



•Pion decay constant



Fixing Model Parameters

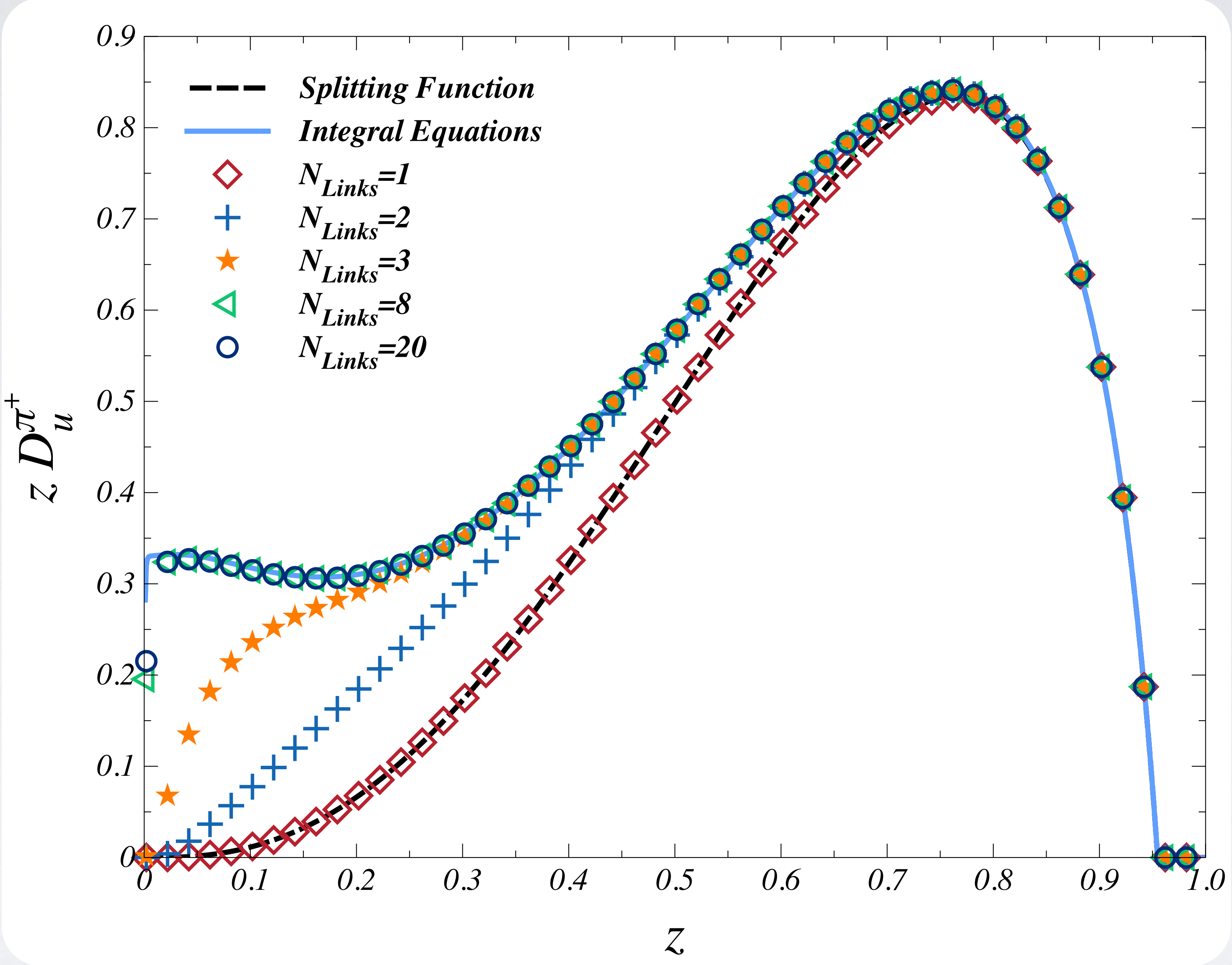
•Use Lepage-Brodsky Invariant Mass cut-off regularisation scheme.

$$M_{12} \leq \Lambda_{12} = \sqrt{\Lambda_3^2 + M_1^2} + \sqrt{\Lambda_3^2 + M_2^2}$$

• Choose a $M_{u(d)}$ and use physical f_π, m_π, m_K to fix model parameters $\Lambda_3,\ G, M_s$ and calculate g_{hqQ}.

DEPENDENCE ON NUMBER OF EMITTED HADRONS

- Restrict the number of emitted hadrons, N_{Links} in MC.

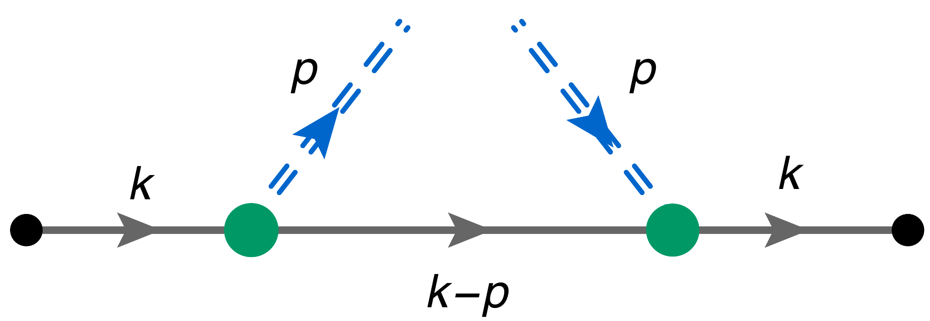


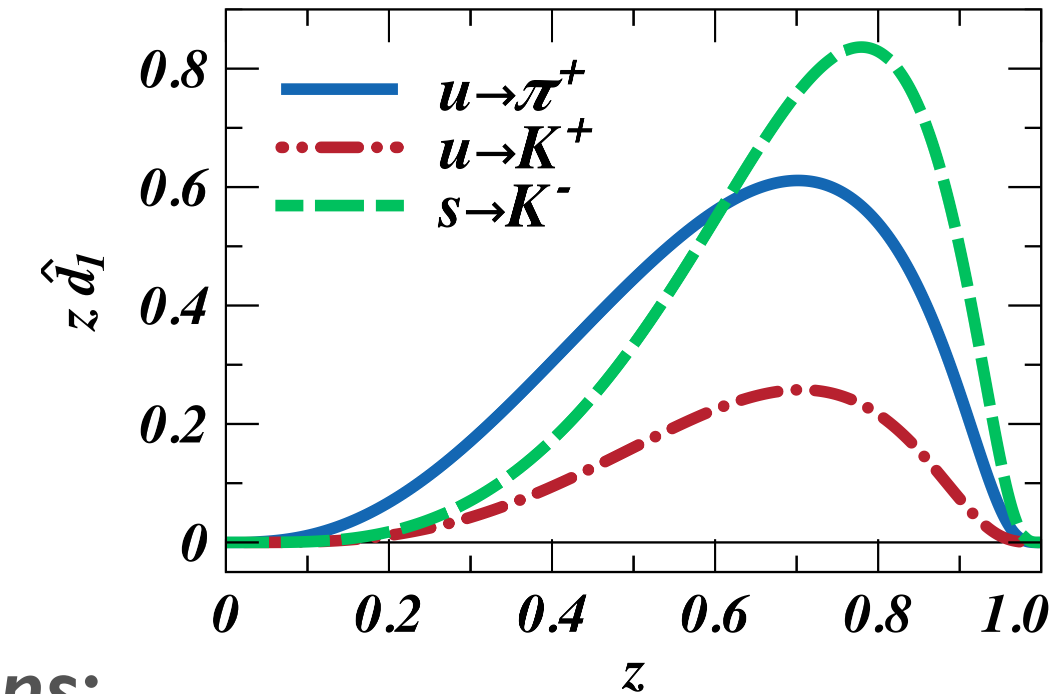
- We reproduce the splitting function and the full solution perfectly.
- The low z region is saturated with just a few emissions.

SOLUTIONS OF THE INTEGRAL EQUATIONS

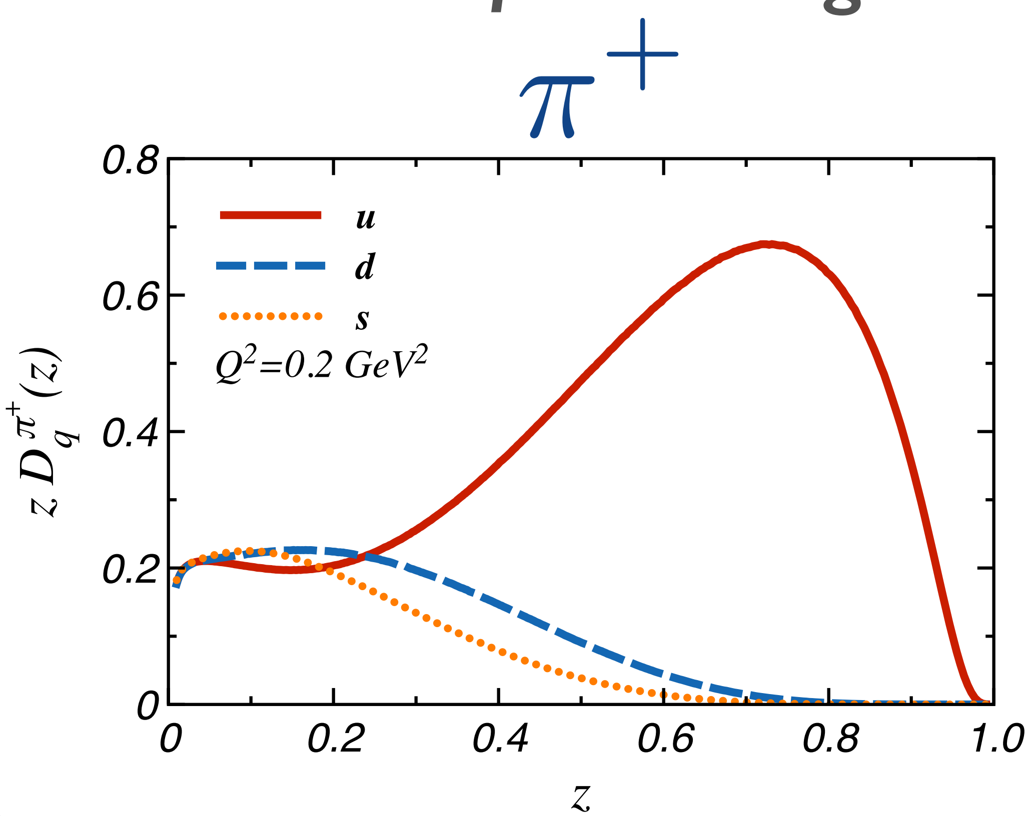
H.M., Thomas, Bentz, PRD. 83:074003, 2011

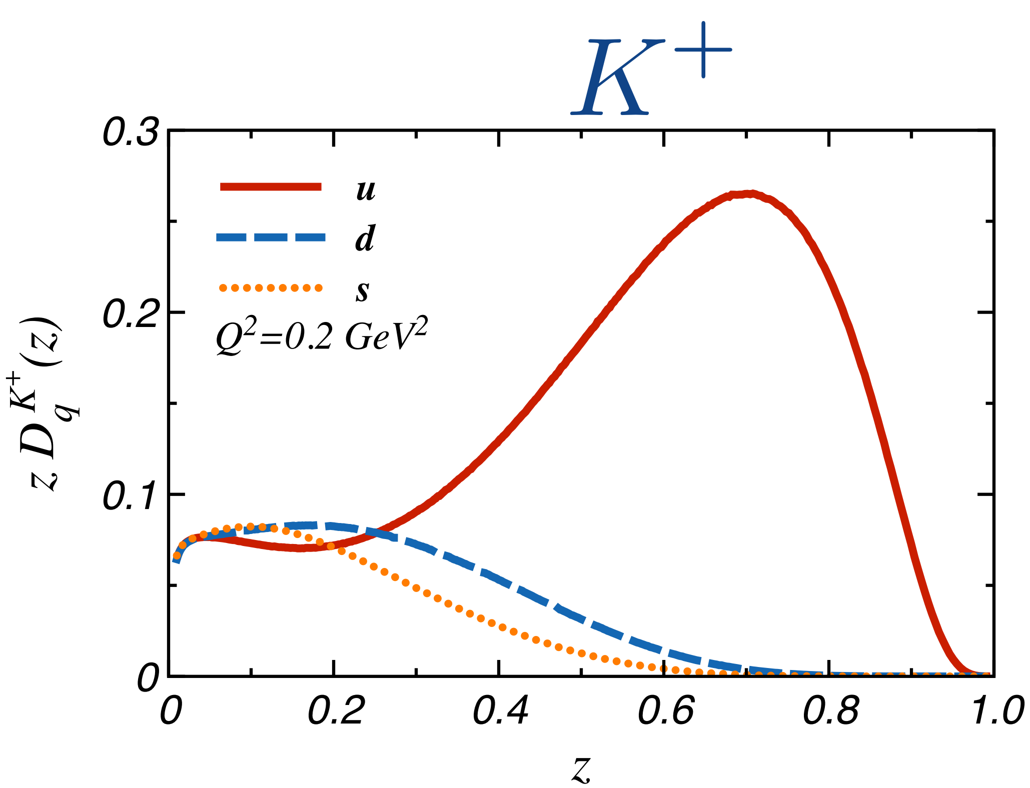
✦ ***Input elementary probabilities from NJL:***





✦ ***Solutions of the integral equations:***





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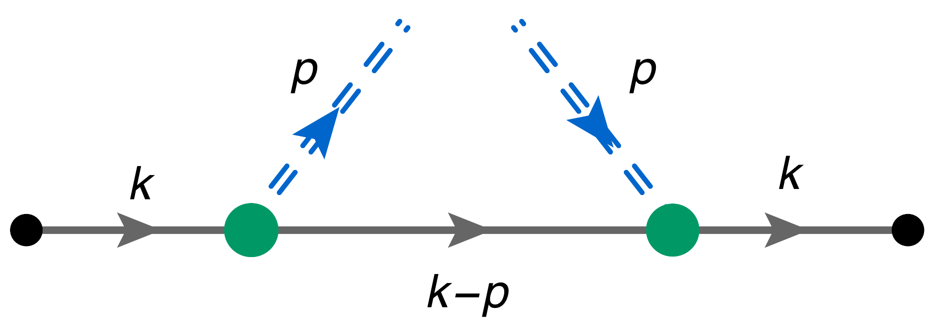

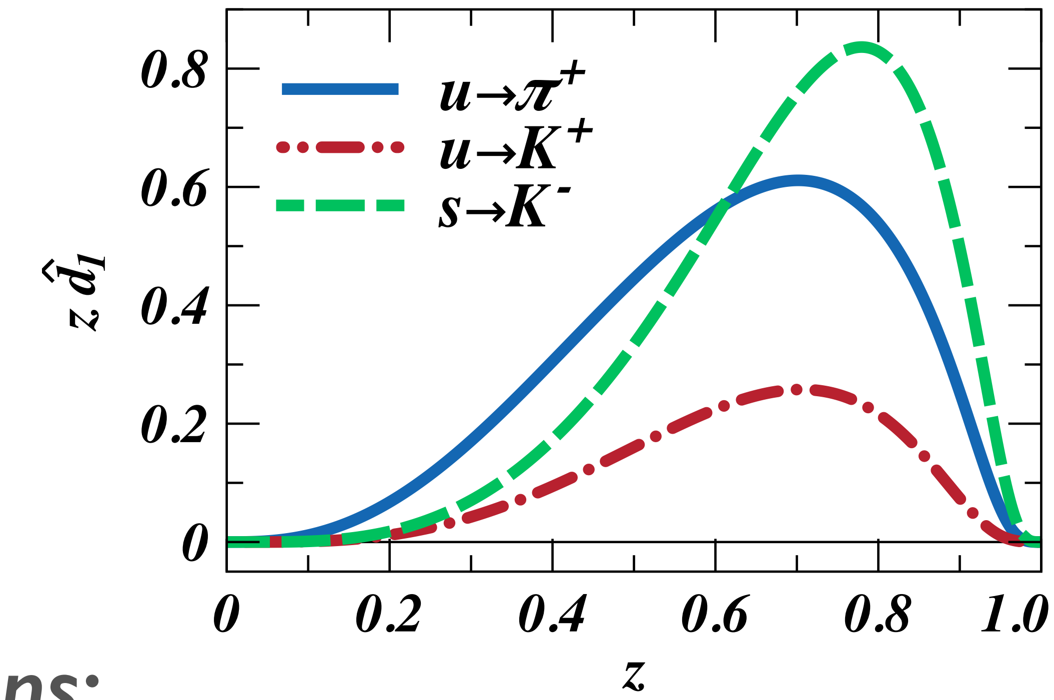

✦ *Solutions of the integral equations:*