3D Parton Distributions: path to the LHC
LNF, Frascati 29/11 - 2/12/2016

Recursive model for jets from polarized quarks

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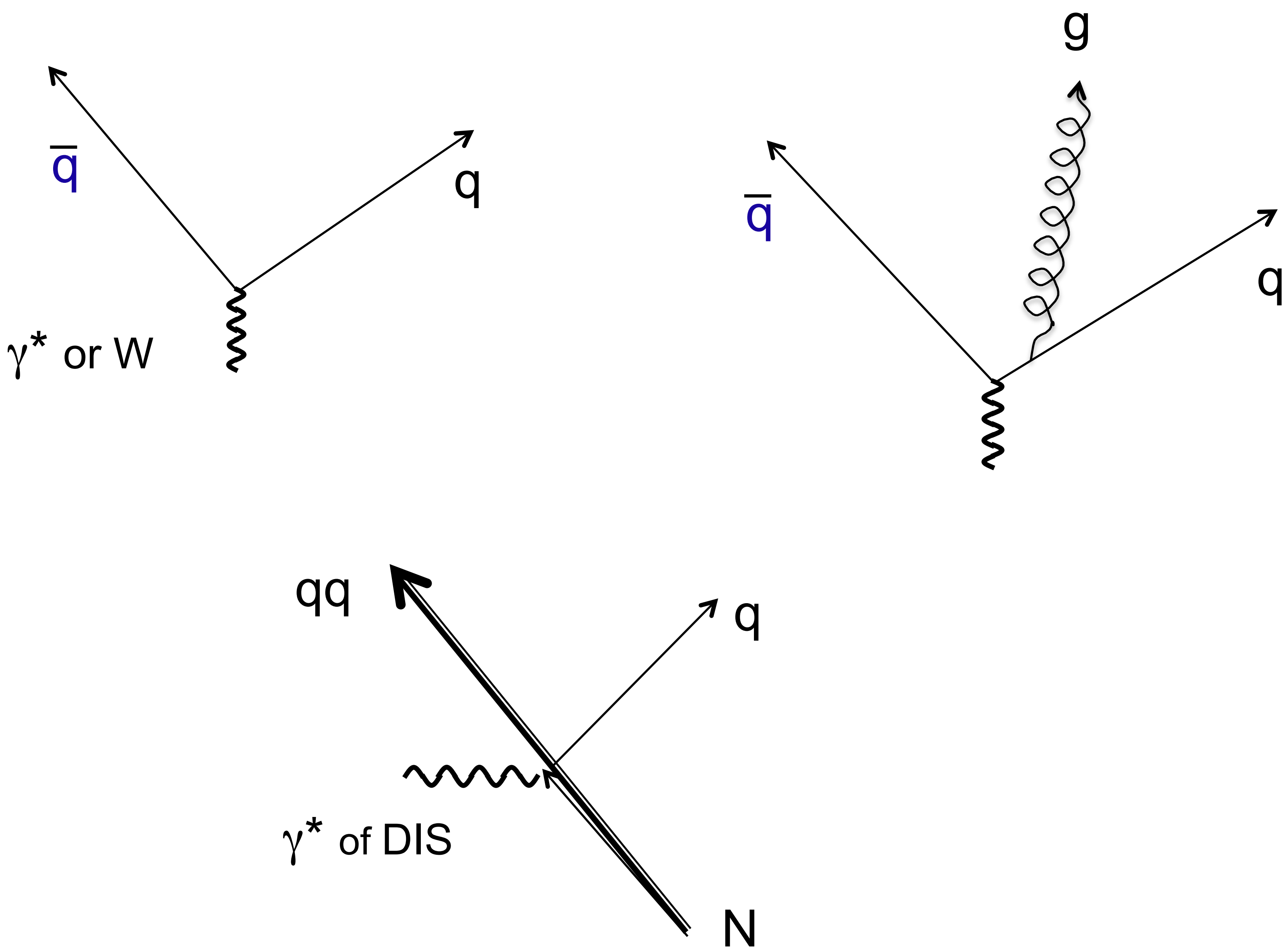
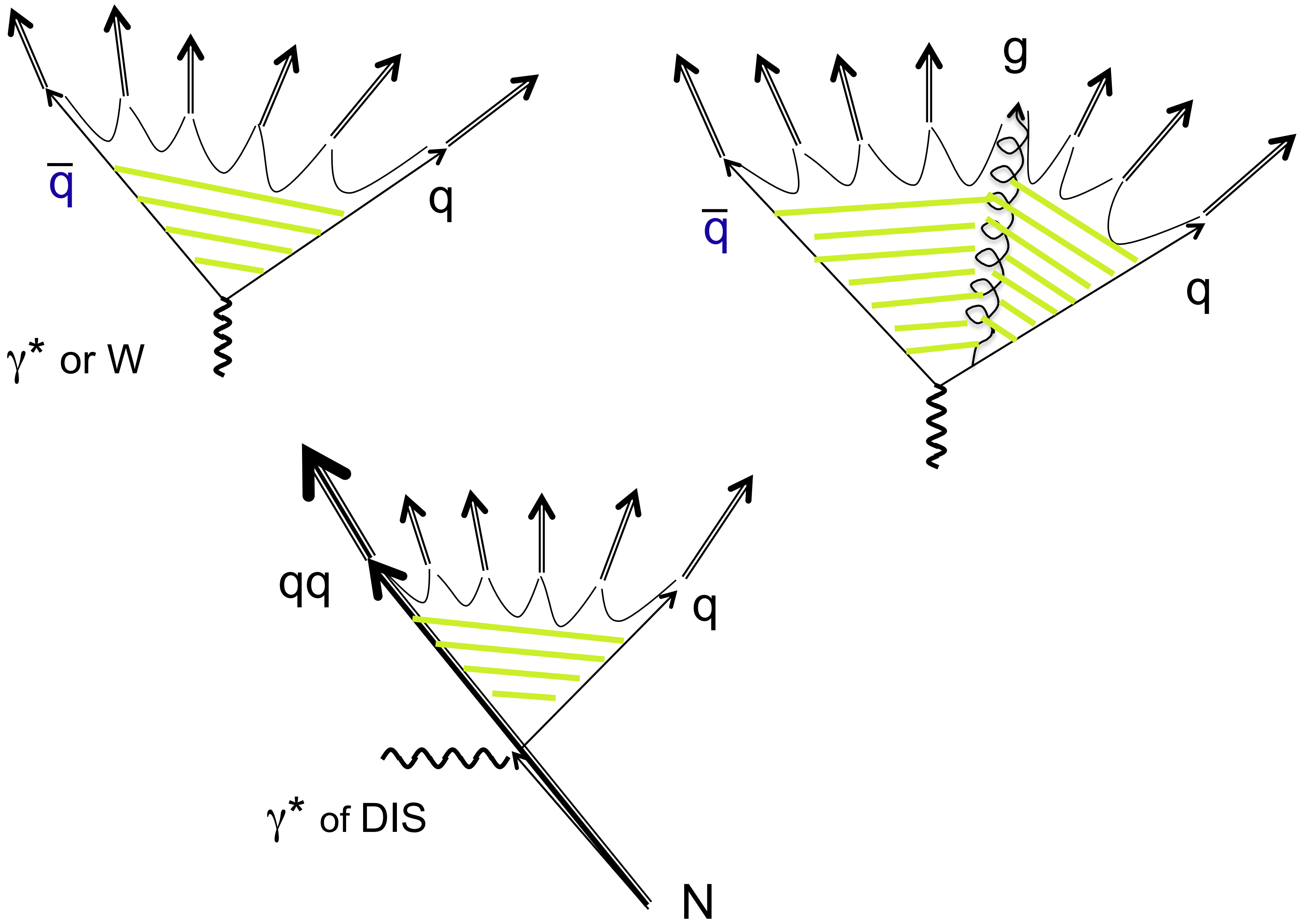
Outlines

- *Quark-Multiperipheral* model (QMPM)
  and *String Fragmentation* model (SFM)
- Recursive models: cutoffs; quark line reversal
- recall on the SFM
- The ***string* + $^3P_0$** mechanism of Collins effect
- Semi-quantization of the SFM, including spin
- The “renormalized vertex” approach
- Implementation in a recursive Monte-Carlo code
- Resuts from simulations

naked final partons



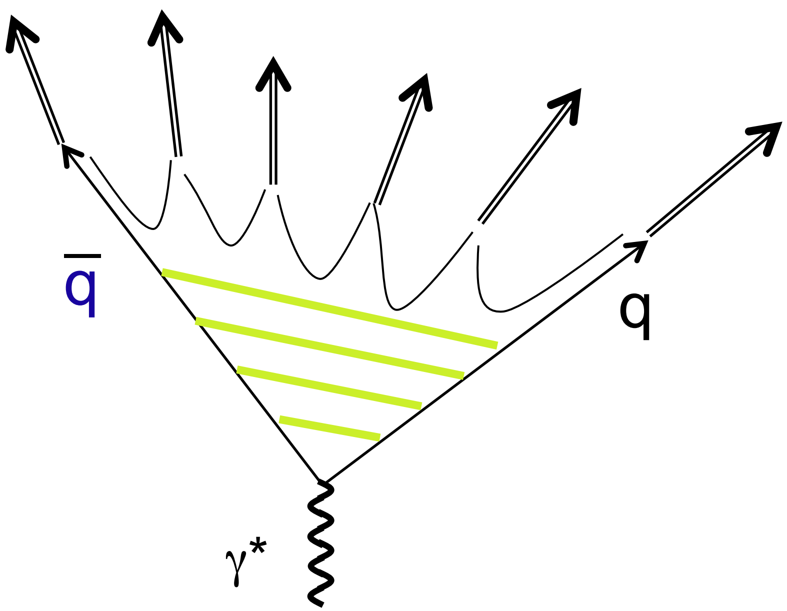
hadronization



Two models of hadronization

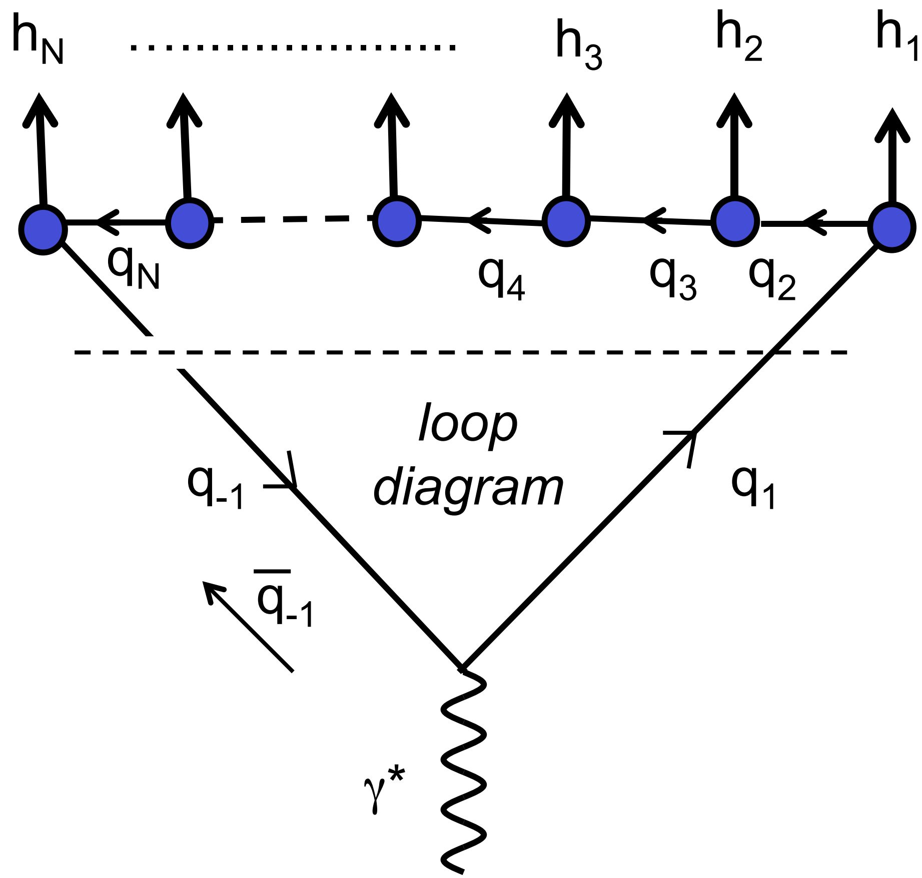
example: $e^+e^- \rightarrow q+\bar{q}$bar $\rightarrow$ hadrons

String Fragmentation (SFM)



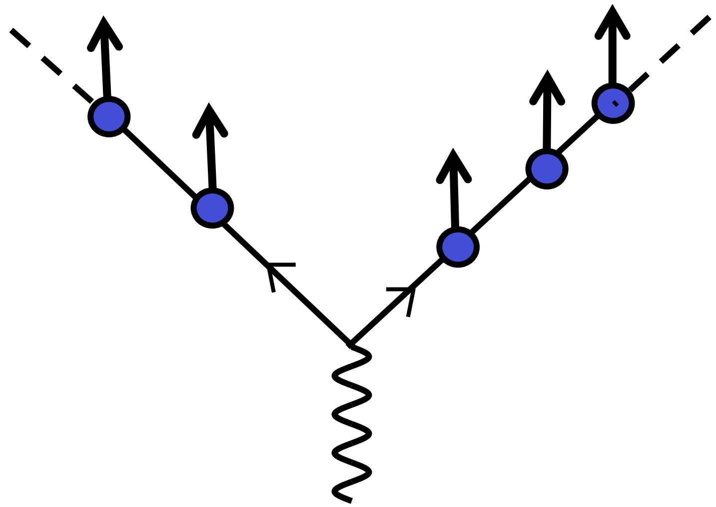
confinement built-in

Quark Multiperipheral (QMPM)



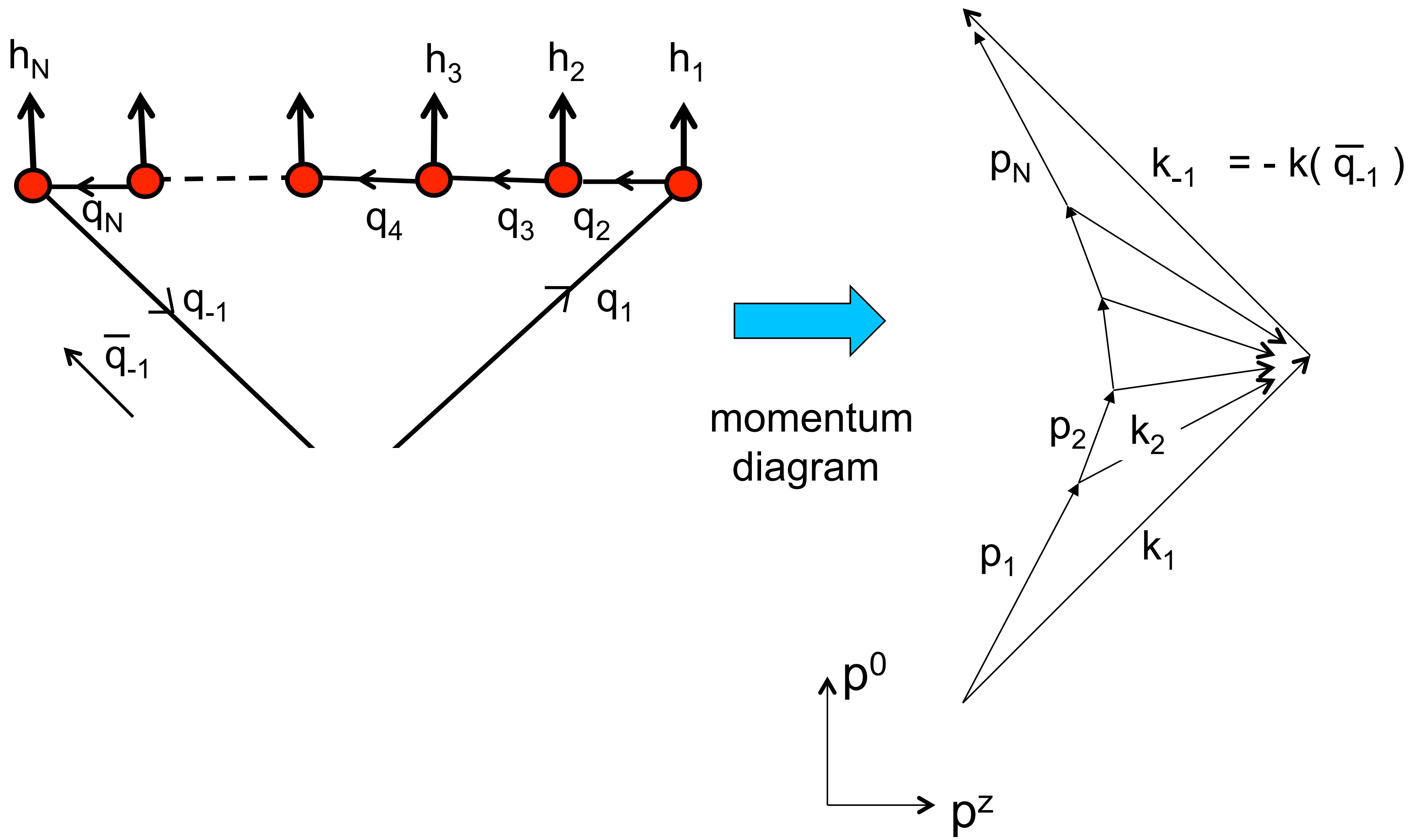
A bad model :

two **independent** cascades of "quark decays"



(no confinement)

Multiperipheral dynamics



The **two** cutoffs in the quark momenta

$$\boxed{k^2 = -|k^+k^-| - \mathbf{k}_T^2}$$

1) $\mathbf{k_T}$ cutoff $\underset{\not\longleftarrow}{\longrightarrow}$ $\mathbf{p_T}$ cuttof

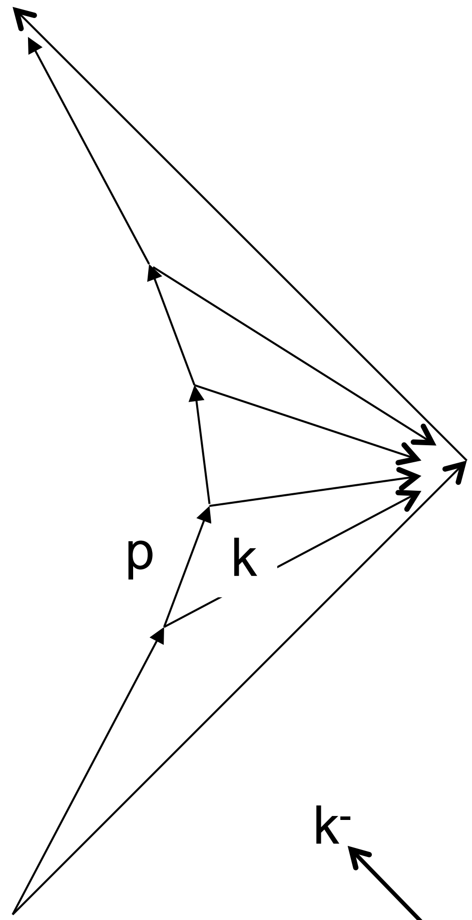
$\rightarrow$ **L**ocal **C**ompensation of the $\mathbf{p_T}$ ‘s

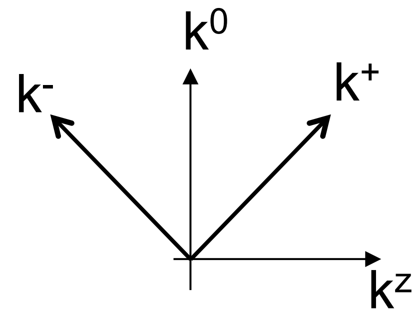
2) $|k^+k^-|$ cutoff

$\rightarrow$ cuttof in 1/z

$\rightarrow$ **S**trong **O**rdering in **R**apidity (hyperbolic chain of p’s)

- In the SFM these cutoffs are ***independent***





Recursive fragmentation without spin

(Petersson, Feynman & Field, …)

The TMD **splitting** distribution

for $k \to p + k'$: $F(k,k')\, dZ\, d^2\mathbf{k'}_T$

($Z = p^+ / k^+$)

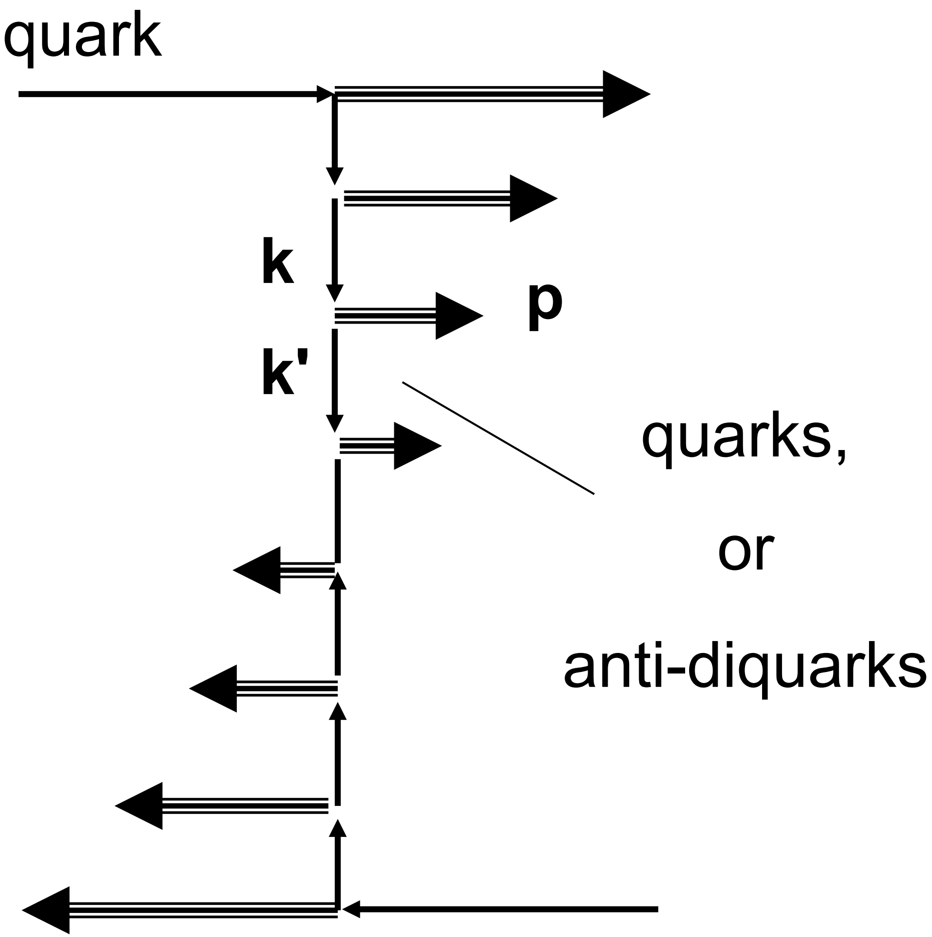
Feynman–Field,… : $F(Z, \mathbf{k'}_T)$

Lund-symmetric: $F(Z, \mathbf{k}_T, \mathbf{k'}_T)$

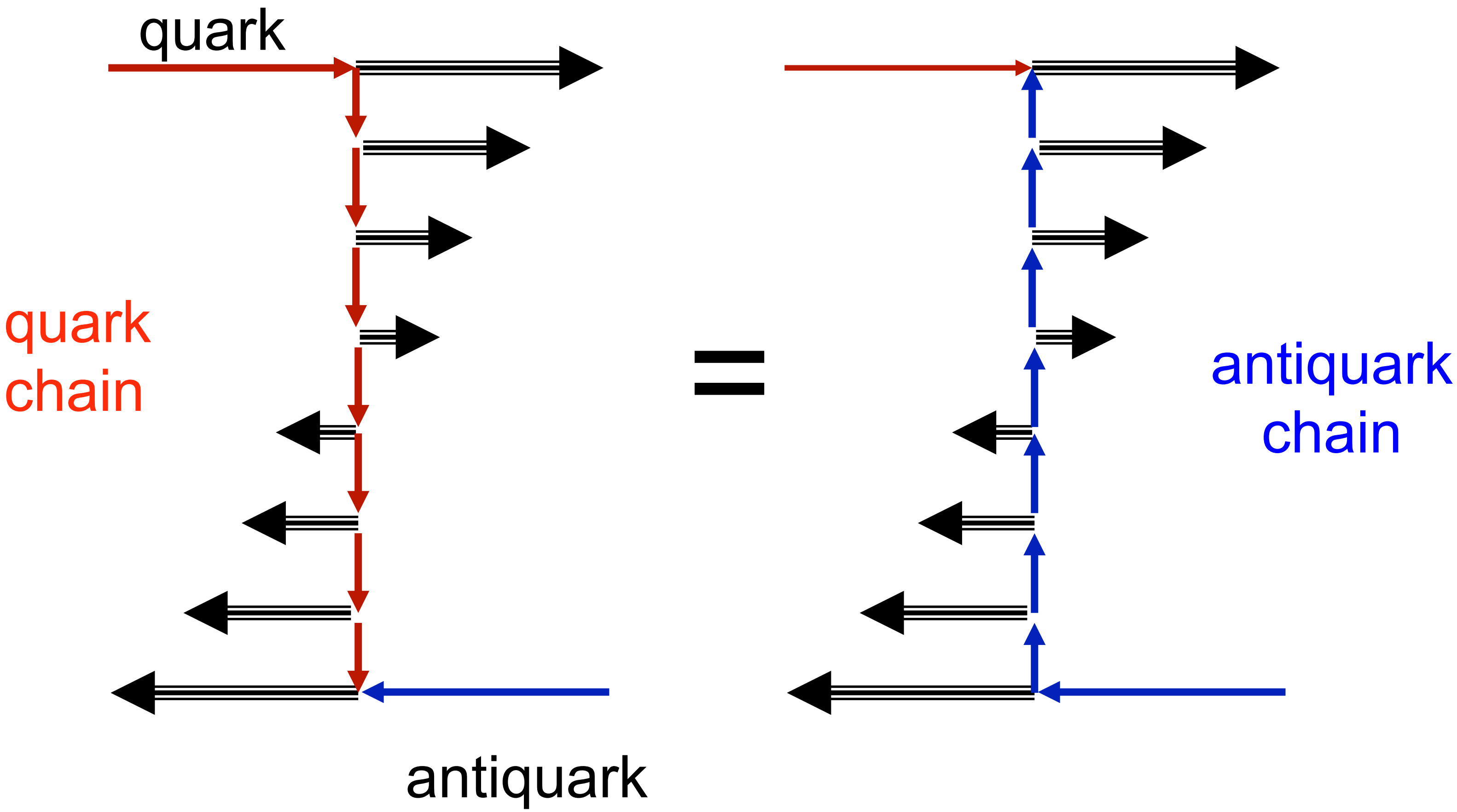
$F(Z, \mathbf{p}_T) =$ bad !

Implicit assumption:

F does not depend on k^-



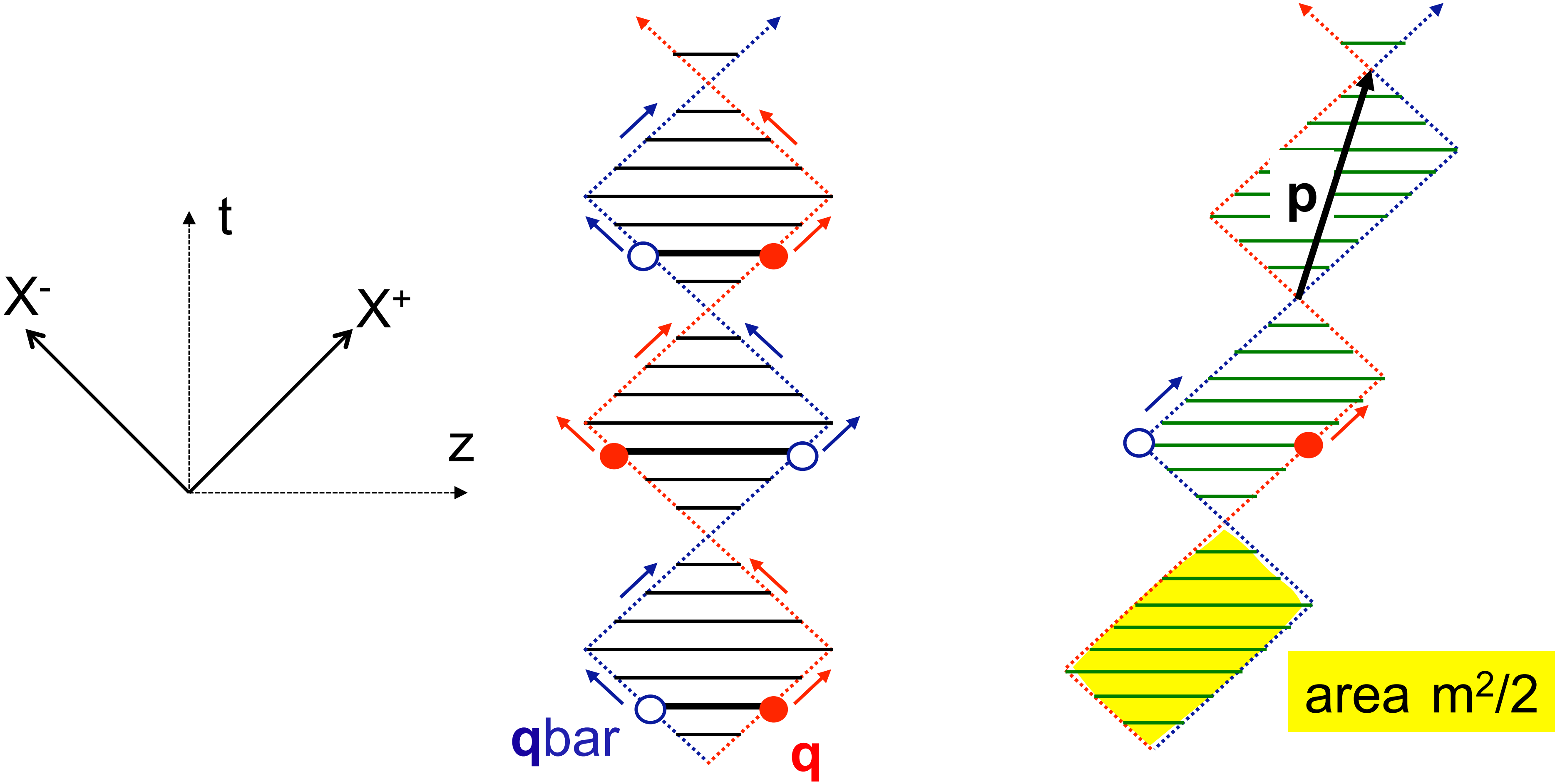
An important constraint :
Symmetry of ***quark line reversal***



→ Lund-symmetric model

Recall on the string fragmentation model (SFM)

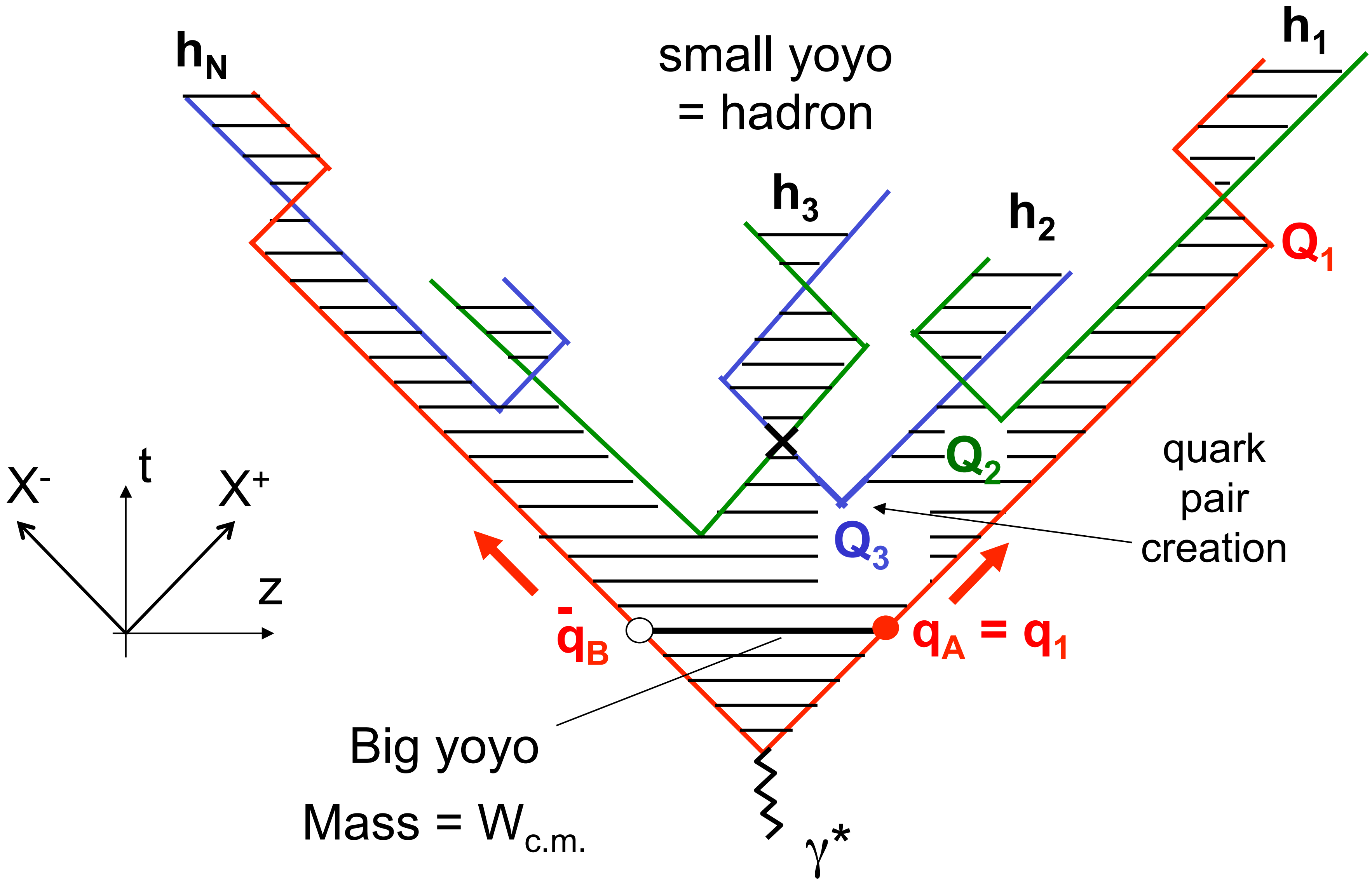
Simplest string motion : the relativistic yo-yo



yo-yo "at rest" yo-yo in motion

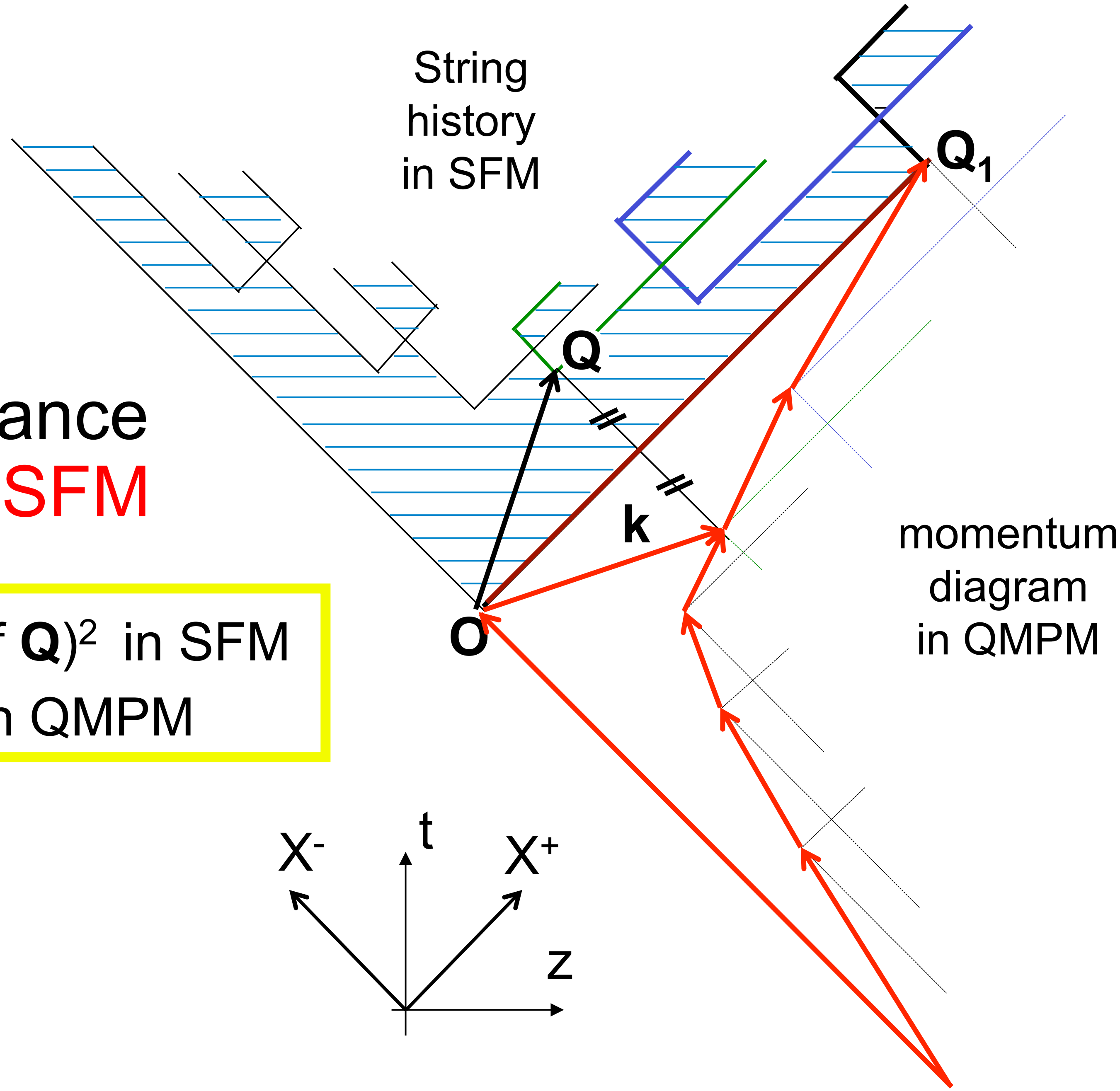
The string tension $\kappa \approx 1$ GeV/fermi is taken as unity

Space-time history of string fragmentation



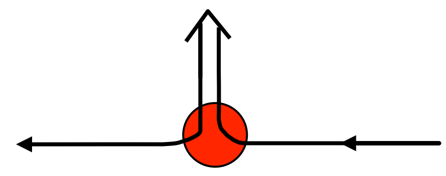
Correspondance
QMPM $\longleftrightarrow$ SFM

(proper time of **Q**)^2 in SFM
= $|k^+k^-|$ in QMPM



The PYTHIA splitting function

$$F(q\to h+q') = (\pi b_T)^{-1}\ \exp(-b_T \mathbf{k}'^2_T)$$
$$\times\ Z^{-1} \times (1-Z)^a\ \times\ \exp\{-\ Z^{-1}\, b_L\, (m_h^2+\mathbf{p}_T^2)\}$$
$$\times\ N^{-1}(m_h^2+\mathbf{p}_T^2)$$



The **PYTHIA algorithm :**

- draw $\mathbf{k}'_T$ first, with the $\exp(-b_T\mathbf{k}'^2_T)$ distribution
- draw Z with the distribution on the 2nd line

(the factor $N^{-1}(m_h^2+\mathbf{p}_T^2)$ normalizes this distribution to unity)

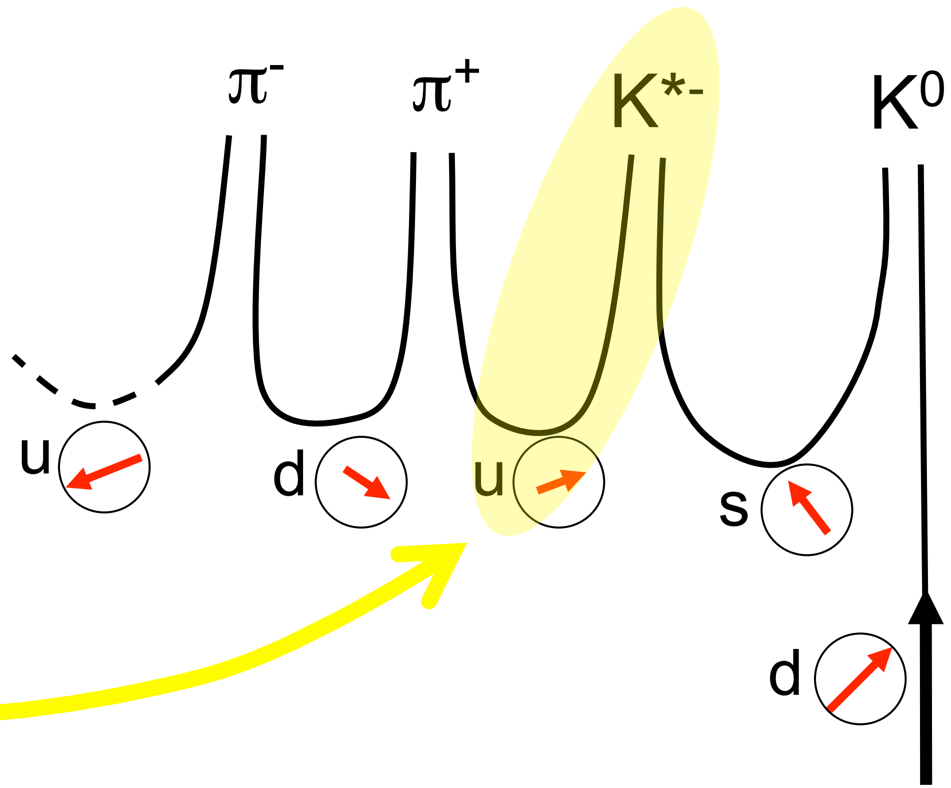
- no ($\mathbf{k}_T$, $\mathbf{k}'_T$) correlation. There should be a ***dynamical*** correlation, due to the factor $\exp\{-b_L(m_h^2+\mathbf{p}_T^2)/Z\}$ which penalizes large $|\mathbf{k}_T - \mathbf{k}'_T|$.
The factor $N^{-1}(m_h^2+\mathbf{p}_T^2)$ cancels it.

- In a version of our model, we reject this factor, thus restore the correlation.

Flavor and **spin** in a recursive Monte-Carlo

Spin density matrices :

$\rho(q) = ½\,(1 + \sigma.\mathbf{S}_q)$



Take care of

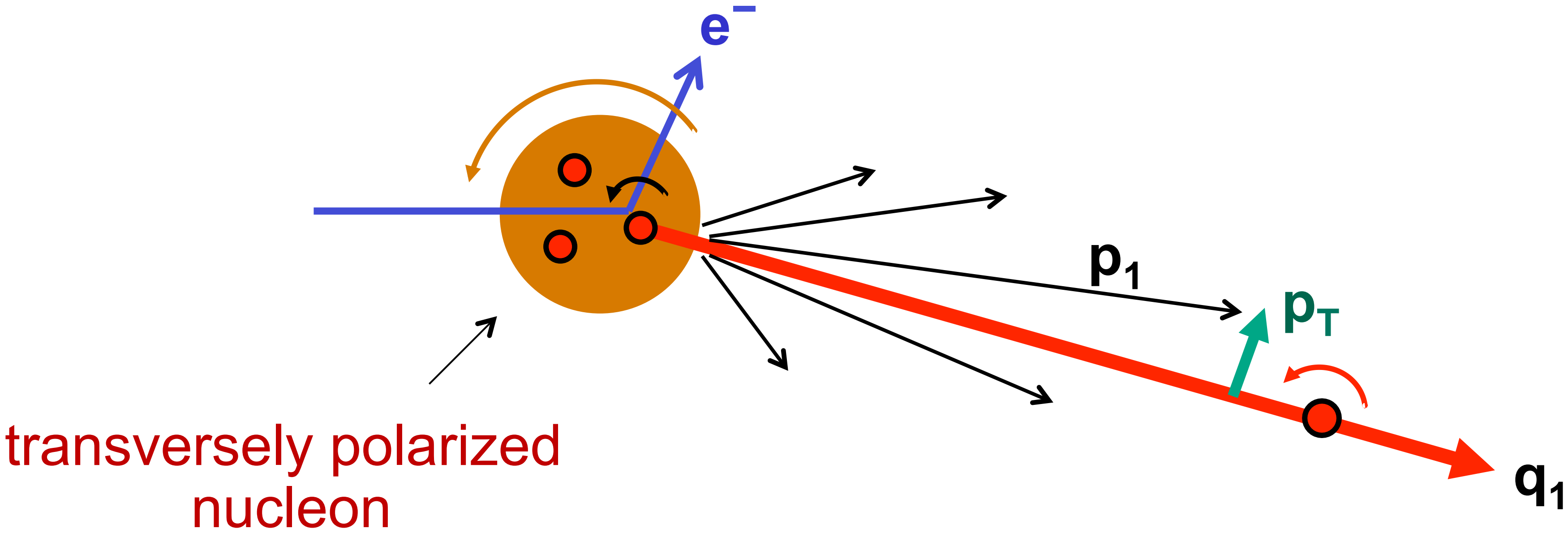
spin *entanglement,*

between K^{*-} and u

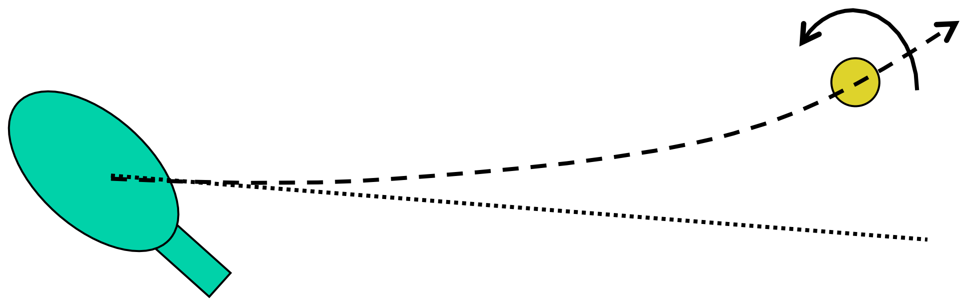
Use the Collins –

Knowles prescription

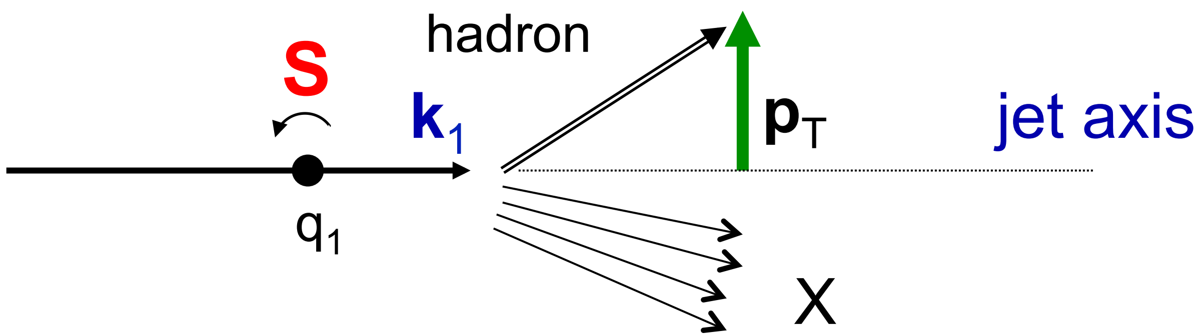
The Collins effect



~ *Magnus effect*
on a ping-pong ball



Collins analyzing power

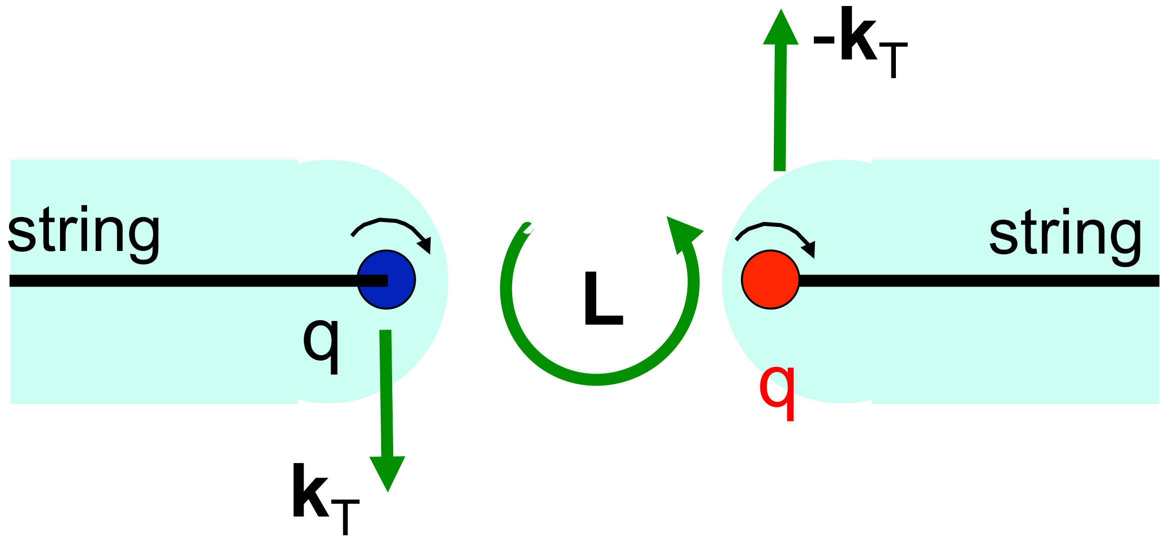


$$D_{q\rightarrow h+X}(z,\, \mathbf{p}_T\, ;\, \mathbf{S}) \;=\; D_{unpol.}(z,p_T)\;\; [1+ A(z,p_T)\; \mathbf{S}.(\mathbf{k}_1\times\mathbf{p}) \,/\, |\mathbf{k}_1\times\mathbf{p}|\;]$$

$A \in [-1,+1]$ = analyzing power

The « ***string + $^3P_0$*** » mechanism of Collins effect

Hypothesis : the (q-qbar) pair is created with the **vacuum quantum numbers**, therefore in the $\mathbf{0^{++}} = \mathbf{{}^3P_0}$ state



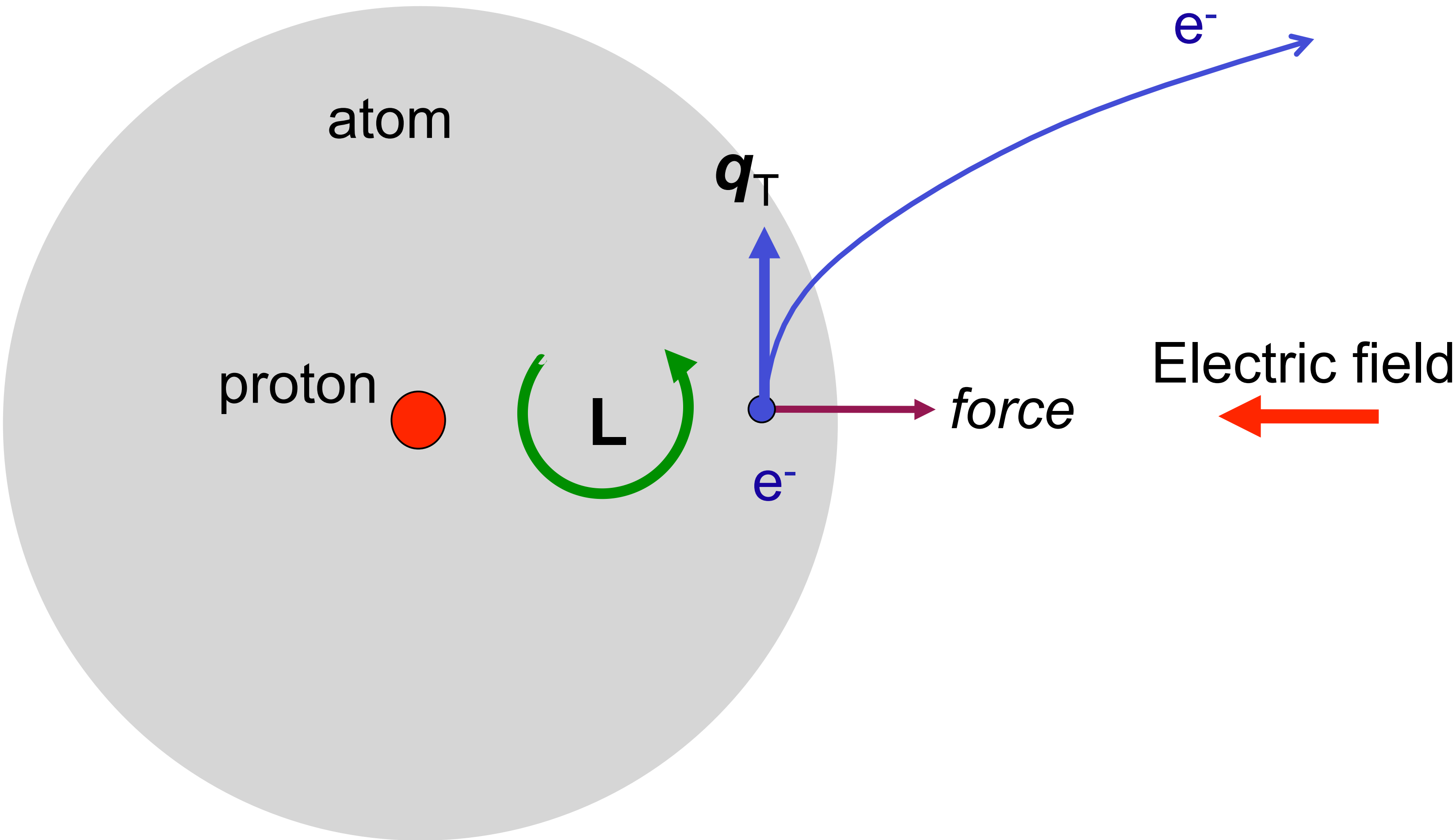
→ azimutal correlation between $\mathbf{k}_T$ and $\mathbf{S}_q$

(= **spin-orbit** effect)

Analogue in atomic physics :

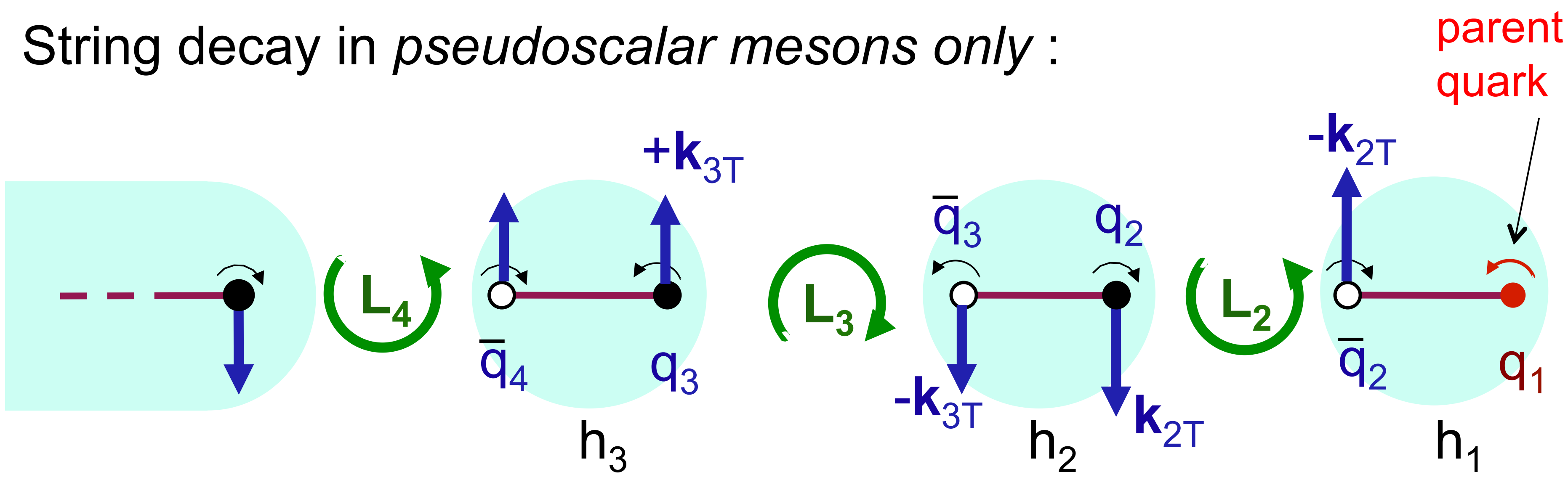
Strong field ionisation of a hydrogen (by tunnel effect)

[X.A. and E. Redouane-Salah, Phys. Rev. A **93** (2016)]



Iteration of the *string* + $^3P_0$ mechanism

String decay in *pseudoscalar mesons only* :

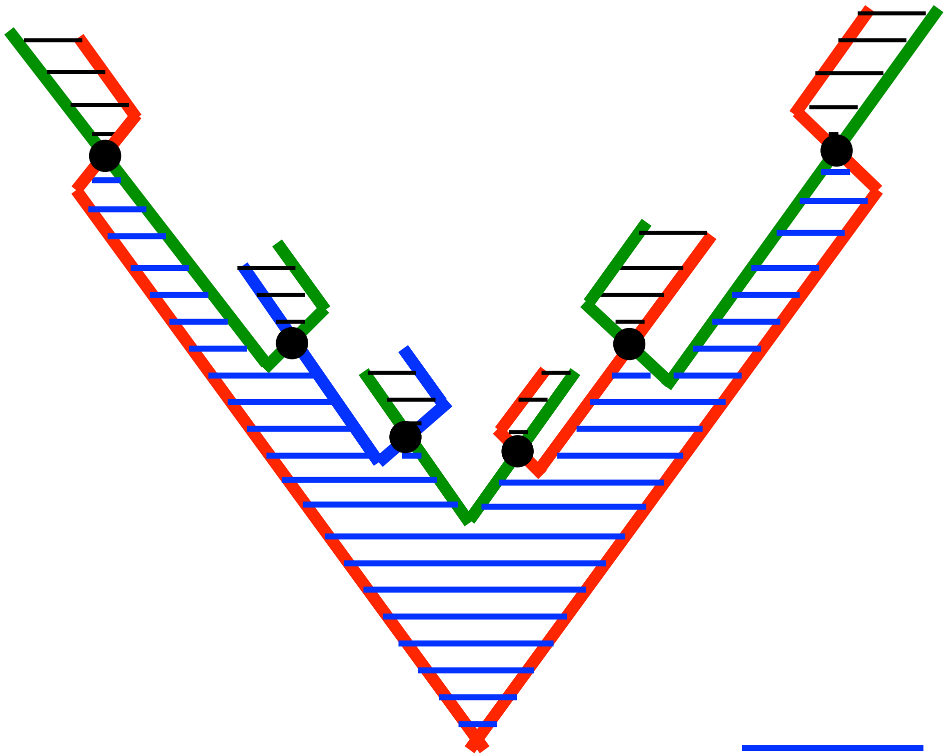


→ large Collins asymmetry for the "unfavored" fragmentation

→ $\langle \mathbf{p}_T^2 \rangle_{\textbf{vector meson}} < 2 \langle \mathbf{k}_T^2 \rangle_{\textbf{quark}} < \langle \mathbf{p}_T^2 \rangle_{\textbf{pion}}$

= "hidden spin" effect.

Semi-quantization of the SFM, *incuding spin*



Following Feynman,

to each *classical history*

we associate a

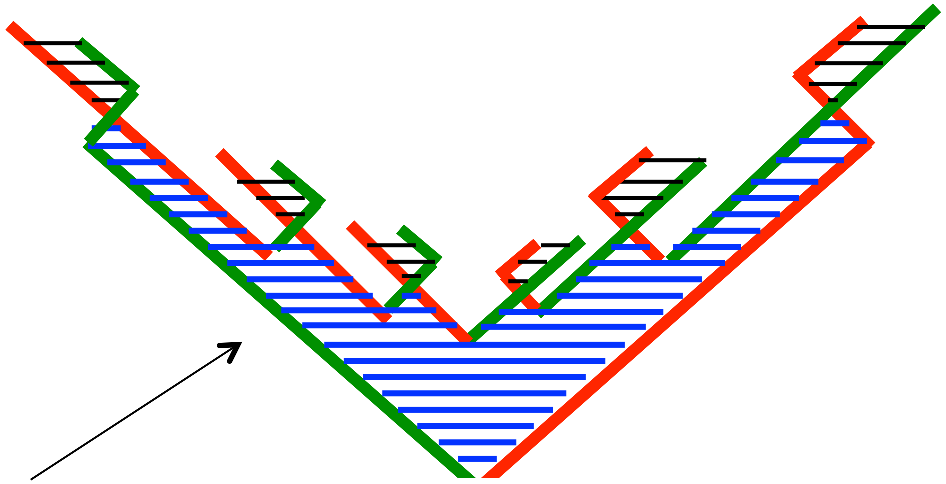
quantum amplitude

$M_N(k_{-1},k_N, \ldots k_2,k_1) = \exp\{ i \times (\text{string action}) \}$

$\times$ quark propagators

$\times$ vertex matrices

String action

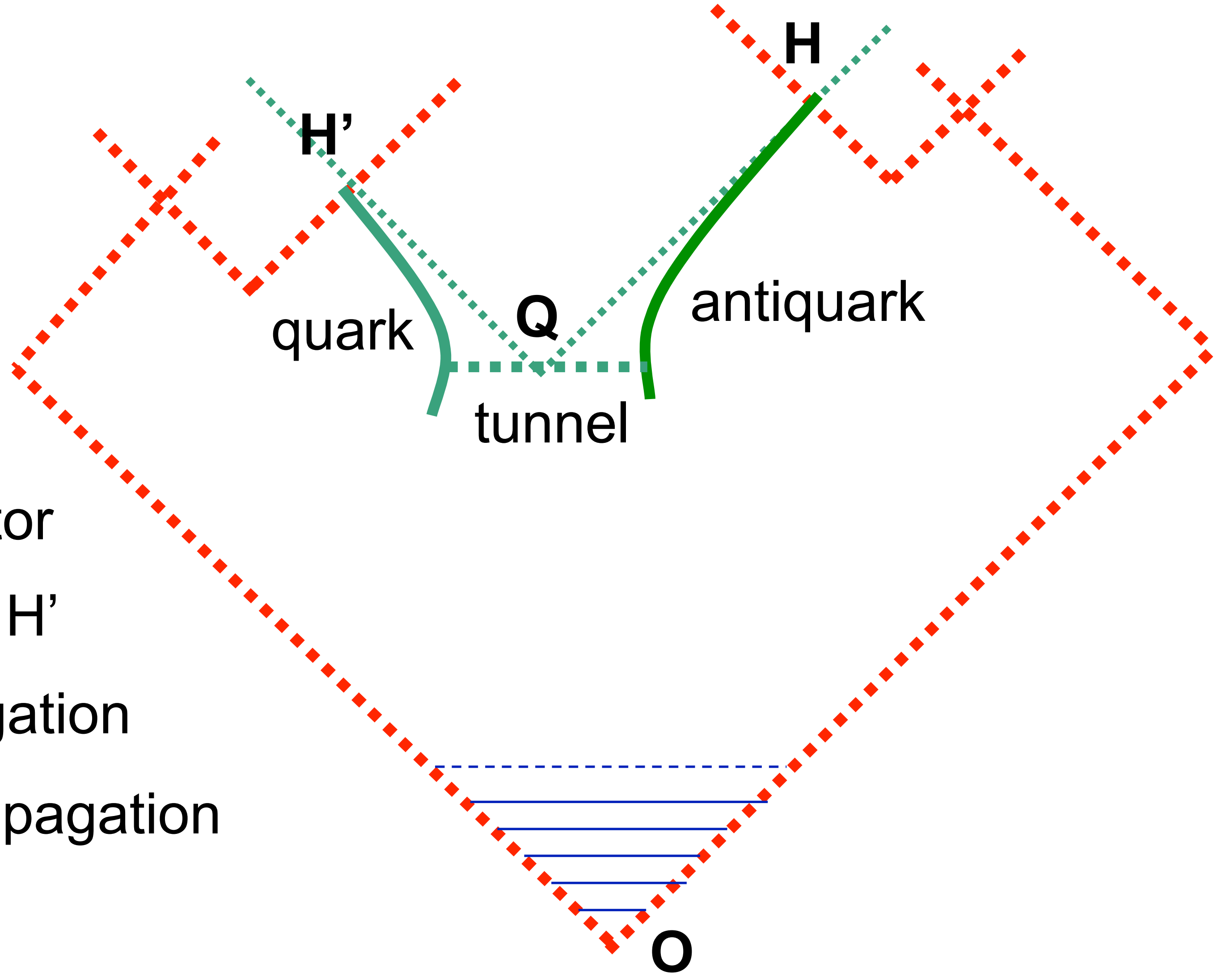


$A_{string} = - \ \kappa_C \times$ (blue-hatched area)

κ_C = complex string tension = $\kappa - i\, P/2$ $\quad$ P = *string fragility*

It gives the factor $\exp\{- Z^{-1}\, b_L\, (m_h^{\,2}+\mathbf{p}_T^{\,2})\}$ of the Lund symmetric model, with $b_L = P/(2\kappa^2)$

Quark propagator (1/3)



Quark propagator

between H and H’

= quark propagation

× antiquark propagation

× tunnel factor

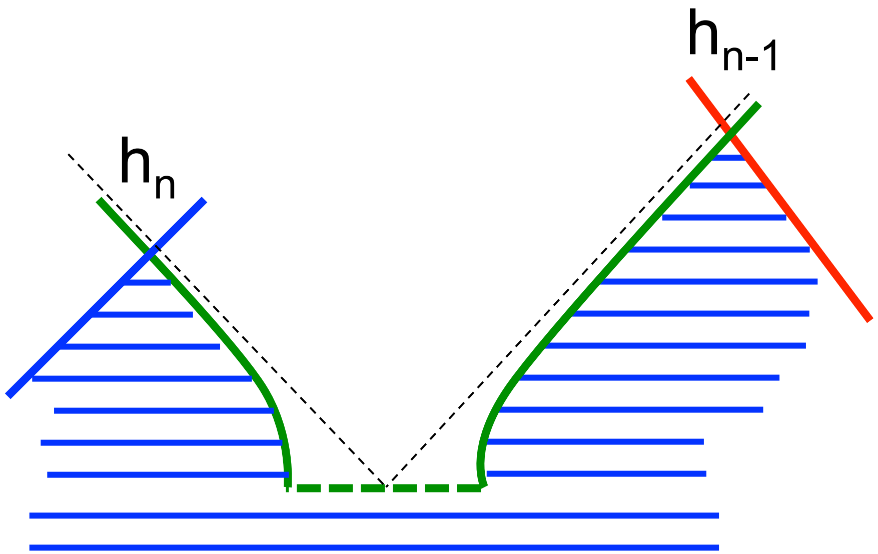
× spin-dependent “Feynman numerator”

quark propagator (2/3): spin-independent part

Schwinger mechanism :

$D = (p^+_n \, p^-_{n+1})^\alpha \;\; \exp\{-\pi(m_q^{\,2}+k_T^{\,2})/\kappa\}$

with $\alpha = (m_q^{\,2}+k_T^{\,2})/\kappa$



tunnelling

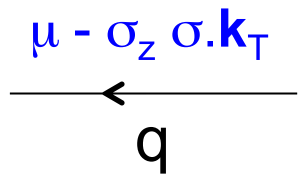
It supresses the heavy quarks

and gives the factors $(1-Z)^a \times \exp(-b_T \mathbf{k}'^{\,2}_T)$

of the Lund splitting function

Quark propagator (3/3): spin-dependent part

We use **Pauli** instead of **Dirac** spinors by projecting the Dirac spinors on the 2D-subspace $\alpha_z\ \psi = \psi$



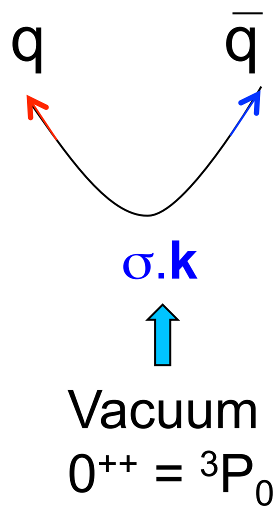
Motivation : $<\alpha_z> = v_z \approx +1$ for a fast quark.

Doing so, we have lost 1 of the 2 q-bits of information carried by an off-mass-shell Dirac spinor.

The Feynman numerator $\gamma{\cdot}k+m_q$ becomes $\mu - \sigma_z\ \sigma.\mathbf{k}_T$

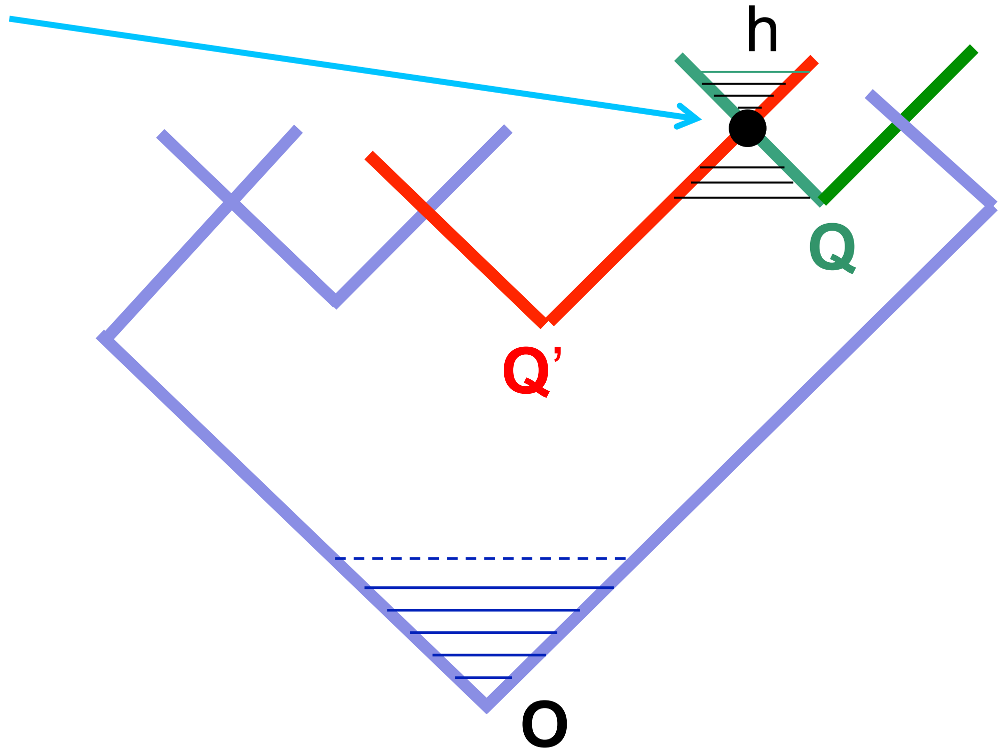
The results of the classical $^3P_0$ model are recovered in the quantum model if $\mathrm{Im}(\mu\ ) > 0$.

We also get a *jet handedness.*



Initial motivation : the $^3P_0$ amplitude for a q-qbar pair is
$\sigma.\mathbf{k} = \sigma_z\ (\ k_z\ + \sigma_z\ \sigma.\mathbf{k}_T)$ in the rest frame of the pair.

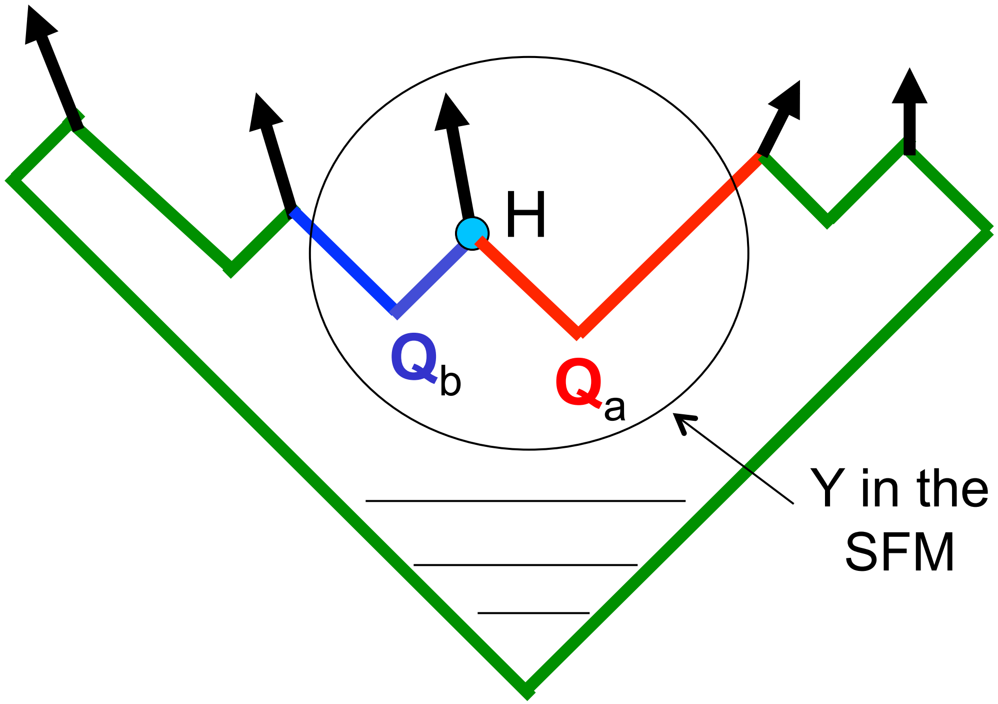
Vertex matrices

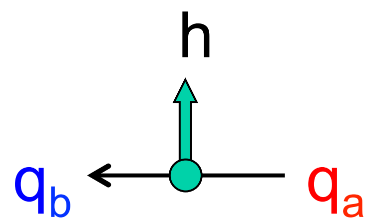


Pseudo-scalars (π, K, η^0) : $V = \sigma_z$ (analogue of γ_5)

Vector meson : $V = G_L A_z + G_T \sigma_T.\mathbf{A}_T \sigma_z$

New approach : start from a “renormalized vertex” Y, which includes the adjacent propagators.



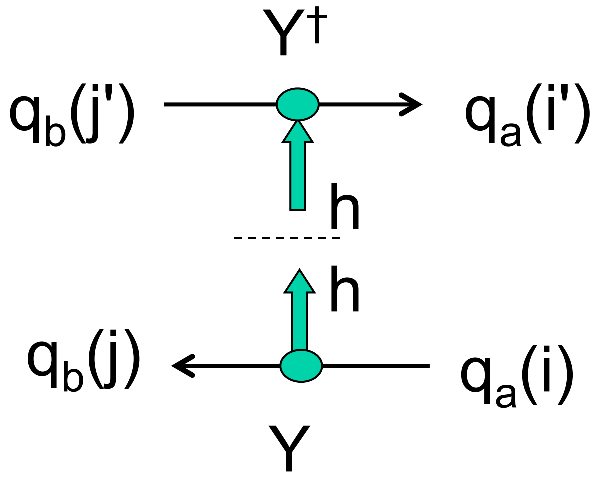


1) SFM without spin, $\mathbf{k}_T$ and flavor :

$$W(k_b,k_a) = |Y|^2 = \exp(-b_L\, Q_b^+ Q_a^-) \times (Q_b^+ Q_a^-)^a \times \delta(p^2 - m_h^2)$$

$W(k_b,k_a)\,\delta(p^2 - m_h^2)$ = *double density in* Q_a and Q_b

2) including quark spin and $\mathbf{k}_T$



Case of pseudoscalar mesons:

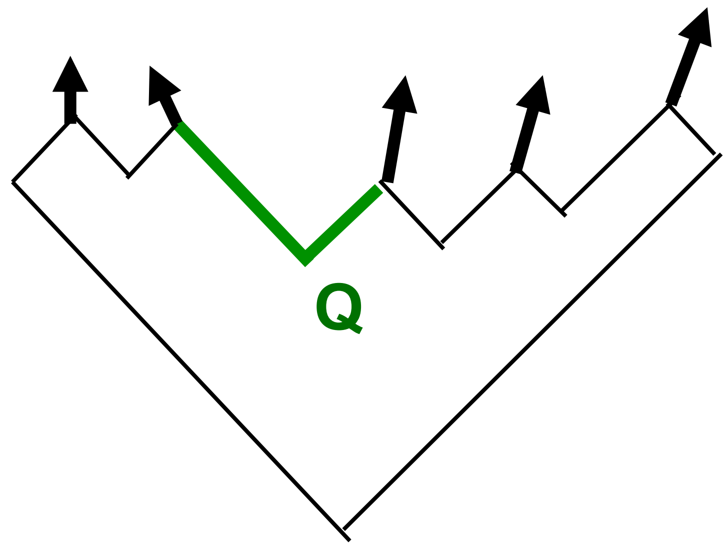
$$Y = Y(\text{without spin and } \mathbf{k}_T) \times \exp(-b_T\, \mathbf{k}_T^2)$$

$$\times\ (\mu - \sigma_z\, \sigma.k_{aT})\ \sigma_z\ (\mu - \sigma_z\, \sigma.k_{bT}\ )$$

$$W(k_b,k_a) = Y^\dagger(k_b,k_a)\ Y(k_b,k_a)$$

W is a ***density matrix*** w.r.t. the spins of q_a and q_b .

Preliminary task: calculate the ***single-quark*** density U(k)



by partial integration of the double-quark density W :

$$\langle i' \mid U(k) \mid i \rangle = \int d^4k_b \, \delta(p^2 - m_h^2) \, \sum_j \, \langle i'j \mid W(k,k_a) \mid ij \rangle$$

$$U(k) = \underbrace{\exp(-b_L \, Q^+Q^-) \times (Q^+Q^-)^a}_{\text{SFM without spin and } \mathbf{k}_T} \times [\, \beta(\mathbf{k}_T^2) + \gamma(\mathbf{k}_T^2)\, \sigma \cdot (\mathbf{z} \times \mathbf{k}_T) \,]$$

Monte-Carlo algorithm for the splitting $q_a(k_a) \rightarrow h(p) + q_b(k_b)$

We know the momentum k_a and the spin density matrix $\rho(q_a)$.

- **step 1 :** draw k_b with the splitting distribution

$$F(q_a \rightarrow h+X) \quad \propto \quad \mathrm{Trace}\{\mathbf{T}\, \rho(q_a)\, \mathbf{T}^\dagger\}$$

with $\mathbf{T}(k_b,k_a) = Y(k_b,k_a)\, U^{-1/2}(k_a)$

- **step 2 :** calculate $\rho(q_b) = \mathbf{T}\, \rho(q_a)\, \mathbf{T}^\dagger$

- **iterate steps 1 and 2**

Results 1) without the *dynamical* ($\mathbf{k}_T$, $\mathbf{k}'_T$) correlation.

The code is easy to program in FORTRAN

Parameters : a=0.5 ;

b_L=6 ; b_T=25 (GeV^{-2})

Re(μ)=0.1 ; Im(μ)=0.2 ; $m_\pi=m_K=m_\eta$=0.14 (GeV)

We generated 2. 10^5 jets, each of 20 particles (π^{+-0}, K^{+-0}, η).

r.m.s.($\mathbf{k}_T$) = 0.24 ; r.m.s.($\mathbf{p}_T$) = 0.38

$\langle \mathbf{k}_T . \mathbf{k}'_T \rangle / \langle \mathbf{k}_T^2 \rangle$ = - 0.27 (correlation mediated by quark spin)

$\langle \mathbf{p}_T . \mathbf{p}'_T \rangle / \langle \mathbf{p}_T^2 \rangle$ = - 0.68 (local compensation of $\mathbf{p}_T$)

Results without the *dynamical* $\mathbf{k}_T$ correlation (continued)

Collins effect :

$\langle p_x \rangle$ (rank 1) = + 0.09
$\langle p_x \rangle$ (rank 2) = - 0.14
$\langle p_x \rangle$ (rank 3) = + 0.08

For all ranks, we used the *estimator*

$$C = \langle E \rangle / \mathrm{sqrt} \langle E^2 \rangle$$

with

$$E = p_x / (|\mu|^2 + \mathbf{p}_T^2), \text{ for a quark polarized along } \mathbf{y}.$$

We get : C(positive hadrons) = + 0.14,
C(negative hadrons) = - 0.09

Results - 2) with the *dynamical* ($\mathbf{k}_T$, $\mathbf{k}'_T$) correlation.

This version needs the preliminary task. It was programmed by A. Kerbizi.

Left figure: A_C (z) (simulated) for π^+, π^- , K^+, K^-

Middle figure: A_C (p_T) (simulated) for π^+ and π^-

Right figure: comparison between simulation and Belle data for

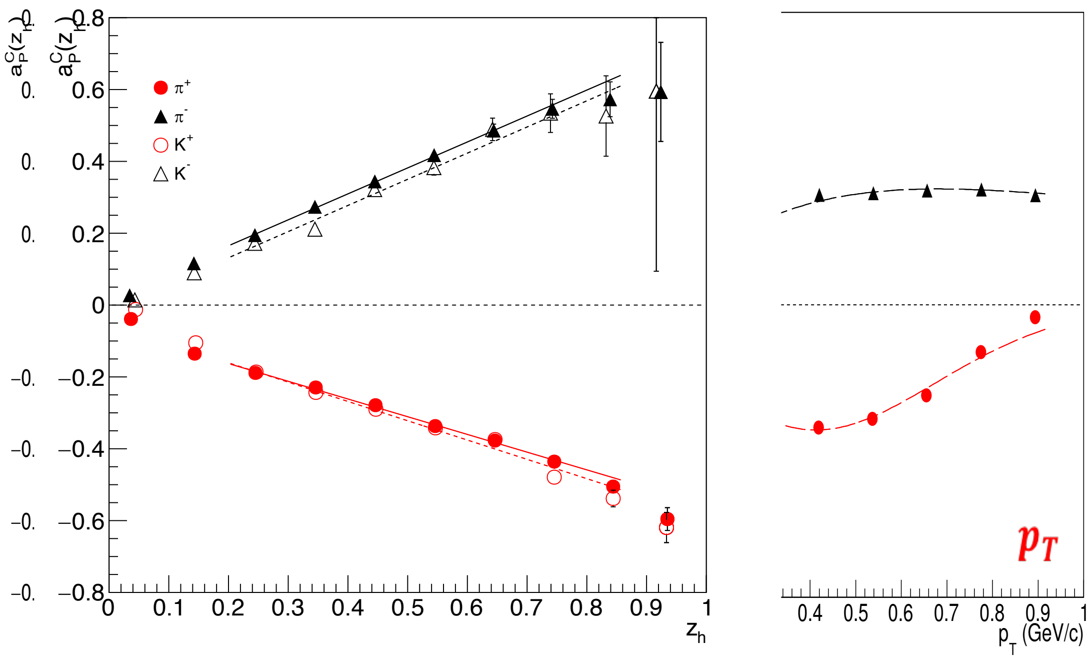
< $A_{Collins}$ ($\pi^+\pi^-$ pairs)> × $A_{Collins}$ (invariant $\pi^+\pi^-$ mass).

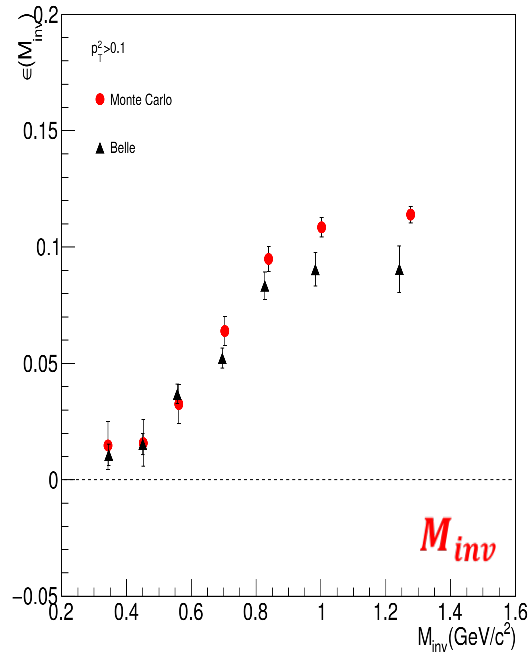
With the estimator

$$A_C = < \sin [\text{azimuth}(\mathbf{p}_T/p_T - \mathbf{p}'_T/p'_T) - \text{azimuth(spin)}] >$$

Cuts: $z > 0.1$ and $p_T^2 > 0.1$ GeV^2

Simulations of the fragmentation process of transversely pol. $\boldsymbol{u}$ quarks: Collins analyzing power for single hadrons and hadron pairs [1]





- Transverse initial polarization
- No primordial k_T
- Fixed initial energy
- Restriction to pseudoscalar mesons $(\pi^{\pm}, K^{\pm} \dots)$

- The sign of the analyzing power is opposite to the hadrons charge
- Linear z_h dependence: stronger slope for negative hadrons
- Stronger p_T dependece for positive hadrons

- $\epsilon(M_{inv}) = \langle a_P^{u\to\pi^+\pi^-} \rangle a_P^{u\to\pi^+\pi^-}(M_{inv})$
- $z_h > 0.1,\ p_T^2 > 0.1 \left(\frac{GeV}{c}\right)^2$ for each pion
- Good comparison with Belle data [2]

Conclusion

We have started to develop and test a recursive jet model which includes the spin degree of freedom, in the string framework. The results are encouraging.
Before implementing in codes like PYTHIA, much work remain to be done :

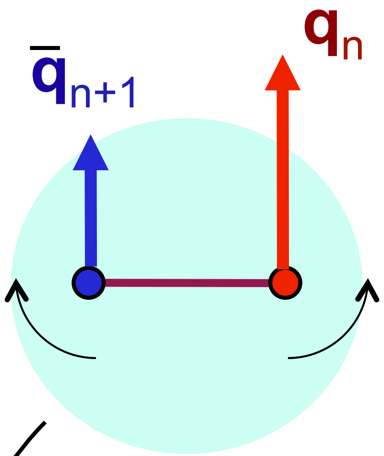
- Include the resonances (at least the vector mesons)
- Include the baryons

Many theoretical questions arised, e.g. the forgotten q-bit.

Thank you for attention !

Hidden spin effects (effect without external polarization)

The $\mathbf{S}_q$ - $\mathbf{q}_T$ correlation acts upon the $\langle \mathbf{p}_T^2 \rangle$ of the *unfavored* hadrons.

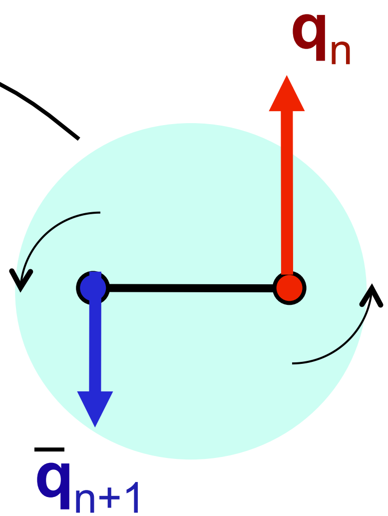


$$\langle \mathbf{p}_T^2 \rangle_{\mathbf{pion}} > 2 \, \langle \mathbf{q}_T^2 \rangle_{\mathbf{quark}}$$

For a *vector meson,* of linear polarization

- *along Oz* : $\langle \mathbf{p}_T^2 \rangle_{\mathbf{v.m.}} < 2 \, \langle \mathbf{q}_T^2 \rangle_{\mathbf{quark}}$

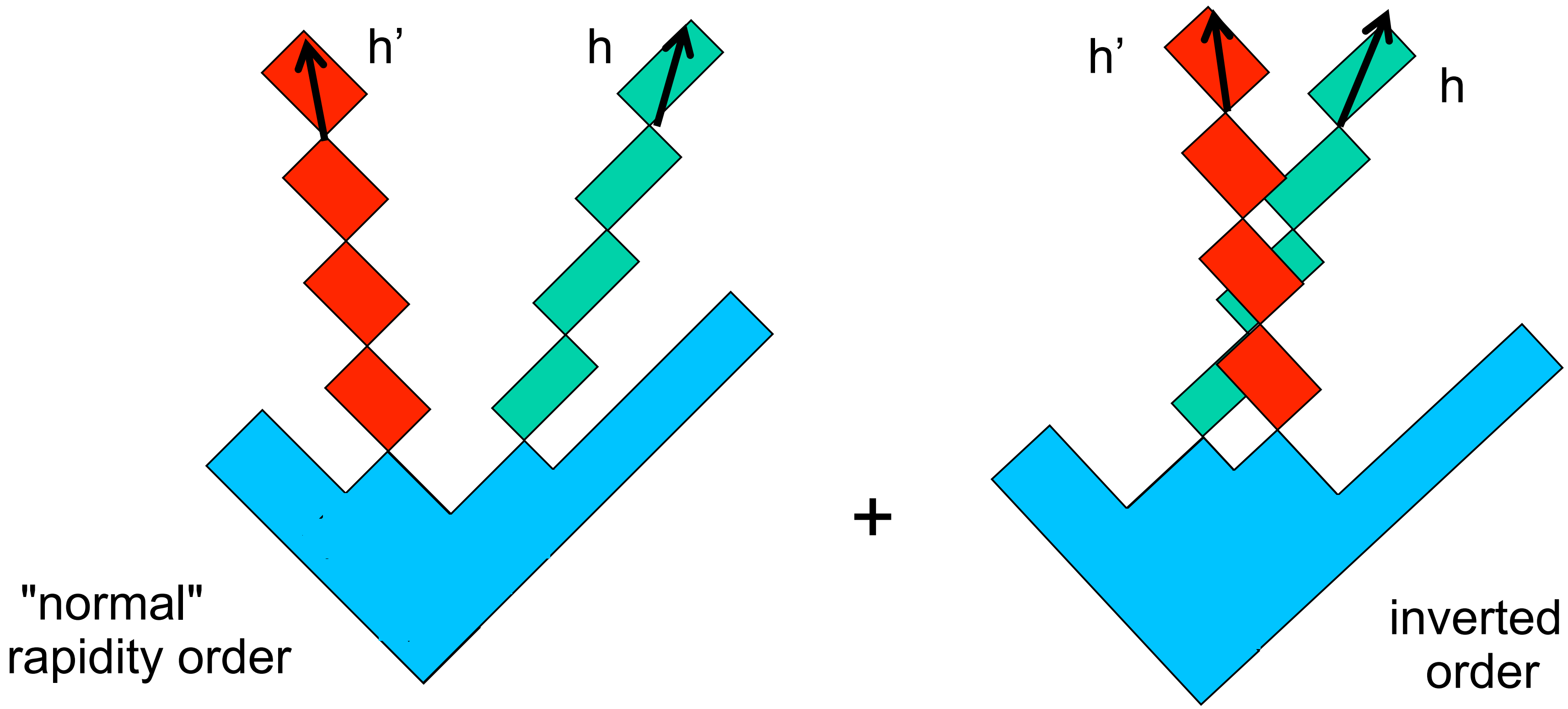
- *along Ox* :



$$\langle \mathbf{p}_{\mathbf{x}}^2 \rangle_{\mathbf{v.m.}} < \langle \mathbf{q}_T^2 \rangle_{\mathbf{quark}} < \langle \mathbf{p}_{\mathbf{y}}^2 \rangle_{\mathbf{v.m.}}$$

On the average, $\langle \mathbf{p}_T^2 \rangle_{\mathbf{v.m.}} < \langle \mathbf{p}_T^2 \rangle_{\mathbf{pion}}$

Interference between permuted *diagrams*

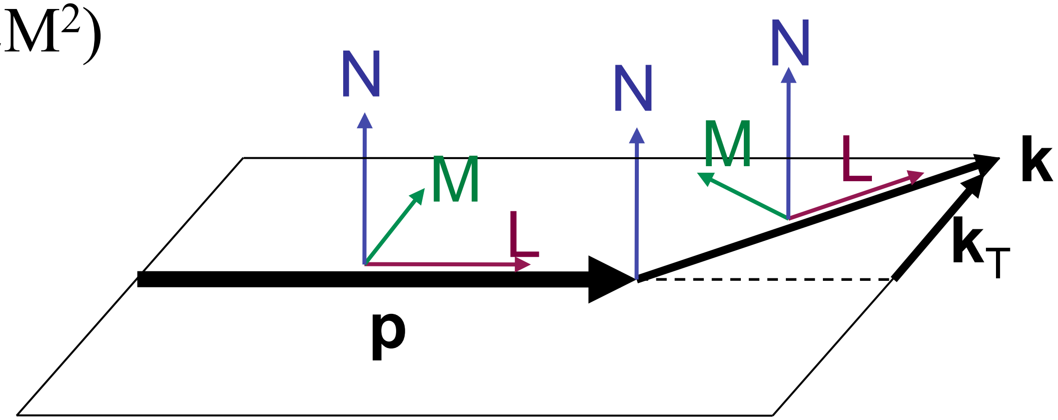


- The amplitude has a factor exp {− i(κ−i b) × bleu area}.
The different blue areas lead to a phase difference between the two amplitudes.
Their interference can give a 2-particle relative Collins effect.
- For identical hadrons, it gives a Bose-Einstein correlation
[Anderson & Hoffman].

(parenthesis) TMD-PDF's in Cartesian parameters of the Argonne convention

$$f_1\, C_{N0} = f_{1T}^{\perp} \times k_T/M \qquad \text{(Sivers)}$$
$$f_1\, C_{0N} = -\, h_1^{\perp} \times k_T/M \qquad \text{(Boer - Mulders)}$$
$$f_1\, C_{LL} = g_1$$
$$f_1\, C_{NN} = h_1 - h_{1T}^{\perp} \times (k_T^2/2M^2)$$
$$f_1\, C_{MM} = h_1 + h_{1T}^{\perp} \times (k_T^2/2M^2)$$
$$f_1\, C_{ML} = g_{1T} \times k_T/M$$
$$f_1\, C_{LM} = h_{1L}^{\perp} \times k_T/M$$

$$C_{ij} \in [-1,+1]$$



$$h_1 / f_1 = (C_{NN} + C_{MM})/2$$
$$h_{1T}^{\perp} / f_1 = (C_{NN} - C_{MM}) \times (M/k_T)^2 \qquad \text{(Kotzinian-Mulders-Tangerman)}$$