Quark transverse dynamics from Lattice QCD

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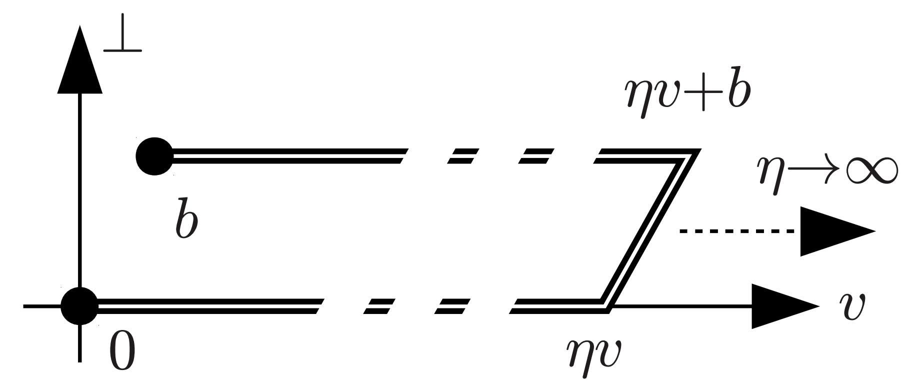
Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \ldots, b]\ q(b)\ |P, S\rangle$$

$$\Phi^{[\Gamma]}(x, k_T, P, S, \ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi)P^+} \exp\left(ix(b \cdot P) - ib_T \cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots)}{\widetilde{\mathcal{S}}(b^2, \ldots)}\Bigg|_{b^+=0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of Wilson line $\mathcal{U}$
- $\widetilde{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

Gauge link structure motivated by SIDIS



Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \to \infty$. Perturbative evolution equations for large $\hat{\zeta}$.

Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_H} f_{1T}^{\perp}\right] \text{odd}$$

$$\Phi^{[\gamma^+\gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_H^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_H} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij} k_j}{m_H} h_1^{\perp}\right] \text{odd}$$

Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\begin{aligned}
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} &= \widetilde{A}_{2B} + i m_H \epsilon_{ij} b_i S_j \widetilde{A}_{12B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+\gamma^5]}_{\text{unsubtr.}} &= -\Lambda \widetilde{A}_{6B} + i[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)]\widetilde{A}_{7B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[i\sigma^{i+}\gamma^5]}_{\text{unsubtr.}} &= i m_H \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} \\
&\quad - i m_H \Lambda b_i \widetilde{A}_{10B} + m_H[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)] b_i \widetilde{A}_{11B}
\end{aligned}$$

(Decompositions analogous to work by Metz et al. in momentum space)

Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected x-integrated TMDs in Fourier (b_T) space (showing just the ones relevant for Sivers, Boer-Mulders shifts), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

Generalized shifts from amplitudes

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

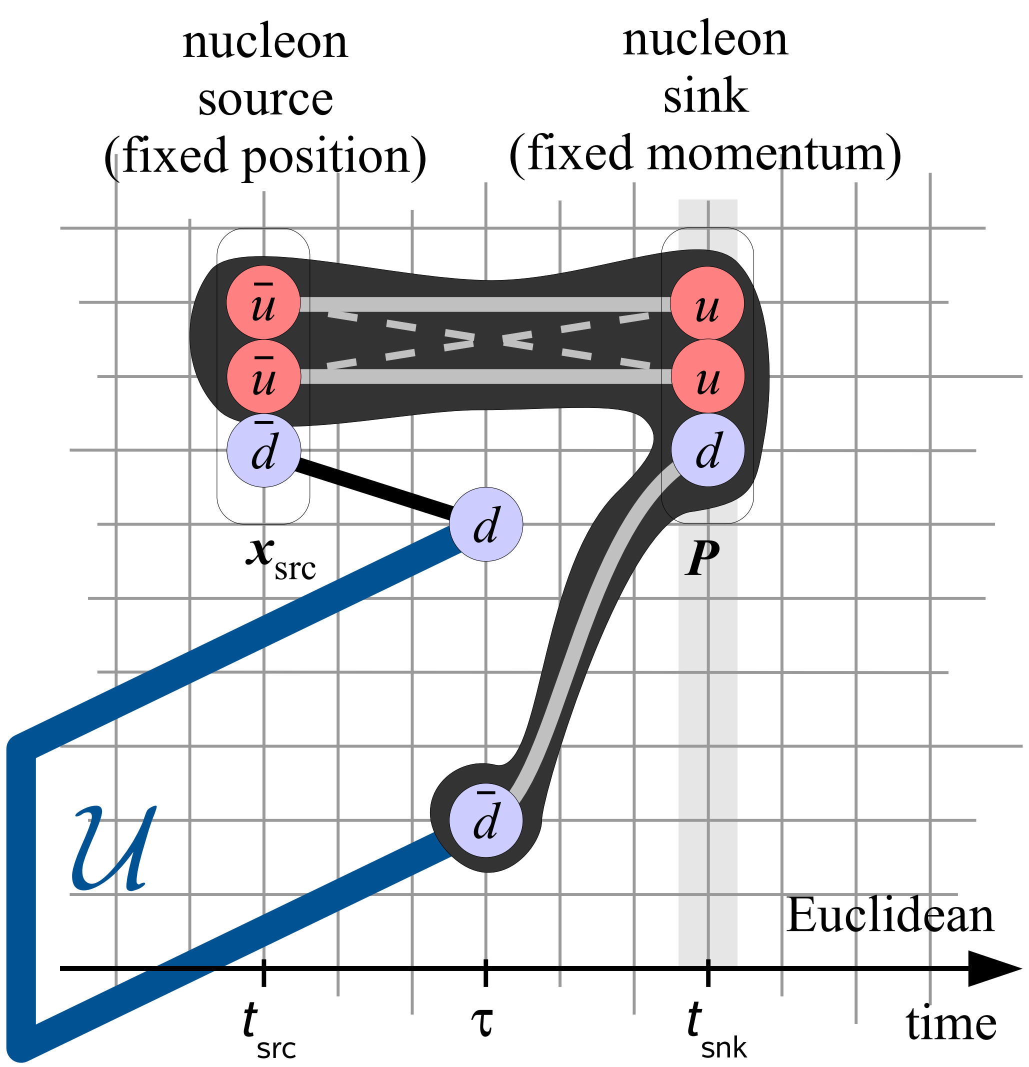
Boer-Mulders shift:

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) \equiv m_H \frac{\tilde{h}_1^{\perp[1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = m_H \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Sivers shift (in a polarized hadron):

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) = -m_H \frac{\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Lattice setup



- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu)$
  $\equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$
- Euclidean time: Place entire operator at one time slice, i.e., b, ηv purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame; form desired $\widetilde{A}_i$ ratios.
- Use variety of P, b, ηv; here $b \perp P$, $b \perp v$ (lowest x-moment, kinematical choices/constraints)
- Extrapolate $\eta \to \infty$, $\hat{\zeta} \to \infty$ numerically.

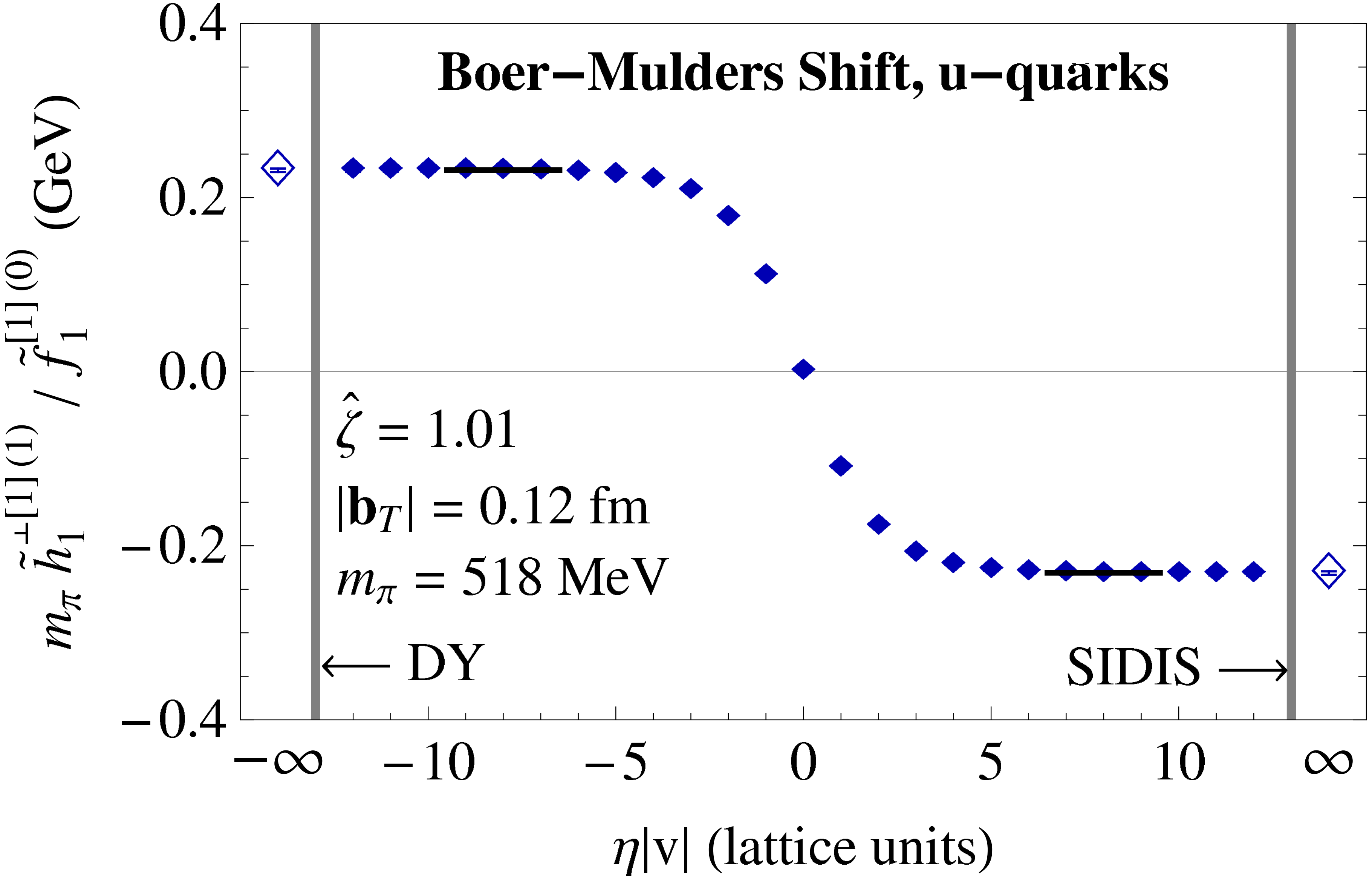
Challenges

- The limit $\hat{\zeta} \to \infty$: Approaching the light cone
- Discretization effects, soft factor cancellation on the lattice in TMD ratios
- Progress toward the physical pion mass

Approaching the light cone (with a pion)

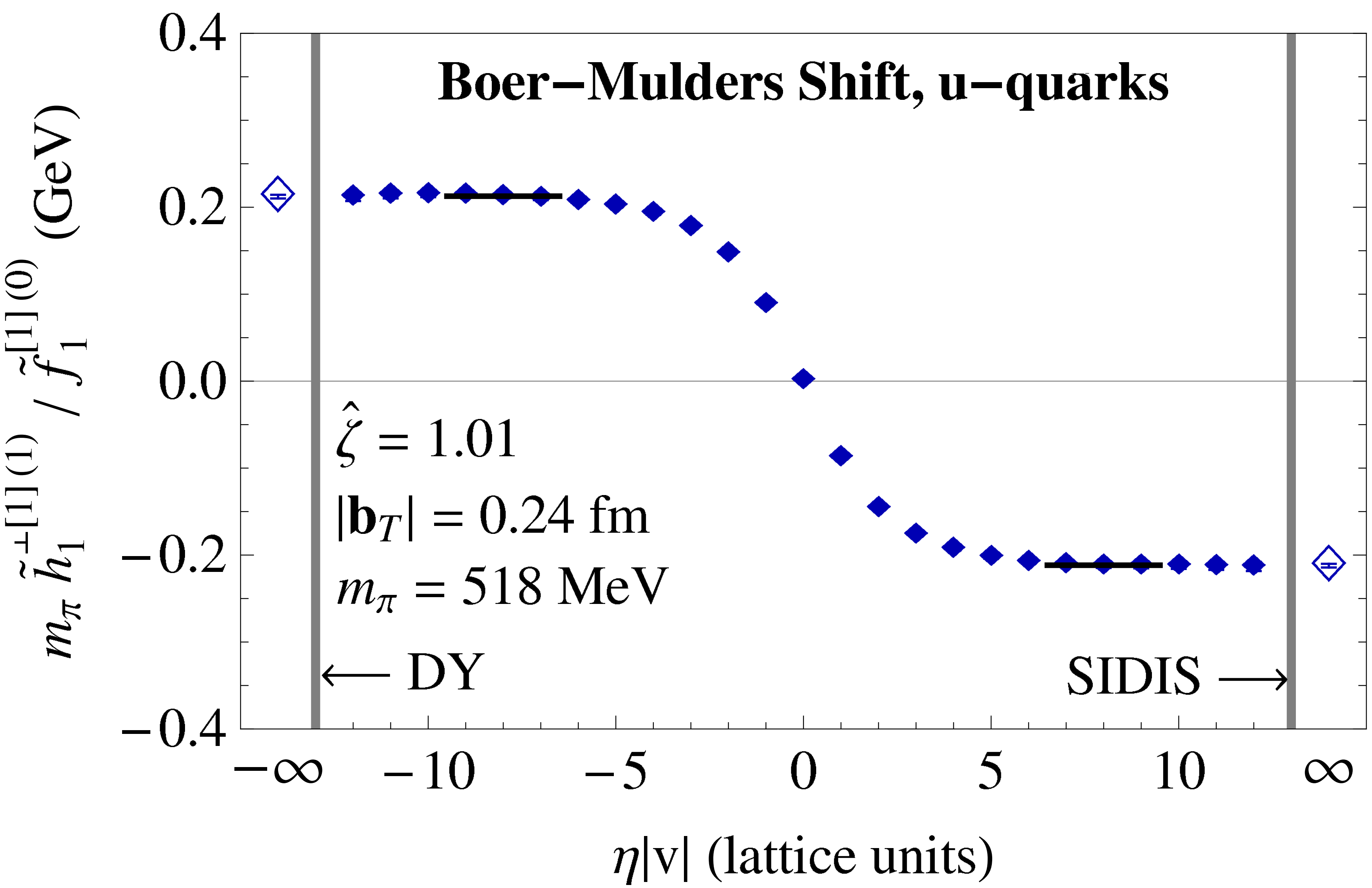
Results: Boer-Mulders shift (pion)

Dependence on staple extent; sequence of panels at different $|b_T|$



Results: Boer-Mulders shift (pion)

Dependence on staple extent; sequence of panels at different $|b_T|$



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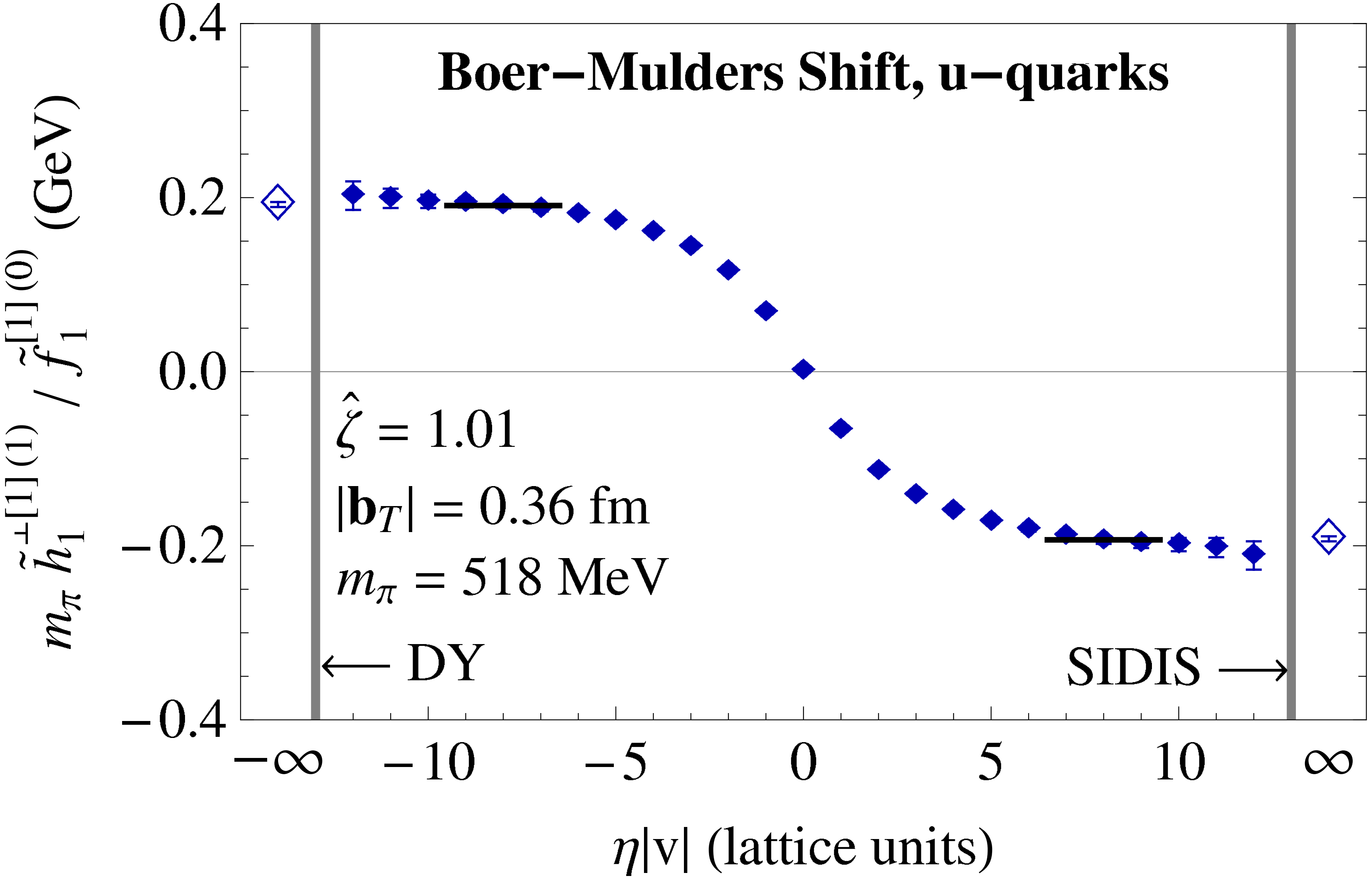
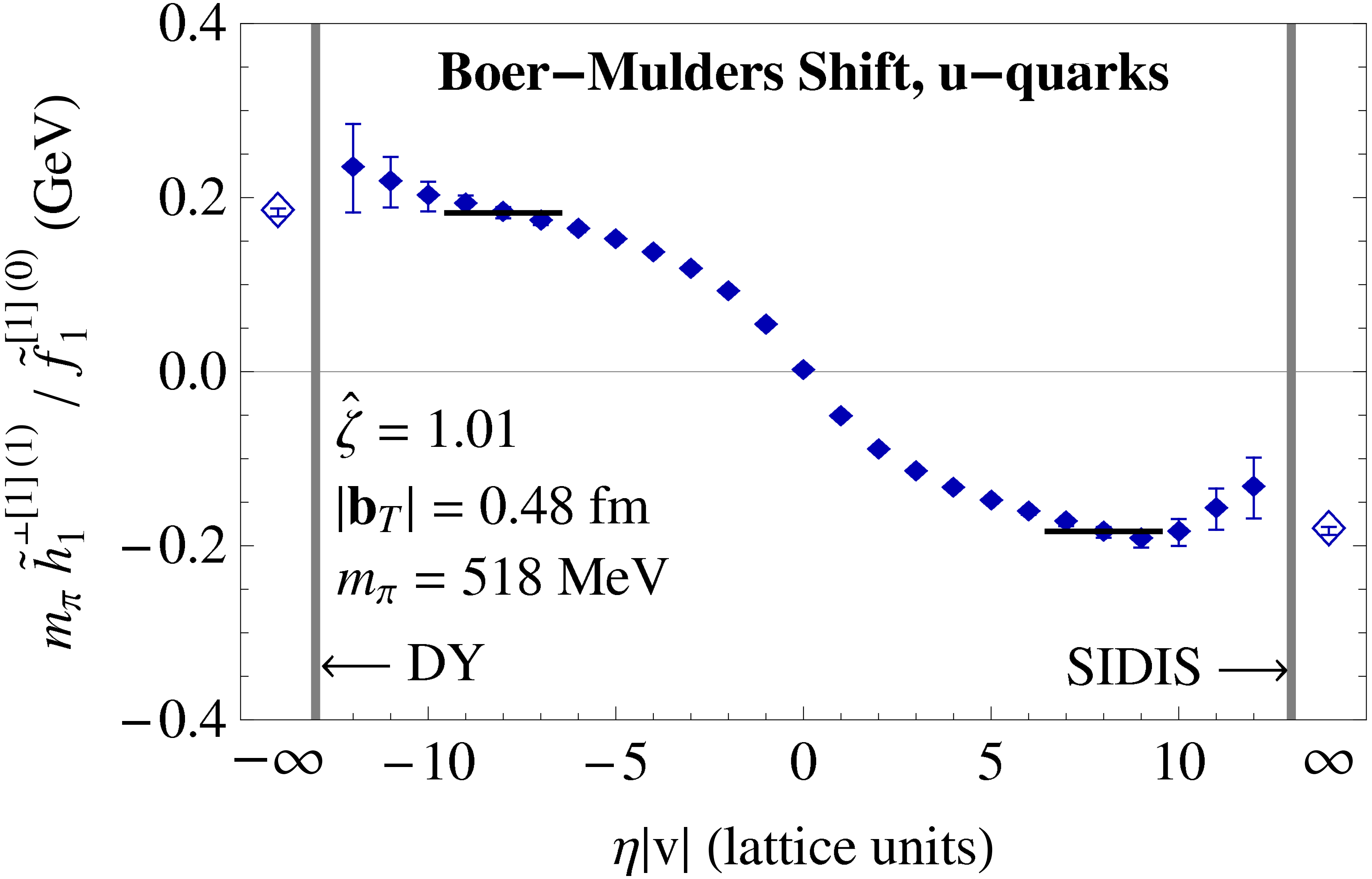


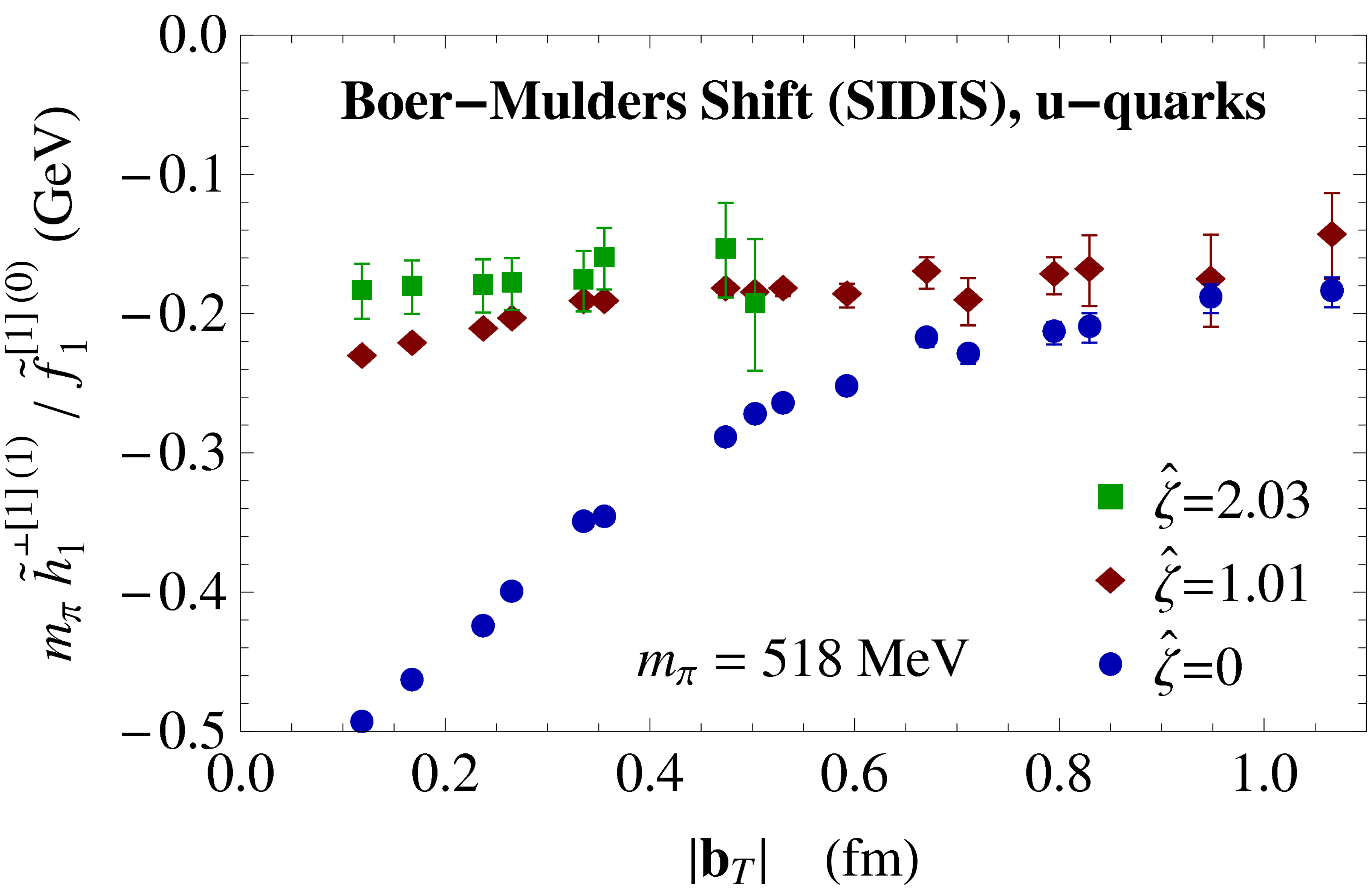
Results: Boer-Mulders shift (pion)

Dependence on staple extent; sequence of panels at different $|b_T|$



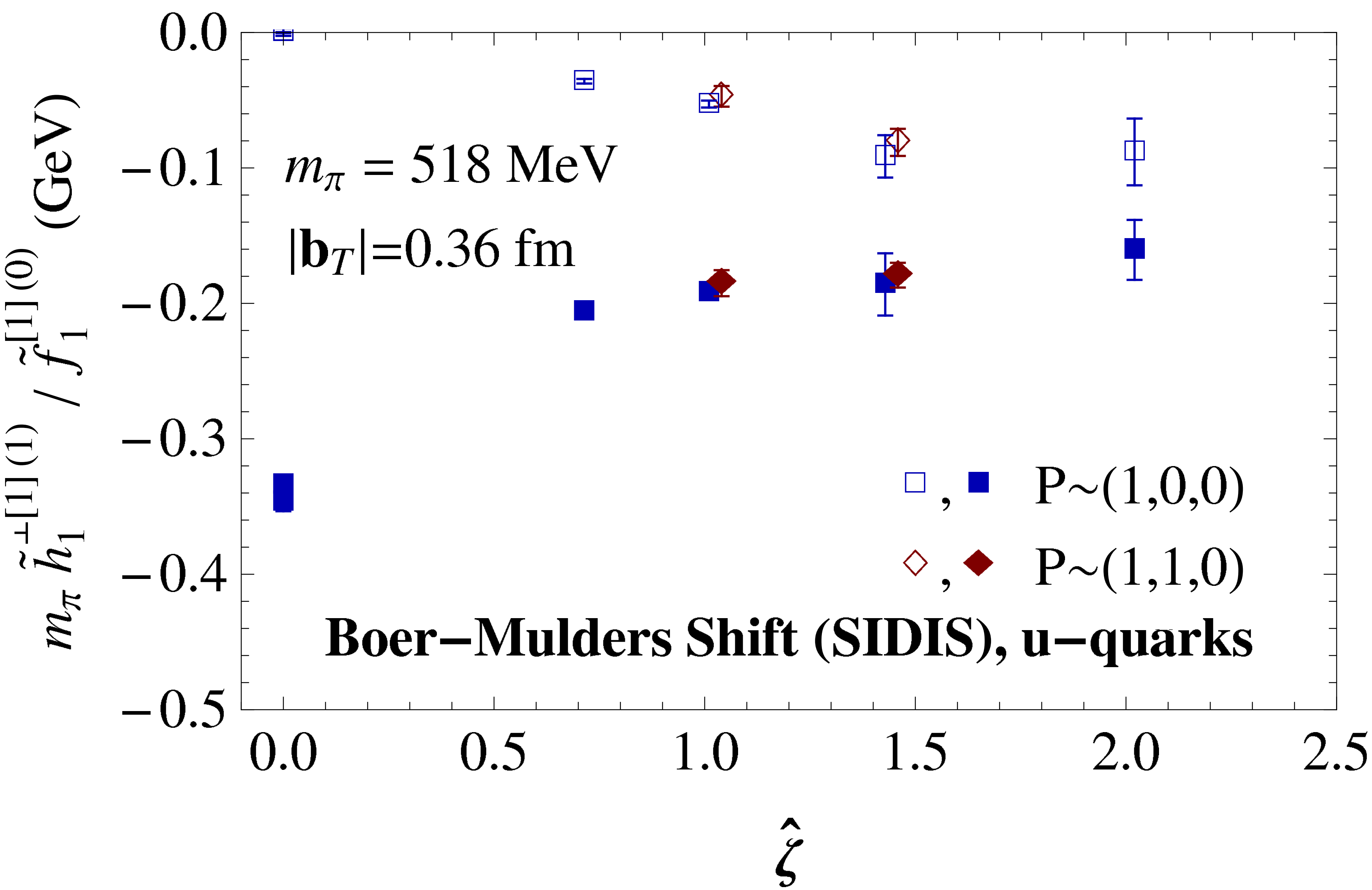
Results: Boer-Mulders shift (pion)

Dependence of SIDIS limit on $|b_T|$



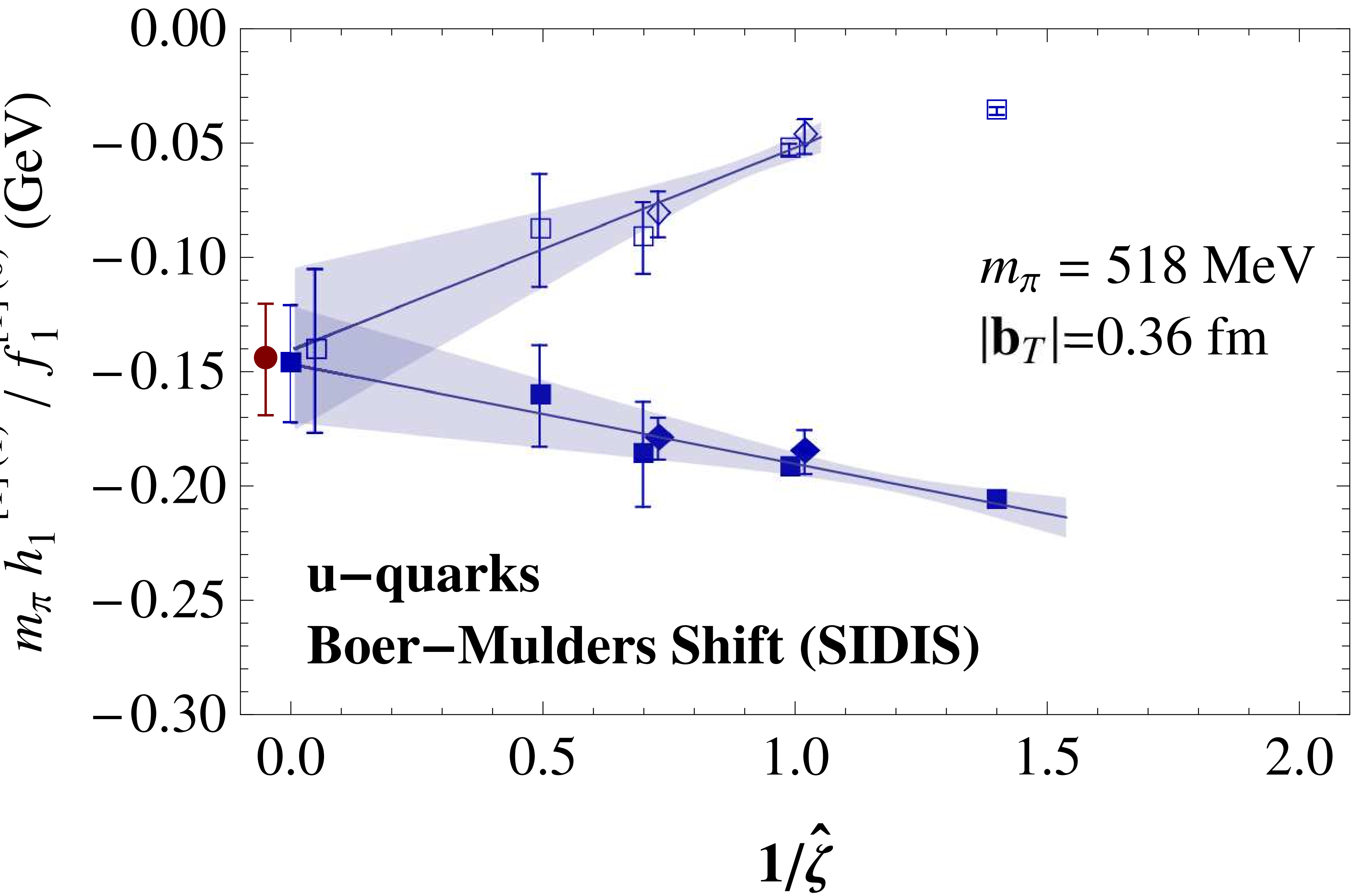
Results: Boer-Mulders shift (pion)

Dependence of SIDIS limit on $\hat{\zeta}$; open symbols: contribution $\widetilde{A}_4$ only



Results: Boer-Mulders shift (pion)

Dependence of SIDIS limit on $\hat{\zeta}$; fit function $a + b/\hat{\zeta}$



Discretization effects:

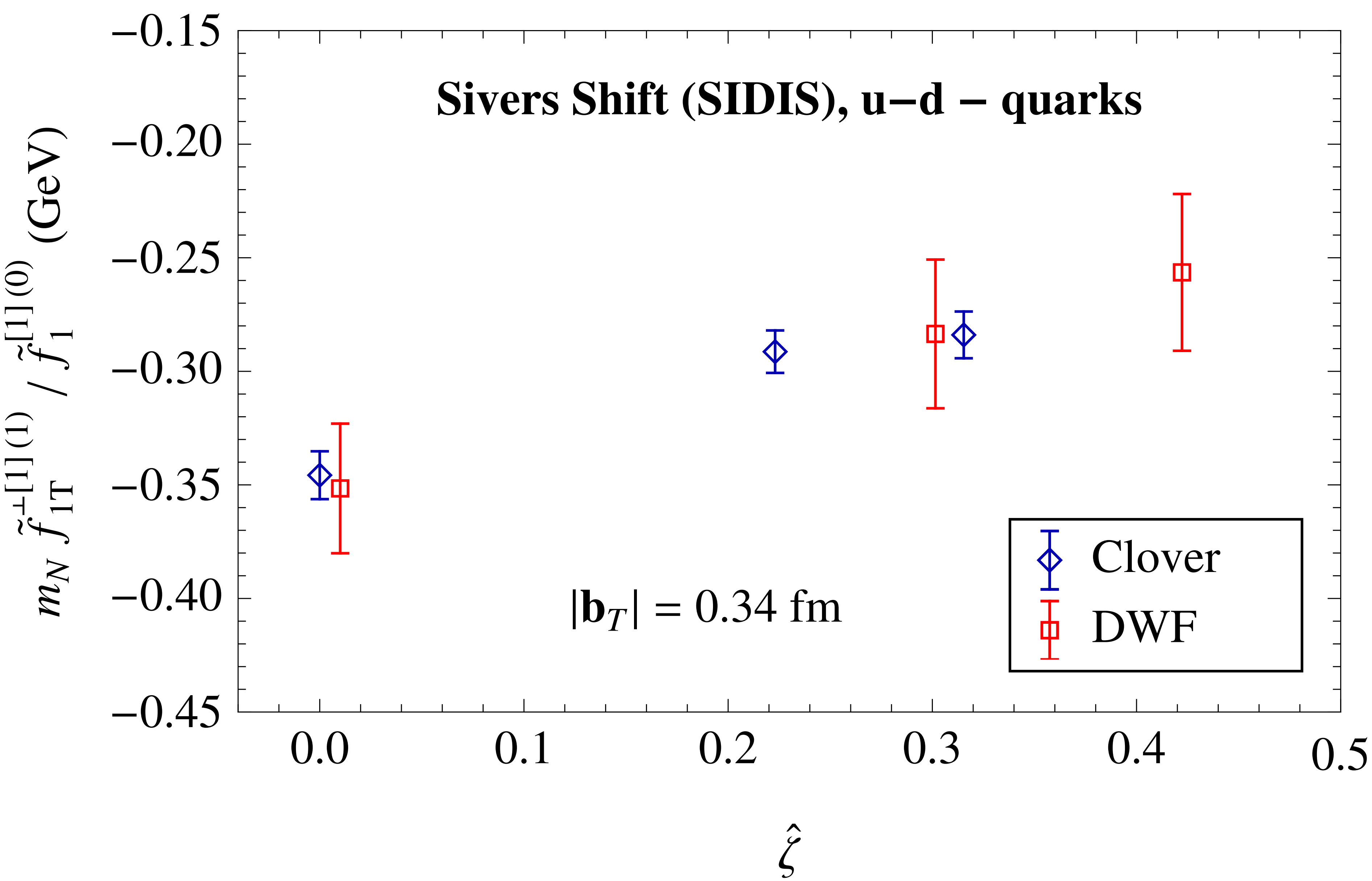
Comparison of

RBC/UKQCD DWF ensemble ($m_\pi = 297$ MeV, $a = 0.084$ fm)

with clover ensemble ($m_\pi = 317$ MeV, $a = 0.114$ fm)
produced by K. Orginos and JLab collaborators

Results: Sivers shift

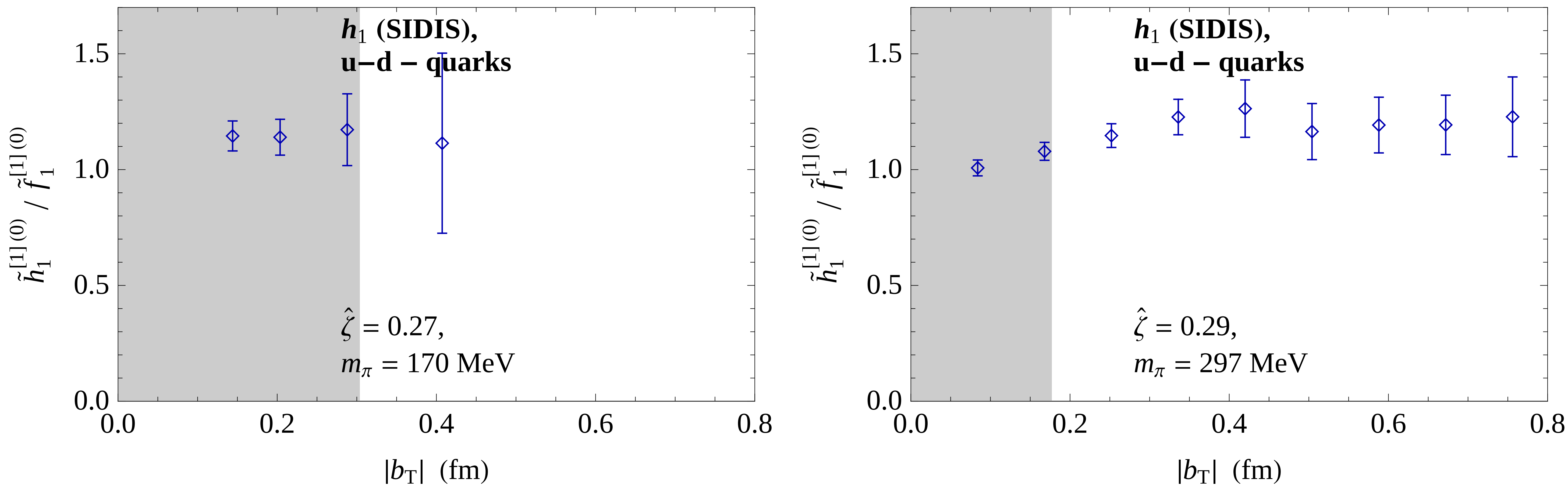
Dependence of SIDIS limit on $\hat{\zeta}$



Dependence on the pion mass

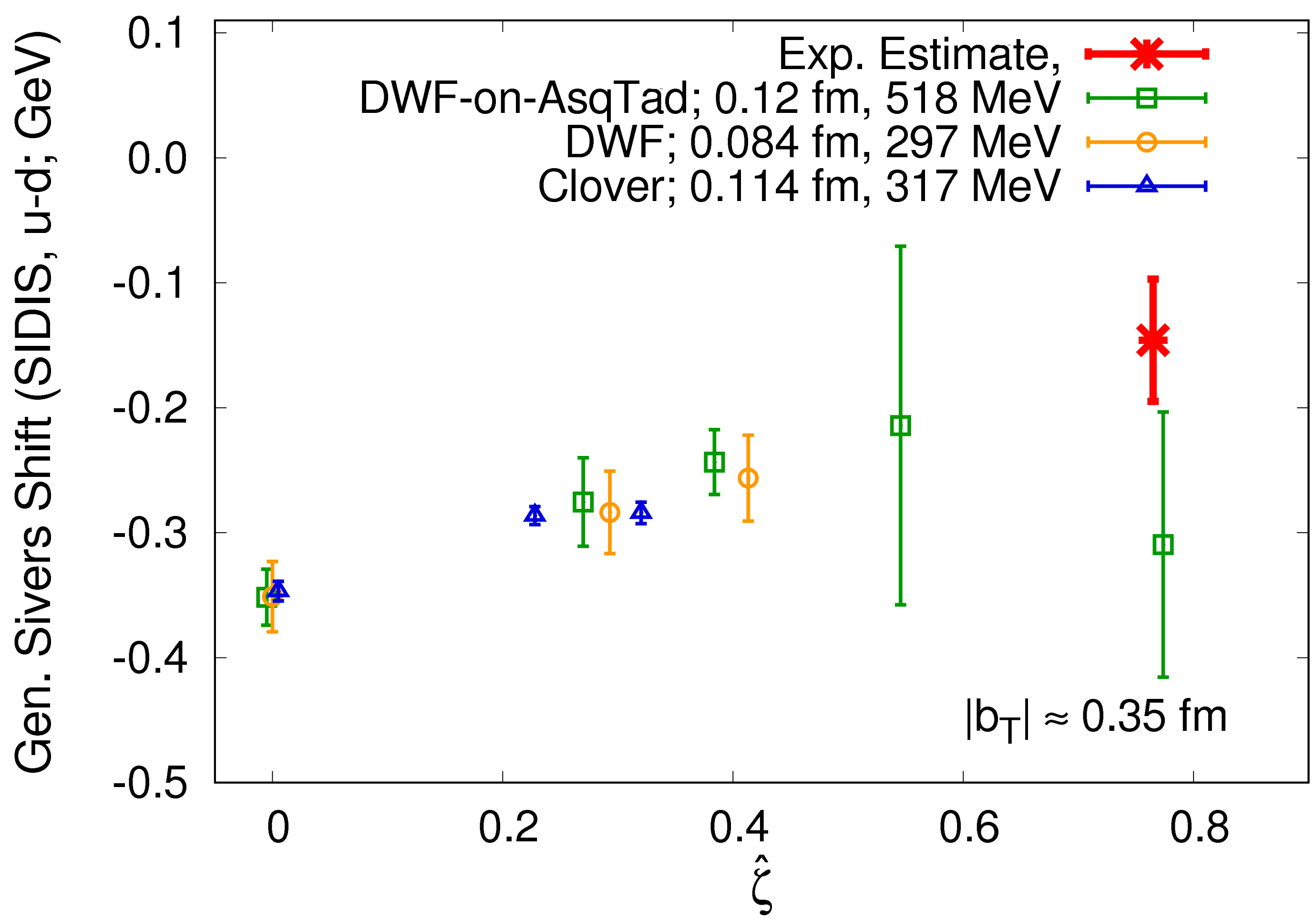
Results: Transversity

Dependence of SIDIS/DY limit on $|b_T|$



Results: Sivers shift summary

Dependence of SIDIS limit on $\hat{\zeta}$



Experimental value from global fit to HERMES, COMPASS and JLab data,
M. Echevarria, A. Idilbi, Z.-B. Kang and I. Vitev, Phys. Rev. D 89 (2014) 074013

Quark Orbital Angular Momentum

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T\, (r_T \times k_T)_3\, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T)$$

Wigner distribution

$$= - \int dx \int d^2k_T \frac{k_T^2}{m^2}\, F_{14}(x, k_T^2, k_T \cdot \Delta_T, \Delta_T^2)\, \Big|_{\Delta_T = 0}$$

Generalized transverse momentum-dependent parton distribution (GTMD)

$$= \epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle P, S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid P', S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}$$

Y. Hatta, X. Ji, M. Burkardt:
Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM
Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM

Connection to GTMDs – A. Metz, M. Schlegel, C. Lorcé, B. Pasquini . . .

Lorentz invariance and equation of motion relations

Straight gauge link

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{\text{straight}} = \widetilde{E}_{2T} + H + E$$

$$-x\widetilde{E}_{2T} = \widetilde{H} - \int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{straight}} + \widetilde{\mathcal{M}}^{\text{straight}}$$

Staple-shaped gauge link

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{\text{staple}} = \widetilde{E}_{2T} + H + E + \mathcal{A}$$

$$-x\widetilde{E}_{2T} = \widetilde{H} - \int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{staple}} + \widetilde{\mathcal{M}}^{\text{staple}}$$

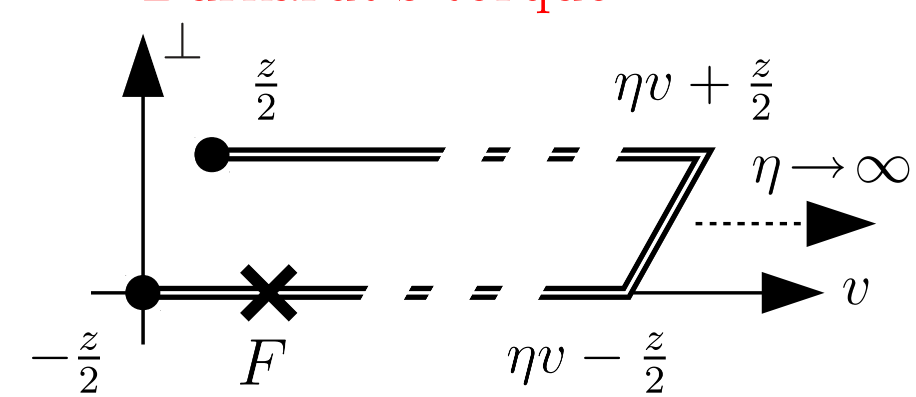
A. Rajan, A. Courtoy, M.E., S. Liuti (arXiv:1601.06117)

$$\mathcal{A} = \frac{d}{dx}\left(\int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{staple}} - \int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{straight}}\right)$$

$$= \frac{d}{dx}\left(\widetilde{\mathcal{M}}^{\text{staple}} - \widetilde{\mathcal{M}}^{\text{straight}}\right)$$

$\longrightarrow$ Burkardt's torque

(here: $z_T \to 0$ limit)



Quark orbital angular momentum in units of the number of valence quarks

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle P, S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid P', S \rangle \Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T \to 0}}{\langle P, S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid P', S \rangle \Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T \to 0}}$$

$P = p - \Delta_T/2$, $P' = p + \Delta_T/2$, p, S in 3-direction, $p \to \infty$

Y. Hatta, M. Burkardt:
Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM
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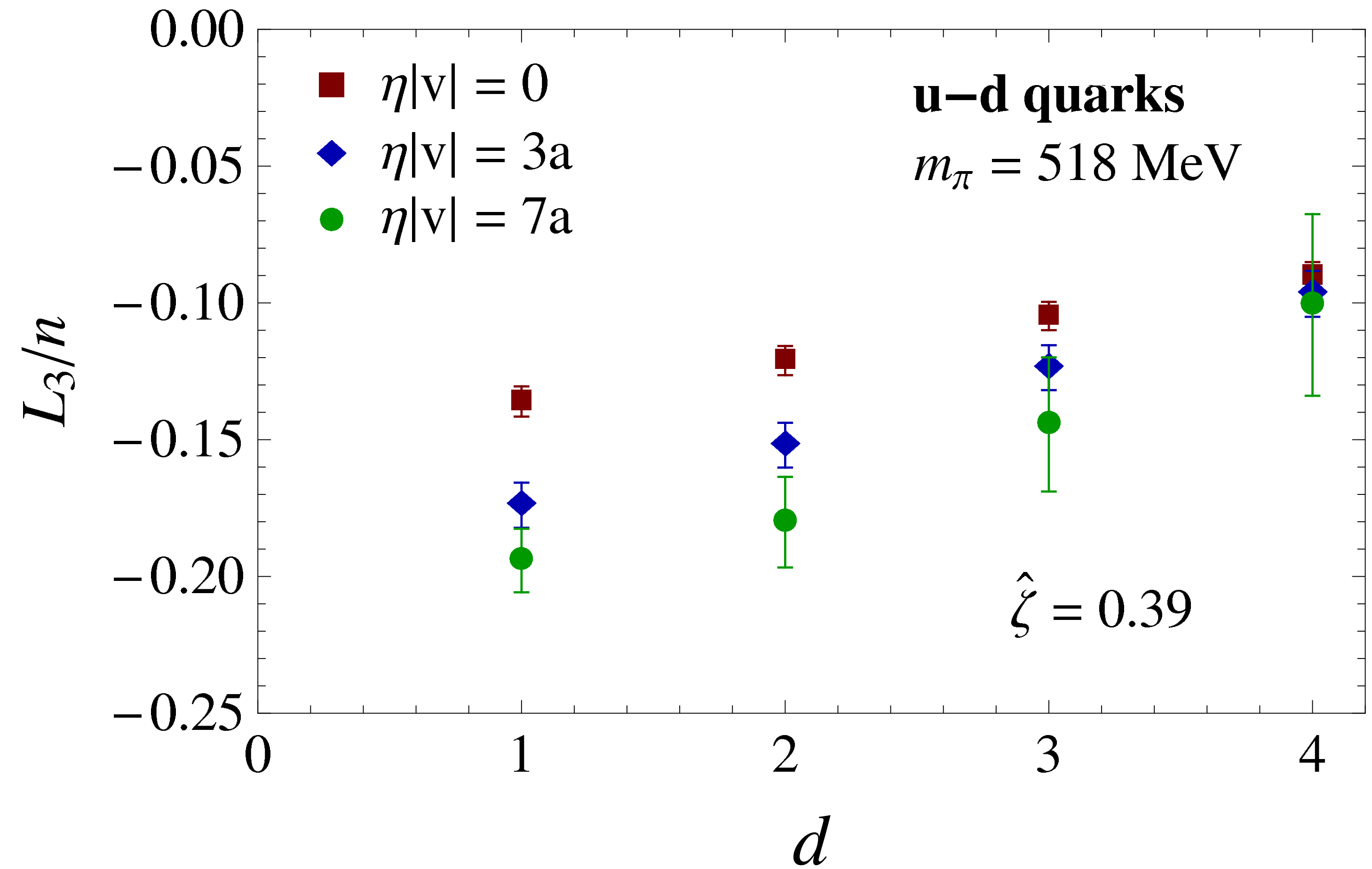
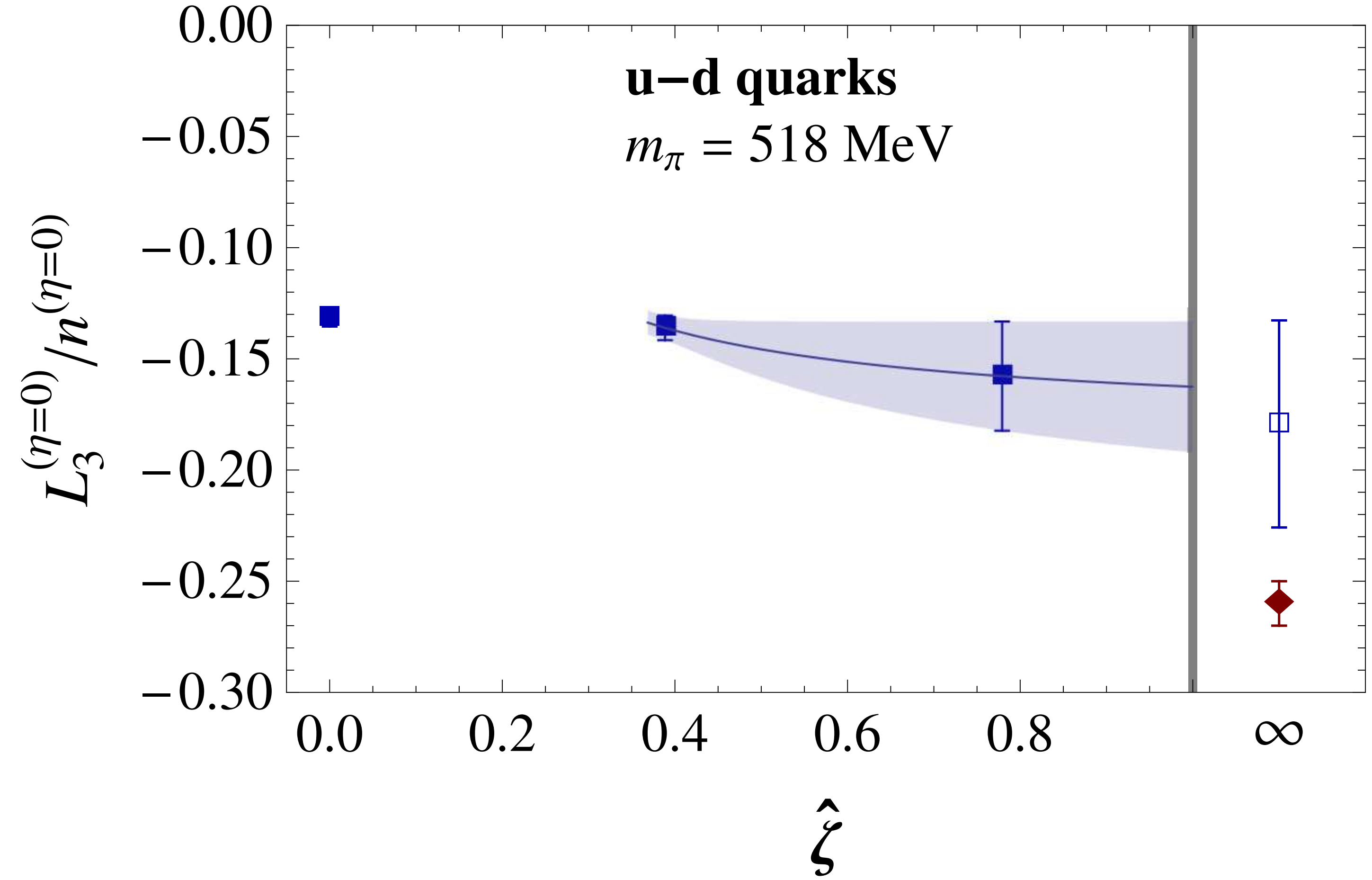
Direct evaluation of quark orbital angular momentum



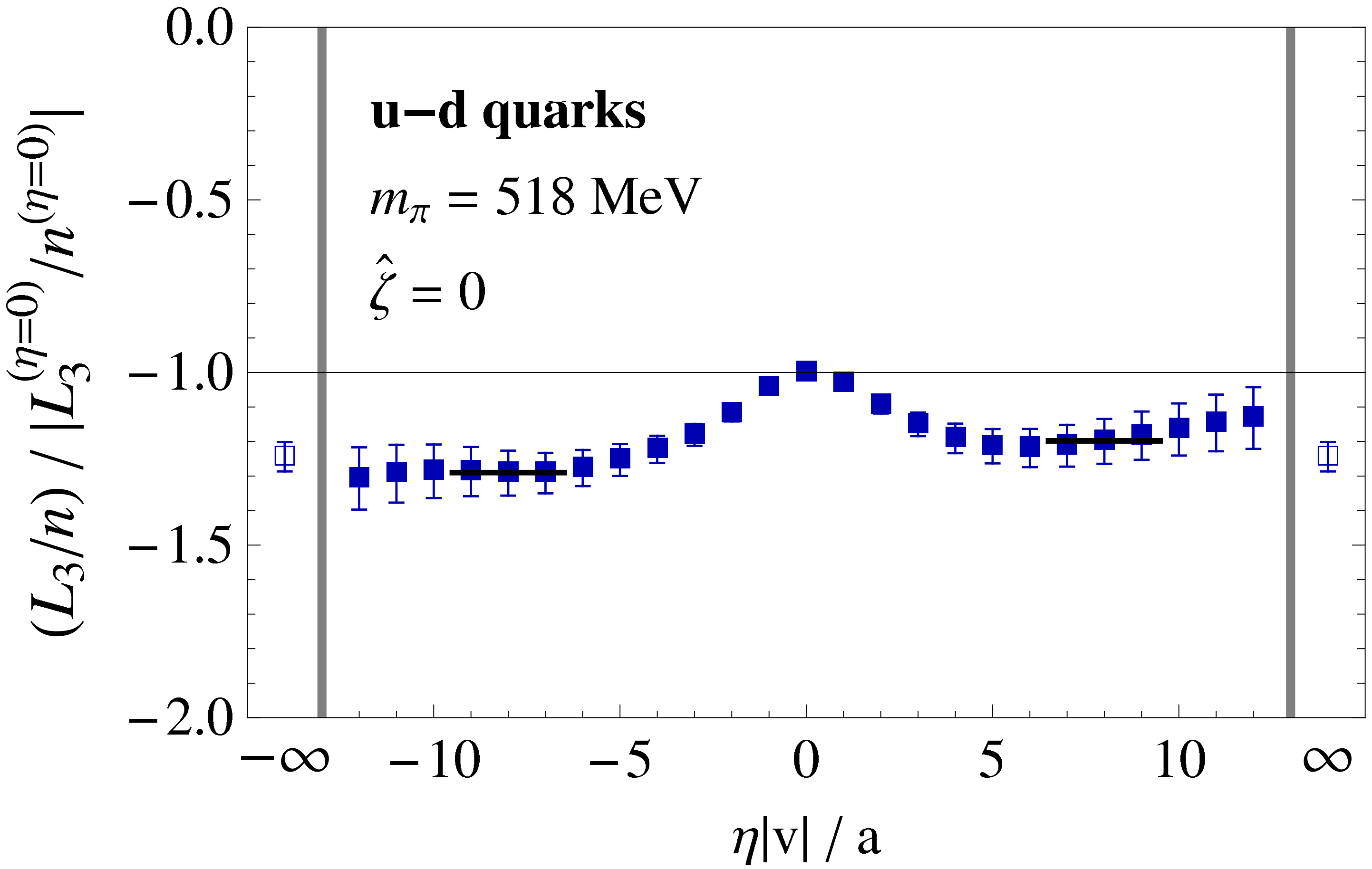
$$\left.\frac{\partial f}{\partial z_{T,i}}\right|_{z_{T,i}=0} = \frac{1}{2da}(f(dae_i) - f(-dae_i))$$

Ji quark orbital angular momentum: $\eta = 0$

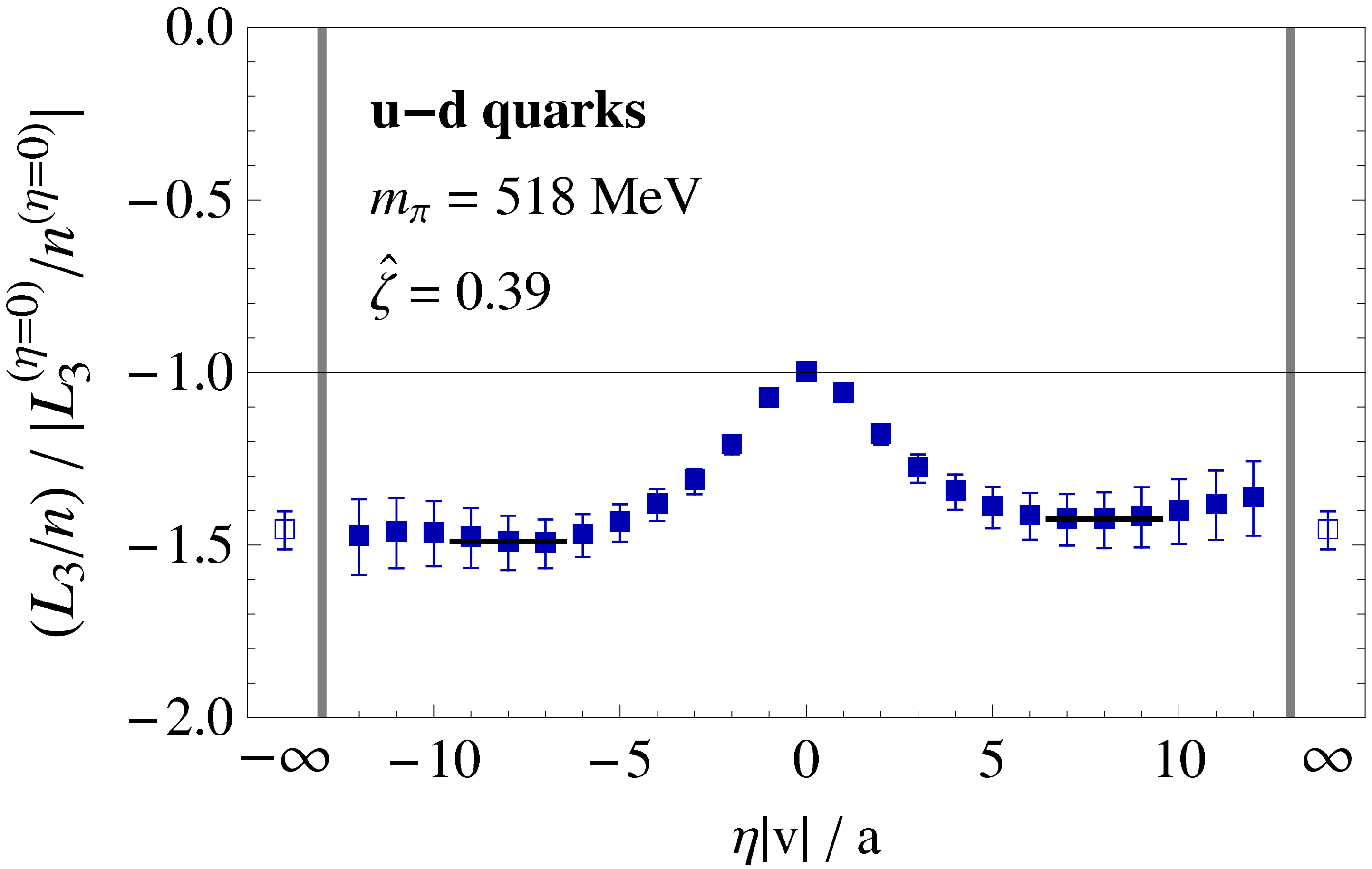


$\longrightarrow$ Signature of underestimate of $\partial f / \partial \Delta_T$

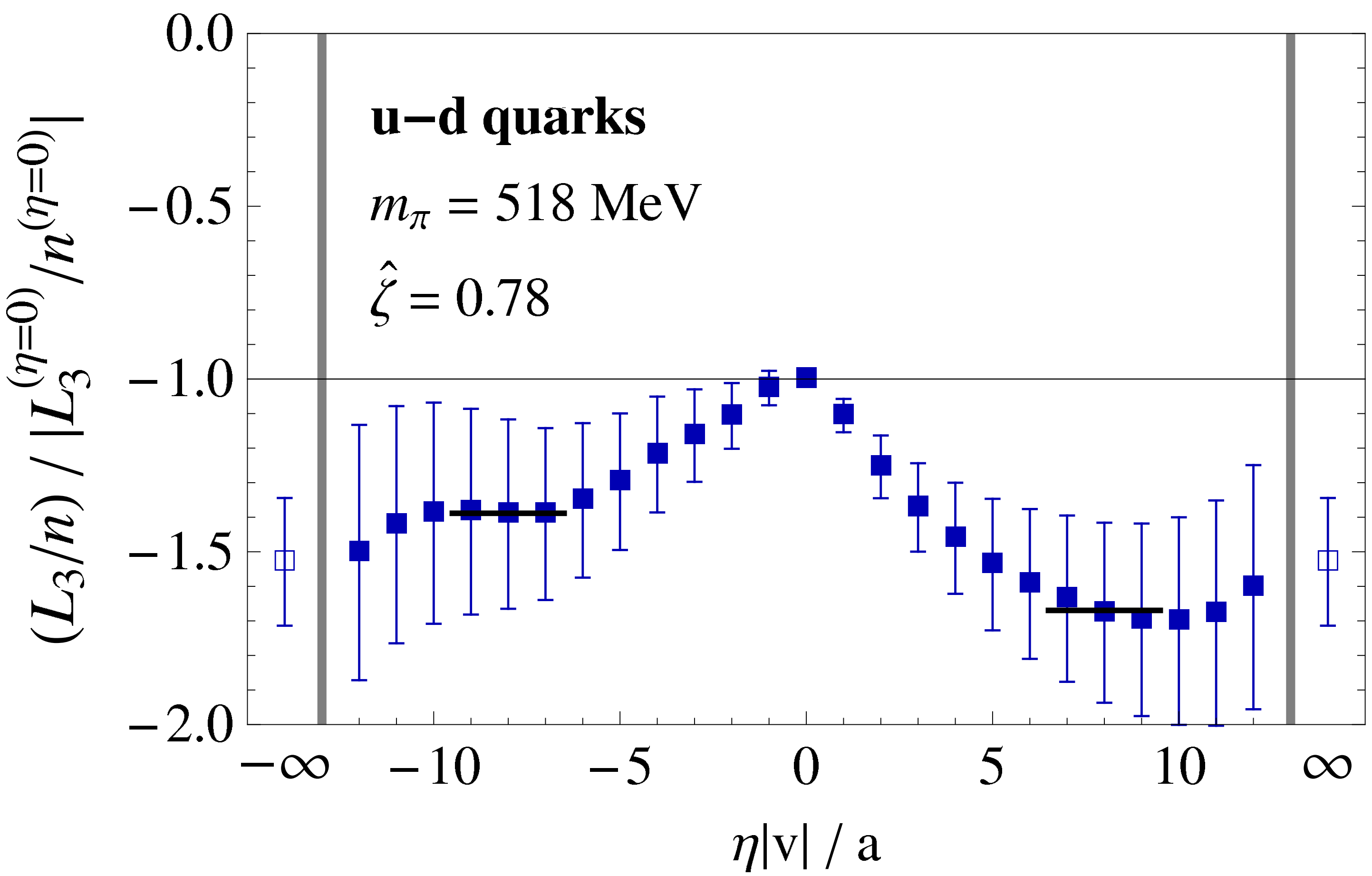
From Ji to Jaffe-Manohar quark orbital angular momentum



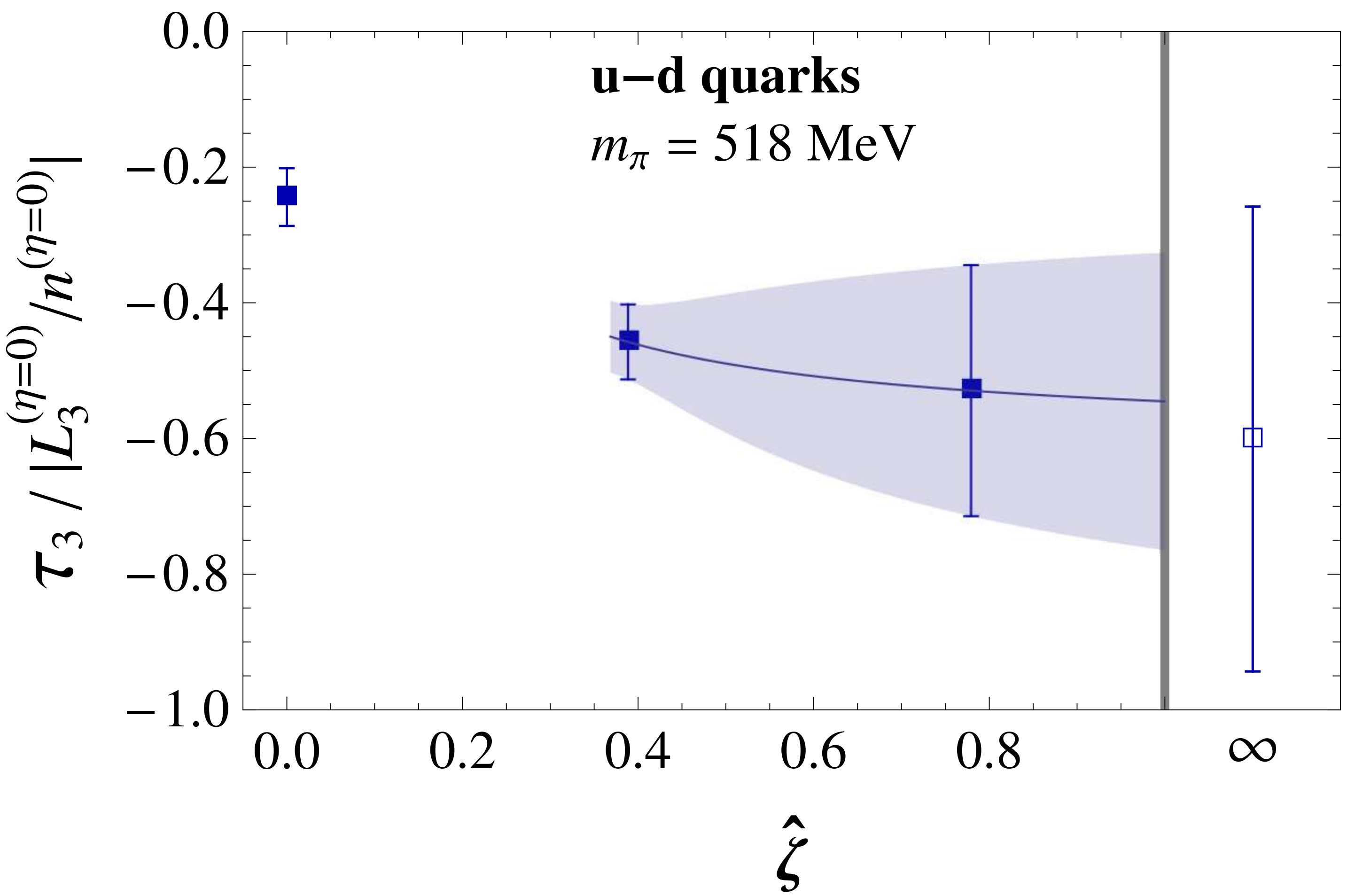
From Ji to Jaffe-Manohar quark orbital angular momentum



From Ji to Jaffe-Manohar quark orbital angular momentum



Burkardt's torque – extrapolation in $\hat{\zeta}$



$$\tau_3 = (L_3^{(\eta=\infty)}/n^{(\eta=\infty)}) - (L_3^{(\eta=0)}/n^{(\eta=0)})$$

Integrated torque accumulated by struck quark leaving proton

Conclusions and Outlook

- Continued exploration of TMDs in Lattice QCD; exploration of diverse challenges.
- Considered appropriate ratios of Fourier-transformed TMDs ("shifts") in which soft factors, multiplicative renormalization constants cancel.
- Generalization to mixed transverse momentum / transverse position observables (Wigner functions) gives direct access to quark orbital angular momentum; can continuously interpolate between the Ji and Jaffe-Manohar definitions. Clear signal for Burkardt's torque.
- Current production: x-dependence of the Sivers shift, lighter pion masses, improved OAM calculation.
- Planned: OAM via GTMDs F_{27}, $F_{28} \longrightarrow$ GPD $\tilde{E}_{2T}$, spin-orbit correlations from GTMD G_{11}.