Description of non-perturbative dynamics in high energy hard inclusive processes

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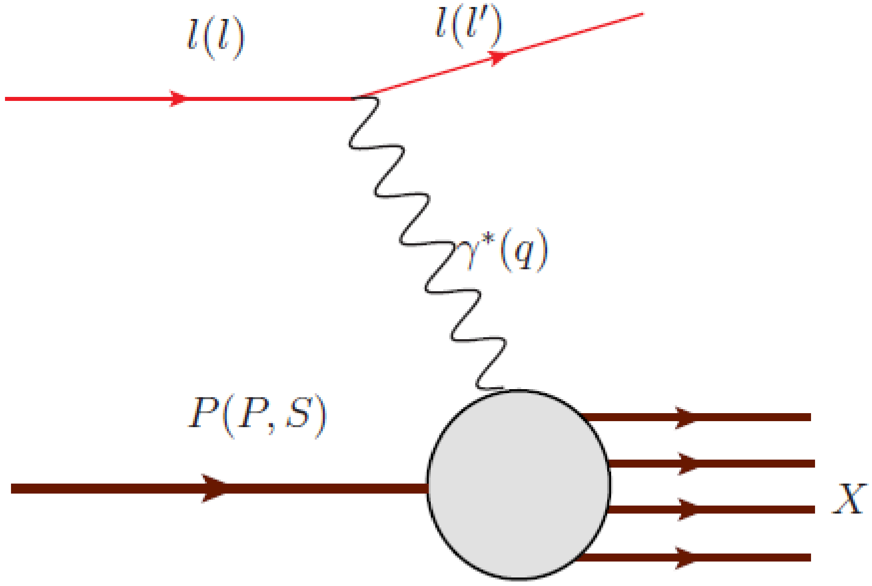
- **Spin and Transverse Momentum Dependent non-perturbative inputs (STMDs)**
  - Only leading twist STMDs
  - Processes with electromagnetic hard probe
- Parton Distribution Functions: STMD PDF
  - DIS, DY, SIDIS, high p_T hadron production in pp collisions
- Parton Fragmentation Functions: STMD FF
  - Hadron production in e^+e^- annihilation: SIA, SIDIS, high p_T hadron production in pp collisions
- STMD Fracture Functions
  - SIDIS
- String Fragmentation
  - LEPTO, PHYTIA

QCD factorization: DIS



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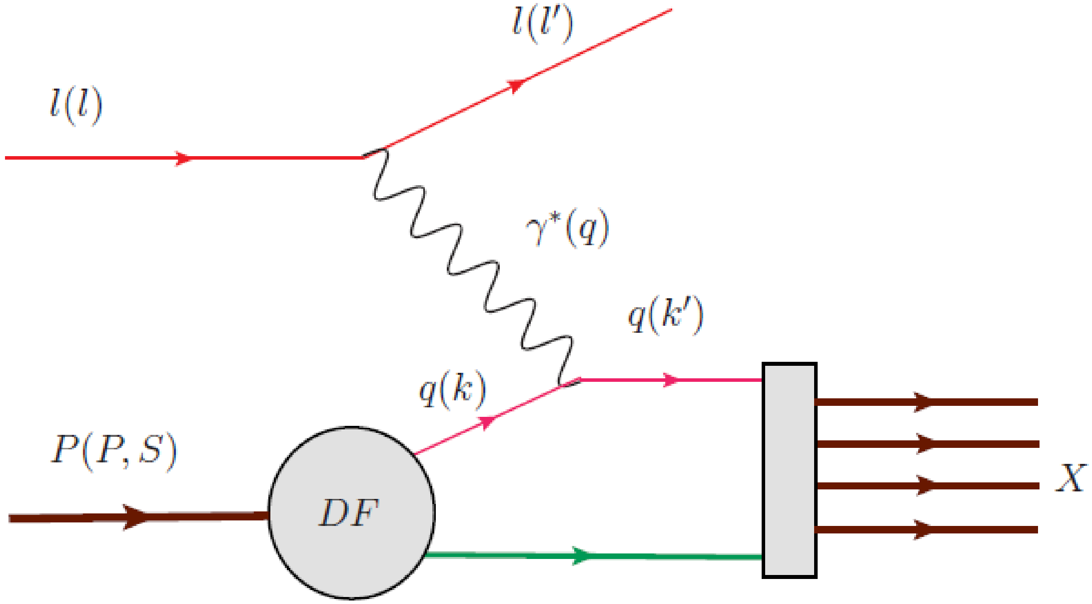
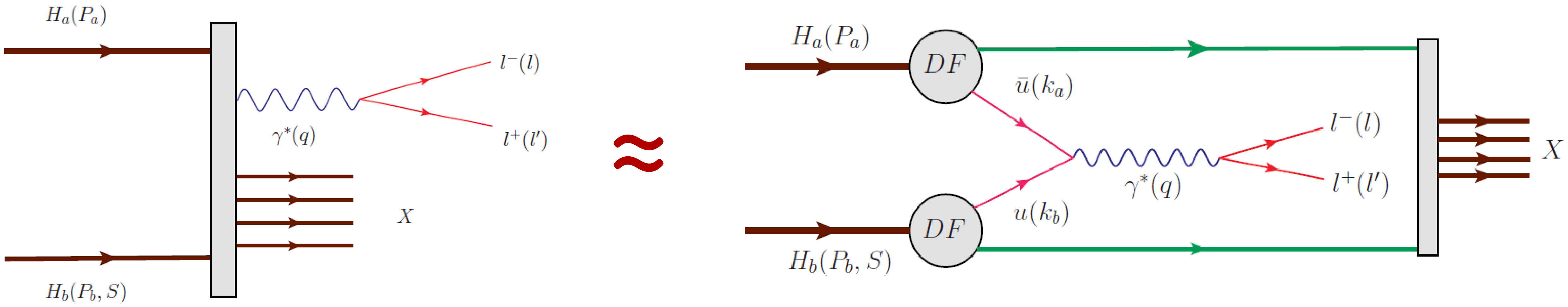


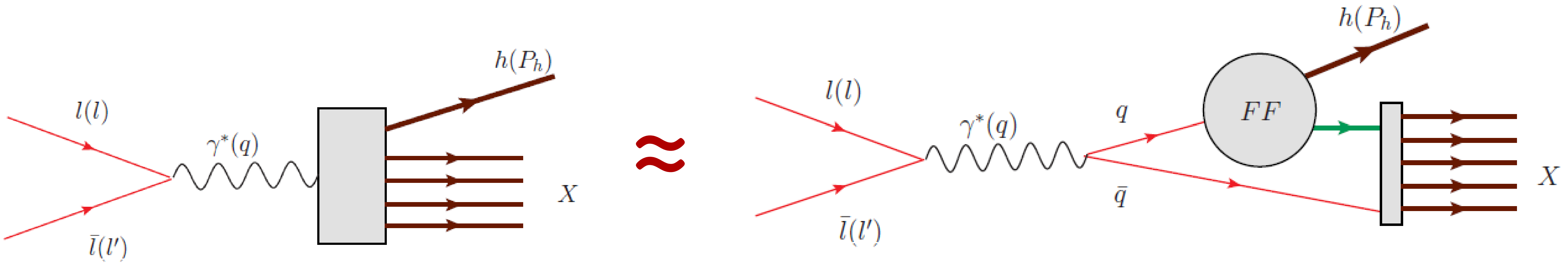
Access to nucleon $f_1^{q+\bar{q}}(x)$ and $g_1^{q+\bar{q}}(x)$ leading twist PDFs

QCD TMD factorization: DY processes



Access to nucleon, pion and kaon $f_1(x, k_T^2)$, $g_1(x), h_1(x, k_T^2)$ and $h_1^{\perp}(x, k_T^2)$ leading twist PDFs

QCD TMD factorization: SIA



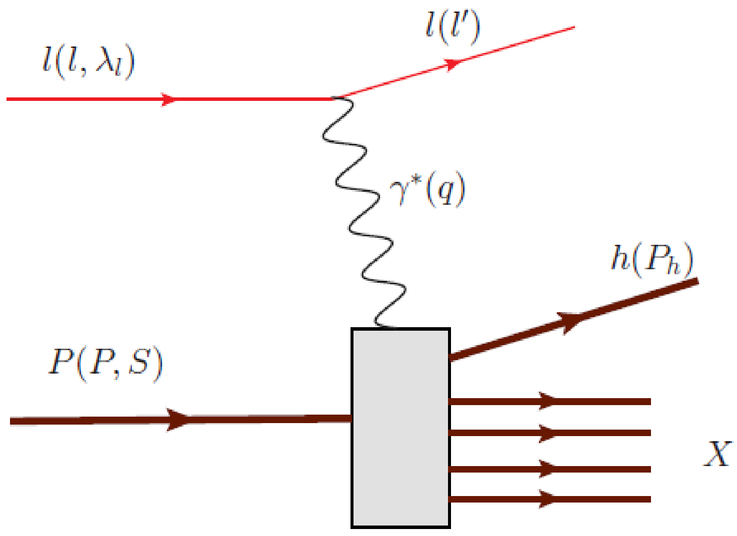
Access to $q+\bar{q}$ fragmentation finctions $D^h_{q+\bar{q}}(\mathrm{z},\mathrm{p}_\perp^2)$

Two hadron production in opposite hemispheres: acces to Collins FF $H^h_{1q}(\mathrm{z},\mathrm{p}_\perp^2)$

Two di-hadron production in opposite hemispheres:

acces to $H_q^{\sphericalangle}(\mathrm{z})$, $H_q^{\perp}(\mathrm{z})$ and $\mathrm{G}_q^{\perp}(\mathrm{z})$

QCD TMD factorization



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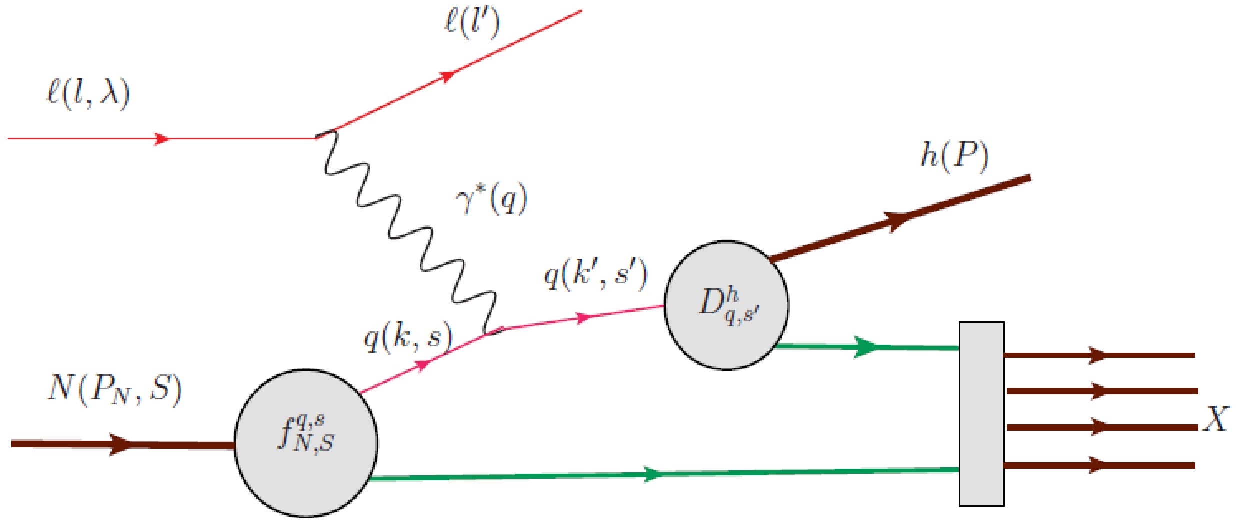
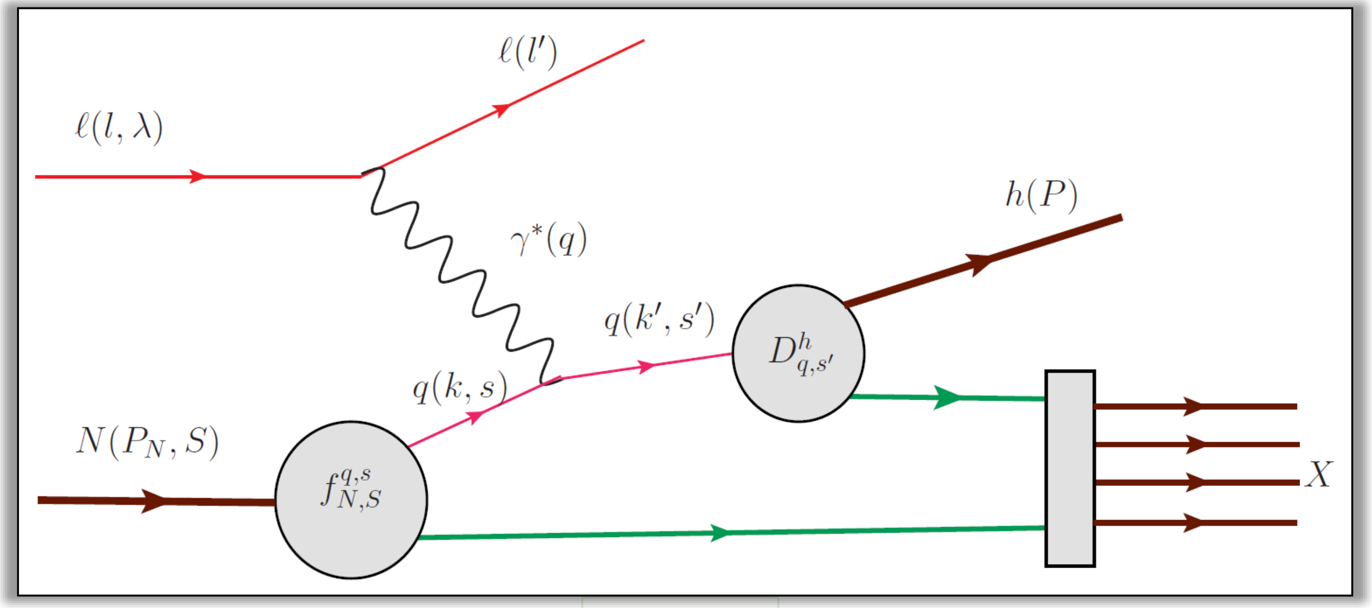


Access to nucleon $f_1^q(x)$, $g_1^q(x)$ and $h_1^q(x)$ leading twist PDFs and Collins FF H_1

SIDIS: CFR



$x_F > 0$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

← Measured in semi inclusive $e^+e^- \to h+X$ annihilation (SIA)

Twist-2 STMD qDFs

Nucleon Polarization \ Quark polarization	U	L	T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij} k_T{}^j}{M} h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M} \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x,k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\left[\begin{array}{l} F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+ \\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+ \\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(\begin{array}{l}F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+\\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+ \\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) \end{array}\right]$$

$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

8 terms out of 18 Structure Functions, 6 azimuthal modulations

4 terms are generated by Collins effect in fragmentation

Recursive FF

Field, Feynman PRD 15(1977)2590, NPB 136(1078)1 (**A PARAMETRIZATION OF THE PROPERTIES OF QUARK JETS**)

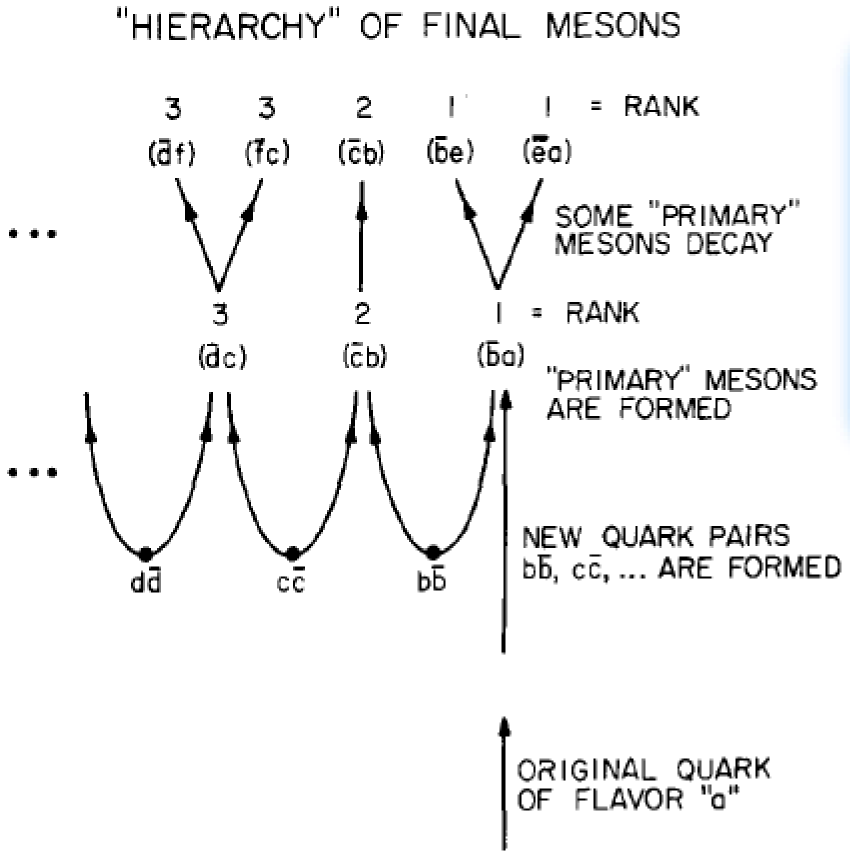


Fig. 1. Illustration of the "hierarchy" structure of the final mesons produced when a quark of type "a" fragments into hadrons. New quark pairs $b\bar{b}$, $c\bar{c}$, etc., are produced and "primary" mesons are formed. The "primary" meson $\bar{b}a$ that contains the original quark is said to have "rank" one and primary meson $\bar{c}b$ rank two, etc. Finally, some of the primary mesons decay and we assign all the decay products to have the rank of the parent. The order in "hierarchy" is *not* the same as order in momentum or rapidity.

assumed that for very high momenta, all distributions scale so that they depend only on ratios of the hadron momenta to the quark momenta. Given these assumptions, complete knowledge of the structure of a quark jet is determined by one unknown function $f(\eta)$ and three parameters describing flavor, primary meson spin, and transverse momentum to be discussed later. The function $f(\eta)$ is defined by

$f(\eta)\,d\eta$ = the probability that the first hierarchy (rank-1) primary meson *leaves* the fraction of momentum η to the remaining cascade, (2.1)

f(η) – elementary q→q' fragmentation or splitting function

Recursive FF: integral equation

$$S = 1 + x + x^2 + x^3 + \cdots = 1 + x(1 + x + x^2 + \cdots) = 1 + x \cdot S$$

2.2. Single-particle decay distribution $F(z)$

The above ansatz leads to an obvious and simple Monte Carlo calculation of a jet as well as to a straightforward recursive integral equation. For example, if we define a single-particle distribution in the quark jet as

$F(z)\,dz$ = the probability of finding any primary meson (independent of hierarchy) with fractional momentum z within dz in a quark jet, (2.4)

then $F(z)$ must satisfy the following integral equation (take $W_0 = 1$)

$$F(z) = f(1-z) + \int_z^1 f(\eta) F(z/\eta)\, d\eta/\eta\,, \qquad (2.5)$$

where the limits are automatic since we define $f(1-z) = 0$ and $F(z) = 0$ for $z > 1$ or $z < 0$. Eq. (2.5) arises because the primary meson might be the first in rank (with probability $f(1-z)\,dz$) or if not, then the first-rank primary meson has left a momentum fraction η with probability $f(\eta)\,d\eta$, and in this remaining cascade the probability to find z in dz is $F(z/\eta)\,dz/\eta$ by the scaling principle. Dividing out the dz leaves eq. (2.5).

Only longitudinal scaled momentum flow is taken into account

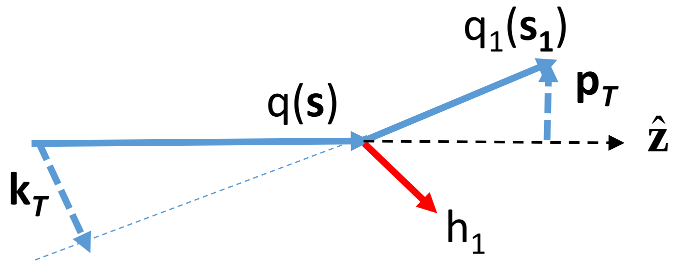
Generalization to STMD FFs

First MC study with constant spin transfer: Matevosyan, AK, Thomas **PLB 731(2014)208**

Framework: Bentz, AK, Matevosyan, Ninomiya, Thomas, Yazaki, **PR D94 (2016)034004** ← Field-Feynman approach + polarization and TM flow

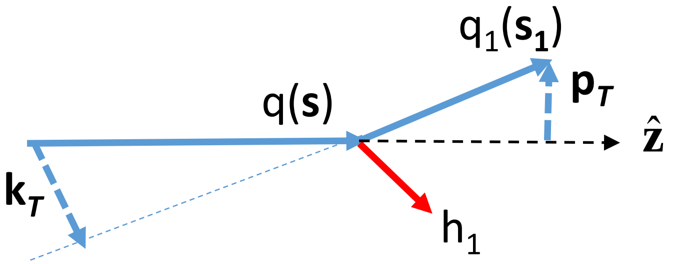
MC study: Matevosyan, AK, Thomas **arXiv:1610.05624**

Initial quark polarization \ Final quark polarization	U	L	T
U	$D(z, p_T^2)$		$-\frac{\mathbf{k}_T \times \hat{\mathbf{z}}}{\mathcal{M}} D_T^{\perp}(z, p_T^2)$
L		$s_L G_L(z, p_T^2)$	$s_L \frac{\mathbf{k}_T}{\mathcal{M}} G_T(z, p_T^2)$
T	$-\frac{(\mathbf{k}_T \times \mathbf{s}_T) \cdot \hat{\mathbf{z}}}{\mathcal{M}} H^{\perp}(z, p_T^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{s}_T}{\mathcal{M}} H_L^{\perp}(z, p_T^2)$	$\mathbf{s}_T H_T(z, k_T^2) + \frac{\mathbf{k}_T}{\mathcal{M}} \frac{(\mathbf{k}_T \cdot \mathbf{s}_T)}{\mathcal{M}} H_T^{\perp}(z, p_T^2)$



$$\mathbf{k}_T = -\mathbf{p}_T / z$$

STMD splitting function probabilities



Polarized quark to polarized quark splitting functions

$$F^{q\to q_1}\left(z,\mathbf{p}_\perp;\mathbf{s}_1,\mathbf{s}\right) = D(z,\mathbf{p}_\perp^2) - \frac{1}{M}\left(\mathbf{k}_T \times \mathbf{s}_{1T}\right)\cdot\hat{\mathbf{z}}\,D_T^\perp(z,\mathbf{p}_\perp^2)$$

$$+\left(\mathbf{s}_T\cdot\mathbf{s}_{1T}\right)H_T(z,\mathbf{p}_\perp^2) + \frac{1}{M}s_{1L}\left(\mathbf{k}_T\cdot\mathbf{s}_T\right)H_L^\perp(z,\mathbf{p}_\perp^2)$$

$$+\frac{1}{M^2}\left(\mathbf{s}_{1T}\cdot\mathbf{k}_T\right)\left(\mathbf{s}_T\cdot\mathbf{k}_T\right)H_T^\perp(z,\mathbf{p}_\perp^2) - \frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_T\right)\cdot\hat{\mathbf{z}}H^\perp(z,\mathbf{p}_\perp^2)$$

$$+\left(s_{1L}\,s_L\right)G_L(z,\mathbf{p}_\perp^2) + \frac{1}{M}s_L\left(\mathbf{s}_{1T}\cdot\mathbf{k}_T\right)G_T(z,\mathbf{p}_\perp^2)$$

Polarized quark to unpolarized hadron splitting functions

$$F^{q\to h_1}\left(z,\mathbf{p}_\perp;\mathbf{s}\right) = F^{q\to q_1}\left(1-z,-\mathbf{p}_\perp;\mathbf{s}_1=0,\mathbf{s}\right) = D(1-z,\mathbf{p}_\perp^2) + \frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_T\right)\cdot\hat{\mathbf{z}}H^\perp(1-z,\mathbf{p}_\perp^2)$$

Quark polarization after hadron emission

$$F^{q\to q_1}\left(z,\mathbf{p}_\perp;\mathbf{s}_1,\mathbf{s}\right)=\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)+\boldsymbol{\beta}\left(z,\mathbf{p}_\perp;\mathbf{s}\right)\cdot\mathbf{s}_1$$

$$\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=D(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_T\right)\cdot\hat{\mathbf{z}}H^\perp(z,\mathbf{p}_\perp^2)$$

$$\beta_L\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=s_L G_L(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\cdot\mathbf{s}_T\right)H_L^\perp(z,\mathbf{p}_\perp^2)$$

$$\boldsymbol{\beta}_\perp\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=-\frac{\mathbf{k}_T^{'}}{M}D_T^\perp(z,\mathbf{p}_\perp^2)+s_L\frac{\mathbf{k}_T}{M}G_T(z,\mathbf{p}_\perp^2)$$
$$+\mathbf{s}_T H_T(z,\mathbf{p}_\perp^2)+\frac{\mathbf{k}_T}{M^2}\left(\mathbf{s}_T\cdot\mathbf{k}_T\right)H_T^\perp(z,\mathbf{p}_\perp^2)$$

α and $\boldsymbol{\beta}$ are

linear functions of $\mathbf{s}$

$\mathbf{k}_T^{'}=(-k_y,k_x)$

The final quark spin is completely determined by elementary splitting functions and depends on z, $\mathbf{p}_\perp$ and initial quark polarization **s**

$$\langle\mathbf{s}_1\rangle=\frac{\boldsymbol{\beta}\left(z,\mathbf{p}_\perp;\mathbf{s}\right)}{\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)}$$

Integral equations for hadron production

Inserting everything into (III.29) we obtain the following two coupled integral equations [10]:

$$D^{(q\to\pi)}(z,\mathbf{p}_\perp^2) = \hat{d}^{(q\to\pi)}(z,\mathbf{p}_\perp^2)$$
$$+2\int\mathcal{D}^2\eta\int\mathcal{D}^4p_\perp\,\delta(z-\eta_1\eta_2)\delta^{(2)}(\mathbf{p}_\perp-\mathbf{p}_{2\perp}-\eta_2\mathbf{p}_{1\perp})$$
$$\times\Big[\hat{d}^{(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)\,D^{(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)+\frac{1}{Mm_\pi z}$$
$$\times\,(\mathbf{p}_{1\perp}\cdot\mathbf{p}_{2\perp})\,\hat{d}_T^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)\,H^{\perp(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)\Big]\,, \qquad \text{(III.39)}$$

$$(\mathbf{p}_\perp\times\mathbf{s}_T)^3\,H^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2) = (\mathbf{p}_\perp\times\mathbf{s}_T)^3\,\hat{h}^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2)$$
$$+2\int\mathcal{D}^2\eta\int\mathcal{D}^4p_\perp\,\delta(z-\eta_1\eta_2)\delta^{(2)}(\mathbf{p}_\perp-\mathbf{p}_{2\perp}-\eta_2\mathbf{p}_{1\perp})$$
$$\times\Big[\frac{m_\pi}{M}\,\eta_2\,(\mathbf{p}_{1\perp}\times\mathbf{s}_T)^3\,\hat{h}^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)D^{(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)$$
$$+\Big(\eta_1\,(\mathbf{p}_{2\perp}\times\mathbf{s}_T)^3\,\hat{h}_T^{(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)-\frac{1}{M^2\eta_1}(\mathbf{s}_T\cdot\mathbf{p}_{1\perp})$$
$$\times\,(\mathbf{p}_{1\perp}\times\mathbf{p}_{2\perp})^3\,\hat{h}_T^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)\Big)\,H^{\perp(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)\Big]\,.$$

$$\int\mathcal{D}^N\eta\equiv\int_0^1 d\eta_1\int_0^1 d\eta_2\cdots\int_0^1 d\eta_N\,,$$
$$\int\mathcal{D}^{2N}p_\perp\equiv\int d^2p_{1\perp}\int d^2p_{2\perp}\cdots\int d^2p_{N\perp}$$

Longitudinal and transverse momentum (Schäfer-Teryaev) sum rules

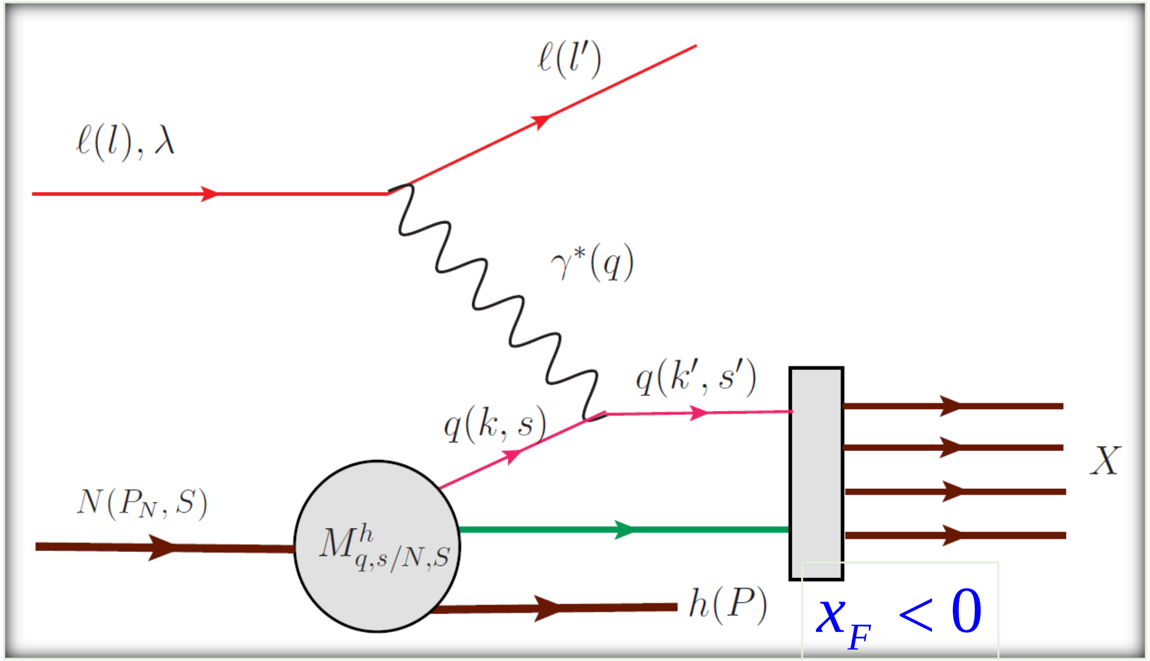
$$\sum_h\gamma_h\int_0^1 dz\,z\int d^2p_\perp\,D^{(q\to h)}(z,\mathbf{p}_\perp^2)=1\,, \qquad \text{(II.22)}$$

$$\sum_h\gamma_h\int_0^1\frac{dz}{2z\,M_h}\int d^2p_\perp\cdot\mathbf{p}_\perp^2\,H^{\perp(q\to h)}(z,\mathbf{p}_\perp^2)=0\,, \qquad \text{(II.23)}$$

where γ_h is the spin degeneracy factor of the hadron and M_h its mass. A similar derivation can be given for the

For further details see Matevosyan's talk

SIDIS: TFR



Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
....

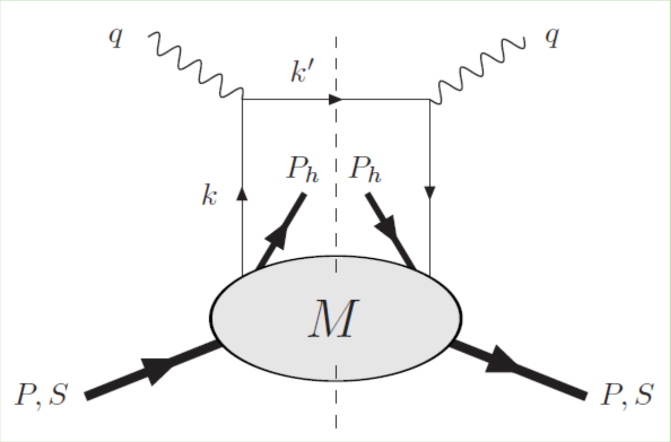
Anselmino, Barone and AK, PL B 699 (2011)108**; 706** (2011)46**; 713** (2012)317

Nucleon and quark polarization and produced hadron and quark transverse momentum are not integrated over.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Quark correlator



$$\mathcal{M}^{[\Gamma]}(x_B, \vec{k}_\perp, \zeta, \vec{P}_{h\perp}) = \frac{1}{4\zeta}\int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{i(x_B P^- \xi^+ - \vec{k}_\perp \cdot \vec{\xi}_\perp)} \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \times$$

$$\times \langle P,S|\bar{\psi}(0)\Gamma|P_h,S_h;X\rangle\langle P_h,S_h;X|\psi(\xi^+,0,\vec{\xi}_\perp)|P,S\rangle$$

$$\Gamma = \gamma^-, \quad \gamma^-\gamma_5, \quad i\sigma^{i-}\gamma_5$$

At LO 16 STMD fracture functions. Probabilistic interpretation at LO: Conditional probability of finding a quark $q(x,k_\perp)$ in the fast moving proton fragmenting to $h(\zeta,P_{h\perp})$ moving in same direction ⇒ STMD CPDFs

STMD Fracture Functions for spinless hadron production

Nucleon Polarization	Quark polarization: U	L	T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^j}{m_h} \hat{t}_1^h + \frac{\epsilon_T^{ij} k_T^j}{m_N} \hat{t}_1^\perp$
L	$\frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^h + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^\perp$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^h + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^\perp$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^h + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^\perp$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T(\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T(\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T(\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

STMD fracture functions

depend on

$$x, k_T^2, \zeta, P_T^2, \mathbf{k}_T \cdot \mathbf{P}_T$$

$$\mathbf{k}_T \cdot \mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q)$$

azimuthal dependence

in fracture functions

Sum Rules

$$\sum_h \int \zeta d\zeta \int d^2P_T\, \hat{u}_1 = (1-x)\, f_1(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{u}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{u}_{1T}^{h} \right) = -(1-x)\, f_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T\, \hat{l}_{1L} = (1-x) g_{1L}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{l}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{l}_{1T}^{h} \right) = (1-x) g_{1T}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1L}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1L}^{h} \right) = (1-x) h_{1L}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1}^{h} \right) = -(1-x) h_{1}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1T}^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2(\mathbf{k}_T \cdot \mathbf{P})^2 - k_T^2 P_T^2}{k_T^4} \hat{t}_{1T}^{hh} \right) = (1-x) h_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1T} + \frac{k_T^2}{2m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{P_T^2}{2m_h^2} \hat{t}_{1T}^{hh} \right) = (1-x) h_1(x,k_T^2)$$

Nonzero fracture functions u,l,t. Useful for modeling.

LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T}=\frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2\times$$

$$\times\left[\begin{array}{l}\tilde{u}_1\left(x,\zeta,P_T^2\right)-S_T\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)+\\ \lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right)+S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\end{array}\right]$$

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{u}_1$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{u}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{l}_{1L}$$

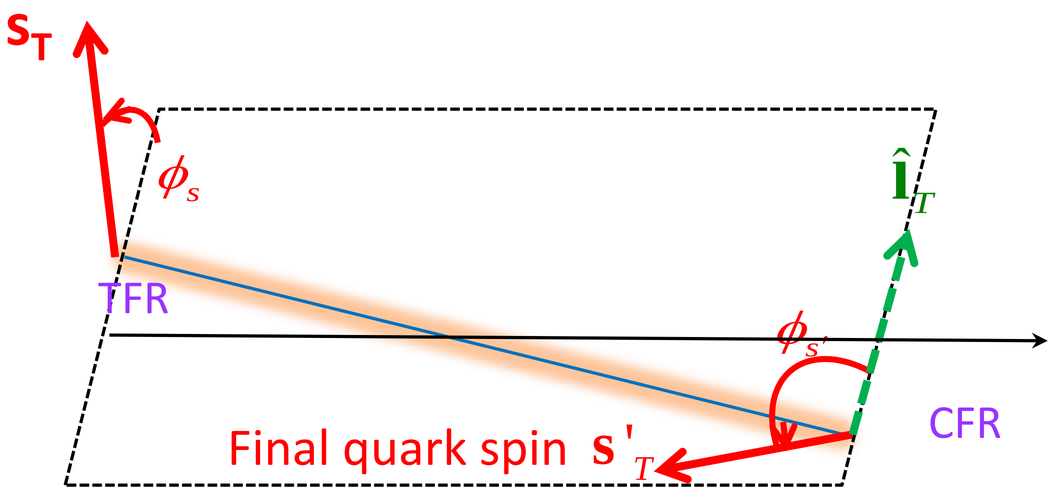
$$\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{l}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

At LO only 4 terms out of 18 Structure Functions,
Only 2 azimuthal modulations

No Collins-like $\sin(\phi_h-\phi_S)$ modulation

No access to quark transverse polarization

Quark spin in hard *l-q* scattering



AK, Yerevan Transversity Workshop, 2009

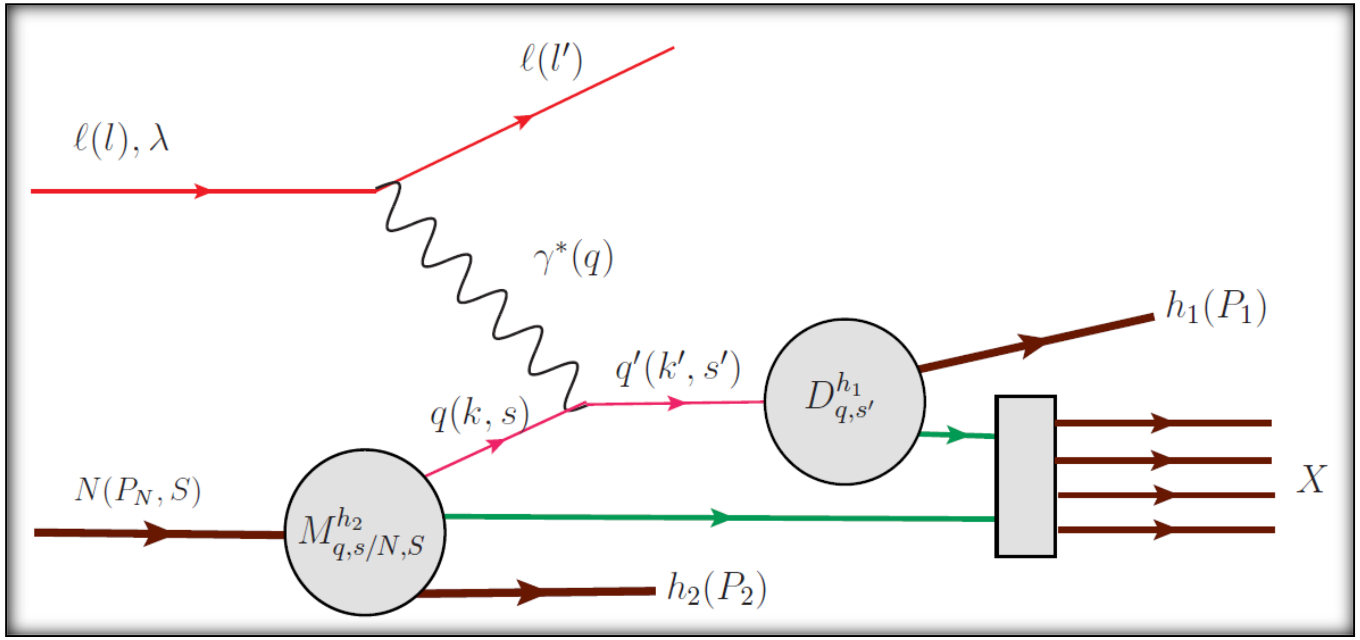
$$\frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} = e_q^2 \frac{2\pi\alpha^2}{\breve{s}^2}\frac{1}{Q^4}\Big((\breve{s}^2+\breve{u}^2)(1+s_L s'_L)+(\breve{s}^2-\breve{u}^2)\lambda(s_L+s'_L)$$
$$-2\breve{s}\breve{u}(\mathbf{s}_T\cdot\mathbf{s}'_T)-4\breve{u}(\mathbf{s}_T\cdot\mathbf{l}_T)(\mathbf{s}'_T\cdot\mathbf{l}'_T)-4\breve{s}(\mathbf{s}_T\cdot\mathbf{l}'_T)(\mathbf{s}'_T\cdot\mathbf{l}_T)\Big)$$

$\breve{s}$ and $\breve{u}$ are usual Mandelstam variables

$$s'_T = D_{nn}(y)s_T, \quad D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'} = \pi - \phi_s$$

CPDFs \`do not know' about final quark spin sideway component flip

Double hadron production in DIS (DSIDIS): TFR & CFR



$$x_{F2} < 0, \quad x_{F1} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z, p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z, p_T^2)$$

Unintegrated DSIDIS cross-section

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l} \hat{u}^{h_2}\otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2}\otimes D_1^{h_1} \\ +\hat{t}^{h_2}\otimes \dfrac{\mathbf{p}_T\times\mathbf{s}'_T}{m_{h_1}} H_1^{h_1} \end{array}\right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l} \sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \\ \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{LT}\right) \end{array}\right)$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms, similarly to 1h SIDIS cross section.

DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1+\phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right)\cos(2\phi_2) \right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

Structure functions $F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi), \text{ with } \Delta\phi = \phi_1 - \phi_2$$

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$+D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

σ_{UT}

$$
\begin{aligned}
\sigma_{UT} = & -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1 - \phi_S) \\
& -\left(\frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1} \right) \sin(\phi_2 - \phi_S) \\
& + D_{nn}(y) \Bigg[\left(\frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1 m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + \phi_S) \\
& \quad + \left(\frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1 m_2 m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}}{m_1 m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_2 + \phi_S) \\
& \quad + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(3\phi_1 - \phi_S) \\
& \quad + \left(\frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2 - \phi_S) \\
& \quad + \left(\frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + 2\phi_2 - \phi_S) \\
& \quad - \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(2\phi_1 - \phi_2 + \phi_S) \\
& \quad - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(\phi_1 - 2\phi_2 - \phi_S) \\
& \quad + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(2\phi_1 + \phi_2 - \phi_S) \Bigg]
\end{aligned}
$$

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{LL} = F_0^{\hat{l}_1 \cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

A_{LU} asymmetry

Anselmino, Barone and AK, 713 (2012)317

$F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi),$ with $\Delta\phi = \phi_1 - \phi_2$

One can choose as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$

Integrating σ_{UU} and σ_{lU} over ϕ_2 we obtain

$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = $$

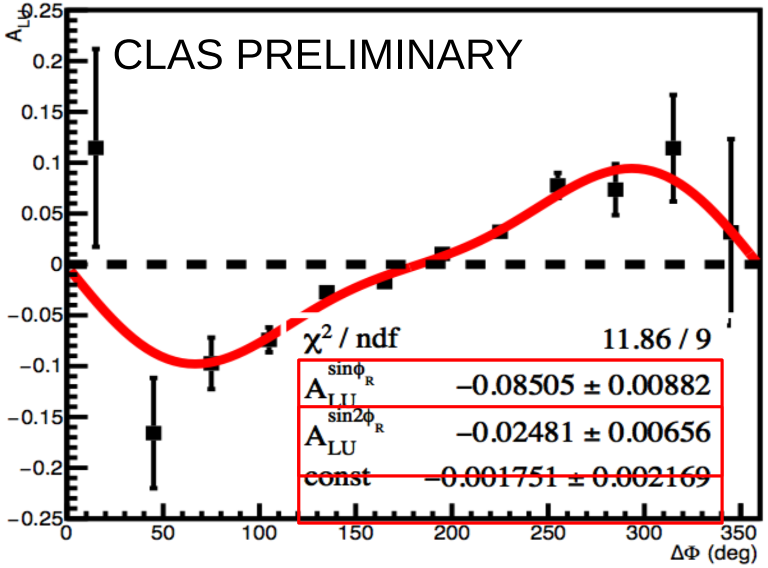
$$= \frac{-\dfrac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)\sin(\Delta\phi)}{F_0^{\hat{u}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)}$$

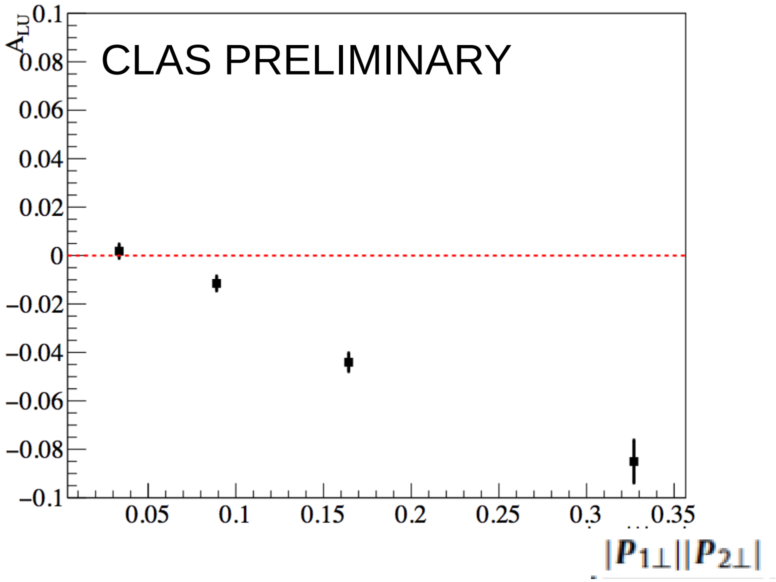
A_{LU} @ CLAS

$$A_{LU} = \frac{\sigma_{LU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{LU1}\cos(\Delta\phi)+a_{LU2}\cos(2\Delta\phi)+\cdots\right)\sin(\Delta\phi)}{\sigma_{UU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{UU1}\cos(\Delta\phi)+a_{UU2}\cos(2\Delta\phi)+\cdots\right)} \approx$$

$$\approx p_1 \sin(\Delta\phi) + p_2 \sin(2\Delta\phi) + \cdots$$

Courtesy of S.Pisano & H.Avakian





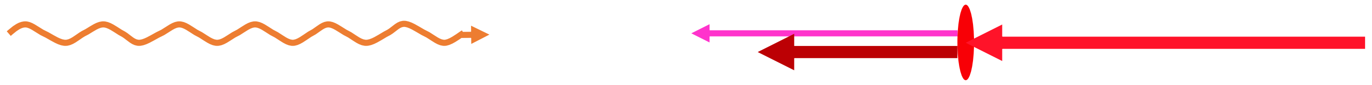
Presence of higher harmonics indicate that $\sigma_{LU}(\Delta\Phi) \neq \sigma_{UU}(\Delta\Phi)$

Hadronization in MC even generators

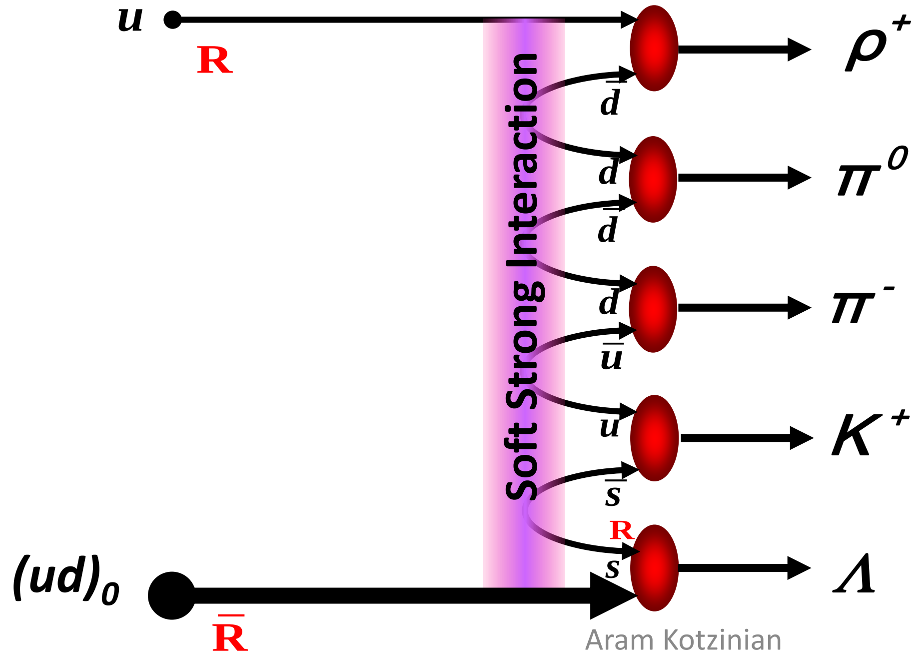
PDF , hard scattering and hadronization are factorized :

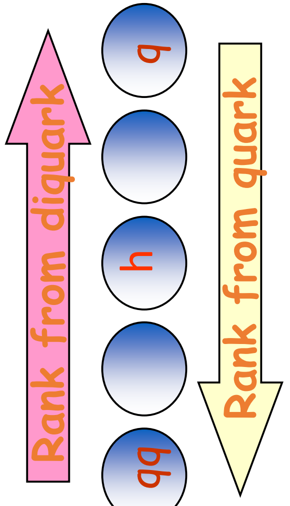
$$d\sigma^{lN\to lhX} = \sum_q f_q(x,\mathbf{k_T},\mathbf{s}_q;\mathbf{S}_N) \otimes d\sigma^{lq\to lq} \otimes H^q_{h/N}(x,\mathbf{k_T},\mathbf{s}_q;x_F,\mathbf{p}^h_\mathbf{T};\mathbf{S}_N)$$

- Before



- After hard scattering





Sivers effect

It is possible to modify existing MC event generators including the nucleon STMD PDF
Example: Sivers effect (not affecting hadronisation)
Matevosyan, AK, Ashenauer, Avakian,Thomas **PR D**92, 054028 (2015)

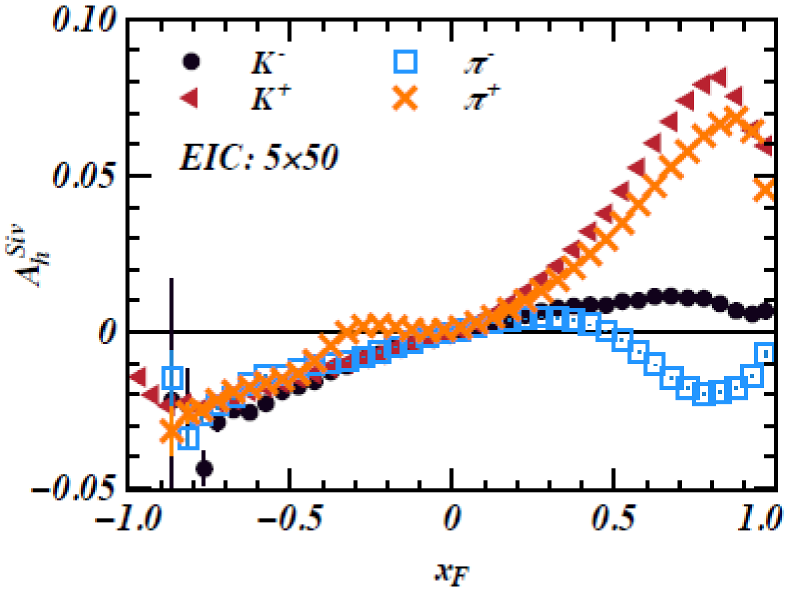


FIG. 13. EIC model SSAs for 5×50 SIDIS kinematics for charged pions and kaons vs x_F. The Sivers asymmetry is present both in the current and target fragmentation regions.

Target remnant in Polarized SIDIS

JETSET is based on SU(6) quark-diquark model

$$p^+ = \frac{1}{\sqrt{18}}\{u^+[3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2}u^-(ud)_{1,1} - \sqrt{2}d^+(uu)_{1,0} + 2d^-(uu)_{1,1}\}$$
$$n^+ = \frac{1}{\sqrt{18}}\{d^+[3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2}d^-(ud)_{1,1} - \sqrt{2}u^+(dd)_{1,0} + 2u^-(dd)_{1,1}\}$$

$$\Delta q(x) = q_+(x) - q_-(x)$$

$$u_+(x) \rightarrow p^+ \ominus u^+ \Longrightarrow \begin{cases} \{(ud)_{0,0} \cdots\cdots u^+\}, & w = 0.9 \\ \{(ud)_{1,0} \cdots\cdots u^+\}, & w = 0.1 \end{cases}$$

90% scalar

$$u_-(x) \rightarrow p^- \ominus u^+ \Longrightarrow \{(ud)_{1,-1} \cdots\cdots u^+\}, \quad w = 1$$

100% vector

$$d_+(x) \rightarrow n^+ \ominus u^+ \Longrightarrow \{(dd)_{1,0} \cdots\cdots u^+\}, \quad w = 1$$

$$d_-(x) \rightarrow n^- \ominus u^+ \Longrightarrow \{(dd)_{1,-1} \cdots\cdots u^+\}, \quad w = 1$$

Probabilities of different string spin configurations depend on quark and target polarizations, target type and process type

Fragmentation functions in LEPTO
Dependence on target remnant spin state

AK EPJ C44, 211 (2005)

Example: valence *u*-quark is removed from proton. Default LEPTO: the remnant ***(ud)*** diquark is in 75% (25%) of cases scalar (vector)

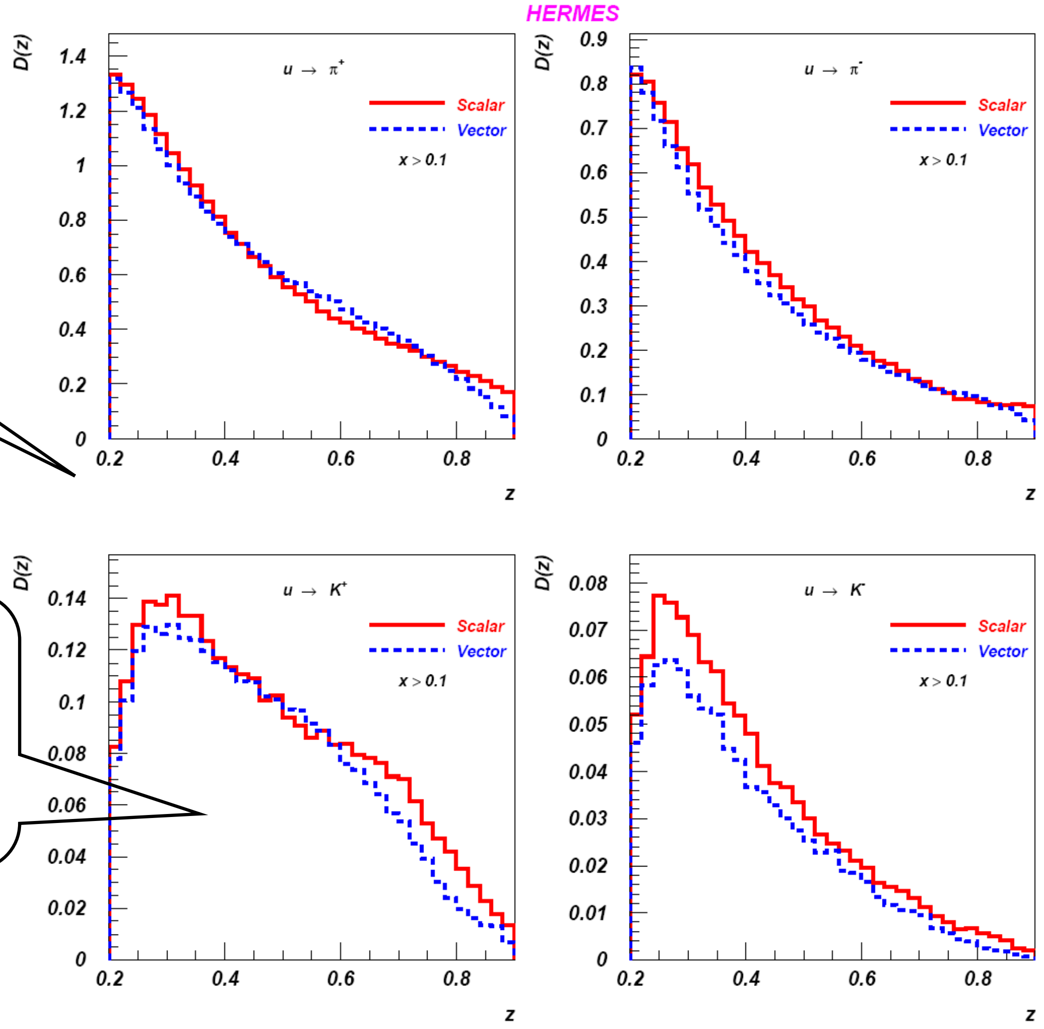
$\{(ud)_0 \cdots\cdots u\}, \quad w=1.$

$\{(ud)_1 \cdots\cdots u\}, \quad w=1.$

Even in unpolarized LEPTO there is a dependence on target remnant spin state

(ud)$_0$: first rank Λ is possible

(ud)$_1$: first rank Λ is impossible



More general approach: hadronization function

- x-z factorization was not checked
  - Extract unknown integrals of fragmentation functions in different subsets of x-bins and compare them
- Missing term in the (polarized) SIDIS equation related to polarization dependent hadronization

Neglected

$$A_1^h(x,z,Q^2)=\frac{\sum_q e_q^2 q(x,Q^2)H_{q/N}^h(x,z,Q^2)\left(\frac{\Delta q(x,Q^2)}{q(x,Q^2)}+\boxed{\frac{\Delta H_{q/N}^h(x,z,Q^2)}{H_{q/N}^h(x,z,Q^2)}}\right)}{\sum_q e_q^2 q(x,Q^2)H_{q/N}^h(x,z,Q^2)\left(1+\boxed{\frac{\Delta q(x,Q^2)\Delta H_{q/N}^h(x,z,Q^2)}{q(x,Q^2)H_{q/N}^h(x,z,Q^2)}}\right)}$$

Asymmetry

$$A_{1N}^{h,Exp}(x,z,Q^2) = \frac{\sum_q e_q^2 \left(\Delta q(x,Q^2) H_{q/N}^h(x,z,Q^2) + q(x,Q^2)\Delta H_{q/N}^h(x,z,Q^2)\right)}{\sum_q e_q^2 \left(q(x,Q^2) H_{q/N}^h(x,z,Q^2) + \Delta q(x,Q^2)\Delta H_{q/N}^h(x,z,Q^2)\right)}$$

$$\approx A_{1N}^{h,Std}(x,z,Q^2) + \boxed{\frac{\sum_q e_q^2 q(x,Q^2)\Delta H_{q/N}^h(x,z,Q^2)}{\sum_q e_q^2 q(x,Q^2) H_{q/N}^h(x,z,Q^2)}}$$

$$\cong A_{1N}^{h,Std}(x,z,Q^2) + \varepsilon(x,z,Q^2)$$

The standard expression for SIDIS asymmetry is obtained when

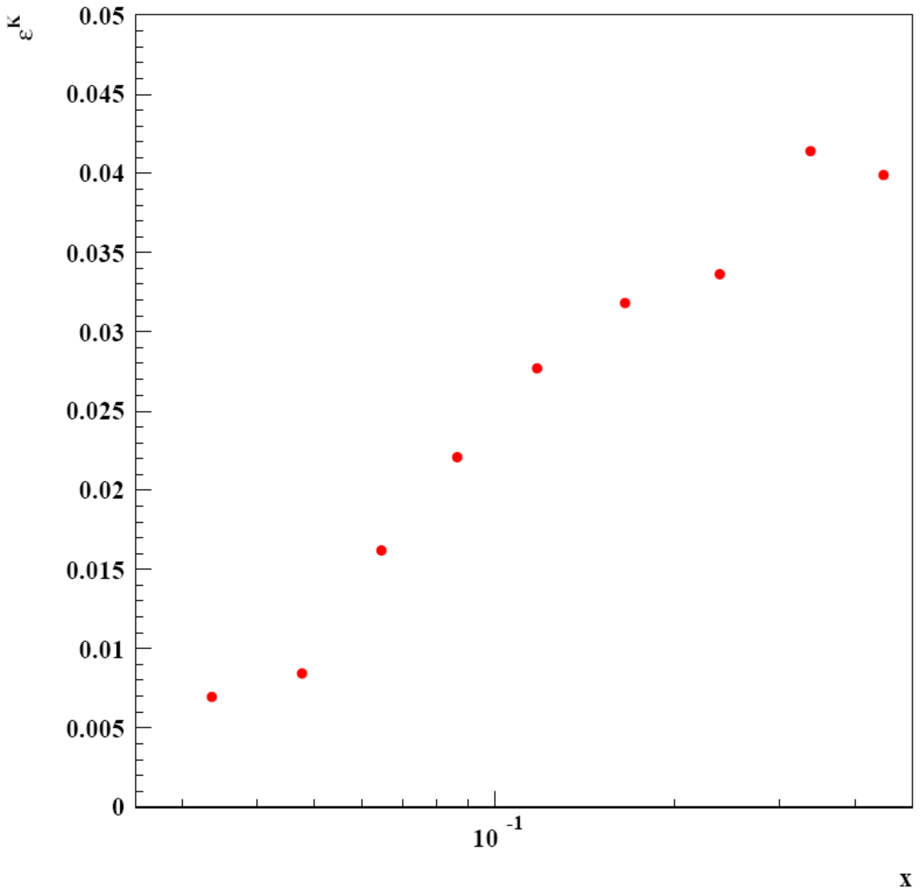
$$H_{q/N}^h(x,z,Q^2) \to D_q^h(z,Q^2) \qquad \Delta H_{q/N}^h(x,z,Q^2) \to 0$$

Only standard part of expression for asymmetry contains quark polarizations

$$A_{1N}^{h,Std}(x,z,Q^2) = A_{1N}^{h,Exp}(x,z,Q^2) - \varepsilon(x,z,Q^2)$$

ε(x)

LEPTO with
HERMES
tuning and cuts
CTEQ6 LO
K^+, K^- production
off deuterium target

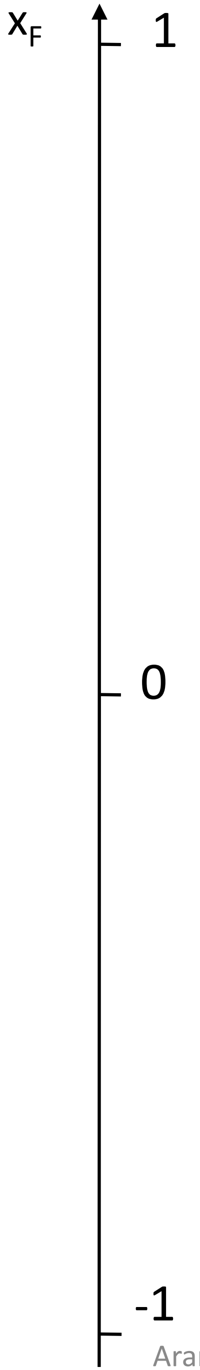


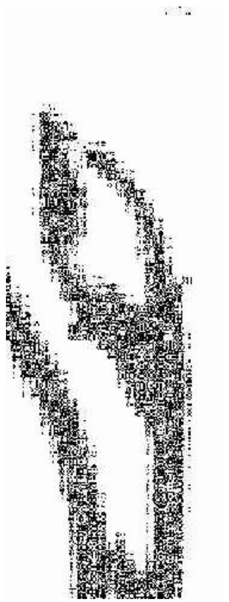
ε(x)≠0 can be considered as correction to factorized cross-section due to finite W, Q^2.
The presence of spin-dependent hadronozation can be important in extraction of helicity PDFs

Conclusions

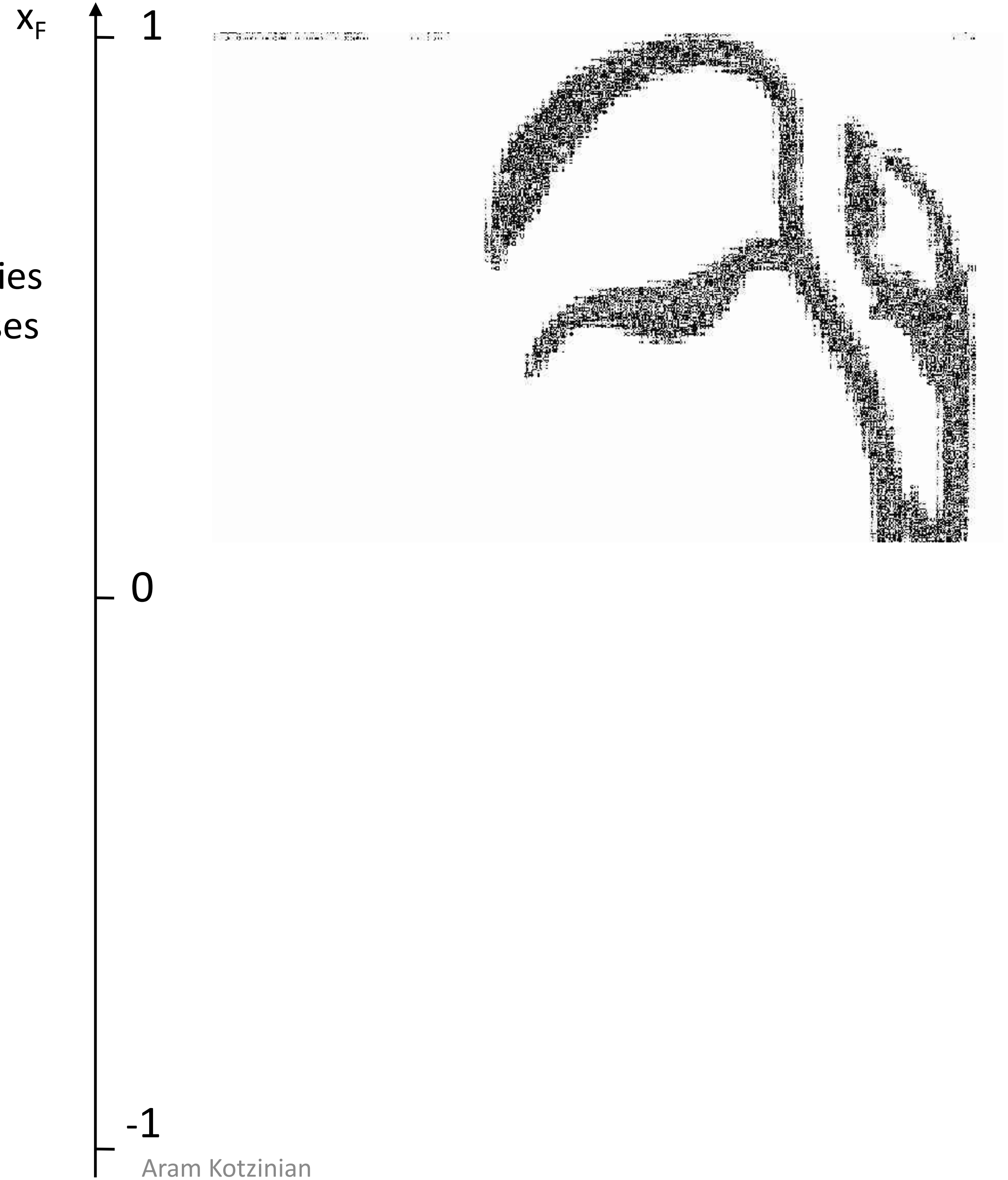
- Lot of progress …
- Still lot of to do

CFR





Larger phase space
higher W, z, x
different STMD asymmetries
Inputs from other processes



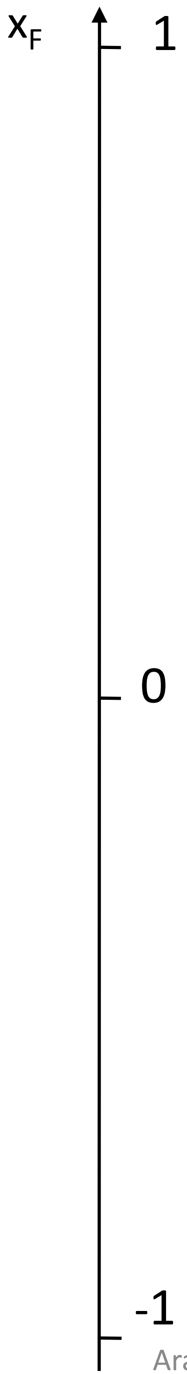
x_F

1

Better resolution,
higher statistics
new theory inputs

0

-1



TFR with good
resolution etc

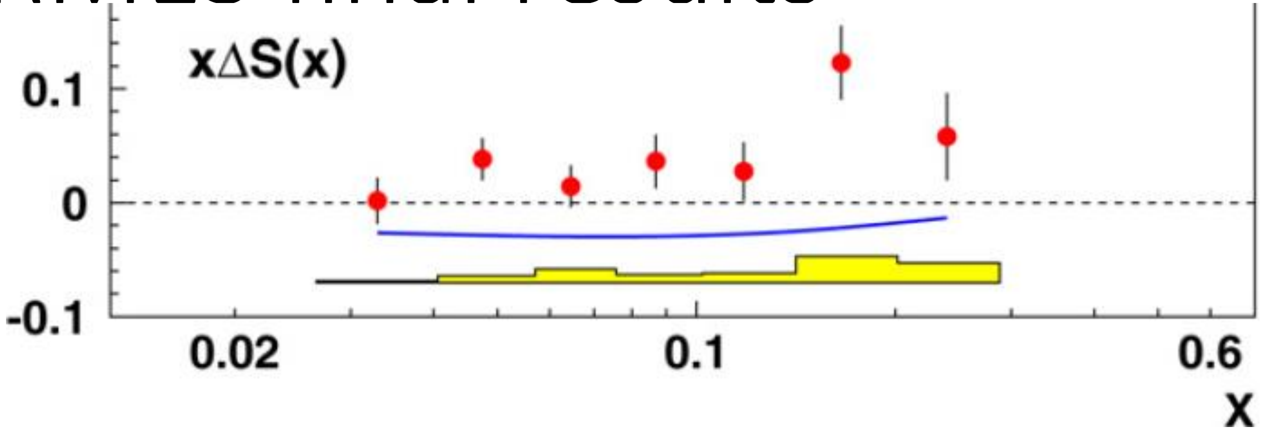
Full picture can be
surprising and
beautiful

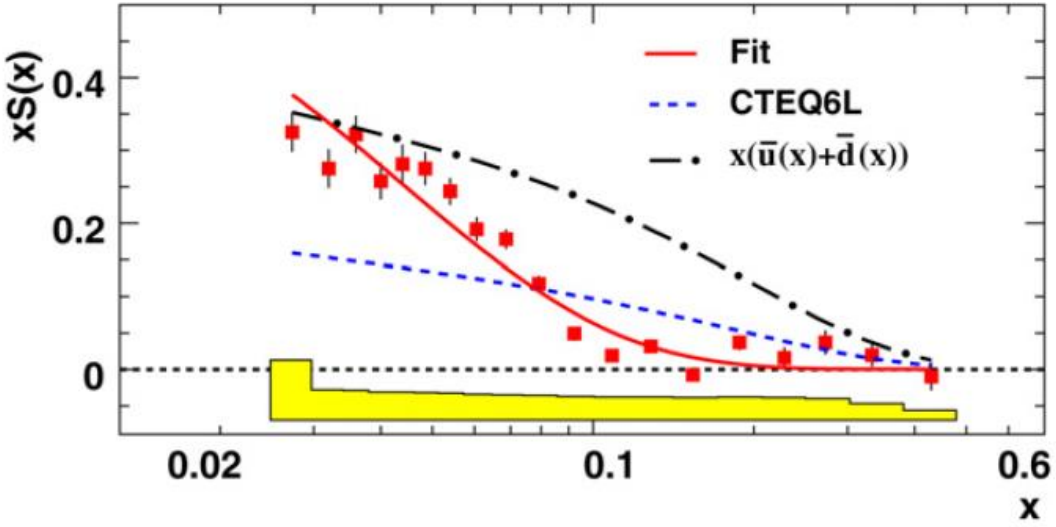
Work in progress

Thank You

Backup

HERMES final results





Modeling ε in LEPTO

$$\varepsilon(x,z,Q^2) = \frac{\sum_q e_q^2 q(x,Q^2)\Delta H^h_{q/N}(x,z,Q^2)}{\sum_q e_q^2 q(x,Q^2) H^h_{q/N}(x,z,Q^2)}$$

LEPTO: HERMES tuning

parl(4)=probability of scalar diquark

$$parl(4) = 0.9 \Rightarrow N^{K/N}_{++} \propto \left(1+(1-y)^2\right)\sum_q e_q^2 q(x) H^{K/N}_{++},$$

$$parl(4) = 0.0 \Rightarrow N^{K/N}_{+-} \propto \left(1+(1-y)^2\right)\sum_q e_q^2 q(x) H^{K/N}_{+-}$$

$$\varepsilon^K_d(x,z,Q^2) = \frac{N^{K/p}_{++} + N^{K/n}_{++} - N^{K/p}_{+-} - N^{K/n}_{+-}}{N^{K/p}_{++} + N^{K/n}_{++} + N^{K/p}_{+-} + N^{K/n}_{+-}}$$

Strange quarks polarization 2

Data from Ahmed El Alaoui PhD thesis, 2006

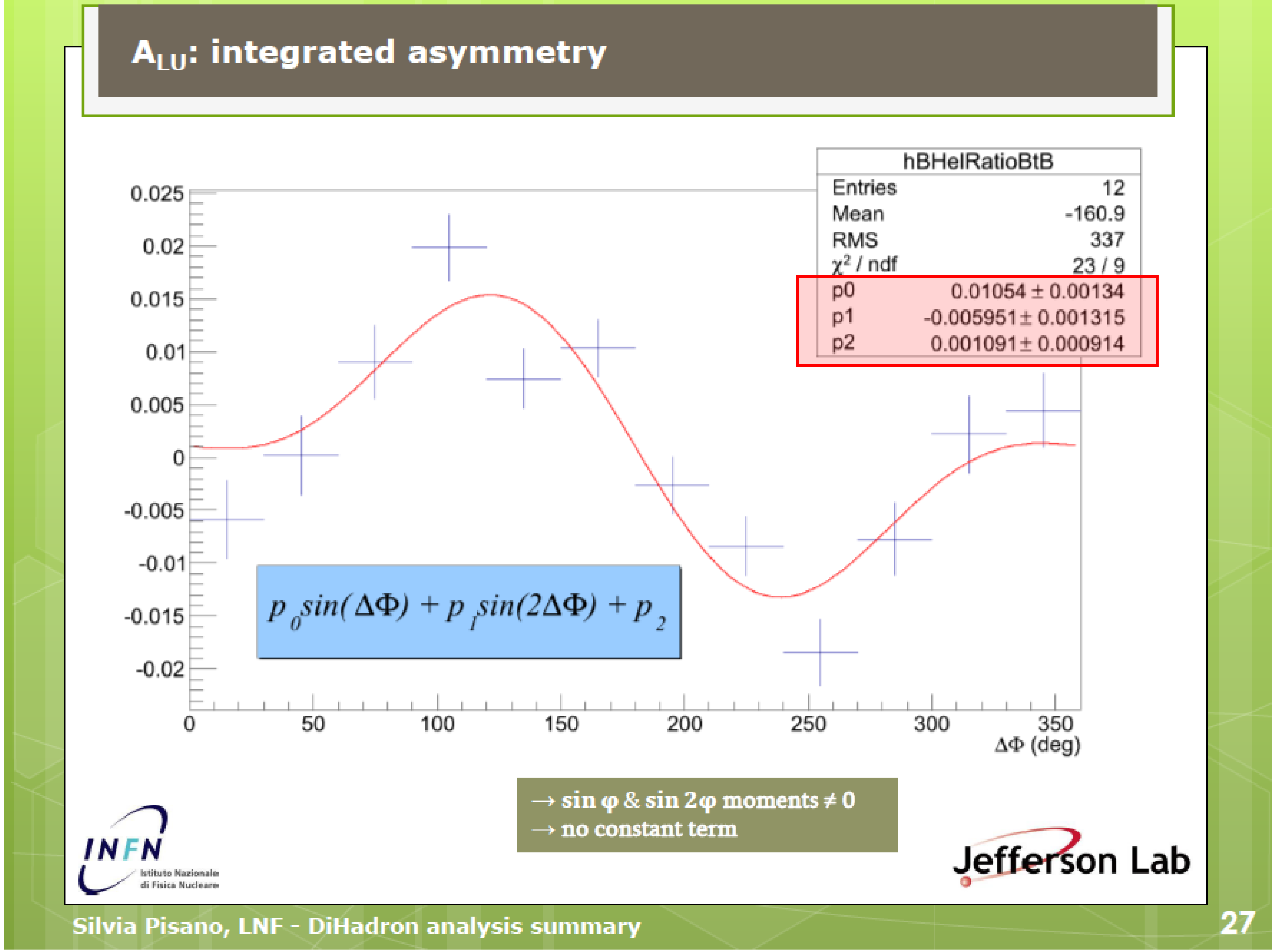
$$\Delta S_{HERMES} = \int_{0.02}^{0.3} dx \Delta S(x) = \sum_{i=1}^{7} \frac{\Delta S}{S}(x_i)_{HERMES} \int_{x_i}^{x_{i+1}} dx S(x) = 0.0055$$

$$\Delta S_{\varepsilon-corr} = \int_{0.02}^{0.3} dx \Delta S(x) = \sum_{i=1}^{7} \frac{\Delta S}{S}(x_i)_{\varepsilon-corr} \int_{x_i}^{x_{i+1}} dx S(x) = -0.027$$

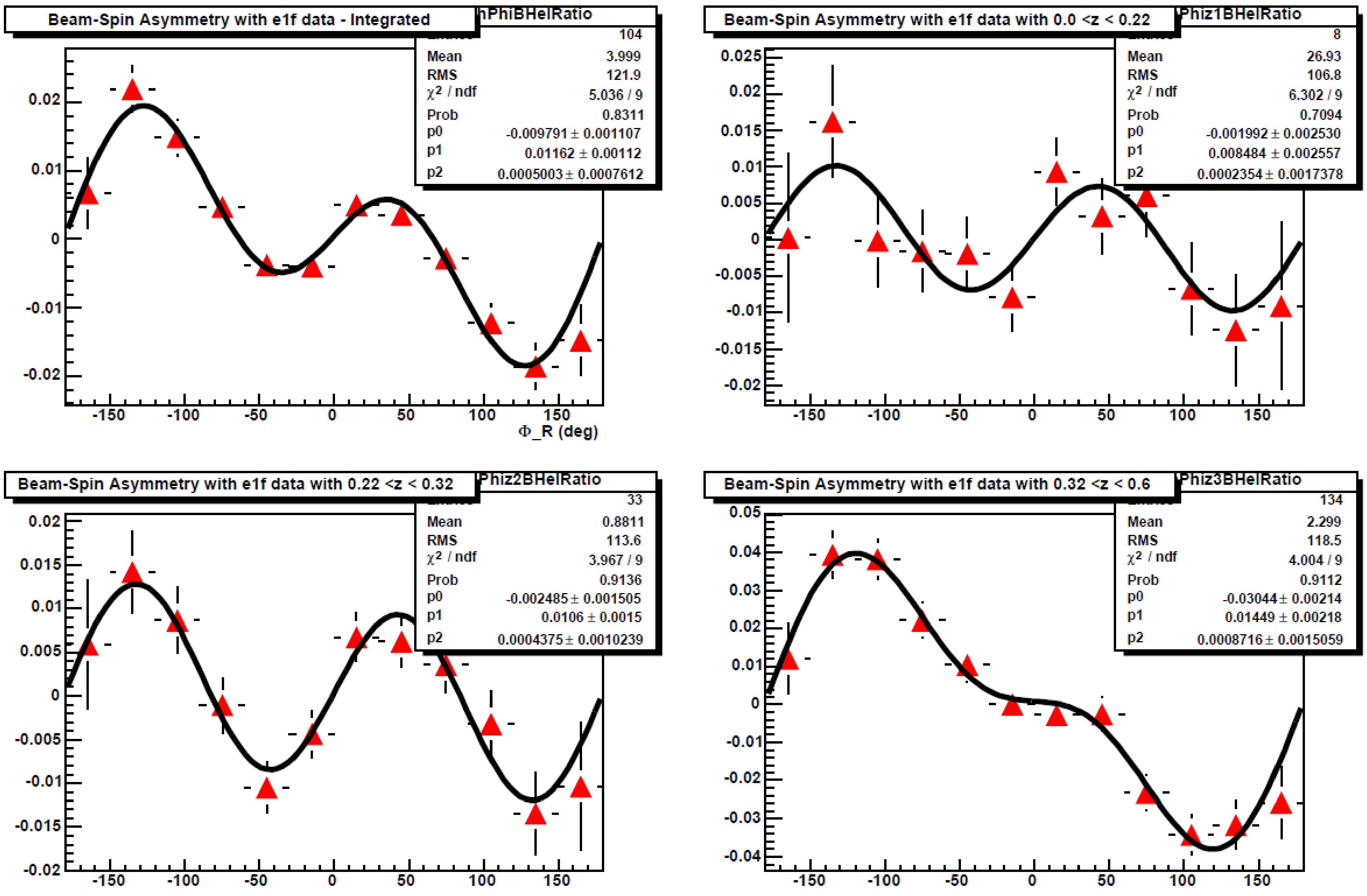
New value is more than one standard deviation away

JLab CLAS, preliminary (**courtesy of Silvia Pisano**)

π^+ in CFR, π^- in TFR,



JLab Very Preliminary



Presence of higher harmonics indicate that $\sigma_{LU}(\Delta\Phi) \neq \sigma_{UU}(\Delta\Phi)$