

Angular Distributions of the Drell-Yan Process

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Workshop on
“3D Parton Distributions: path to the LHC”
Laboratori Nazionali di Frascati
November 29 – December 2, 2016

Based on the paper of JCP, Wen-Chen Chang, Evan McClellan,
Oleg Teryaev, Phys. Lett. B758 (2016) 384, arXiv: 1511.08932

The Drell-Yan Process

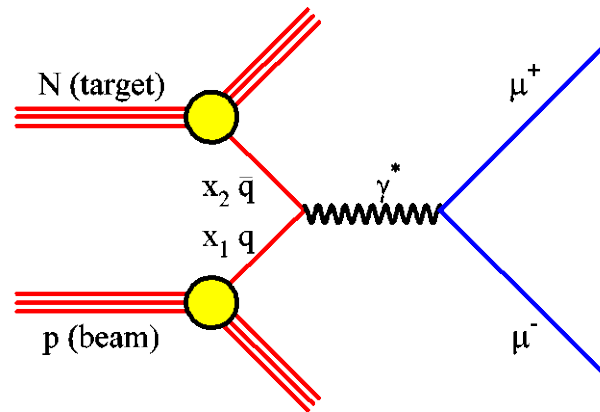
MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \rightarrow \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \rightarrow 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function νW_2 near threshold.



$$\left(\frac{d^2\sigma}{dx_1 dx_2} \right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2} \sum_a e_a^2 [q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1)q_a(x_2)]$$

Angular Distribution in the “Naïve” Drell-Yan

VOLUME 25, NUMBER 5

PHYSICAL REVIEW LETTERS

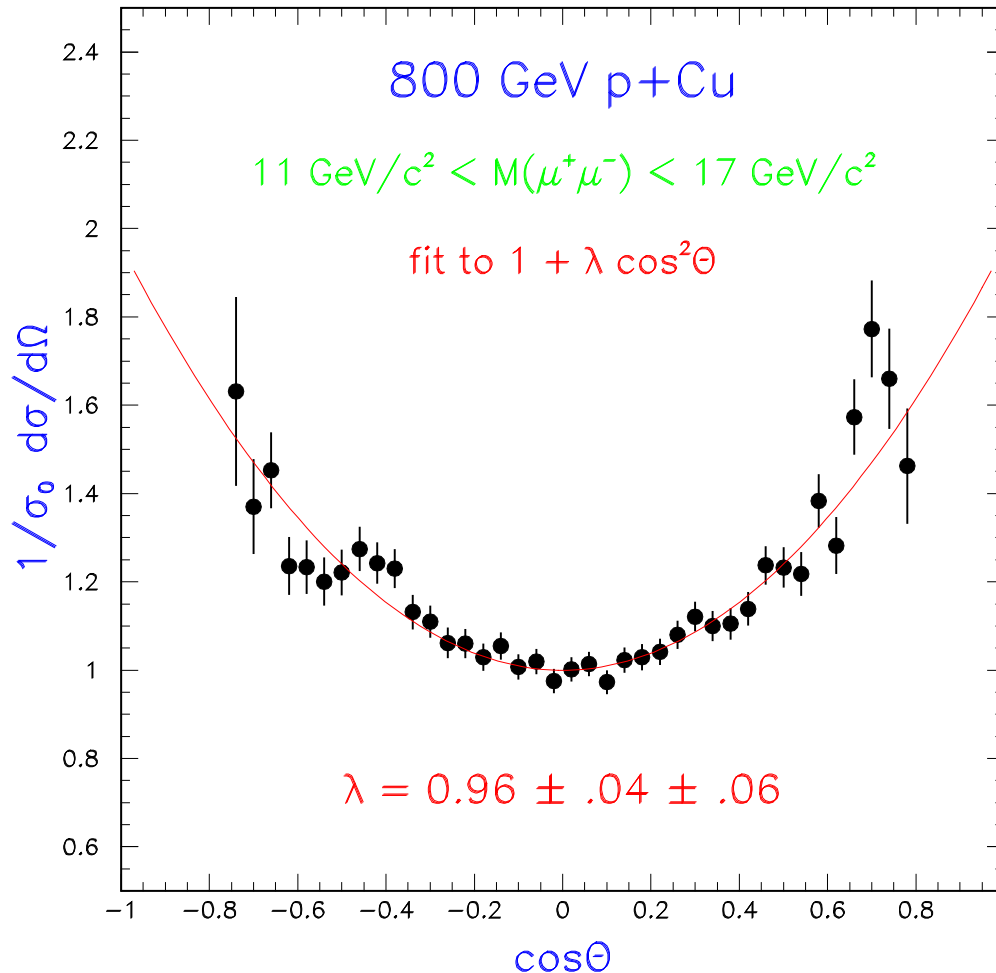
3 AUGUST 1970

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin- $\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1 + \cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's¹⁰ vector-dominance model, where θ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

Drell-Yan angular distribution

Lepton Angular Distribution of “naive” Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0(1 + \lambda \cos^2 \theta); \quad \lambda = 1$$

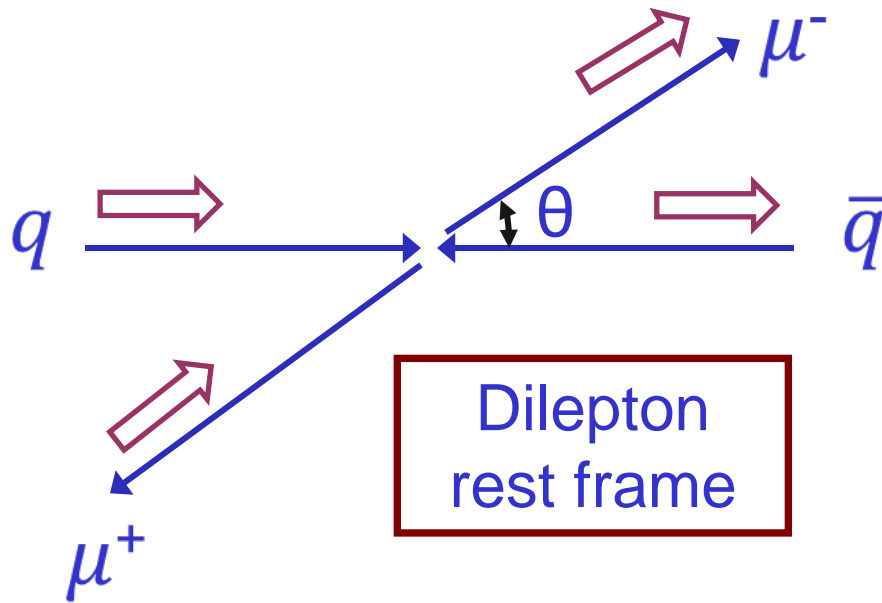


Data from Fermilab
E772

(Ann. Rev. Nucl. Part.
Sci. 49 (1999) 217-253)

Why is the lepton angular distribution $1 + \cos^2 \theta$?

Helicity conservation and parity



Adding all four helicity configurations:

$$d\sigma \sim 1 + \cos^2 \theta$$

$$RL \rightarrow RL$$

$$d\sigma \sim (1 + \cos \theta)^2$$

$$RL \rightarrow LR$$

$$d\sigma \sim (1 - \cos \theta)^2$$

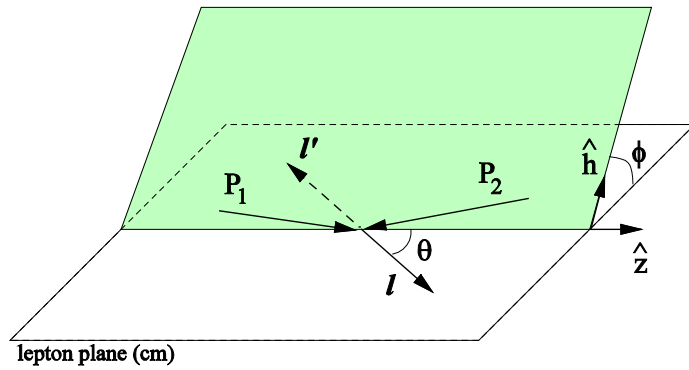
$$LR \rightarrow LR$$

$$d\sigma \sim (1 + \cos \theta)^2$$

$$LR \rightarrow RL$$

$$d\sigma \sim (1 - \cos \theta)^2$$

Drell-Yan lepton angular distributions



Θ and Φ are the decay polar and azimuthal angles of the μ^- in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

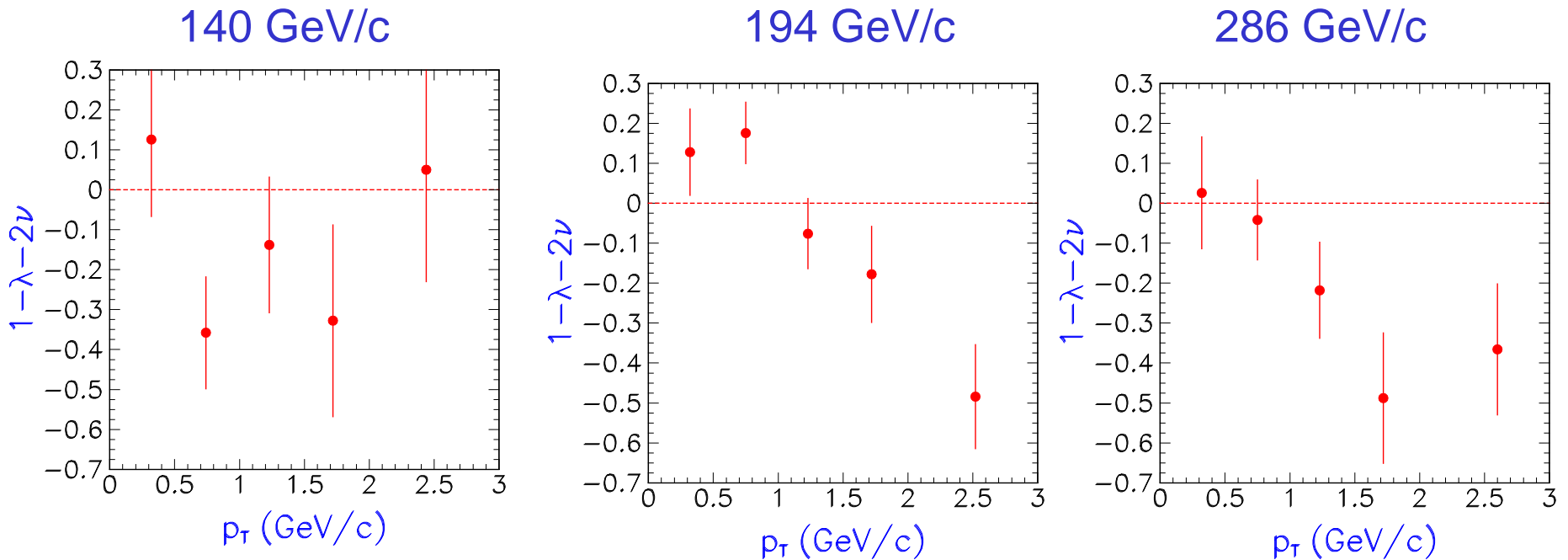
$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right) = \left[\frac{3}{4\pi}\right] \left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi\right]$$

Lam-Tung relation: $1 - \lambda = 2\nu$

- Reflect the spin-1/2 nature of quarks
(analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

Decay angular distributions in pion-induced Drell-Yan

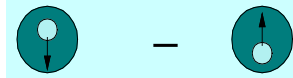
Is the Lam-Tung relation violated?



Data from NA10 (Z. Phys. 37 (1988) 545)

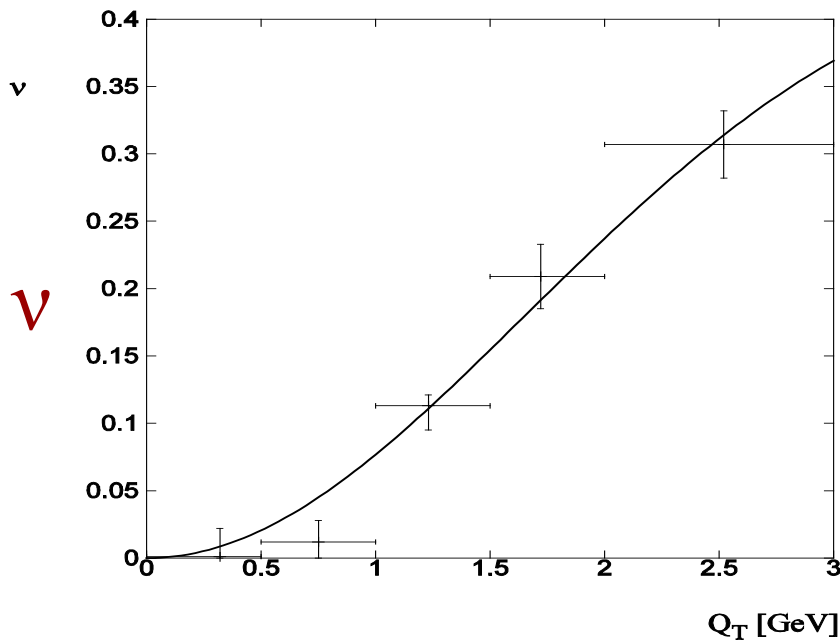
Violation of the Lam-Tung relation suggests interesting new origins
(Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar,
Hoyer, Vântinnen, Vogt, etc.)

Boer-Mulders function h_1^\perp



- Boer pointed out that the $\cos 2\phi$ dependence can be caused by the presence of the Boer-Mulders function.

- h_1^\perp can lead to an azimuthal dependence with $v \propto \left(\frac{h_1^\perp}{f_1}\right) \left(\frac{\bar{h}_1^\perp}{\bar{f}_1}\right)$



$$h_1^\perp(x, k_T^2) = \frac{\alpha_T}{\pi} c_H \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$

$$v = 16\kappa_1 \frac{Q_T^2 M_C^2}{(Q_T^2 + 4M_C^2)^2}$$

Boer, PRD 60 (1999) 014012

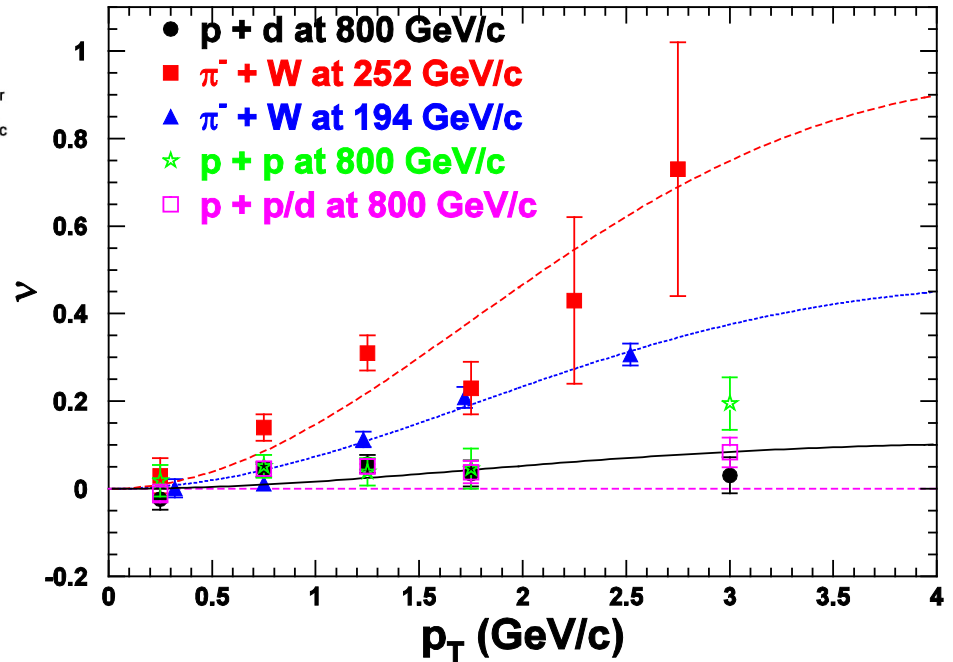
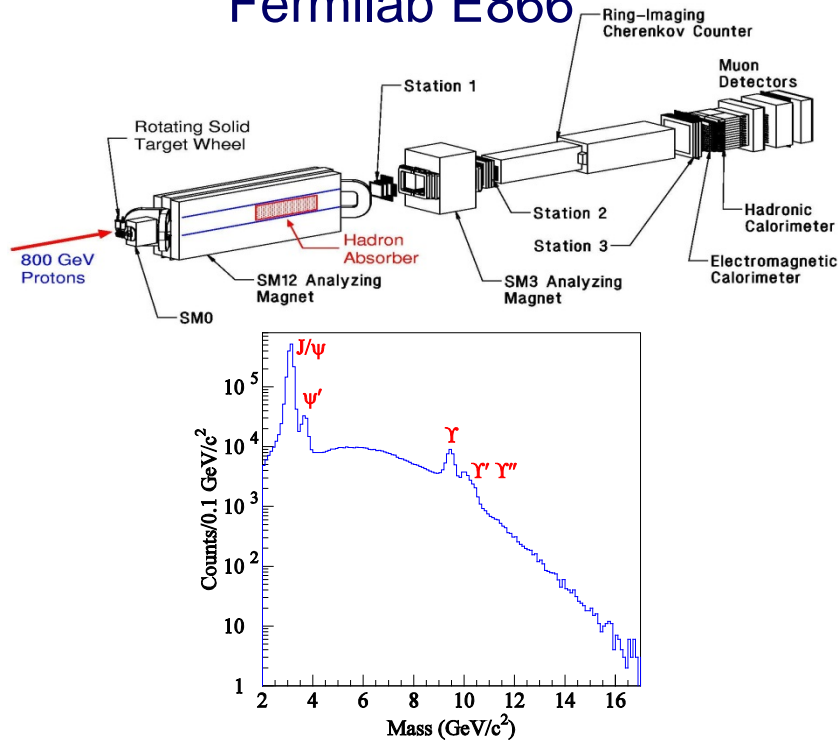
$$\kappa_1 = 0.47, M_C = 2.3 \text{ GeV}$$

$v > 0$ implies valence BM functions for pion and nucleon have same signs

Azimuthal $\cos 2\Phi$ Distribution in p+d Drell-Yan

Lingyan Zhu et al., PRL 99 (2007) 082301;
PRL 102 (2009) 182001

Fermilab E866



With Boer-Mulders function h_1^\perp :

$$v(\pi^- W \rightarrow \mu^+ \mu^- X) \sim [\text{valence } h_1^\perp(\pi)] * [\text{valence } h_1^\perp(p)]$$

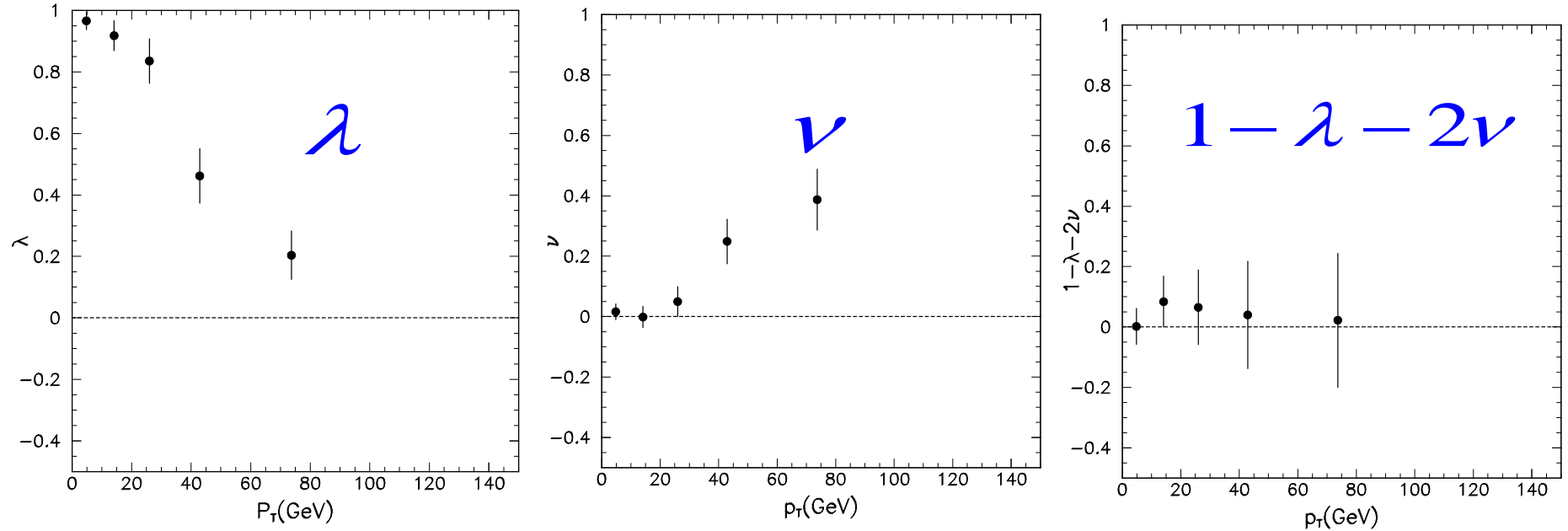
$$v(pd \rightarrow \mu^+ \mu^- X) \sim [\text{valence } h_1^\perp(p)] * [\text{sea } h_1^\perp(p)]$$

Sea-quark BM function is much smaller than valence BM function

Lam-Tung relation from CDF Z-production

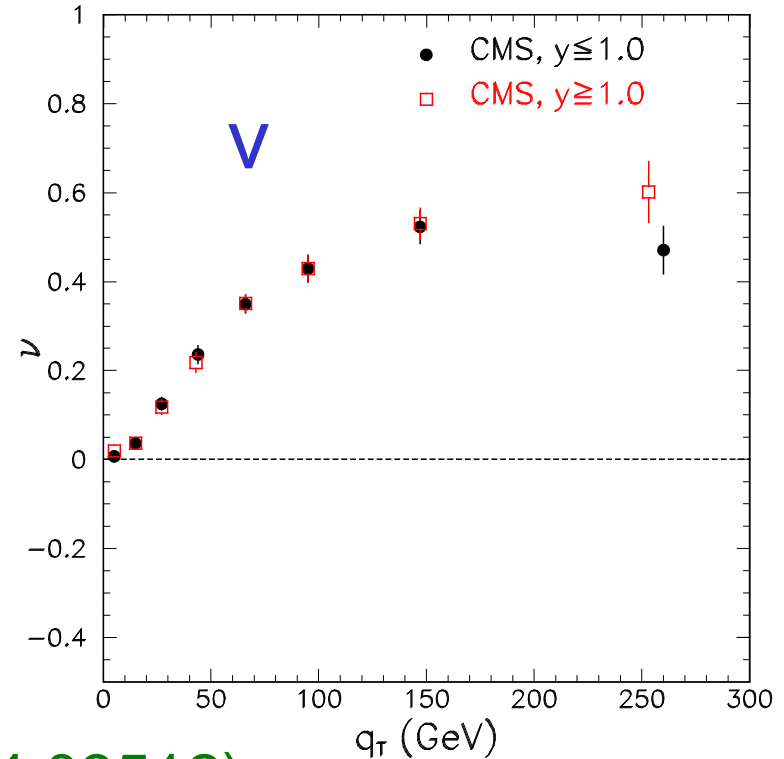
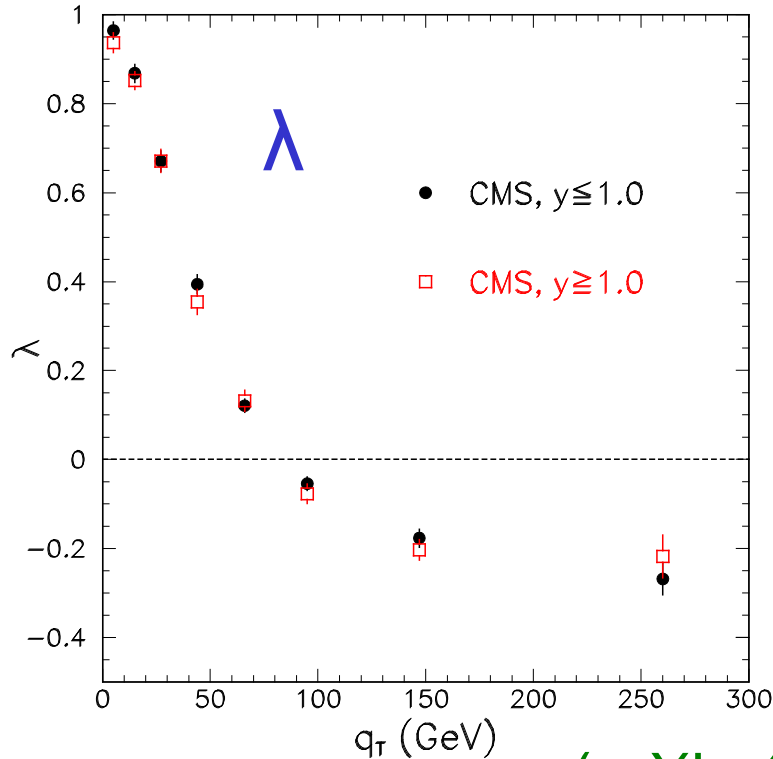
$$p + \bar{p} \rightarrow e^+ + e^- + X \text{ at } \sqrt{s} = 1.96 \text{ TeV}$$

arXiv:1103.5699



- Strong p_T (q_T) dependence of λ and ν
- Lam-Tung relation ($1 - \lambda = 2\nu$) is satisfied within experimental uncertainties

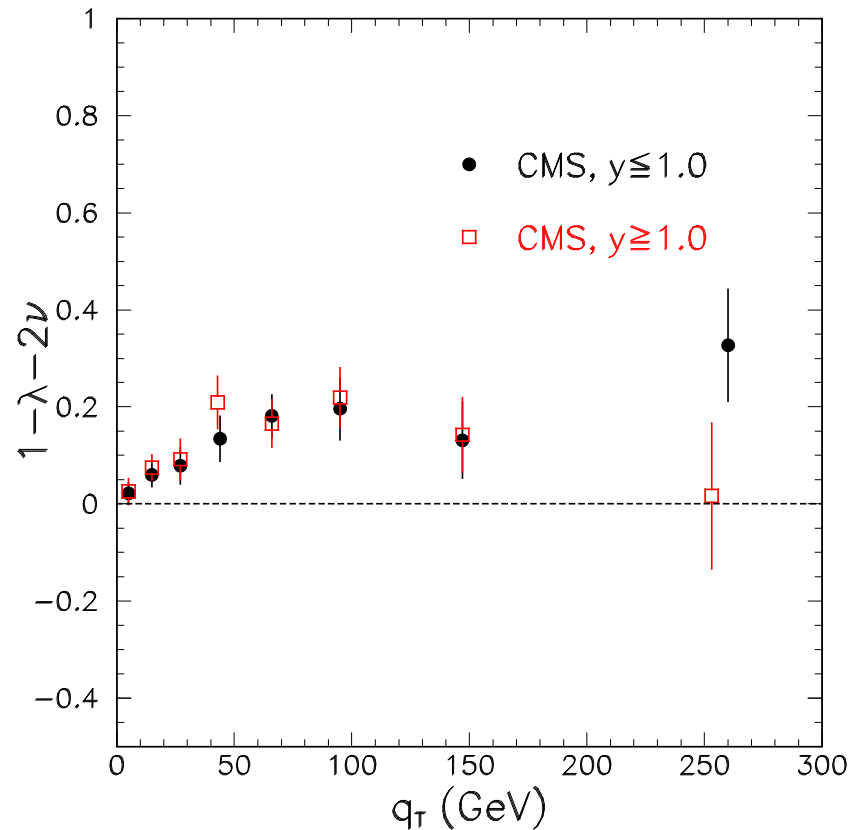
Recent CMS data for Z-boson production in $p+p$ collision at 8 TeV



(arXiv:1504.03512)

- Striking q_T dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

Recent data from CMS for Z-boson production in $p+p$ collision at 8 TeV



- Yes, the Lam-Tung relation is violated ($1 - \lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation?

Interpretation of the CMS Z-production results

$$\begin{aligned} \frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3\cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi \end{aligned}$$

Questions:

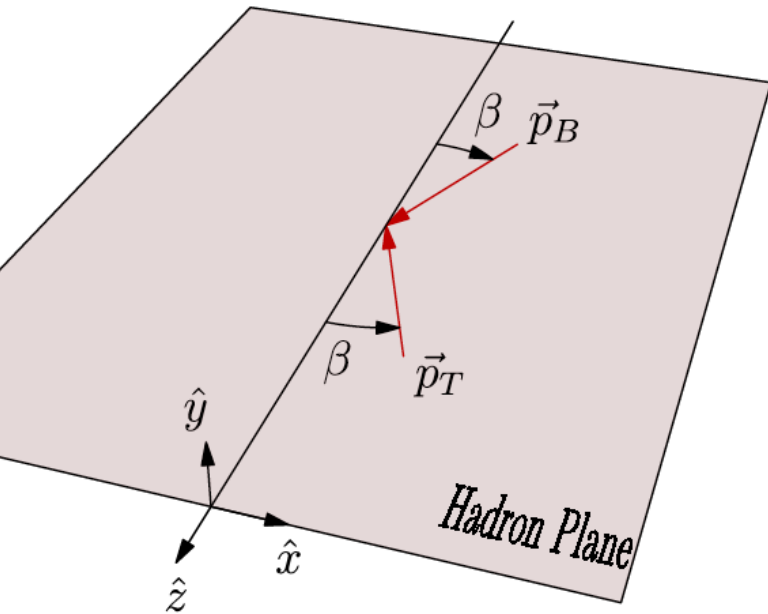
- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the Q_T dependence of A_0, A_1, A_2 , etc?
- Can one understand the origin of the violation of Lam-Tung relation?

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame

1) Hadron Plane

- Contains the beam \vec{P}_B and target \vec{P}_T momenta
- Angle β satisfies the relation $\tan \beta = q_T / Q$



How is the angular distribution expression derived?

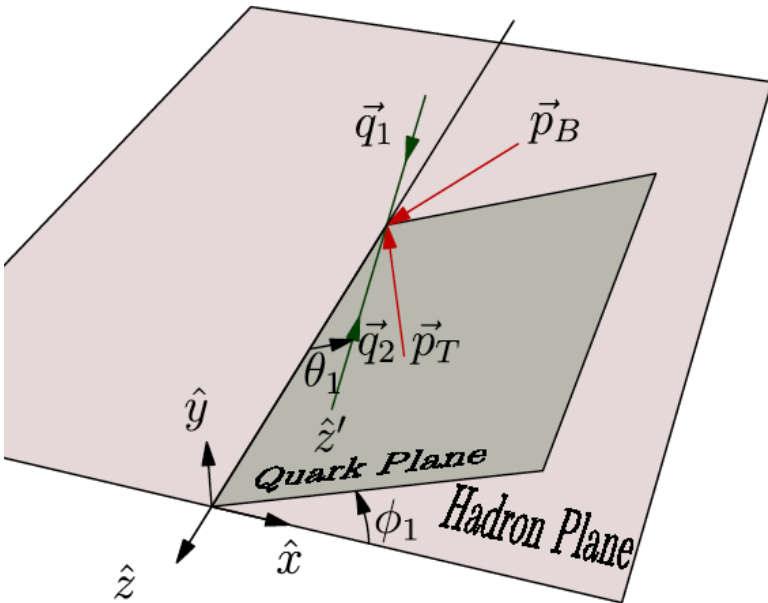
Define three planes in the Collins-Soper frame

1) Hadron Plane

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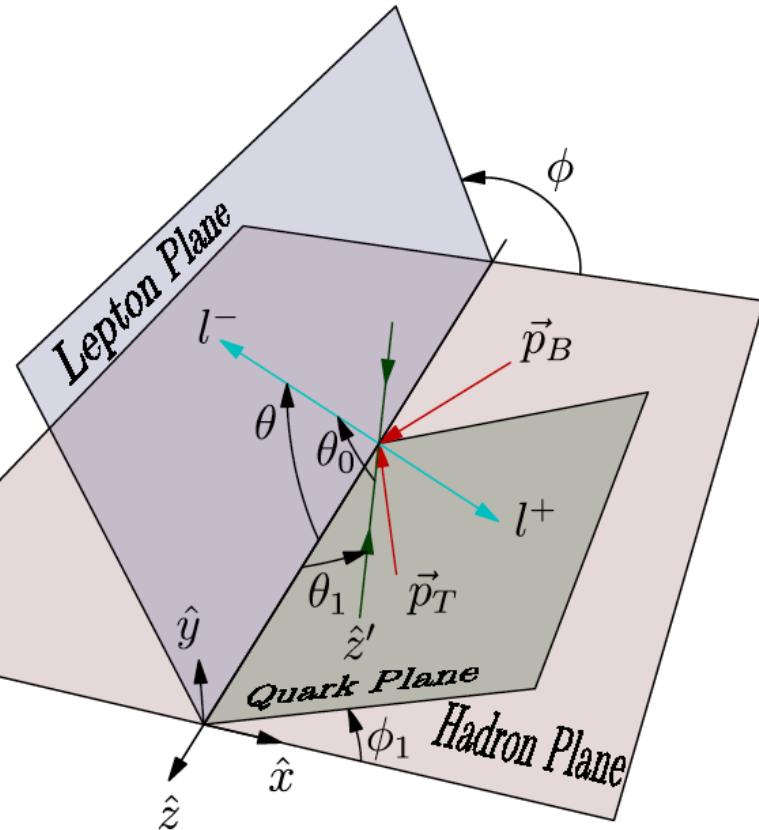
2) Quark Plane

- q and \bar{q} have head-on collision along the \hat{z}' axis
- \hat{z}' axis has angles θ_1 and ϕ_1 in the C-S frame



How is the angular distribution expression derived?

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2) Quark Plane

- q and \bar{q} have head-on collision along the \hat{z}' axis
- \hat{z}' axis has angles θ_1 and ϕ_1 in the C-S frame

3) Lepton Plane

- l^- and l^+ are emitted back-to-back with equal $|\vec{P}|$
- l^- is emitted at angle θ and ϕ in the C-S frame

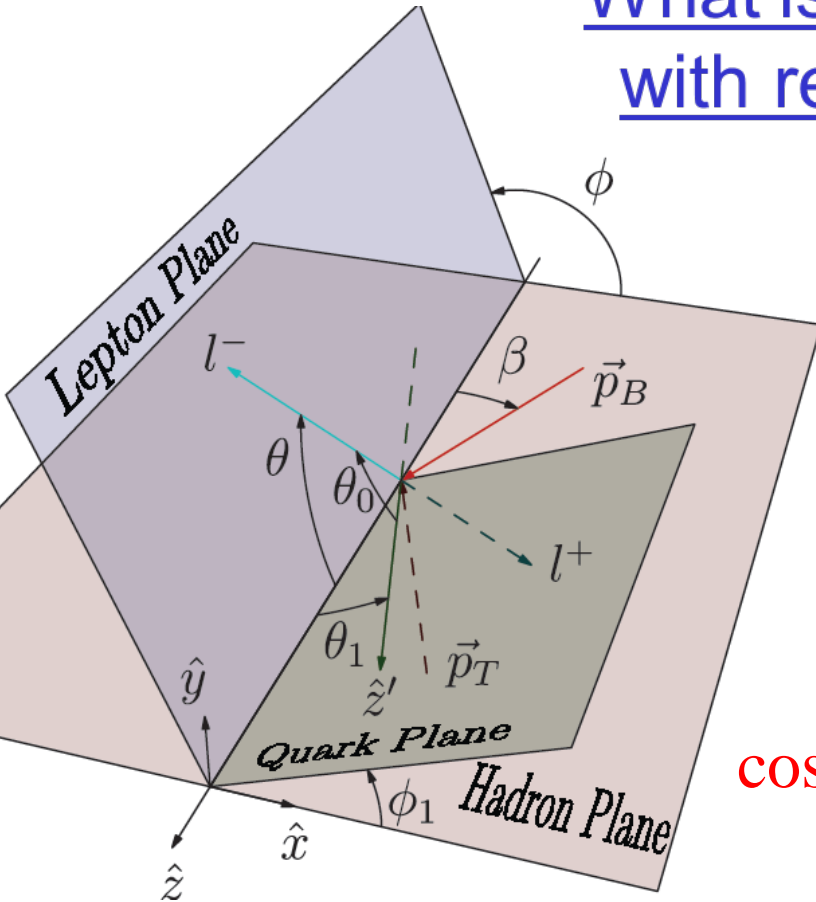
How is the angular distribution expression derived?

What is the lepton angular distribution with respect to the \hat{z}' (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0$$

How to express the angular distribution in terms of θ and ϕ ?

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1)$$



How is the angular distribution expression derived?

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{\sin^2 \theta_1}{2} (1 - 3 \cos^2 \theta) \\ & + \left(\frac{1}{2} \sin 2\theta_1 \cos \phi_1\right) \sin 2\theta \cos \phi \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1\right) \sin^2 \theta \cos 2\phi \\ & + (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1\right) \sin^2 \theta \sin 2\phi \\ & + \left(\frac{1}{2} \sin 2\theta_1 \sin \phi_1\right) \sin 2\theta \sin \phi \\ & + (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi.\end{aligned}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta) \\ & + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi \\ & + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi \\ & + A_6 \sin 2\theta \sin \phi \\ & + A_7 \sin \theta \sin \phi\end{aligned}$$

$A_0 - A_7$ are entirely described by θ_1 , ϕ_1 and a

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

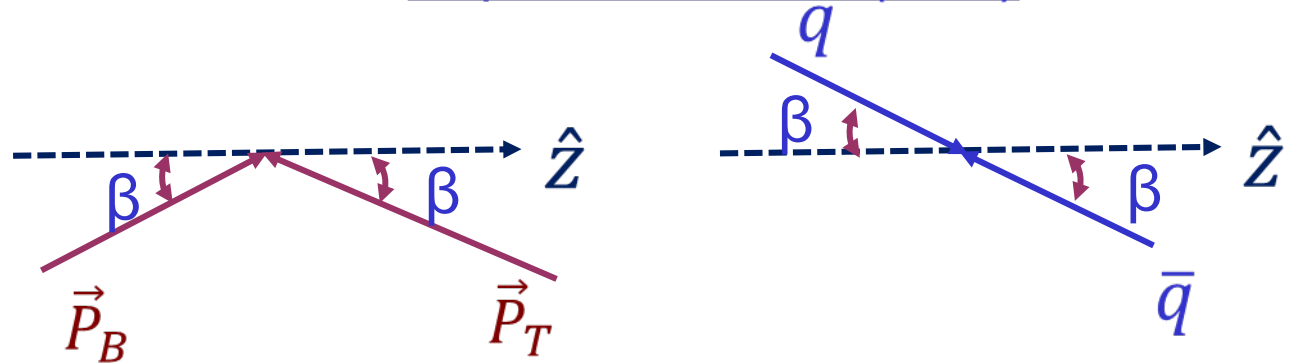
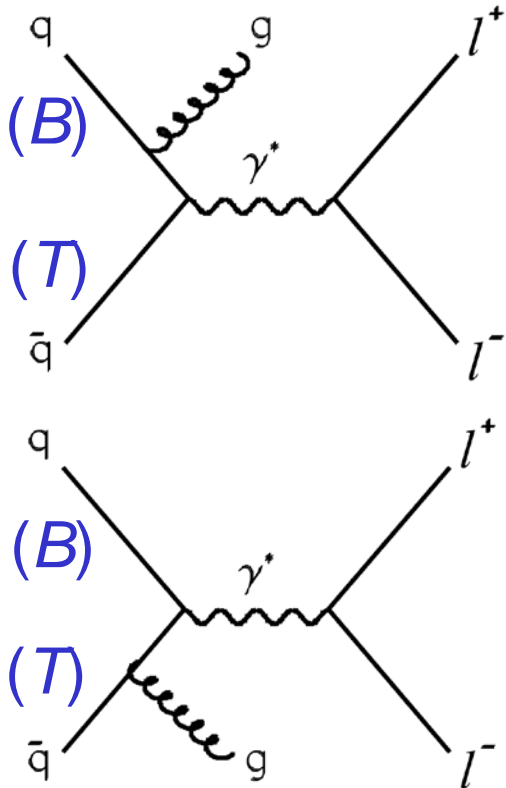
$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, a , is reduced by a factor of $\langle \cos \theta_1 \rangle$ for A_4
- A_5, A_6, A_7 are odd function of ϕ_1 and must vanish from symmetry consideration
- Some equality and inequality relations among $A_0 - A_7$ can be obtained

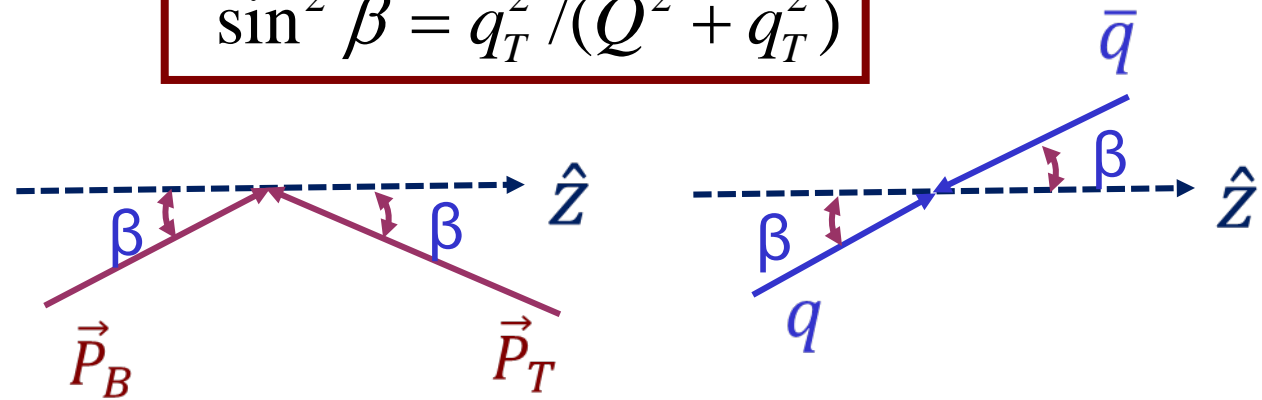
What are the values of θ_1 and ϕ_1 at order α_s ?

1) $q\bar{q} \rightarrow \gamma^*(Z^0)g$

In γ^* rest frame (C-S)



$$\sin^2 \beta = q_T^2 / (Q^2 + q_T^2)$$



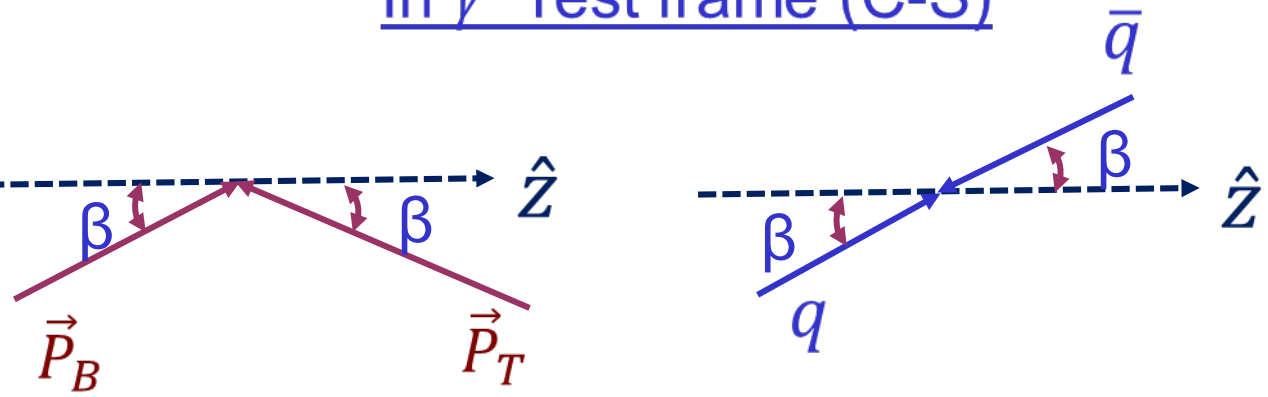
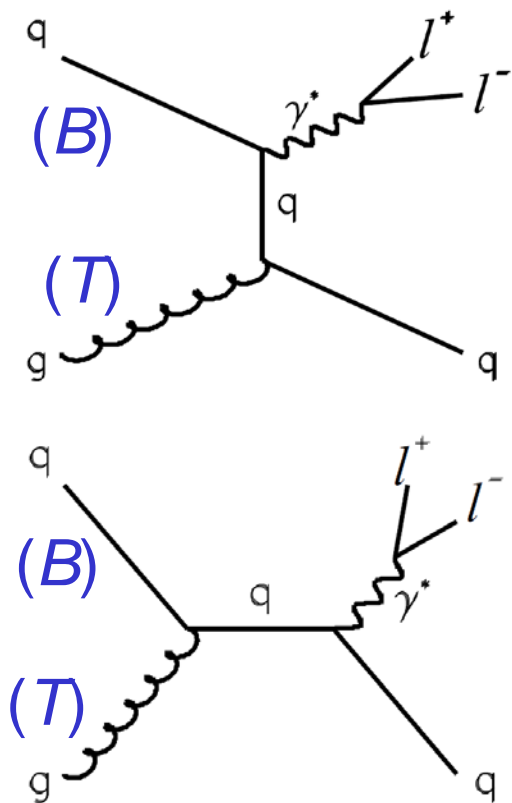
$$\theta_1 = \beta \text{ and } \phi_1 = 0; \quad A_0 = A_2 = \sin^2 \beta$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \quad \nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

What are the values of θ_1 and ϕ_1 at order α_s ?

2) $qg \rightarrow \gamma^*(Z^0)q$

In γ^* rest frame (C-S)



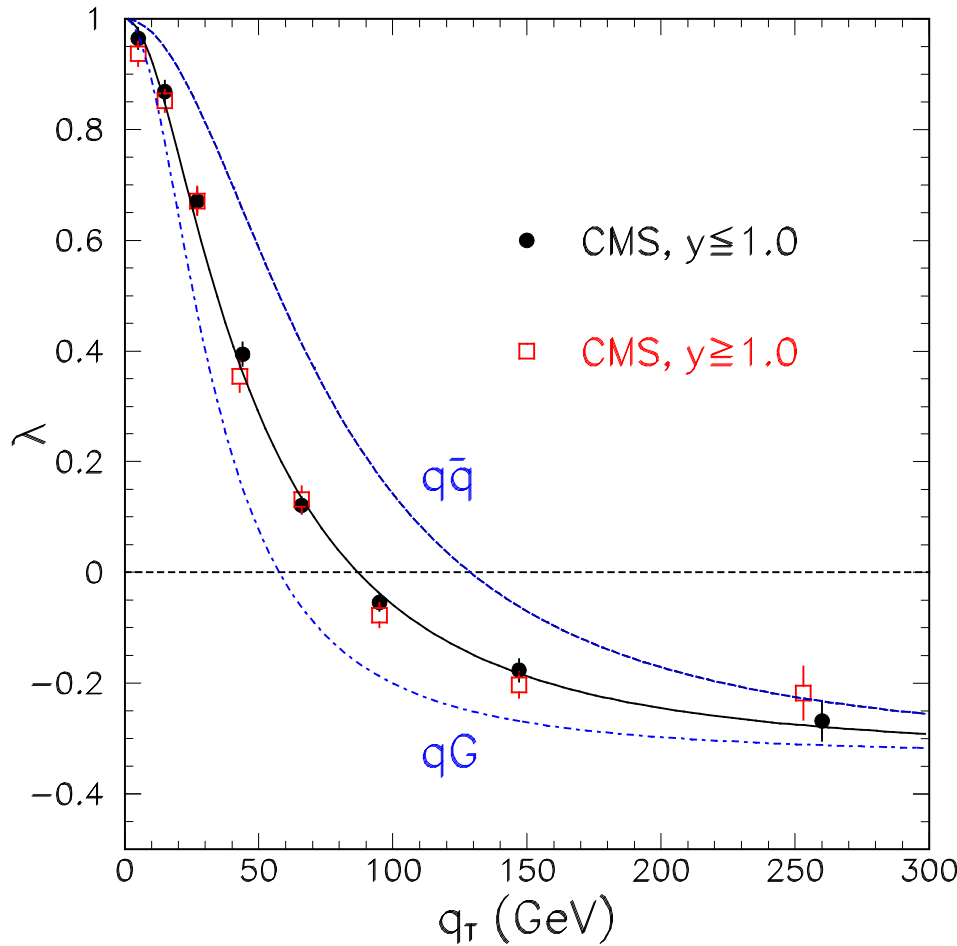
$\theta_1 = \beta$ and $\phi_1 = 0$

$\theta_1 > \beta$ and $\phi_1 = 0$; $A_0 = A_2 \approx 5q_T^2 / (Q^2 + 5q_T^2)$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \quad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

Compare with CMS data on λ

(Z production in $p+p$ collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for } q\bar{q} \rightarrow Zg$$

$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for } qG \rightarrow Zq$$

For both processes

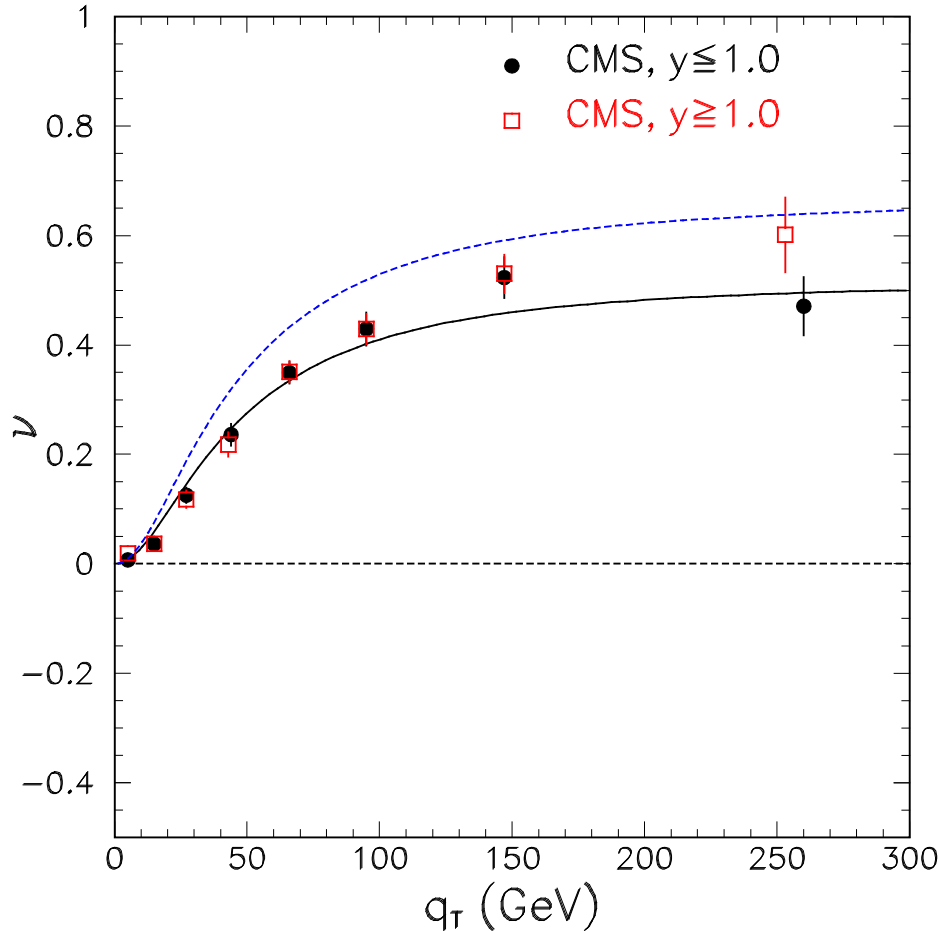
$$\lambda = 1 \text{ at } q_T = 0 \quad (\theta_1 = 0^\circ)$$

$$\lambda = -1/3 \text{ at } q_T = \infty \quad (\theta_1 = 90^\circ)$$

Data can be well described
 with a mixture of 58.5% qG
 and 41.5% $q\bar{q}$ processes

Compare with CMS data on ν

(Z production in $p+p$ collision at 8 TeV)



$$\nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for } q\bar{q} \rightarrow Zg$$

$$\nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for } qG \rightarrow Zq$$

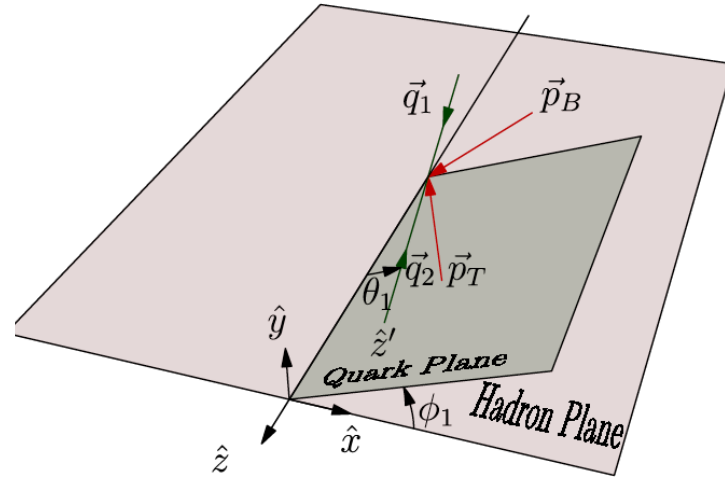
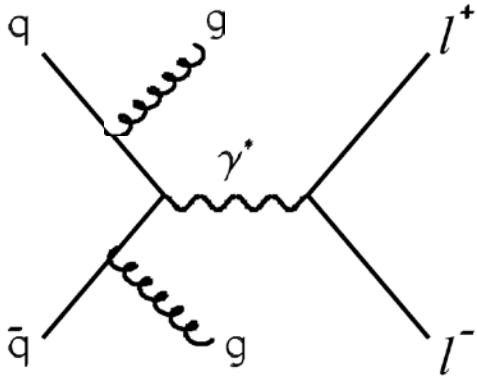
Dashed curve corresponds to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

Solid curve corresponds to $\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.77$

$q - \bar{q}$ axis is non-coplanar relative to the hadron plane

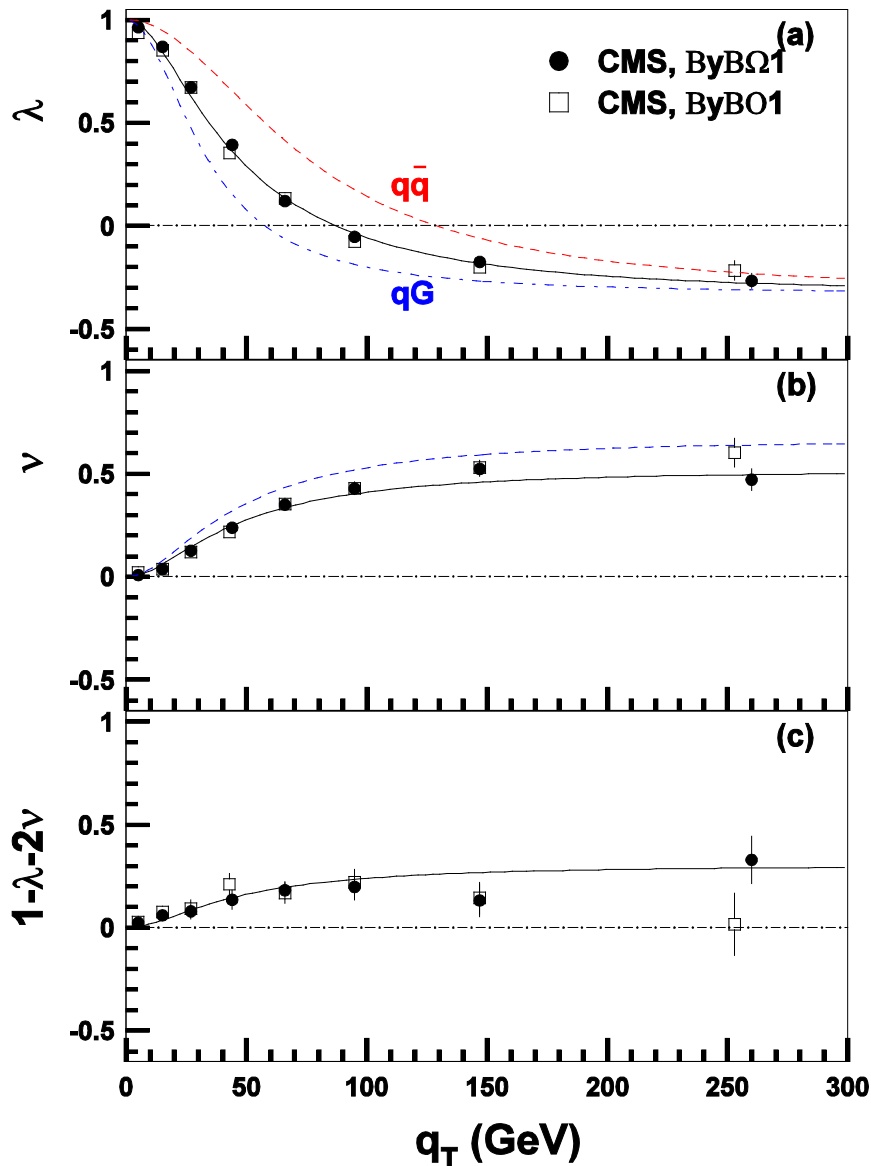
Origins of the non-coplanarity

1) Processes at order α_s^2 or higher



2) Intrinsic k_T from interacting partons

Compare with CMS data on Lam-Tung relation



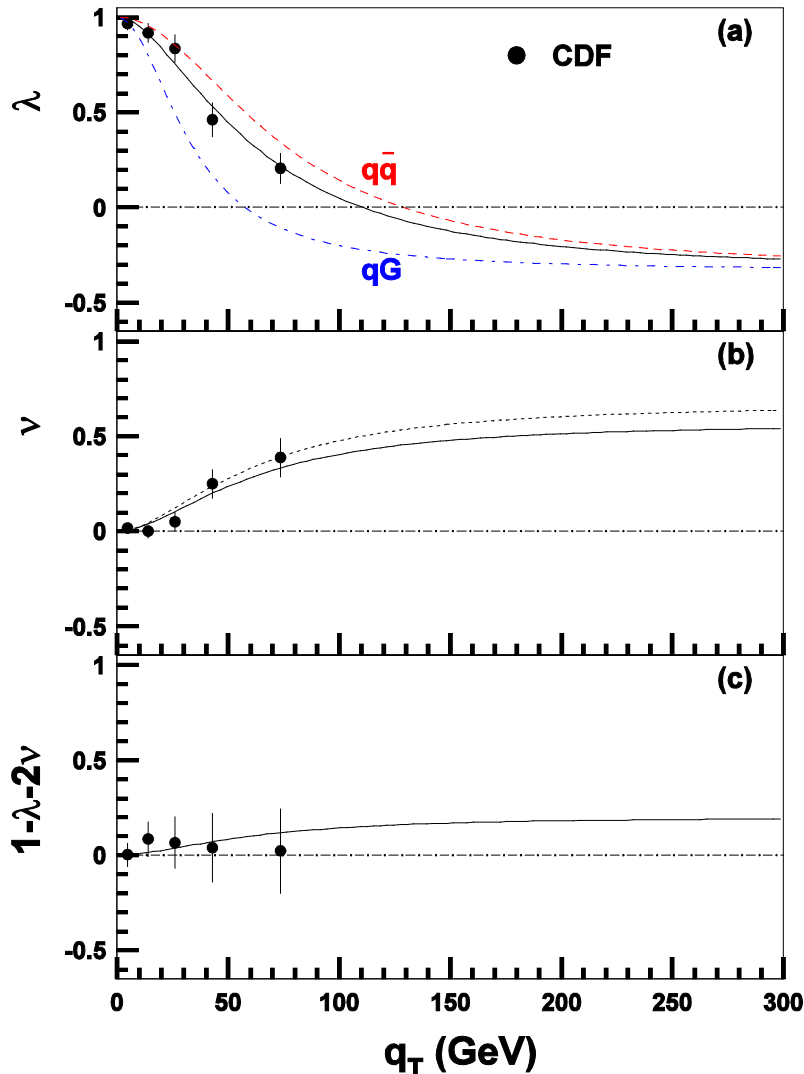
Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes, and

$$\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.77$$

Violation of Lam-Tung relation is well described

Compare with CDF data

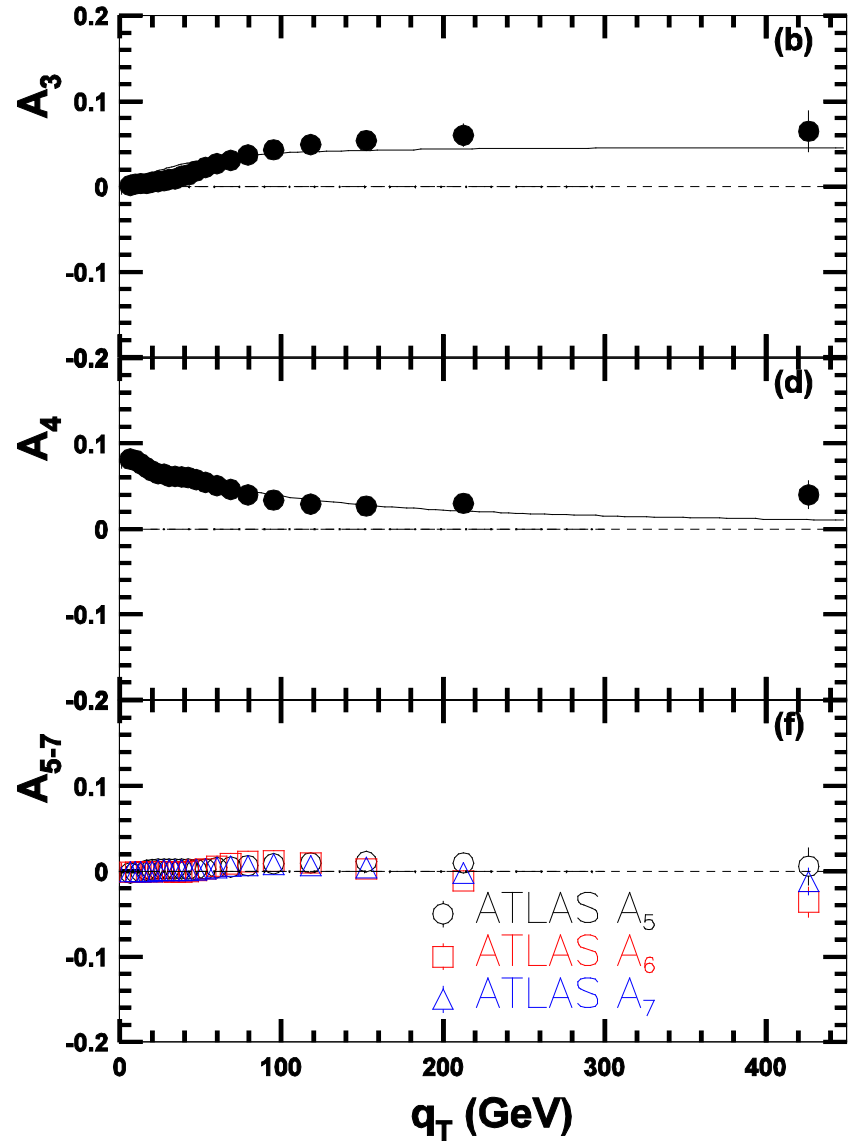
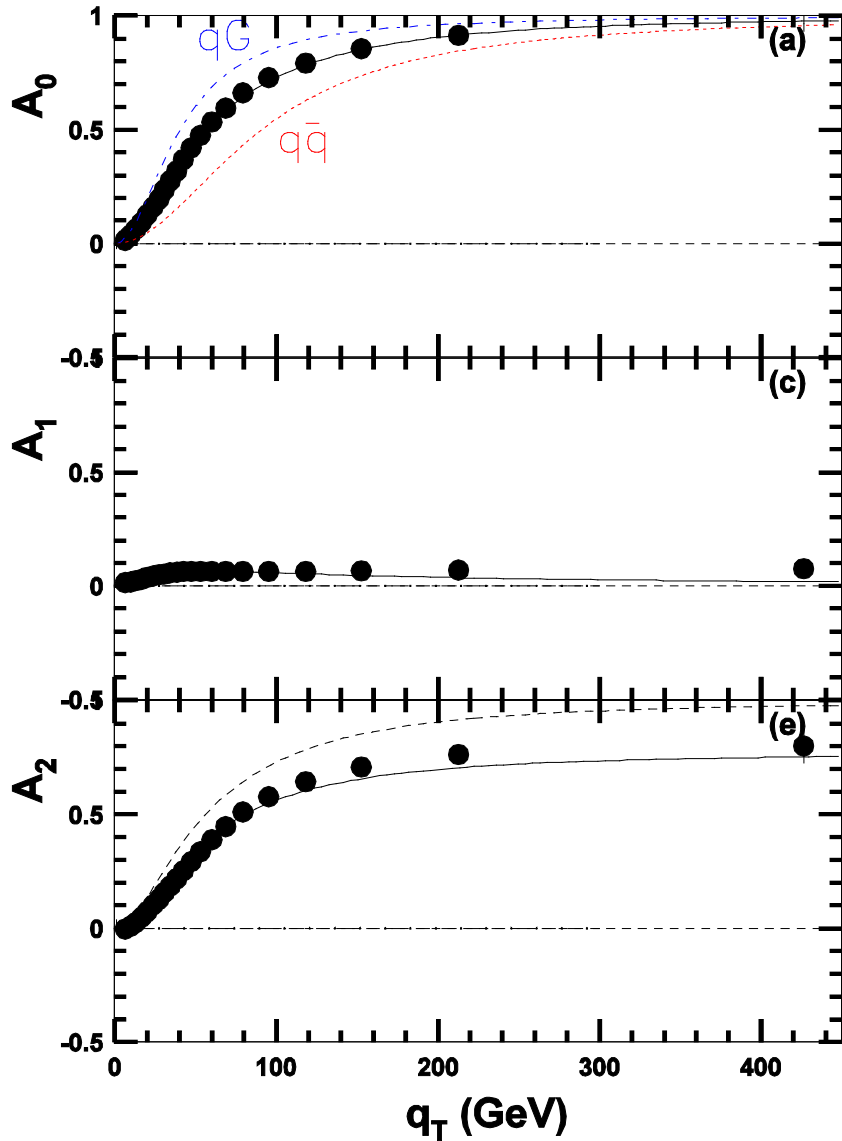
(Z production in $p + \bar{p}$ collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5% qG and 72.5% $q\bar{q}$ processes, and $\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.85$

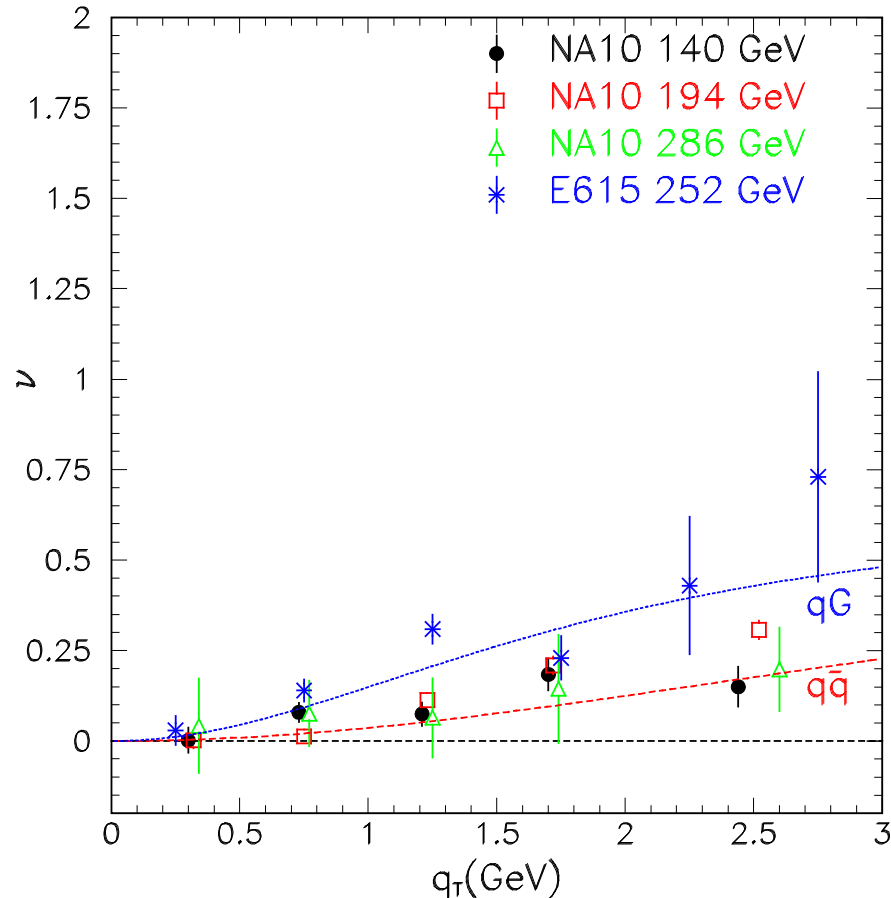
Violation of Lam-Tung relation is not ruled out

Compare with ATLAS data on A_0 - A_7



(Peng, Chang, McClellan, Teryaev, to be published)

Pion-induced D-Y



See Lambertsen
and Vogelsang,
arXiv: 1605.02625

- The ν data should be between the $q\bar{q}$ and qG curves, if the effect is entirely from pQCD. The $q\bar{q}$ process should dominate.
- Surprisingly large pQCD effect is predicted!
- Extraction of the B-M functions must remove the pQCD effect.

Summary

- The lepton angular distribution coefficients A_0 - A_7 are described in terms of the polar and azimuthal angles of the $q - \bar{q}$ axis.
- The striking q_T dependence of A_0 (or equivalently, λ) can be well described by the mis-alignment of the $q - \bar{q}$ axis and the Collins-Soper z -axis.
- Violation of the Lam-Tung relation ($A_0 \neq A_2$) is described by the non-coplanarity of the $q - \bar{q}$ axis and the hadron plane. This can come from order α_s^2 or higher processes or from intrinsic k_T .
- This study can be extended to fixed-target Drell-Yan data.