3D Parton Distributions – Path to the LHC (LNF, Frascati)
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Gluon TMDs at low x and diffractive processes

Piet J Mulders

mulders@few.vu.nl

European Research Council

Abstract

Abstract

We discuss the momentum distributions of gluons and consider the dependence of the gluon parton distribution functions (PDFs) on both fractional (longitudinal) momentum x and transverse momentum p_T, referred to as the gluon TMDs. Looking at the operator structure of the TMDs, we are able to unify various descriptions at small-x including the dipole picture and the notions of pomeron and odderon exchange The results also may be used in diffractive processes.

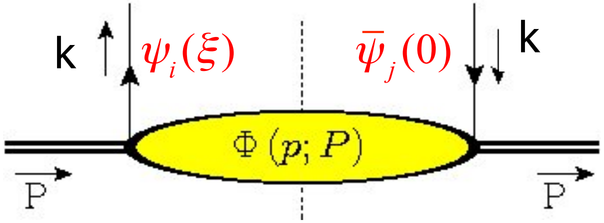
1. TMD correlators and their operator structure, color gauge invariance
2. Rank of TMD and operator structure
3. The Wilson loop correlator unifying ideas on diffraction, dipole picture and small-x behavior

Standard TMDs

- TMDs incorporate hadron structure

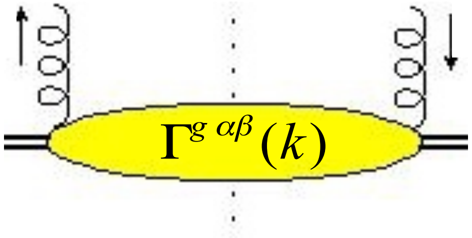
$$u_i(k)\overline{u}_j(k) \sim (\not{k})_{ij} \Longrightarrow \Phi_{ij}(k;P,S)$$

$$\epsilon^{\alpha}(k)\epsilon^{\beta *}(k) \sim -g_T^{\alpha\beta} \Longrightarrow \Gamma^{\alpha\beta}(k;P,S)$$



- High energies (lightlike n = P′/P.P′ and P.n=1) and including transverse momenta

$$k^\mu = xP^\mu + k_T^\mu + (k^2 - k_T^2)n^\mu$$



$$\Longrightarrow \quad \Phi(x,k_T;P,S) \quad \text{and} \quad \Gamma^{\alpha\beta}(x,k_T;P,S)$$

- Polarized targets provide opportunities and challenges

$$M\,S^\mu = S_L\,P^\mu + M\,S_T^\mu + M^2\,S_L\,n^\mu$$

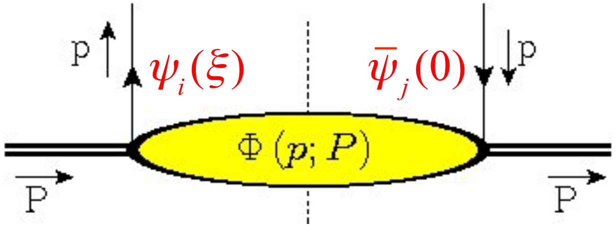
$$M^2\,S^{\mu\nu} = -S_{LL}\,P^\mu P^\nu + \tfrac{1}{2}P^{\{\mu}S_{LT}^{\nu\}} + \tfrac{1}{2}M^2\,S_{TT}^{\mu\nu} + \ldots$$

- At high energies x linked to scaling variables (e.g. x = $Q^2/2P.q$) and convolutions of transverse momenta are linked to azimuthal asymmetries (noncollinearity) requiring semi-inclusivity and/or polarization

Matrix elements for TMDs

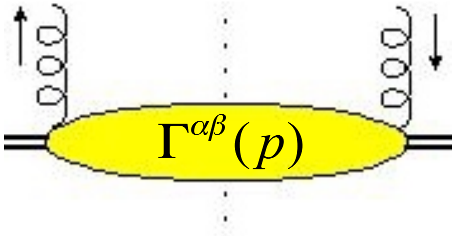
- quark-quark

$$\Phi_{ij}(x,p_T;n) = \int \frac{d\,\xi\cdot P\, d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\langle P,S|\overline{\psi}_j(0)\,\psi_i(\xi)|P,S\rangle\,\Big|_{\xi\cdot n=0}$$

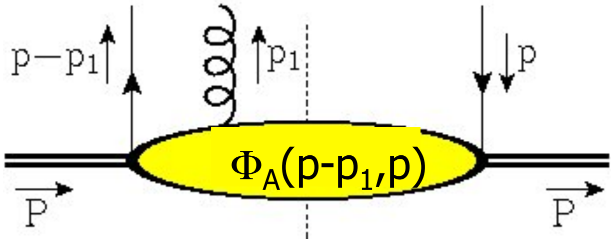


- gluon-gluon

$$\Gamma^{\mu\nu}(x,p_T;n) = \int \frac{d\,\xi\cdot P\, d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,F^{n\nu}(\xi)\,|P,S\rangle\Big|_{\xi\cdot n=0}$$



- quark-gluon-quark



$$\Phi^{\alpha}_{D;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\overline{\psi}_j(0)D^{\alpha}(\eta)\,\psi_i(\xi)\middle|P\right\rangle$$

$$\Phi^{\alpha}_{F;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\overline{\psi}_j(0)F^{n\alpha}(\eta)\,\psi_i(\xi)\middle|P\right\rangle$$

TMDs and color gauge invariance

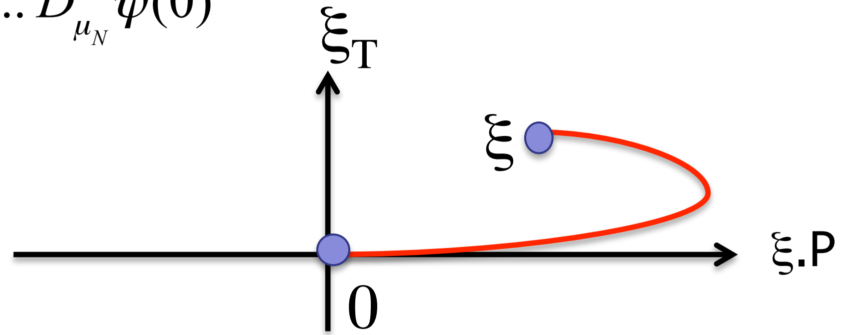
- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

$$U(0,\xi) = \mathcal{P}\exp\left(-ig\int_0^{\xi} ds^{\mu}A_{\mu}\right)$$

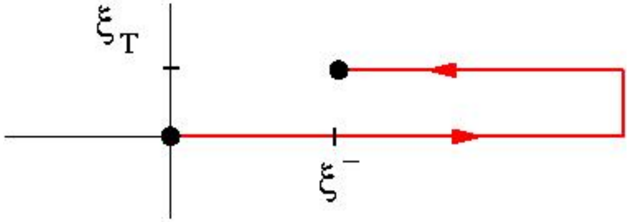
$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence for Φ(x,p_T)



- ‘Dominant’ paths: along lightcone connected at lightcone infinity (staples)

- Reduces to ‘straight line’ for Φ(x)



$$\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$$

Matrix elements for TMDs

- quark-quark

$$\Phi_{ij}^{[U]}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\langle P,S|\overline{\psi}_j(0)\,U_{[0,\xi]}\psi_i(\xi)|P,S\rangle\,\Big|_{\xi\cdot n=0}$$

- gluon-gluon

$$\Gamma^{[U,U']\,\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,U_{[0,\xi]}\,F^{n\nu}(\xi)\,U'_{[\xi,0]}\,|P,S\rangle\Big|_{\xi\cdot n=0}$$

- … and even single Wilson loop correlator

$$\delta(x)\,\Gamma_0^{[U,U']}(p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P,S|U_{[0,\xi]}\,U'_{[\xi,0]}\,|P,S\rangle\Big|_{\xi\cdot n=0}$$

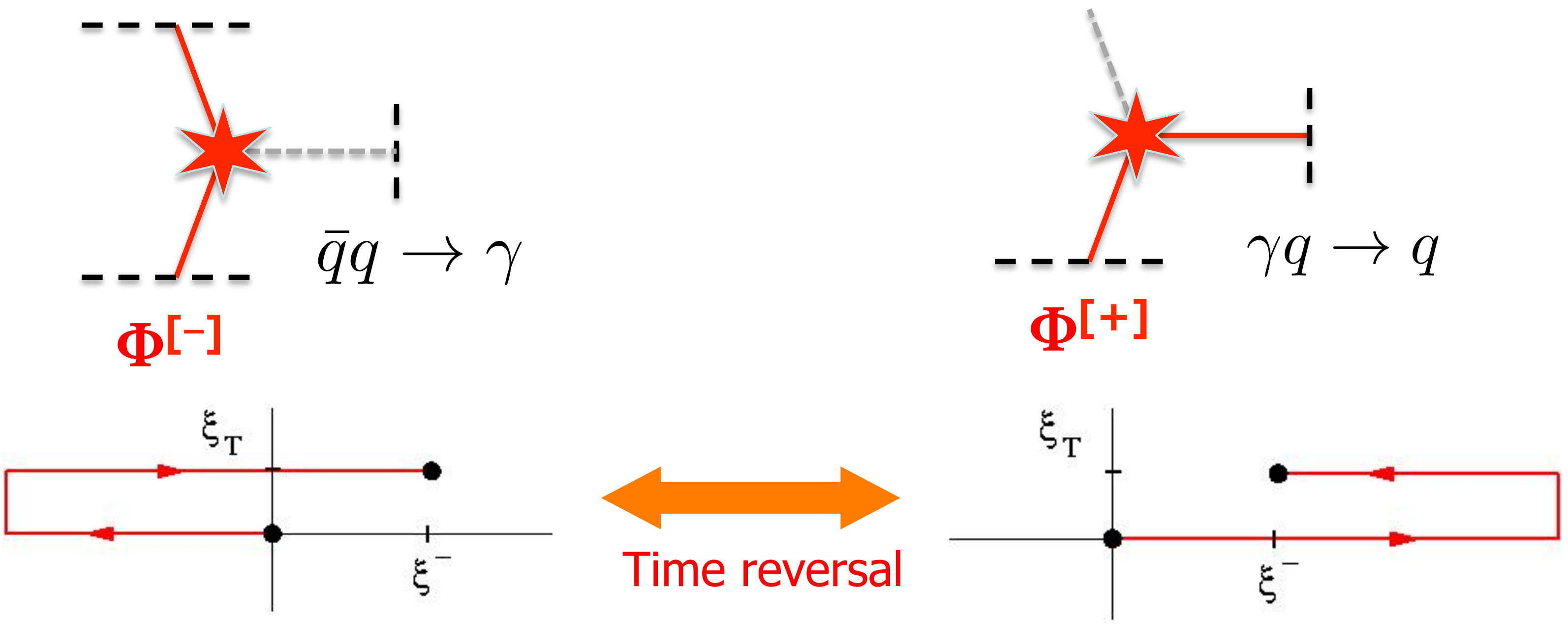
Quark correlators and gauge links

$$\Phi^{q[C]}_{ij}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{ip.\xi} \langle P|\bar{\psi}_j(0)U^{[C]}_{[0,\xi]}\psi_i(\xi)|P\rangle_{\xi.n=0}$$

TMD

path dependent gauge link

- Gauge links associated with resummation of dimension zero (not suppressed!) collinear A^n = A$^+$ gluons, leading for TMD correlators to process-dependence:

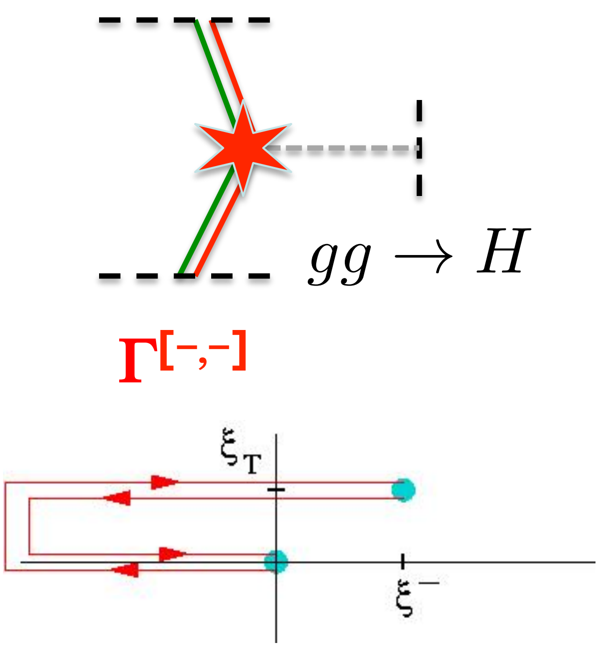


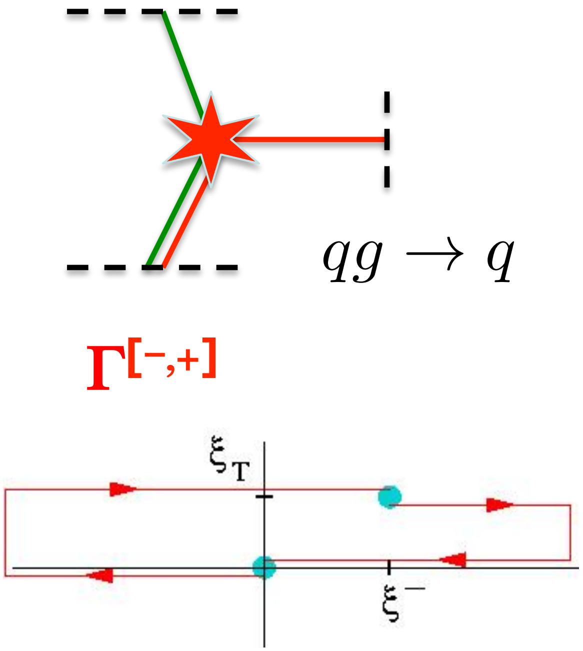
Belitsky, Ji, Yuan, 2003; Boer, M, Pijlman, 2003

Gluon correlators and gauge links

$$\Gamma^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| U^{[C]}_{[\xi,0]} F^{n\alpha}(0) U^{[C']}_{[0,\xi]} F^{n\beta}(\xi) \middle| P \right\rangle_{\xi.n=0}$$

◆ The TMD gluon correlators need two links, which can have different paths. N



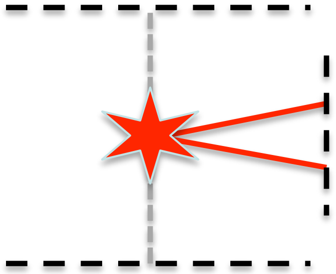


◆ Note presence of transverse gluons in the perturbative expansion of $\Gamma^{\alpha\beta[U]}$

Bomhof, M, Pijlman, 2006

Single Wilson loop correlator in diffraction

$$\boxed{\delta(x)\,\Gamma_0^{[U,U']}(p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|U_{[0,\xi]}\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}}$$



(relevant at small x, t = p_T^2)

$$q_1 = \left[\alpha\frac{M_X}{\sqrt{2}},(1-\alpha)\frac{M_X}{\sqrt{2}},-q_T\right]$$

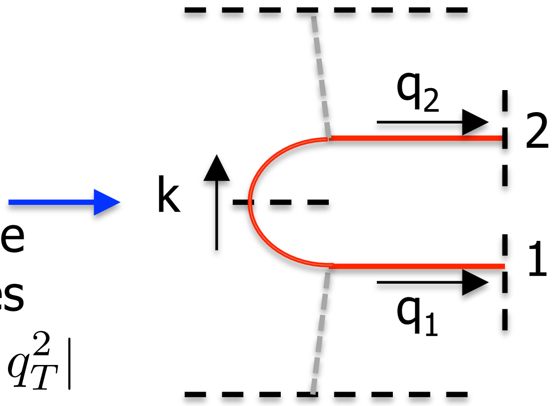
$$q_2 = \left[(1-\alpha)\frac{M_X}{\sqrt{2}},\alpha\frac{M_X}{\sqrt{2}},q_T\right]$$

$$q = \left[\frac{M_X}{\sqrt{2}},\frac{M_X}{\sqrt{2}},0_T\right]$$

$$k = \left[-\alpha\frac{M_X}{\sqrt{2}},\alpha\frac{M_X}{\sqrt{2}},q_T\right]$$

soft in diffractive processes

$k^2 \approx -|q_T^2|$

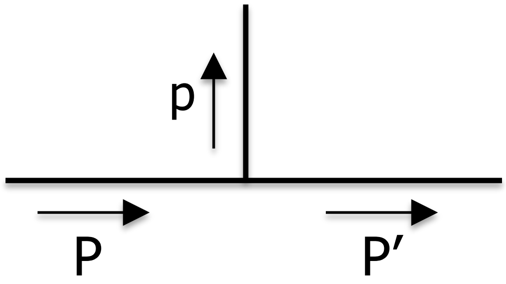


$\Gamma_0^{[-,+]}$

$$\boxed{\begin{aligned} M_X^2 &= \frac{|q_T^2|}{\alpha(1-\alpha)} \approx \frac{|q_T^2|}{\alpha} \\ \Delta\eta_{12} &\approx -\ln\alpha \end{aligned}}$$

Cf: Gieseke, Qiao, Bartels 2000; Dominguez, Xiao, Yuan 2011

Collinear momentum fraction x → 0 and diffraction



$$p = \left[\frac{t - p_T^2}{2xP^+}, xP^+, p_T\right]$$

$$P = \left[\frac{M^2}{2P^+}, P^+, 0_T\right]$$

$$P' = \left[\frac{M'^2 - p_T^2}{2(1-x)P^+}, (1-x)P^+, -p_T\right]$$

$$P^+ \Longrightarrow \frac{M_X}{x\sqrt{2}}$$

$$p_T \sim q_T$$

$$t - \frac{p_T^2}{1-x} = -x\left(\frac{M'^2}{1-x} - M^2\right)$$

At small x (and small t): $$\boxed{t = p_T^2 - x\left(M'^2 - M^2\right)}$$

$$x = 0 \iff t = p_T^2$$

Parametrization of gluon correlators

- Unpolarized target

$$\Gamma^{ij[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}\, f_1^{[U]}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp\,[U]}(x,k_T^2)\right\}$$

- Vector polarized target

$$\Gamma_L^{ij[U]}(x,k_T) = \frac{x}{2}\left\{i\epsilon_T^{ij}\, S_L g_1^{[U]}(x,k_T^2) + \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, S_L h_{1L}^{\perp\,[U]}(x,k_T^2)\right\}$$

$$\begin{aligned}\Gamma_T^{ij[U]}(x,k_T) = \frac{x}{2}\Bigg\{ & \frac{g_T^{ij}\,\epsilon_T^{kS_T}}{M}\, f_{1T}^{\perp[U]}(x,k_T^2) - \frac{i\epsilon_T^{ij}\,k_T{\cdot}S_T}{M}\, g_{1T}^{[U]}(x,k_T^2) \\ & - \frac{\epsilon_T^{k\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,k_T^2) - \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp}(x,k_T^2)\Bigg\}\end{aligned}$$

Definite rank TMDs

- Expansion in constant tensors in transverse momentum space

$$g_T^{\mu\nu} = g^{\mu\nu} - P^{\{\mu}n^{\nu\}} \qquad\qquad \epsilon_T^{\mu\nu} = \epsilon^{Pn\mu\nu} = \epsilon^{-+\mu\nu}$$

- … or traceless symmetric tensors (of definite rank)

$$k_T^i$$

$$k_T^{ij} = k_T^i k_T^j - \tfrac{1}{2} k_T^2\, g_T^{ij}$$

$$k_T^{ijk} = k_T^i k_T^j k_T^k - \tfrac{1}{4} k_T^2 \left(g_T^{ij} k_T^k + g_T^{ik} k_T^j + g_T^{jk} k_T^i \right)$$

- Simple azimuthal behavior: $k_T^{i_1 \ldots i_m} \longleftrightarrow |k_T|\, e^{\pm i m \varphi}$

  functions showing up in cos(mϕ) or sin(mϕ) asymmetries (wrt e.g. ϕ_T)

- Simple Bessel transform to b-space (relevant for evolution):

$$F_m(x, k_T) = \int_0^\infty b db\;\, J_m(k_T b)\, F_m(x, b)$$

$$F_m(x, b) = \int_0^\infty k_T\, dk_T\;\, J_m(k_T b)\, F_m(x, k_T)$$

Structure of gluon **TMD PDFs** in polarized target

- TMDs $\Gamma\ldots(x,k_T^2)$

		PARTON SPIN		
	GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
TARGET SPIN	U	f_1^g		$h_1^{\perp g}$
	L		g_1^g	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$

- Integrated (collinear) correlator: only circled ones survive
- Collinear functions are spin-spin correlations
- TMDs also momentum-spin correlations (spin-orbit) including also T-odd (single-spin) functions (appearing in single-spin asymmetries)

PJM & Rodrigues, PR D63 (2001) 094021

Meissner, Metz and Goeke, PR D76 (2007) 034002

Structure of gluon **TMD PDFs** in spin 1 target

		PARTON SPIN		
	GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \dots$
TARGET SPIN	U	f_1^g		$h_1^{\perp g}$
	L		g_1^g	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$
	LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
	LT	f_{1LT}^g	g_{1LT}^g	h_{1LT}^g $h_{1LT}^{\perp g}$
	TT	f_{1TT}^g	g_{1TT}^g	h_{1TT}^g $h_{1TT}^{\perp g}$ $h_{1TT}^{\perp\perp g}$

Jaffe & Manohar, Nuclear gluonometry, PL B223 (1989) 218

D Boer, S Cotogno, T van Daal, PJM, A Signori, Y Zhou, ArXiv 1607.01654

Untangling operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\,\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) (\partial^n_\xi)^{N-1} U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

x = p.n

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U^{[n]}_{[0,\xi]} (D^n_\xi)^{N-1} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements of operators that all have the same twist since dim(D^n) = 0

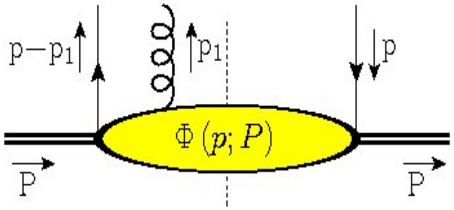
$$\Phi^{(N)} = \left\langle P \middle| \bar{\psi}(0) (D^n)^{N-1} \psi(0) \middle| P \right\rangle$$

- Moments are particularly useful because their anomalous dimensions can be rigorously calculated and these can be Mellin transformed into the splitting functions that govern the QCD evolution.

Transverse moments → operator structure of TMD PDFs

- Operator analysis for [U] dependence (e.g. [+] or [-]) TMD functions: in analogy to Mellin moments consider transverse moments → role for quark-gluon m.e.

$$p_T^\alpha \Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\right|\bar{\psi}(0)\,U_{[0,\pm\infty]}\,iD_T^\alpha\, U_{[\pm\infty,\xi]}\,\psi(\xi)\left|P\right\rangle_{\xi.n=0}$$



calculable

- $$\boxed{\int dp_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\,\Phi_G^\alpha(x)}$$

T-even

T-odd

$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$

T-even (gauge-invariant derivative)

$$\Phi_A^\alpha(x) = PV\int \frac{dx_1}{x_1}\,\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

$$\Phi_D^\alpha(x) = \int dx_1\; \Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\boxed{\Phi_G^\alpha(x) = \pi\,\Phi_F^{n\alpha}(x,0\,|\,x)}$$

T-odd (soft-gluon or gluonic pole, ETQS m.e.)

Efremov, Teryaev; Qiu, Sterman;
Brodsky, Hwang, Schmidt; Boer, Teryaev; Bomhof, Pijlman, M

Gluonic pole factors are calculable

- $C_G^{[U]}$ calculable gluonic pole factors (quarks)

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\square]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

- Complicates life for ‘double p_T’ situation such as Sivers-Sivers in DY, etc.
- In essence the factors would come naturally in perturbative calculations (Gamberg, Kang)

Buffing, Mukherjee, M, PRD86 (2012) 074030, ArXiv 1207.3221

Buffing, Mukherjee, M, PRD88 (2013) 054027, ArXiv 1306.5897

Buffing, M, PRL 112 (2014), 092002

Operator classification of gluon TMDs

factor	**RANK OF GLUON TMDs FOR SPIN ½ HADRON**				
	0	1	2	3	
1	$f_1 \quad g_1$	$g_{1T}^{[\partial]}$	$h_1^{\perp[\partial\partial]}$		
$C_{G,c}^{[U]}$		$f_{1T}^{\perp[Gc]} \quad h_1^{[Gc]}$	$h_{1L}^{\perp[\partial Gc]}$	$h_{1T}^{\perp[\partial\partial Gc]}$	
$C_{GG,c}^{[U]}$	$\delta f_1^{[GGc]} \ldots$	…	$h_1^{\perp[GGc]}$		
$C_{GGG,c}^{[U]}$		…	…	$h_{1T}^{\perp[GGGc]}$	

Process dependence in p_T dependence of TMDs due to gluonic pole operators (e.g. affecting $\langle p_T^2\rangle$)

$$f_1^{[U]}(x,p_T^2) = f_1 + C_{GG,c}^{[U]}\delta f_1^{[GGc]}$$ with $\delta f_1^{[GG\ c]}(x) = 0$

Multiple functions for rank 2 (double gluonic poles, multiple color configurations c)

Boer, Buffing, M, JHEP08 (2015) 053, arXiv:1503.03760

D Boer, S Cotogno, T van Daal, PJM, A Signori Y Zhou, JHEP, ArXiv 1607.01654

Small x physics in terms of TMDs

- The single Wilson-loop correlator Γ_0

$$\Gamma_0(k_T) = \frac{1}{2M^2}\left\{e(k_T^2) - \frac{\epsilon^{k S_T}}{M}\,e_T(k_T^2) + \ldots\right\}$$

factor	RANK OF WILSON LOOP TMDs FOR SPIN ½ HADRON			
	0	1	2	3
1	e			
$C_{G,c}^{[U]}$		$e_T^{[G]}$		
$C_{GG,c}^{[U]}$	…	…		
$C_{GGG,c}^{[U]}$		…		

- Note limit x → 0 for gluon TMDs linked to gluonic pole m.e. of Γ_0

$$\boxed{(2\pi)^2\,\Gamma^{ij\,[U,U']}(0,k_T) \sim C_{GG}^{[U,U']} M^2 \Gamma_{0\,GG}^{ij[U,U']}(k_T) \sim C_{GG}^{[U,U']} \frac{k_T^i\,k_T^j}{M^2}\Gamma_0^{[U,U']}(k_T)}$$

RHS depends in fact on t, which for x = 0 becomes p_T^2

Small x physics in terms of gluon TMDs

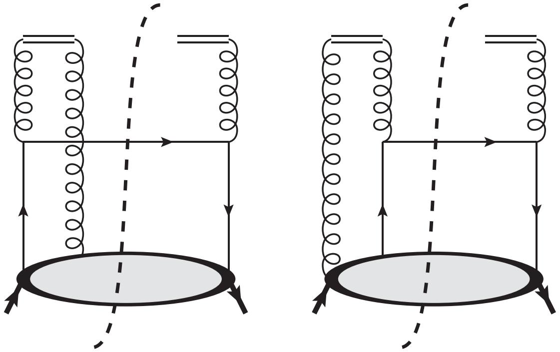
- Note limit x → 0 for gluon TMDs linked to gluonic pole m.e. of Γ_0

$$\pi^2\,\Gamma^{\alpha\beta\,[U,U']}(0,p_T) = C_{GG}^{[U,U']}\,\Gamma_{0\,GG}^{\alpha\beta}(p_T)$$

- Dipole correlators: at small x only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

$$\begin{aligned} x\,f_1^{[+,-]}(x,k_T^2) &\longrightarrow \frac{k_T^2}{2M^2}\,e^{[+,-]}(k_T^2) \\ x\,h_1^{\perp[+,-]}(x,k_T^2) &\longrightarrow e^{[+,-]}(k_T^2) \end{aligned}$$

$$\begin{aligned} x\,f_{1T}^{\perp[+,-]}(x,k_T^2) &\longrightarrow \frac{k_T^2}{2M^2}\,e_T^{[+,-]}(k_T^2) \\ x\,h_1^{[+,-]}(x,k_T^2) &\longrightarrow \frac{k_T^2}{2M^2}\,e_T^{[+,-]}(k_T^2) \\ x\,h_{1T}^{\perp[+,-]}(x,k_T^2) &\longrightarrow e_T^{[+,-]}(k_T^2) \end{aligned}$$


(a) (b)

Agrees for x → 0 with perturbatively generated $e_T^{[GGG]}$

Dominguez, Xiao, Yuan 2011

D Boer, S Cotogno, T van Daal, PJM, A Signori, Y Zhou, ArXiv 1607.01654

D Boer, MG Echevarria, PJM, J Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485

Conclusion

- (Generalized) universality of TMDs studied via operator product expansion, extending the well-known collinear distributions to TMDs, ordered into functions of definite rank.
- Knowledge of operator structure is important (e.g. in lattice calculations) as well as for the small-x limits for gluons
- Multiple operator possibilities for higher rank functions
- The TMD PDFs appear in cross sections with specific calculable factors that deviate from (or extend on) the naïve parton universality for hadron-hadron scattering but can also be addressed in pQCD
- Applications in polarized high energy processes, even for unpolarized hadrons (with linearly polarized gluons)
- Applications in diffractive processes and simplifications at small-x via Wilson loop correlator, confirmed in perturbative calculations