Quark angular and transverse momentum in covariant approach

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(based on collaboration and discussions
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Outline

- Introduction
- Covariant approach:
  - TMDs: calculation, predictions, QCD evolution…
  - spin & OAM, role of gluons
- Summary

Introduction

Intrinsic motion in composite systems is required by QM:

electrons in atom *non-relativistic motion, OAM & spin are decoupled*

$$d \approx 10^{-10}m, \quad p \approx 10^{-3}MeV, \quad m_e \approx 0.5MeV, \quad \beta \approx 0.002$$

nucleons in nucleus *Fermi motion*

$$d \approx 10^{-15}m, \quad p \approx 10^{2}MeV, \quad m_N \approx 940MeV, \quad \beta \approx 0.1$$

<u>quarks in nucleon</u> *relativistic motion, OAM & spin cannot be decoupled*

$$d \approx 10^{-15}m, \quad p \approx 10^{2}MeV, \quad m_e \approx 5MeV, \quad \beta \approx 1$$

Covariant approach

Main results:

- Sum rules: Wanzura-Wilczek (WW), Burhardt-Cottinngham (BC) and Efremov-Leader-Teryaev (ELT)
- Relations between TMDs, PDFs and TMDs (giving predictions for TMDs
- Study and prediction of the role of OAM

[1] P.Zavada, Phys. Lett. B 751, 525 (2015).
[2] A. V. Efremov, O. V. Teryaev and P. Zavada, J.Phys.Conf.Ser. 678 (2016) no.1, 012001, arXiv:1511.01164 [hep-ph].
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[10] A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada, PoS DIS2010, 253 (2010).
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[12] A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada, Phys. Rev. D 80, 014021 (2009).
[13] A. V. Efremov, O. V. Teryaev and P. Zavada, Phys. Rev. D 70, 054018 (2004).

Paradigm of covariant approach

- **Large Q^2:** In the rest frame we have

$$|\mathbf{q}_R|^2 = Q^2 + \nu^2 = Q^2\left(1 + \frac{Q^2}{(2Mx)^2}\right) \Rightarrow |\mathbf{q}_R| \gtrsim \nu = \frac{Q^2}{2Mx} \geq \frac{Q^2}{2M}$$

$$\Rightarrow \Delta\lambda \lesssim \Delta\tau \approx \frac{2Mx}{Q^2}$$

So a space-time domain of lepton -quark QED interaction is limited.

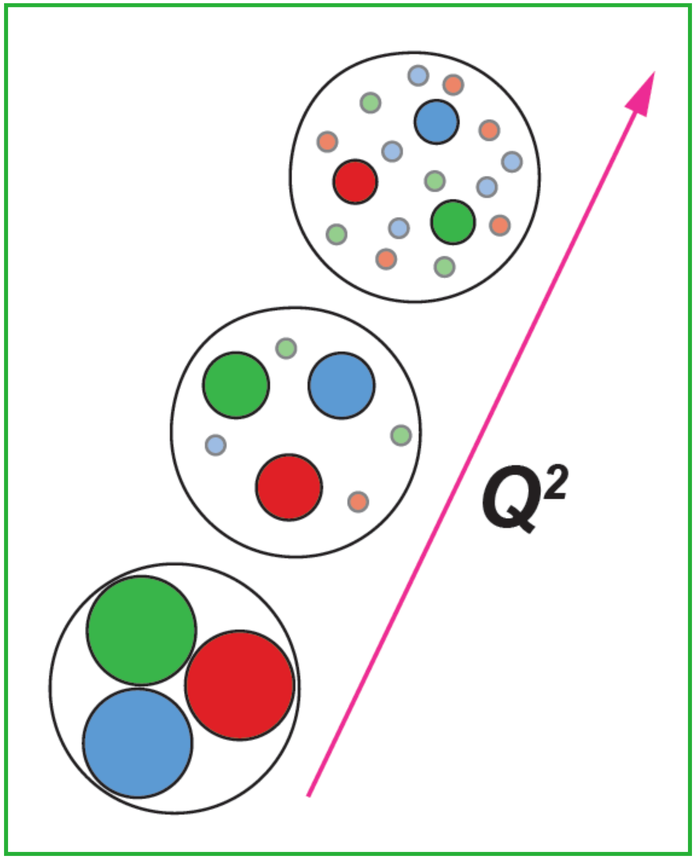
- **Effect of asymptotic freedom:** Limited extend of this domain prevent the quark from an interaction with the rest of nucleon during the lepton-quark interaction – **in any reference frame.**

In fact we assume characteristic time of QCD process accompanying γ absorption much greater than absorption time itself:

$$\Delta\tau \ll \Delta\tau_{QCD}$$

Since Lorentz time dilation is universal, the first relation holds in any reference frame. This is essence of our covariant leading order approach.

$$\Delta T(\beta) = \frac{\Delta T_0}{\sqrt{1-\beta^2}}$$

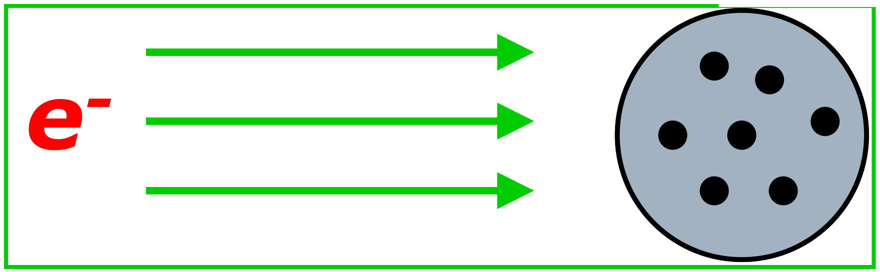


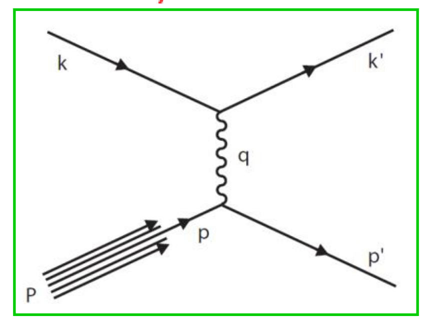
Remarks:

- p_L and p_T are equally important…
- We do not aim to describe complete nucleon dynamic structure, but only a picture of short time interval corresponding to DIS.
- We assume Q^2-dependence of this Lorentz-invariant "effective" picture: $n_q(pP/M,Q^2)$.

Structure functions

General framework: $\Delta\sigma(x,Q^2) \sim |A|^2 = L_{\alpha\beta}W^{\alpha\beta}$





The quarks are represented by the quasifree fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry

$$G_q^{\pm}(p_0)d^3p; \qquad p_0 = \sqrt{m^2 + \mathbf{p}^2},$$

which are expected to depend effectively on Q^2. These distributions measure the probability to find a quark in the state

$$u(p,\lambda\mathbf{n}) = \frac{1}{\sqrt{N}}\begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}} \end{pmatrix}; \qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}},$$

where m and p are the quark mass and momentum, $\lambda = \pm 1/2$ and $\mathbf{n}$ coincides with the direction of target polarization $\mathbf{J}$.

$W^{\alpha\beta} \Rightarrow$

$F_1(x,Q^2)$

$F_2(x,Q^2)$

$g_1(x,Q^2)$

$g_2(x,Q^2)$

Rotational symmetry (rest frame) & Lorentz invariance

Rest frame representation

If one assumes $Q^2 \gg 4M^2x^2$, then:

$$F_2(x) = Mx^2 \int G(p_0)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0}$$

$$g_1(x) = \frac{1}{2}\int \Delta G(p_0)\left(m+p_1+\frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0},$$

$$g_2(x) = -\frac{1}{2}\int \Delta G(p_0)\left(p_1+\frac{p_1^2-p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0}$$

- integrals can be inverted
- study and prediction OAM

… or in terms of conventional distributions:

$$f_1^a(x) = Mx\int G^a(p_0)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0},$$

$$g_1^a(x) = \int \Delta G^a(p_0)\left(m+p_1+\frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0},$$

$$g_2^a(x) = -\int \Delta G^a(p_0)\left(p_1+\frac{p_1^2-p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0}$$

- *G,ΔG* are not known, but integrals imply relations between distributions: WW relation, sum rules WW, BC, ELT; helicity↔transversity, transversity ↔ pretzelosity; unpolarized+SU(6) → polarized
- partial integration (only over p_1) defines p_T – dependent distributions: $f(x) \to f(x, p_T)$
- relations between TMDs, but also TMDs↔PDFs

Relations are generated by LI & RS !

PDF-TMD relations

1. UNPOLARIZED

$$f_1^a(x,\mathbf{p}_T) = -\frac{1}{\pi M^2}\,\frac{d}{dy}\left[\frac{f_1^a(y)}{y}\right]_{y=\xi(x,\mathbf{p}_T^2)}$$

$$\xi(x,\mathbf{p}_T^2) = x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)$$

For details see:
P.Z. Phys.Rev.D **83**, 014022 (2011), **arXiv:0908.2316 [hep-ph]**
A.Efremov, P.Schweitzer, O.Teryaev and P.Z. Phys.Rev.D **83**, 054025(2011)
arXiv:0912.3380 [hep-ph], arXiv:1012.5296 [hep-ph]

The same relation was shortly afterwards obtained independently:
U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D **81**, 036010 (2010),
arXiv:0909.5650 [hep-ph]

we assume $m\to 0$ (if not stated otherwise)

PDF-TMD relations

2. POLARIZED

$$g_1^a(x,\mathbf{p}_T) = \frac{2x-\xi}{2} K^a(x,\mathbf{p}_T) ,$$

$$h_1^a(x,\mathbf{p}_T) = \frac{x}{2} K^a(x,\mathbf{p}_T) ,$$

$$g_{1T}^{\perp a}(x,\mathbf{p}_T) = K^a(x,\mathbf{p}_T) ,$$

$$h_{1L}^{\perp a}(x,\mathbf{p}_T) = -K^a(x,\mathbf{p}_T) ,$$

$$h_{1T}^{\perp a}(x,\mathbf{p}_T) = -\frac{1}{x} K^a(x,\mathbf{p}_T) .$$

Known $f_1(x)$, $g_1(x)$ allow us to predict some unknown TMDs

$$K^a(x,\mathbf{p}_T) = \frac{2}{\pi \xi^3 M^2} \left(2 \int_\xi^1 \frac{dy}{y} \, g_1^a(y) + 3 \, g_1^a(\xi) - x \, \frac{dg_1^a(\xi)}{d\xi} \right)$$

$$\xi(x,\mathbf{p}_T^2) = x \left(1 + \frac{\mathbf{p}_T^2}{x^2 M^2} \right)$$

Numerical results:
(unpolarized)

Another model approaches to TMDs give compatible results:

1. U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D 81, 036010 (2010)

2. C.Bourrely, F.Buccellla, J.Soffer, Phys.Rev. D 83, 074008 (2011); Int.J.Mod.Phys. A28 (2013) 1350026

Input for $f_1(x)$
MRST LO at *4 GeV²*

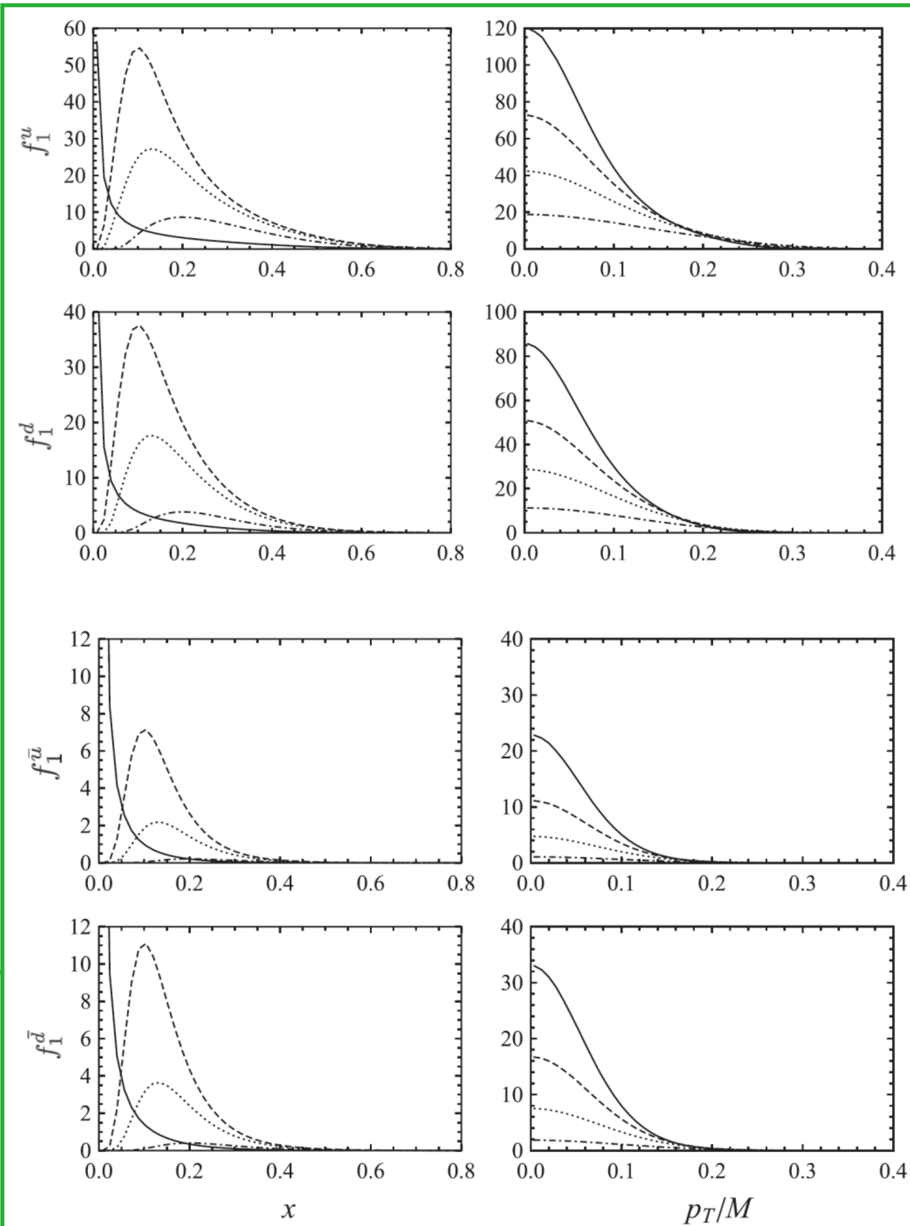


FIG. 1. The TMDs $f_1^a(x, \mathbf{p}_T)$ for u, d (*upper part*) and $\bar{u}$, $\bar{d}$-quarks (*lower part*). *Left panel*: $f_1^a(x, \mathbf{p}_T)$ as a function of x for $p_T/M = 0.10$ (dashed line), 0.13 (dotted line), 0.20 (dash-dotted line). The solid line corresponds to the input distribution $f_1^a(x)$. *Right panel*: $f_1^a(x, \mathbf{p}_T)$ as a function of p_T/M for $x = 0.15$ (solid line), 0.18 (dashed line), 0.22 (dotted), 0.30 (dash-dotted line).

$\langle p_T \rangle < 0.1$GeV, $p_T/M < 0.5$

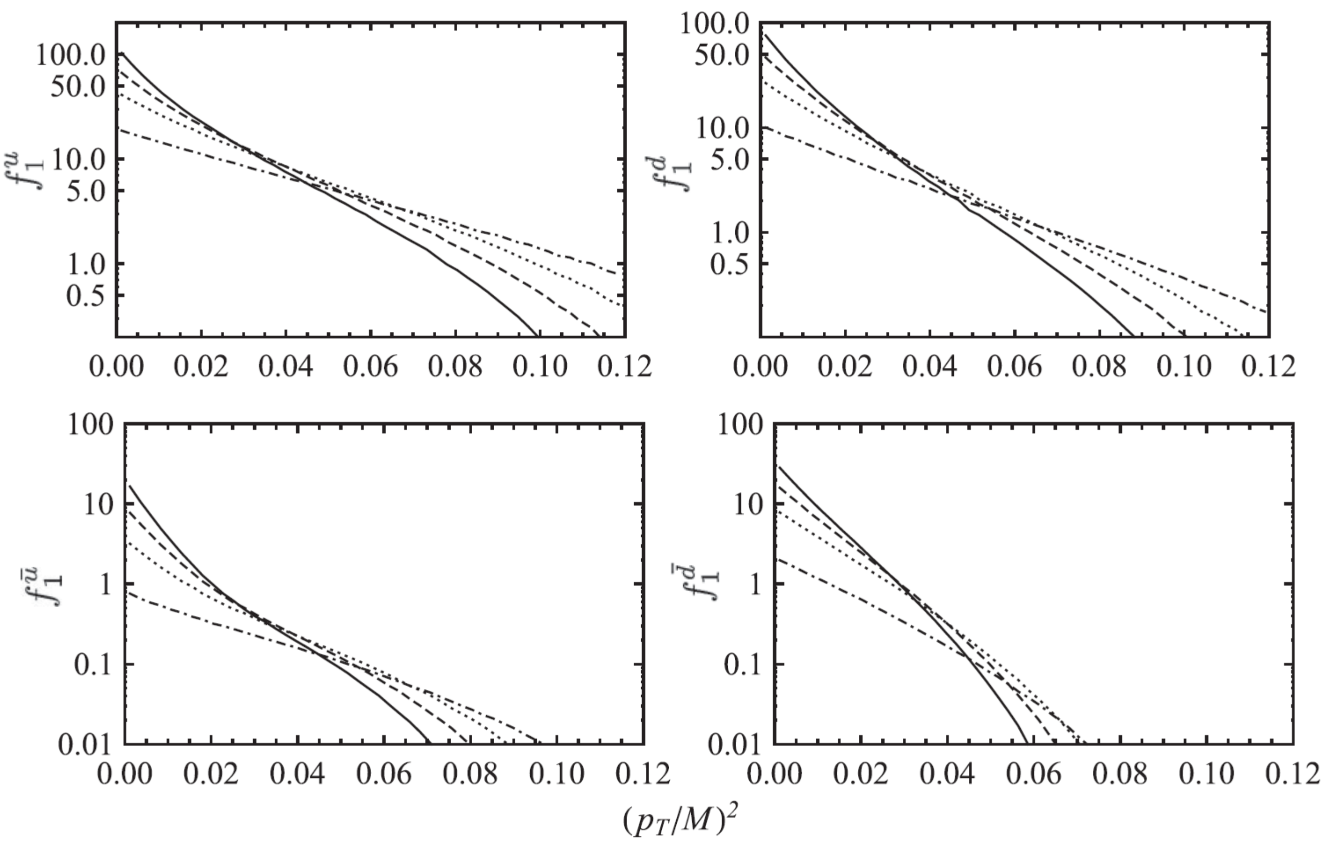


FIG. 2. $f_1^a(x, \mathbf{p}_T)$ as a function of $(p_T/M)^2$ for $x = 0.15$ (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dash-dotted line).

- Gaussian shape – is supported by phenomenology
- $\langle p_T^2 \rangle$ depends on x , is smaller for sea quarks

Numerical results:

(polarized)

Input for g_1 : LSS LO at *4 GeV²*

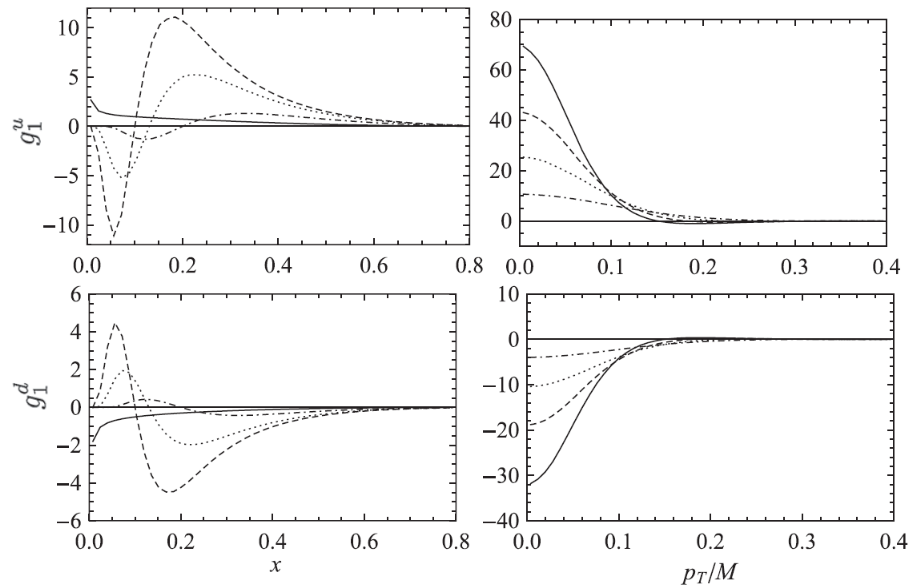
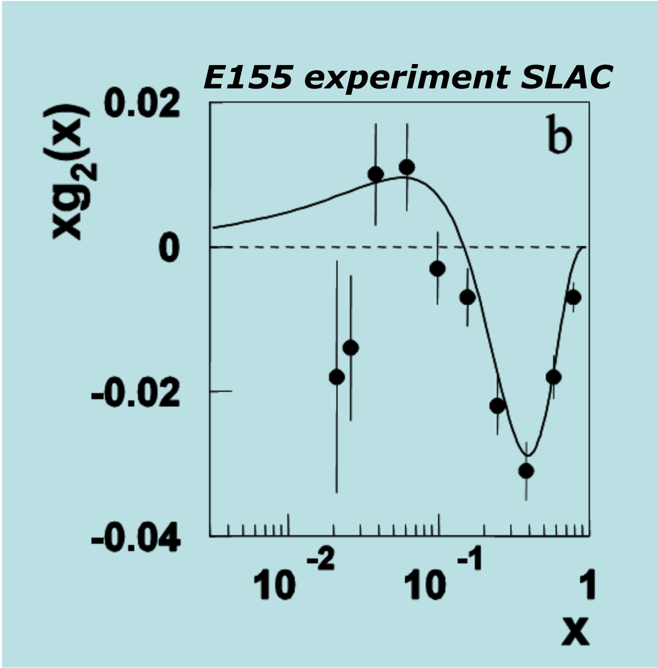


FIG. 3. $g_1^q(x, \mathbf{p_T})$ for *u*- (*upper panel*) and *d*-quarks (*lower panel*). *Left panel*: $g_1^q(x, \mathbf{p_T})$ as a function of x for $p_T/M =$ 0.10 (dashed line), 0.13 (dotted line), 0.20 (dash-dotted line). The solid line corresponds to the input distribution $g_1^q(x)$. *Right panel*: $g_1^q(x, \mathbf{p_T})$ as a function of p_T/M for $x = 0.15$ (solid line), 0.18 (dashed line), 0.22 (dotted line), 0.30 (dash-dotted line).

… can be compared to $g_2(x)$:
In both cases the sign is correlated with the sign of p_L in the rest frame



P.Z. Phys.Rev.D **67**, 014019 (2003)

QCD evolution of TMDs

LI & **RS** generate the relations **TMDs↔PDFs:**

$$f_1^a(x, \mathbf{p}_T) = -\frac{1}{\pi M^2}\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{q(y)}{y}\right]_{y=\xi}; \qquad \xi = x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)$$

The most direct way to introduce evolution is via $q(x, Q^2)$:

$$f_1^a(x, \mathbf{p}_T, Q^2) = -\frac{1}{\pi M^2}\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{q(y, Q^2)}{y}\right]_{y=\xi}; \qquad \xi = x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)$$

for details see A. Efremov, O. Teryaev and P.Z., J.Phys.Conf.Ser. 678 (2016), no.1, 012001, arXiv:1511.01164 [hep-ph]. (in progress)

TMDs - numerical results:

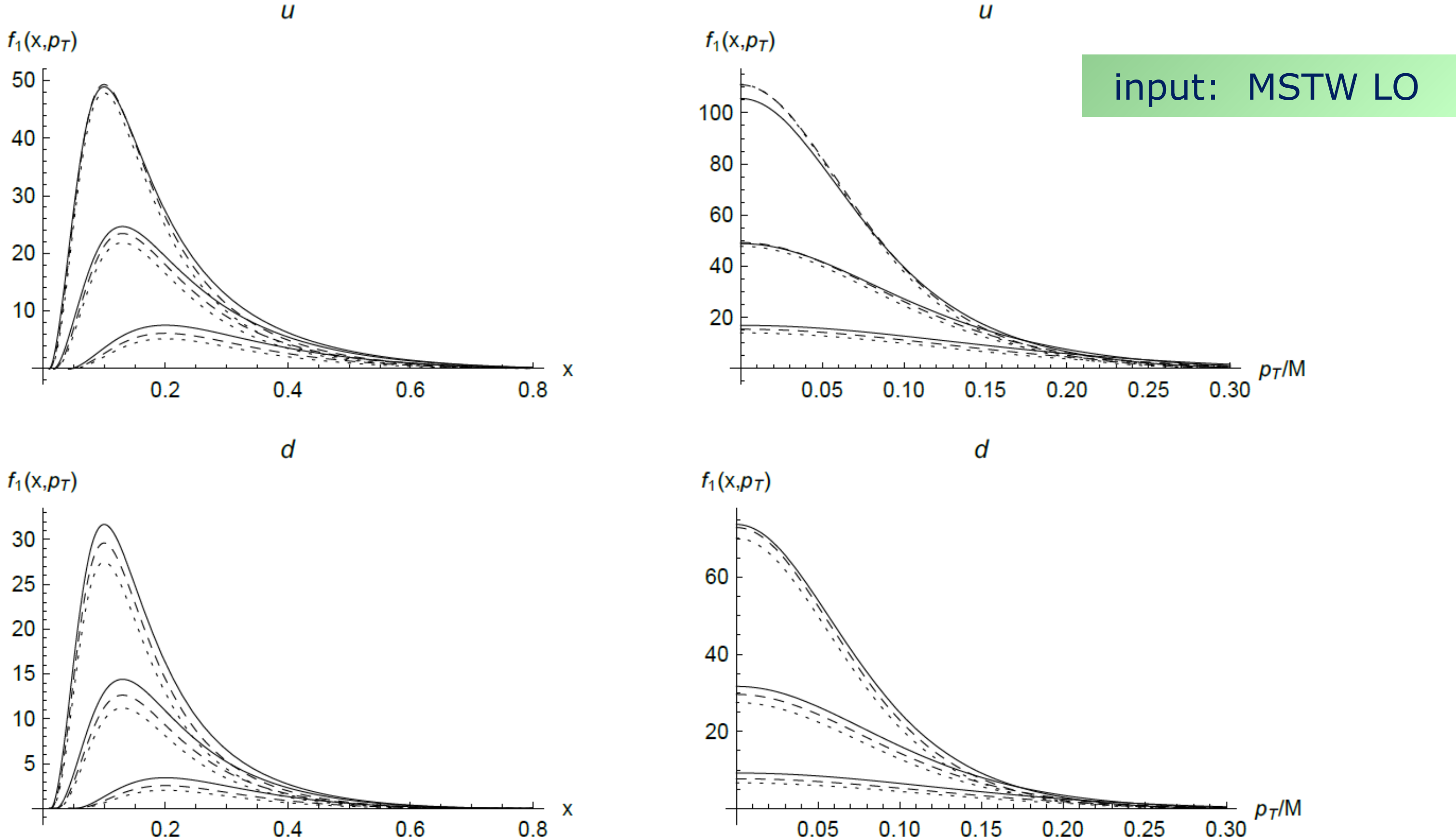


FIG. 3: TMD at different scales: $Q^2 = 4, 40, 400 GeV$ (solid, dashed, dotted curves) for u and d quarks. Sets of curves in left panels (from top) correspond to fixed $p_T/M = 0.1, 0.13, 0.20$. The curves in left panels (from top) correspod to fixed $x = 0.18, 0.22, 0.30$.

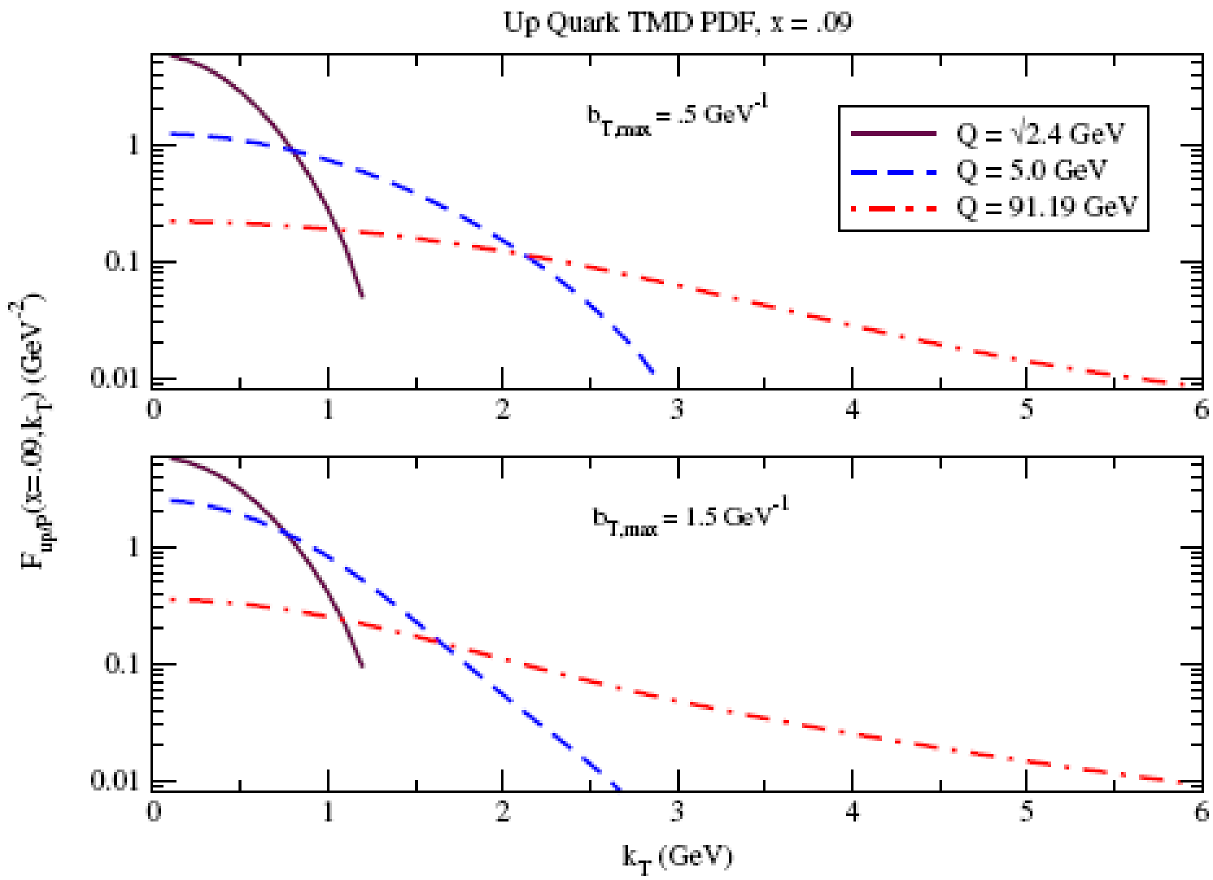
LI & **RS** $\Longrightarrow p_T \leq M/2$

a weak scale dependence…

PHYSICAL REVIEW D **83,** 114042 (2011)

Transverse momentum dependent parton distribution and fragmentation functions with QCD evolution

S. Mert Aybat[1,2,*] and Ted C. Rogers[2,†]



Why the results of the calculations differ so much?

Comparison:

- **pQCD evolution:** p_T can exceed ≈1 GeV
  correct dynamics (QCD) + reduced kinematic
  (no covariance, no rest frame sphericity…)

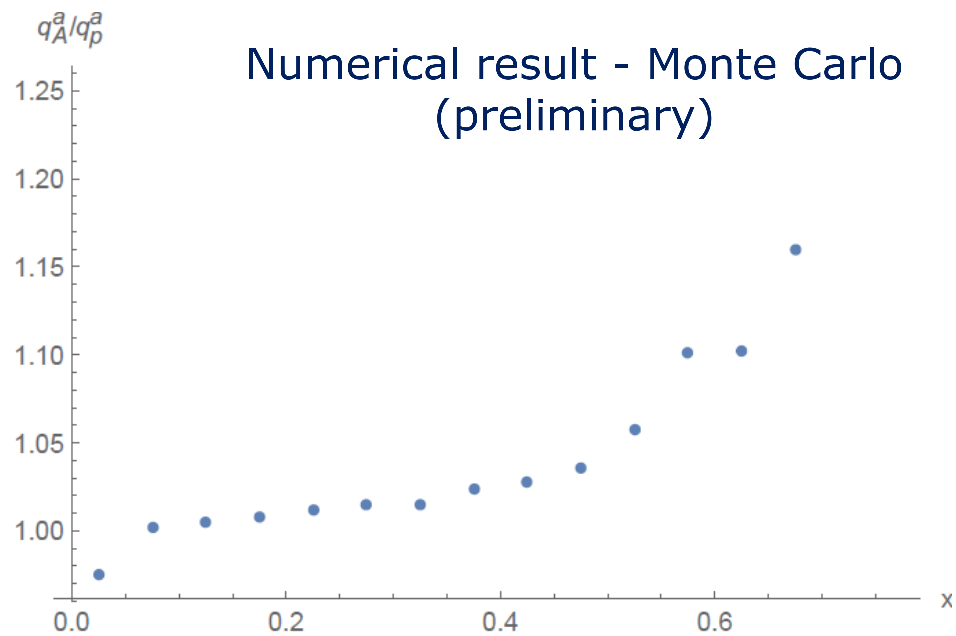
- **Covariant approach:** p_T ≈0.1 GeV
  simplistic model + correct 3D kinematics
  (constrained by LI & RS)

- **Correct answer:**
  may come from JLab experiments ?

Next step: TMDs and nuclei

Covariant approach:

$q(x) \Rightarrow G(p)$
$f(x, p_T)$



3D convolution with nuclear Fermi motion $\Rightarrow G_A(p) \Rightarrow$ $q_A(x)$, $f_A(x, p_T)$

Spin & OAM

Eigenstates of angular momentum

Usual plane-wave spinors are replaced by spinor spherical harmonics (both in momentum representation):

$$u(\mathbf{p},\lambda\mathbf{n}) = \frac{1}{\sqrt{N}}\begin{pmatrix}\phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}}\end{pmatrix}$$

$$\frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}}\,, \qquad N = \frac{2p_0}{p_0+m}$$

$\Rightarrow$

$$|j,j_z\rangle = \Phi_{jl_pj_z}(\omega) = \frac{1}{\sqrt{2\epsilon}}\begin{pmatrix}\sqrt{\epsilon+m}\,\Omega_{jl_pj_z}(\omega) \\ -\sqrt{\epsilon-m}\,\Omega_{j\lambda_pj_z}(\omega)\end{pmatrix}$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix}\sqrt{\frac{j+j_z}{2j}}Y_{l_p,j_z-1/2}(\omega) \\ \sqrt{\frac{j-j_z}{2j}}Y_{l_p,j_z+1/2}(\omega)\end{pmatrix}; \quad l_p = j-\frac{1}{2},$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix}-\sqrt{\frac{j-j_z+1}{2j+2}}Y_{l_p,j_z-1/2}(\omega) \\ \sqrt{\frac{j+j_z+1}{2j+2}}Y_{l_p,j_z+1/2}(\omega)\end{pmatrix}; \quad l_p = j+\frac{1}{2}$$

where ω represents the polar and azimuthal angles (θ,φ) of the momentum p with respect to the quantization axis, $l_p = j \pm 1/2$ and $\lambda_p = 2j - l_p$ (l_p defines parity).

New representation is convenient for general discussion about role of OAM. The rest frame of the composite system is a starting reference frame.

P. Z. Phys. Rev. D 89, 014012 (2014)

Spinor spherical harmonics $|j, j_z>$

- SSH represent solutions of the free Dirac equation, which reflects the known QM rule:

In relativistic case spin and OAM are not decoupled (separately conserved), but only sums *j* and $j_z = s_z + l_z$ are conserved.

- However, one can always calculate the mean values of corresponding operators:

$$s_z = \frac{1}{2}\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}, \qquad l_z = -i\left(p_x \frac{\partial}{\partial p_y} - p_y \frac{\partial}{\partial p_x}\right)$$

result:

$$\langle s_z \rangle_{j,j_z} = \frac{1 + (2j+1)\,\mu}{4j\,(j+1)} j_z, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1 + (2j+1)\,\mu}{4j\,(j+1)}\right) j_z,$$

where $\mu = m/\varepsilon$.

Non-relativistic limit (*μ=1*):

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{2j}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{2j}\right) j_z$$

j≥1/2

$l_p = j-1/2$

Relativistic case (*μ → 0*):

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{4j\,(j+1)}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{4j\,(j+1)}\right) j_z$$

⬇

$$\left|\langle s_z \rangle_{j,j_z}\right| \le \frac{1}{4\,(j+1)} \le \frac{1}{6}, \qquad \frac{\left|\langle s_z \rangle_{j,j_z}\right|}{\left|\langle l_z \rangle_{j,j_z}\right|} \le \frac{1}{4j^2+4j-1} \le \frac{1}{2}$$

… and for *j=1/2* :

$$\left|\langle s_z \rangle_{j,j_z}\right| = \frac{1}{6} \qquad \frac{\langle s_z \rangle_{j,j_z}}{\langle l_z \rangle_{j,j_z}} = \frac{1}{2}$$

Remark:
The ratio *μ=m/ε* plays a crucial role, since it controls a "contraction" of the spin component which is compensated by the OAM. It is an **QM effect of relativistic kinematics.**

In other words, lower component can play an important role!
cf. Bo-Qiang Ma, DSPIN2015 talk

Many-fermion states

Composition of one-particle states (SSH) representing composed particle with spin ***J=J$_z$=1/2:***

$$|(j_1, j_2, ...j_n)_c J, J_z\rangle = \sum_{j_{z1}=-j_1}^{j_1} \sum_{j_{z2}=-j_2}^{j_2} ... \sum_{j_{zn}=-j_n}^{j_n} c_j \, |j_1, j_{z1}\rangle \, |j_2, j_{z2}\rangle \, ... \, |j_n, j_{zn}\rangle$$

where c_j's consist of Clebsch-Gordan coefficients:

$$c_j = \langle j_1, j_{z1}, j_2, j_{z2} \,|J_3, J_{3z}\,\rangle \, \langle J_3, J_{z3}, j_3, j_{z3} \,|J_4, J_{z4}\,\rangle \, ... \, \langle J_n, J_{zn}, j_n, j_{zn} \,|J, J_z\,\rangle$$

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} = \langle s_{z1} + s_{z2} + ... + s_{zn} \rangle_c \, , \qquad \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \langle l_{z1} + l_{z2} + ... + l_{zn} \rangle_c$$

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} + \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \frac{1}{2},$$

$$|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6}, \qquad \frac{|\langle \mathbb{S}_z \rangle|}{|\langle \mathbb{L}_z \rangle|} \leq \frac{1}{2} \qquad J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}$$

for $\mu \rightarrow 0$

Spin structure functions: explicit form

For $Q^2 \gg 4M^2x^2$ we get (in terms of rest frame variables)

$x = Q^2/2Pq$

$$g_1(x) = \frac{1}{2}\int \left(\mathrm{u}(\epsilon)\left(p_1 + m + \frac{p_1^2}{\epsilon+m}\right) + \mathrm{v}(\epsilon)\left(p_1 - m + \frac{p_1^2}{\epsilon-m}\right)\right)\delta\left(\frac{\epsilon+p_1}{M} - x\right)\frac{d^3p}{\epsilon},$$

$$g_2(x) = -\frac{1}{2}\int \left(\mathrm{u}(\epsilon)\left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon+m}\right) + \mathrm{v}(\epsilon)\left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon-m}\right)\right)\delta\left(\frac{\epsilon+p_1}{M} - x\right)\frac{d^3p}{\epsilon}.$$

where u, v are functions related to the polarization tenzor, which is defined by the initial state $\Psi_{1/2}$

This result is exact for SFs generated by (free) many-fermion state **J=1/2** represented by the spin spherical harmonics. For given state $\Psi_{1/2}$ we have checked calculation:

$$\langle \mathbb{S}_z \rangle = \left\langle \Psi_{1/2} \left| \mathbb{S}_z \right| \Psi_{1/2} \right\rangle = \left\langle s_{z1} + s_{z2} + \ldots + s_{zn} \right\rangle$$

$$\Gamma_1 = \int_0^1 g_1(x)\, dx$$

give equivalent results! ✓

Proton spin structure

The SSH formalism can be used for proton description in conditions of DIS. We assume:

- The proton state can be at each Q^2 represented by a superposition of Fock states:

$$\Psi = \sum_{q,g} a_{qg} \left|\varphi_1, ...\varphi_{n_q}\right\rangle \left|\psi_1, ...\psi_{n_g}\right\rangle$$

- In a first step we ignore possible contribution of gluons, then:

$$\Psi = \sum_{q} a_{q} \left|\varphi_1, ...\varphi_{n_q}\right\rangle$$

where the quark states $\left|\varphi_1, ...\varphi_{n_q}\right\rangle$ are represented by eigenstates:

$$J = J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}$$

Proton spin content

We have shown the system **J=1/2** composed of (quasi) free fermions $\mu \to 0$ satisfies:

$$|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},$$

(or the same in terms of Γ_1)

Reduced spin is compensated by OAM

$$\langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}$$

and equality takes place for a simplest configuration:

$$j_1 = j_2 = j_3 = \ldots = j_{n_q} = \frac{1}{2}$$

If we change notation

$$|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},$$

$$\Delta\Sigma \lesssim 1/3$$

this result is well compatible with the data
(cf. experiments [30-32]):

$$\Delta\Sigma = 0.32 \pm 0.03(stat.)$$

[30] M. G. Alekseev et al. [COMPASS Collaboration], Phys. Lett. B 693, 227 (2010)].
[31] V. Y. Alexakhin et al. [COMPASS Collaboration], Phys. Lett. B 647, 8 (2007) .
[32] A. Airapetian et al. [HERMES Collaboration], Phys. Rev. D 75, 012007 (2007).
[33] C. Adolph et al. [COMPASS Collaboration], Phys. Lett. B 718, 922 (2013) .
[34] A. Airapetian et al. [HERMES Collaboration], JHEP 1008, 130 (2010) .

Comment:

Until now we assumed:

- Proton spin S=1/2 is generated only by quarks S_q+L_q
- During DIS the quarks can be considered (quasi)free in any ref. frame, $\Delta\tau \ll \Delta\tau_{QCD}$. DIS is mediated by one photon exchange.
- In the proton rest frame the quarks are relativistic, $\mu=m/\varepsilon \to 0$

Then:

$$|\langle \mathbb{S}_z^q \rangle| = \frac{1}{6} \qquad \frac{\langle \mathbb{S}_z^q \rangle}{\langle \mathbb{L}_z^q \rangle} = \frac{1}{2}$$

Role of gluons in proton spin

- Until now we assumed the simplest scenario: $\mu = m/\varepsilon \to 0$ and $\mathbf{J_g}=0$, which gave $\Delta\Sigma \approx 1/3$. This complies with the data very well, for both, quarks and gluons.
- However, the recent data from RHIC may suggest $\mathbf{J_g}>0$. Such value does not contradict our approach. If one admits also $\mu = m/\varepsilon > 0$, then instead of

$$|\langle \mathbb{S}_z^q \rangle| = \frac{1}{6} \qquad \frac{\langle \mathbb{S}_z^q \rangle}{\langle \mathbb{L}_z^q \rangle} = \frac{1}{2}$$

we have

$$|\langle \mathbb{S}_z^q \rangle| = \frac{1+2\tilde{\mu}}{6} \qquad \frac{\langle \mathbb{S}_z^q \rangle}{\langle \mathbb{L}_z^q \rangle} = \frac{1+2\tilde{\mu}}{2-2\tilde{\mu}} \qquad J^q = \langle \mathbb{S}_z^q \rangle + \langle \mathbb{L}_z^q \rangle \qquad \tilde{\mu} = \left\langle \frac{m}{\epsilon} \right\rangle$$

At the same time:

$$\frac{1}{2} = J^q + J^g \quad \Rightarrow \quad \Delta\Sigma = \frac{1}{3}\left(1 - 2J^g\right)\left(1 + 2\tilde{\mu}\right)$$

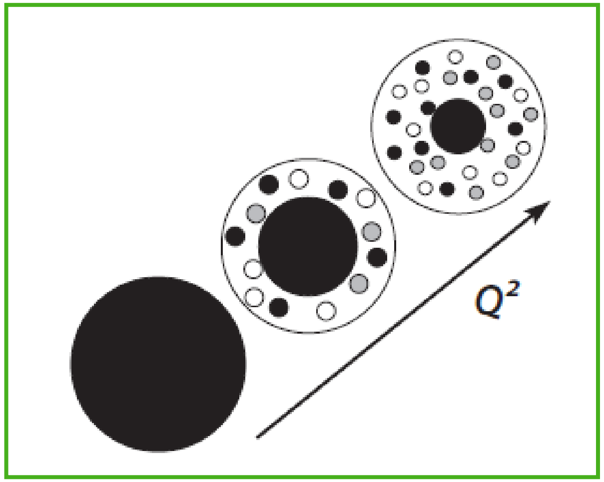
for details see P.Z. Phys. Lett. B 751, 525 (2015).

SPIN OF THE PARTICLE IN ITS SCALE DEPENDENT PICTURE

Two questions:

❑ How much do the virtual particles surrounding bare particle contribute to the spin of corresponding real, dressed particle?

❑ How much do the virtual particles mediating binding of the constituents of a composite particle contribute to its spin?



The **electron**, as a Dirac particle, in its rest frame has AM defined by its spin, $s = 1/2$. This value is the same for the dressed electron (as proved experimentally) and for the bare one (as defined by the QED Lagrangian). **So, can the AM contribution of virtual cloud $J^{\gamma}(Q^2)$ differ from zero and how much?**

For similarly motivated studies see:
Bo-Qiang Ma; talk for DSPIN-15
Tianbo Liu, Bo-Qiang Ma; Phys.Rev. D91 (2015) 017501
S. J. Brodsky, Dae Sung Hwang, Bo-Qiang Ma, I. Schmidt ; Nucl. Phys. B 593 (2001) 311–335
Matthias Burkardt and Hikmat BC; Phys.Rev. D79 (2009) 071501(R)
Xinyu Zhang, Bo-Qiang Ma; Phys.Rev. D85 (2012) 114048
A. Bacchetta, L. Mantovani and B. Pasquini, Phys.Rev. D93 (2016) 013005

Semiclassical calculation of stationary electromagnetic field in the frame defined by spinor spherical harmonic:

$$\Phi_{jl_pj_z}(\mathbf{r}) = \frac{1}{\sqrt{2\epsilon}} \begin{pmatrix} \sqrt{\epsilon+m} R_{kl_p} \Omega_{jl_pj_z}(\omega) \\ -\sqrt{\epsilon-m} R_{k\lambda_p} \Omega_{j\lambda_pj_z}(\omega) \end{pmatrix}$$

Our reference frame is the rest frame of the composite system of these states.

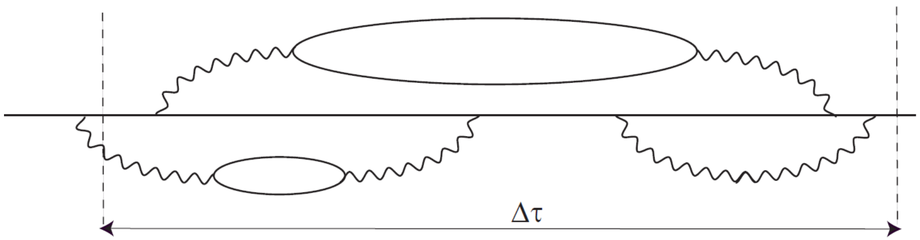
$$I_\mu = (I_0, \mathbf{I}) = \Phi^\dagger_{jl_pj_z}(\mathbf{r})\, \gamma^0 \gamma_\mu \Phi_{jl_pj_z}(\mathbf{r})$$

$$\mathbf{E}(\mathbf{r}) = \int I_0(\mathbf{r}') \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3/2}} d^3\mathbf{r}'$$

$$\mathbf{H}(\mathbf{r}) = \int \mathbf{I}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3/2}} d^3\mathbf{r}'$$

$$\mathbf{J}^\gamma = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{H}) d^3\mathbf{r} \quad \Rightarrow \quad \mathbf{J}^\gamma = 0$$

This result represents a mean value, which is not influenced by the fluctuations generated by single γ.



Can we do a similar calculation for the color field ?

Summary

<u>**Covariant approach:**</u>

- **Constrains on LI & RS are crucial!**
- **TMDs: relations, calculation, predictions, QCD evolution, nuclear TMDs**
- **Interplay of spin & OAM, role of gluons…**
- **Agreement with the data, particularly as for $\Delta\Sigma$, is a strong argument for this approach**

Thank you for your attention!

Backup slides

F_1, F_2 – EXACT AND MANIFESTLY COVARIANT FORM:

$$F_1(x) = \frac{M}{2}\left(\frac{B}{\gamma} - A\right), \qquad F_2(x) = \frac{Pq}{2M\gamma}\left(\frac{3B}{\gamma} - A\right),$$

where

$$A = \frac{1}{Pq}\int G\left(\frac{Pp}{M}\right)[m^2 - pq]\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$B = \frac{1}{Pq}\int G\left(\frac{pP}{M}\right)\left[\left(\frac{Pp}{M}\right)^2 + \frac{(Pp)(Pq)}{M^2} - \frac{pq}{2}\right]\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$\gamma = 1 - \left(\frac{Pq}{Mq}\right)^2.$$

conventional collinear approach: $p_\mu \to xP_\mu$

… SIMILARLY FOR G_1, G_2 :

$$g_1 = Pq\left(G_S - \frac{Pq}{qS}G_P\right), \qquad g_2 = \frac{(Pq)^2}{qS}G_P,$$

where

$$G_P = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[\frac{pS}{pP+mM}1 + \frac{1}{mM}\left(pP - \frac{pu}{qu}Pq\right)\right] \times \delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$G_S = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[1 + \frac{pS}{pP+mM}\frac{M}{m}\left(pS - \frac{pu}{qu}qS\right)\right] \times \delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0};$$

$$u = q + (qS)S - \frac{(Pq)}{M^2}P.$$