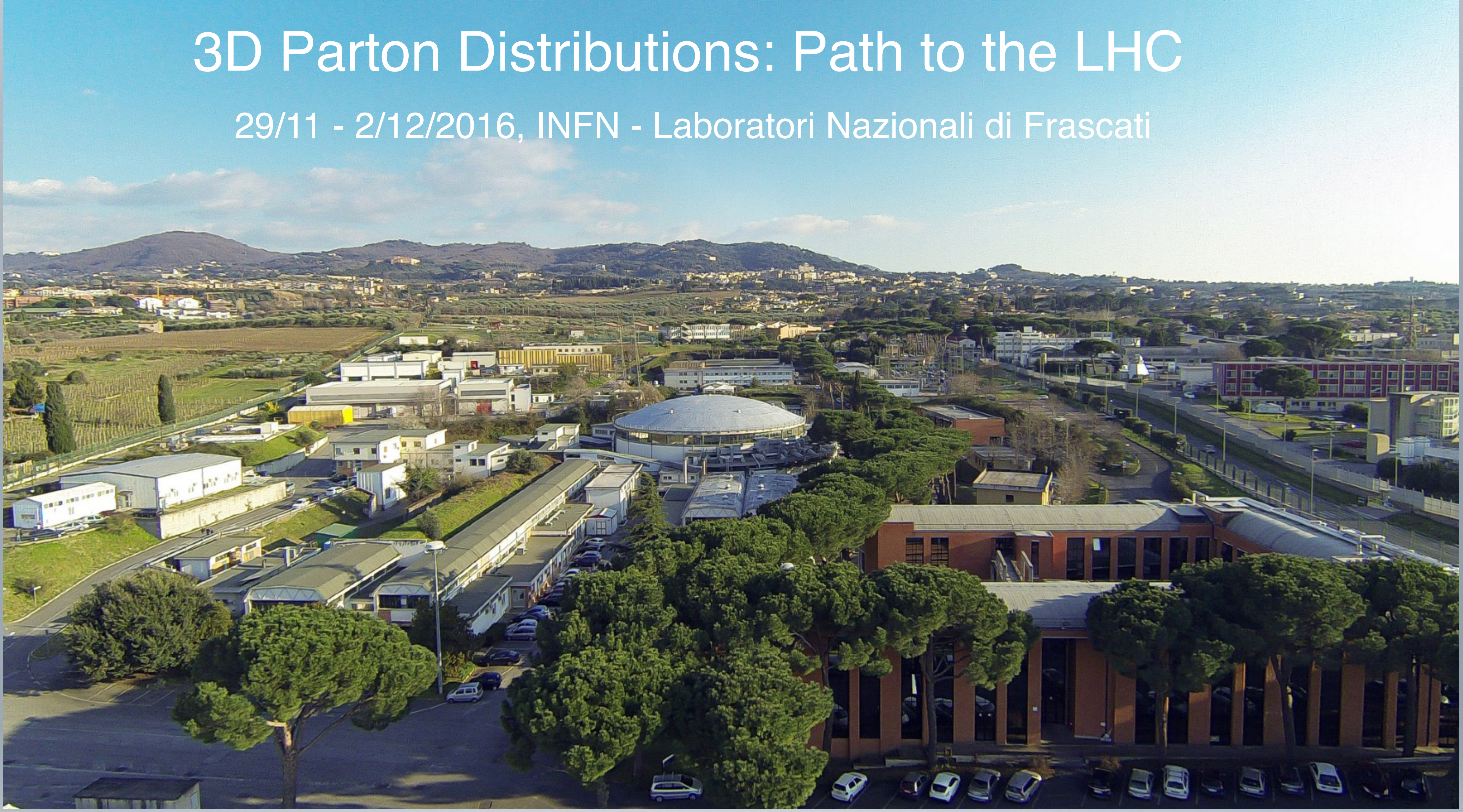
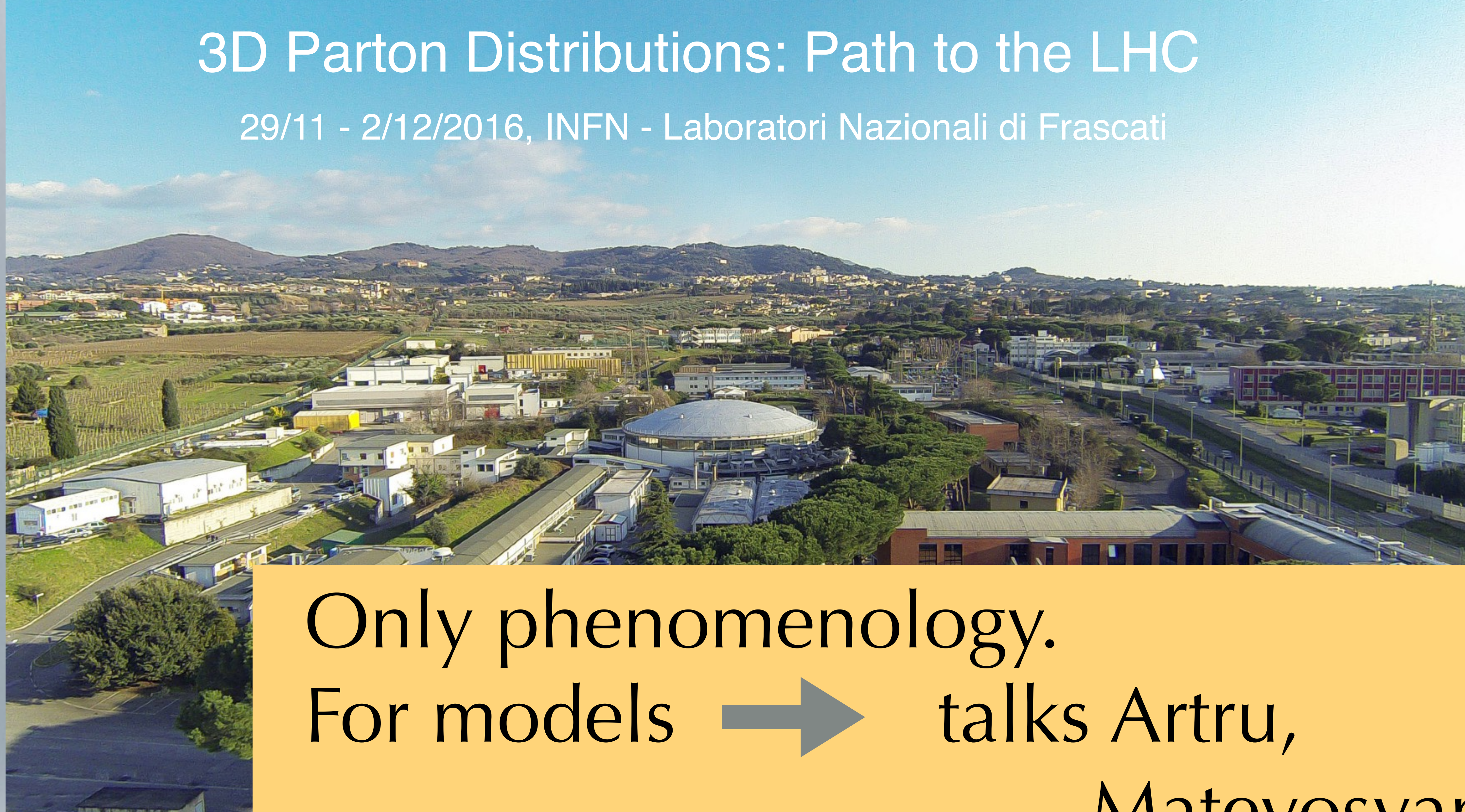


Marco Radici
INFN - Pavia

Fragmentation Functions : status and perspectives



Only phenomenology.
For models → talks Artru, Matevosyan

Marco Radici
INFN - Pavia

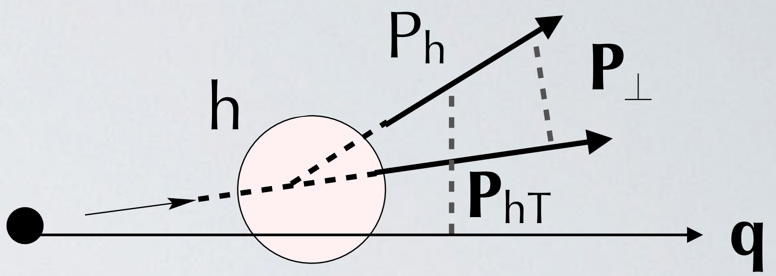
Istituto Nazionale di Fisica Nucleare

Fragmentation Functions : status and perspectives

TMD FF map

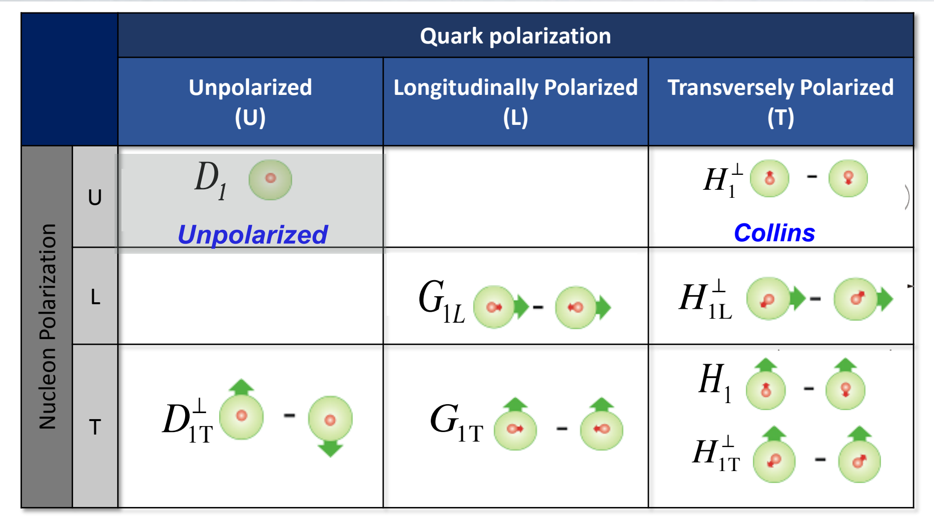
next talk by Liang
TMD FF up to S_h=1
including twist 3

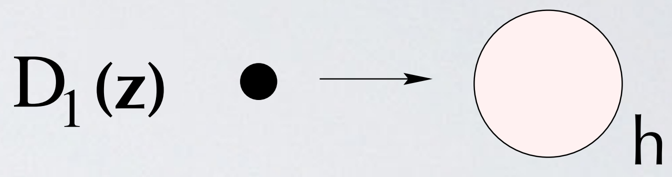
leading twist
$S_h \leq 1/2$



Hadron Polarization		Quark polarization: Unpolarized (U)	Quark polarization: Longitudinally Polarized (L)	Quark polarization: Transversely Polarized (T)
	U	D_1 *Unpolarized*		$H_1^\perp$ - *Collins*
	L		G_{1L} -	$H_{1L}^\perp$ -
	T	$D_{1T}^\perp$ -	G_{1T} -	H_1 - $H_{1T}^\perp$ -

collinear 1h FF : fresh news





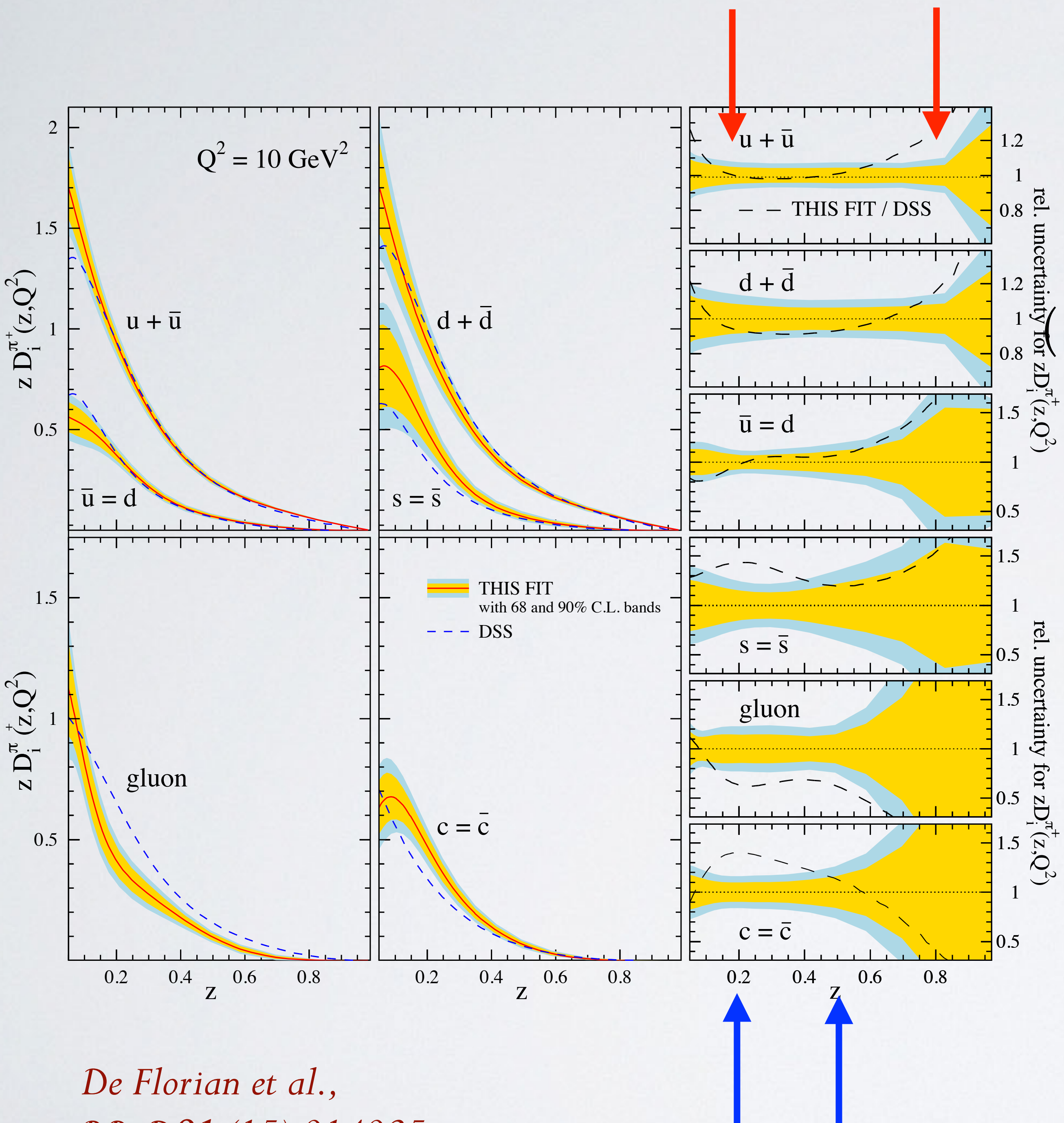
*De Florian et al., P.R. D**91** (15) 014035*

1. DSS (2007) → major update DSS 2015 (only for q→ h=π)

- more/better data for e^+e^- (Belle, BaBar)
- SIDIS (Hermes, Compass)
- RHIC (STAR)
- LHC (Alice)
- new error analysis
- **global χ^2/dof ~ 2.2 → 1.2**

first time use of LHC data

collinear 1h FF : DSS 2015

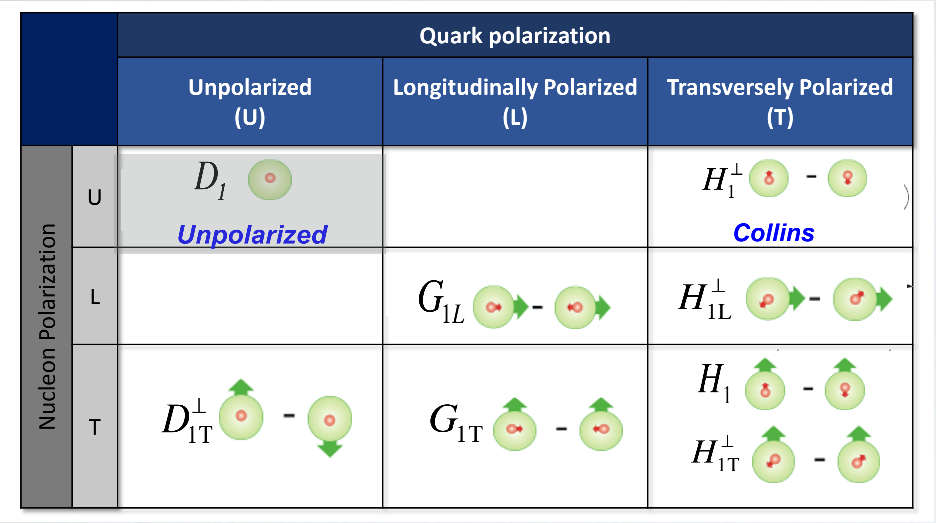


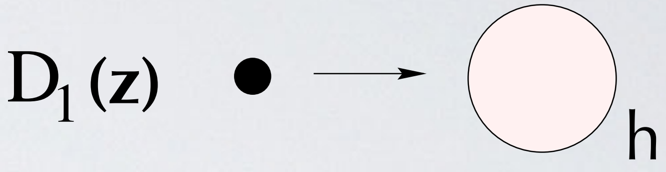
De Florian et al.,
*P.R. D**91** (15) 014035*

caveat

- major improvement only for total up & down channels: rel. uncertainty ≲10% for 0.2< z< 0.8
- for other channels, improvement upon DSS 2007 only for 0.2< z< 0.5
- Compass data for SIDIS multiplicities for deuteron target only
- Kaon fragmentation data not included

collinear 1h FF : JAMFF



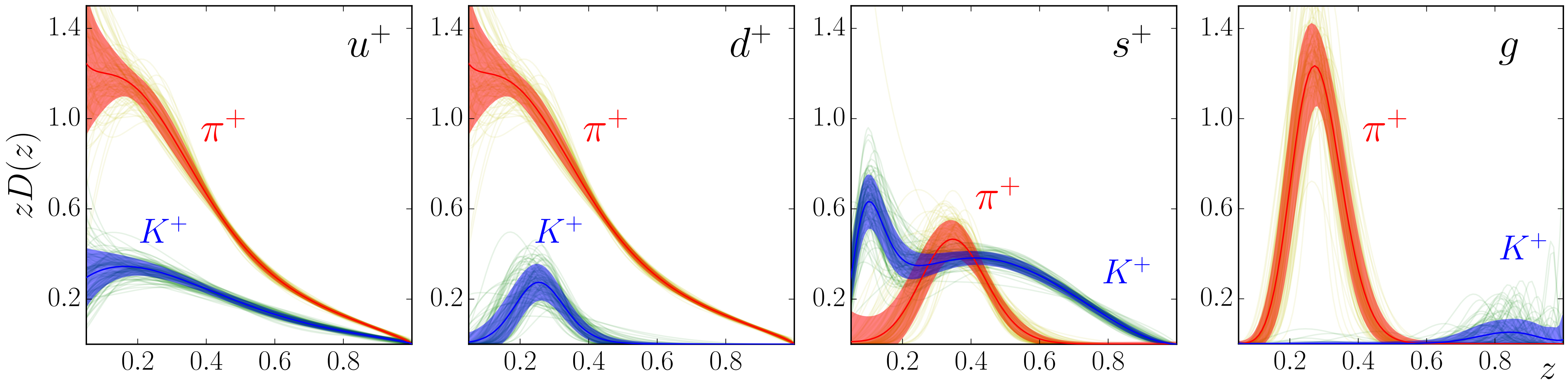


Sato et al., arXiv:1609.00899

2. new fit from JAM collaboration: JAMFF (for q→ h=π, K)

- only S.I. e^+e^- data
- 18 parameters for π, 24 for K
- Iterative Monte Carlo methodology
- **global χ^2/dof ~ 1.3 (π) , 1.01 (K)**

collinear 1h FF : JAMFF



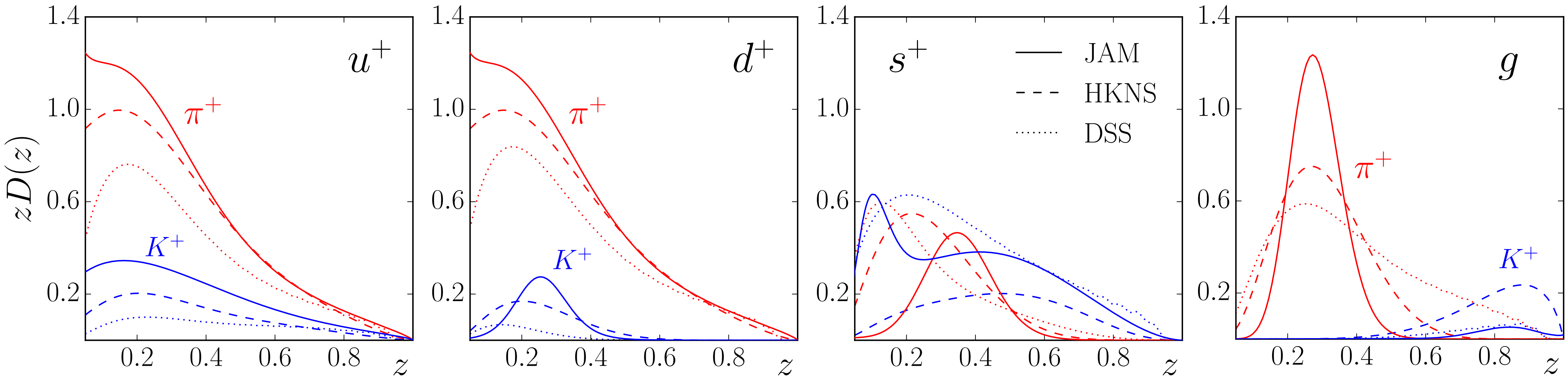
$$q^+ = q + \bar{q}$$

$Q^2 = 1$ GeV^2

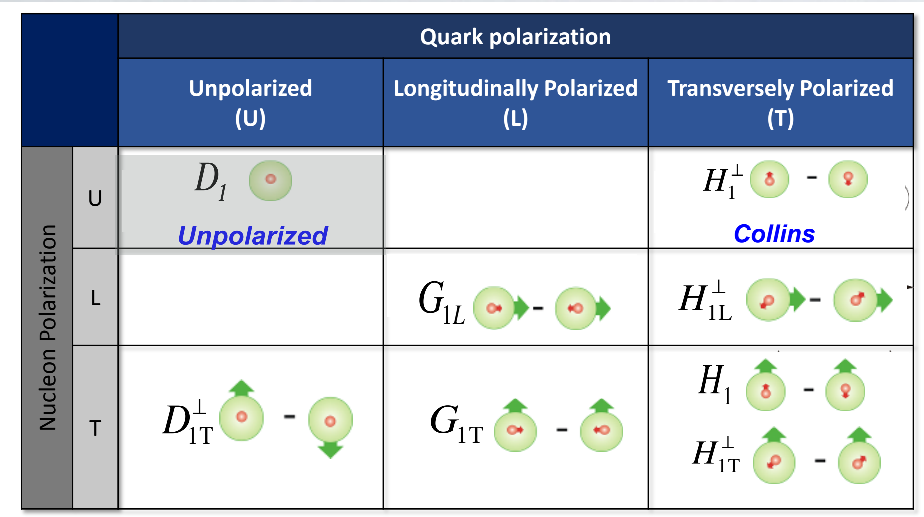
—— JAM *Sato et al., arXiv:1609.00899*

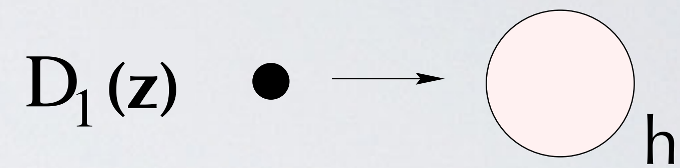
- - - - - HKNS *Hirai et al., P.R.D**75** (07) 094009*

······ DSS 2007 *De Florian et al., P.R.D**75** (07) 114010*



collinear 1h FF : fresh news





New extractions from NNLO analysis
($q \rightarrow h=\pi$ only)

3. Anderle, Ringer, Stratmann

P.R. D92 (15) 114017

- only S.I. $e^+ e^-$ data
- old SLAC & LEP + Belle + BaBar data (288)
- 16 parameters
- **global χ^2/dof : LO=0.89 → NNLO=0.64**

4. NNPDF Collaboration: NNFF1.0

E. Nocera, talk at QCD-N16 (Bilbao)

- more or less same data set
- neural network methodology
- **global χ^2/dof : LO=1.14 → NNLO=0.91**

collinear 1h FF : fresh news

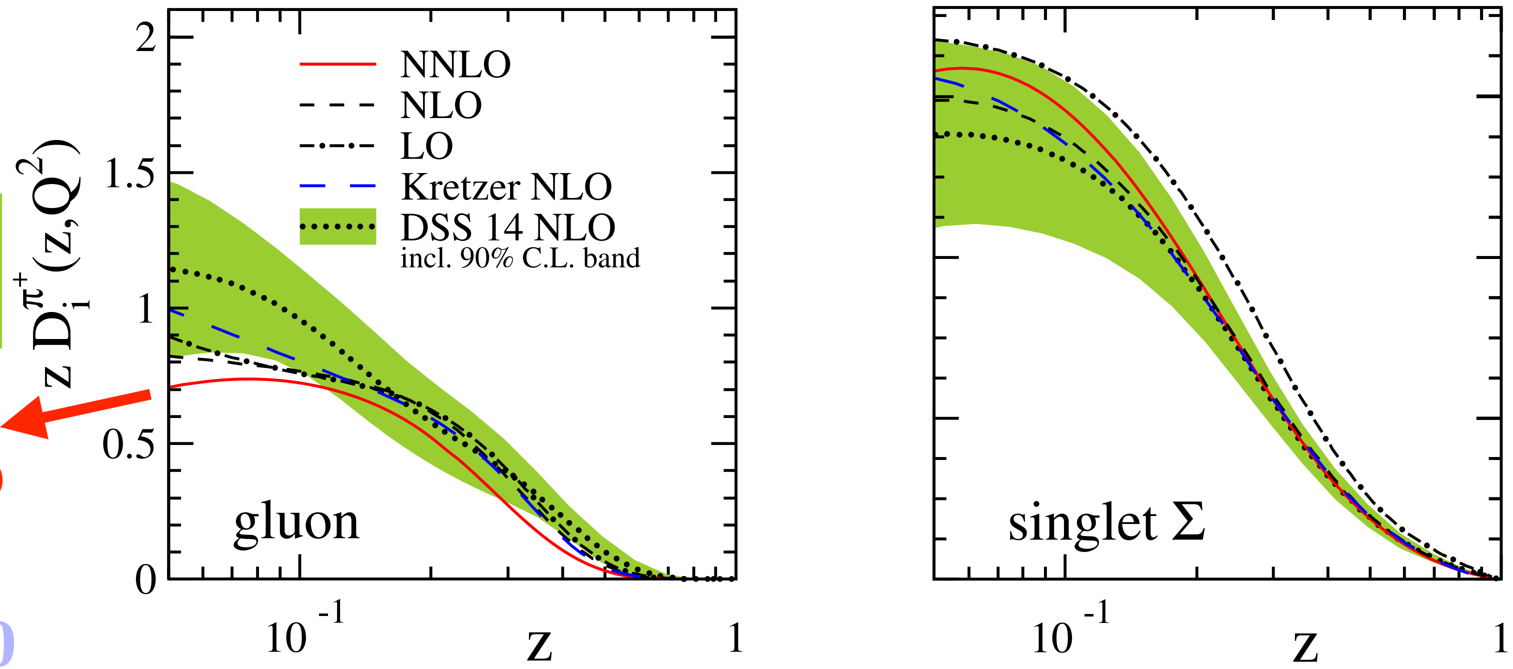
gluon

$$\Sigma = \sum_{q=1}^{N_f} \left[D_1^{q\to h}(z) + D_1^{\bar{q}\to h}(z) \right]$$

Anderle, Ringer, Stratmann,
P.R. D92 (15) 114017

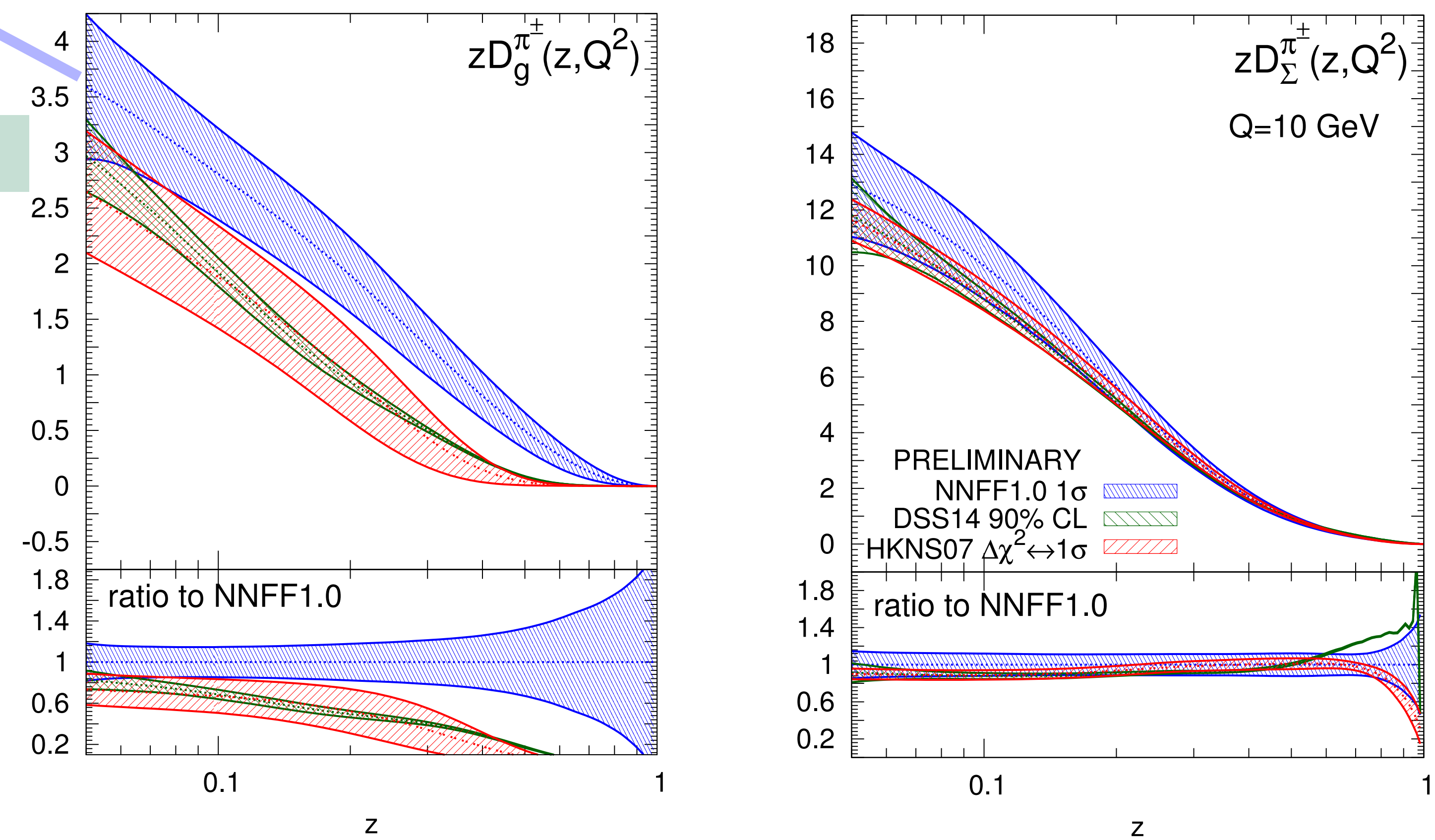
DSS 2015
NLO

ARS15
NNLO



$Q^2 = 10\ GeV^2$

NNFF1.0



$Q^2 = 100\ GeV^2$

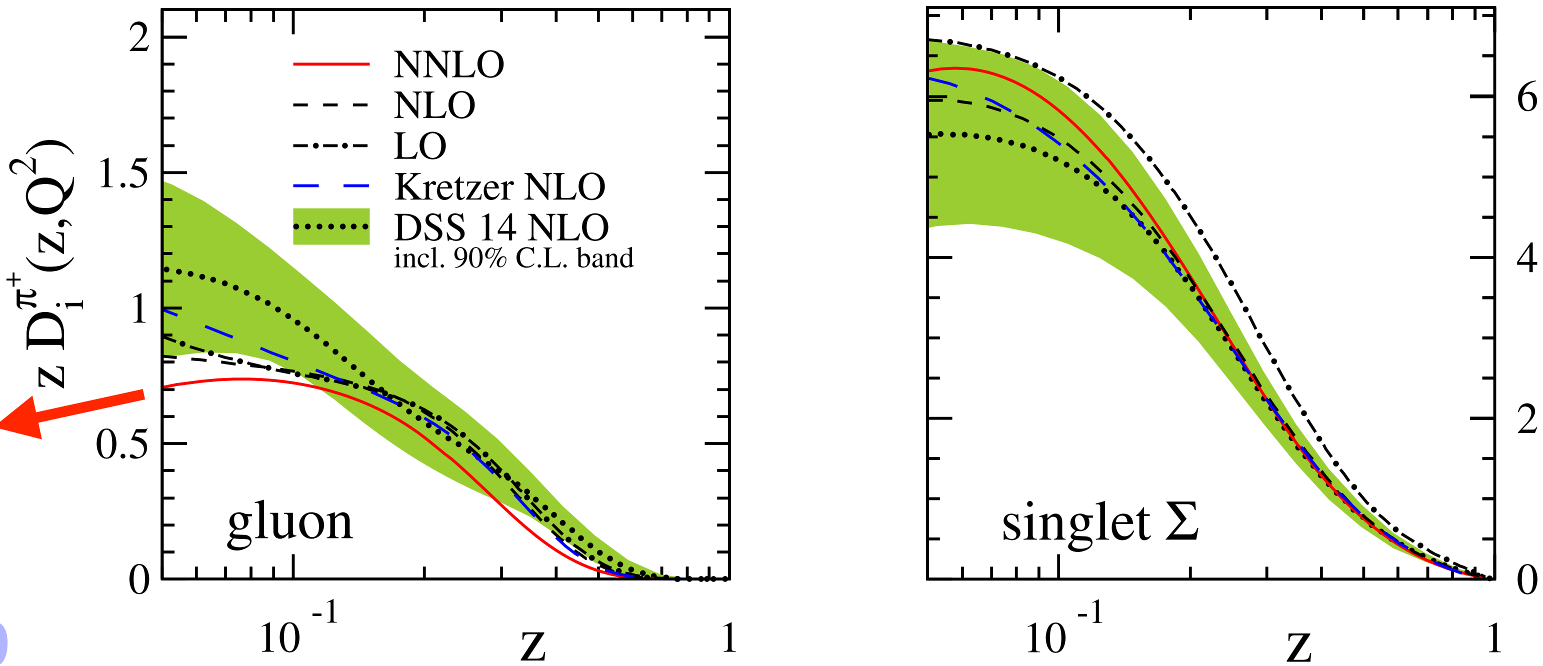
E. Nocera,
talk at QCD-N16 (Bilbao)

collinear 1h FF : fresh news

gluon

$$\Sigma = \sum_{q=1}^{N_f} \left[D_1^{q\to h}(z) + D_1^{\bar{q}\to h}(z) \right]$$

Anderle, Ringer, Stratmann,
P.R. D92 (15) 114017

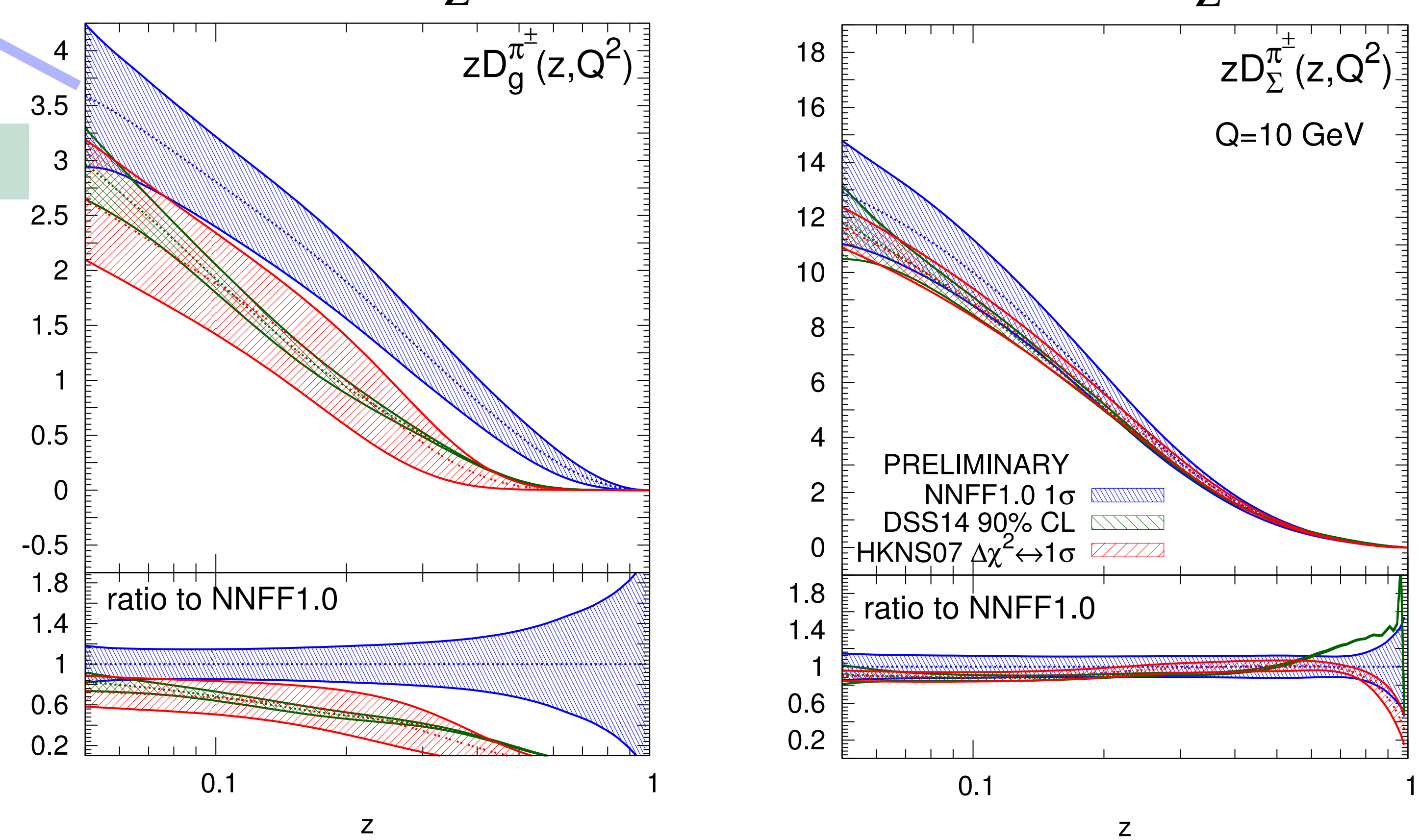


Q^2 = 10 GeV^2

ARS15 NNLO

DSS 2015 NLO

NNFF1.0



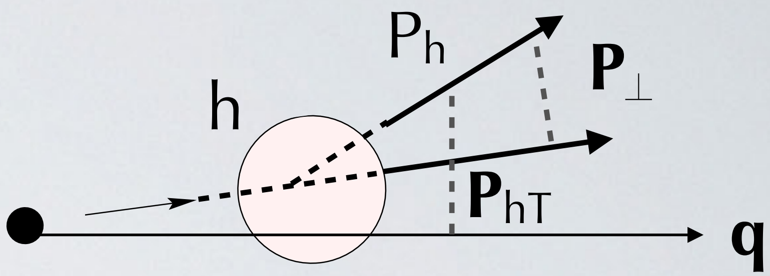
Q^2 = 100 GeV^2

difficult comparison,
but at NNLO
still evident
discrepancies for gluon
at not very small z…

unpolarized TMD FF

Nucleon Polarization	Quark polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	D_1 *Unpolarized*		$H_1^{\perp}$ *Collins*
L		G_{1L}	$H_{1L}^{\perp}$
T	$D_{1T}^{\perp}$	G_{1T}	H_1, $H_{1T}^{\perp}$

$D_1\ (z, \mathbf{P}_{hT})$

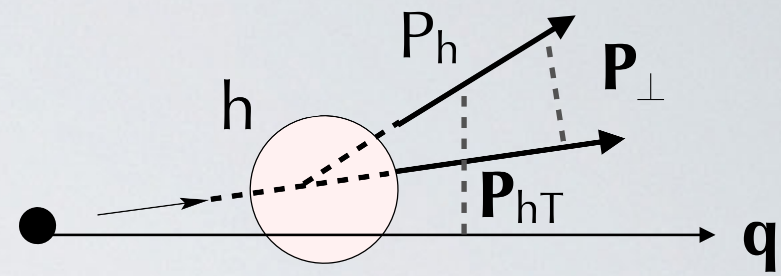


What do we know about the $\mathbf{P}_{hT}$ dependence ?

unpolarized TMD FF

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 Unpolarized		$H_1^\perp$ Collins
	L		G_{1L}	$H_{1L}^\perp$
	T	$D_{1T}^\perp$	G_{1T}	H_1, $H_{1T}^\perp$

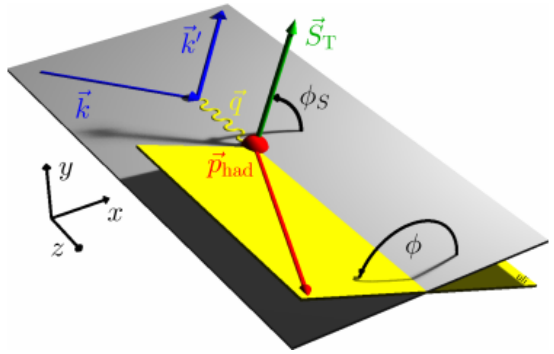
$D_1\,(z,\,\mathbf{P}_{hT})$



What do we know about the $\mathbf{P}_{hT}$ dependence ?

1. Does the $\mathbf{P}_{hT}$ dependence change with **flavor ?**
2. Does the $\mathbf{P}_{hT}$ dependence change with **z ?**
3. Does the $\mathbf{P}_{hT}$ dependence change with **energy $\sqrt{s}$?**
4. Does the $\mathbf{P}_{hT}$ dependence change with **scale Q^2 ?**

D_1 from unintegrated SIDIS multiplicities

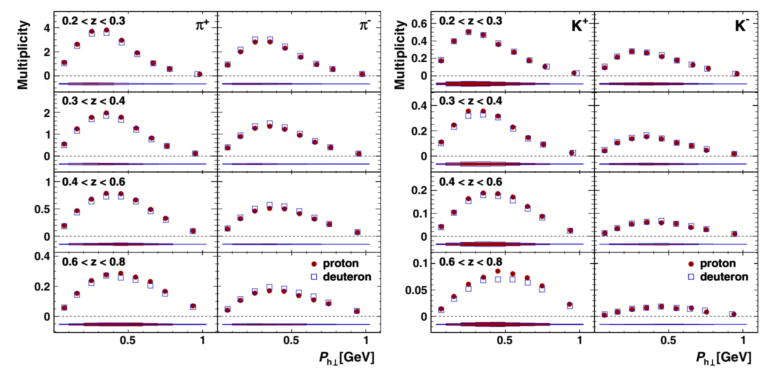


$$M_N^h = \frac{d\sigma_N^h / dx\, dz\, d\boldsymbol{P}_{hT}^2\, dQ^2}{d\sigma_{\mathrm{DIS}}/dx\, dQ^2} \approx \frac{\sum_q e_q^2\, [f_1^q \otimes D_1^q]\,(x, z, \boldsymbol{P}_{hT}^2; Q^2)}{\sum_q e_q^2\, f_1^q(x; Q^2)}$$

$$\boldsymbol{P}_{hT}^2 / z \ll Q^2$$

Present

- target: proton, deuteron
- final state: π^+, π^-, K^+, K^-
- 2688 points

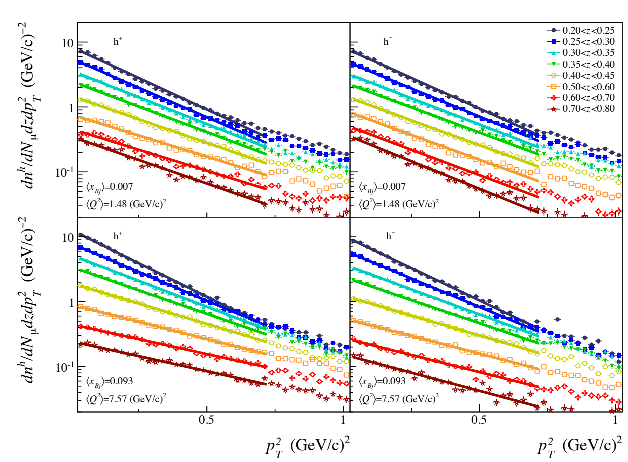


*Airapetian et al., P.R. D**87** (13) 074029*

talk Schnell

about 20000 data points (!)

- target: deuteron
- final state:
  - h^+, h^- (run 2004)
  - π^+,π^-,K^+,K^- (run 2006)

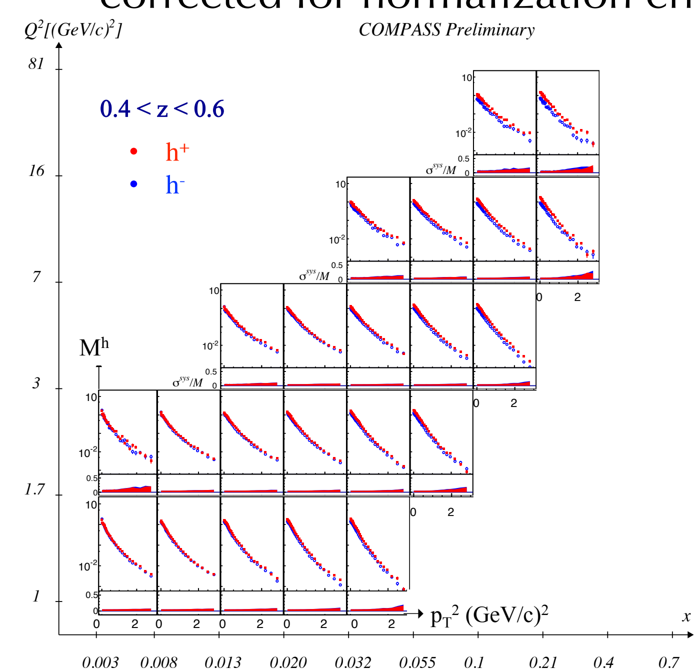


*Adolph et al., E.P.J. **C73** (13) 2531*

*Erratum: E.P.J. **C75** (15) 94*

talk Bressan

corrected for normalization error



N. Makke, talk at SPIN2016

D_1 from unintegrated SIDIS multiplicities

available fits

	Framework	Hermes	Compass	# points
Pavia 2013 *Bacchetta et al., JHEP* ***1311*** *(13) 194*	Gaussian $\langle \mathbf{p}_T^2 \rangle_q$ (z) 7 parameters no evolution	✔	✘	1538
Torino 2014 *Anselmino et al., JHEP* ***1404*** *(14) 005*	Gaussian $\langle \mathbf{p}_T^2 \rangle$ (1 parameter) only collinear DGLAP evolution N_y=A+By (y=Q^2/xs) **(C)**	✔ separately	✔ separately	576 (H) 6284 **(C)**
	↓ Framework of TMD evolution ↓			
EIKV 2014 *Echevarria et al., P.R.D****89*** *(14) 074013*	TMD framework, NLL level not a real fit	1 bin	(x,Q^2)	(?)
Pavia 2016 in preparation	TMD framework, NLL level first global fit (includes Drell-Yan and Z^0)	✔	✔	8156

→ talk Bacchetta

What do we know about $D_1(z, P_{hT})$?

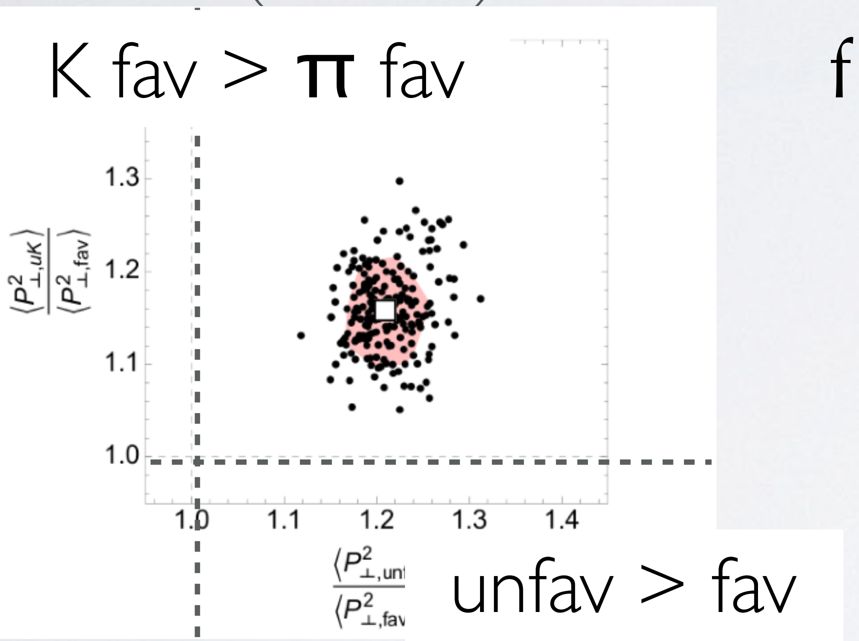
1. does $\mathbf{P}_{hT}$ dependence change with flavor ?

What do we know about $D_1(z, P_{hT})$?

I. does P_{hT} dependence change with flavor ?

Pavia 2013

(Hermes)



$\chi^2/\text{dof} =$ **1.63**

Torino 2014

(Hermes)

flavor indep. → $\chi^2/\text{dof} =$ **1.69**

unfav > fav → $\chi^2/\text{dof} =$ **1.60**

A. Signori, talk at QCD-N16 (Bilbao)

Pavia 2016

(global)

flavor indep.

global $\chi^2/\text{dof} \sim$ **1.55**

(flavor dep. in progress)

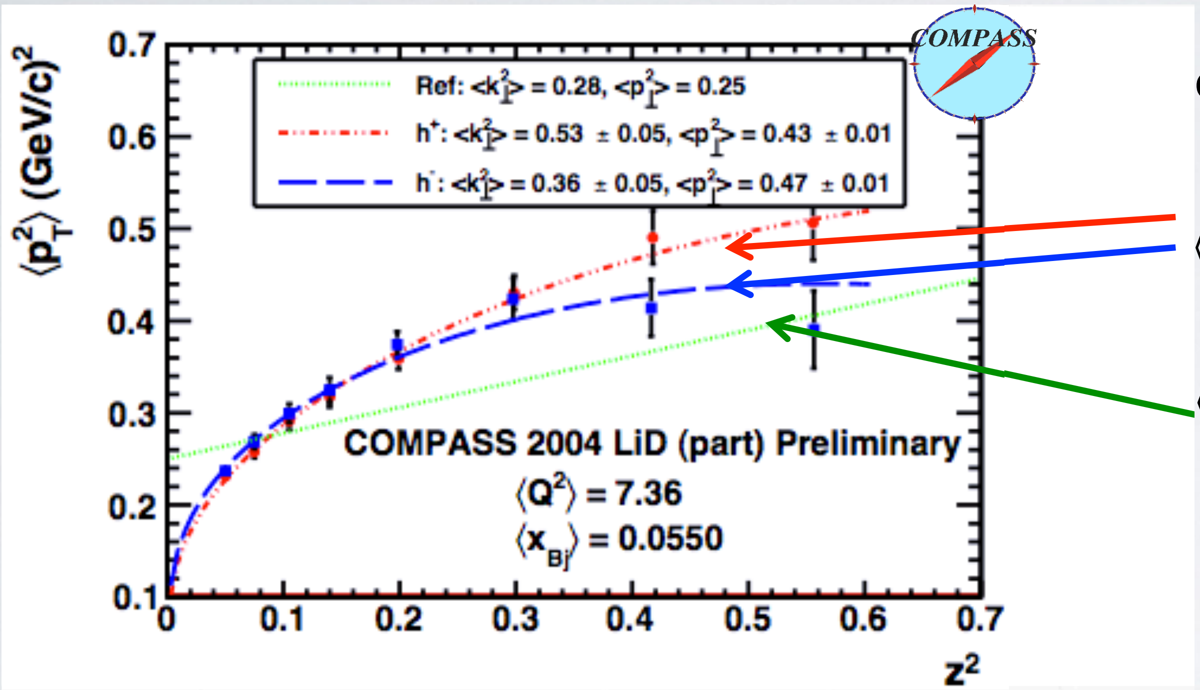
Answer : maybe…

What do we know about $D_1 (z, P_{hT})$?

2. does $\mathbf{P}_{hT}$ dependence change with z ?

What do we know about $D_1(z, P_{hT})$?

2. does P_{hT} dependence change with z ?



Adolph et al., E.P.J. ***C73*** *(13) 2531*
C. Marchand, talk at DIS2011

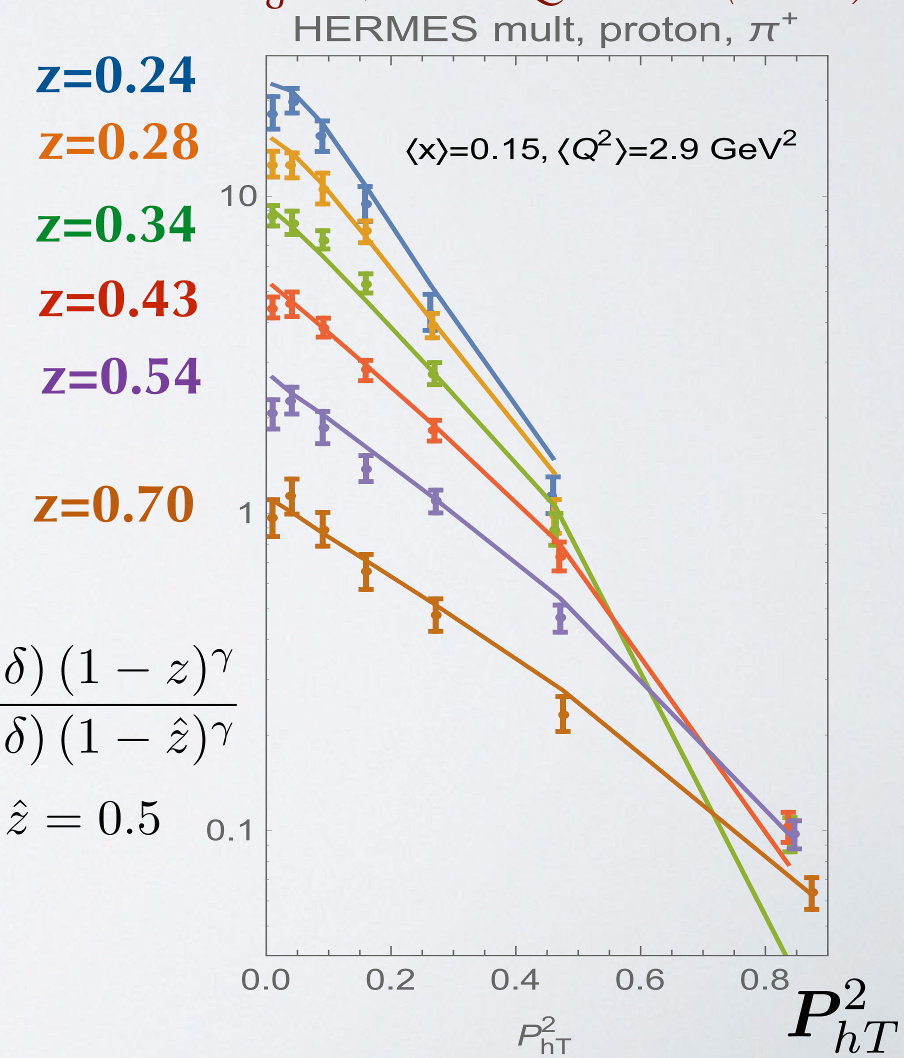
detected hadron — fragm. parton — initial parton

$$\langle \boldsymbol{P}_{hT}^2\rangle = z^\alpha (1-z)^\beta \langle \boldsymbol{P}_\perp^2\rangle + z^2 \langle \boldsymbol{k}_\perp^2\rangle \qquad \alpha=0.5,\ \beta=1.5$$

$$\langle \boldsymbol{P}_{hT}^2\rangle = \langle \boldsymbol{P}_\perp^2\rangle + z^2 \langle \boldsymbol{k}_\perp^2\rangle$$

Gaussian

A. Signori, talk at QCD-N16 (Bilbao)



Answer: it is likely..

Pavia 2016

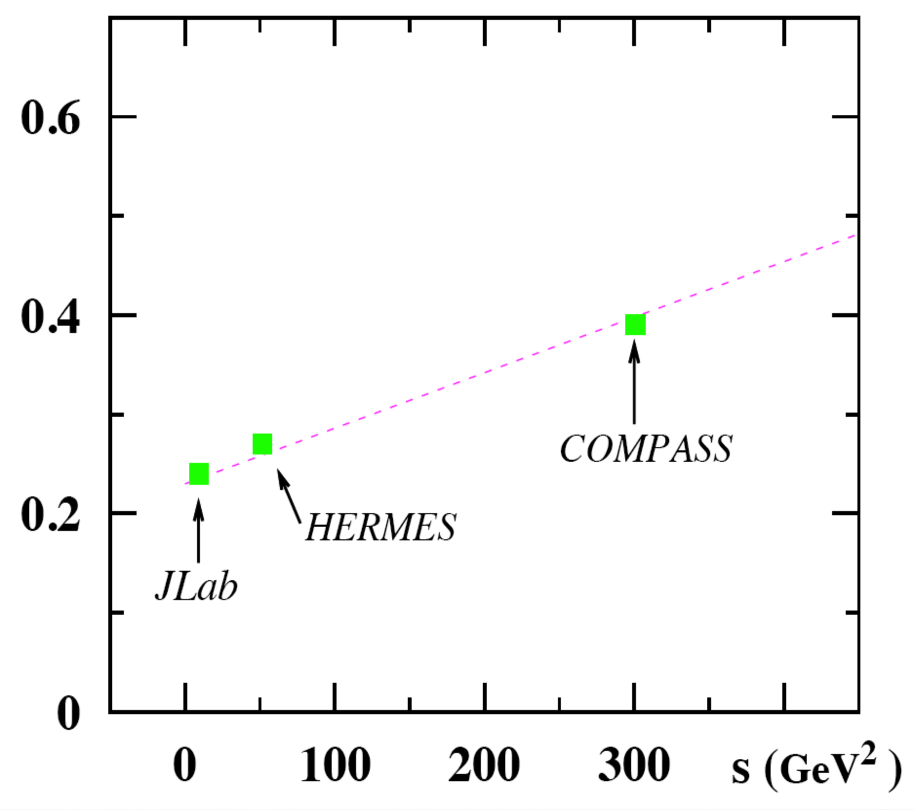
$$\langle \boldsymbol{P}_{hT}^2\rangle(z) = \langle \widehat{\boldsymbol{P}_{hT}^2}\rangle \frac{(z^\beta+\delta)(1-z)^\gamma}{(\hat{z}^\beta+\delta)(1-\hat{z})^\gamma}$$

$\beta = 2.68 \pm 0.08$ $\quad \hat{z} = 0.5$
$\delta = 3.36 \pm 0.12$
$\gamma = 0.04 \pm 0.004$

What do we know about $D_1(z, P_{hT})$?

3. does P_{hT} dependence change with energy $\sqrt{s}$?

average width of
transverse momentum
Gaussian distributions



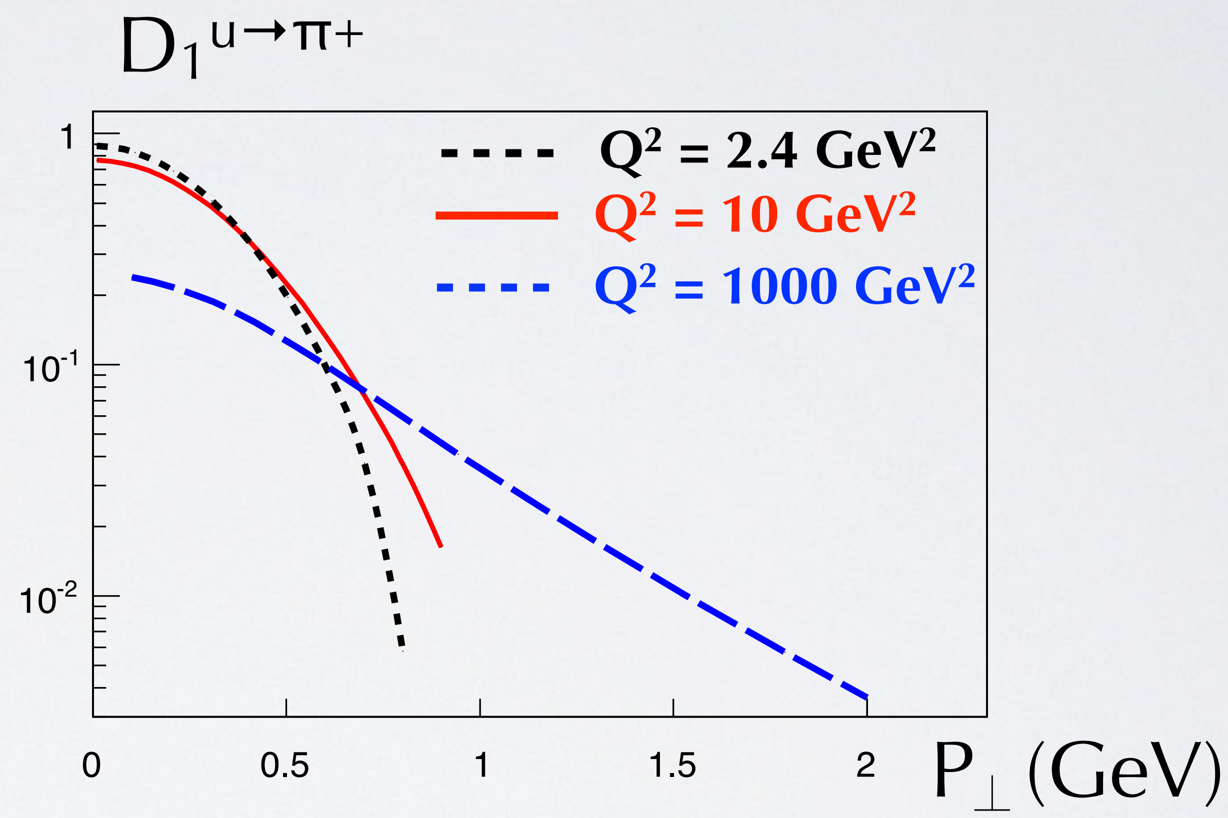
Schweitzer, Teckentrup, Metz,
P.R. D81 (10) 094019

Answer: it is likely,
but need processes at much higher s → e^+e^-

What do we know about $D_1(z, P_{hT})$?

4. does P_{hT} dependence change with scale Q^2 ?

effects of
TMD evolution



*Kang, Prokudin, Sun, Yuan, P.R. D**93** (16) 014009*

strong dependence predicted at much larger scales

What do we know about $D_1(z, P_{hT})$?

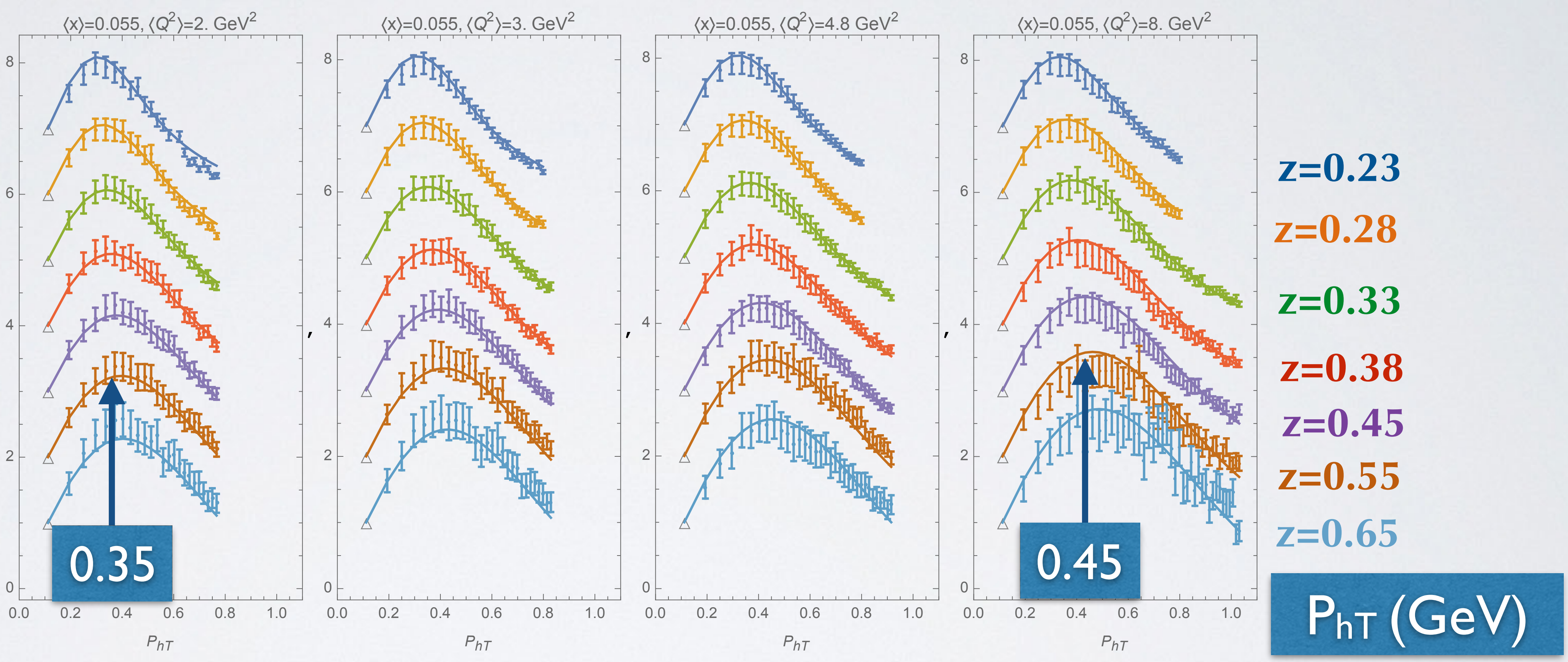
4. does P_{hT} dependence change with scale Q^2 ?

COMPASS $D(e,e' h^+)$

Pavia 2016

A. Signori, talk at QCD-N16 (Bilbao)

$\langle x \rangle = 0.055$
$\chi^2/\text{dof} = 1.49$



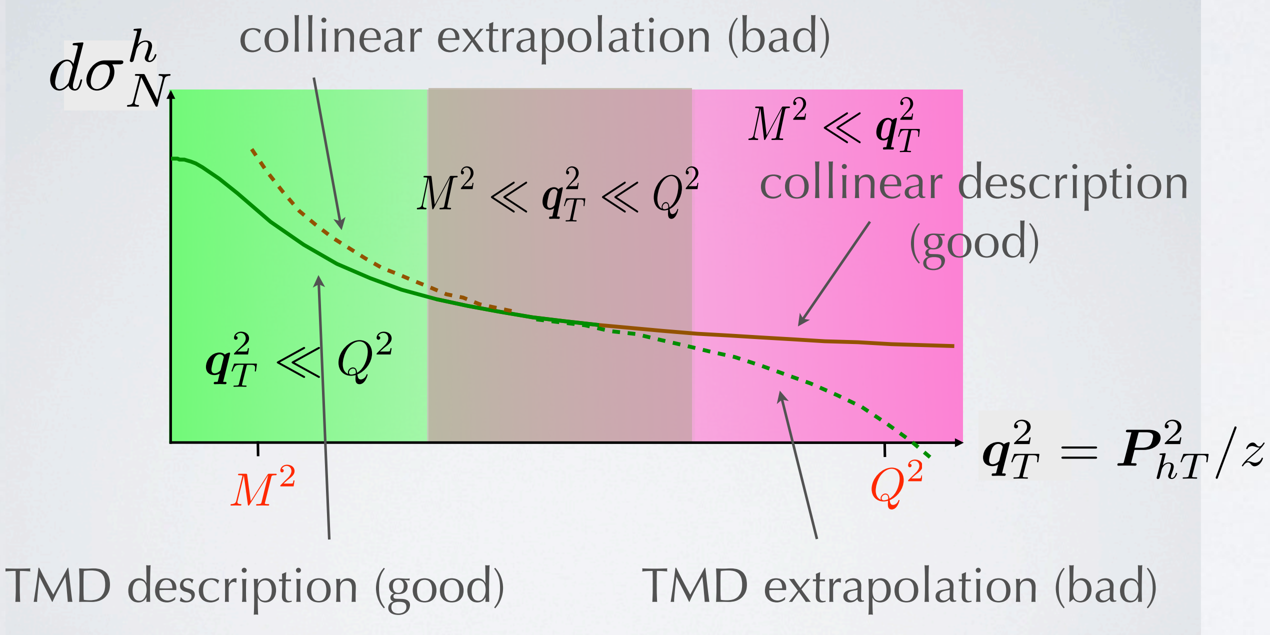
$\langle Q^2 \rangle = 2 \rightarrow \ \ldots \ \ldots \rightarrow 8 \ \text{GeV}^2$

Answer: at SIDIS scales, very moderately

The matching problem in SIDIS

SIDIS unpolarized cross section
$$\frac{d\sigma_N^h}{dx\,dz\,d\boldsymbol{P}_{hT}^2\,dQ^2} \approx \sum_q e_q^2\,[f_1^q \otimes D_1^q]\,\mathcal{H}_q(Q^2) + Y(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

$$\boldsymbol{q}_T^2 = \boldsymbol{P}_{hT}^2/z$$



need to match collinear (fixed-order) description such that

$$\int_0^\infty d\boldsymbol{P}_{hT}^2\,\frac{d\sigma_N^h}{dx\,dz\,d\boldsymbol{P}_{hT}^2\,dQ^2} = \frac{d\sigma_N^h}{dx\,dz\,dQ^2}$$

➡ talk Gamberg

The matching problem in SIDIS

SIDIS unpolarized cross section

$$\frac{d\sigma_N^h}{dx\,dz\,d\boldsymbol{P}_{hT}^2\,dQ^2} \approx \sum_q e_q^2\,[f_1^q \otimes D_1^q]\,\mathcal{H}_q(Q^2) + Y(Q^2,\boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

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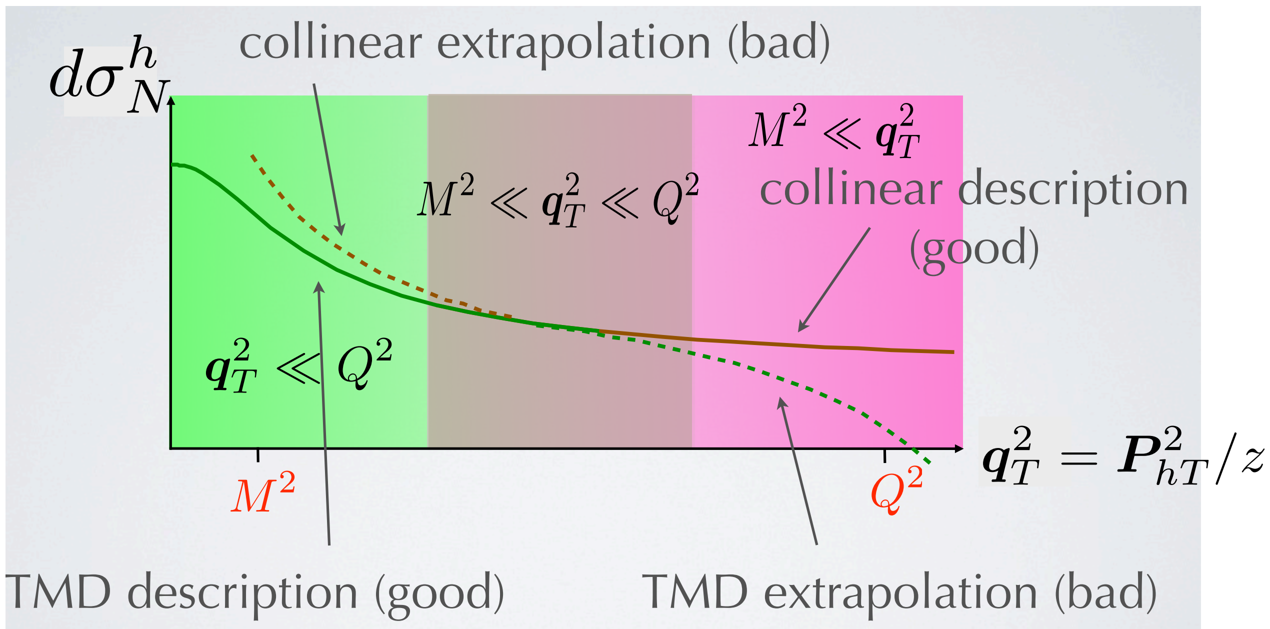

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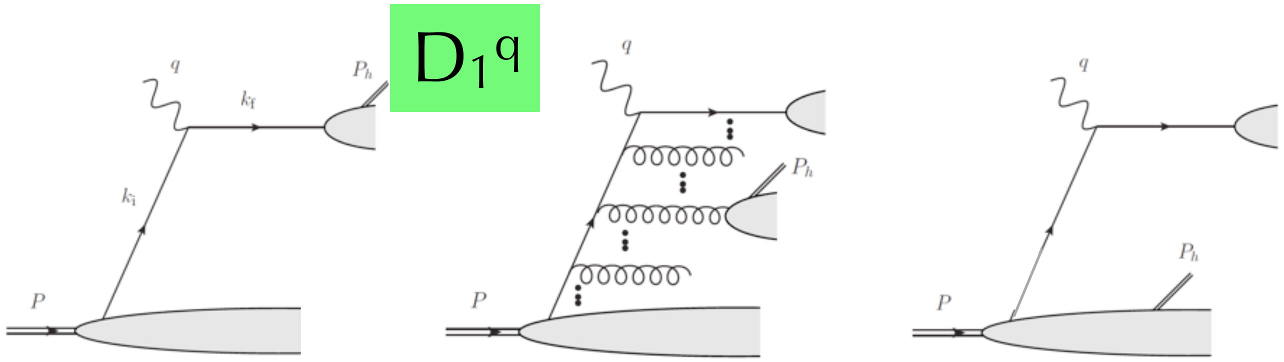
$$\int_0^\infty d\boldsymbol{P}_{hT}^2\,\frac{d\sigma_N^h}{dx\,dz\,d\boldsymbol{P}_{hT}^2\,dQ^2} = \frac{d\sigma_N^h}{dx\,dz\,dQ^2}$$

➡ talk Gamberg

Moreover, we need to check that we are in the current fragm. region in order to define $D_1{}^q$



The matching problem in SIDIS

factorization th.'s for (current) fragmentation and (target) fracture functions assume that current and target regions are well separated in rapidity $y_h = \frac{1}{2} \log \frac{P_h^+}{P_h^-}$

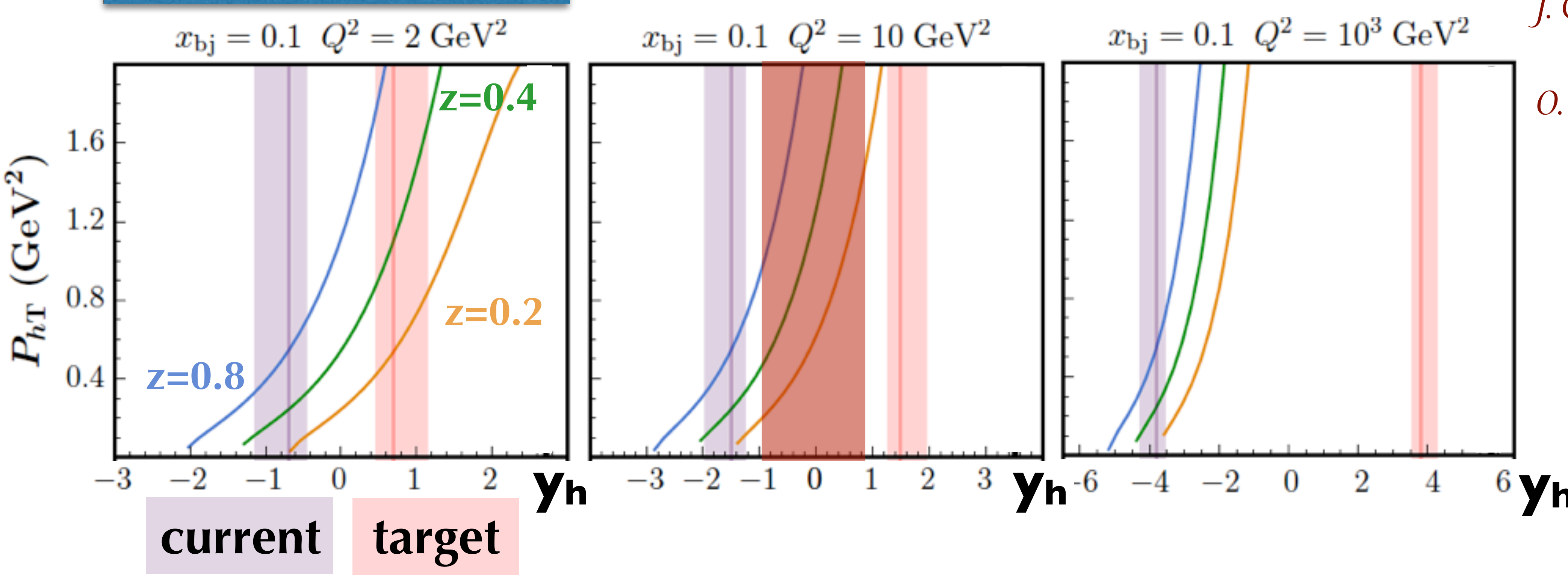
at hermes COMPASS it's true for few P_{hT}

at JLab12 still Y term is relevant

at EIC it's ok

J. Collins, arXiv:1610.09994

O. Gonzalez, talk at POETIC16



The matching problem in SIDIS

factorization th.'s for (current) fragmentation and (target) fracture functions
assume that current and target regions are well separated in rapidity $y_h = \frac{1}{2} \log \frac{P_h^+}{P_h^-}$

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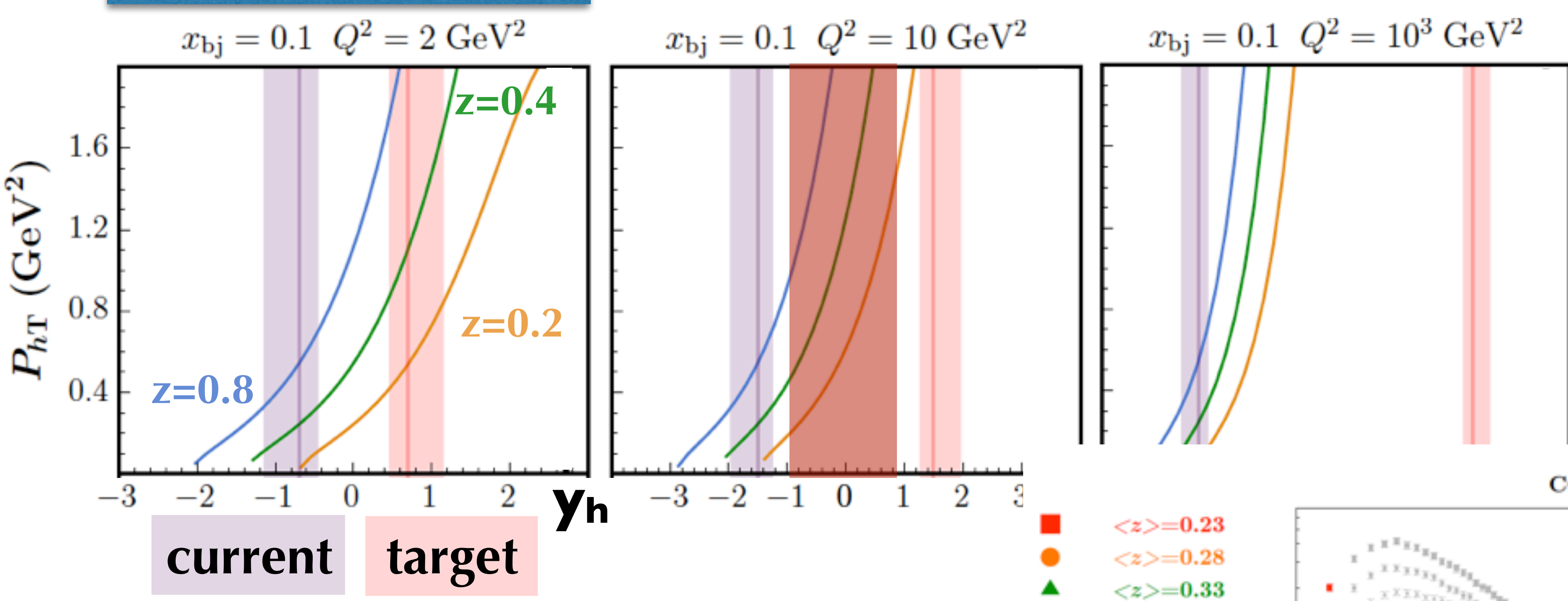
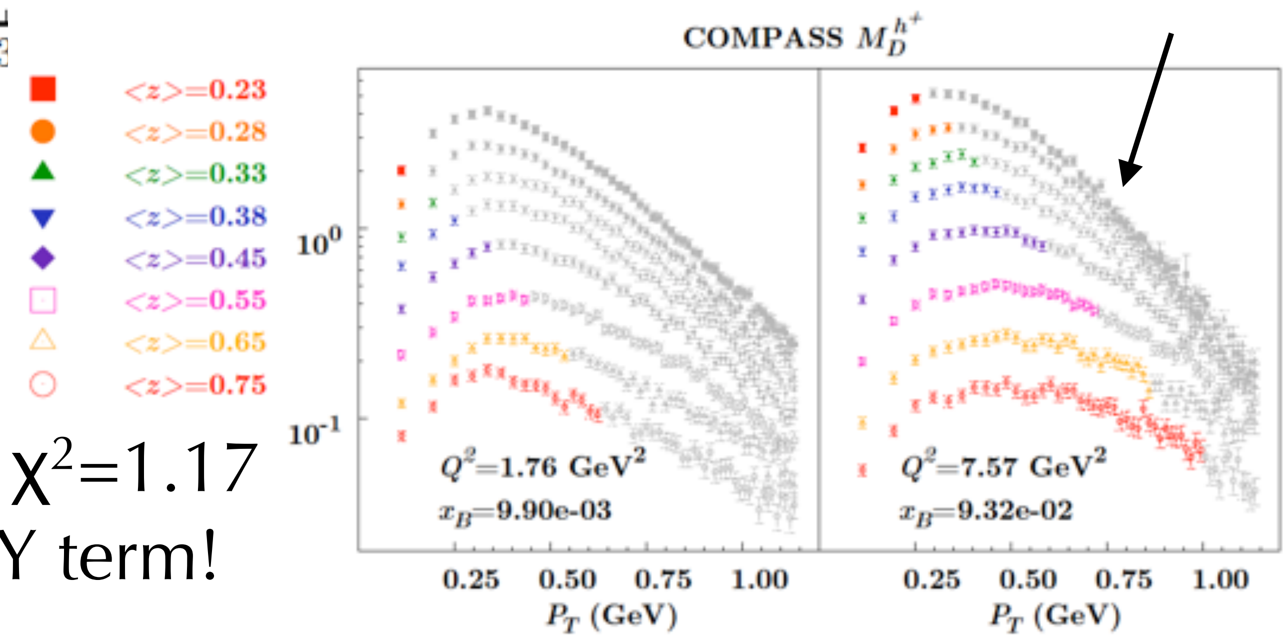


grey points are outside cut



define cut in rapidity

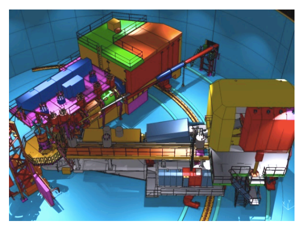
Example: χ^2=1.17 with no Y term!

D_1 from unintegrated SIDIS multiplicities

Future

$$M_N^h = \frac{d\sigma_N^h / dx\, dz\, d\boldsymbol{P}_{hT}^2\, dQ^2}{d\sigma_{\mathrm{DIS}} / dx\, dQ^2} \approx \frac{\sum_q e_q^2 \, [f_1^q \otimes D_1^q]\,(x, z, \boldsymbol{P}_{hT}^2; Q^2)}{\sum_q e_q^2 \, f_1^q(x; Q^2)}$$

$$\boldsymbol{P}_{hT}^2 / z \ll Q^2$$



SHMS

Hall C : E12-09-017 $H_2/D_2(e,e'\pi)$

will test factorization hypothesis

Hall B
- E12-06-112 $H_2(e,e'\pi)$
- E12-09-007 $D_2(e,e'\pi/K)$

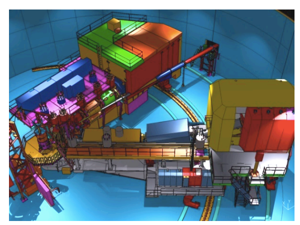
→ talk Rossi

D_1 from unintegrated SIDIS multiplicities

Future

$$M_N^h = \frac{d\sigma_N^h / dx\, dz\, d\boldsymbol{P}_{hT}^2\, dQ^2}{d\sigma_{\mathrm{DIS}}/dx\, dQ^2} \approx \frac{\sum_q e_q^2\, [f_1^q \otimes D_1^q]\,(x, z, \boldsymbol{P}_{hT}^2; Q^2)}{\sum_q e_q^2\, f_1^q(x; Q^2)}$$

$$\boldsymbol{P}_{hT}^2/z \ll Q^2$$



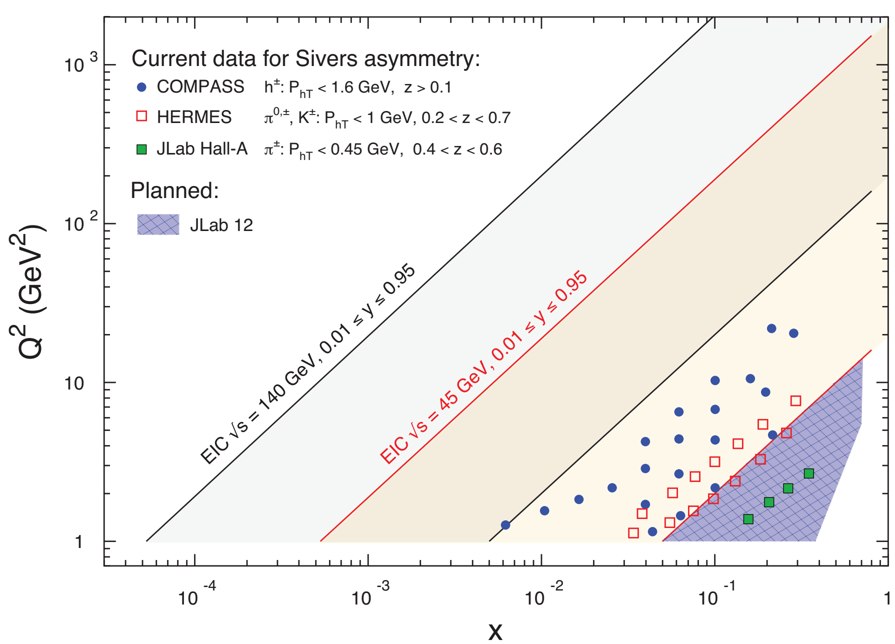
SHMS

Hall C : E12-09-017 $H_2/D_2(e,e'\pi)$

will test factorization hypothesis

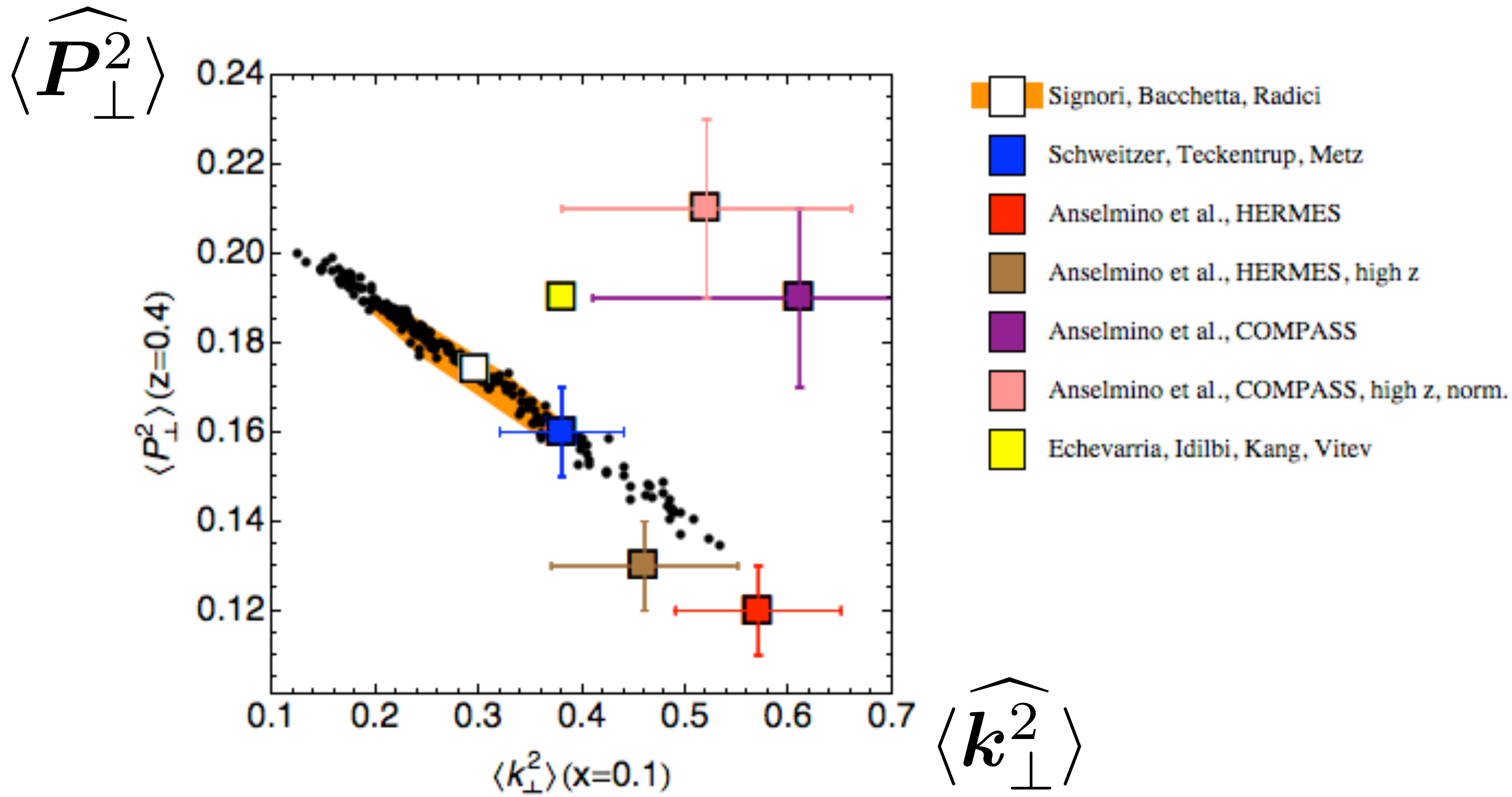
Hall B
- E12-06-112 $H_2(e,e'\pi)$
- E12-09-007 $D_2(e,e'\pi/K)$

→ talk Rossi



large coverage in (x,Q^2)

Another problem: SIDIS anticorrelations



Pavia 2013

*Bacchetta et al., JHEP **1311** (13) 194*

*Schweitzer, Teckentrup, Metz, P.R.D**81** (10) 094019*

Torino 2014 *Anselmino et al., JHEP **1404** (14) 005*

Hermes

" (high z)

Compass

" (high z)

EIKV 2014

*Echevarria et al., P.R.D**89** (14) 074013*

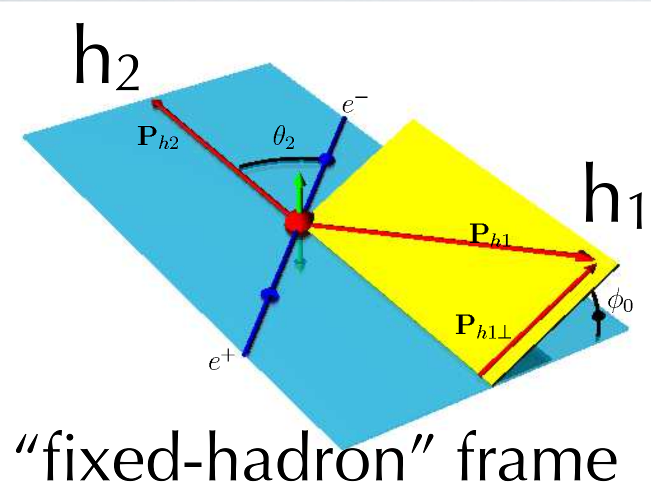
$D_1(z, P_{hT})$ from e^+e^-

1. only way to break anticorrelation in SIDIS

2. we need large e^+e^- scales ≫ SIDIS scales to study how $\langle \mathbf{P}_{hT}^2 \rangle$ changes with Q^2 and s

e^+e^- unintegrated multiplicity

$e^+ e^- \to h_1\, h_2\, X$

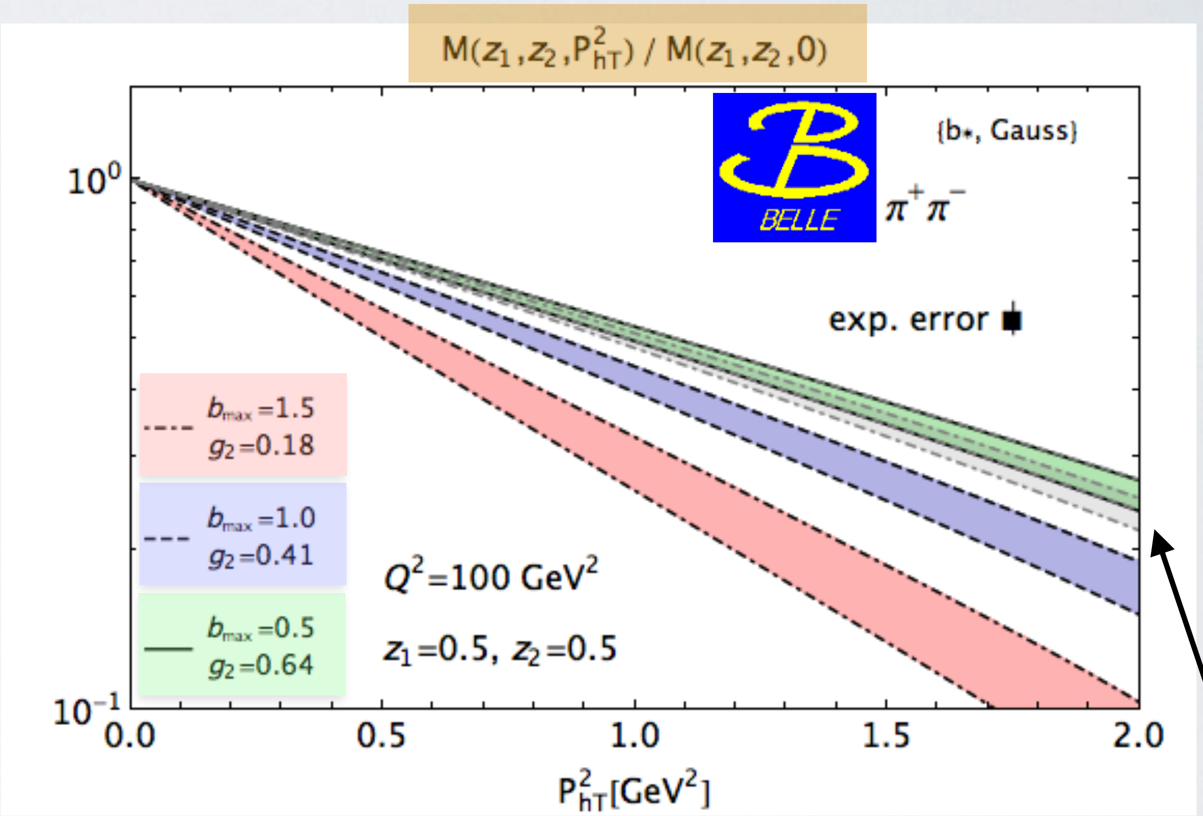


$$M = \frac{d\sigma^{h_1 h_2}/dz_1\, dz_2\, d\boldsymbol{P}^2_{h_1 T}\, dy}{d\sigma_{\rm incl}/dz_2\, dy} \approx \frac{\sum_q e_q^2\, [D_1^q \otimes D_1^{\bar q}](z_1, z_2, \boldsymbol{P}^2_{h_1 T}; Q^2)}{\sum_q e_q^2 D_1^q(z_2; Q^2)}$$

$$\frac{\boldsymbol{P}^2_{h_1 T}}{z_1} \ll Q^2$$

$$D_1^q(z, \boldsymbol{b}_T; Q^2) = R(Q^2, \mu_b^2(b_T, b_{\rm max}))\, e^{-\frac{1}{2} g_2 b_T^2 \log\frac{Q}{Q_0}}\, D_1^q(z, \boldsymbol{b}_T; \mu_b)$$

parameters of nonperturbative evolution



Bacchetta et al., JHEP ***1511*** *(15) 076*

sensitivity to C_1 in $\mu_b = C_1 / b_T$: $C_1/2 < C_1 < 2\, C_1$

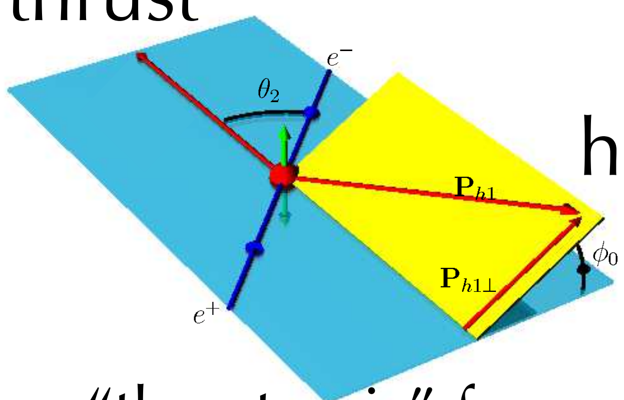
a 7% error at Belle scale can constrain nonperturbative parameters $\{b_{max}, g_2\}$

e^+e^- cross section

$e^+ e^- \rightarrow h\,X$

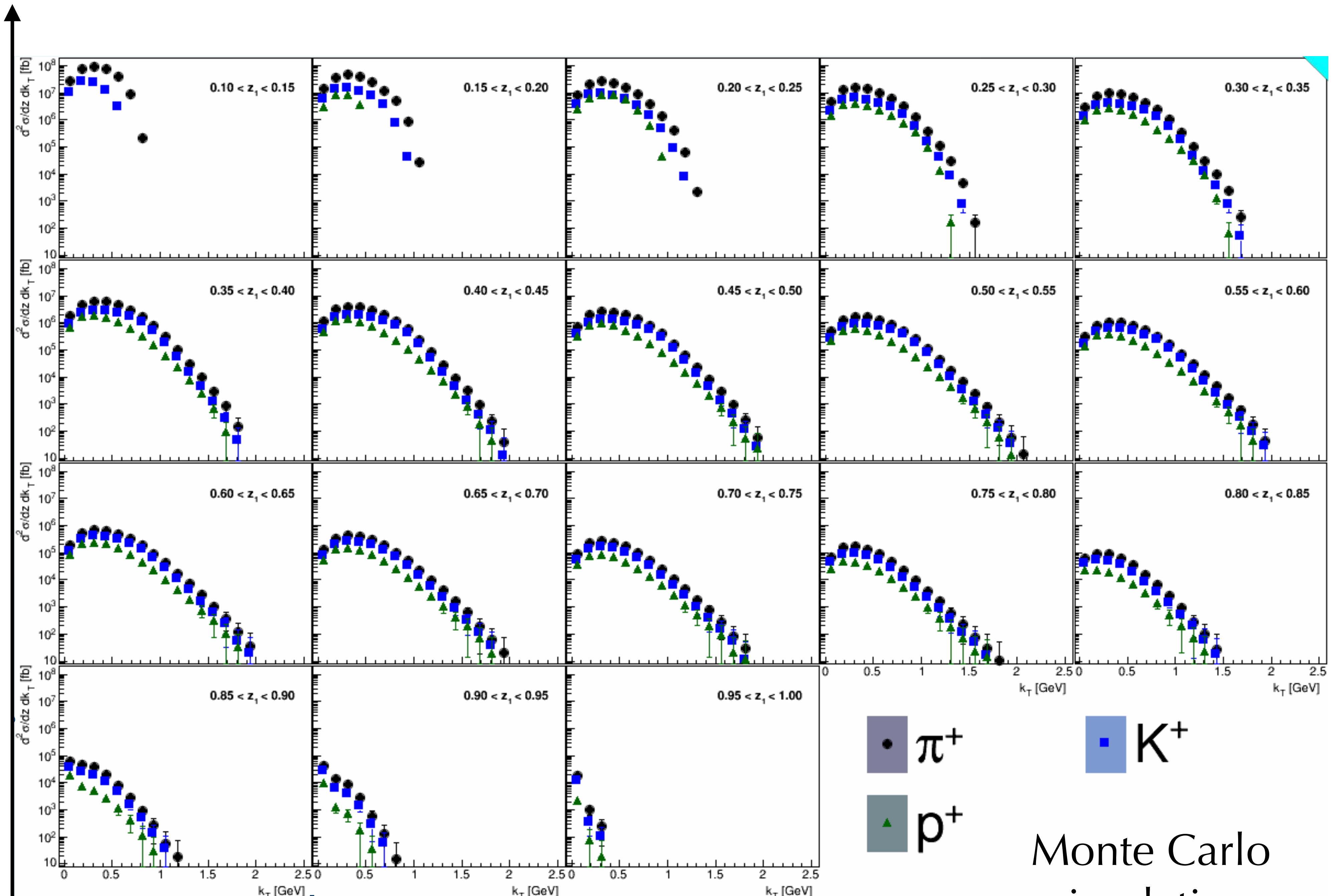
thrust

h



"thrust-axis" frame

$$\frac{d\sigma}{dz\,d\boldsymbol{P}_{hT}}$$



$|\boldsymbol{P}_{hT}|$

Monte Carlo simulation

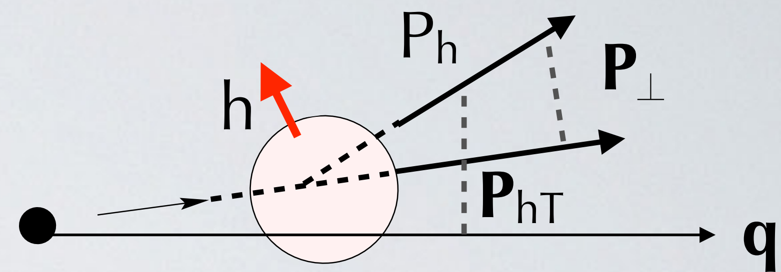
R. Seidl, talk at SPIN2016

upcoming Belle data for unintegrated e^+e^- cross section

Access to $D_{1T}^{\perp}$

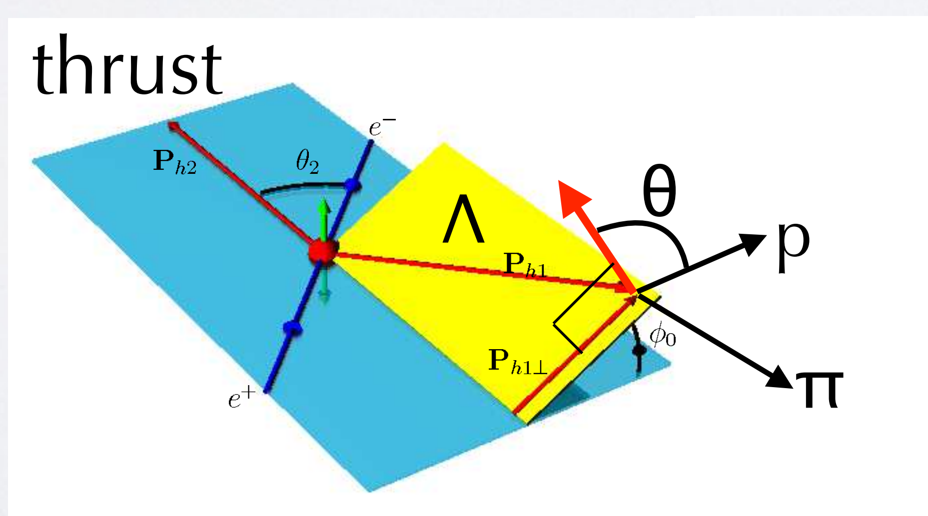
Nucleon Polarization		Quark polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
	U	D_1 Unpolarized		$H_1^{\perp}$ Collins
	L		G_{1L}	$H_{1L}^{\perp}$
	T	$D_{1T}^{\perp}$	G_{1T}	H_1 $H_{1T}^{\perp}$

$D_{1T}^{\perp}(z, \mathbf{P}_{hT})$



encodes "spontaneous" polarization of h

new data from Belle on
$e^+ e^- \to \Lambda/\bar{\Lambda} + X$
$e^+ e^- \to \Lambda/\bar{\Lambda} + \pi/K + X$
with full $(z_\Lambda, P_{\Lambda T})$ dependence

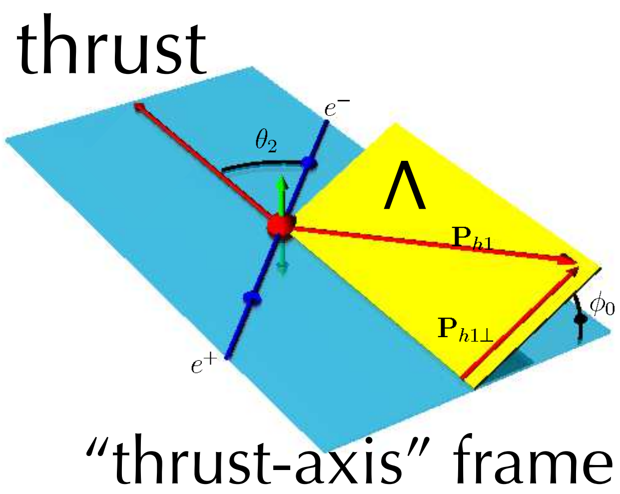


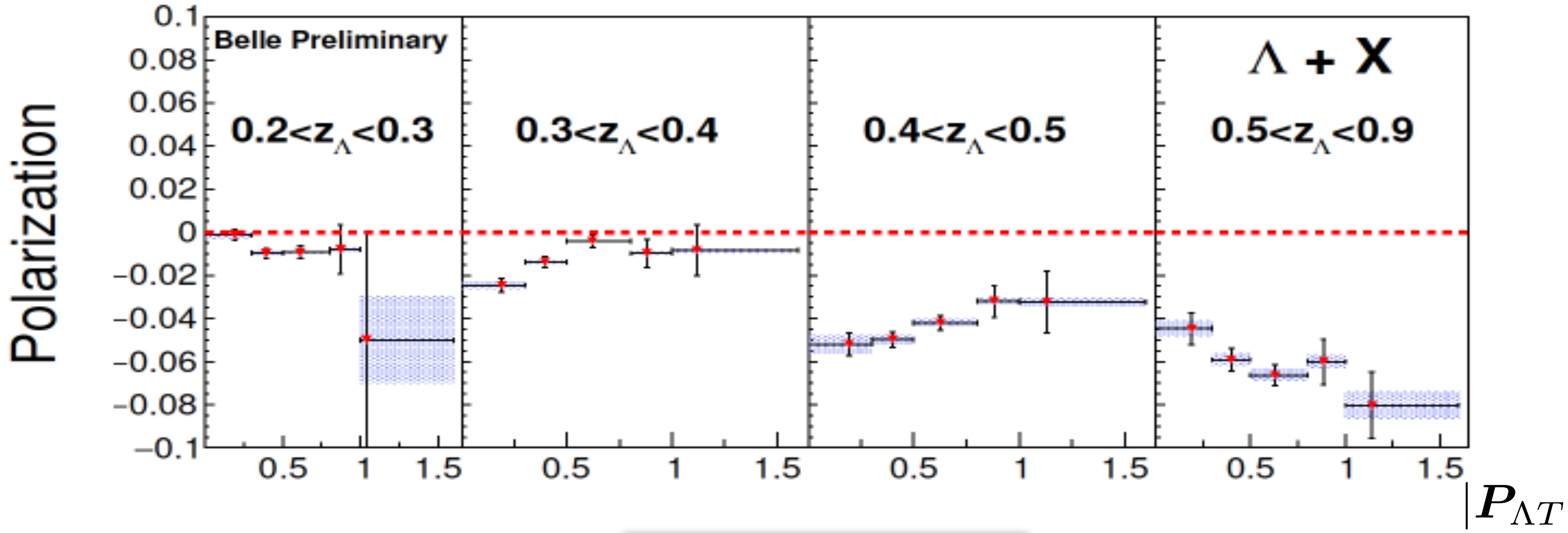
polarization *P*

$$\frac{1}{N}\frac{dN}{d\cos\theta} = 1 + \alpha\, P\, \cos\theta$$

Λ polarization data

$e^+ e^- \to \Lambda + X$





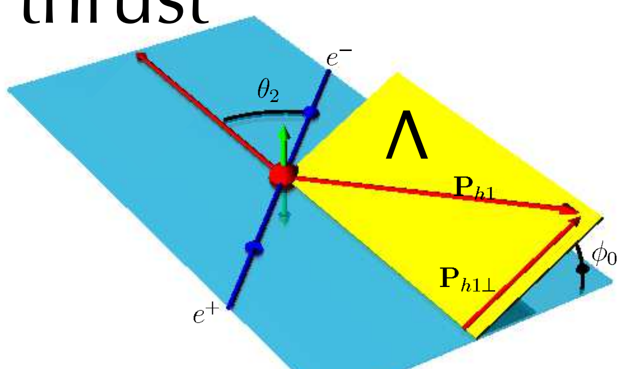
rising |P| decreasing |P| rising |P|

Y. Guan, talk at SPIN2016

Λ polarization data

$e^+ e^- \rightarrow \Lambda + X$

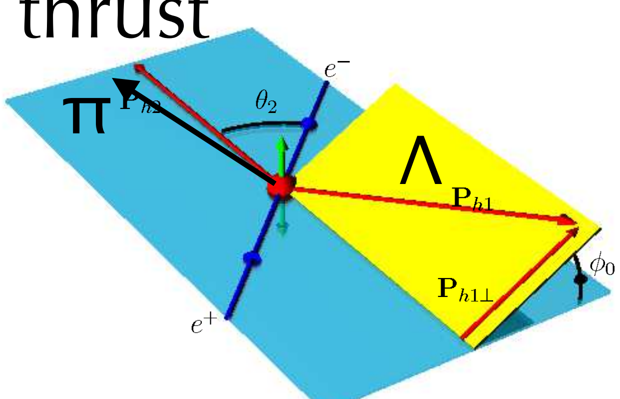
thrust



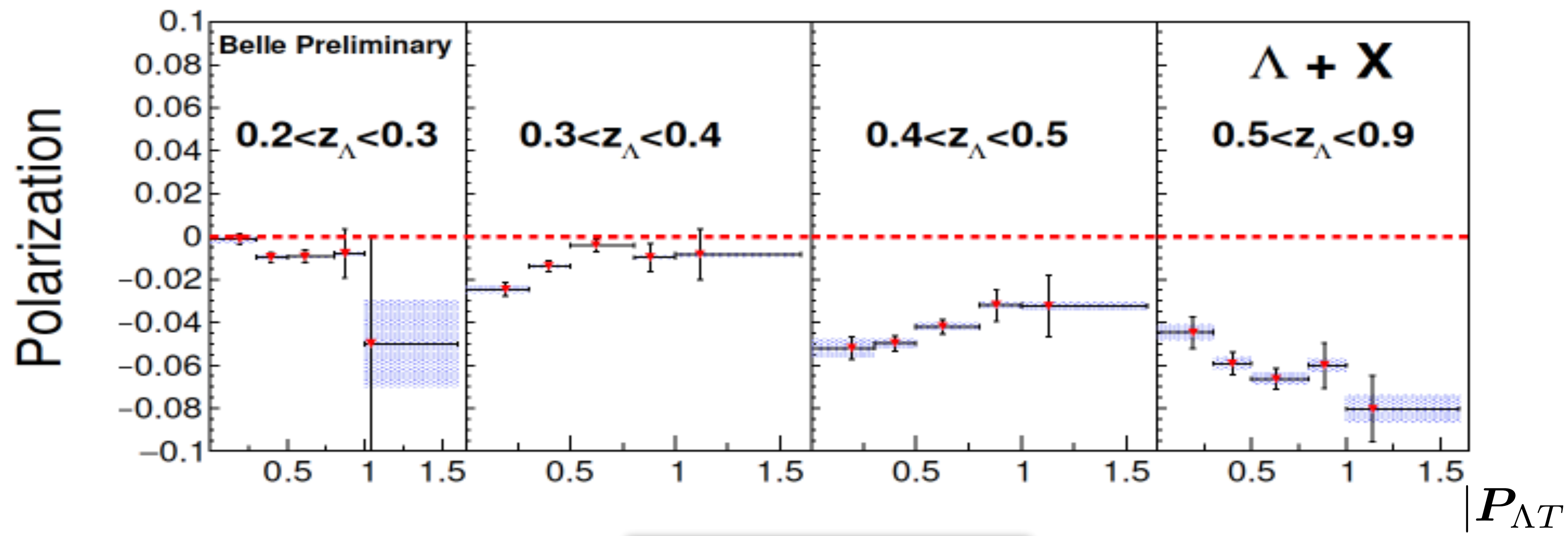
“thrust-axis” frame

Y. Guan, talk at SPIN2016

thrust



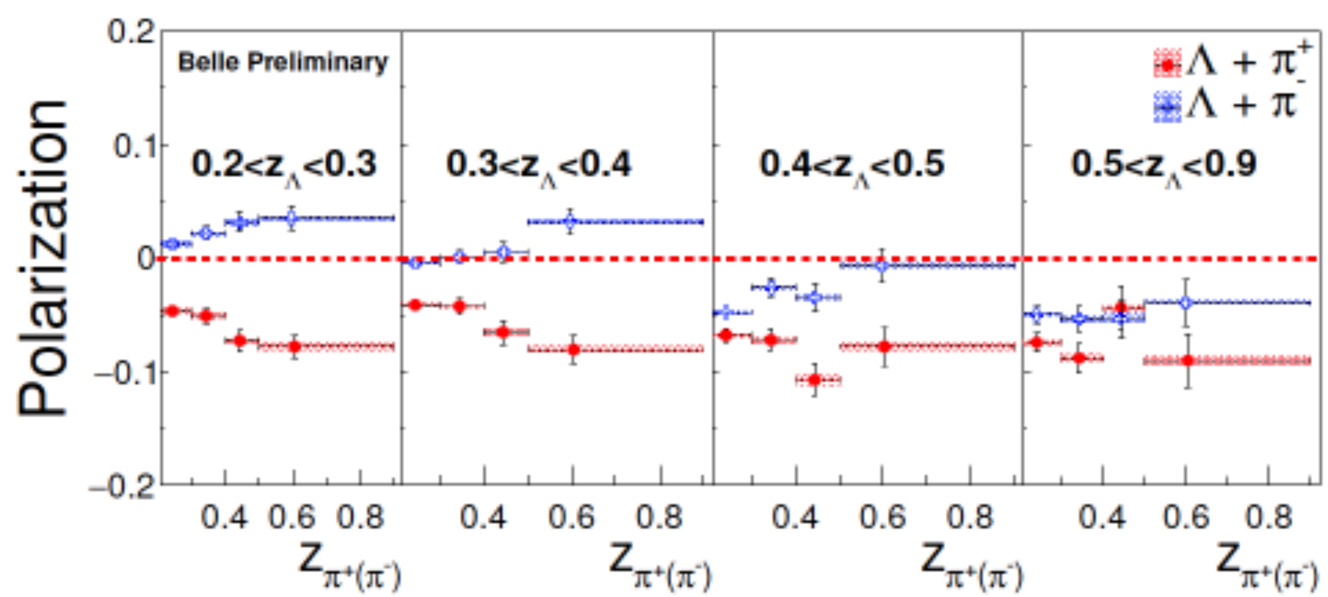
“thrust-axis” frame



rising |P|

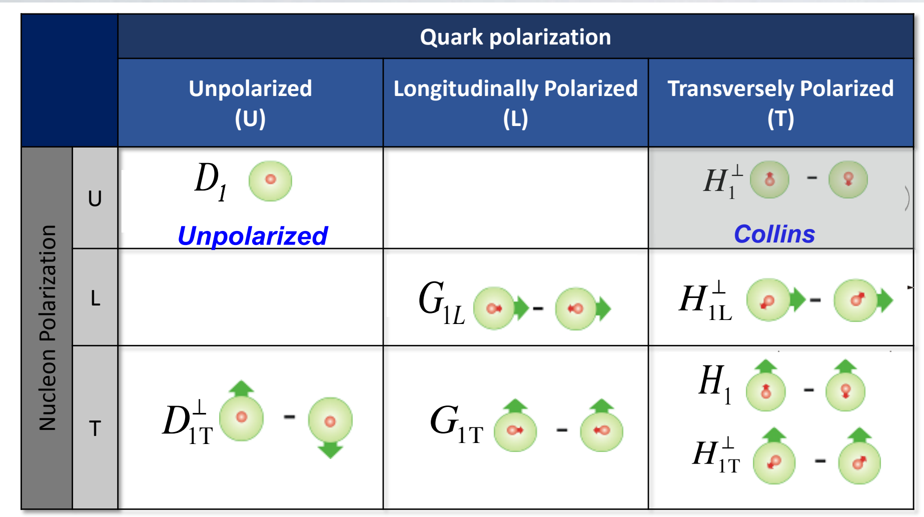
decreasing |P|

rising |P|



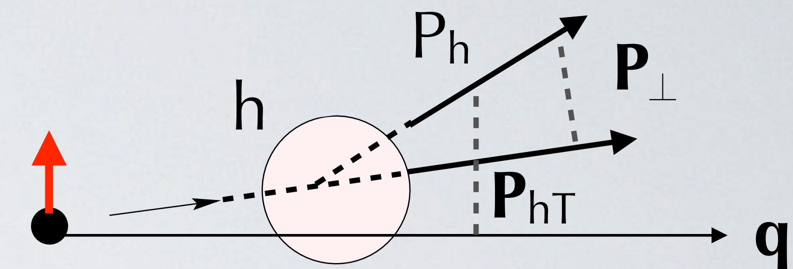
at large z_Λ polarization changes sign

Collins function



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 *Unpolarized*		$H_1^{\perp}$ - *Collins*
	L		G_{1L} -	$H_{1L}^{\perp}$ -
	T	$D_{1T}^{\perp}$ -	G_{1T} -	H_1 - $H_{1T}^{\perp}$ -

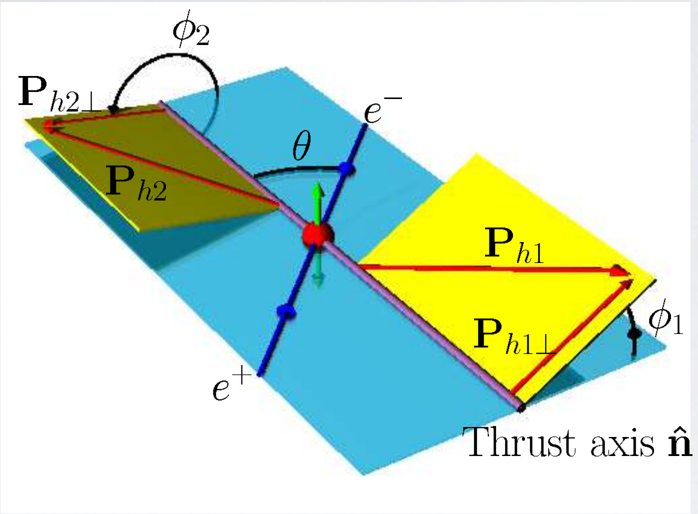
$$H_1^{\perp}(z, \mathbf{P}_{hT})$$



What do we know about the $\mathbf{P}_{hT}$ dependence ?

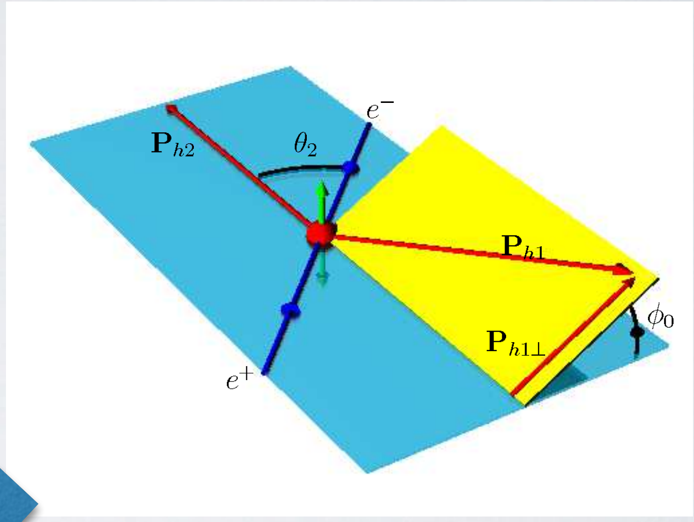
e⁺e⁻ Collins effect

"thrust axis" frame : A_{12}



not obvious QCD generalization of TMD factorization formula because of thrust axis definition

"fixed hadron" frame : A_0



$$A_0^{e^+e^-} \sim \frac{\sin^2\theta}{1+\cos^2\theta}\,\cos 2\phi_0\,\frac{\sum_q e_q^2\left[H_{1,q}^{\perp}(z_1,\boldsymbol{p}_{1T}^2)\otimes H_{1,\bar{q}}^{\perp}(z_2,\boldsymbol{p}_{2T}^2)\right](z_1,z_2,\boldsymbol{P}_{1\perp}^2)}{\sum_q e_q^2\left[D_1^q(z_1,\boldsymbol{p}_{1T}^2)\otimes D_1^{\bar{q}}(z_2,\boldsymbol{p}_{2T}^2)\right](z_1,z_2,\boldsymbol{P}_{1\perp}^2)}$$

$$\ldots\otimes\ldots\to\int d\boldsymbol{p}_{1T}\,d\boldsymbol{p}_{2T}\,\delta\left(\boldsymbol{p}_{1T}+\boldsymbol{p}_{2T}+\frac{\boldsymbol{P}_{1\perp}}{z_1}\right)\ldots$$

$$R_{\rm exp}\equiv\frac{d\sigma}{d\sigma_0}$$

Unlike | Like

$\frac{\text{Unlike-sign}}{\text{Like-sign}}$ $$\frac{R^U_{\rm exp}}{R^L_{\rm exp}}\approx 1+A_0^{e^+e^-}\left(\frac{\pi^+\pi^-+\pi^-\pi^+}{\pi^+\pi^-+\pi^-\pi^+}\right)-A_0^{e^+e^-}\left(\frac{\pi^+\pi^++\pi^-\pi^-}{\pi^+\pi^++\pi^-\pi^-}\right)$$

$\frac{\text{Unlike-sign}}{\text{Charged}}$ $$\frac{R^U_{\rm exp}}{R^C_{\rm exp}}\approx 1+A_0^{e^+e^-}\left(\frac{\pi^+\pi^-+\pi^-\pi^+}{\pi^+\pi^-+\pi^-\pi^+}\right)-A_0^{e^+e^-}\left(\frac{\text{all}\quad\pi\pi}{\text{all}\quad\pi\pi}\right)$$

to kill false asymmetries

Data for e^+e^- Collins effect

BELLE

$s = Q^2 = 112\ GeV^2$

Abe et al., P.R.L. ***96*** *(06) 232002*
*Seidl et al., P.R. D****78*** *(08) 032011*
*D****86*** *(12) 039905(E)*

$A_{12}^{U/L/C}(z_1, z_2)$

$A_0^{U/L/C}(z_1, z_2)$

BABAR

$s = Q^2 = 112\ GeV^2$

*Lees et al., P.R. D****90*** *(14) 052003*

$A_{12}^{U/L/C}(z_1, z_2, P_{1T}, P_{2T})$

$A_0^{U/L/C}(z_1, z_2, P_{1T})$

*Lees et al., P.R. D****92*** *(15) 111101*

$A_{12}^{U/L/C}(z_1, z_2)$

$A_0^{U/L/C}(z_1, z_2)$

KK and Kπ pairs

BESIII

$s = Q^2 = 13\ GeV^2$

Ablikim et al., P.R.L. ***116*** *(16) 042001*

$A_0^{U/L/C}(z_1, z_2, P_{1T})$

Data for e^+e^- Collins effect

Abe et al., P.R.L. ***96*** *(06) 232002*
*Seidl et al., P.R. D****78*** *(08) 032011*
*D****86*** *(12) 039905(E)*

$A_{12}^{U/L/C}(z_1, z_2)$

$A_0^{U/L/C}(z_1, z_2)$

$s = Q^2 = 112$ GeV^2

---

*Lees et al., P.R. D****90*** *(14) 052003*

$A_{12}^{U/L/C}(z_1, z_2, P_{1T}, P_{2T})$

$A_0^{U/L/C}(z_1, z_2, P_{1T})$

*Lees et al., P.R. D****92*** *(15) 111101*

$A_{12}^{U/L/C}(z_1, z_2)$

$A_0^{U/L/C}(z_1, z_2)$

KK and Kπ pairs

$s = Q^2 = 112$ GeV^2

Ablikim et al., P.R.L. ***116*** *(16) 042001*

$A_0^{U/L/C}(z_1, z_2, P_{1T})$

$s = Q^2 = 13$ GeV^2

phase transition :
direct access to transverse dynamics of fragmenting parton
and to its QCD evolution

e^+e^- Collins effect

available fits of $H_1^\perp$

both perform global fits (SIDIS + e^+e^-) with χ^2/dof in [0.85 - 1.2]

	Framework	Belle	BaBar $A_0(z_1,z_2,P_{1T})$ U / L / C	# points	BaBar A_{12} U / L / C	BESIII $A_0(z_1,z_2,P_{1T})$ U / L / C
Torino 2015 *Anselmino et al., P.R.D92 (15) 114023*	Gaussian, fixed width various params. for fav (z) unfav (z) = N_{unf} D_1(z) **only chiral-odd collinear DGLAP evolution** 5 parameters	✔	✔	122	**predicted**	**predicted**
KPSY 2015 *Kang et al., P.R.D93 (16) 014009*	**TMD evolution in CSS scheme** at NLO + NLL level * $\hat{H}^{(3)}(z) \propto D_1(z)$ fav (z) ≠ unfav (z) only homogeneous evo eqs. 7 parameters	✔	✔	122	✘	**predicted**

$$* \quad H_1^{\perp\, q}(z,\boldsymbol{b}_T;Q^2) = \sum_i \left(\delta C_{q/i} \otimes \hat{H}^{(3)\, i}\right)(z,b_*;\mu_b)\, e^{S(b_*,\mu_b;Q)}\, e^{S_{\rm NP}(b_T;Q)}\, H^q_{\rm NP}(z,\boldsymbol{b}_T;Q_0^2)$$

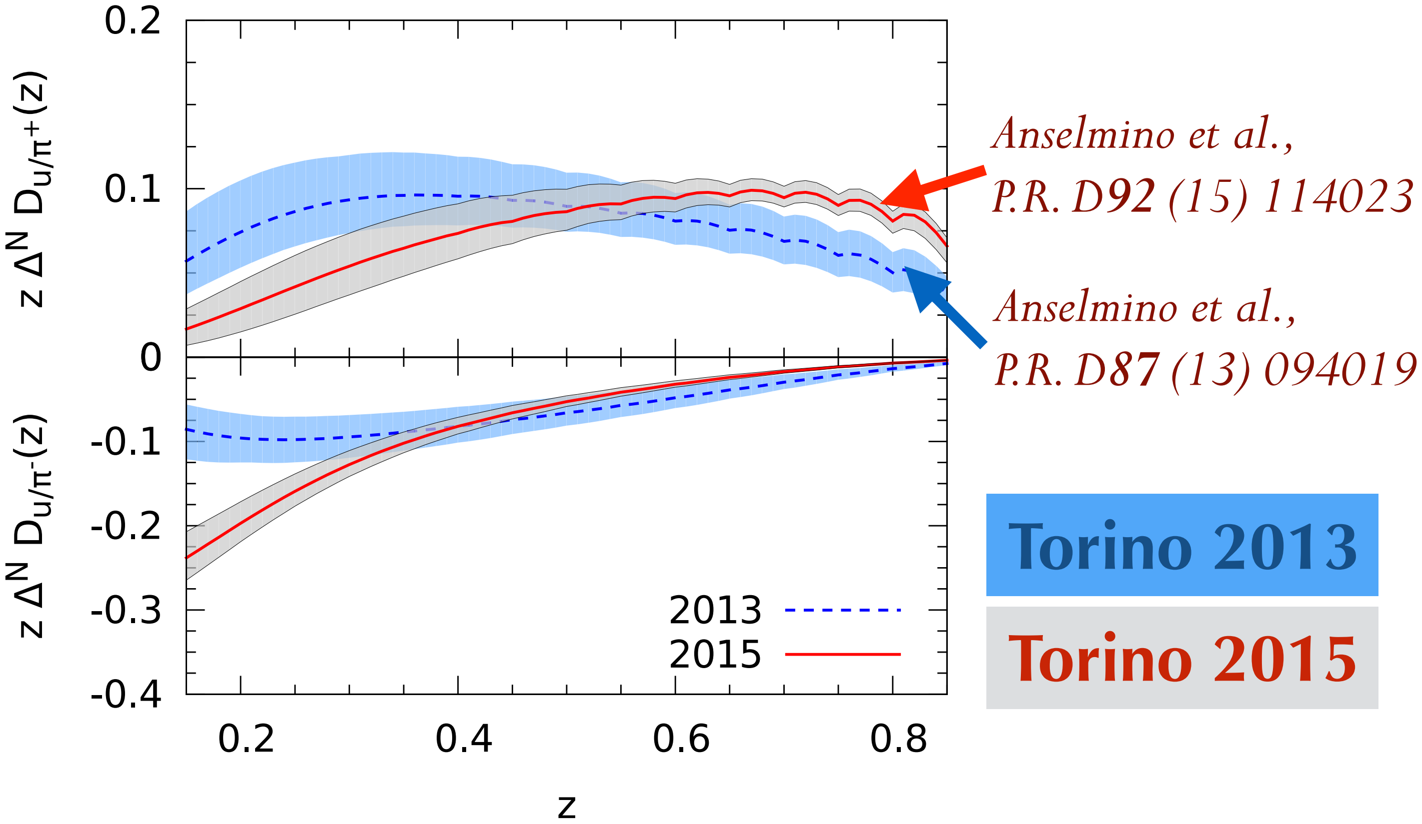
Moments of Collins function : z dep.

favored

$$\Delta^N D_{h/q}(z) = 4\, H_1^{\perp(1/2)q\to h}(z)$$

$$H_1^{\perp(n)}(z) = \int d\boldsymbol{P}_\perp \, \frac{1}{2} \left(\frac{\boldsymbol{P}_\perp^2}{z^2 m_h^2}\right)^n H_1^\perp(z, \boldsymbol{P}_\perp^2)$$

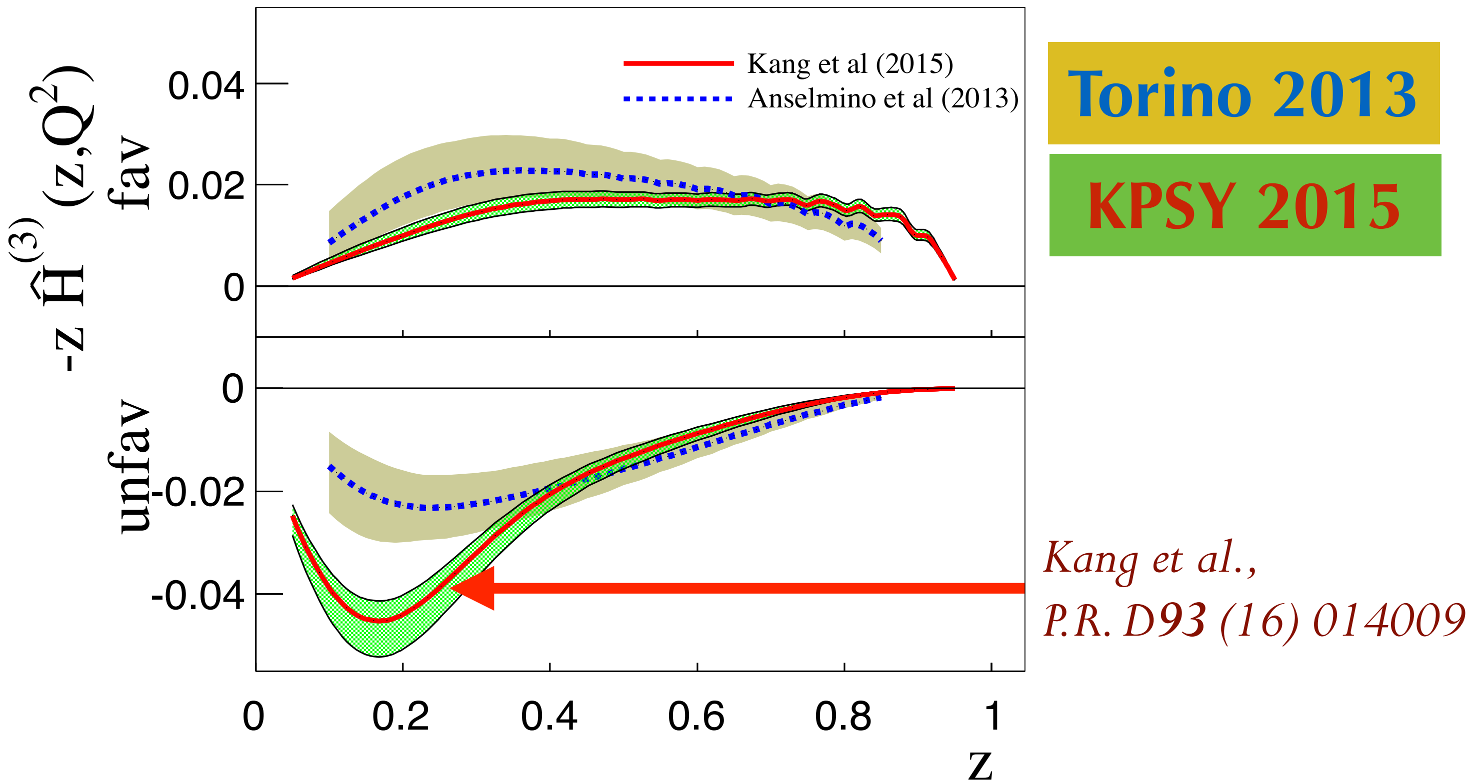
unfavored



$$\hat{H}^{(3)}(z) = -2zM_h\, H_1^{\perp\,(1)}(z)$$

- similar results
- both very good χ^2/dof

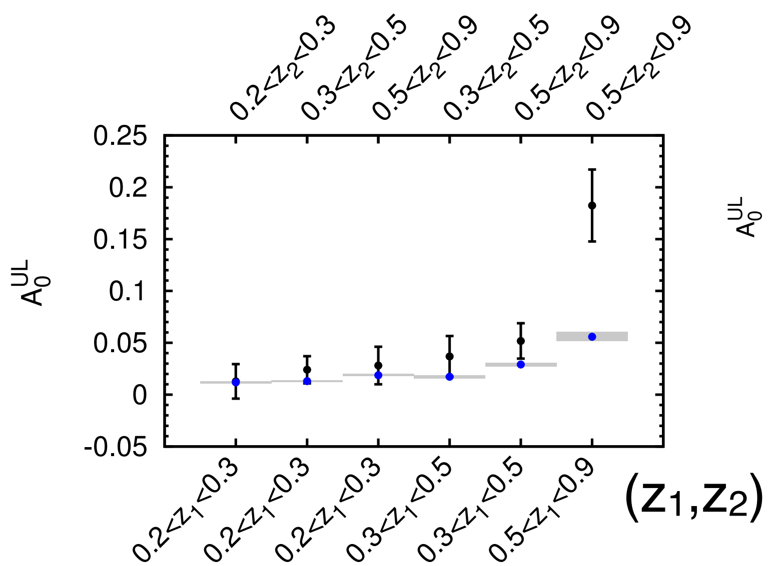
$\rightarrow$ R^U/R^L or R^U/R^C not very sensitive to TMD evolution

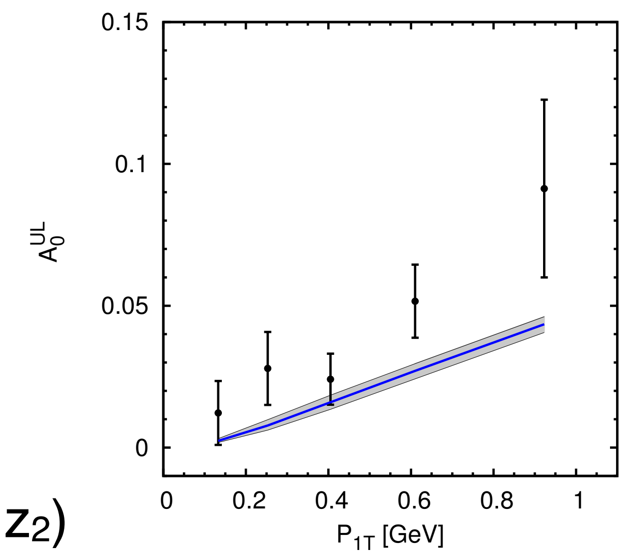


Predicting the BESIII asymmetry

A_0^{UL}

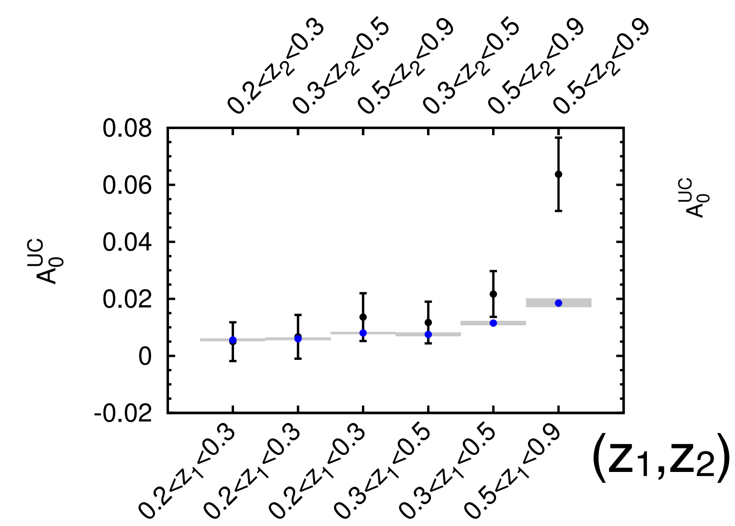
$$A_0^{e^+e^-}\left(\frac{\pi^+\pi^- + \pi^-\pi^+}{\pi^+\pi^- + \pi^-\pi^+}\right) - A_0^{e^+e^-}\left(\frac{\pi^+\pi^+ + \pi^-\pi^-}{\pi^+\pi^+ + \pi^-\pi^-}\right)$$

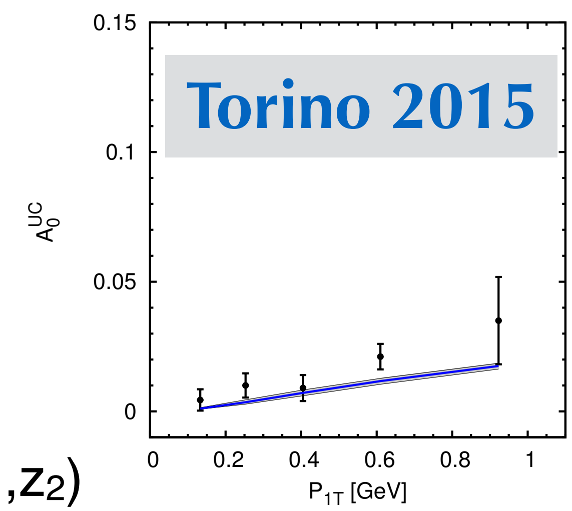




A_0^{UC}

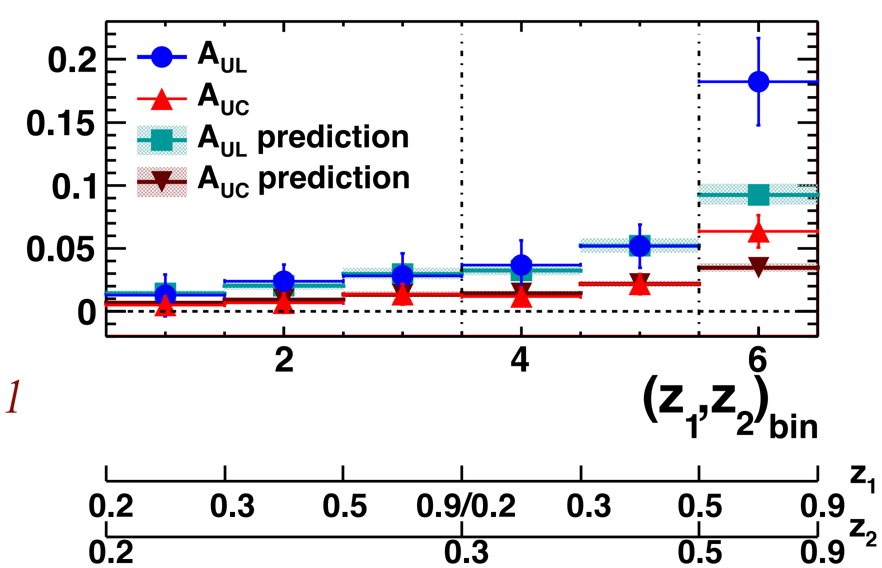
$$A_0^{e^+e^-}\left(\frac{\pi^+\pi^- + \pi^-\pi^+}{\pi^+\pi^- + \pi^-\pi^+}\right) - A_0^{e^+e^-}\left(\frac{\text{all }\pi\pi}{\text{all }\pi\pi}\right)$$

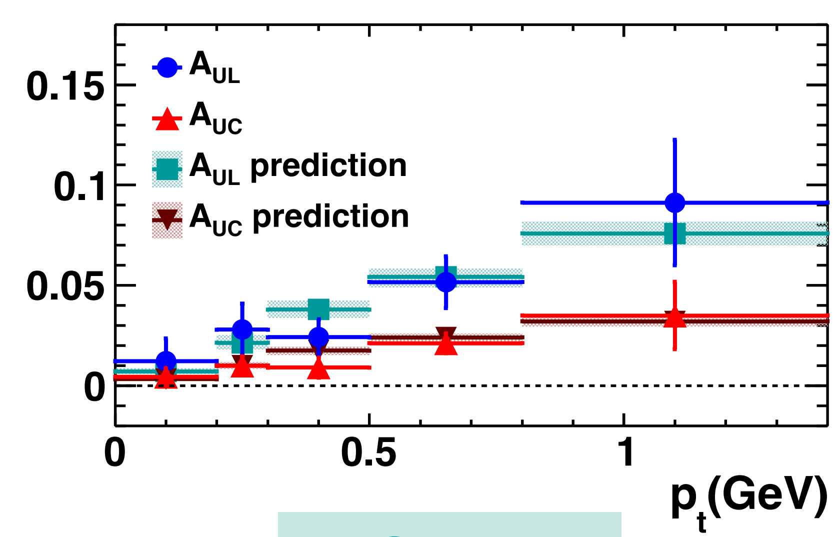




*Anselmino et al., P.R. D**92** (15) 114023*

Ablikim et al.,
*P.R.L. **116** (16) 042001*

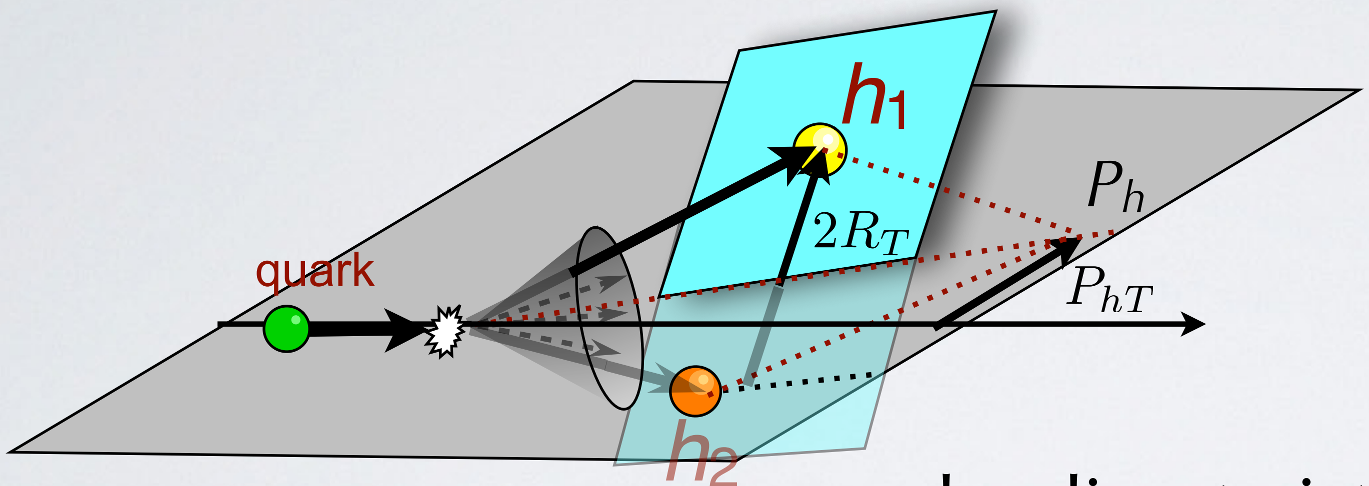




predictions = A_0^{UL} **KPSY 2015** A_0^{UC} **KPSY 2015**

Di-hadron Fragmentation Functions (DiFF)

Bianconi et al., P.R. D62 (00) 034008



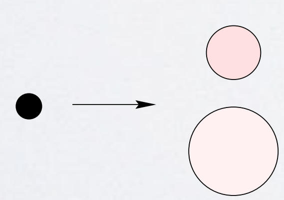
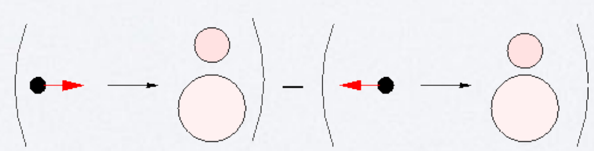
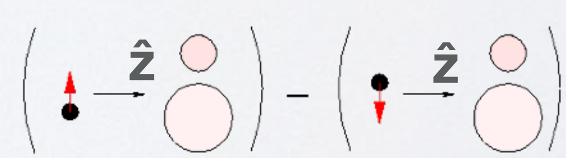
$$P_h^\mu = P_1^\mu + P_2^\mu$$

$$R^\mu = (P_1^\mu - P_2^\mu) / 2$$

$$R_T^2 = \frac{z_1 z_2}{z^2} M_h^2 - \frac{z_2}{z} M_1^2 - \frac{z_1}{z} M_2^2$$

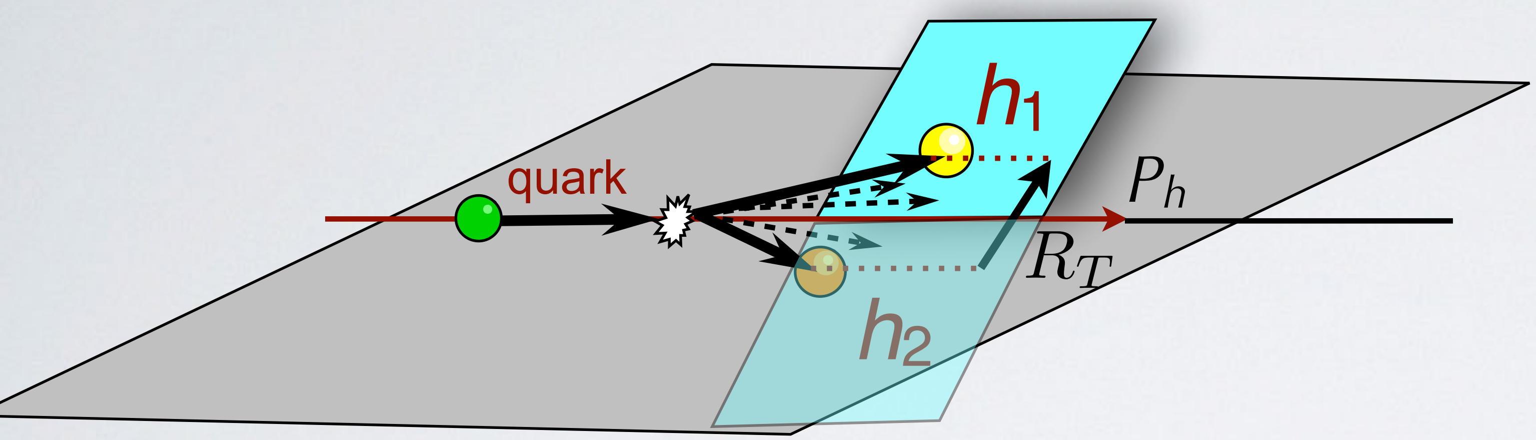
Bacchetta & Radici, P.R.D67 (03) 094002

leading twist, $R_T^2 \ll Q^2$

Hadron polarization	Quark polarization: Unpolarized (U)	Quark polarization: Longitudinally Polarized (L)	Quark polarization: Transversely Polarized (T)
U	D_1 	$G_1^\perp$ $\mathbf{S_L} \cdot \mathbf{P_{hT}} \times \mathbf{R_T}$ 	$H_1^{\sphericalangle}$ $\hat{\mathbf{z}} \cdot \mathbf{S_T} \times \mathbf{R_T}$ $H_1^\perp$ $\hat{\mathbf{z}} \cdot \mathbf{S_T} \times \mathbf{P_{hT}}$ 

Di-hadron Fragmentation Functions (DiFF)

Bianconi et al.,
P.R. D62 (00) 034008



$P_h^\mu = P_1^\mu + P_2^\mu$

$R^\mu = (P_1^\mu - P_2^\mu) / 2$

$\int dP_{hT}$ collinear framework

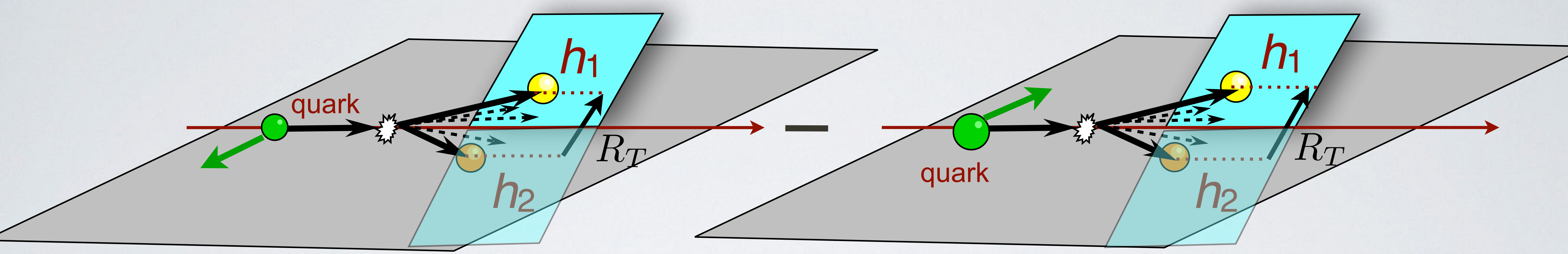
leading twist, $R_T^2 \ll Q^2$

		Quark polarization		
		Unpolarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Hadron polarization	**U**	D_1	$G_1^\perp$ $S_L \cdot P_{hT} \times R_T$	$H_1^{\sphericalangle}$ $\hat{z} \cdot S_T \times R_T$ $H_1^\perp$ $\hat{z} \cdot S_T \times P_{hT}$

chiral-odd partner of transversity

Access to transversity via DiFF

Collins, Heppelman, Ladinsky, N.P. ***B420*** *(94)*



correlation between

quark polarization and $\boldsymbol{R}_T = (z_2 \boldsymbol{P}_{1T} - z_1 \boldsymbol{P}_{2T}) / z$

or, equivalently, azimuthal orientation of (h_1, h_2) plane

(only if $h_1 \neq h_2$)

effect encoded in $h_1(x)\; H_1^{\sphericalangle}(z, M_h^2)$

$z = z_1 + z_2$

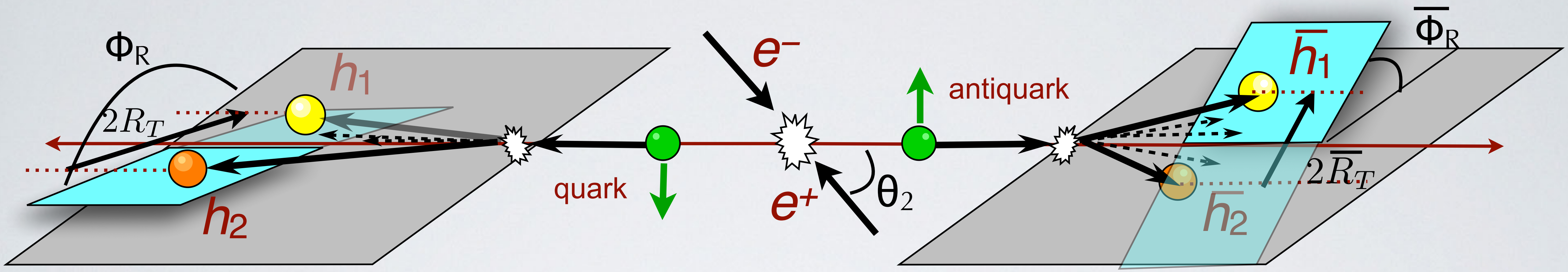
$P_h^2 = M_h^2 \iff \boldsymbol{R}_T^2$

alternative to Collins effect

*Radici, Jakob, Bianconi, P.R.D****65*** *(02) 074031*
*Bacchetta & Radici, P.R.D****67*** *(03) 094002*

talk Courtoy

extraction of DiFF from e^+e^-



back-to-back hadron pairs → $\cos(\Phi_R+\bar{\Phi}_R)$ modulation

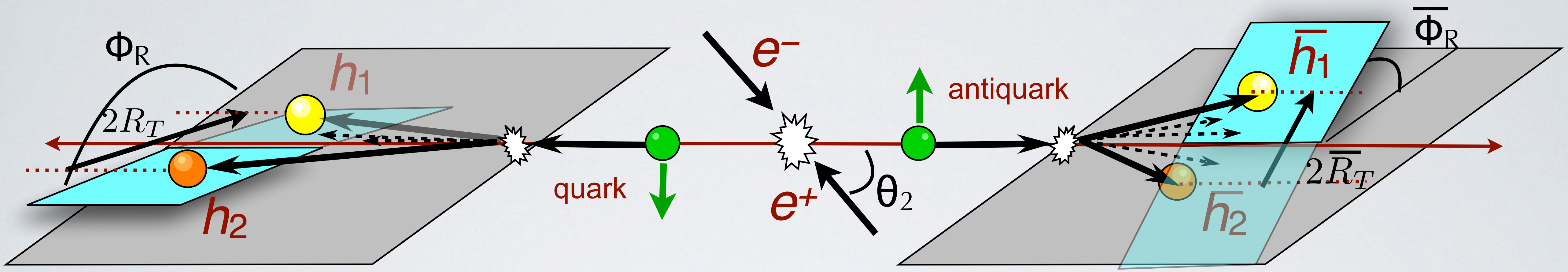
Artru & Collins, Z.Ph. C ***69*** *(96) 277*

Boer, Jakob, Radici, P.R.D ***67*** *(03) 094003*

$$A^{\cos(\phi_R+\bar{\phi}_R)} = \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,\frac{|\boldsymbol{R}_T|}{M_h}\,\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\,\frac{\sum_q e_q^2\, H_1^{\sphericalangle\, q}(z,M_h^2)\ \bar{H}_1^{\sphericalangle\, \bar{q}}(\bar{z},\bar{M}_h^2)}{\sum_q e_q^2\, D_1^q(z,M_h^2)\ \bar{D}_1^{\bar{q}}(\bar{z},\bar{M}_h^2)}$$

same as in SIDIS

extraction of DiFF from e^+e^-



back-to-back hadron pairs → $\cos(\Phi_R+\bar{\Phi}_R)$ modulation

*Artru & Collins, Z.Ph. C**69** (96) 277*

*Boer, Jakob, Radici, P.R.D**67** (03) 094003*

$$A^{\cos(\phi_R+\bar{\phi}_R)} = \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,\frac{|\boldsymbol{R}_T|}{M_h}\,\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\,\frac{\sum_q e_q^2\, H_1^{\sphericalangle\, q}(z,M_h^2)\ \bar{H}_1^{\sphericalangle\, \bar{q}}(\bar{z},\bar{M}_h^2)}{\sum_q e_q^2\, D_1^q(z,M_h^2)\ \bar{D}_1^{\bar{q}}(\bar{z},\bar{M}_h^2)}$$

same as in SIDIS

for D_1

Belle data for $A^{\cos(\Phi_R+\Phi_R)}$ → first extraction of DiFF, but using PYTHIA

*Vossen et al., P.R.L. **107** (11) 072004*

*Courtoy et al., P.R.D**85** (12) 114023*

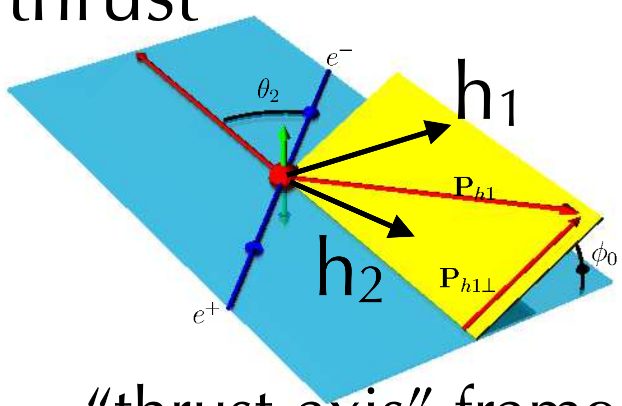
*Radici et al., JHEP **1505** (15) 123*

upcoming Belle data for unpolarized cross section

e^+e^- cross section for $(\pi\pi)$ in same hemisphere

$e^+ e^- \rightarrow (h_1 h_2)\, X$

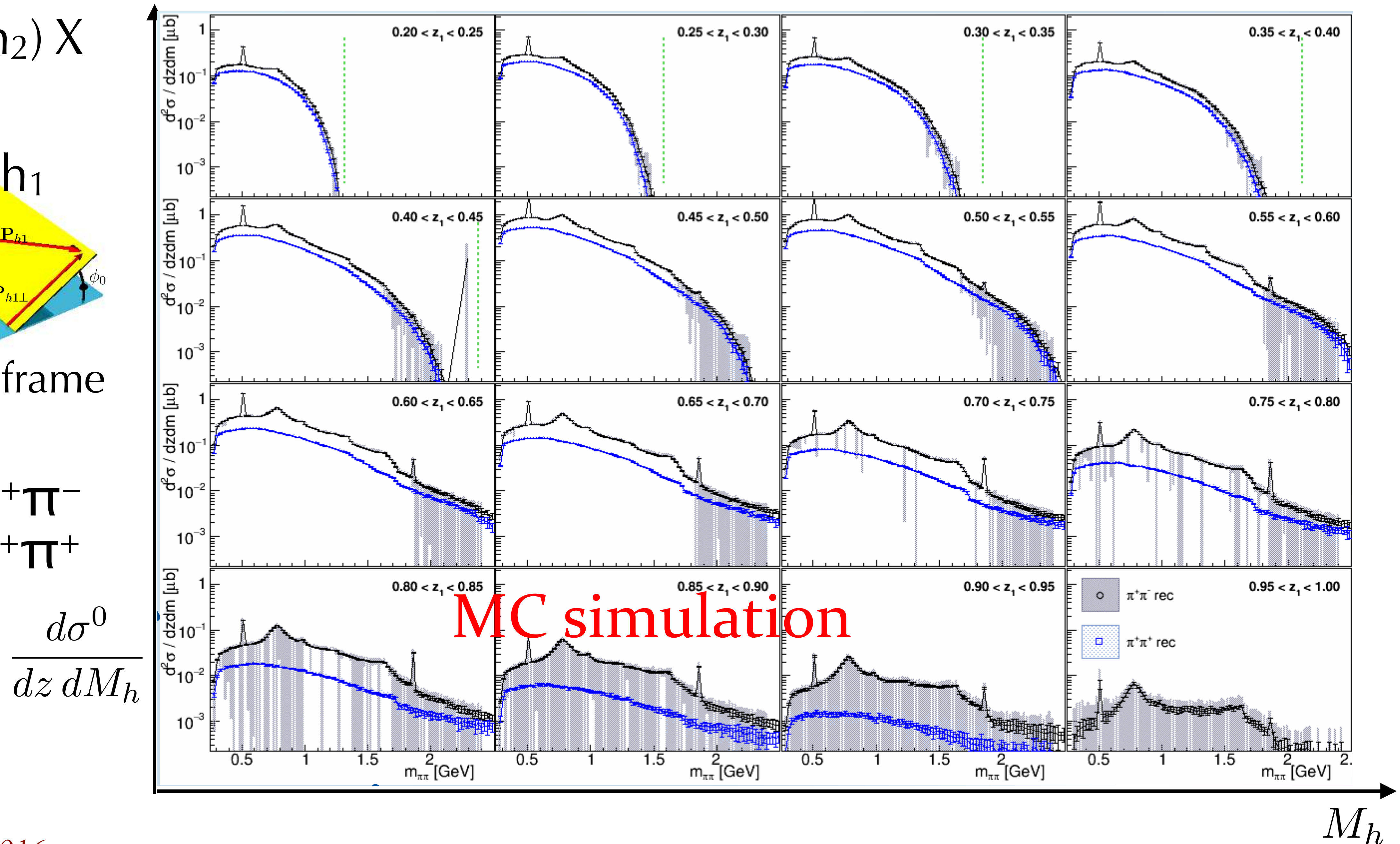
thrust



"thrust-axis" frame

● $\pi^+\pi^-$

● $\pi^+\pi^+$



R. Seidl, talk at SPIN2016

upcoming Belle data for (z, M_h) binning of unpolarized di-hadron e^+e^- cross section

The power of DiFF

collinear framework → factorization theorems

$e^+e^- \to (\pi^+\pi^-)\,(\pi^+\pi^-)\,X$

$$A_{e^+e^-} \sim \frac{H_1^{\sphericalangle}\ \overline{H_1^{\sphericalangle}}}{D_1\ \overline{D_1}}$$

prediction

*Boer, Jakob, Radici, P.R. D**67** (03) 094003*

extraction

*Courtoy et al., P.R. D**85** (12) 114023*

SIDIS: $e\,p^\uparrow \to e'\,(\pi^+\pi^-)\,X$

$$A_{\mathrm{SIDIS}} \sim \frac{h_1\ H_1^{\sphericalangle}}{f_1\ D_1}$$

prediction

*Radici, Jakob, Bianconi, P.R.D**65** (02) 074031*

*Bacchetta & Radici, P.R. D**67** (03) 094002*

extraction

*Bacchetta, Courtoy, Radici, P.R.L. **107** (11) 012001*

*Bacchetta, Courtoy, Radici, JHEP **1303** (13) 119*

*Radici et al., JHEP **1505** (15) 123*

universality of DiFF and usual DGLAP evolution

*Ceccopieri, Radici, Bacchetta, P.L.**B650** (07) 81*

$p\,p^\uparrow \to (\pi^+\pi^-)\,X$

$$A_{pp} \sim \frac{f_1 \otimes h_1 \otimes H_1^{\sphericalangle}}{f_1 \otimes f_1 \otimes D_1}$$

prediction

*Bacchetta and Radici, P.R. D**70** (04) 094032*

test universality

*Radici et al., P.R.D**94** (16) 034012*

The power of DiFF

collinear framework → factorization theorems

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prediction
*Boer, Jakob, Radici, P.R. D**67** (03) 094003*
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prediction
*Bacchetta and Radici, P.R. D**70** (04) 094032*
test universality
*Radici et al., P.R.D**94** (16) 034012*

SIDIS: $e\,p^{\uparrow} \to e'\,(\pi^+\pi^-)\,X$

$$A_{\rm SIDIS} \sim \frac{h_1\ H_1^{\sphericalangle}}{f_1\ D_1}$$

prediction
*Radici, Jakob, Bianconi, P.R.D**65** (02) 074031*
*Bacchetta & Radici, P.R. D**67** (03) 094002*

extraction
*Bacchetta, Courtoy, Radici, P.R.L. **107** (11) 012001*
*Bacchetta, Courtoy, Radici, JHEP **1303** (13) 119*
*Radici et al., JHEP **1505** (15) 123*

subleading twist: access to e(x)

Jefferson Lab

$$A_{LU} \sim \frac{e\ H_1^{\sphericalangle}\quad (+\,f_1\ \tilde{G}^{\sphericalangle})}{f_1\ D_1}$$

is twist-3 DiFF small?

prediction
*Bacchetta and Radici, P.R. D**69** (04) 074026*

talk Courtoy

a phase transition in 3D studies

1D
(standard parton distribution functions - PDFs)

Parton model

QCD analysis + data

Global fits

3D
(transverse momentum distributions - TMDs)

Parton model

Phase 1

QCD analysis + data

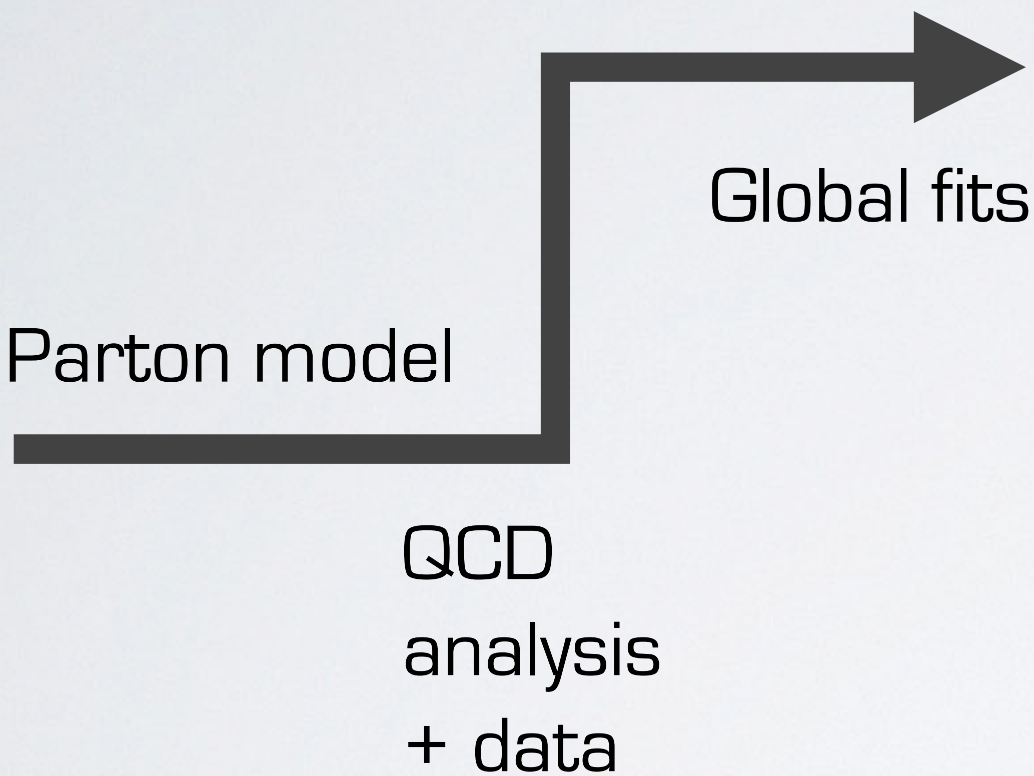
(where we are now)

Phase 2

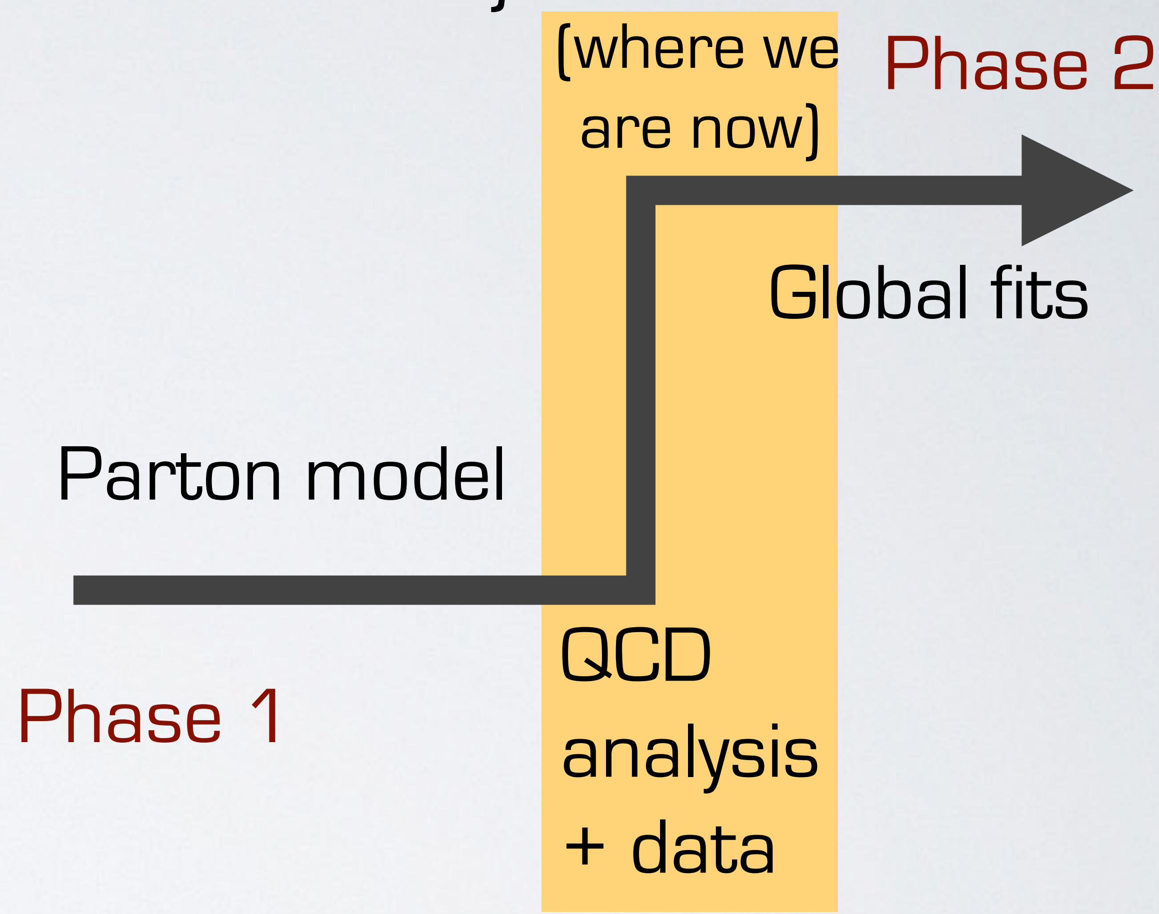
Global fits

a phase transition in 3D studies

1D
(standard parton distribution functions - PDFs)



3D
(transverse momentum distributions - TMDs)



with TMD FF (and DiFF) we are a little step behind but with the upcoming data for unintegrated e^+e^- cross sections we are well underway to fill the gap…