# Matching TMD factorization and collinear factorization

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3D Parton Distributions: path to the LHC

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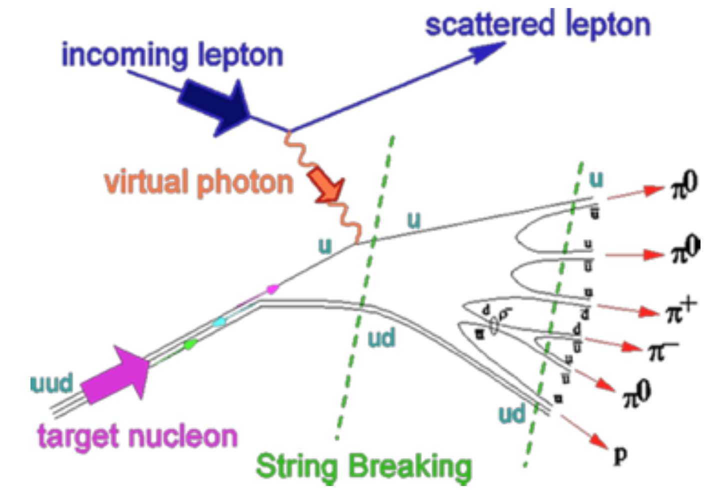
Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

# Outline




- Review work on improved implementation for combining transverse-momentum-dependent (TMD) factorization and collinear factorization in semi-inclusive DIS

  **Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang**

- The result is a modified version of the "$W$+$Y$" prescription traditionally used in the Collins-Soper-Sterman (CSS) formalism

- Address the "standard matching prescription" traditionally used in the CSS formalism relating low and high $q_T$ behavior of cross section @ moderate Q

  - Collins Soper Sterman NPB 1985
  - A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)

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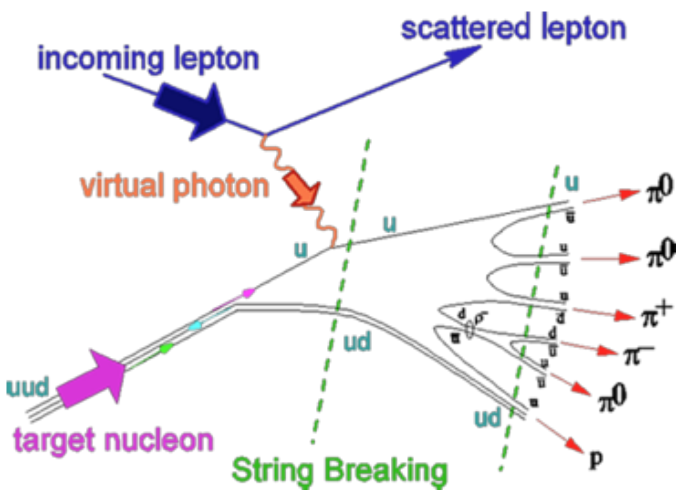



- In particular address the role of $Y$ term matching of low and high $q_T$ behavior of cross section @ moderate Q
  - Collins Soper Sterman NPB 1985
  - A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)

- Introduce method to combine TMD and Collinear Factorization formalism

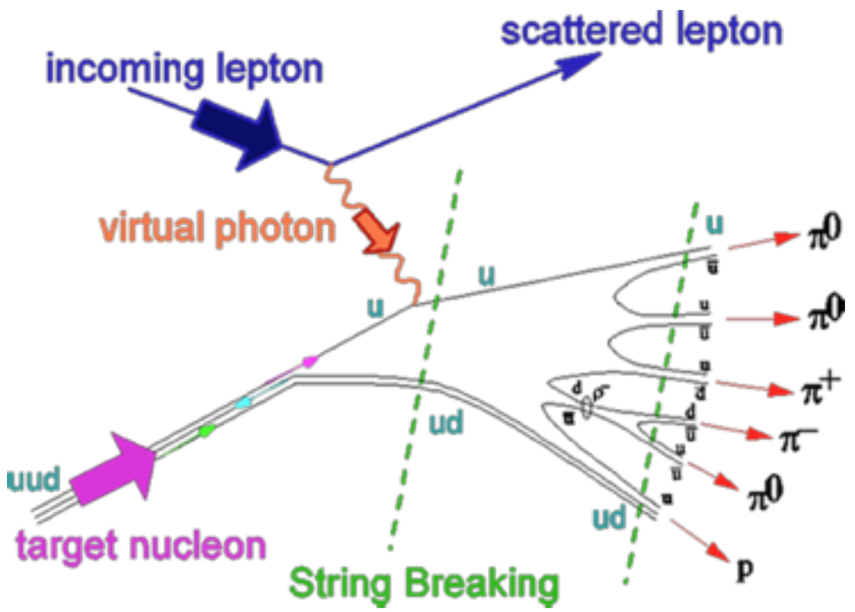
- We briefly discuss how an EIC/LHC could help to further our study of matching between the TMD approach and collinear factorization

# Comments Message

- The standard $W + Y$ prescription was arranged to apply also for intermediate $q_T$; in particular it keeps full accuracy when $m << q_T << Q$, a situation in which both pure TMD and pure collinear factorization have degraded accuracy
- **However it did not specifically address the issue of matching to collinear factorization for the cross section integrated over $q_T$**

$$\int dq_T \; d\sigma(q_T, Q)$$

- We develop a prescription to which matches the integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two which are suppressed by powers of 1/Q
- **Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations**.
- We preserve transverse-coordinate space version of the $\boldsymbol{W_{\mathrm{TMD}}}$ term, but only modify the way in which it is used.




$$\frac{d\sigma(q_T, Q)}{d^2q_T dQ \ldots} \equiv d\sigma(q_T, Q)$$

Short hand notation throughout talk

# Start w/ review of CSS $W + Y$ definition

Birds eye view

✦ Collins Soper Sterman NPB 1985

- Standard CSS formalism separates the cross section into a sum of two terms *W* & *Y* such that *their sum* gives the cross section up to an error that **relative to the cross section is** power suppressed

$$O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

# Start w/ review of CSS $W + Y$ definitions

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- $W$ describes the small transverse momentum behavior $q_T \ll Q$ and an additive correction term $Y$ accounts for behavior at $q_T \sim Q$

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- $W$ is written in terms of TMD pdfs and/or TMD ffs and is *designed* to be an accurate description in the limit of $q_T/Q \ll 1$. It includes all non-perturbative transverse momentum dependence

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- $W$ is written in terms of TMD pdfs and/or TMD ffs and is constructed to be an accurate description in the limit of $q_T/Q \ll 1$. It includes all non-perturbative transverse momentum dependence
- The " $Y$-term " is described in terms of "collinear approximation" to the cross section: it is the correction term for large $q_T \sim Q$

# Start w/ review of CSS $W + Y$ definitions

- Collins Soper Sterman NPB 1985

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- The CSS construction of $W + Y$ and the specific approximations are applied, thru the **operations-approximators** $\boldsymbol{T}_{\mathbf{TMD}}$ and $\boldsymbol{T}_{\mathbf{coll}}$ that apply only in ***"design" regions*** $q_T \ll Q$ and $q_T \sim Q$ respectively which we emphasize by the ***range*** of the argument above

$$m << q_T << Q$$

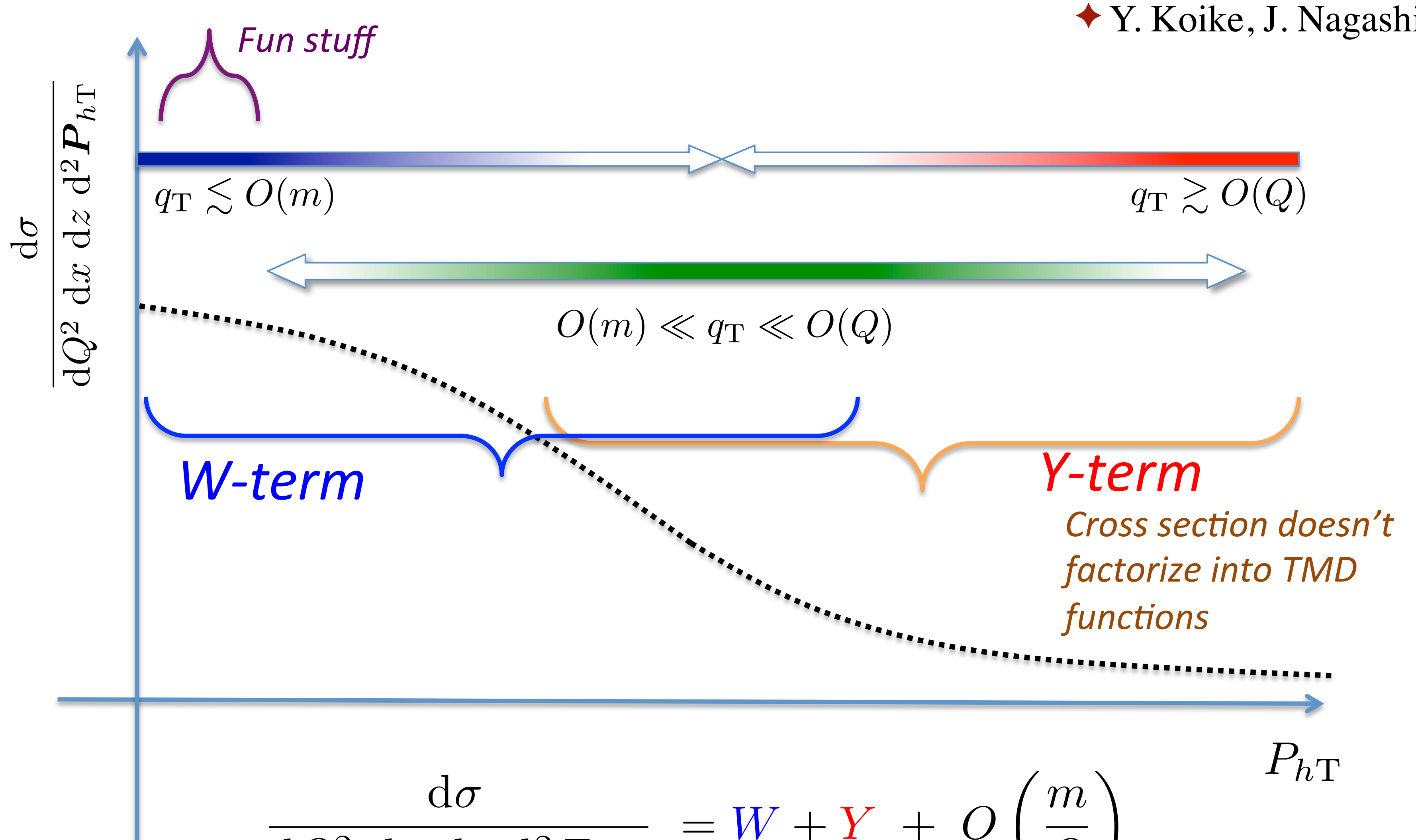
# Matching and $W + Y$-schematic

- This was *designed* with the aim to have a formalism that is valid to leading power in $m/Q$ uniformly in $q_T$, where $m$ is a typical hadronic mass scale
- and where there is a broad intermediate range of transverse momentum characterized by $m \ll q_T \ll Q$

Implementations/studies

- ✦ Nadolsky Stump C.P. Yuan PRD 1999 **HERA data**
- ✦ Y. Koike, J. Nagashima, W. Vogelsang NPB (2006) **eRHIC**

## From Ted Rogers **W + Y**




$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

note $P_{hT} = z q_T$

# Matching and $W + Y$-studies

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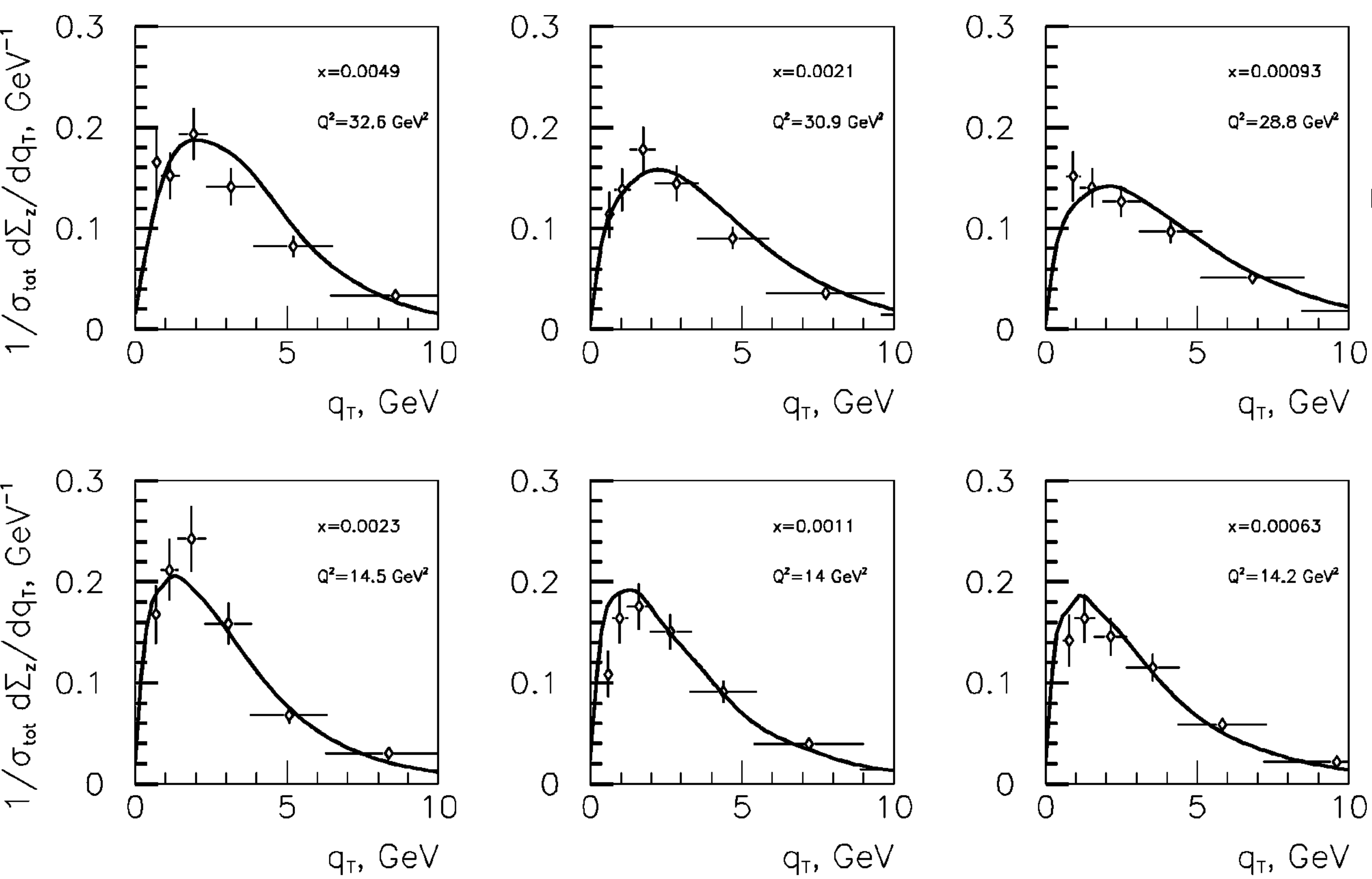
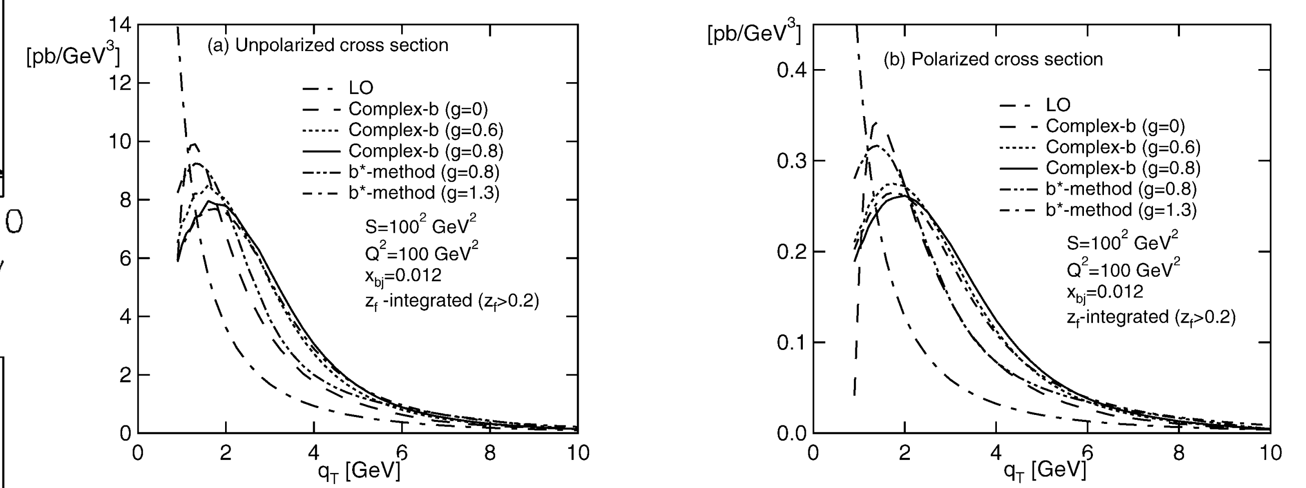
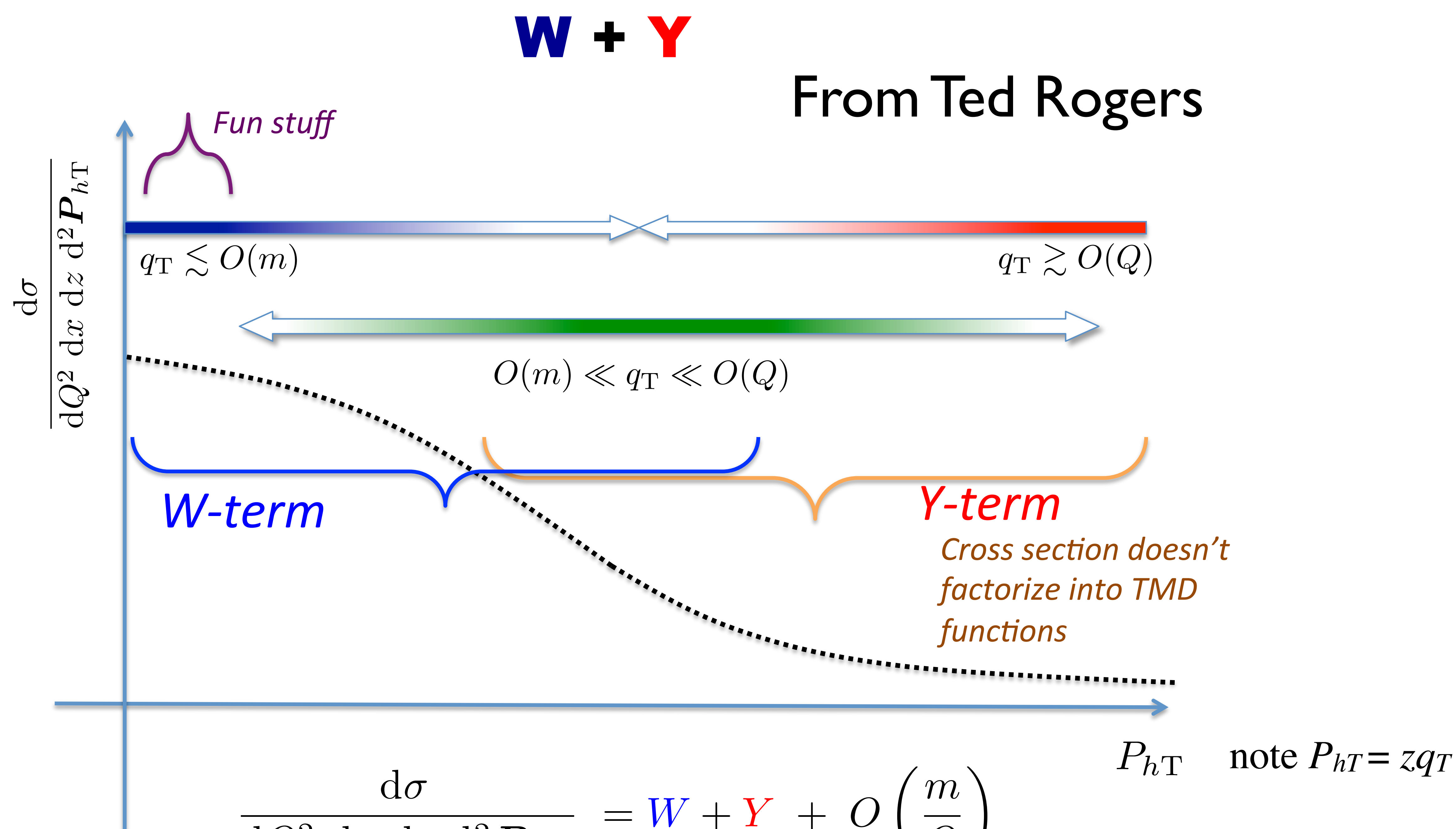


# Matching and $W + Y$-schematic

- However at lower phenomenologically interesting values of Q, neither of the ratios $q_T/Q$ or $m/q_T$ are necessarily very small and matching can be problematic




$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{h\mathrm{T}}} = W + Y + O\left(\frac{m}{Q}\right)$$

# Matching and $W + Y$-schematic

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**W + Y**

From Ted Rogers


note $P_{hT} = zq_T$

$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

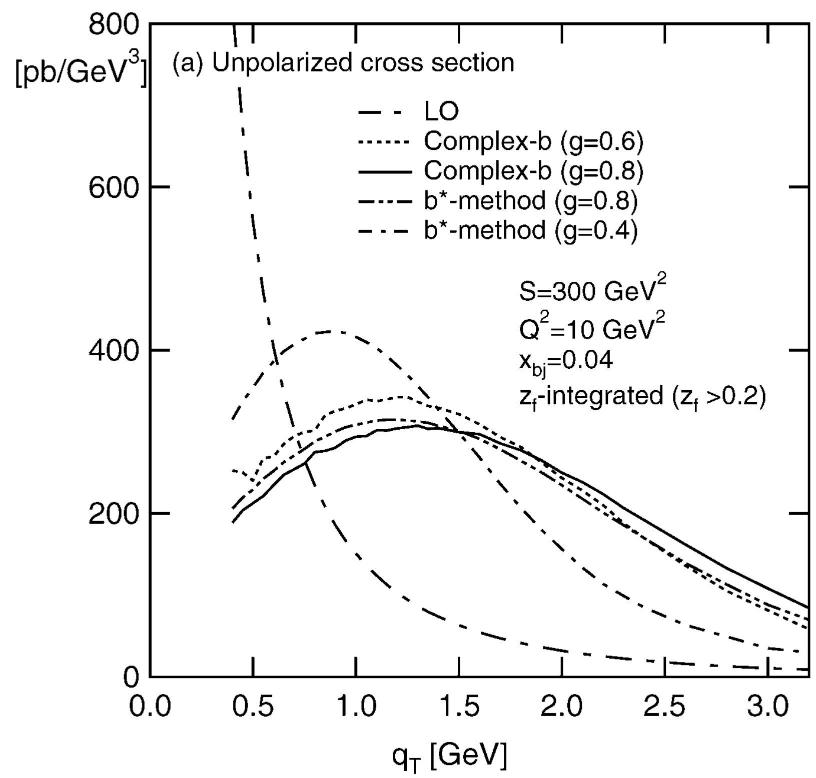
# Matching and $W + Y$ -studies

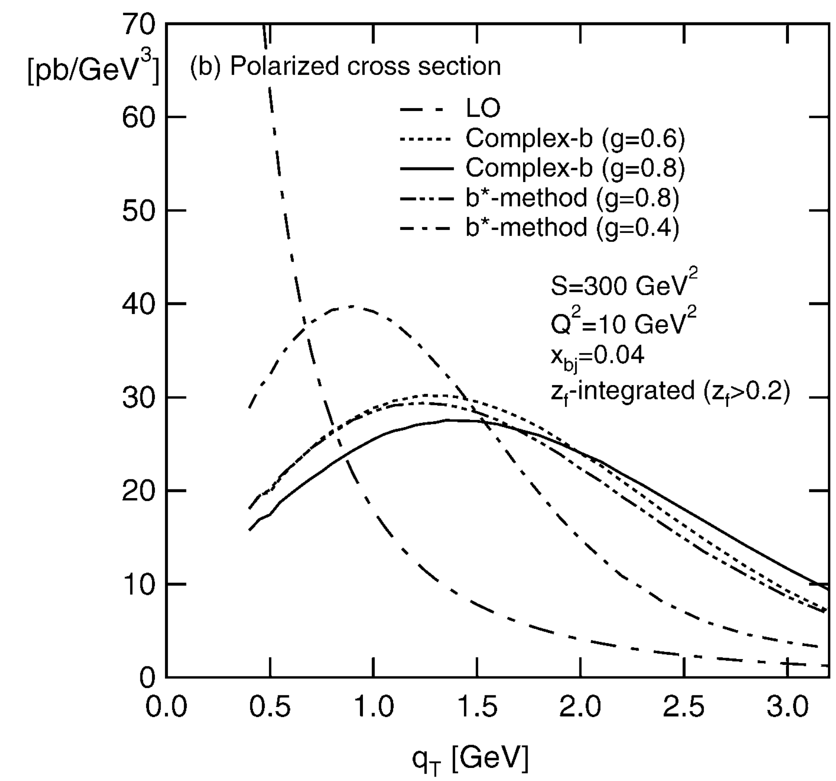
This impacts studies of non-perturbative nucleon structure @ COMPASS & JLAB !!!

$$m \lesssim q_T \lesssim Q$$

Implementations

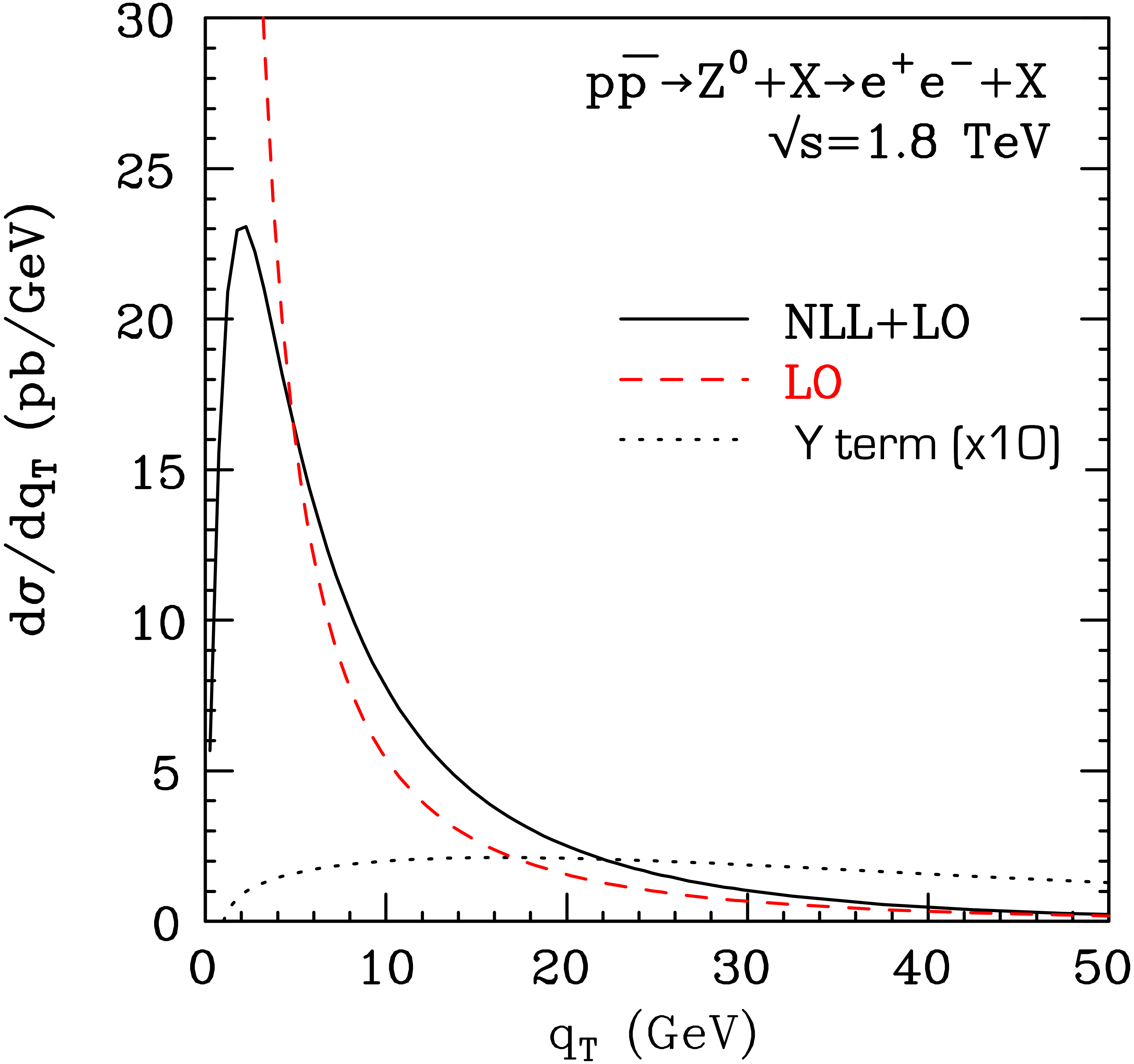
✦ Y. Koike, J. Nagashima, W. Vogelsang NPB (2006) **COMPASS no data yet**



# Y term in Z boson production

*Bozzi et al. arXiv:0812.2862*



# Matching and $W + Y$ -studies

## Compass Example

Boglione Prokudin et al. JHEP 2015

- When $q_T$ is above some small fraction of Q, $W$ deviates a lot from $d\sigma(q_T, Q)$

- Then it becomes negative and "asymptotes" to $\frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$

  **Nadolsky et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. B744, 59 (2006)**

- At large $q_T$ $W+Y$ is then a difference of large terms and *truncation errors* can be augmented

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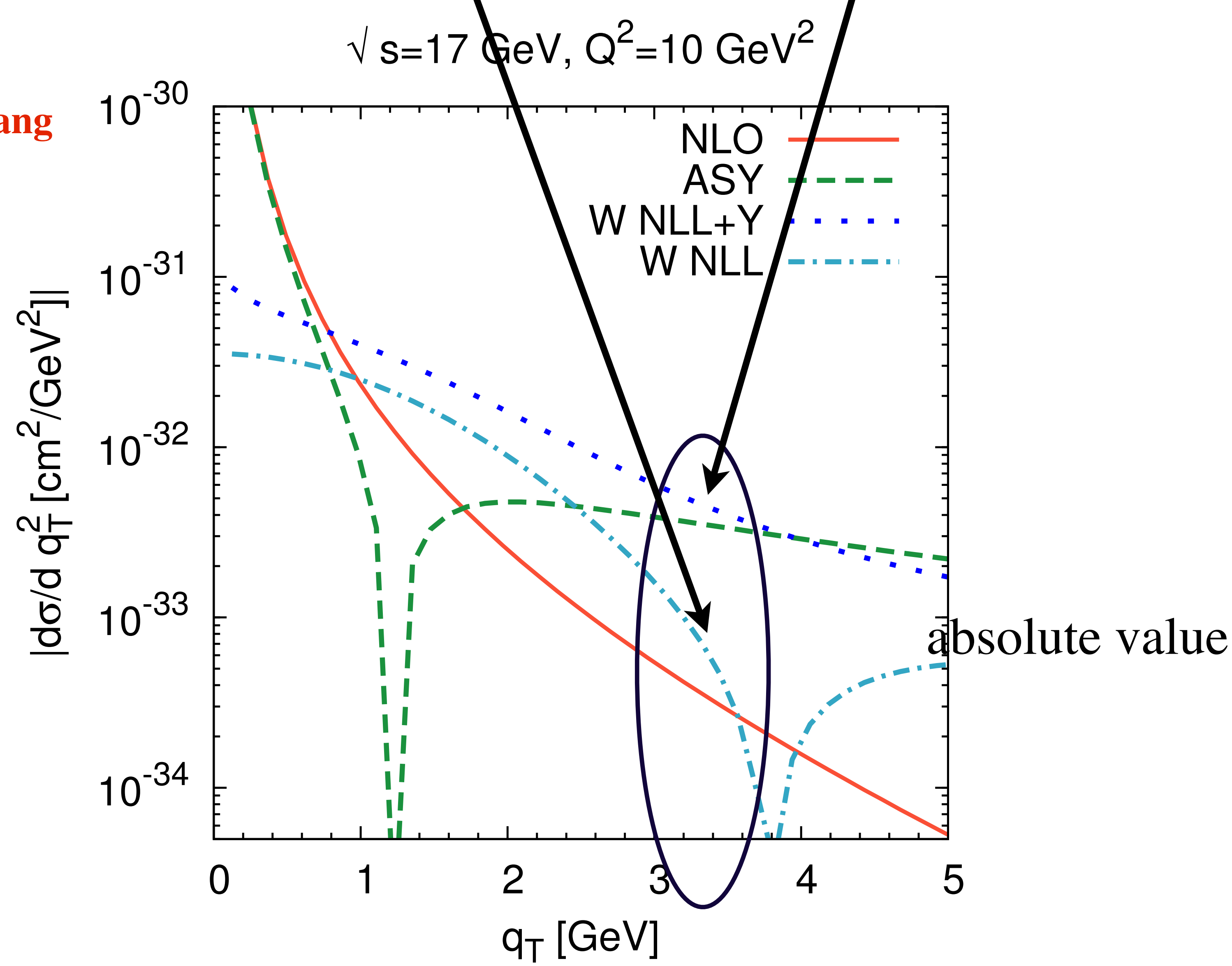
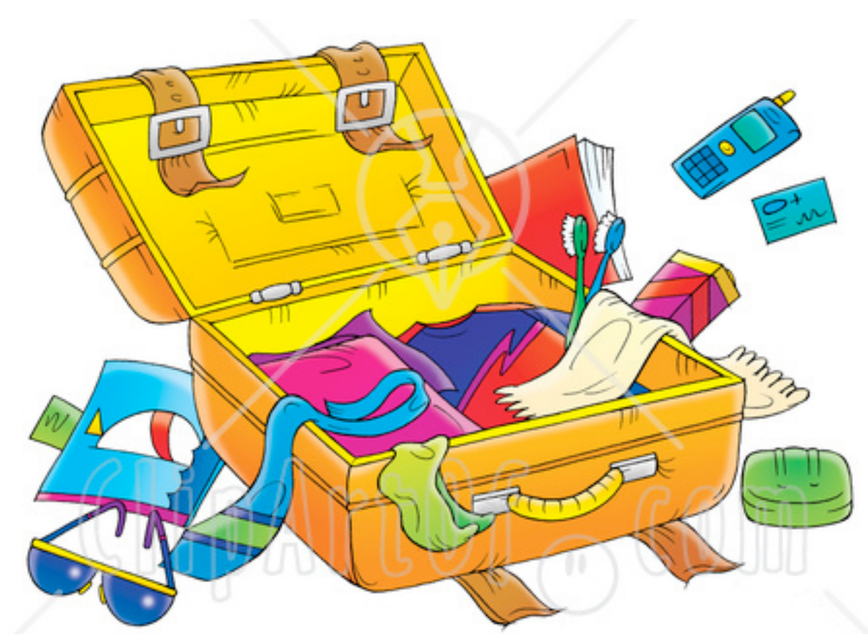
**Matching becomes a problem COMPASS/Jlab like energies**



- To get a sense of these *truncation errors* we further "unpack" *W+ Y* via their "*Approximators*" and its *construction in terms of* W, Y, FO, ASY <u>terms</u>



# Review of Region Analysis "Approximators"
# W, Y, FO, ASY Definitions

Original CSS definition of $W$ is given by instruction to carryout an approximation of the *cross section* designed to be good in the region $q_T << Q$ up to powers of $q_T/Q$ and $m/Q$

$$T_{TMD} d\sigma(q_T, Q) \approx d\sigma(q_T \ll Q, Q) + O\left(\frac{q_T}{Q}\right)^a d\sigma(q_T, Q) + O\left(\frac{m}{Q}\right)^{a'} d\sigma(q_T, Q)$$

$$W(q_T, Q) \equiv T_{TMD} d\sigma(q_T, Q)$$

# Review of Region Analysis "Approximators"
# W, Y, FO, ASY Definitions

Another approximator for the design "region" of $q_T \sim Q$ defines $FO$ up to powers of $m/q_T$

$$T_{coll} d\sigma(q_T, Q) \approx d\sigma(q_T \gtrsim Q, Q) + O\left(\frac{m}{q_T}\right)^b d\sigma(q_T, Q)$$

$$FO(q_T, Q) \equiv T_{coll} d\sigma(q_T, Q)$$

# Review of Region Analysis "Construction"

- CONSTRUCTION: one starts with smallest-size region which is in a neighborhood of $q_T = 0$, where $T_{TMD}$ gives a very good approximation adding and subtracting the $T_{TMD}$ approximation

$$d\sigma(q_T, Q) = T_{TMD}\, d\sigma(q_T, Q) + [d\sigma(q_T, Q) - T_{TMD}\, d\sigma(q_T, Q)]$$

- The error in the bracket is order $(q_T/Q)^a$ and is only unsuppressed at $q_T >> m$

- **Now, *extend* the range of $q_T$ . . .**

# Review of Region Analysis "Construction"
# W, Y, FO, ASY Definitions

- Extending $q_T$, one then applies $T_{coll}$ to the bracket & uses the fixed order (FO) perturbative expansion

**The Result is the combination**

$$d\sigma(m \lesssim q_T \lesssim Q, Q) \approx T_{TMD}\, d\sigma(q_T, Q) + T_{coll}\left[ d\sigma(q_T, Q) - T_{TMD}\, d\sigma(q_T, Q)\right]$$

$$+ O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$$d\sigma(m \lesssim q_T \lesssim Q, Q) \approx W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$q_T/Q << 1$ $\quad$ $q_T \sim Q$ or $m/q_T << 1$

# Now we see the definition of the $Y$ term via "approximators"

$$Y(q_T, Q) \equiv T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}\, d\sigma(q_T, Q)$$

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

- It is the difference of the cross section calculated with collinear pdfs and ffs at fixed order FO and the asymptotic contribution of the cross section
- ***At small $q_T$ the FO and ASY are dominated by the same diverging terms***

$$\frac{1}{q_T^2} \quad \text{and} \quad \frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$$

- ***Thus its expected that the Y term is small or zero leaving***

$$d\sigma(q_T \ll Q, Q) \approx W(q_T, Q)$$

# The Asymptotic piece of the NLO cross section in detail

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

$$\left(\frac{d\sigma_{BA}}{dx\,dz\,dQ^2\,dq_T^2\,d\phi}\right)_{\text{asym}} = \frac{\sigma_0 F_l}{S_{eA}} \frac{\alpha_s}{\pi} \frac{1}{2q_T^2} \frac{A_1(\psi,\phi)}{2\pi}$$

$$\times \sum_j e_j^2 \Big[ D_{B/j}(z,\mu)\{(P_{qq}\otimes f_{j/A})(x,\mu) + (P_{qg}\otimes f_{g/A})(x,\mu)\}$$

$$+\{(D_{B/j}\otimes P_{qq})(z,\mu) + (D_{B/g}\otimes P_{gq})(z,\mu)\} f_{j/A}(x,\mu)$$

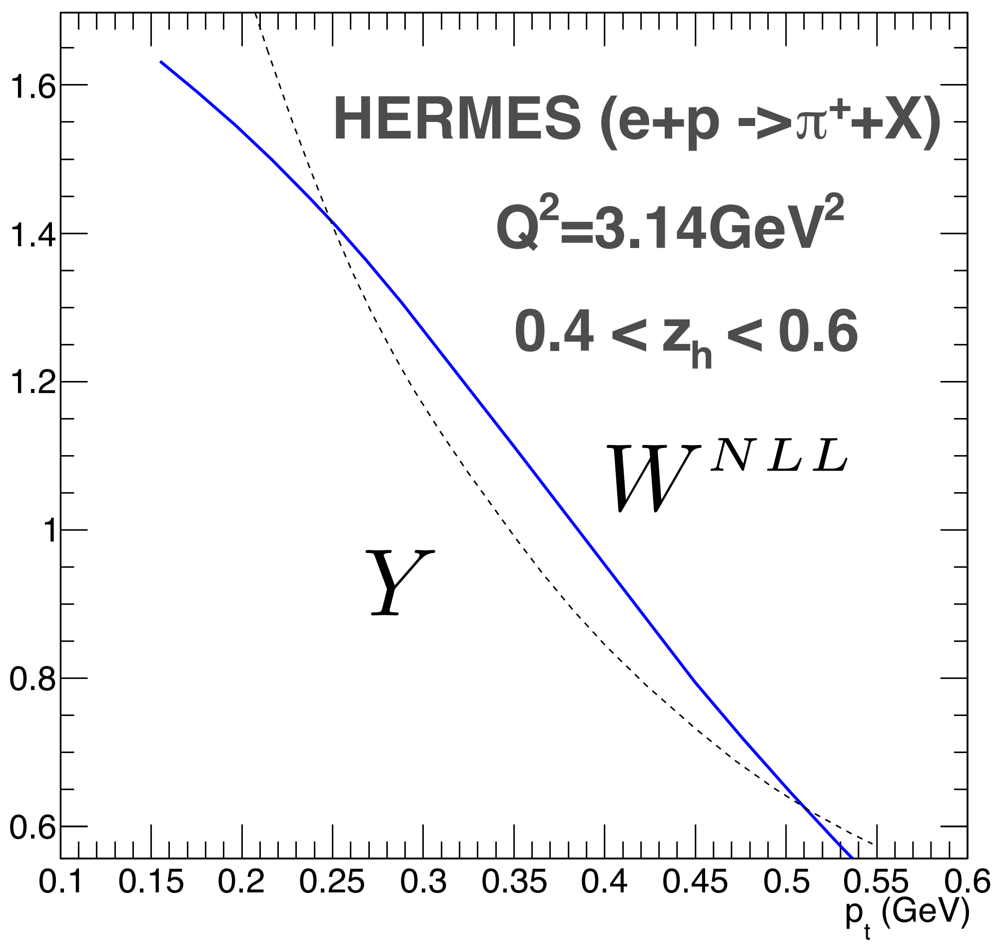
$$+2D_{B/j}(z,\mu) f_{j/A}(x,\mu)\left\{ C_F \log\frac{Q^2}{q_T^2} - \frac{3}{2} C_F \right\} + \mathcal{O}\left(\frac{\alpha_s}{\pi}, q_T^2\right)\Big].$$

- Nadowsly et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. B744, 59 (2006),

# Matching and $W + Y$ -studies

- At small $q_T$ the $Y$ term is in principle suppressed: it is the difference of the FO perturbative calculation of the cross section and the asymptotic contribution of $W$ for small $q_T$
- But again there can be a difference of of large terms and truncation errors are augmented: **Here the Y term is larger than W?!**

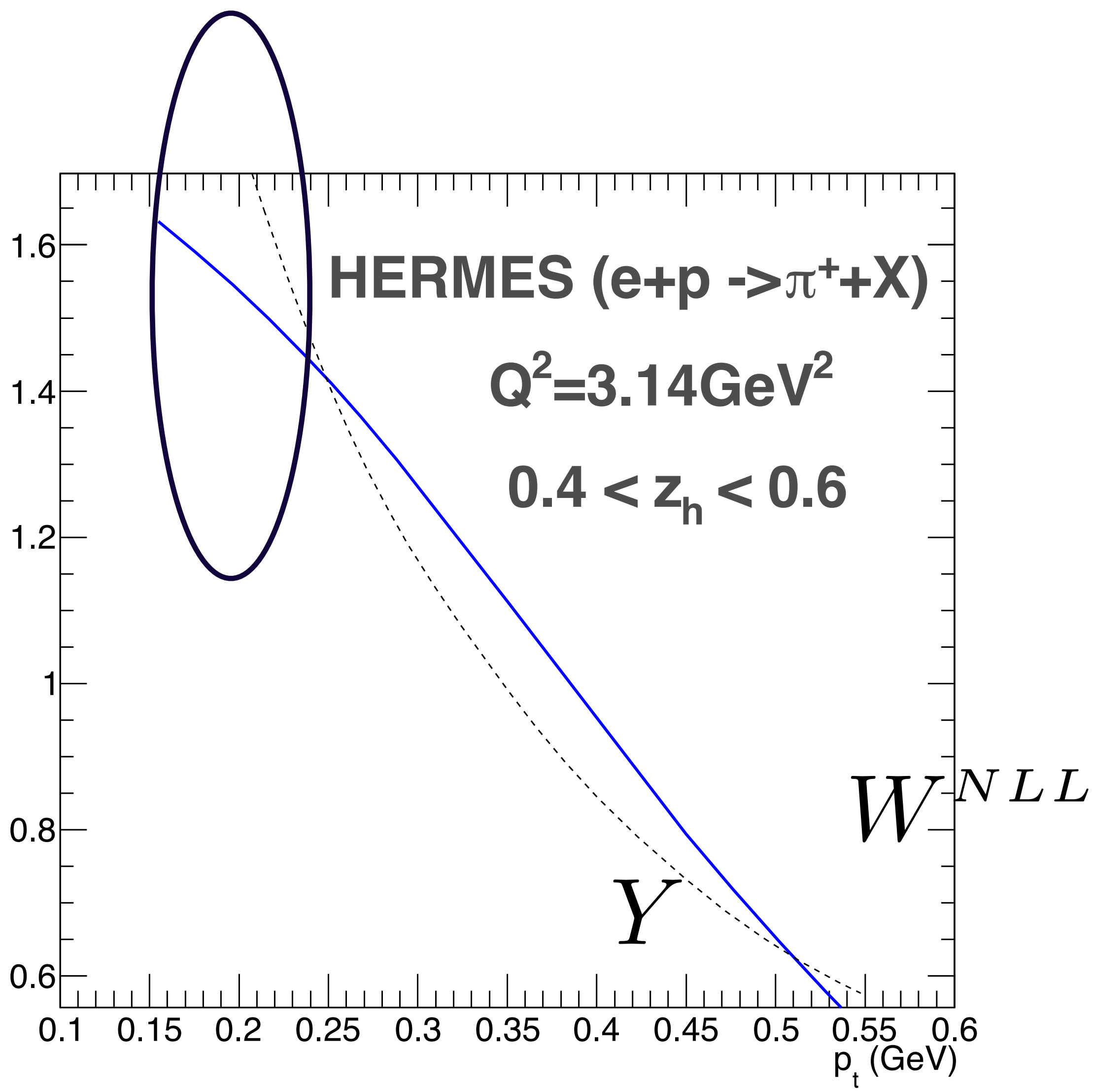
P. Sun F. Yuan et al arXiv: 1406.3073

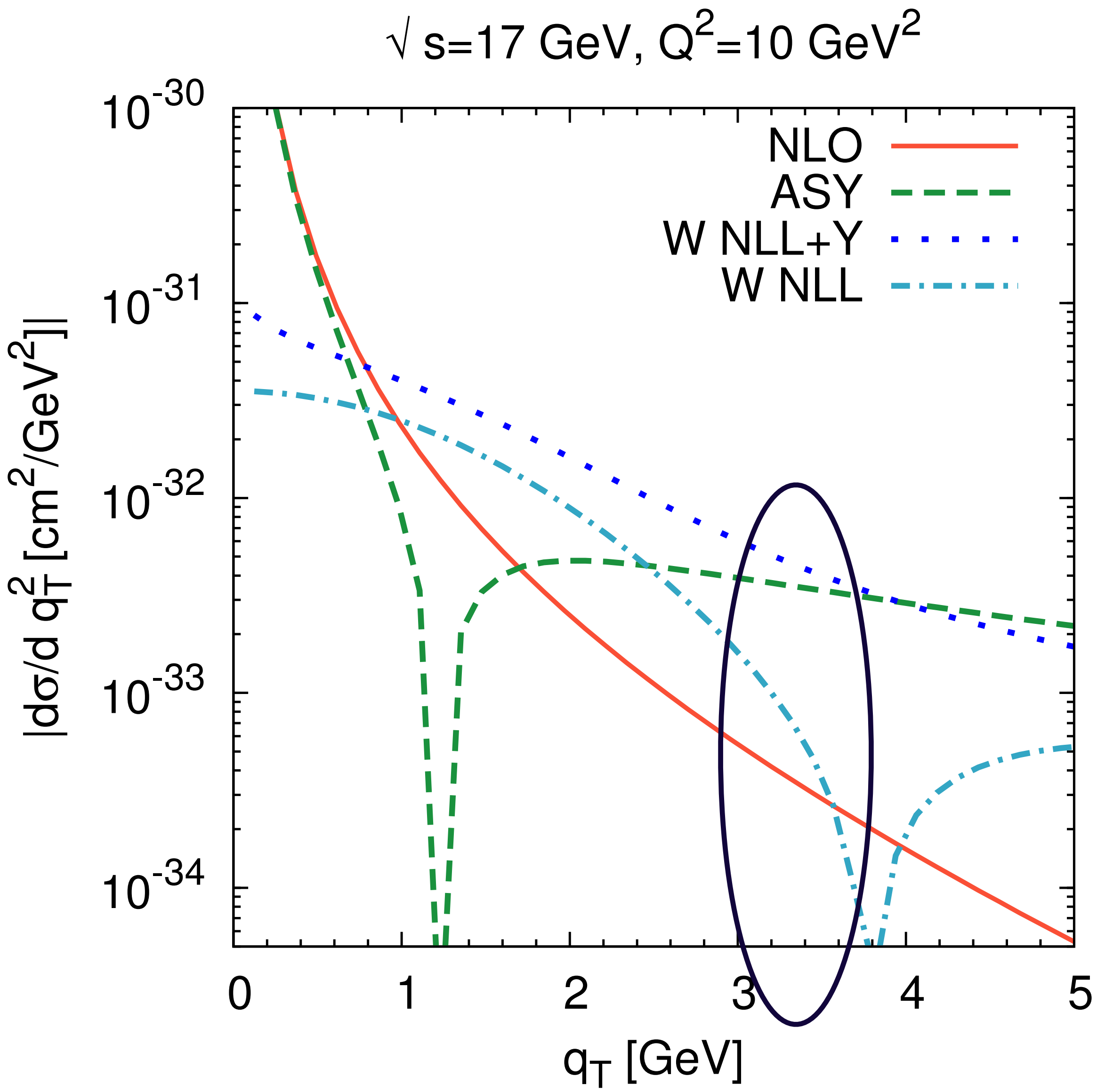



$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

# Matching and $W + Y$-schematic

- **Thus the region *between* large and small $q_T$ needs special treatment if errors are to be strictly power suppressed point-by-point in $q_T$**



# Extend formalism to

Phys.Rev. D 94 Collins, L.G, Prokudin, Sato, Rogers, Wang

$$q_T \lesssim m \quad \text{and} \quad q_T \gtrsim Q$$

# Extend formalism to

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$$q_T \lesssim m$$

- For $q_T \lesssim m$ collinear factorization is not applicable for the differential cross section. But this region is actually where the *W-term* in has its highest validity. So one simply must ensure that the *Y -term* is sufficiently suppressed in Eq. (10) for $q_T \lesssim m$

- Modify *Y*

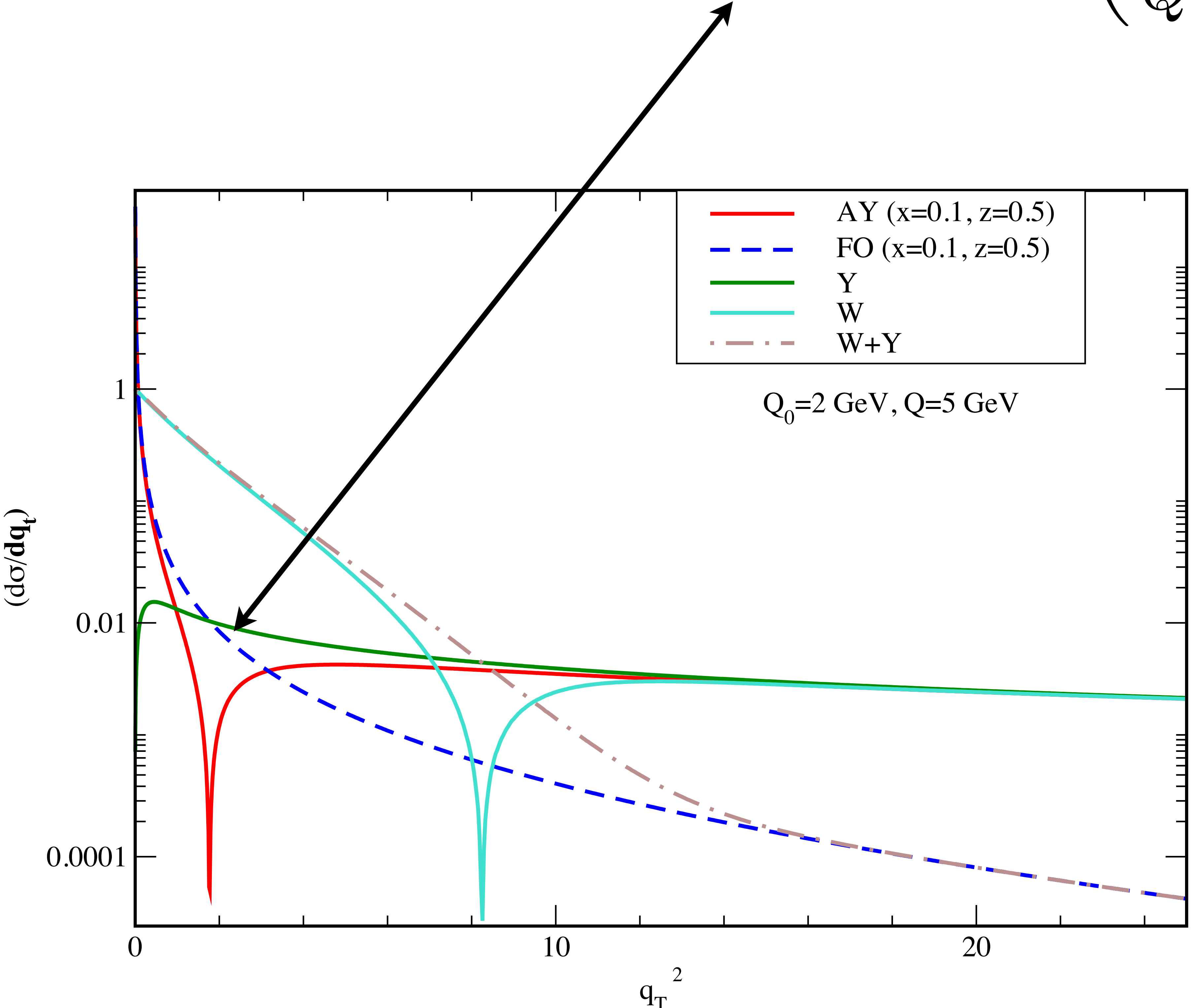
$$Y(q_T, Q) = \{FO(q_T, Q) - ASY(q_T, Q)\}\, X(q_T/\lambda)$$

with small $q_T$ cutoff

$$X(q_\mathrm{T}/\lambda) = 1 - \exp\{-(q_\mathrm{T}/\lambda)^{a_X}\}$$

- Now we can extend the power suppression error estimate down to $q_T = 0$ to get

$$d\sigma(q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$



# Extend formalism to

Phys.Rev. D 94 Collins, L.G, Prokudin, Sato, Rogers, Wang

$$q_T \gtrsim Q$$

Modification of the cross section leaves the standard treatment of TMD factorization only slightly modified.

In particular the op. definitions along with evolution properties are the same as in the usual formalism

**We do this in two steps however now we need explicit expression for *W from JCC* formalism**

**see Collins Rogers PRD 2015**

# Summary of elements of TMD factorization

$$W(q_T,Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T\cdot b_T} \tilde{W}(b_T,Q)$$

- Factorization and TMD evolution in $b_T$ *space*
- Solve the CSS & RG evolution Eqs for W term in SIDIS with "boundary condition" to freeze $b_T$ above some $b_{max}$ and with BCs

$$b_*(b_T) = \sqrt{\frac{b_T^2}{1+b_T^2/b_{max}}}$$

$$\tilde{W}(q_T,Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T\cdot b_T} \tilde{W}^{OPE}(b_*(b_T),Q)\, \tilde{W}_{NP}(b_T,Q;b_{max})$$

$$\tilde{W}_i^{OPE}(b_*(b_T),Q) = H_i(Q)\ \tilde{C}^{pdf}_{i/i'}(x_A/\hat{x}, b_* b_\star) \otimes \tilde{f}_{i'/A}(\hat{x},\mu_{b_\star})\ \tilde{C}^{ff}_{j'/i}(z_B/\hat{z}, b_*) \otimes \tilde{d}_{B/i'}(\hat{z},\mu_b) e^{-S^{pert}(b_*,Q)}$$

**Collinear pdfs**

$$\tilde{W}_{NP}(b_T,Q;b_{max}) = e^{-S_{NP}(b_T,Q;b_{max})}$$

Aidala, Field, Gamberg, Rogers PRD 2015

$$g_K(b_T;b_{\max}) = \frac{g_2(b_{\max}) b_{\rm NP}^2}{2} \ln\left(1+\frac{b_T^2}{b_{\rm NP}^2}\right)$$

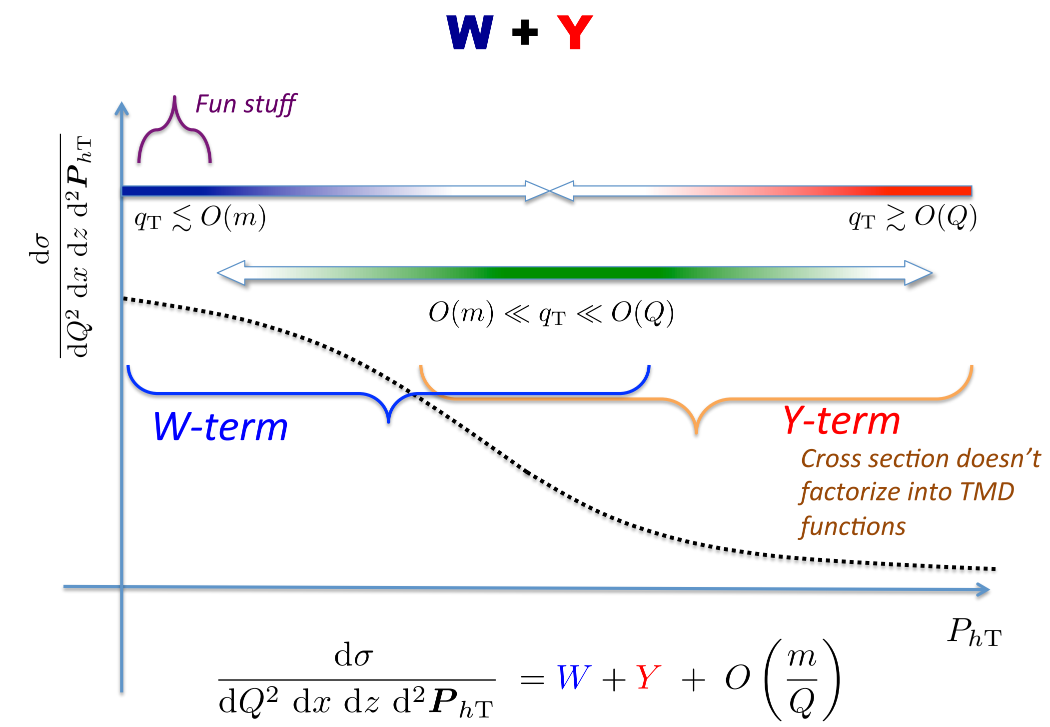
$$S_{NP}(b_T,Q;b_{max}) = g_A(x_A,b_T;b_{max}) + g_B(z_B,b_T;b_{max}) - 2g_K(b_T;b_{max}) \ln\left(\frac{Q}{Q_0}\right)$$

**Fourier Transforms of TMDs and universal soft function $g_K$**

# Two modifications

## a) Introduce small b-cuttoff

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$




## b) Introduce large $q_T$-cuttoff so that $W_{\mathrm{New}}$ vanishes at large $q_T$

$$\Xi\left(\frac{q_T}{Q}, \eta\right) = \exp\left[-\left(\frac{q_T}{\eta Q}\right)^{a_\Xi}\right]$$

$$\tilde{W}_{New}(q_T, Q; \eta, C_5) = \Xi\left(\frac{q_T}{Q}, \eta\right) \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T}\, \tilde{W}^{OPE}\left(b_*(b_c(b_T)), Q\right) \tilde{W}_{NP}(b_c(b_T)), Q; b_{max})$$

## B.C.

$$b_*(b_c(b_{\mathrm{T}})) \longrightarrow \begin{cases} b_{\min} & b_{\mathrm{T}} \ll b_{\min} \\ b_{\mathrm{T}} & b_{\min} \ll b_{\mathrm{T}} \ll b_{\max} \\ b_{\max} & b_{\mathrm{T}} \gg b_{\max}\,. \end{cases}$$

# i) Semi-inclusive to Collinear integrate over $q_T$

- Parton Model W-term

$$W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) \int d^2k_T f_{j'/A}(x, k_T) d_{B/i'}(z, q_T + k_T)$$

$$\int d^2q_T W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

Underlies Model building
w/ and w/o evolution using TMD and collinear
evolution approach   Anselmino et al. 2005-2016

- Standard CSS W-term

$$W_{CSS}(q_T, Q) = \int \frac{d^2b_T}{(2\pi)^2} e^{iq_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2q_T W_{CSS}(q_T, Q) = 0 \quad !$$

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**See appendix for details Phys.Rev. D 94 (2016)**
**J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang**

$$W_{CSS}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = \int \delta^2(b_T)\, b_T^a \times \text{logarithmic corrections}$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = 0 \quad !$$

**For details Phys.Rev. D 94 (2016)**
**Collins, Gamberg, Prokudin, Sato, Rogers, Wang**

$$W_{New}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{New}(b_T, Q)$$

$$\int d^2 q_T W_{New}(q_T, Q) = \tilde{W}(b_{c\ min}, Q)$$

$$\int d^2 q_T \, W_{New}(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \; + O(\alpha_s(Q))$$

**Has a normal collinear factorization in terms of collinear pdfs**

$$\int d^2 q_T \, W_{New}(q_T, Q) + Y(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \; + O(\alpha_s(Q))$$

$$+ \text{ terms dominated by large } q_T \text{ contribution to } Y \text{ term}$$

***Has implications for modeling TMD and fitting***

## Large $q_T$-cuttoff so on $W_{\rm New}$ vanishes at large $q_T$

b) Introduce large $q_T$-cuttoff so that $W_{\rm New}$ vanishes at large $q_T$

$$\Xi\left(\frac{q_T}{Q},\eta\right) = \exp\left[-\left(\frac{q_T}{\eta Q}\right)^{a_\Xi}\right]$$

$$\tilde{W}_{New}(q_T,Q;\eta,C_5) = \Xi\left(\frac{q_T}{Q},\eta\right)\int\frac{d^2b_T}{(2\pi)^2}e^{iq_T\cdot b_T}\tilde{W}^{OPE}\left(b_*(b_c(b_T)),Q\right)\tilde{W}_{NP}(b_c(b_T)),Q;b_{max})$$

$$b_*(b_c(b_{\rm T})) \longrightarrow \begin{cases} b_{\rm min} & b_{\rm T} \ll b_{\rm min} \\ b_{\rm T} & b_{\rm min} \ll b_{\rm T} \ll b_{\rm max} \\ b_{\rm max} & b_{\rm T} \gg b_{\rm max}\,. \end{cases}$$

# Now $Y$ term is further modified

$$Y_{New}(q_T, Q) = [T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}^{New}\, d\sigma(q_T, Q)]\, X(q_T/\lambda)$$

$$= [FO(q_T, Q) - ASY_{New}(q_T, Q)]\, X(q_T/\lambda)$$

## Putting all together

$$d\sigma(q_T,Q) \approx T_{TMD}^{New} d\sigma(q_T,Q) + T_{coll}\left[\, d\sigma(q_T,Q) - T_{TMD}^{New}\, d\sigma(q_T,Q)\right]$$

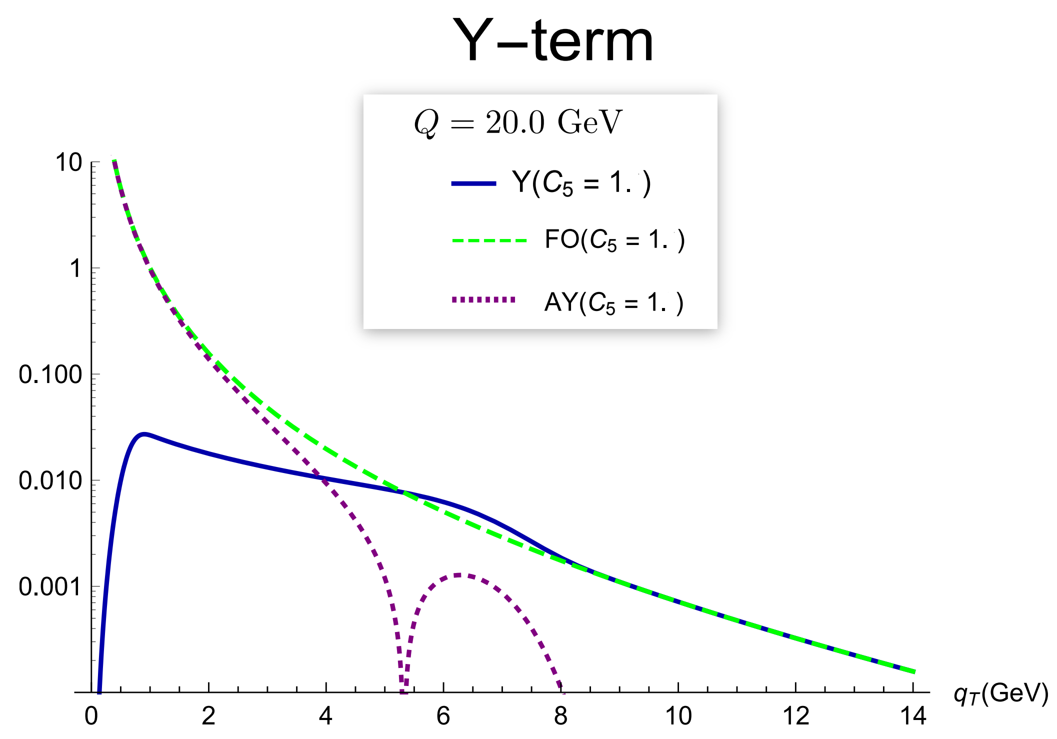
$$+ O\left(\frac{m}{Q}\right)^c d\sigma(q_T,Q)$$

or

$$d\sigma(q_T,Q) \approx W_{New}(q_T,Q) + Y_{New}(q_T,Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T,Q)$$

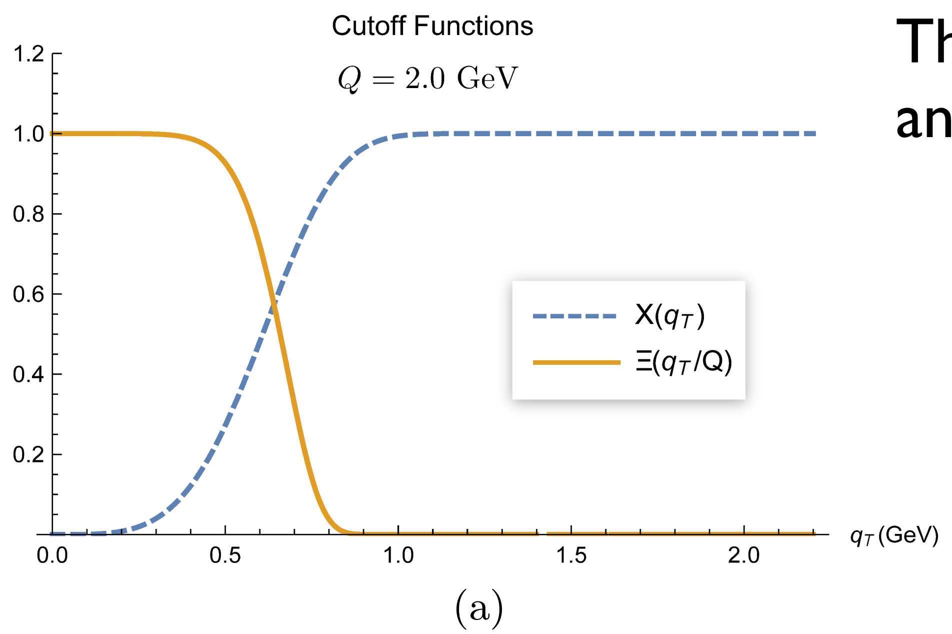
# Putting all together demonstration

To illustrate the steps above, we have performed sample calculations of the Y -term using analytic approximations for the collinear pdfs and collinear ffs. We consider only the target up-quark gamma q to qg channel, and for the running alpha_s we use the two-loop beta function f = 3 since we are mainly interested in the transition to low Q. Thus we use Lambda_QCD = 0.339 GeV [27]. To further simplify our calculations, we use analytic expressions for the collinear correlation functions, taken from appendix A1 of Ref. [28] for the up-quark pdf and from Eq. (A4) of Ref. [29] for the up-quark-to-pion fragmentation function.

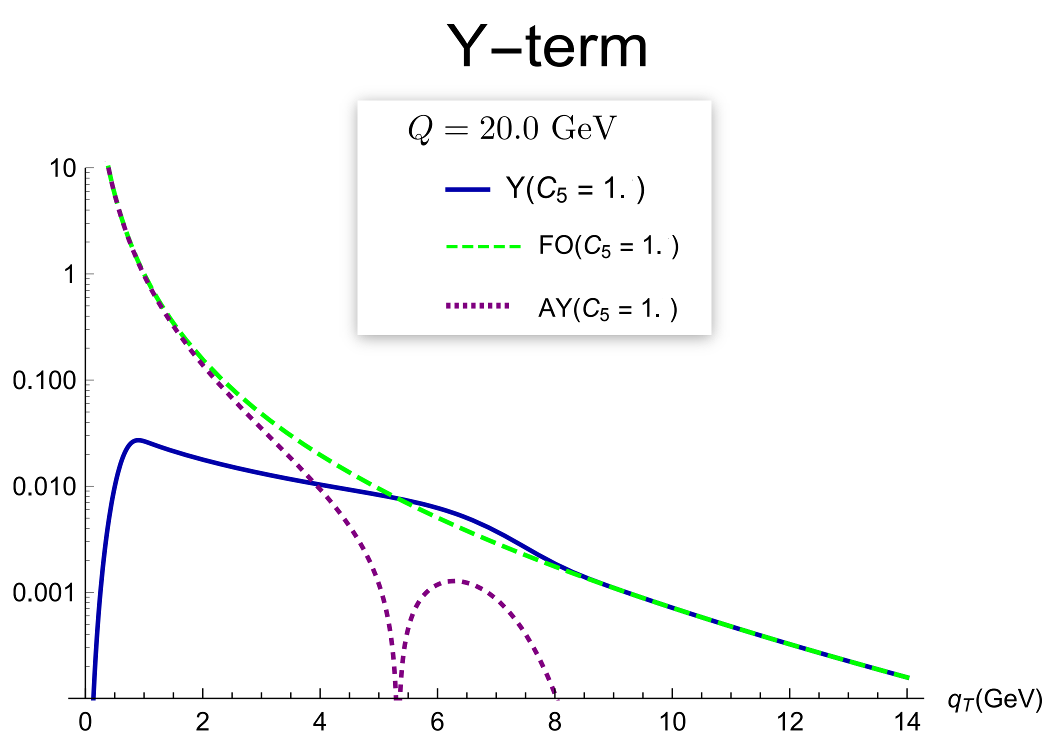



$\lambda$

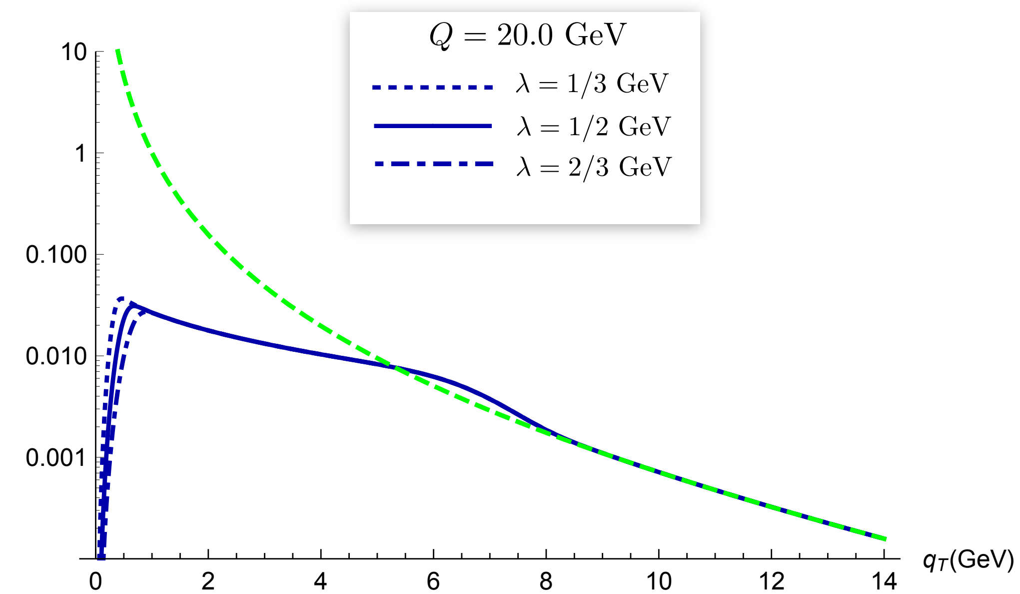
# Putting all together demonstration




The cutoff functions in for low $q_T$/lambda (blue dashed line) and large qT/Q (brown solid line) for Q = 20.0 GeV




## Insenstive to $\lambda$

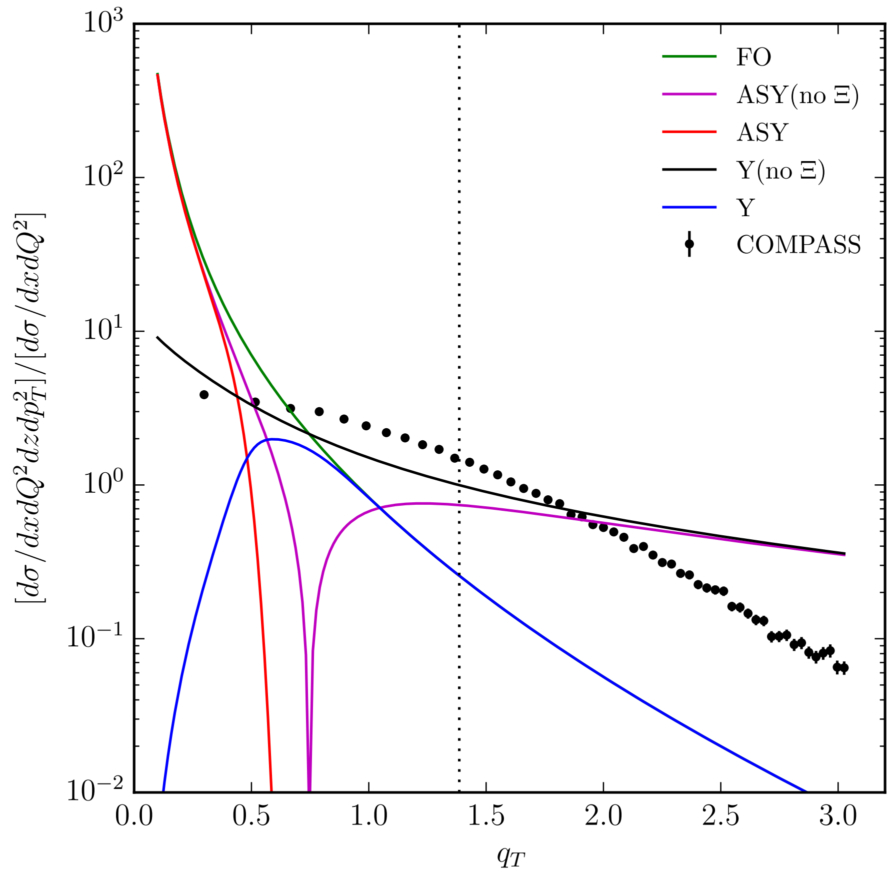


# Comments

- With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two being suppressed by powers of 1/Q.
- Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used.
- This work has dealt only with unpolarized cross sections.
- We are studying the analogous topic applied to polarized phenomena.
- This is central to the EIC and studying the 3-D momentum and spatial structure of the nucleon and further exploring the connection between TMD and collinear factorization

# Matching with fixed-order calculations

*Collins et al., arXiv: 1605.00671*

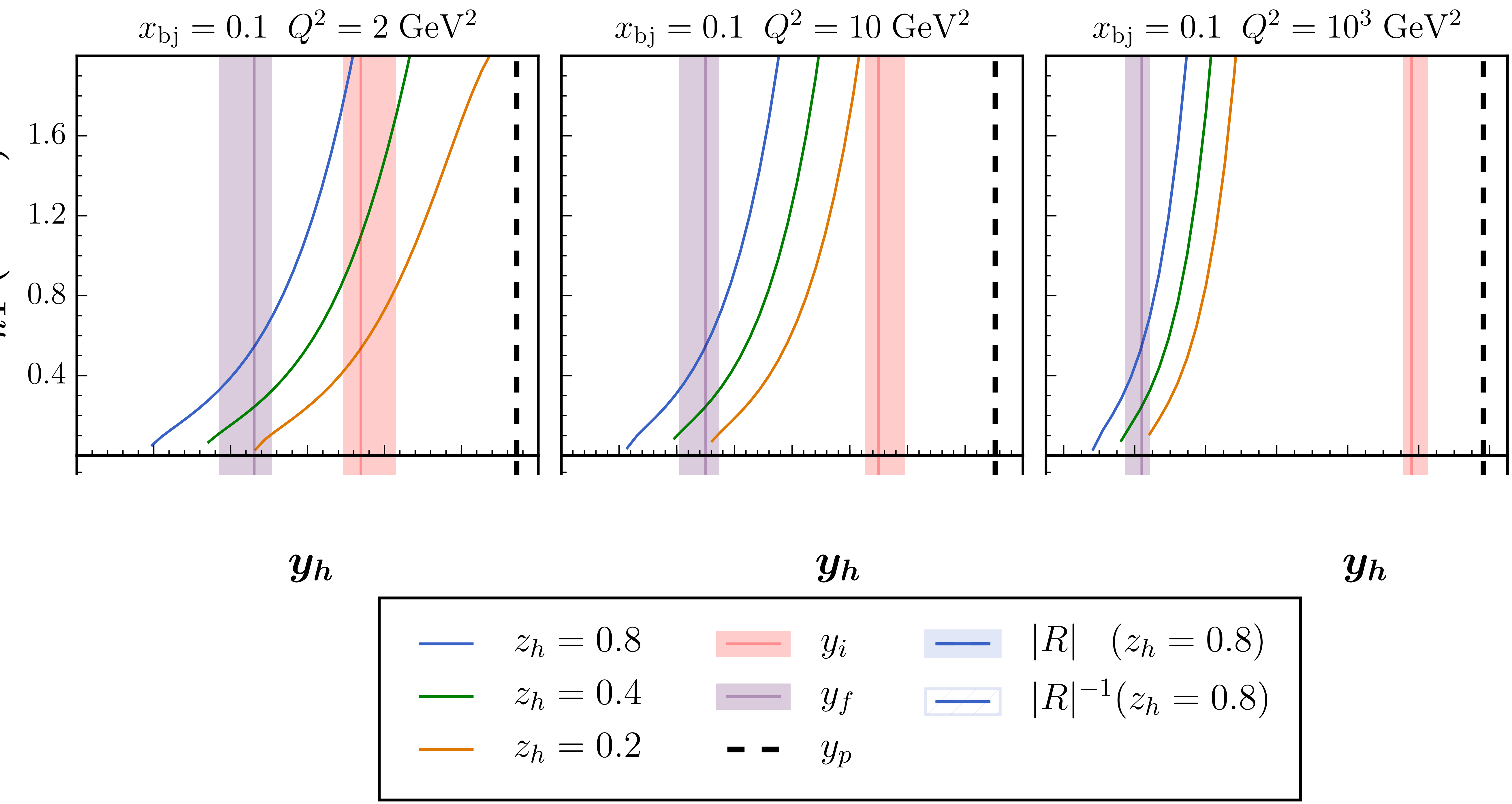



$Q^2 = 1.92\text{GeV}^2, x = 0.0318, z = 0.375$

The collinear calculation (green line) is much smaller than data
Standard Y term is bigger than data (black line) ➞ modifications needed (blue line)

# EXTRA Slides

**Kinematics of Current Region Fragmentation in Semi-Inclusive Deeply Inelastic Scattering**
**M. Boglione, Collins, Gamberg, Gonzalez-Hernandez, Rogers, Sato**
**To appear today/tomorrow ...**



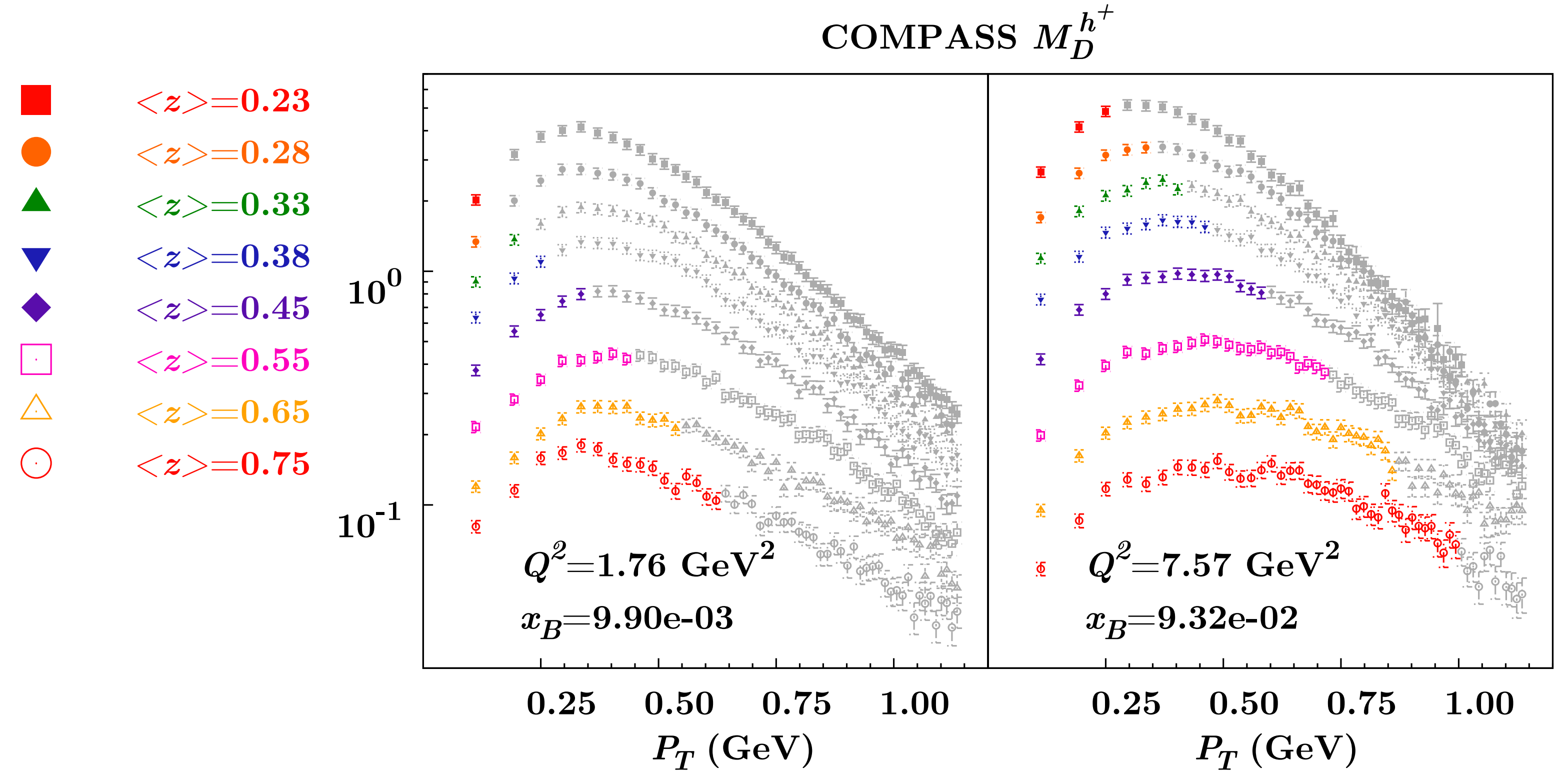



Figure 4: A selection of COMPASS data from [23]. The colored points correspond to the hadron moving with rapidity smaller than some maximum value, which has been chosen to be a quarter-way between the largest estimate of $y_{\rm f}$ and the value of $y_{\rm h}$ for which $R = 1$. This ensures that for $Q^2 \sim 10\,{\rm GeV}^2$, $R \lesssim 0.25$. Within our rough order of magnitude estimate, grey points are likely to receive important contributions from non-current regions. For detailed phenomenological calculations, it is important to improve the estimates of Eq. (26) by more precise constraints on $M_{\rm iT}$ and $M_{\rm fT}$, and also to use a range of rapidity cutoffs.

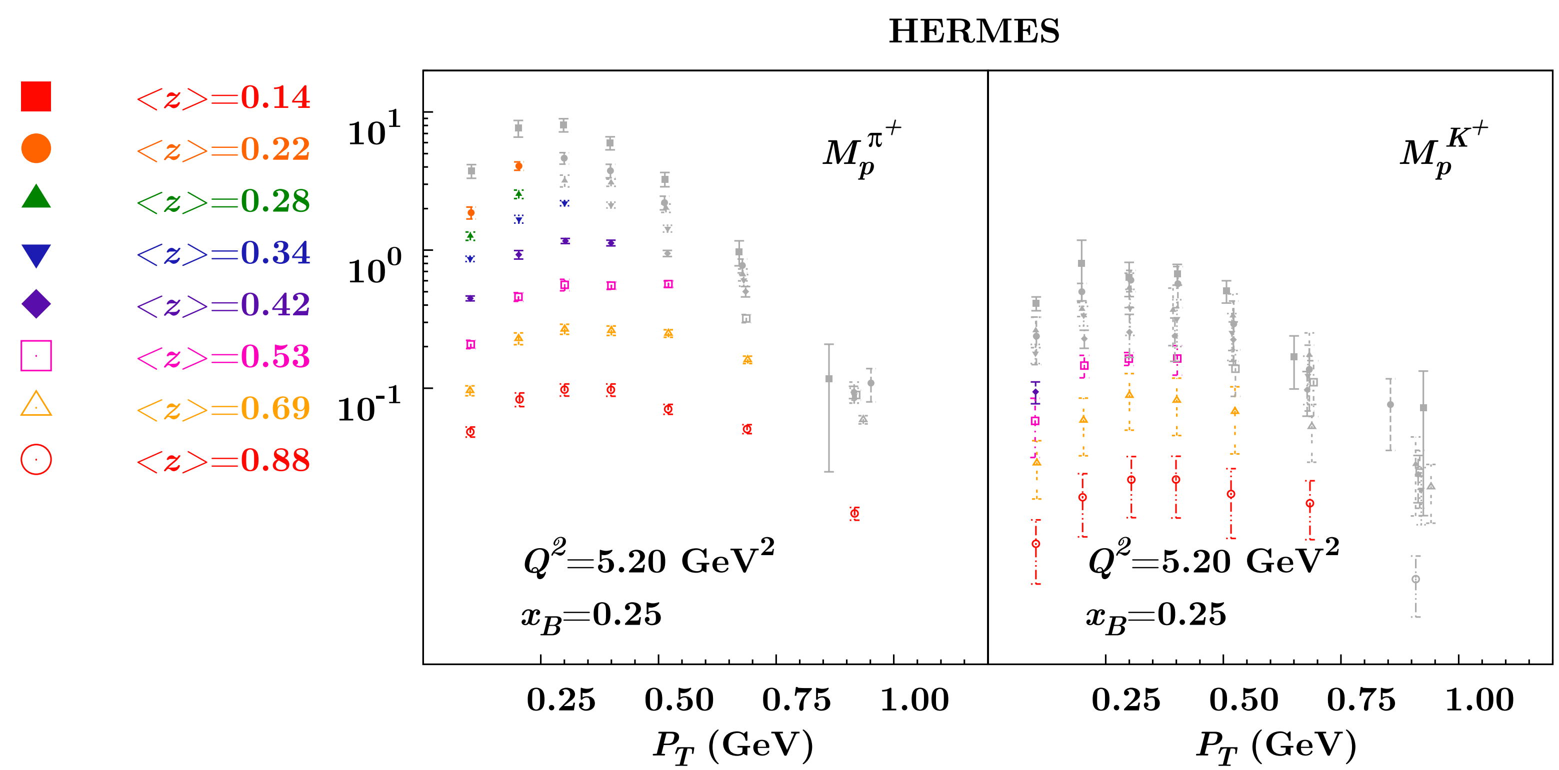
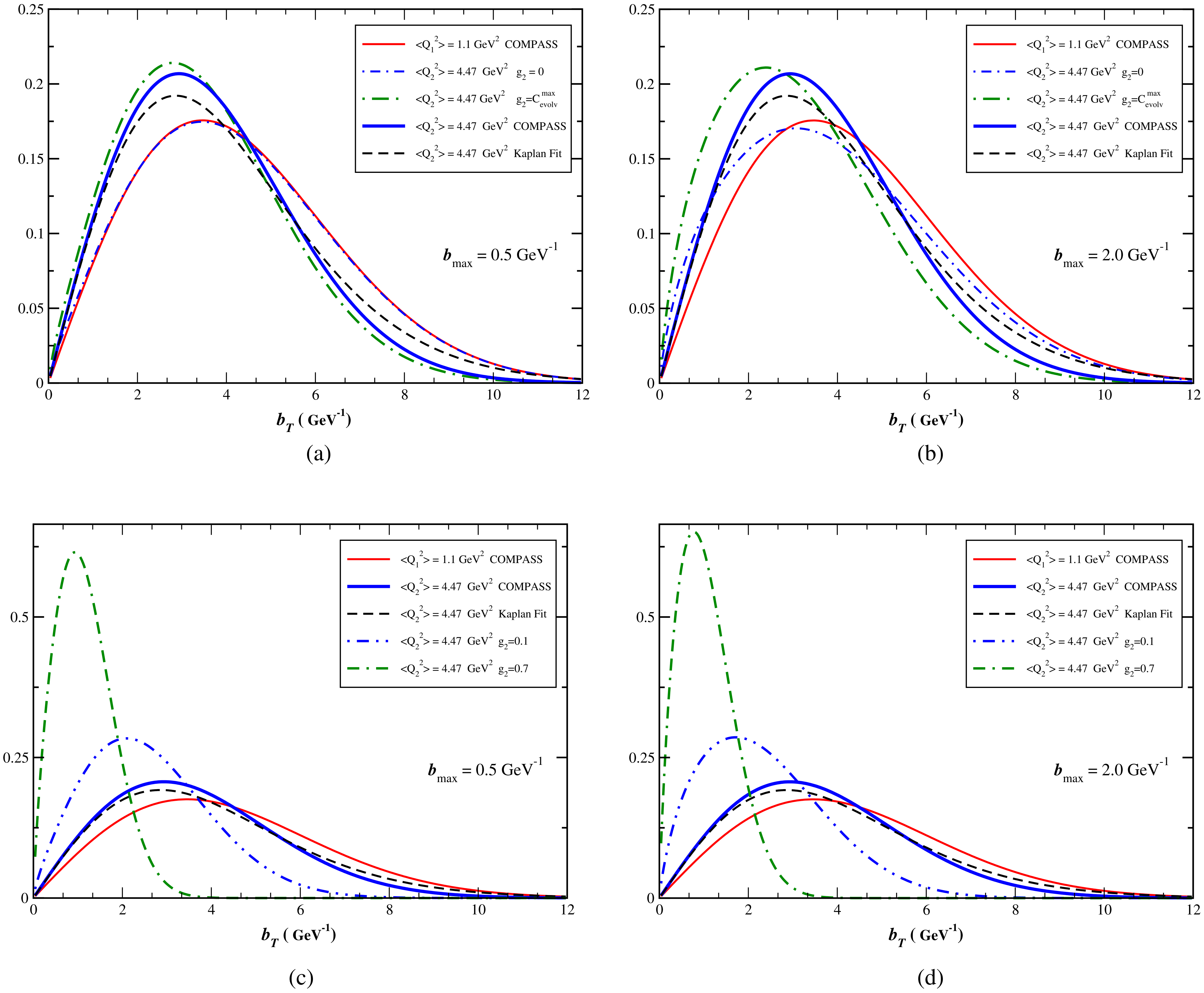



Figure 5: A selection of HERMES data from [24]. Points are as described in Fig. 4. The larger mass of the kaon results in a larger number of points that are likely to receive significant contributions from the non-current regions, within our rough order of magnitude estimate. For detailed phenomenological calculations, it is important to improve the estimates of Eq. (26) by more precise constraints on $M_{\rm iT}$ and $M_{\rm fT}$, and also to use a range of rapidity cutoffs.

# TMD Evolution and COMPASS Data

Aidala, Field, Gamberg, Rogers PRD 2015

$$g_K(b_T; b_{\max}) = \frac{g_2(b_{\max}) b_{\rm NP}^2}{2} \ln\left(1 + \frac{b_T^2}{b_{\rm NP}^2}\right).$$



# Expression for $W(b_c,Q)$

$$\tilde{W}(b_c(b_{\rm T}),Q) = H(\mu_Q,Q)\sum_{j'i'}\int_{x_A}^{1}\frac{d\hat{x}}{\hat{x}}\tilde{C}^{\rm pdf}_{j/j'}(x_A/\hat{x},b_*(b_c(b_{\rm T}));\bar{\mu}^2,\bar{\mu},\alpha_s(\bar{\mu}))f_{j'/A}(\hat{x};\bar{\mu})\times$$
$$\times\int_{z_B}^{1}\frac{d\hat{z}}{\hat{z}^3}\tilde{C}^{\rm ff}_{i'/j}(z_B/\hat{z},b_*(b_c(b_{\rm T}));\bar{\mu}^2,\bar{\mu},\alpha_s(\bar{\mu}))d_{B/i'}(\hat{z};\bar{\mu})\times$$
$$\times\exp\left\{\ln\frac{Q^2}{\bar{\mu}^2}\tilde{K}(b_*(b_c(b_{\rm T}));\bar{\mu})+\int_{\bar{\mu}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1)-\ln\frac{Q^2}{{\mu'}^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$
$$\times\exp\left\{-g_A(x_A,b_c(b_{\rm T});b_{\rm max})-g_B(z_B,b_c(b_{\rm T});b_{\rm max})-2g_K(b_c(b_{\rm T});b_{\rm max})\ln\left(\frac{Q}{Q_0}\right)\right\}$$

## Boundary conditions

$$b_*(b_c(b_{\rm T}))\longrightarrow\begin{cases}b_{\rm min} & b_{\rm T}\ll b_{\rm min}\\ b_{\rm T} & b_{\rm min}\ll b_{\rm T}\ll b_{\rm max}\\ b_{\rm max} & b_{\rm T}\gg b_{\rm max}\,.\end{cases}$$