Beam-Beam in Lepton Colliders

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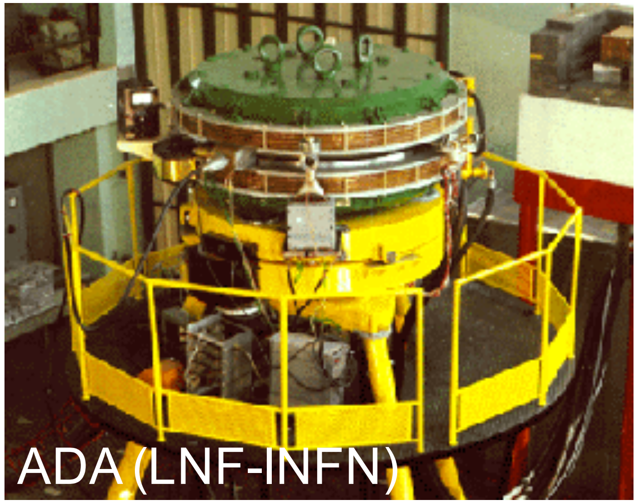
Colliders

Colliding beams stored in circular accelerators is an idea dating back roughly to 1960 when the first test accelerator machines have been built:

electron/electron, Princeton-Stanford, 1957
ADA, the first electron/positron collider, LNF-INFN, 1962
VEP1**,** electron/electron, Novosibirsk, 1964



Since then a lot of efforts in order to achieve highest Energy and Luminosity frontiers.
Large part of these studies have been addressed to understand model and keep under control **Beam-Beam interaction**

Colliders

Colliders are built and used to implement small impact parameter crashes between beams in order to produce elementary particles

Beams consist of huge ensemble of particles

- only few of them collide and produce new physics
- largest part of them experience perturbations with respect to their original motion due to electromagnetic forces -> ***Beam-Beam Interaction***

Main parameters characterizing a collider are:

energy **E**

kind of particles collided (leptons, hadrons or mixed)

Luminosity ***L***

Strength of beam-beam interaction ξ

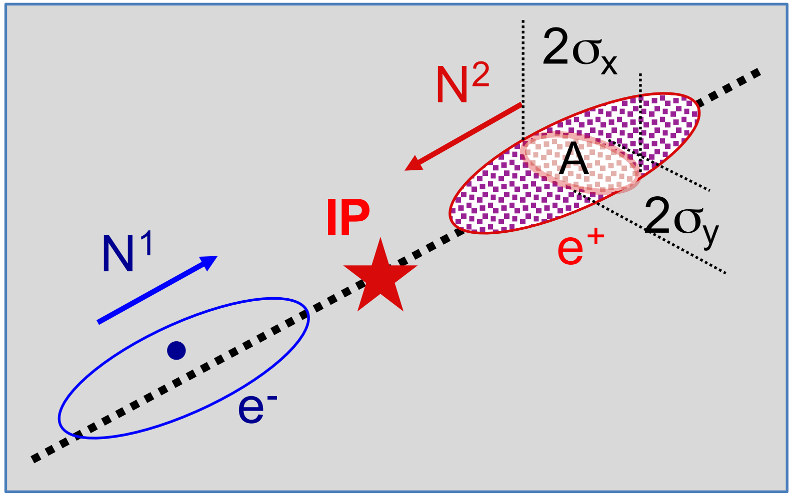
Luminosity

Considering $\boldsymbol{\sigma_p}$ the cross-section of the process of interest the the rate of the particles produced by a collider $\dot{N}_p$ is

$$\dot{N}_p = \sigma_P L \qquad L\left[\frac{1}{cm^2 s}\right] = L\left[\frac{10^{33}}{nb\ s}\right]$$

L summarizes how the collider performs

Processes with $\boldsymbol{\sigma_p} \ll 1$ are studied that is why higher and higher luminosities are required



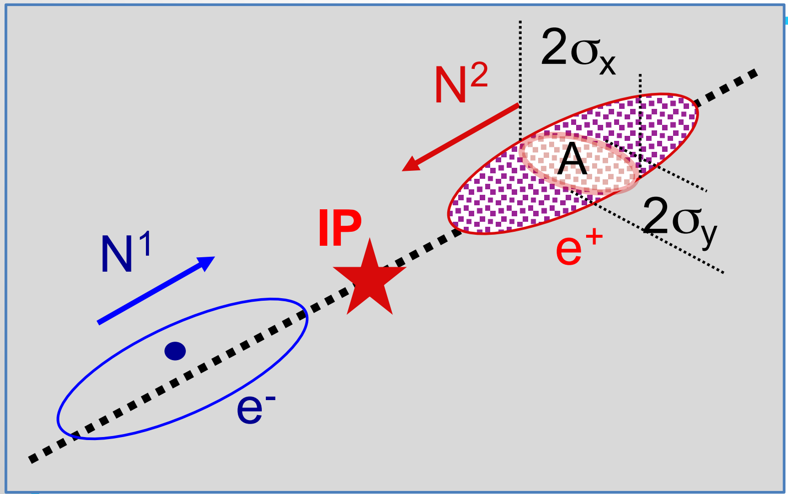
Assuming:

- head on collisions
- two-dimensional Gaussian distribution
- e^+ and e^- beam have the same $\sigma^*_{x,y}$ and same velocity $|v_1| = |v_2|$
- particles longitudinally distributed are projected onto a transverse section **A**

$$\frac{\partial^2 N_2}{\partial x \partial y} = \frac{N_2}{2\pi\sigma_x^*\sigma_y^*} e^{\left(-\frac{x^2}{2\sigma^{*2}_x} - \frac{y^2}{2\sigma^{*2}_y}\right)}$$

e^+ surface density on A

Luminosity



$$\frac{\partial^2 N_2}{\partial x \partial y} = \frac{N_2}{2\pi\sigma_x^*\sigma_y^*} e^{\left(-\frac{x^2}{2\sigma^{*2}_x}-\frac{y^2}{2\sigma^{*2}_y}\right)}$$

e^+ surface density on A

dW is the probability a particle on *dA = dxdy* of the e^- beam collides with an e^+

$$dW = \sigma_P \frac{\partial^2 N_2}{\partial x \partial y}$$

$d\dot{N}_1$ is the e^- number crossing dA surface of the e^+ beam per unit time

$$d\dot{N}_1 = \frac{bf_r N_1}{2\pi\sigma_x^*\sigma_y^*} e^{\left(-\frac{x^2}{2\sigma^{*2}_x}-\frac{y^2}{2\sigma^{*2}_y}\right)} dxdy$$

$$d\dot{N}_p = \sigma_p \frac{bf_r N_1 N_2}{(2\pi)^2 \sigma_x^{*2}\sigma_y^{*2}} e^{\left(-\frac{x^2}{2\sigma^{*2}_x}-\frac{y^2}{2\sigma^{*2}_y}\right)} dxdy$$

differential event rate and by integration

$$\dot{N}_p = \sigma_p \frac{bf_r N_1 N_2}{4\pi\sigma_x^*\sigma_y^*}$$

$$L = \frac{bf_r N_1 N_2}{4\pi\sigma_x^*\sigma_y^*}$$

$$L = \frac{1}{4\pi e^2 f_r b} \frac{I_1 I_2}{\sigma_x^* \sigma_z^*}$$

High L requires:

- high beam currents
- small transverse beam sizes

Space Charge effect

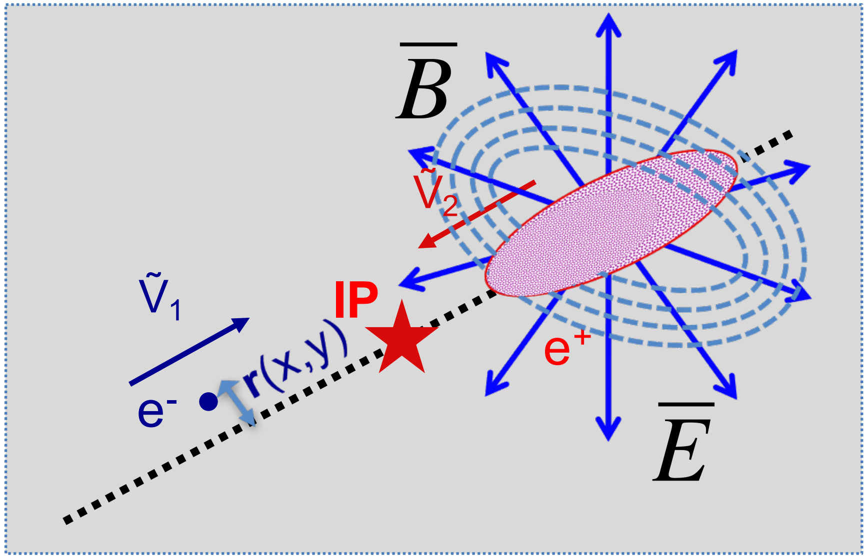
A bunch is an ensemble of charged particles it generates an electromagnetic (EM) potential acting on other charged particles

In the center of mass frame of the bunch $\mathcal{F}'$ only an electrostatic field is generated

Moving to the laboratory frame $\mathcal{F}$ Lorentz transformation gives rise to both Electric and Magnetic field

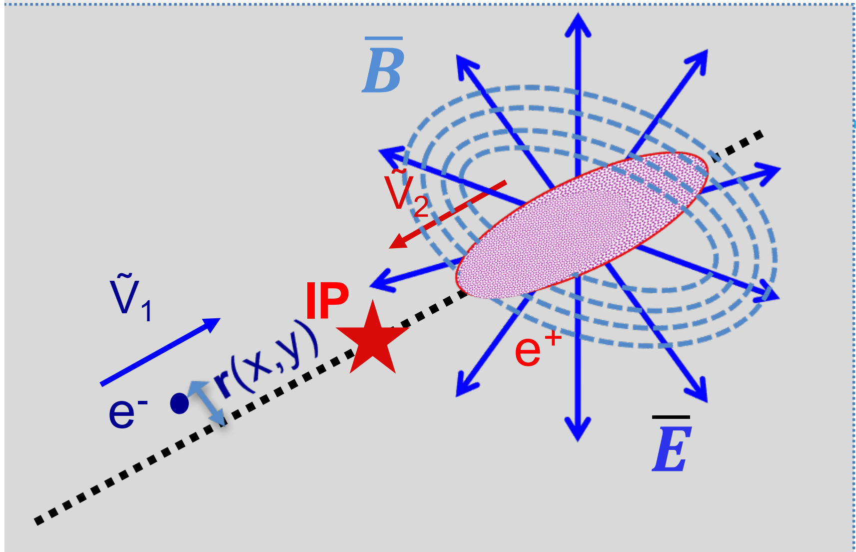
The EM field created by a bunch acts on:

- bunch itself (***main space charge effect***)
- opposite bunch (***beam-beam effect***)



Since early years Space Charge effect has been recognized has a main source of current limitation in colliders and initially named **Amman-Ritson effect**

Beam-Beam Force



EM field of a single e^- moving from $\mathcal{F}'$ to $\mathcal{F}$ reference system

$$E_\perp = \gamma E'_\perp \qquad E_{//} = E'_{//}$$

$$B_\perp = \frac{\gamma}{c^2} \mathbf{v}_2 \times E'_\perp \qquad B_{//} = 0$$

$$F_\perp = -e\left(E_\perp + \mathbf{v}_1 \times B_\perp\right) = -e\left(1 + \beta_1\beta_2\right)E_\perp \approx -2eE_\perp$$

bunch Gaussian distribution in $\mathcal{F}'$

$$\rho'(x,y,s') = \frac{eN_2}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_{s'}} e^{\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{(s'-s'_0)^2}{2\sigma_{s'}^2}\right)}$$

$$\sigma_{x,y} = \sigma'_{x,y} \qquad \sigma'_s = \gamma\sigma_s \qquad \sigma^*_{x,y}(e^-) = \sigma^*_{x,y}(e^+)$$

In $\mathcal{F}'$ $\sigma'_s >> \sigma_{x,y}$

E field at the e^- position is easily evaluated by using charge density and assuming round beam

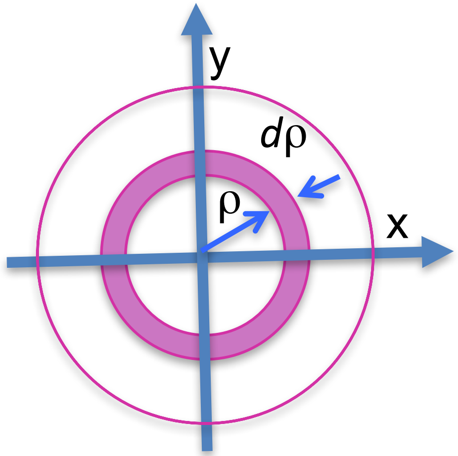
$$\sigma_x = \sigma_y \qquad r^2 = x^2 + y^2$$

$$\rho'(r,s') = A(s')e^{-\frac{r^2}{2\sigma^2}} \qquad A(s') = \frac{eN_2}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma'_s} e^{\left(-\frac{(s'-s'_0)^2}{2\sigma'^2_s}\right)}$$

The charge ***dq*** in a cylindrical shell is:

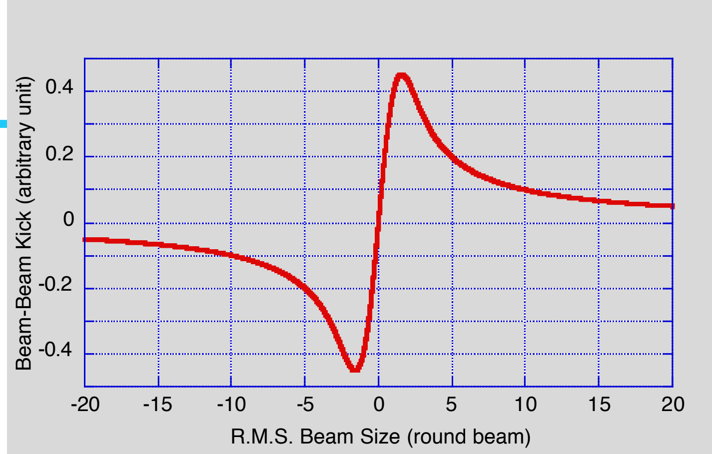
$$dq = 2\pi\rho'(r,s')\rho d\rho ds'$$

then the charge ia a cylinder of length ***Δs'*** and radius ***r***



integrating and using the Gauss's theorem ..

Beam-Beam Force



$$E_\perp(r,s) = \gamma E'_\perp(r,s) = \frac{eN_2}{(2\pi)^{3/2}\varepsilon_0 r\sigma_s} e^{\left(-\frac{(s-s)^2}{2\sigma_s^2}\right)} \frac{1}{r}\left(1 - e^{\left(-\frac{r^2}{2\sigma^2}\right)}\right)$$

$$F_\perp(r,s) = -\frac{2e^2N_2}{(2\pi)^{3/2}\varepsilon_0 r\sigma_s} e^{\left(-\frac{(s-s)^2}{2\sigma_s^2}\right)} \frac{1}{r}\left(1 - e^{\left(-\frac{r^2}{2\sigma^2}\right)}\right)$$

$\approx 1 - \frac{r^2}{2\sigma^2}$

first order in r

$$F_\perp(r,s) = -\frac{e^2N_2 r}{(2\pi)^{3/2}\varepsilon_0 r\sigma^2\sigma_s} e^{\left(-\frac{(s-s)^2}{2\sigma_s^2}\right)}$$

The total change of the e^- transverse momentum *$\Delta p_\perp$* due to ***$E_\perp$*** *is*

$$dp_\perp(r) = -\frac{e^2N_2 r}{(2\pi)^{3/2}\varepsilon_0 r\sigma^2\sigma_s} e^{\left(-\frac{(s-s)^2}{2\sigma_s^2}\right)} ds \;\Rightarrow\; \Delta p_\perp(r) = -\frac{e^2N_2}{2\pi\varepsilon_0 c}\frac{r}{2\sigma^2}$$

BB kick causes a change in the e^- trajectory acting as a focusing quadrupole in both directions

B-B deflection

$$\Delta x' = \frac{\Delta p_x}{p} = -\boxed{\frac{e^2N_2}{2\pi\varepsilon_0 pc}\frac{1}{2\sigma^2}}\,x$$

$$\Delta y' = \frac{\Delta p_y}{p} = -\boxed{\frac{e^2N_2}{2\pi\varepsilon_0 pc}\frac{1}{2\sigma^2}}\,y$$

Integrated quadrupole strength

$$\Delta x' = \kappa l\, x$$

Beam-Beam focal length

$$f_{x,y} = \frac{e^2 N_2}{2\pi\varepsilon_0 pc} \frac{1}{2\sigma^2} \rightarrow f_{x,y} = \frac{N_2 r_0}{2\pi\varepsilon_0} \frac{1}{2\sigma^2}$$

$f_{x,y}$ *is* ***focusing*** *in both transverse directions for colliding beams having* ***opposite charge*** *and* ***defocusing*** *for beams having the* ***same charge***

Incoherent Beam-Beam Tune shifts

Linear BB kick modifies the particle one turn map **M -> $\mathbf{M_{BB}}$**

$$M_{BB} = \begin{pmatrix} \cos[2\pi(Q+\Delta Q)] & \beta^* \cdot \sin[2\pi(Q+\Delta Q)] \\ -\dfrac{\sin[2\pi(Q+\Delta Q)]}{\beta^*} & \cos[2\pi(Q+\Delta Q)] \end{pmatrix}$$

$$M_{BB} = Q_{BB} M = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(2\pi Q) & \beta \cdot \sin(2\pi Q) \\ -\sin(2\pi Q)/\beta & \cos(2\pi Q) \end{pmatrix}$$

$$M_{BB} = \begin{pmatrix} \cos(2\pi Q) & \beta \cdot \sin(2\pi Q) \\ -\dfrac{\sin(2\pi Q)}{\beta} - \dfrac{\cos(2\pi Q)}{f} & \cos(2\pi Q) - \dfrac{\beta \sin(2\pi Q)}{f} \end{pmatrix}$$

$$2 \cdot \cos[2\pi(Q+\Delta Q)] = 2\cos(2\pi Q) - \beta \frac{\sin(2\pi Q)}{f}$$

for $\Delta Q \ll 1$

$$2 \cdot \cos[2\pi(Q+\Delta Q)] \approx 2\cos(2\pi Q) - 4\pi \Delta Q \sin(2\pi Q)$$

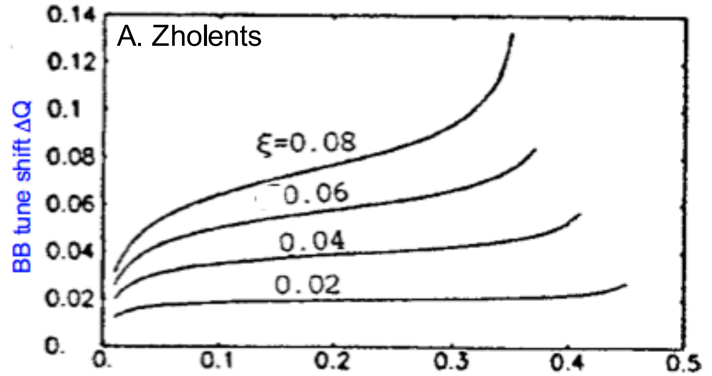
$$\Delta Q_{x,y} \approx \frac{\beta_{x,y}}{4\pi f} = \xi_{x,y}$$

$$\xi_{x,y} = \frac{e^2 N_2}{2\pi\varepsilon_0 pc} \frac{\beta_{x,y}}{2\sigma^2}$$

ξ is the linear BB parameter
(round beam)

$$\frac{\beta}{\beta^*} = \frac{\sin(2\pi Q)}{\sin(2\pi(Q+\Delta Q))}$$

Dynamic β effect



Tune shift dependence on ξ and Q

Q=ν/2π

Stability of the e^- particle

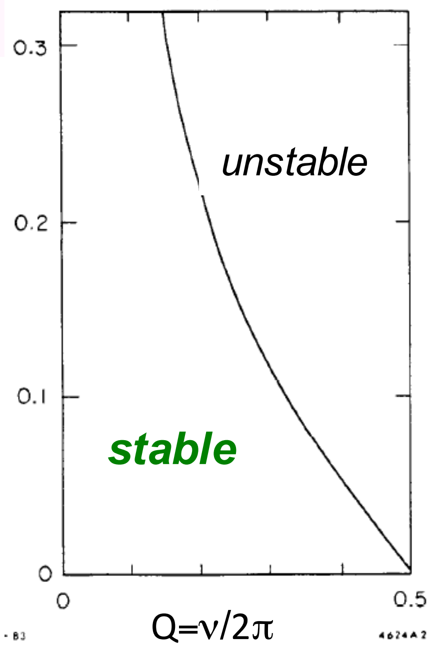
Stability of the particle motion motion requires

$$|Tr(M_{BB})| < 2$$

$$\xi < \begin{cases} \dfrac{1}{2\pi}\cot(\pi Q) & n < \nu < n+0.5 \quad n \in Z \\ -\dfrac{1}{2\pi}\tan(\pi Q) & n+0.5 < \nu < n+1 \quad n \in Z \end{cases}$$

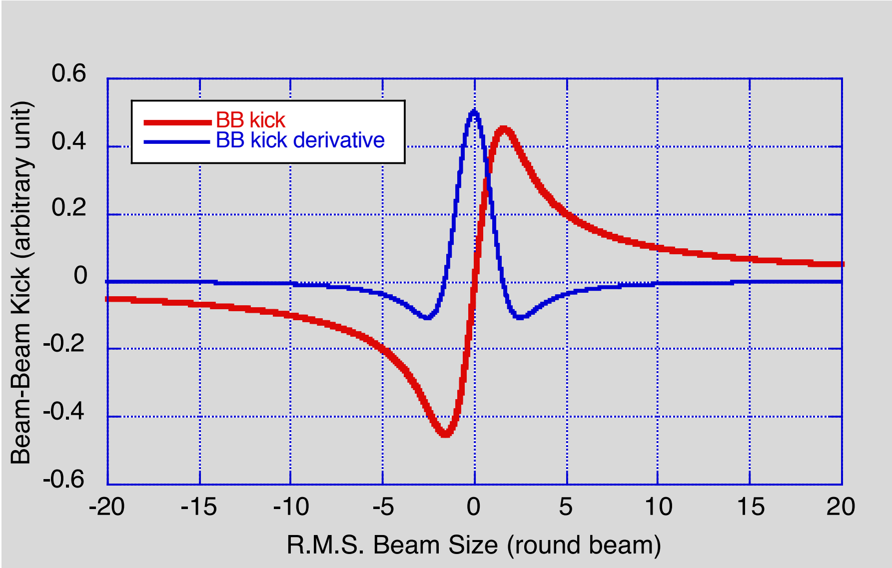
e^- beam is most unstable if Q between collisions is below half integer and it is most stable when is above half integer

Stability condition reverses in case of beams with the same charge



Tune Spread

Horizontal and vertical tune shifts $\Delta Q_{x,y}$ are related to the slope of the BB force $F_{\perp}$



$\Delta Q_{x,y}$ of the e^- is computed averaging the slope of $F_{\perp}$ over the e^- oscillation amplitude

A small amplitude particle experiences linear focusing and

$$\Delta Q_{x,y} = \xi_{x,y}$$

very large amplitude particles have almost no tune shift

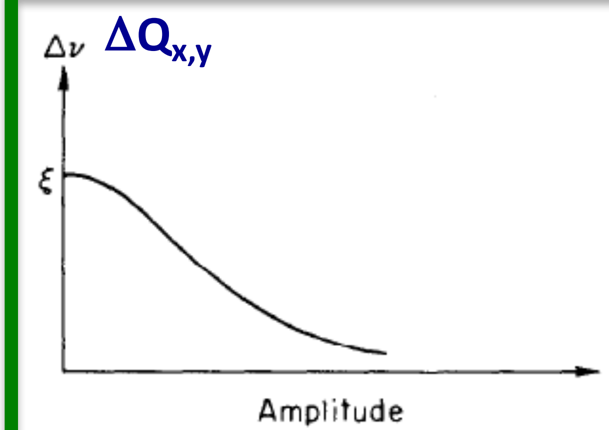
If instead of one e^- the beam contains many particles each of them will have its own tune shift and the tune shifts values will be distributed in the range

$$0 \leq \Delta Q_{x,y} \leq \xi_{x,y}$$

this ***tune spread*** is a direct consequence of

- *non linearity* of the BB interaction
- transverse oscillation amplitude of the particles in the bunch are distributed over a range

Detuning with amplitude



Linear Beam-Beam Parameter

$$\xi_x = -\frac{r_0 N_2}{2\pi\gamma} \frac{\beta^*{}_x}{\sigma_x{}^*(\sigma_x{}^* + \sigma_y{}^*)}$$

$$\xi_y = -\frac{r_0 N_2}{2\pi\gamma} \frac{\beta^*{}_x}{\sigma_y{}^*(\sigma_x{}^* + \sigma_y{}^*)}$$

Linear BB parameter for flat beams
$\xi_{x,y}$ are used to quantify the strength of BB interaction although it does not describe its intrinsic non-linear character

	Energy (GeV)	ξ_x - ξ_y	L (10^{30} $cm^{-2}s^{-1}$)
VEPP-2000	1 GeV	0.075 – 0.075	100
VEPP-4M	6	0.05	20
BEBC	2.5	0.035	5 – 12.6
BEPC-II	1.89 – 2.3	0.0327	649
DAΦNE (Crab-Waist)	0.510	0.044	453
LEP	100 – 104.6	0.083	24 at Z peak 100 > 90 GeV
KEKB	8 (e^-) – 3.5 (e^+)	0.129 – 0.09 (e^-) 0.127 – 0.129 (e^+)	21083
PEP-II	9 (e^-) – 3.1 (e^+)	0.07 – 0.0498 (e^-) 0.051 – 0.073 (e^+)	12069
SuperKEKB	7 (e^-) – 4 (e^+)	0.001 – 0.081 (e^-) 0.003 – 0.088 (e^+)	800000

Data from high energy collider parameters 2013

Tune Spread Modifies the Tune Plane

Unperturbed tunes Q_{x0} Q_{y0} (ν_{x0} ν_{y0}) evolution

$$a = \frac{\sigma_y}{\sigma_x}$$

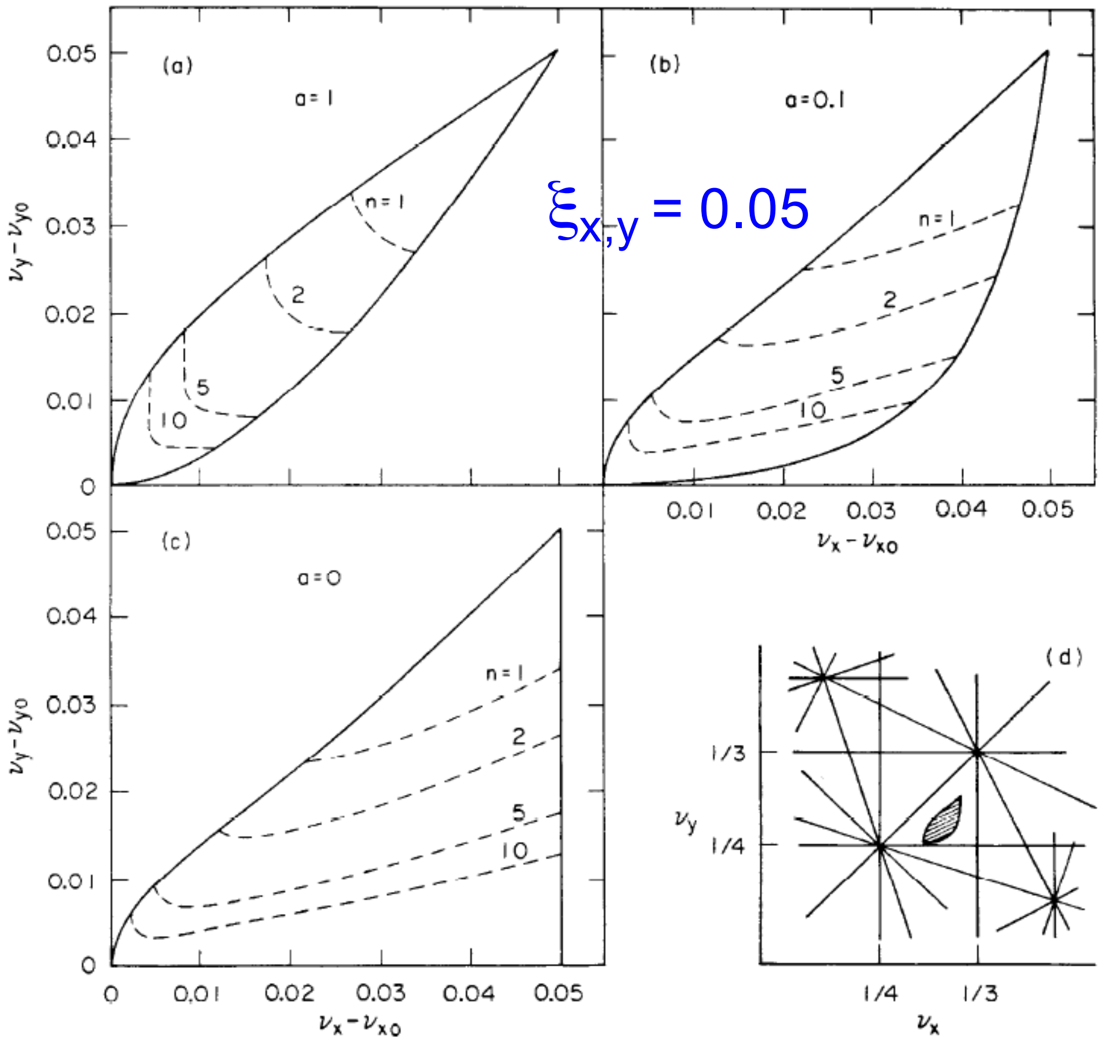
For small amplitude particles

$$Q_{x0} = Q_{x0} + \xi_x$$
$$Q_{y0} = Q_{y0} + \xi_y$$

Large amplitude particles are almost unperturbed

Tune spread leads working point to occupy a wide area

Tune spread must be done as small as necessary to keep the working area confined in a resonance free region



$\xi_{x,y} = 0.05$

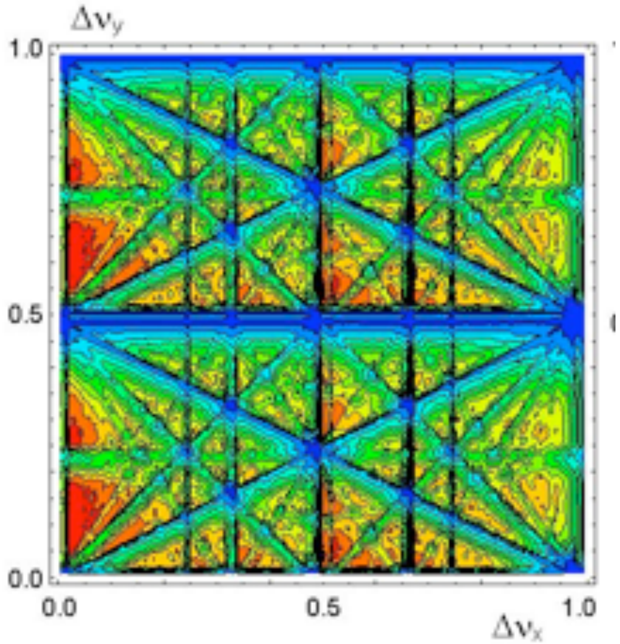
(A. Chao)

Weak Strong Resonances

Synchrotron oscillations and the chromatic dependence of the tunes on energy determine *betatron tune modulations*

Particle motion diffuses in the transverse phase space and some particles can move and remain *trapped* close to the machine physical aperture

When all the resonances are taken into account the tune plane is almost all filled



Higher order resonances are usually weak but their widths may overlap resulting in strong perturbations leading to unstable motion

Strong resonances within the tune spread modify the distribution of the particles in the beam leading to the appearance of non Gaussian tails

These effects are responsible for:

- ***dynamical aperture*** reduction
- poor ***lifetime***
- ***background*** on the detector

Strong Strong BB Interaction

- Perturbation of one beam affects in turn the other beam
- Beam distributions are no longer Gaussian
- The simplest method to approach this case consists in assuming still Gaussian beams and considering *rms* beam sizes at IP dependent on dynamic beta β^* which implies β and ξ depend on one another

Many experimental issues featuring operating collider can be explained in the framework of Strong-Strong BB interaction only

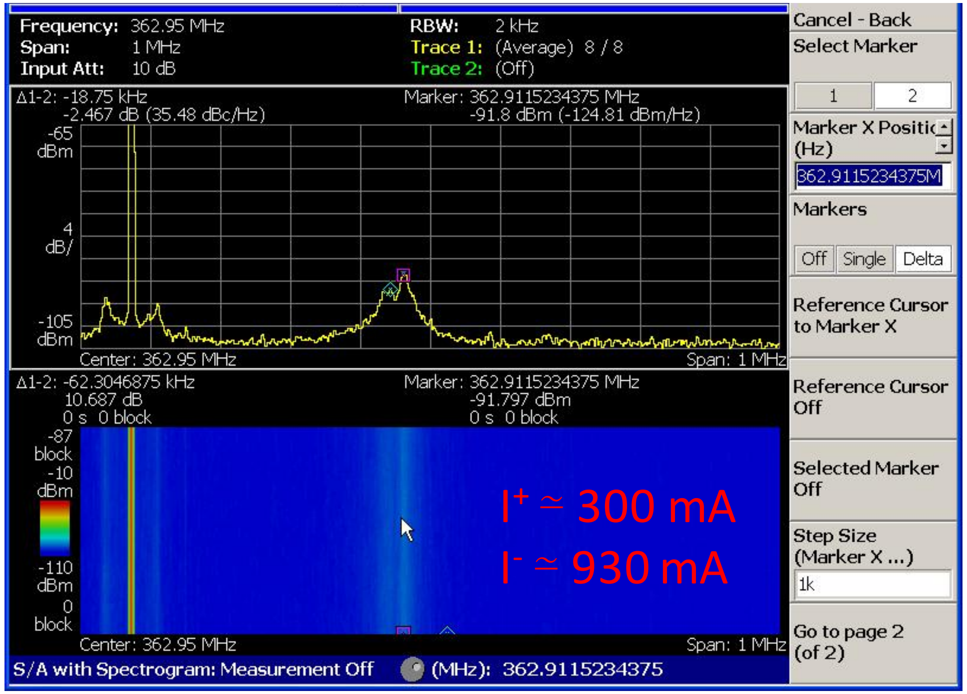
- ***Blow-up*** of σ_y leading to

  $L \propto N$ and $\xi \propto N$
- ***Flip-fop*** effect
- ***Coherent beam centroid motion (0 and π modes)***

Numerical Codes are required to study in a reliable a systematic way such complex interaction

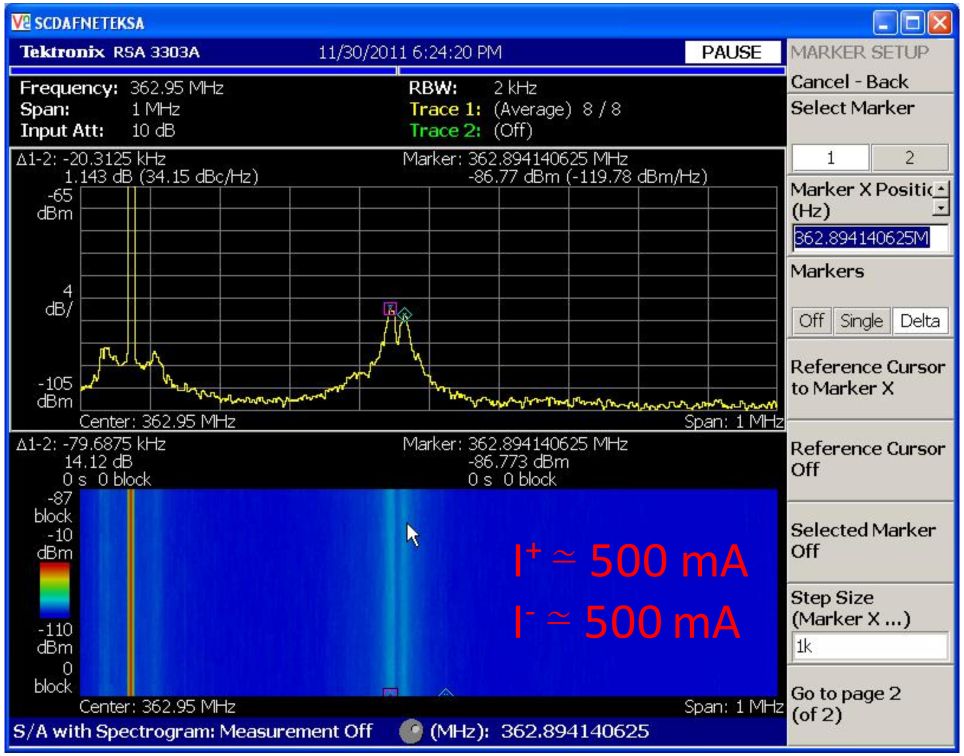
Tune Shift measurement

Horizontal tune shift of the weak **e⁺** beam as measured at DAΦNE by using a spectrum analyzer



Tune Shift measurement

Horizontal tune shift of the weak **e^+** beam as measured at DAΦNE by using a spectrum analyzer



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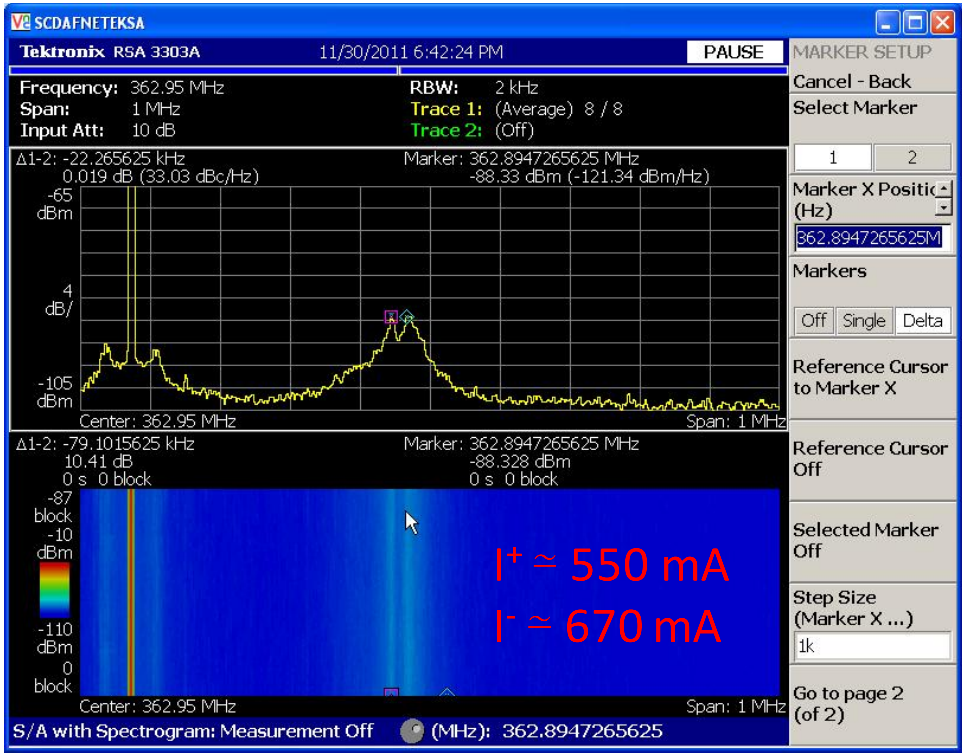

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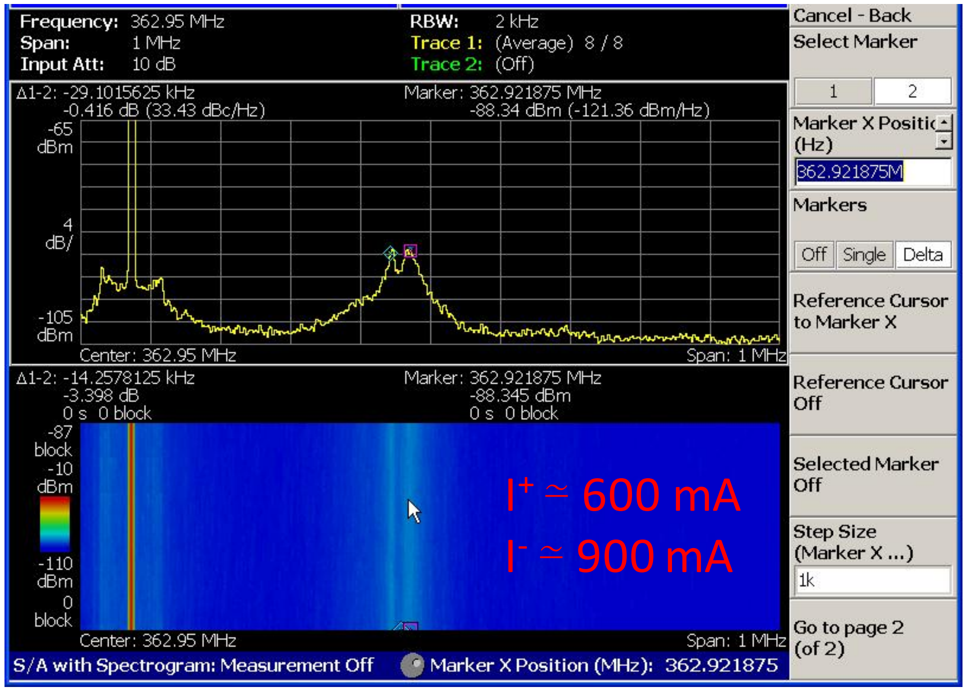


L and ξ

Luminosity as a function of the linear BB parameter

$$L = \frac{2bf_r N \xi_y \gamma}{r_0 \beta_y}\left(1+\frac{\sigma_y}{\sigma_x}\right) \qquad \xi_y \propto N$$

at low current:

$L \propto N^2$ $\sigma_{x,y}$ constant

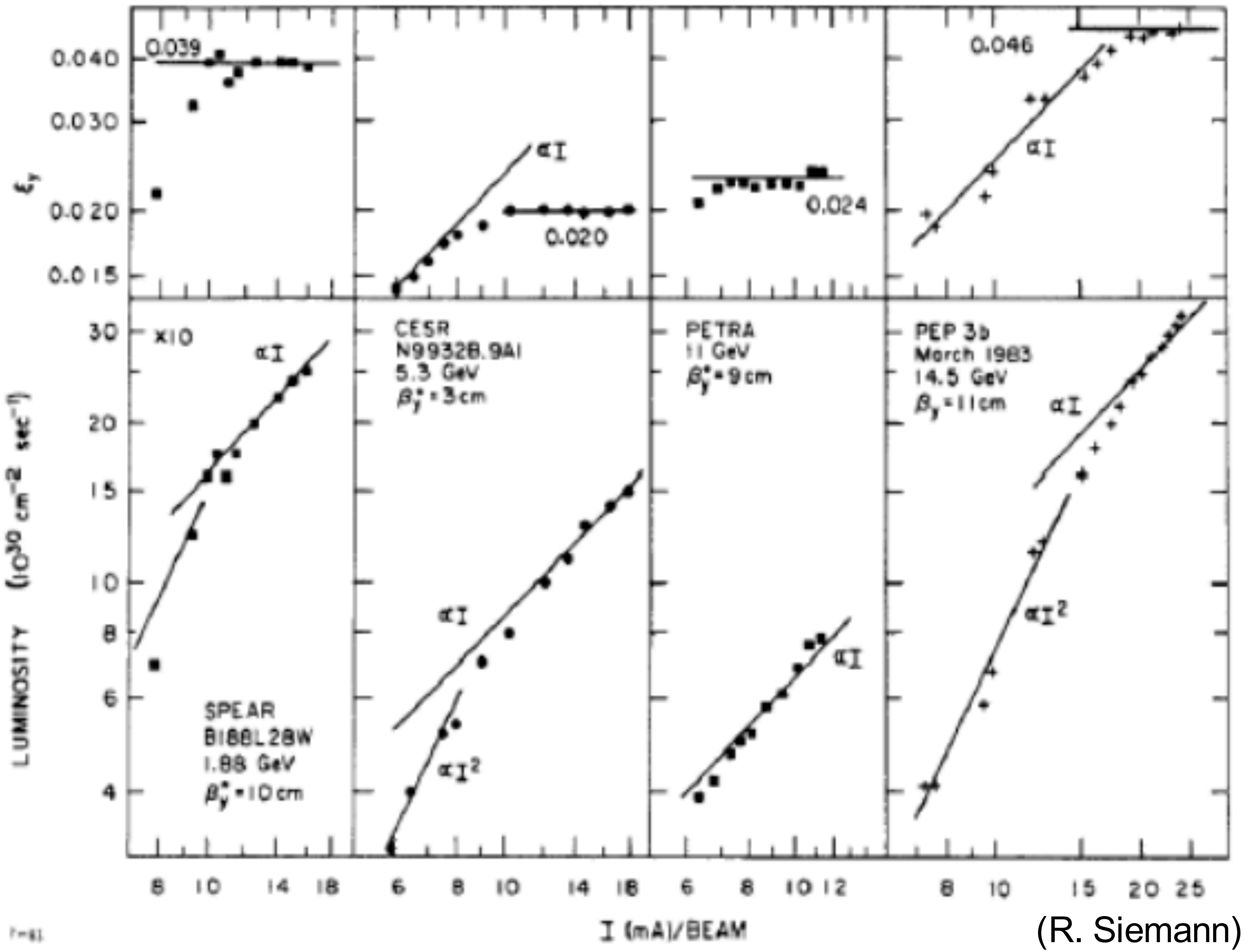
above a given beam current I_{BB}

ξ_y saturates and force $\sigma_y \propto N$

$$L \propto N$$

non Gaussian transverse tails appear and increase linearly till to reach the machine aperture limit with a consequent reduction in τ

Beam-Beam limit



(R. Siemann)

BB Interaction and Other Effects

Beam-Beam interaction interferes with

- *other collective effects typical of colliding beams as the ones induced by:*
  - *vacuum*
  - *ring impedance*
  - *noise due to Feedbacks and RF systems*
- *nonlinearities in the ring lattice*

This additional phenomena make experimental study of BB interaction quite difficoult
There is no BB code including all these additional aspects

Vacuum effects on e^+ beam *e-cloud*

At DAFNE the highest current storable in the e^+ beam is considerably lower than the e^- one

$I^-_{MAX} = 2.4$ A

$I^+_{MAX} = 1.4$ A

Anomalous pressure rises are measured in the e^+ ring and the beam shows:

- vertical beam size increase
- tune spread along the bunch train
- strong horizontal instability

Cure

Electrode for e-cloud mitigation installed inside the dipole vacuum chamber

- Different bunches along the train have different Q_x^0 and Q_y^0
- Instabilities due to e-cloud add up and interfere with the ones proper of the lattice and coming from BB interaction

Tune Spread along the batch due to ***e-cloud***

DAΦNE e^+ beam:

100 bunches, spaced by 2.7 ns

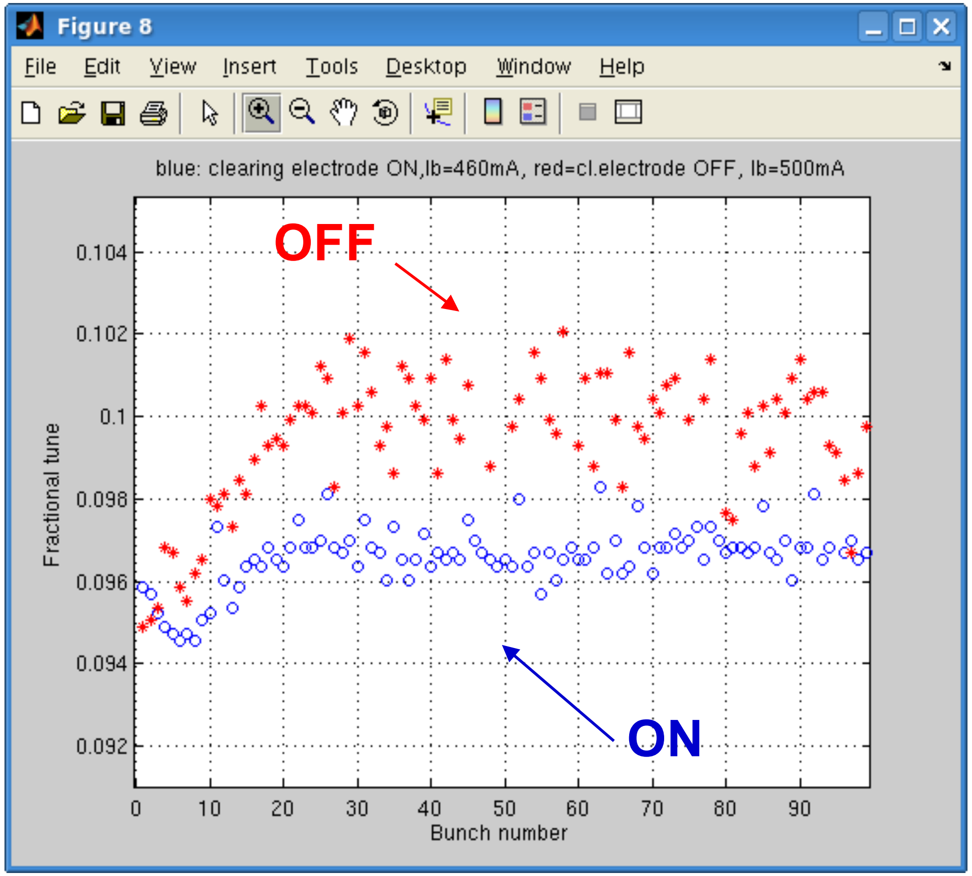
20 buckets gap

Turning some electrodes off
the **horizontal tune spread (over different bunches) is almost halved**

$\Delta\nu^x_{1\text{-}100}$ ~ 0.006 (off)

$\Delta\nu^x_{1\text{-}100}$ ~ 0.003 (on)

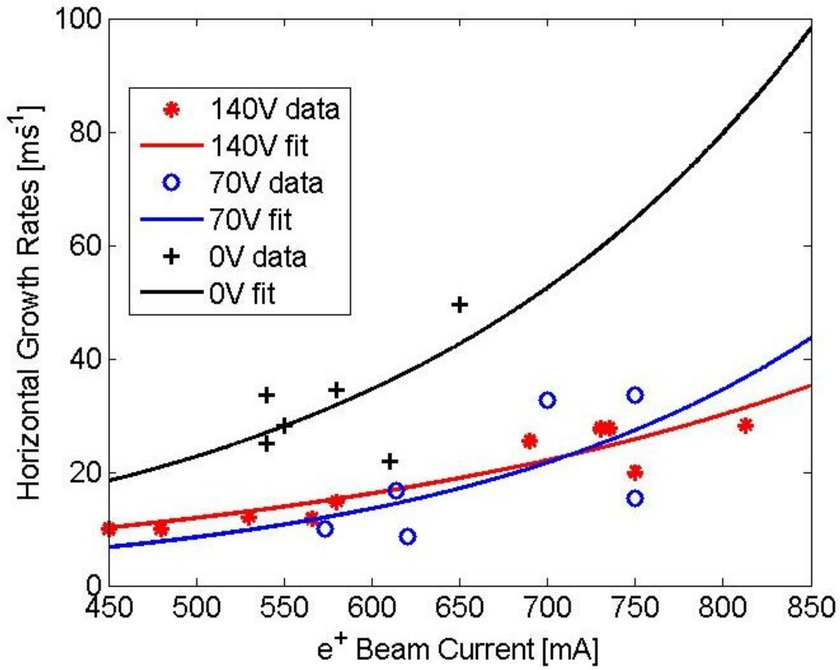
$<\Delta\nu^x>$ ~ 0.0065 (on/off)



E-cloud Instability

Horizontal instability growth rate measured by the front end of the bunch-by-bunch feedback at DAΦNE

Electrode voltages:
0 V, 70 V and 140 V



- Without electrodes the instability growth rate increases with the stored beam currents

Vacuum Chamber Impedance

Stored beam induces image charges and currents on the conducting wall of the vacuum pipe which act back on the beam itself

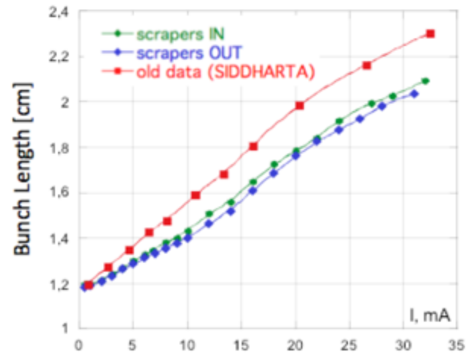
Under certain conditions this effect can cause microwave instabilities which above a given threshold introduce

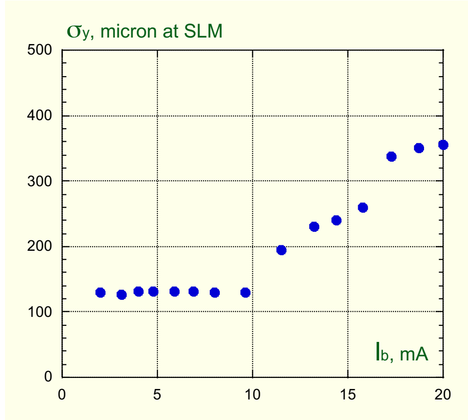
- bunch lengthening with the bunch current
- transverse beam size growth

Instabilities and transverse beam blow-up due to microwave instability threshold add up and interfere with the ones coming from *BB interaction*

Cure

Push microwave instability threshold toward higher single bunch current values by higher α_c and higher chromaticity values





Vacuum effects on e⁻ beam Ion Trapping

Poor vacuum, under certain conditions, can determine ion trapping by the e⁻ beam resulting in

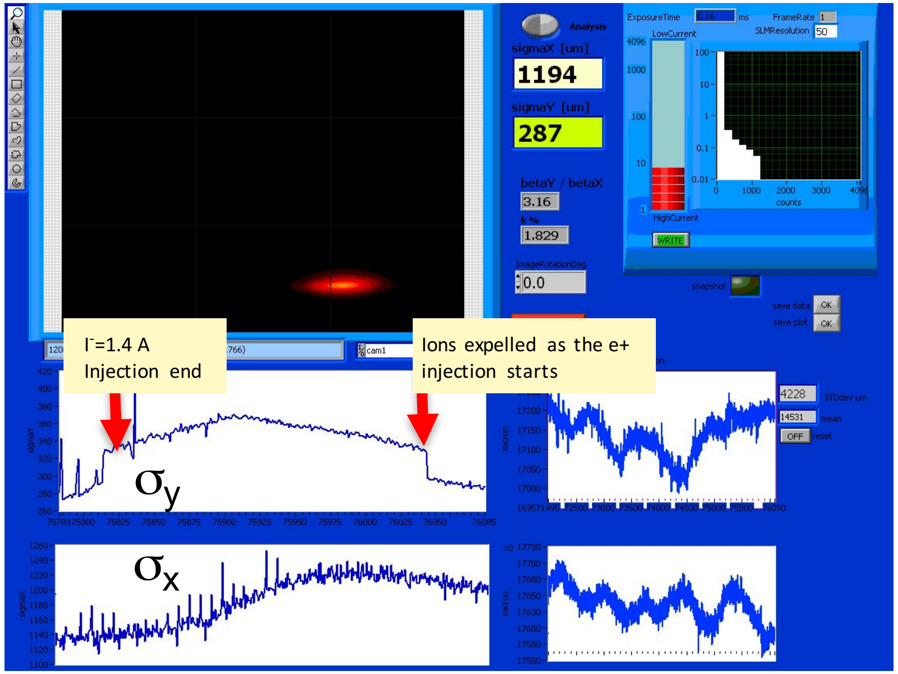
- sudden variation in the transverse beam size
- tune shift in both planes
- instabilities

Ion trapping effects become more harmful as the e⁻ current increases

Cure

proper lattice configuration

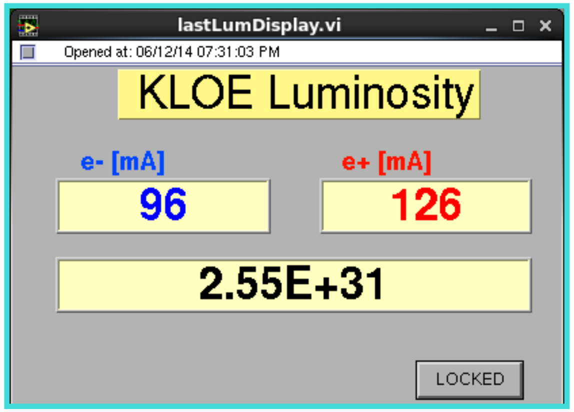
gap in the e⁻ bunch train

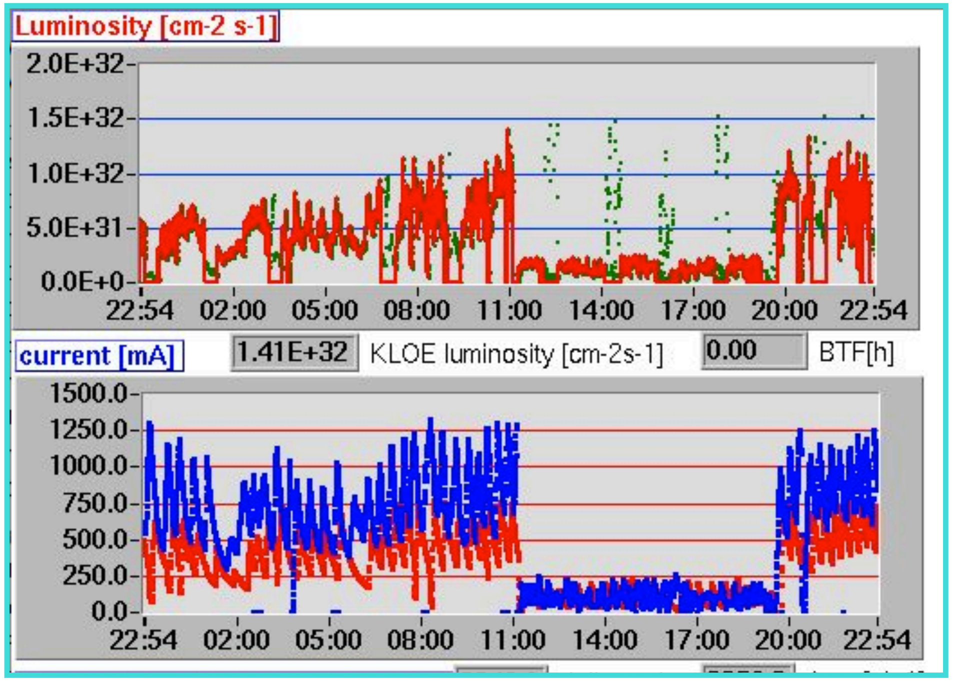


Instabilities and transverse beam blow-up due to Ion Trapping add up and interfere with the ones coming from *BB interaction*

10 Bunches Luminosity Measurement

Aiming at minimizing the impact of collective effects on *L*



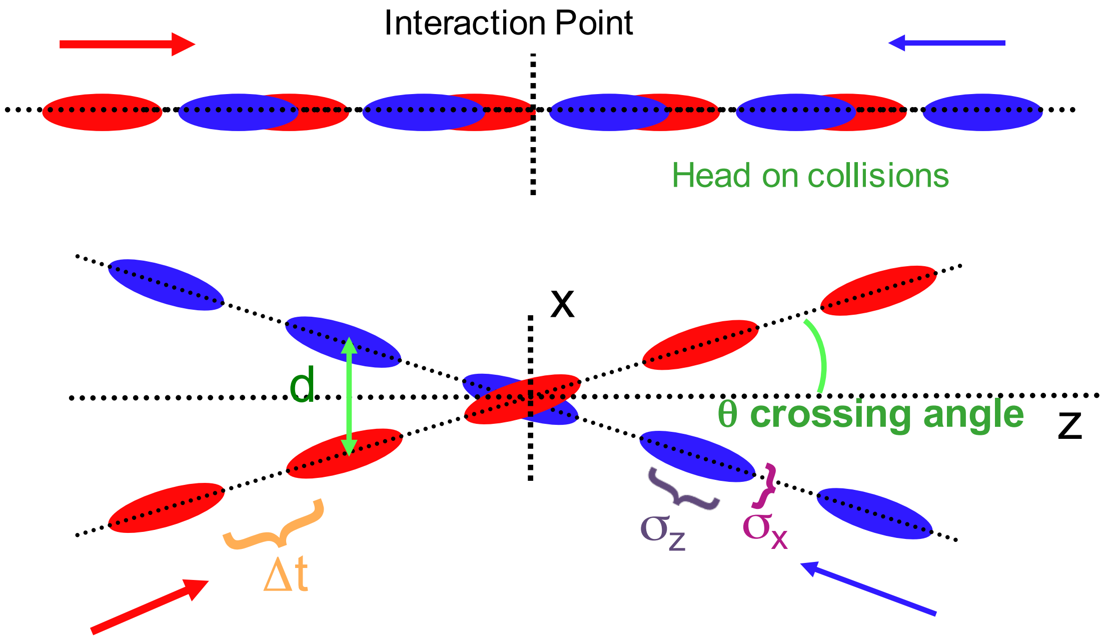


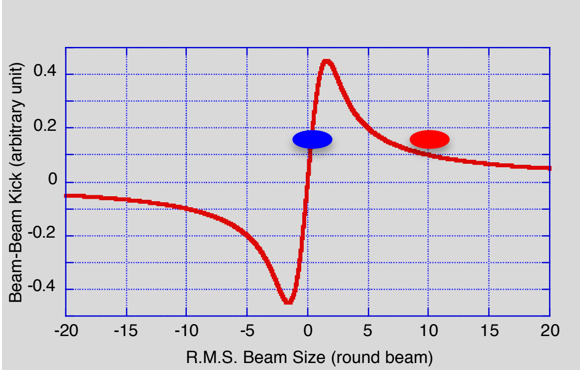
***L_{peak} ~ 2.5 10^{32} cm^{-2} s^{-1}** might be achieved by using 100 bunches*

•Beam-beam is not a limiting factor

Parasitic Crossings

Colliding beams consist of many bunches
Head on collisions determine many parasitic crossings
Crossing angle is introduced to minimize *parasitic crossings*





$$\Phi \approx \frac{\sigma_z}{\sigma^*_x} tg\left(\frac{\theta}{2}\right) < 1$$

Still ***Long Range Beam-Beam (LRBB) interactions*** is not negligible in fact it cause:

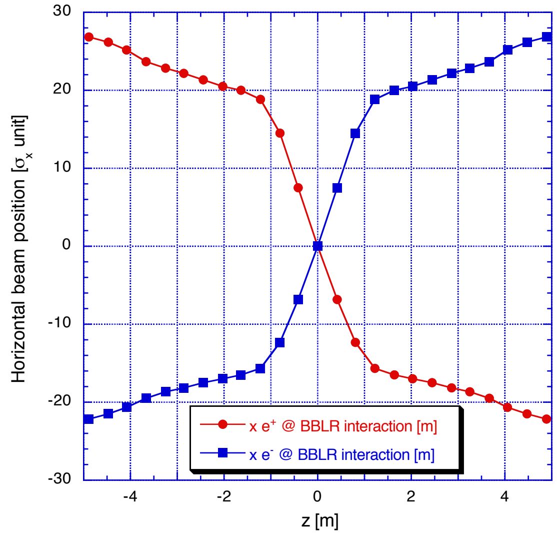
- closed orbit distortion
- correlation between the transverse and longitudinal motion
- excite dangerous resonances

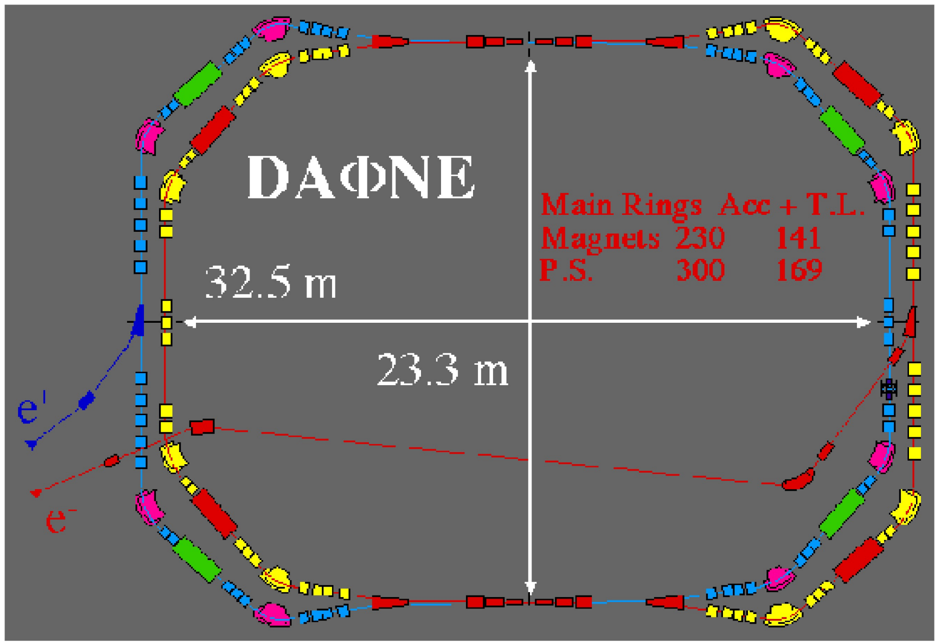
Long Range Beam-Beam Interaction at DAΦNE

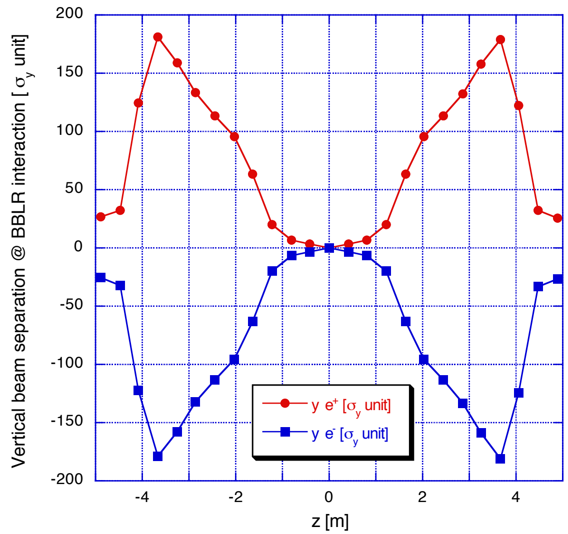
In the DAΦNE *original* configuration

- e^+ and e^- stored in 105 - 111 bunches
- 25 [mrad] crossing angle
- *2.7 [nsec] bunch spacing !!!!*
- 5 [m] long common IR
- ε 2.5 10^{-6} [m]

24 LRBB interactions



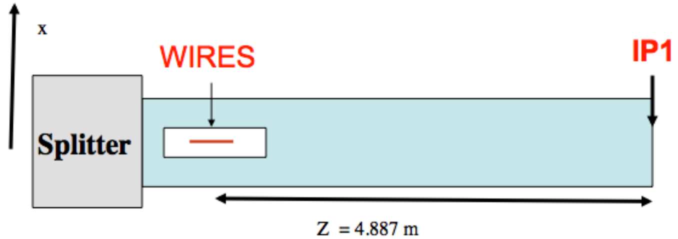


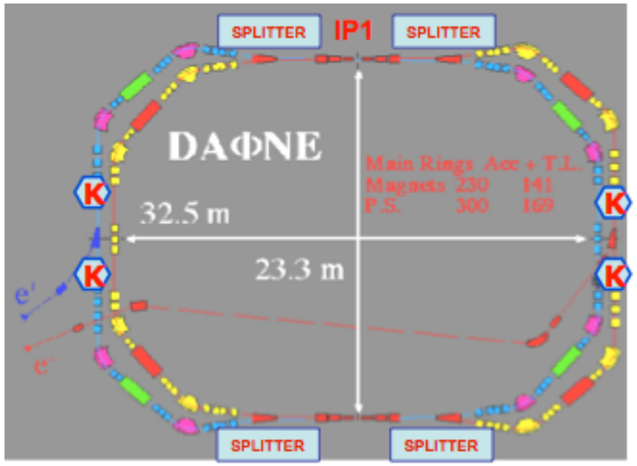


Wires for LRBB compensation at DAΦNE

LRBB were causing

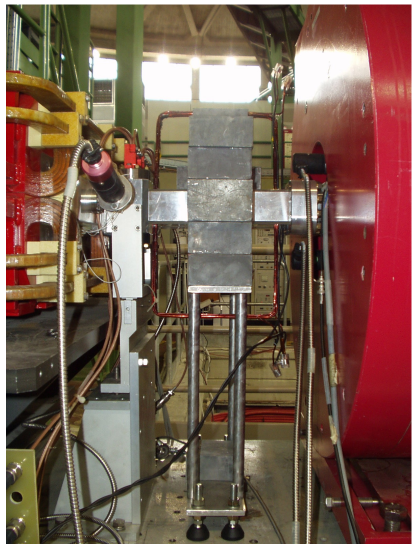
- Orbit distortion
- Beam lifetime reduction both during inject and coasting resulting in a limitation on
  - maximum storable current
  - peak and integrated ***L***





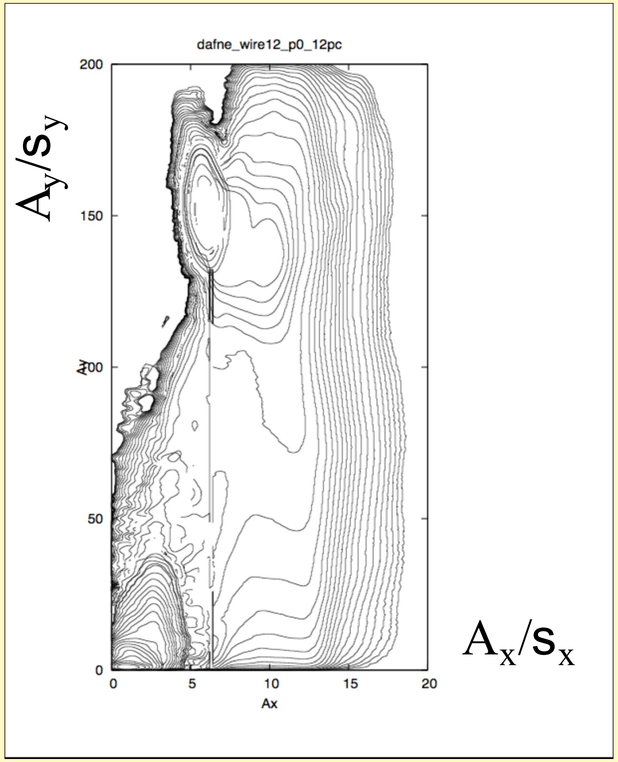
$$\Delta r' = \frac{2Nr_0}{\gamma r}$$ **LRBB deflection**

- Wires were installed outside the vacuum chamber using a short section in IR1, just before the splitters, where the vacuum pipes were separated.
- The wires carried a tuneable DC current, and produced a stationary magnetic field (1/r) with a shape similar to the one created by the opposite beam

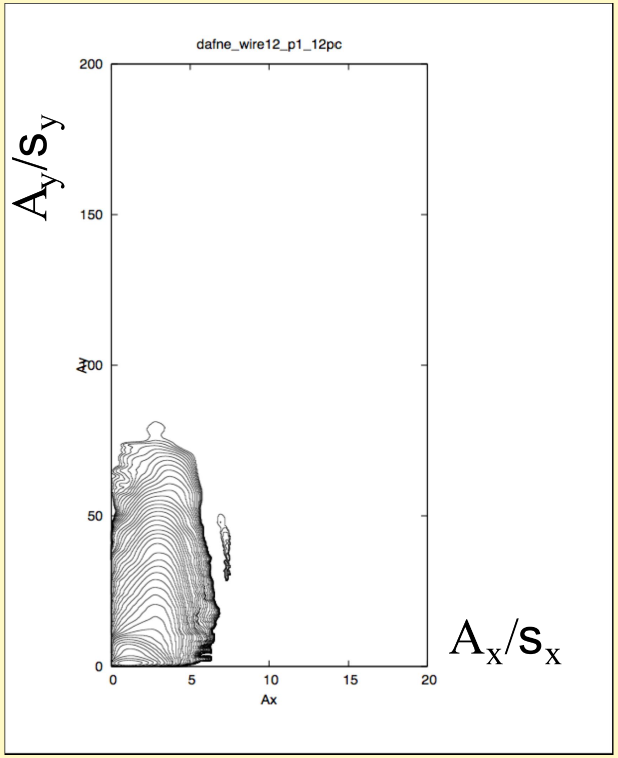


LIFETRACK simulations

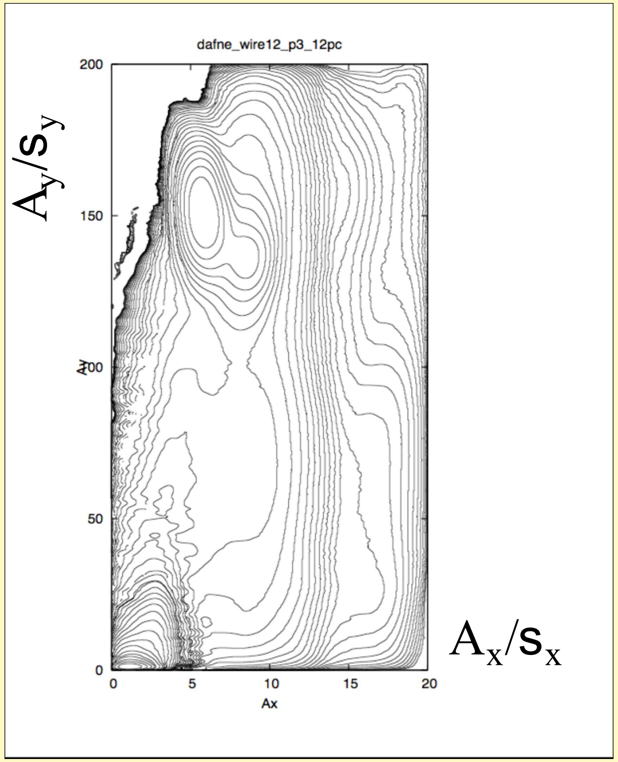
Particle equilibrium density in the transverse space of the normalized betatron oscillation amplitudes



Wires OFF



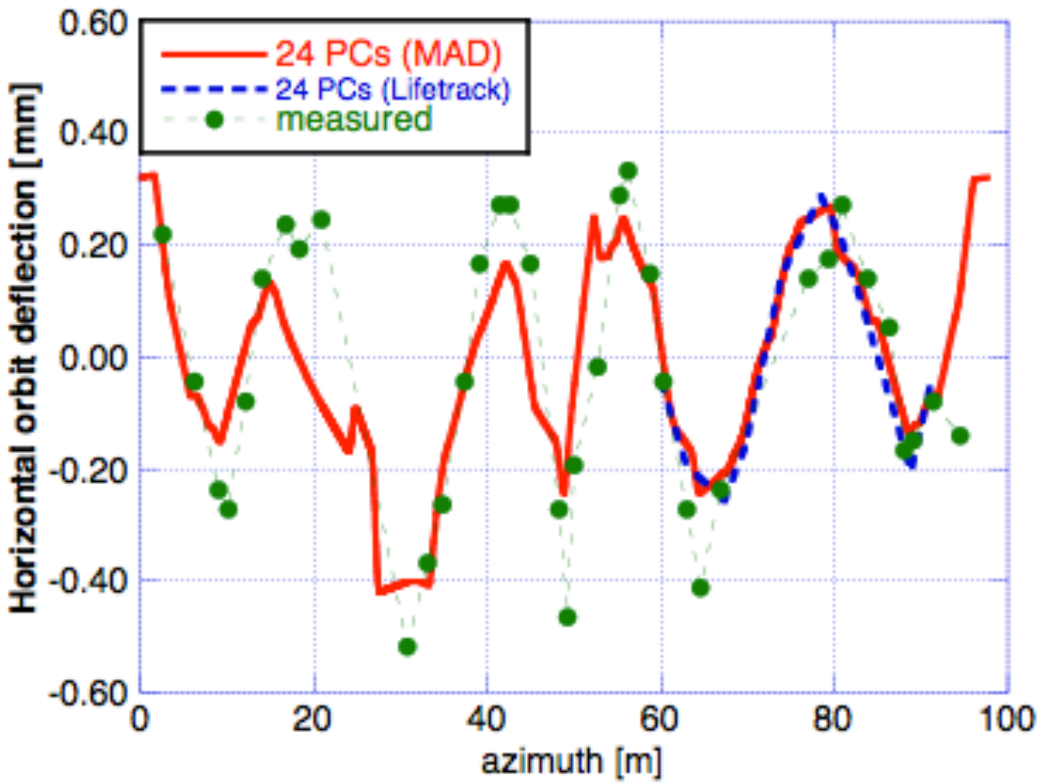
Wires ON



Wires ON (wrong polarity)

Beam-Beam Orbit deflection

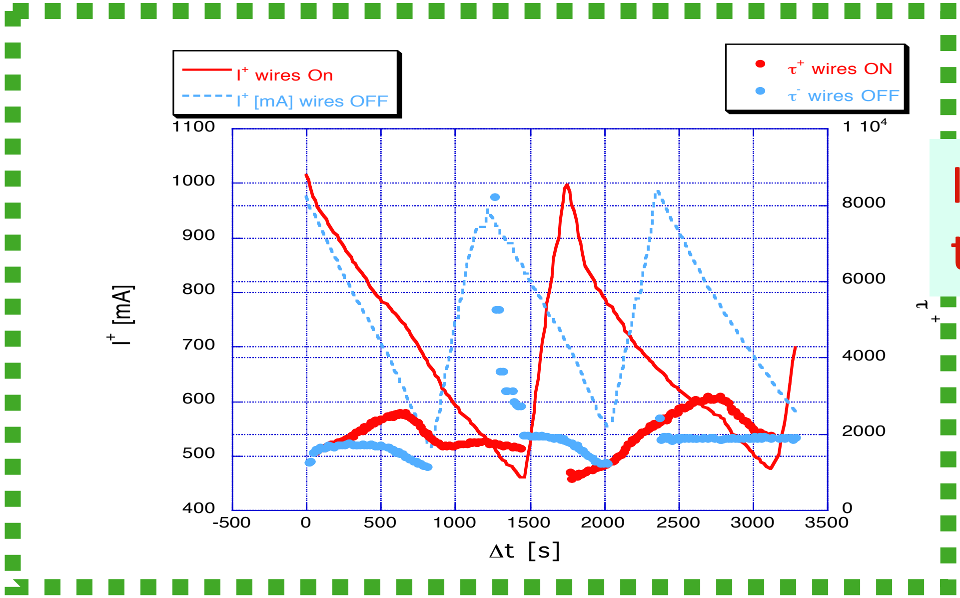
Comparison between orbit deflections due to main collision at IP + 24 BBLR interactions computed by MAD and by Lifetrack.



computed orbit deflection due to main collision + 24 BBLR interactions for a positron bunch colliding with an electron beam of 10 mA/bunch

Experimental Results Using Wires at DAΦNE

- Switching on and off the wires we obtain the same luminosity while colliding the same beam currents.
- The *positron lifetime is on average higher when wires are on*, while the electron one is almost unaffected.
- The beam blow-up occurring from time to time at the end of beam injection, corresponding to a sharp increase in the beam lifetime, almost disappear.
- It is possible to deliver the same integrated luminosity injecting the beam two times only instead of three in the same time integral, or to increase the integrated luminosity by the same factor keeping the same injection rate.
- A higher τ means less background on the experimental detector.
- It is possible to optimize the collision at maximum current

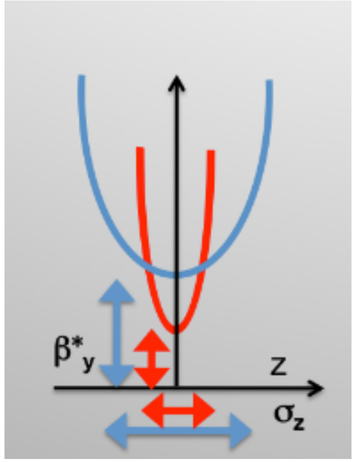


It's possible to improve the τ^+ of the 'weak' e^+ beam in collision.

The Frascati Approach to *BB* Interaction Optimization

A new collision scheme has been designed and implemented on the DAΦNE collider, the *Crab-Waist* collision scheme to overcome limitation in ***L*** due to:

- hourglass effect $\beta^*_y \sim \sigma_z$
- ***LRBB*** interactions
- beam transverse sizes enlargement due to ***BB* interaction**



Crab-Waist is based on:

Large Piwinski angle Φ

$$\Phi \approx \frac{\sigma_z}{\sigma^*_x} tg\left(\frac{\theta}{2}\right) >> 1$$

large θ
small σ^*_x ➜ ***L*** gain with N; low ξ_x; ξ_y decrease with Y oscillation amplitude

β^*_y comparable with overlap area

$$\beta^*_y \approx 2\sigma^*_x / \theta$$

➜ ***L*** geometrical gain; lower ξ_y; Y Synchro-betatron resonances suppression

Crab-Waist transformation by two Sextupoles

$$y = \frac{xy'}{\theta}$$

➜ ***L*** geometrical gain; lower ξ_y; X-Y Synchro-betatron resonances suppression

L and ξ in terms of Φ

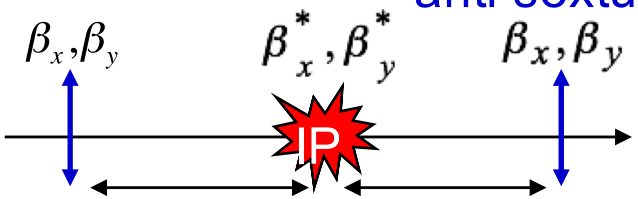
$$L = bf_r \frac{1}{4\pi\sigma_x\sigma_y}\left[\frac{N^2}{\sqrt{1+\Phi^2}}\right]$$

$$\xi_x = \frac{r_0\beta_x}{2\pi\gamma{\sigma^2}_x}\left[\frac{N}{1+\Phi^2}\right] \qquad \xi_y = \frac{r_0\beta_y}{2\pi\gamma\sigma_y\sigma_x}\left[\frac{N}{\sqrt{1+\Phi^2}}\right]$$

Increasing N proportionally to Φ

- *L* grows as Φ
- ξ_y remains constant
- ξ_x decreases as $1/\Phi$

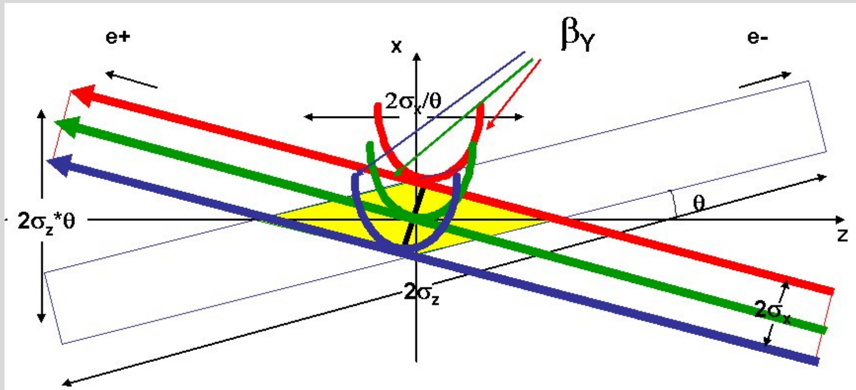
Crab-Waist Transformation



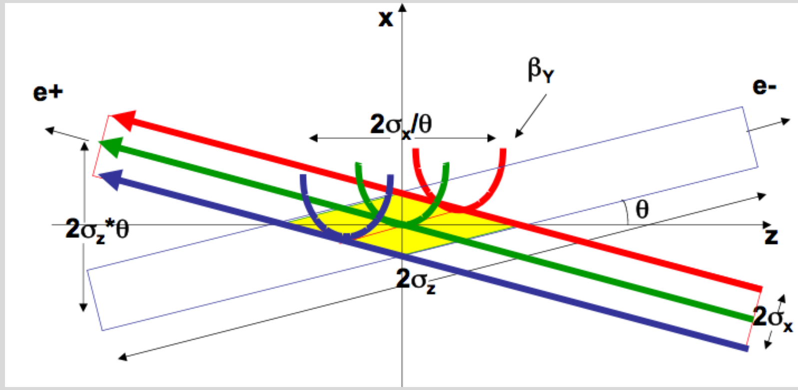
$$\Delta\nu_x = \pi$$
$$\Delta\nu_y = \frac{\pi}{2}$$

Sextupole strength

$$K_s = \frac{\chi}{2\theta}\frac{1}{\beta^*_y \beta^s_y}\sqrt{\frac{\beta^*_x}{\beta^s_x}}$$



Crab OFF

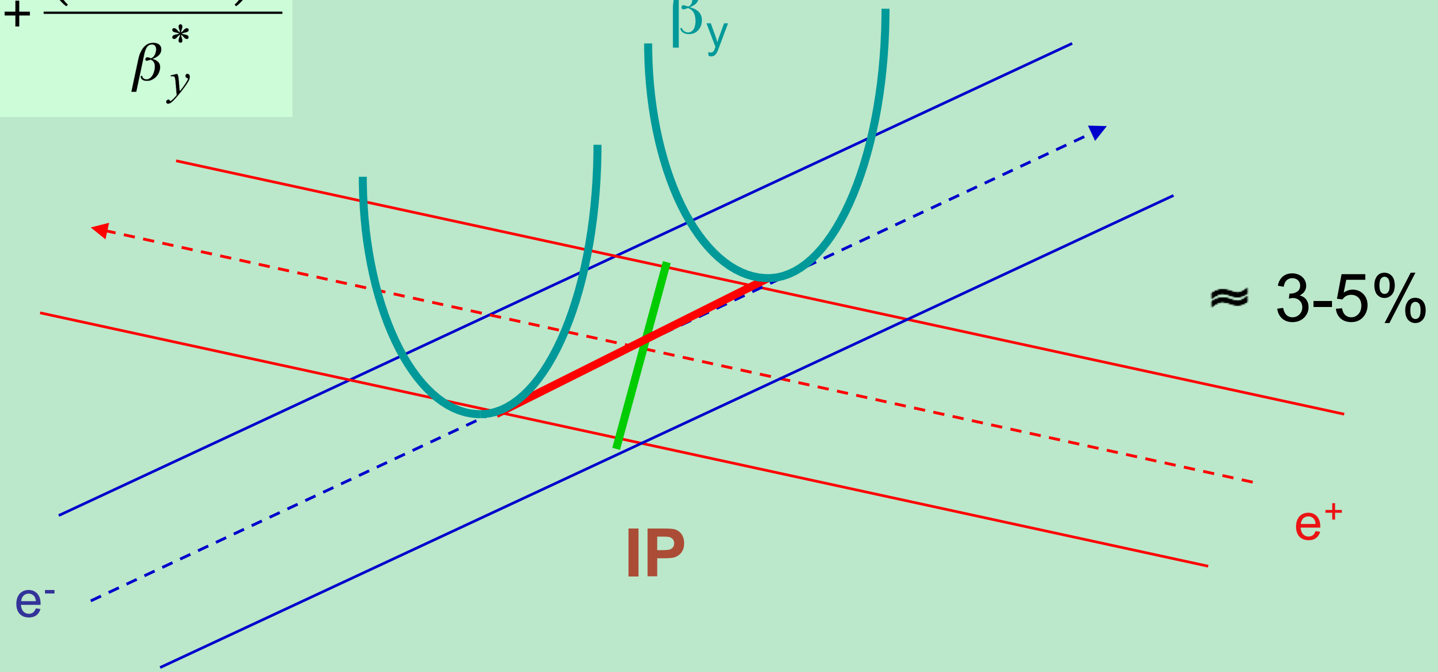


Crab ON

(P. Raimondi, M. Zobov)

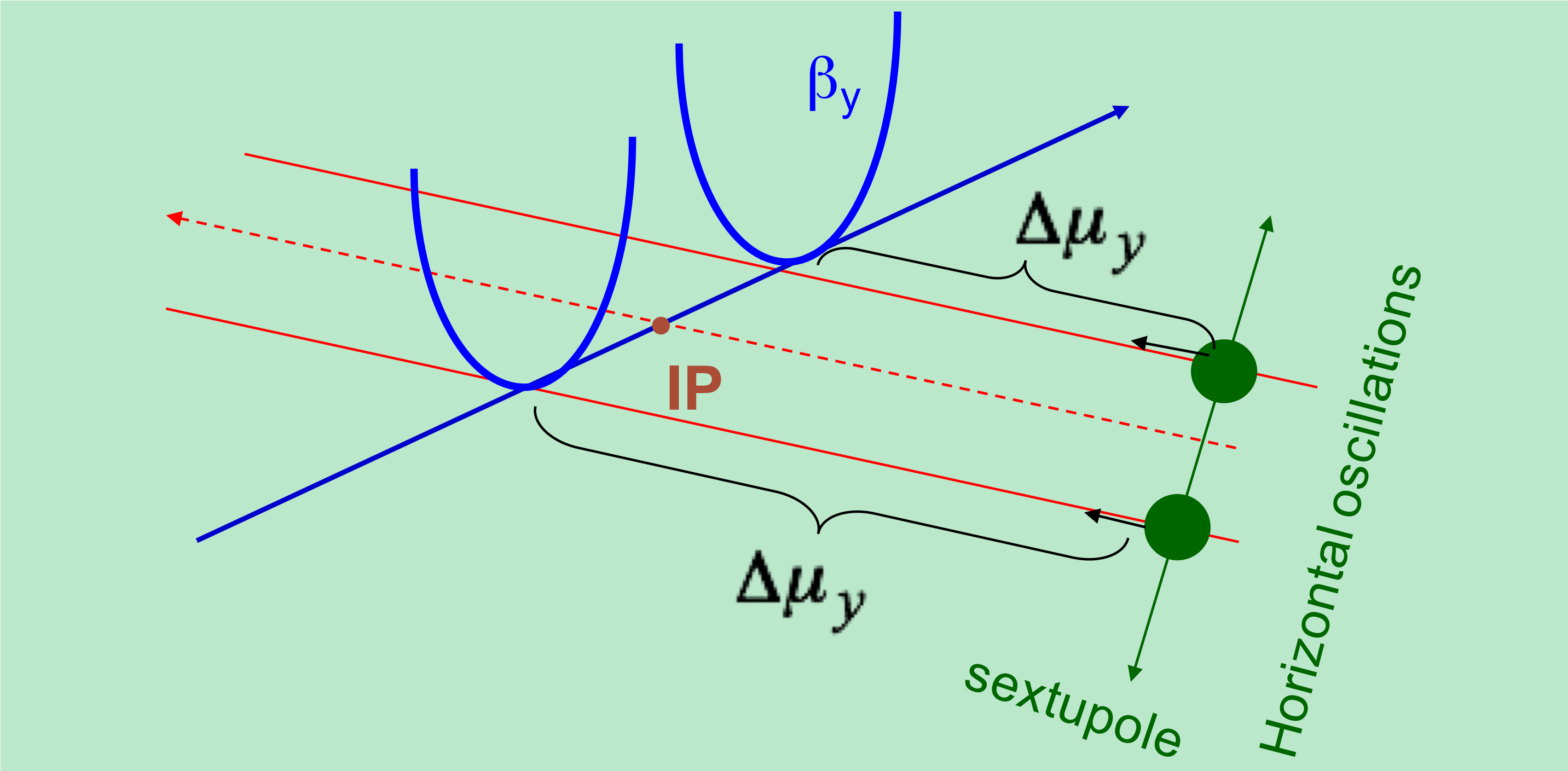
Geometric Factor due to *Crab-Waist* Transformation

$$\beta_y = \beta_y^* + \frac{(s - x/\theta)^2}{\beta_y^*}$$



- Minimum of β_y for e^- beam is along the maximum density of the opposite e^+ beam
- The waist length is oriented along the overlap area. The line of the minimum beta with the *Crab-Waist* (red line) is longer than without it (green line).

Suppression of X-Y Resonances

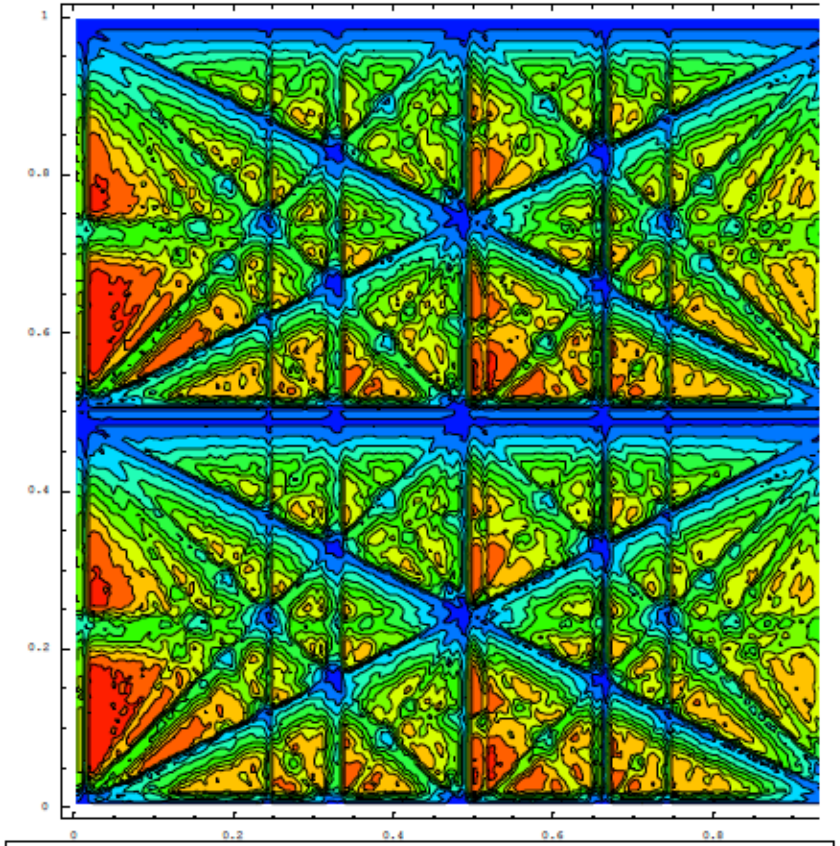


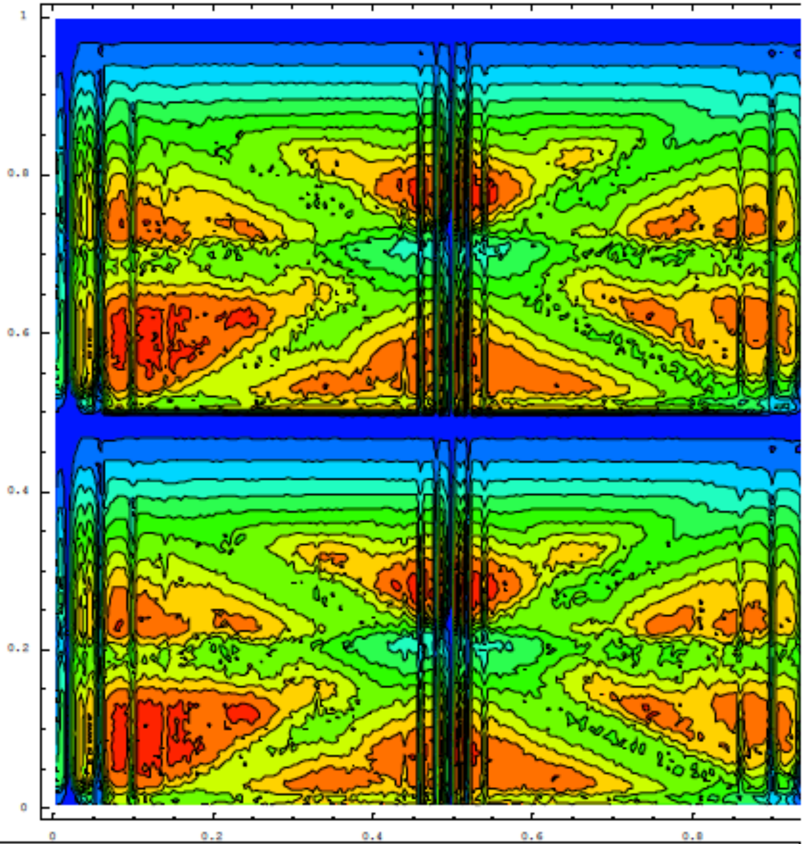
Performing horizontal oscillations:

- Particles see the same density and the same (minimum) vertical beta function
- The vertical phase advance between the sextupole and the collision point remains the same ($\pi/2$)

Suppression of X-Y Resonances

Much higher luminosity!





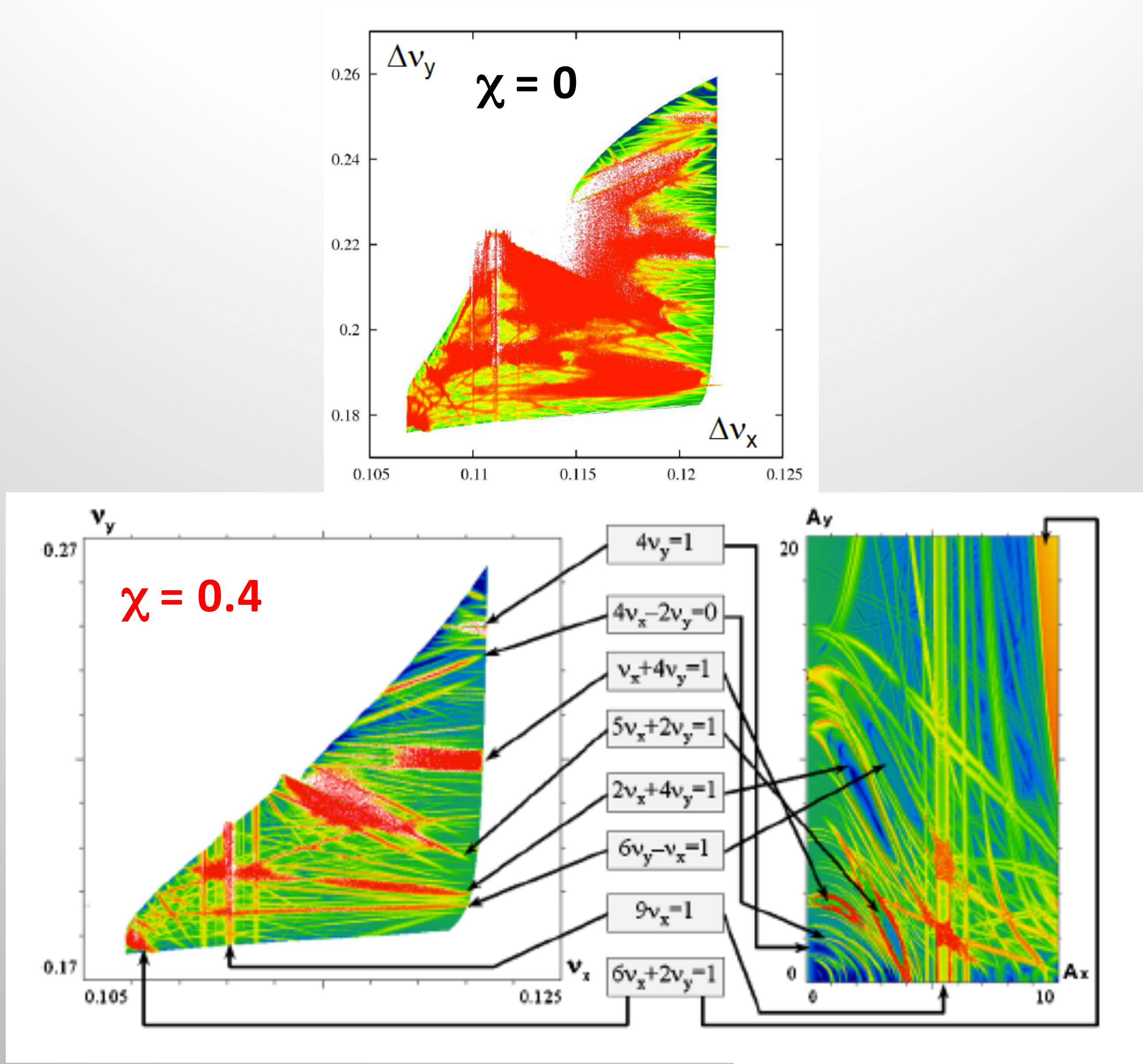
Typical case (KEKB, DAΦNE etc.):

1. low Piwinski angle $\Phi < 1$
2. β_y comparable with σ_z

Crab Waist On:

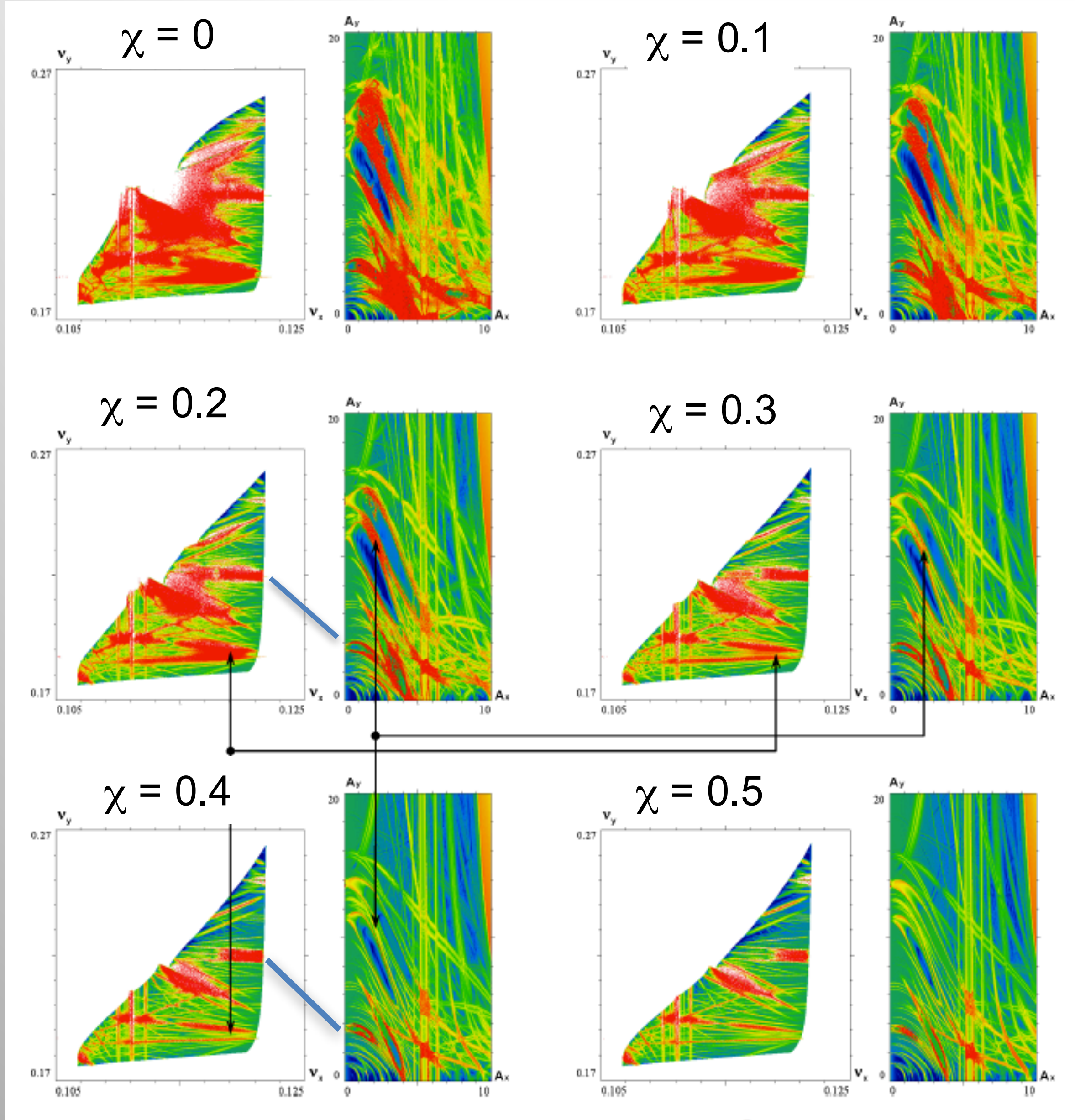
1. large Piwinski angle $\Phi >> 1$
2. β_y comparable with σ_x/θ

Frequency Map Analysis of *BB* Interaction



(E. Simonov, D. Shatilov et al.)

χ Optimization by *FMA*



How resonances are suppressed by CW transformation

Tune and amplitude plane are shown

Let us consider the evolution of two specific resonances

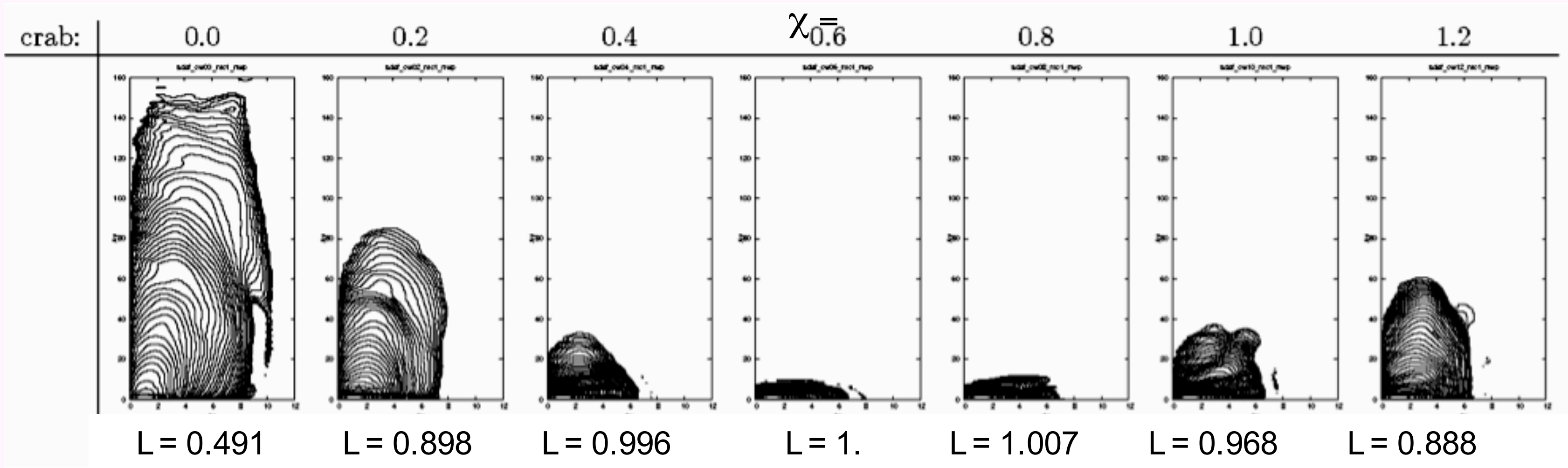
$$\nu_x + 4\nu_y = 1$$
$$2\nu_x + 4\nu_y = 1$$

As χ **-> 0** the two resonances merge and form a wide forbidden area for the beam tunes

As resonances are suppressed the footprint area shrinks

χ Optimization by *LIFETRACK*

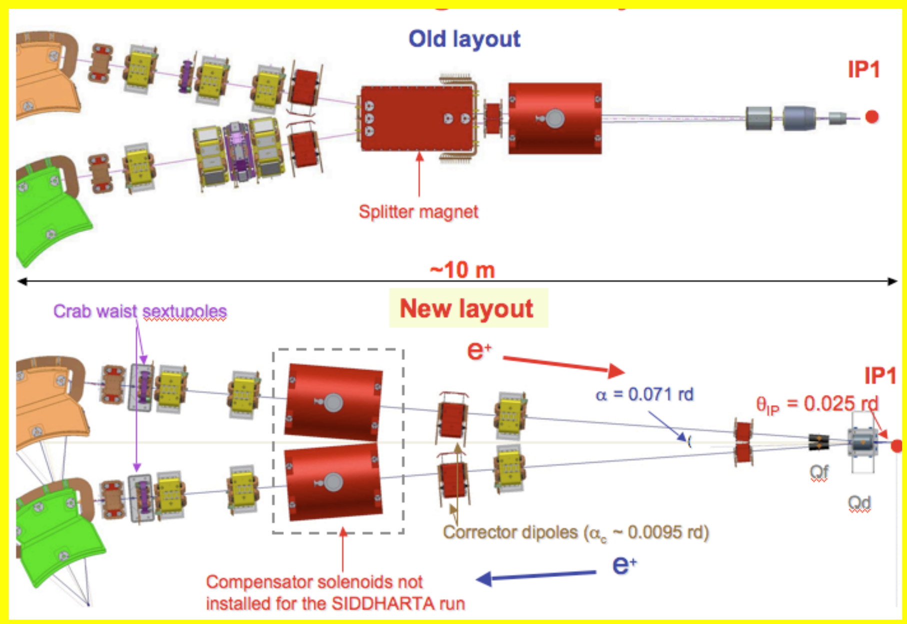
χ nominal 0.6



Luminosity (arbitrary unit) *and **Beam tails versus waist rotation*** χ

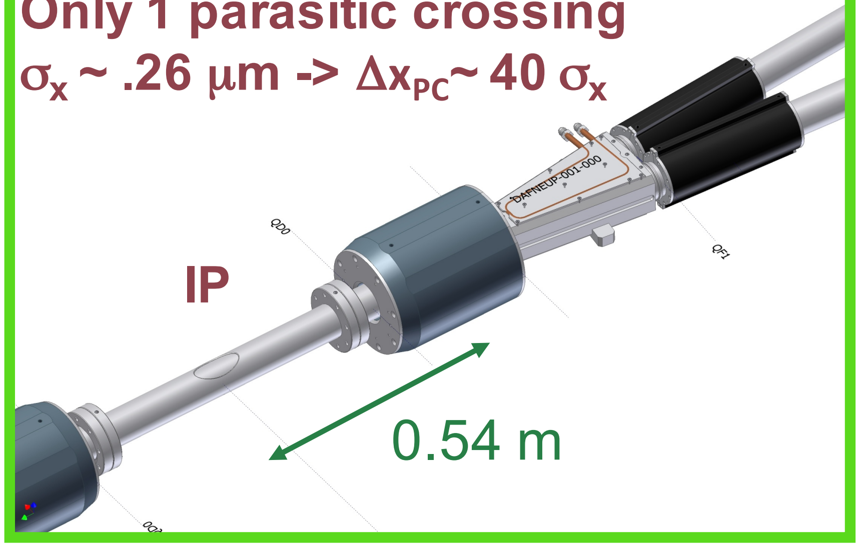
Crab-Waist and LRBB Interactions

New Interaction Region Layout



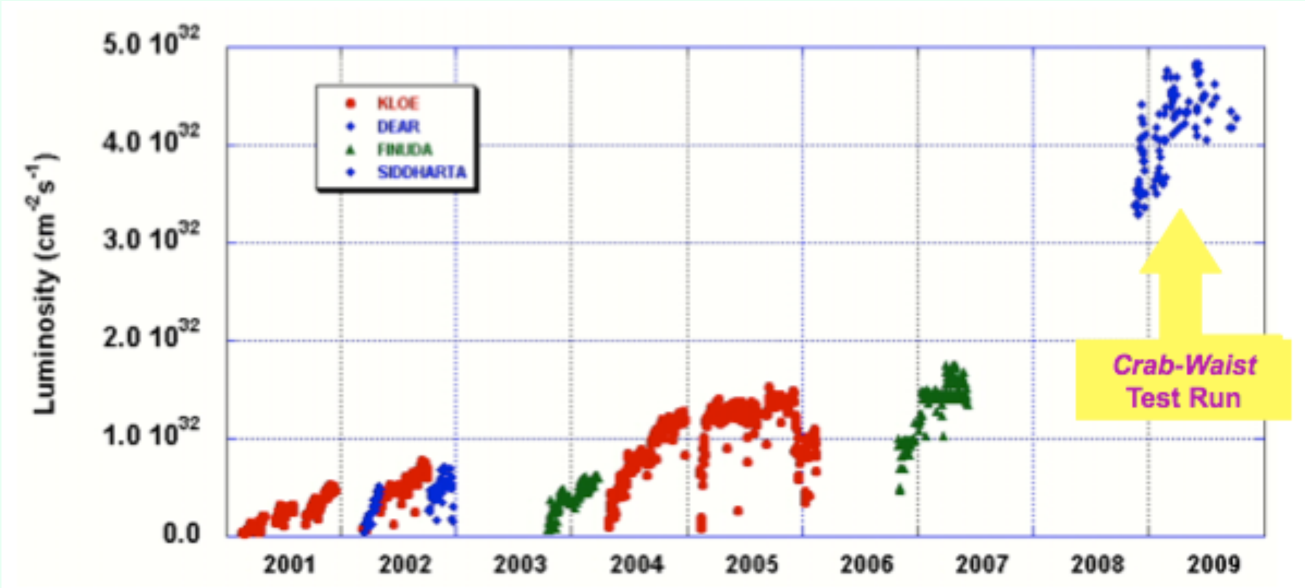
Only 1 parasitic crossing

σ_x ~ .26 μm -> Δx_{PC}~ 40 σ_x



LRBB interactions disappear

L Results During the *CW* Test Run

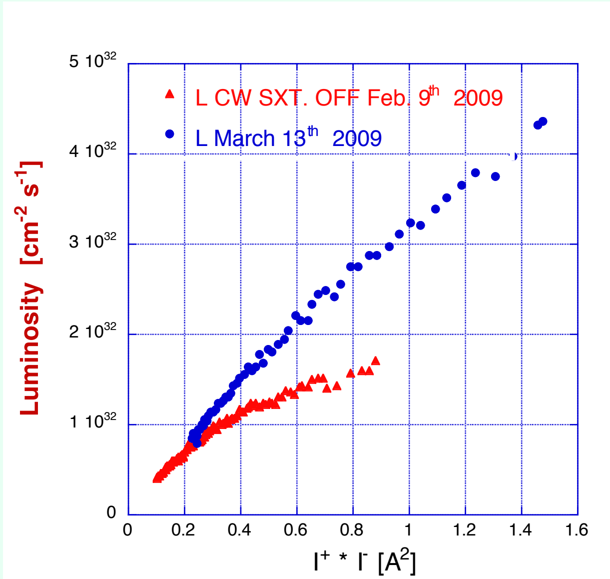


A factor 3 higher luminosity achieved without increasing beam currents

No evidence of vertical BB saturation with CW sextupoles on

$\xi_y = 0.044$

LRBB interaction cancelled



Thank you for your attention