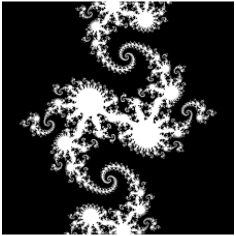
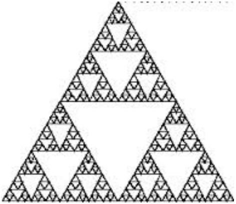
Tsallis statistics in HEP

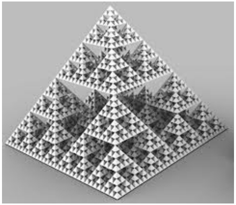
Airton Deppman

Frascati - July 05, 2016

What are fractals?







Intermittency

Normalized Moments:

R.Hwa
PRD41 (1990) 1456

$C_q = \sum_{k_0}^{\infty} k^q P_k / \left(\sum_{k_0}^{\infty} k P_k \right)^q = \delta^{\tau(q)}$

$P_k^q = (Q_k/N)^q = \delta^{\alpha_q}$

$Q_k =$ number of events with k particles in the bin with width δ

$N =$ total number of events

$\tau(q) = q\alpha_q - f(\alpha_q) = (q-1)D_q$

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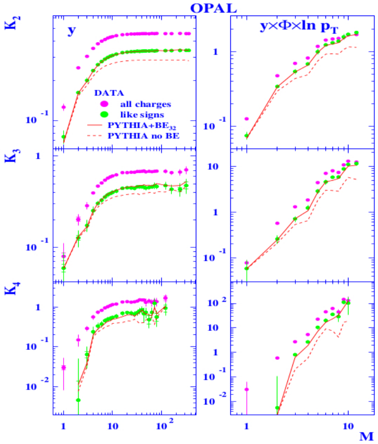
Self-similarity $\rightarrow$
N.G. Antoniou et al.
PRC93, 014908 (2016)

Intermittency
Exponential growth of cummulants
(integrated correlation)

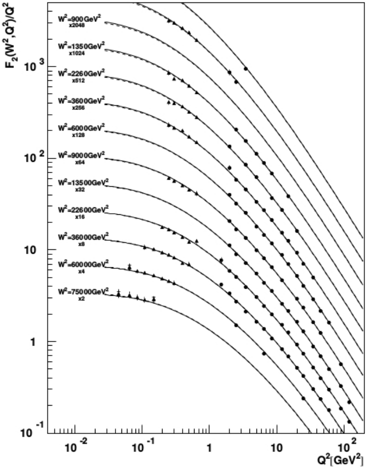
Intermittency data analysis

E. Sarkisyan: arXiv: hep-ex/0209079



Parton Distribution Function

T. Lastovicka EPJC 24(2002) 529



	$\mathcal{D}_0$	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	Q_0^2 [GeV^2]
all fit	0.339	0.073	1.013	−1.287	0.062
	±0.145	±0.001	±0.01	±0.01	±0.01
$\mathcal{D}_2$ fixed	0.523	0.074	*1*	−1.282	0.051
	±0.014	±0.001	const.	±0.01	±0.002

$$logf_i(x,Q^2) = D_1 log(1/x) log(1+Q^2/Q_o^2) + D_2 log(1/x) + D_3 log(1+Q^2/Q_o^2) + D_o^i$$

Fireball and hadron definitions

Hagedorn's defintion for firebal

A fireball is:

→ *a statistical equilibrium (hadronic black-body radiation) of an undetermined number of all kinds of fireballs, each of which, in turn, is considered to be* —

The model we wish to focus on in this paper is the *bootstrap model of hadrons*, in which the hadrons are assumed to be compounds of hadrons. The model can be represented schematically by

Frautischi's defintion for hadrons:

Thermofractal - definition

1. The total energy is given by

$$U = F + E\,,$$

The number of subsystem in N for all thermofractals.

2. $\langle E\rangle/\langle F\rangle$ is constant for all the subsystems. $E/F \to P(E/F)$.
3. At some point n of the hierarchy of subsystems the phase space is so narrow that one can consider

$$P(E_n)d\,E_n = \rho dE_n\,,$$

with ρ being independent of the energy E_n.

Thermofractal - Thermodynamics

For an ideal gas of elementary particles (Landau):

$$P(U)dU = (kT)^{-\frac{3N}{2}} U^{\frac{3N}{2}-1} \exp\left(-\frac{U}{kT}\right) dU \,,$$

Define for a thermofractal:

$$P(U)dU = A\exp(-\alpha F/kT) DFDE$$

with

$$\alpha = 1 + \frac{\varepsilon}{NkT}$$

and

$$\varepsilon = \frac{E}{F} kT \,.$$

$$DF = F^{\frac{3N}{2}-1} dF$$

and for the internal energy it is possible to write

$$DE = \tilde{P}(E)dE \,,$$

Thermofractal - Thermodynamical potential

The thermodynamical potential is given by

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}-1} \exp\left(-\frac{\alpha F}{kT}\right) dF \tilde{P}(\varepsilon) d\varepsilon \,.$$

which, after integration on F results in

$$\Omega = A \int_0^\infty \left[1 + \frac{\varepsilon}{NkT}\right]^{-3N/2} \tilde{P}(\varepsilon) d\varepsilon \,.$$

Second property of thermofractals (self-affine solution):

$$\ln P(U) \propto: \ln \tilde{P}(\varepsilon)$$

$$P(\varepsilon) = A \left[1 + \frac{\varepsilon}{NkT}\right]^{-3Nn/2}$$

Thermofractal and Tsallis

Second property of thermofractals (self-similar solution):

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}-1} \exp\left(-\frac{\alpha F}{kT}\right) dF [\tilde{P}(\varepsilon)]^\nu d\varepsilon \,.$$

$$P(U) := \tilde{P}(\varepsilon)$$

$$P(\varepsilon) = A\left[1 + \frac{\varepsilon}{NkT}\right]^{-\frac{3N}{2}\frac{1}{1-\nu}}$$

Introducing the index q by

$$q - 1 = \frac{2}{3N}(1-\nu)$$

and the effective temperature

$$\tau = \frac{2(1-\nu)}{3} T$$

$$P(\varepsilon) = A\left[1 + (q-1)\frac{\varepsilon}{Nk\tau}\right]^{-\frac{1}{q-1}} ,$$

For an ideal gas of thermofractals Tsallis statistics must be used!

Nonextensive self-consistent theory

$$Z_q(V_o, T) = \int_0^\infty \sigma(E)[1 + (q-1)\beta E]^{-\frac{q}{(q-1)}} dE$$

and

$$\ln[1 + Z_q(V_o, T)] = \frac{V_o}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int_0^\infty dm \int_0^\infty dp\, p^2 \rho(n; m) \times [1 + (q-1)\beta\sqrt{p^2 + m^2}]^{-\frac{nq}{(q-1)}} ,$$

Self-consistency principle:

$$Z_q(V_o, T) = \int_0^\infty \sigma(E)[1 + (q-1)\beta E]^{-\frac{q}{(q-1)}} dE = \exp\left\{ \frac{V_o}{2\pi^2 \beta^{3/2}} \int_0^\infty dm\, m^{3/2} \rho(m)[1 + (q-1)\beta m]^{-\frac{1}{q-1}} \right\} - 1$$

Weak constraint:

$$\ln[\sigma(E)] = \ln[\rho(m)]$$

Self-consistency solution

Self-consistency is obtained if

$$\rho(m) = \frac{\gamma}{m^{5/2}}[1 + (q_o - 1)\beta_o m]^{\frac{1}{q_o - 1}}$$

and

$$\sigma(E) = bE^a \left[1 + (q_o - 1)\beta_o E\right]^{\frac{1}{q_o - 1}}$$

Partition function:

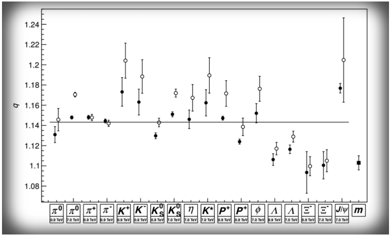
$$Z_q(V_o, T) \to b\Gamma(a+1)\left(\frac{1}{\beta - \beta_o}\right)^{a+1}$$

with

$$a + 1 = \alpha = \frac{\gamma V_o}{2\pi^2 \beta^{3/2}}$$

Limiting temperature: β_o and entropic index: q_o.

Experimental analyses



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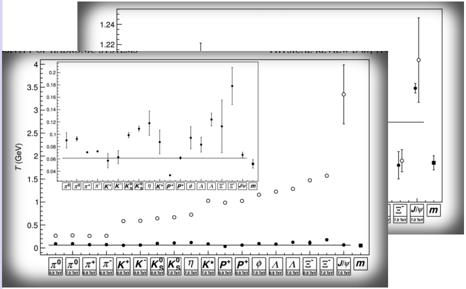
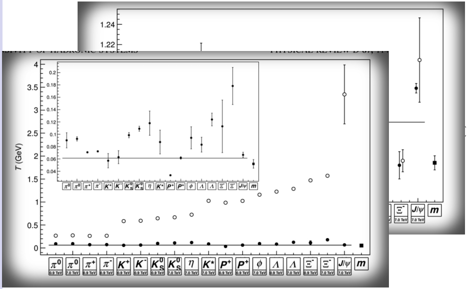
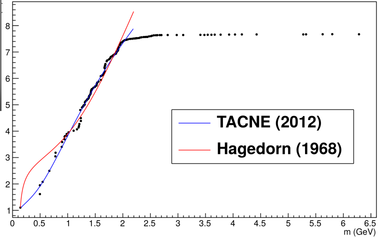
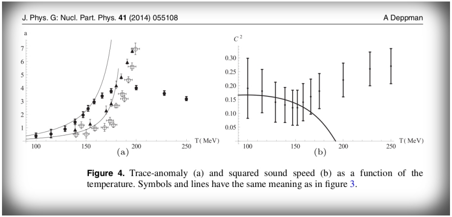


Experimental analyses



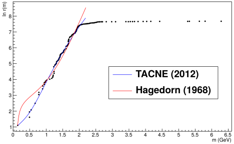


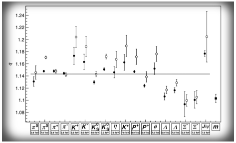


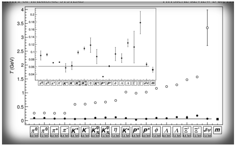
Fireball and hadron definitions

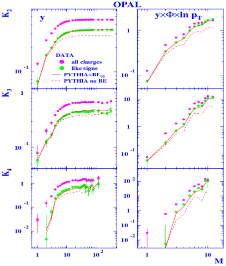
$$D = 1 + \frac{\log N'}{\log R} \qquad N = \frac{1}{(q-1)}\frac{\tau}{T}$$

$$R = \frac{(q-1)N/N'}{3-2q+(q-1)N} \qquad N' = N + 2/3$$









Partition function for a nonextensive ideal gas

Nonextensive thermodynamics for hadronic matter with finite chemical potentials

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The nonextensive thermodynamics of an ideal gas composed by bosons and/or fermions is derived from its partition function for systems with finite chemical potentials. It is shown that the thermodynamical quantities derived in the present work are in agreement with those obtained in previous works when $\mu \leq m$. However some inconsistencies of previous references are corrected when $\mu > m$. A discontinuity in the first derivatives of the partition function and its effects are discussed in details. We show that at similar conditions, the nonextensive statistics provide a harder EOS than that provided by the Boltzmann-Gibbs statistics.

PACS numbers: 05.70.Ce,95.30.Tg,26.60.-c

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$$\log \Xi_q(V,T,\mu) = -\xi V \int \frac{d^3p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \log_q^{(-r)} \left(\frac{e_q^{(r)}(x) - \xi}{e_q^{(r)}(x)} \right), \qquad (6)$$

EOS than that provided by the Boltzmann-Gibbs statistics.

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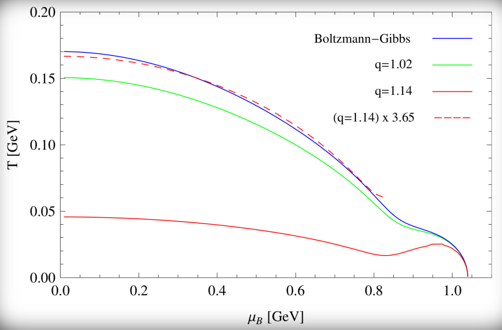

Hadronic Fractal Dimension

$q = 1.14$ and $\tau/T = 0.32$

$N = 2.3$ and $N' = 1.7$

$R = 0.104$ and $D = 0.69$

Intermittency in rapidity distribution for pp: $D = 0.43 - 0.65$

Microscopic origins of S_q

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}} \exp\left(-\frac{F}{kT}\right) dF \left[1 + (q-1)\frac{\varepsilon}{k\tau}\right]^{\nu/(q-1)} d\varepsilon$$

$$\frac{\nu}{q-1} = \frac{1}{q-1} - \frac{3N}{2}$$

$$\Omega = \int_0^\infty \int_0^\infty A F^{\frac{3N}{2}-1} \exp\left(-\frac{F}{kT}\right) dF \left[1 + (q-1)\frac{\varepsilon}{k\tau}\right]^{1/(q-1)} d\varepsilon$$

$$\Omega_o = \int_0^\infty \int_0^\infty \exp\left(-\frac{F}{kT}\right) F^{3N/2} dF$$

$$\Omega = \Omega_o - \int_0^\infty A \exp\left(-\frac{F}{kT}\right) F^{\frac{3N}{2}-1} \times \left[1 - \int_0^\infty \exp\left(-(q-1)\frac{\varepsilon}{Nk\tau}\frac{F}{kT}\right)\left[1 + (q-1)\frac{\varepsilon}{k\tau}\right]^{-\nu/(q-1)} d\varepsilon\right] dF$$

Microscopic origins of S_q

Dashen, Ma, Bernstein (PR 187 1969):

$$\Omega = \Omega_o - \frac{1}{4\pi\beta i}\int_0^\infty \exp(-E/kT)\left(TrS^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S\right)_C$$

Therefore:

$$\left(TrS^{-1}\frac{\partial}{\partial E}S\right)_C = 1 - \int_0^\infty \exp\left(-\frac{(q-1)\varepsilon}{Nk\tau}\frac{F}{kT}\right)\left[1+(q-1)\frac{\varepsilon}{k\tau}\right]^{-\frac{\nu}{q-1}} d\varepsilon$$

Next steps

- Determine the PDF corresponding to the thermofractalstructure proposed, and compare with Lastovicika parametrization.
- Extend NESCT to nucleus-nucleus collisions.
- Study possible relations between the fractal structure and non perturbative QCD (fractal diagram).
- Study the relation between the scattering matrix S and q.

Conclusions

1) Thermofractal structure + NESCT $\rightarrow$ unified description of p_T distribution, hadron mass spectrum, intermittency.
2) The parameters T_o and q_o are the only free parameters that needs to be obtained from experimental data.
3) It is possible that Parton Distribution Functions can be connected with the thermodynamical theory as well.
4) Contribution to the understanding of nonperturbative QCD through *S-matrix* connection.

Grazie