Non-supersymmetric SO(10): status and prospects

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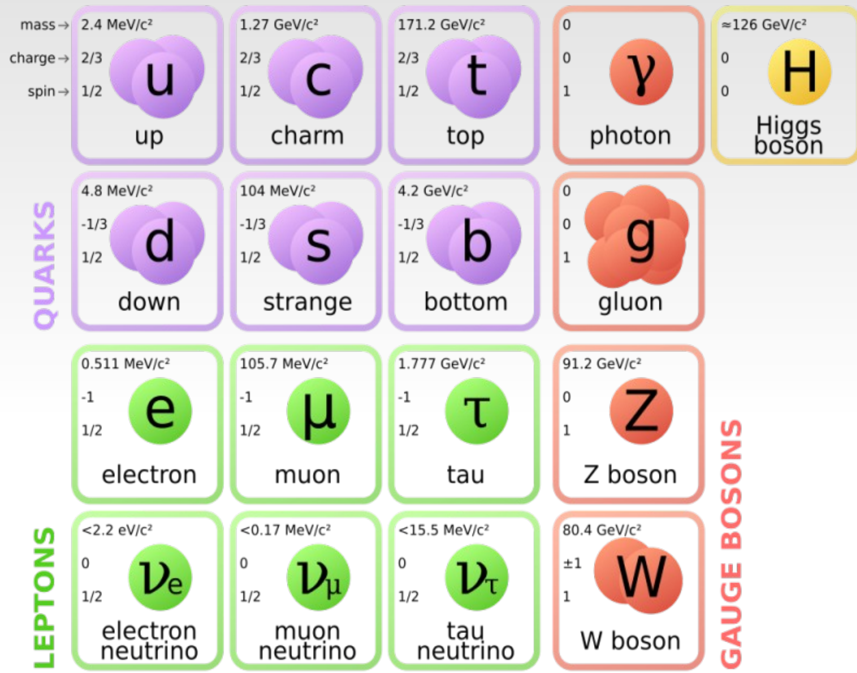
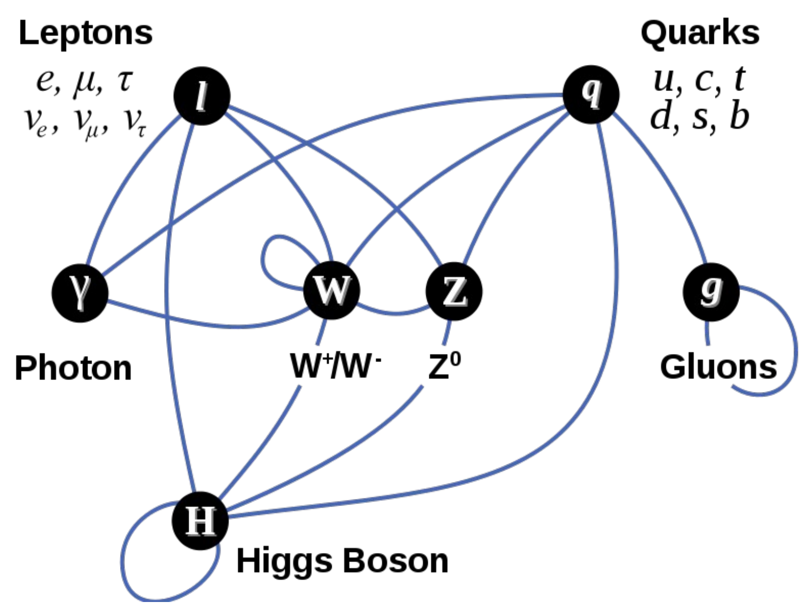
Standard Model of Particle Physics



Particles and interactions



Some open questions...

- Charge Equality : $|1 - Qe/Qp| < 10^{-21}$

- Dark Matter:
  the total mass energy of the known universe contains 4.9% ordinary matter, 26.8% dark matter. The SM does not have any candidate for it

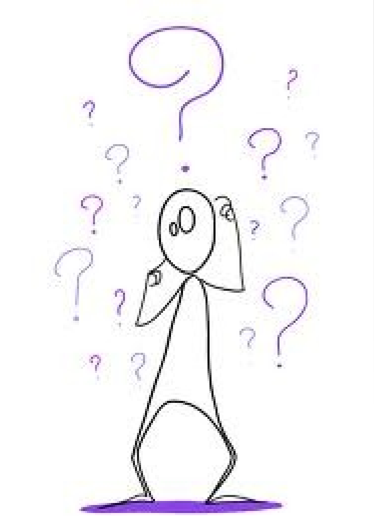
- Neutrino and Quark Oscillations:
  fermions of different flavors oscillate into each other due to non-zero masses and mixing angles

- Stability of the Higgs potential
  in the SM, the Higgs quartic coupling becomes negative around 10^{11} GeV...an interesting energy scale

→ Some guiding principle for Physics Beyond the SM?

Naturalness

- *Absence of new-physics signals casts some doubts on the relevance of our concept of naturalness*



"Let us consider a theory valid up to a maximum energy and make all its parameters dimensionless by measuring them in units of Λ . The naturalness criterion states that one such parameter is allowed to be much smaller than unity only if setting it to zero increases the symmetry of the theory. If this does not happen, the theory is unnatural"

G. Giudice, arXiv:0801.2562

- It worked in the past

Naturalness - an example

- Mass difference between K_L^0 and K_S^0 states

computed in an effective theory valid at energies of the order of the kaon mass

experimental value

$$\frac{m_{K_L^0} - m_{K_S^0}}{m_{K_L^0}} = \frac{G_F^2 f_K^2}{6\pi^2} \sin^2\theta_C \Lambda^2 = 7\times 10^{-15}$$

$\Lambda < 2$ GeV

before reaching this energy scale a new particle (the c-quark with mc ≈ 1.2 GeV) modifies the short-distance behavior of the theory

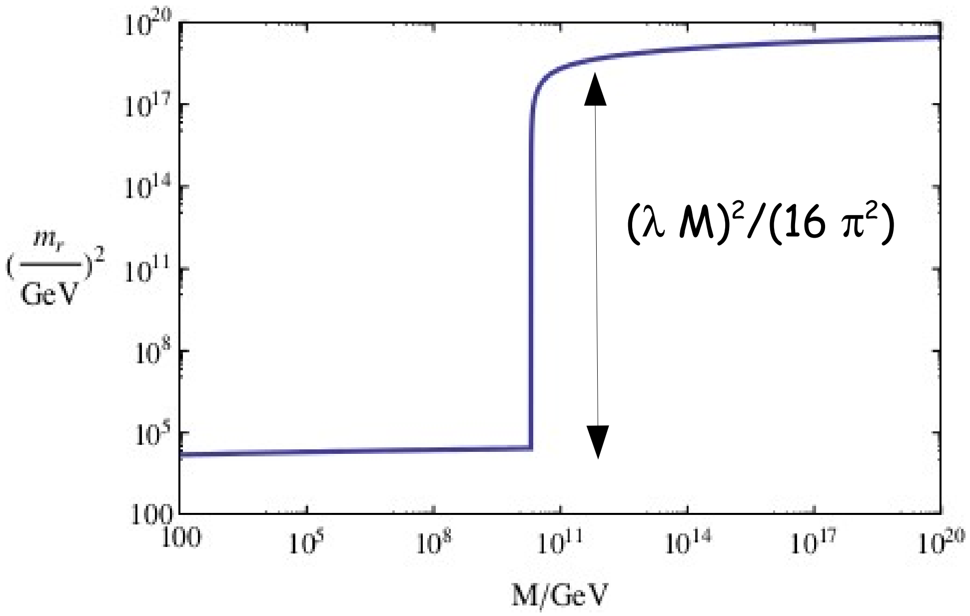
For the Higgs mass...

$$\delta m_H^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_H^2\right)\Lambda^2 \quad \Longrightarrow \quad \Lambda \le O(1)\,TeV$$

New physics expected at these energies

Otherwise strong dependence on the cut-off of the theory

Barbieri,1309.3473v1



running Higgs mass squared versus the scale M in the SM

new particle of mass $M_{new} = 10^{10}$ GeV and a gauge invariant dimensionless coupling to the Higgs boson of strength $\lambda = 1$

"initial condition" on m_r at some short distance scale, $M \gg M_{new}$, has been chosen with great accuracy to reproduce at $M = m_H$ the observed physical Higgs mass

Which direction?

Building models where naturalness is restored not so far from the weak scale

Models with large fine tunings that disregard the naturalness principle in part or even completely

This scenario will be analyzed in the following

Requirements for Beyond the Standard Model scenarios

non-SUSY GUT

- Unification of couplings at a large scale compatible with proton decay
- A Yukawa sector compatible with all data on flavour physics, fermion masses and mixings
- Agreement with leptogenesis as the origin of the baryon asymmetry
- An axion suitable to solve the strong CP problem and account for the observed Dark Matter

Requirements for Beyond the Standard Model scenarios

- The group rank must be larger than that of the SM (=4);

  breaking patterns must exist to go down to the SM

- If we believe in true unification of forces, an unique coupling constant must govern all gauge interactions (no parial unification)

- Irreducible representations of the group must accommodate the fermions of the SM

- Renormalizable Yukawa operators must be built as singlets of the group

An impressive features of SO(10)

special orthogonal rotations in 10 dimensions

- SM fermions in the (3 replica of) 16 spinorial representation

Saki Khan, talk at Santa Fe 2014

3 (u) + 3 ($\bar{u}$) +
3 (down) + 3 ($\bar{d}$) +
1 (e) + 1 ($\bar{e}$) +
1 (ν_L) + 1 (ν_R)

$u_r: \{-++\ +-\}$	$d_r: \{-++\ -+\}$	$u_r^c: \{+--\ ++\}$	$d_r^c: \{+--\ --\}$
$u_b: \{+-+\ +-\}$	$d_b: \{+-+\ -+\}$	$u_b^c: \{-+-\ ++\}$	$d_b^c: \{-+-\ --\}$
$u_g: \{++-\ +-\}$	$d_g: \{++-\ -+\}$	$u_g^c: \{--+\ ++\}$	$d_g^c: \{--+\ --\}$
$\nu: \{---\ +-\}$	$e: \{---\ -+\}$	$\nu^c: \{+++\ ++\}$	$e^c: \{+++\ --\}$

The first 3 indicates color spin and last two weak spin.

part of the irreducible representation

Not a singlet as in SU(5)

An interesting feature of SO(10)

G.Altarelli & DM, JHEP 1308 (2013) 021

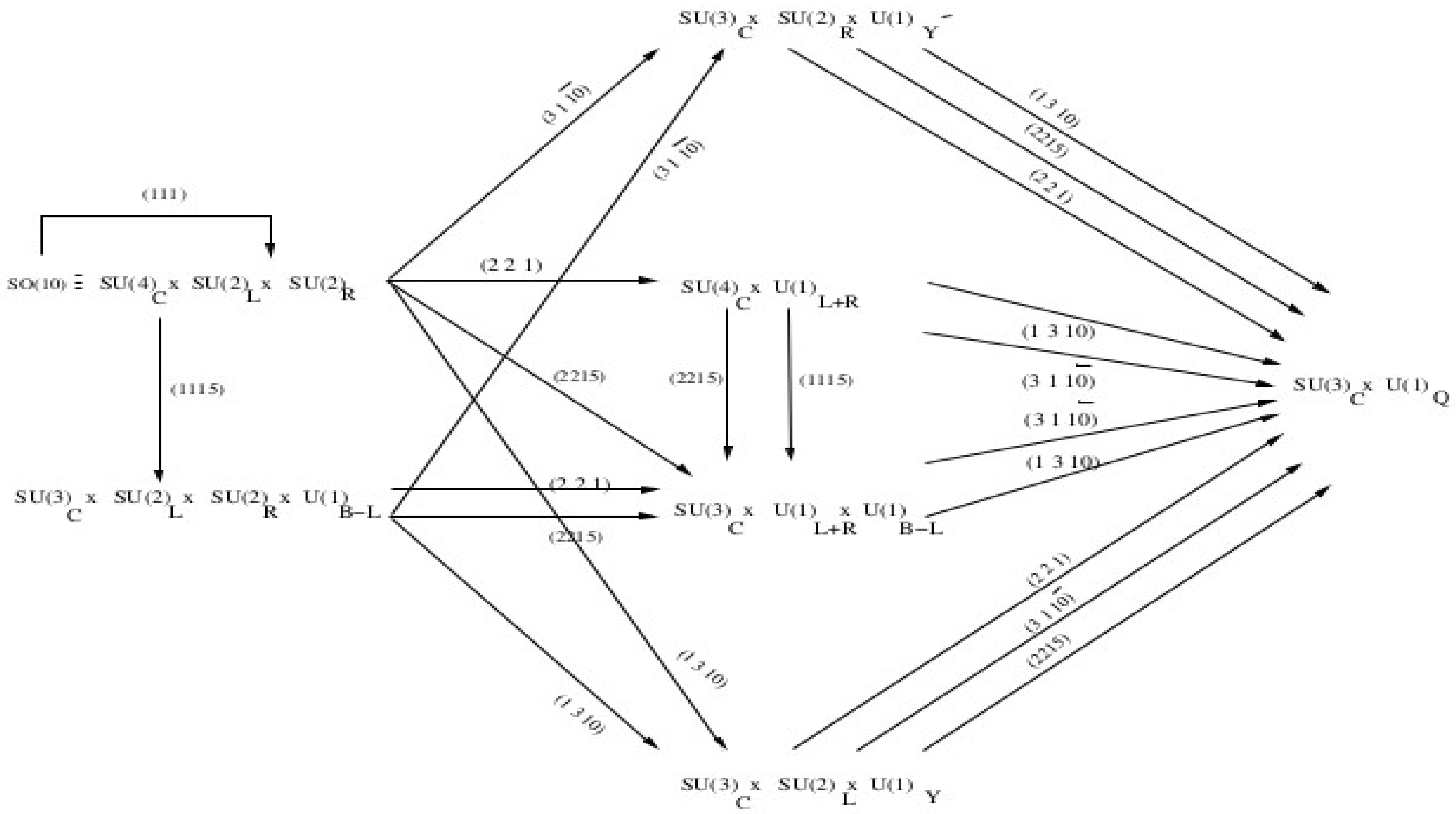
- All these different phenomena can be satisfied with a single intermediate scale → let us focus on simplicity

$$M_I \sim 10^{11}\,GeV$$

See-saw and leptogenesis compatible with M_I
(mνlight proportional to $1/M_I$)

M_I also suitable for the axion to reproduce the correct Dark Matter abundance
(Fa proportional to M_I)

An interesting feature of SO(10)



A less interesting feature of SO(10)...

The prize to pay:

- very large Higgs representations
- relevant amount of fine-tuning in the scalar and Yukawa sectors

- Can we do everything with more parameters?

  - NO!
    Not all SO(10) models work equally well
    for instance:
    -mass matrices are strongly correlated
    -GUT scale and intermediate scale mostly fixed in a 1-step breaking

A possible-simple breaking chain

M_{GUT} — SO(10)

210

breaking SO(10)

$PS = SU(4) \times SU_L(2) \times SU_R(2)$

M_I

$\overline{\mathbf{126}}$ **- 45**

for breaking Pati-Salam and give fermion masses

for a viable axion candidate

$SM = SU(3) \times SU_L(2) \times U(1)$

M_Z

10

for breaking SM and give fermion masses

Lazarides and Shafi, Nucl. Phys., B350:179-192 (1991).
Babu and Mohapatra, Phys. Rev. Lett. 70, 2845 (1993)

The role of the Higgs states

- The role of the $\overline{126}$

PS (4_C - 2_L - 2_R) quantum numbers

$$\overline{126} = (6,1,1) \oplus (\overline{10},3,1) \oplus (10,1,3) \oplus (15,2,2)$$

colored states: must be at M_{GUT}

mass at M_{GUT}

useful for see-saw type-II; not used here

mass at M_{GUT}

contain color singlet: used for breaking PS → SM

mass at M_I

vev at the EW scale: involved in the fermion masses and in the evolutions PS->M_{GUT}

mass at M_I

The role of the Higgs states

- The role of the 10

$$10 = (6,1,1) \oplus (1,2,2)$$

colored states:
must be at M_{GUT}

vev at the EW scale:
involved in the evolution
SM-> M_{GUT}

- The 45

$$45 = (6,2,2) \oplus (1,3,1) \oplus (15,1,1) \oplus (1,1,3)$$

contain color
singlet: used for
breaking PS → SM

Where are the dangerous colored states?

- Extended survival hypothesis:

it is the assumption that at any scale, the only scalar multiplets present are those that develop VEVs at smaller scales

We will relax this later

	210	$\overline{126}$	45	10
M_{GUT}	All components	(6,1,1) ($\overline{10}$,3,1)	(1,3,1) (6,2,2) (15,1,1)	(6,1,1)
M_I	—	(10,1,3) (15,2,2)	(1,1,3)	—
EW	-	-	-	(1,2,2)

M_{GUT} and M_I from gauge coupling unification

no freedom

- To 1-loop accuracy

$$\alpha_i^{-1}(M_2) = \alpha_i^{-1}(M_1) - \frac{a_i}{2\pi}\log\frac{M_2}{M_1}$$

$$a_i = \frac{4}{3}n_g - \frac{11}{3}N + \frac{1}{3}\eta S(R_p)$$

$$S(R_p) \equiv T(R_p)\frac{d(R)}{d(R_p)}$$

number of fermion generations

order of the SU(N) group

1 for complex
½ for real
representations

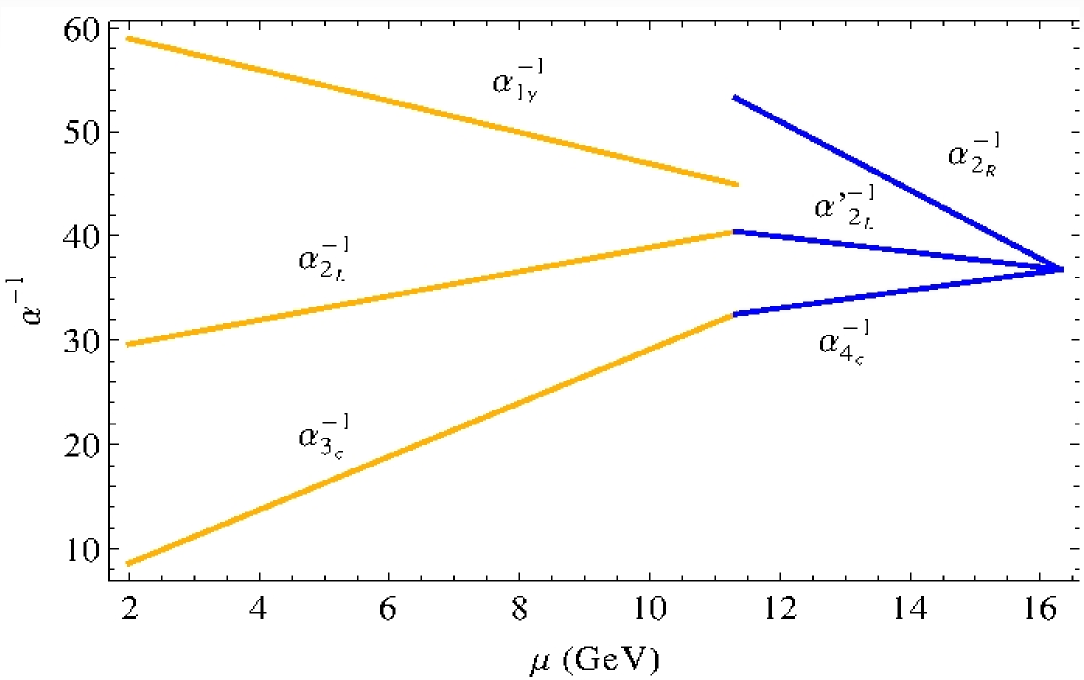
Dynkin index of the representation R_p

dimension of the representation R_p

M_{GUT} and M_I from gauge coupling unification
no freedom

- To 1-loop accuracy

$$\alpha_i^{-1}(M_2) = \alpha_i^{-1}(M_1) - \frac{a_i}{2\pi}\log\frac{M_2}{M_1}$$



a_3	a_{2L}	a_Y	a_4	a'_{2L}	a_{2R}
-7	-19/6	41/10	-7/3	2	28/3

$$M_I = (1.3 \pm 0.2)\cdot 10^{11}\, GeV$$

$$M_{GUT} = (1.9 \pm 0.6)\cdot 10^{16}\, GeV$$

$$\alpha_{GUT} \sim 0.027$$

Proton decay

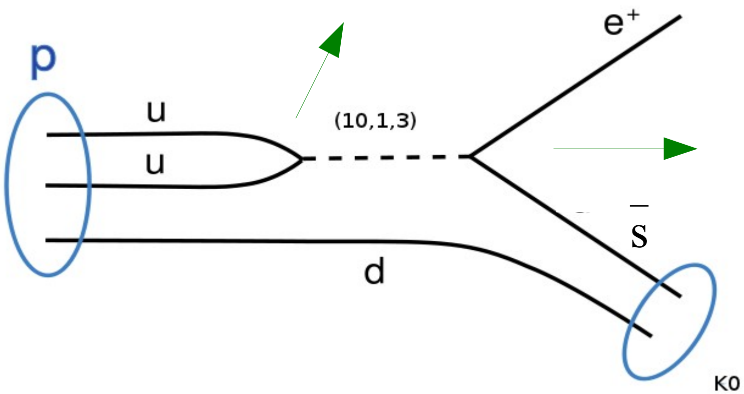
- naïve estimate

$$\tau \sim \frac{M_{GUT}^4}{\alpha_{GUT}^2 m_p^5} \sim 5 \cdot 10^{36}\, y \gg \tau^{\exp} \equiv 10^{34}\, y$$

- Potential problems from colored scalar triplets contained in the (10,1,3) of 126 with masses around M_I

p-> K^0 e^+

$(m_u{}^2/v_{15}{}^2)^{1/2}$



$(m_s m_d/v_{15}{}^2)^{1/2}$

$$\Gamma \sim \frac{m_u^2 m_d m_s \sin^2\theta_C}{v_{15}^4} \left(\frac{m_p^5}{M_T^4} \right)$$

$$M_T \geq 10^{10-11}\, GeV \sim M_I$$

The Yukawa sector

- Yuwaka Lagrangian

$$L_Y = 16_F \left(h\,10 + f\,\overline{126} \right) 16_F$$

h,f complex
symmetric matrices

- Let us analyze the first term: 16.16.10

$$10 = (6,1,1) \oplus (1,2,2) \xrightarrow{\text{decomposition under SU(3)xSU(2)xU(1)}} (1,2,2) = \left(1,2,\frac{1}{2}\right) \oplus \left(1,2,-\frac{1}{2}\right) \equiv H_u \oplus H_d$$

- In the SM: $H^*_u = H_d \rightarrow$ in the limit $V_{cb}=0$ we would get $m_\tau/m_b \sim 1$

contradiction with the experimental fact $m_\tau/m_b \ll 1$

Bajc (2006)

The Yukawa sector

- one assumes a 10 with complex components → H_u different from H_d

$$k_{u,d} = \langle (1,2,2)_{u,d} \rangle_{10}$$

- To rely on minimality, the term 16.16.10* must be avoided

  U(1) symmetry a la Peccei-Quinn

$$16_F \rightarrow e^{i\alpha} 16_F\,,\, 10 \rightarrow e^{-2i\alpha} 10,\, \overline{126} \rightarrow e^{-2i\alpha} \overline{126}$$

No need for a new coupling h' !

- For the $(15,2,2)_{126}$:

$$v_{u,d} = \langle (15,2,2)_{u,d} \rangle_{126}$$

Mass matrices

$$\left.\begin{aligned} M_u &= h k_u + f v_u \\ M_d &= h k_d + f v_d \end{aligned}\right\} \text{quarks}$$

$$\left.\begin{aligned} M_l &= h k_d - 3 f v_d \\ M_{\nu_D} &= h k_u - 3 f v_u \\ M_\nu^M &= f v_R \end{aligned}\right\} \text{leptons}$$

$$v_R = \langle (10,1,3) \rangle_{126} \neq 0$$

for see-saw type-I & leptogenesis

- See-saw mechanism for the light neutrino masses

$$m_{\nu_{light}} = M_{\nu_D}^T \left(M_\nu^M \right)^{-1} M_{\nu_D}$$

O(EW)   O(lepton number violation)

Baryon asymmetry through Leptogenesis

- Let us introduce the baryon-to-photon number ratio as a fit observable

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (5.7 \pm 0.6) \times 10^{-10}$$

Iocco et al., Phys. Rept.472, 1 (2009)

- <u>Leptogenesis</u>: a mechanism to generate dynamically the baryon asymmetry through a lepton asymmetry via the out of equilibrium decays of the lightest right-handed neutrino

Sphalerons then convert the lepton asymmetry into the baryon asymmetry

Baryon asymmetry through Leptogenesis

To get dynamically generated Lepton asymmetry:

Sakharov conditions

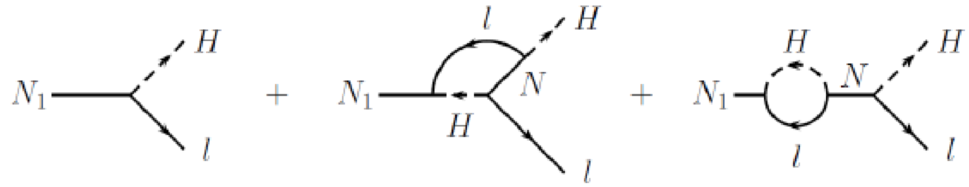
- Violation of L: guaranteed if neutrinos are Majorana particles
- C and CP violation: guaranteed if the neutrino Yukawa couplings contain physical phases
- Departure from thermal equilibrium: guaranteed, due to the expansion of the Universe

⇓

to avoid wash-out of the generated asymmetry

Baryon asymmetry through Leptogenesis

Lectures by A.Ibarra



$$\epsilon_1 = \frac{\Gamma(N_1 \to lH) - \Gamma(N_1 \to l^c H^c)}{\Gamma(N_1 \to lH) + \Gamma(N_1 \to l^c H^c)}$$

$$\varepsilon_{i\alpha} = \frac{3}{16\pi(h^\dagger h)_{ii}} \sum_{j\neq i} \left\{ \mathrm{Im}\left[h^\star_{\alpha i} h_{\alpha j} (h^\dagger h)_{ij}\right] \frac{\xi(x_j/x_i)}{\sqrt{x_j/x_i}} + \frac{2}{3(x_j/x_i - 1)} \mathrm{Im}\left[h^\star_{\alpha i} h_{\alpha j} (h^\dagger h)_{ji}\right] \right\}, \tag{9}$$

Dirac mass matrices

where $x_i \equiv (M_i/M_1)^2$ and

$$\xi(x) = \frac{2}{3} x \left[(1+x) \ln\left(\frac{1+x}{x}\right) - \frac{2-x}{1-x}\right]. \tag{10}$$

Majorana masses

Blanchet and Di Bari, JCAP 0703, 018 (2007)

Fit results

- We have to estimate 15 real parameters:

  12 in Md, 3 different vevs

- 15 observables at the GUT scale (extrapolated in SM-like):

  6 quark masses, 4 in the CKM, 3 in the PMNS, η_B, $\Delta m_{sol}/\Delta m_{atm}$

Obs.	fit	pull	Obs.	fit	pull
mu	0.49	0.03	$\|V_{us}\|$	0.225	0.038
md	0.78	0.75	$\|V_{cb}\|$	0.042	-0.208
ms	32.5	-1.5	$\|V_{ub}\|$	0.0038	-0.659
mc	0.287	-1.49	J	3.1×10^{-5}	0.589
mb	1.11	-2.77	$\sin^2\theta_{12}$	0.318	0.611
mt	71.4	0.7	$\sin^2\theta_{23}$	0.353	-1.548
r	0.031	0.1	$\sin^2\theta_{13}$	0.0222	-0.758
η_B	5×10^{-10}	-0.001			

Fit results

$$\chi^2_{min} = 17.4$$

- All data reproduced within 3σ
- The largest contribution from the atmospheric angle → The tendency to drift toward smaller values is due to the stringent requirements imposed by η_B (otherwise χ^2 ~0.95)

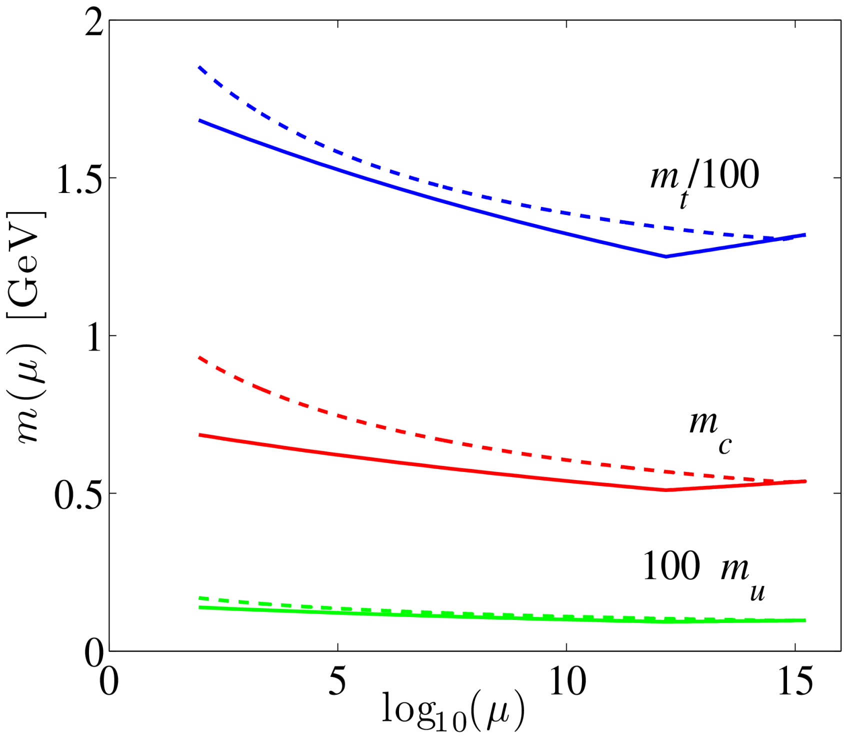
predictions

Light ν masses (eV)	Heavy ν masses (10^{11} GeV)	Phases (°)	m_{ee} (eV)	Σm_i (eV)
0.0046	1.00	δ=88.6	5×10^{-4}	0.065
0.0098	1.09	ϕ_1=-33.2		
0.0504	21.4	ϕ_2=15.7		

→ compact RH spectrum

Effects of M_I on the Yukawa RGE's

- Important observation: the change of the gauge groups is relevant on the running of the Yukawa's
- Exercise: find **h** and **f** at the GUT scale that gives a good description of the observables at the EW scale



Solid lines: including the change of the gauge group at MI

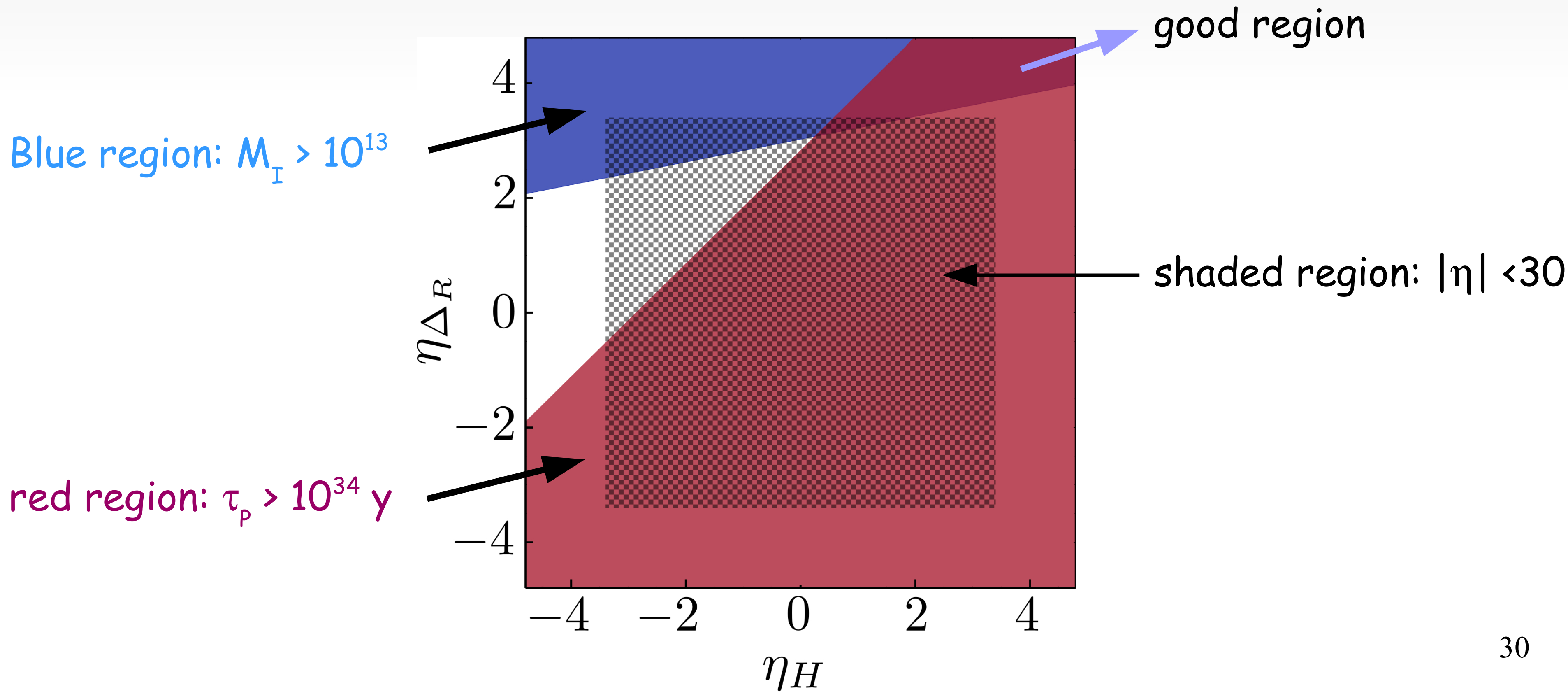
Dashed lines: same starting point at GUT but not including the change of the gauge group at MI

Effects of threshold corrections on M_I

- More freedom: not all Higgs boson masses at the same scale.

  Let us consider $\eta = \mathrm{Log}[m_H/M_I]$

- Exercise: move the Δ_R=(10,1,3) and Δ_H=(15,2,2) masses to give $M_I > 10^{13}$ GeV and $\tau_p > 10^{34}$ y



The request for an axion candidate

The strong interaction sector admits a term that violates both CP and P

$$L_{CP} = \bar{\theta} \frac{\alpha_s}{8\pi} G \cdot \tilde{G}$$

$\bar{\Theta}$ can be any value of O(1)

Enormous neutron electric dipole moment $d_n \sim e \bar{\theta} \frac{m_q}{M_N^2}$ unless: $\bar{\theta} < 10^{-9}$

- Peccei-Quinn: the full Lagrangian invariant under a $U(1)_{PQ}$

⇒ Anomalous $U(1)_{PQ}$ spontaneously broken at a scale F_a

→ $\bar{\theta} = 0$ dinamically

→ PNG in the theory the axion

Weinberg and Wilczek

The request for an axion candidate

- $U(1)_{PQ}$ is anomalous ⇒ $m_a \sim \Lambda^2_{QCD}/F_a$

→ scale associated with the U(1) breaking

Invisible axions:

coupling to pions: $\xi_{a\pi} = \lambda_3 f_\pi / F_a$

coupling to eta: $\xi_{a\eta} = \lambda_0 f_\pi / F_a$

λ_0, λ_3 = model parameters

coupling to photons: $L \sim 1/F_a$

If $F_a \sim M_I \rightarrow$ small mass and small couplings

The request for an axion candidate

- Under U(1):

$$16_F \to e^{i\alpha}\, 16_F\,,\, 10 \to e^{-2i\alpha}\, 10,\, \overline{126} \to e^{-2i\alpha}\, \overline{126}$$

- It is expected that the $U(1)_{PQ}$ be broken by $\langle \overline{126} \rangle \neq 0$ at the scale of $SU(2)_R$ breaking, otherwise the 10 would drive the U(1) breaking to give $M_{PQ} \approx M_W$, which is ruled out by experiments

- $\langle \overline{126} \rangle \neq 0$ is not enough, since a linear combination of $U(1)_{PQ}$ and B-L remains unbroken

Add another Higgs representation

The request for an axion candidate

- 16 Mohapatra and Senjanovic, Z.Phys. C17, 53 (1983)
- another 126 B.Bajc et al.,Phys. Rev. D73, 055001 (2006)
- 45 → focus in this repr. to break the degeneracy

$(1,1,3) \in 45$ with vanishing B-L and α' different from α

⇓

little impact on the coupling constant evolutions

Axions as dark matter particles

- The axion mechanism gives a solution to the strong CP problem without the need of imposing an additional constraint in the fitting procedure

Kim and Carosi, Rev. Mod. Phys. 82 (2010) 557

- mass: $$m_a = \frac{(m_u/m_d)^{\frac{1}{2}}}{1+m_u/m_d} \frac{f_\pi m_\pi}{F_a}$$ $\Rightarrow$ $m_a \sim (4.3-4.7)\times 10^{-5}\, eV$

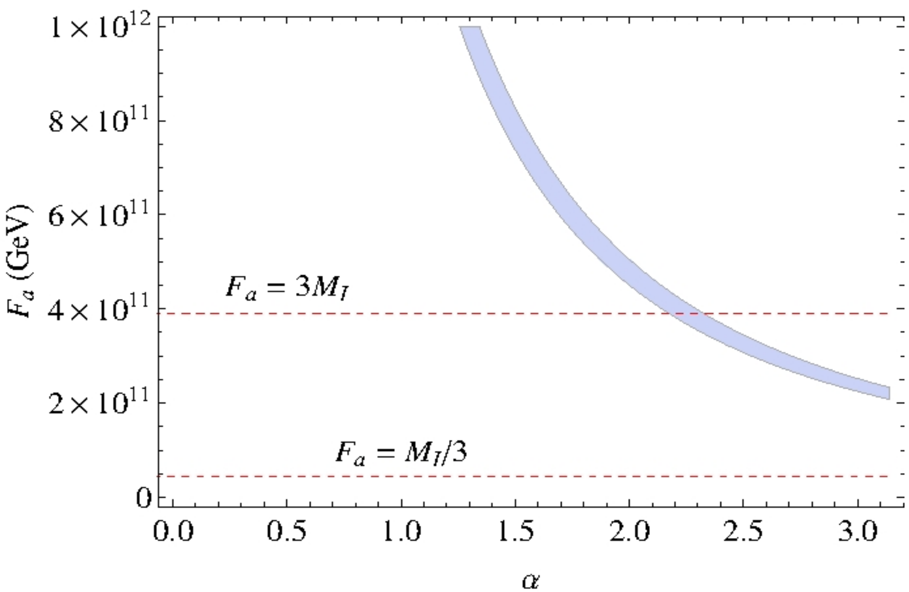
F_a ~ MI

- energy density of cold axions:

$$\Omega_c h^2 \sim 0.7 \left(\frac{F_a}{10^{12}\, GeV}\right)^{7/6} \left(\frac{\alpha}{\pi}\right) = \underbrace{0.1192 \pm 0.0062}_{}$$

PLANCK+WP+BICEP2+BA
arXiv:1403.6462

α=initial misalignment angle



Conclusions

- Non-susy SO(10) gives a viable GUT scenario for beyond SM physics
- A particular breaking chain with $M_I \sim 10^{11}$ GeV is needed to accommodate all compelling phenomena that demand new physics below M_{GUT}
- Price to pay: very large level of fine-tuning

  We have to find out possible mechanisms to reduce it
- Competitive scenarios: non-renormalizable couplings (smaller Higgs representations), fermions in other than the 16 repr., family symmetries...

Backup

Naturalness

- Electromagnetic energy of an electron as a sphere of radius r: α/r

this must be smaller than the total energy $E=m_e$

$\rightarrow r > \alpha/m_e \gg$ atomic radius

Either the different contributions to the total energy mysteriously cancel with a high precision, or some new physics sets in before the energy scale r^{-1}, modifying the EM contribution to the electron mass at short distances and preserving naturalness

the positron has to be included in a consistent relativistic quantum theory

Including leptogenesis

- The important novelty of our approach is the introduction of the baryon-to-photon number ratio as a fit observable

$$\eta_B = (5.7 \pm 0.6) \times 10^{-10}$$

Iocco et al.,
Phys. Rept.472, 1 (2009)

- To compute η_B: implementing the Boltzmann equations

The procedure is really time-expensive

- Alternative way:

W.Buchmuller, P.Di Bari and M.Plumacher,
Annals Phys.315, 305 (2005)

1- work with a given number of flavors and active RH neutrinos
2-implement simplified solutions of the Boltzmann equations
3-check a posteriori that the assumptions in step (1) are correct

Including leptogenesis

4- in the case of a positive answer, use the heavy spectrum and the Dirac mass matrix obtained from the fit to solve numerically the Boltzmann equations and get a more precise determination of η_B

We start assuming:

$$10^9 < M_{\nu_1} < 10^{12}\, GeV$$

τ Yukawa coupling is in equilibrium: two-flavour approach

Blanchet and Di Bari,
JCAP 0703, 018 (2007)
Abada et al.,
JHEP 0609, 010 (2006)

$$\left(M_{\nu_2} - M_{\nu_1}\right) / M_{\nu_1} \sim O(1)$$

N_1 and N_2 contribute to leptogenesis

Davidson, Nardi, Nir,
Phys.Rept.466, 105 (2008)

Di Bari, Riotto,
Phys.Lett. B671 (2009) 462-469;
JCAP 1104 (2011) 037

Including leptogenesis

Blanchet and Di Bari,
JCAP 0703, 018 (2007)

$$\varepsilon_{i\alpha} = \frac{3}{16\pi (h^\dagger h)_{ii}} \sum_{j\neq i} \left\{ \mathrm{Im}\left[h^\star_{\alpha i} h_{\alpha j} (h^\dagger h)_{ij}\right] \frac{\xi(x_j/x_i)}{\sqrt{x_j/x_i}} + \frac{2}{3(x_j/x_i - 1)} \mathrm{Im}\left[h^\star_{\alpha i} h_{\alpha j} (h^\dagger h)_{ji}\right] \right\}, \tag{9}$$

Dirac mass matrices

where $x_i \equiv (M_i/M_1)^2$ and

Majorana masses

$$\xi(x) = \frac{2}{3} x \left[(1+x) \ln\left(\frac{1+x}{x}\right) - \frac{2-x}{1-x} \right]. \tag{10}$$

decay terms

$z=M_1/T$

$$\frac{dN_{N_i}}{dz} = -D_i \left(N_{N_i} - N^{\rm eq}_{N_i}\right)$$

$$\frac{dN_{\Delta_\alpha}}{dz} = \sum_i \varepsilon_{i\alpha} D_i \left(N_{N_i} - N^{\rm eq}_{N_i}\right) - N_{\Delta_\alpha} \sum_i P^0_{i\alpha} W^{\rm ID}_i$$

N_{Δ_α}'s and the N_i's are the abundances per number of N_1's

$B/3-L_\alpha$

Flavor projectors

washout term

Additional contributions to leptogenesis

- Additional decay channels involving the RH gauge bosons and the color singlets in the (10,1,3)
- Let us consider the W_R

$$\Gamma_{N_1} = \frac{\left(M^{dag}_{\nu_D} M_{\nu_D}\right)_{11}}{4\pi v_u^2} M_{\nu_1}\left(1+X\right)$$

Dilution factor

$$M_{\nu_1} > M_{W_R}$$

NO because 2-body decays

$N \rightarrow l\, W_R$ are too fast $\rightarrow X \sim O(10^4\text{-}10^5)$

$$M_{\nu_1} < M_{W_R}$$

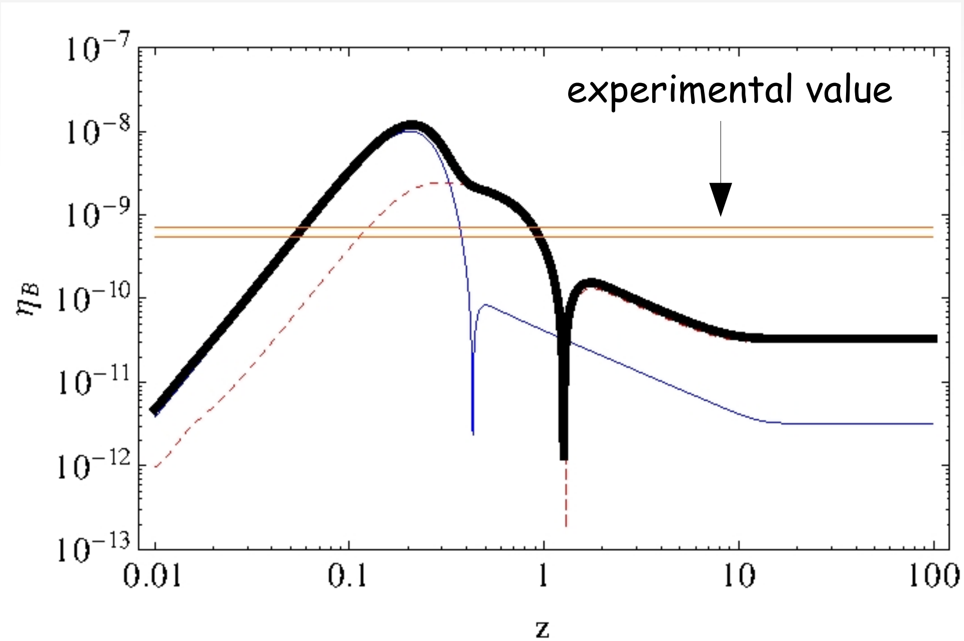
3-body decays $\rightarrow \Gamma_3 < H$ implies

$$M_{\nu_1} > 2\cdot 10^{11} / \left(M_{W_R} / M_{\nu_1}\right)^4$$

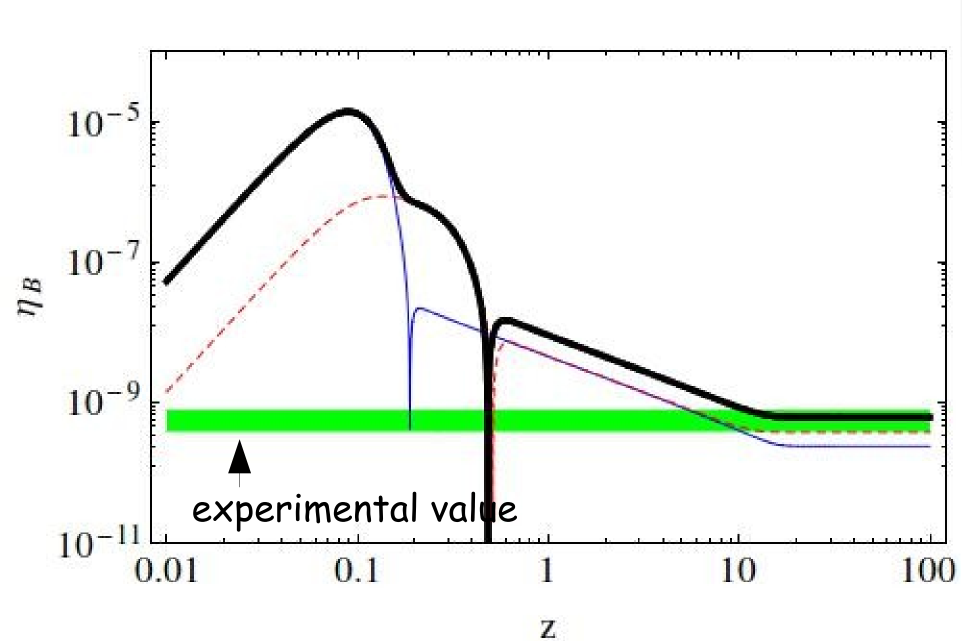
Satisfied for $M_{WR} \sim M\nu_1 \sim M_I$

A comment on leptogenesis

- Leptogenesis not included in the fit



- Leptogenesis included in the fit



$z=M_1/T$

NB: different vertical scales!

A comment on axions

- Lagrangian supplemented with $U(1)_{PQ}$

$$\mathcal{L}_{\rm SM}^{\rm eff} = \mathcal{L}_{\rm SM} + \bar{\theta}\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}_{a\mu\nu} - \frac{1}{2}\partial_\mu a \partial^\mu a + \mathcal{L}_{\rm int.}[\partial^\mu a/f;\psi] + \frac{a}{f}\xi\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}_{a\mu\nu}$$

- Minimum of the potential

$$\left\langle \frac{\partial V_{\rm eff}}{\partial a} \right\rangle = -\frac{\xi}{f}\frac{\alpha_s}{8\pi}\langle G_a^{\mu\nu}\tilde{G}_{a\mu\nu}\rangle \Big|_{\langle a \rangle} = 0 \,.$$

$$\langle a \rangle = -\frac{f}{\xi}\bar{\theta}$$

This solves the strong CP problem, since L^{eff}_{SM} , when expressed in terms of the physical axion field, $a_{phys} = a - \langle a \rangle$, no longer contains the CP violating $\theta G\tilde{G}$ term.

Other breaking chains

$SO(10) \rightarrow 3_c 2_L 2_R 1_X \rightarrow SM$

$(1,2,2,0)$ in 126 + $(1,1,3,0)$ in 45 $\Rightarrow$ $M_I \sim 10^9$ GeV

[or $(1,2,2,-1/2)$ in 16]

$SO(10) \rightarrow 3_c 2_L 2_R 1_X \times P \rightarrow SM$

$M_I \sim (0.4\text{-}1)\ 10^{11}$ GeV

$\tau \sim 10^{-1/-2}\ \tau_{exp}$

$3_c 2_L 2_R 1_X$ not a suitable intermediate scale

How to compute RGE's coefficients

	$d(R_p)$	$T(R_p)$
$SU(2)$	1	0
	2	1/2
	3	2
$SU(4)$	4	1/2
	6	1
	10	3
	15	4

$$(1,2,2) \equiv \Phi\,, \qquad (10,1,3) \equiv \Delta_R\,, \qquad (15,2,2) \equiv \Sigma\,, \qquad (1,1,3) \equiv \sigma$$

- Example: contributions to a4

$$\Sigma: \quad \frac{1}{3} \cdot T(15) \cdot \frac{d[(15,2,2)]}{d(15)} = \frac{1}{3} \cdot 4 \cdot \frac{60}{15} = \frac{16}{3}$$

$$\Delta_R: \quad \frac{1}{3} \cdot T(10) \cdot \frac{d[(10,1,3)]}{d(10)} = \frac{1}{3} \cdot 3 \cdot \frac{30}{10} = 3$$

$$a_4 = \frac{4}{3} \cdot 3 - \frac{11}{3} \cdot 4 + \frac{25}{3} = -\frac{7}{3}\,.$$

Mass matrices

$$M_u = h k_u + f v_u \qquad M_d = h k_d + f v_d \qquad M_{\nu_D} = h k_u - 3 f v_u$$

$$M_l = h k_d - 3 f v_d \qquad M_\nu^M = f v_R$$

→ for see-saw type-I & leptogenesis

- Rewritten in a suitable form for a fit:

Joshipura-Patel 2011
Dueck-Rodejohann 2013

$$M_u = r_\nu \left(\frac{3+s}{4} M_d + \frac{1-s}{4} M_l \right)$$

$$M_\nu^D = r_\nu \left(\frac{3(1-s)}{4} M_d + \frac{1+3s}{4} M_l \right)$$

$$M_\nu^M = r_R^{-1} \left(M_d - M_l \right)$$

$$r_\nu = k_u / k_d$$

$$s = v_u / r_\nu v_d$$

M_d = down-quark mass matrix

M_l = charged lepton mass matrix