Scattering of gravity waves in subcritical flows over an obstacle

Interpreting and conceiving analog gravity experiments

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Based on ongoing collaborations with
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Plan

- the Analogue Gravity Program: a qualitative intro.
- Scattering of water waves in sub-critical flows,
  how do they relate to those in trans-critical flows ?
  which parameters fix their properties ?
- Experiments in Vancouver and Pprime-Poitiers.

Analogue Gravity.

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  - Sound waves in a flowing gaz obey $\Box\phi(t, x) = 0$ in a curved 4D metric.
  - Hence, a statio. **transonic** flow should steadily emit Hawking radiation.
  - One might conceive experiments testing this prediction.

"completed" by incorporating short distance physics.

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  - Short distance physics induces **UV dispersion**,
  - *this might cure the "trans-Planckian" problem,* → **"Horava gravity"**

- 1995. W. Unruh, PRD *"Dumb holes and the effects of high freq. ..."*
  - Curved metric (IR effects) and dispersion (UV effects) combined in a **single wave equation**.
  - Numerically showed the "robustness" of Hawking radiation

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The program: testing Hawking's 1974 predictions

- What can be tested ?
  Which fluid should be used ?
  Which accuracy can be reached ?

- NB.
  Neither the Schwinger effect: $|\beta|^2 \propto e^{-\pi m^2/eE}$,
  nor the Unruh effect: $|\beta_\omega|^2 \propto e^{-2\pi\omega/a}$,
  has been tested so far.

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Hawking's predictions

Solving $\Box\phi(t,x) = 0$ in the near vicinity of a BH horizon, one gets Hawking '74

- I. The mean number of **spontaneously** emitted quanta of freq. ω is

$$n_\omega = 1/(e^{2\pi\omega/\kappa} - 1),$$

where the freq. $\kappa = 1/4M$ is the "surface gravity".

- II. These are **maxim. entangled** to inside partners with **neg. energy**:

$$|0\rangle_{\rm incoming} = \prod_{\omega>0} \left(e^{z_\omega a^\dagger_\omega a^\dagger_{-\omega}} \right) |0\rangle_{\rm outgoing} \otimes |0\rangle_{\rm inside}$$

(product of) 2-mode squeezed states

- Both predictions **directly** follow from a **mode analysis**:

$$\phi_\omega^{\rm incoming} = \alpha_\omega\, \phi_\omega^{\rm outgoing} + \beta_\omega\, (\phi_{-\omega}^{\rm inside\ partner})^*$$

where

$$|\beta_\omega|^2 = n_\omega, \quad |\beta_\omega/\alpha_\omega|^2 = |z_\omega|^2 = e^{-2\pi\omega/\kappa} < 1,$$

and $|\alpha_\omega|^2 - |\beta_\omega|^2 = 1$, hence $U(1,1)$: **anomalous** scattering

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What could be observed ?

- A. in the **stimulated (classical) channel**:
  I. spectral properties, i.e., $|\beta_\omega|^2$, $\arg(\beta_\omega/\alpha_\omega)$, ...

  using classical waves,
  norm/phases of (the **anomalous***) scattering coefficients*
  can be observed as fctions of ω and of the flow parameters fixing κ

  Remember $|\beta_\omega^{\mathrm{Hawking}}|^2 = 1/(e^{2\pi\omega/\kappa} - 1)$ for $0 < \omega < \infty$.

- B. in the **spontaneous (quantum) channel (ultra low temp.)**:
  II. coherence (2-mode entanglement, **non-separability**).

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What has been/is being/ done

- **“ab initio” calculations of the coeff. of the S-matrix in different settings**
  to predict the properties of the emitted radiation by the anomalous scattering $|\beta_\omega|^2$: "analog Hawking radiation".

- since 2008, **experiments**
  - in water tanks, Nice ’08, Vancouver ’11, Poitiers ’14-’16, Nottingham > 2016,
  - in atomic BEC, Technion ’11, ’13 (BH-Laser), ’15 + Institut d’Optique,
  - in glass, in polariton systems Marcoussis, in air Le Mans,...

- theoretically BEC is the neatest/simplest case, ...

Analogue Gravity with shallow water waves

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- 2008. G. Rousseaux et al, NJP. *"Observation of ... "* **Observation** of **NEW** (negative energy waves).
- 2011. S. Weintfurtner et al, PRL *"Measurement of stim. HR ..."* observed
  - the *linear* mode conversion giving rise to **NEW**, $\phi_\omega^{\rm in} \to \alpha_\omega \phi_\omega^{\rm out} + \beta_\omega (\phi_{-\omega}^{\rm part})^* + ...$
  - that $R_\omega = |\beta_\omega|^2/|\alpha_\omega|^2 \sim e^{-\omega/\omega_c}$ follows a Boltzmann law.
  - yet, the flow was **sub-critical**, i.e. $(v/c)|_{\rm max} \sim 0.7$, *no horizon* $\to$ *how to* **understand** *this observation ?*

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Aims: Long term program

- **I.** Understand the **scattering** of surface waves in the **absence** of a "sonic" horizon, i.e., $|(v/c)|_{\max} < 1$.
  To this end, study and numerically solve the linear wave eq.

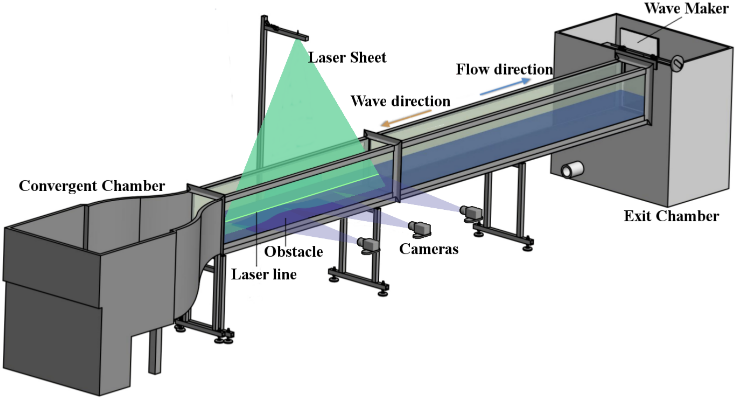
- **II. Interpret** experiments, e.g. behavior of $R_\omega^{\text{Vanc.}}$, and **propose** *"improved"* experiments.

  - study and numerically solve the non-linear eq. to ***design the obstacle*** *determining the* ***background flow***

  - optimize the extraction of scattering coefficients from data

NB. a source of difficulty: R_ω is significative only for very low $\omega \sim 0.2 Hz \to$ very long wave lengths $\sim 10m$.

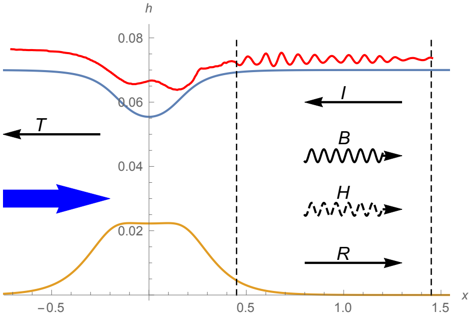
Experimental settings: counter flow



– **Counter-propagating** statio. waves $\delta h_\omega(t,x)$ emitted by wave maker are blocked near the top of the **obstacle**, and reflected/blue shifted

– **Free surface fluct.** $\delta h(t,x)$ illuminated by a laser sheet, photographed by cameras, analyzed in double Fourier space $\rightarrow \tilde{\delta h}(\omega,k)$.

Scattering of counter-propag. modes



In brown the 'designed' obstacle, in red the observed (time-average) surface.
There is a zero-frequency modulation: an undulation.
Vertical lines show the region used to analyzed the mode amplit. $\tilde{\delta h}(\omega, k)$.

The incoming mode I and the four scattered modes T, R, B, H,

$$\phi_\omega^{\leftarrow} \to T_\omega\, \phi_\omega^{\leftarrow} + R_\omega\, \phi_\omega^{\rightarrow} + \alpha_\omega\, \phi_\omega^{\rightarrow, d} + \beta_\omega\, (\phi_{-\omega}^{\rightarrow, d})^*$$

$$|R_\omega|^2 + |T_\omega|^2 + |\alpha_\omega|^2 - |\beta_\omega|^2 = 1, \quad \text{hence } U(1,3)$$

B and H are dispersive modes, H **carries a negative energy (NEW).**

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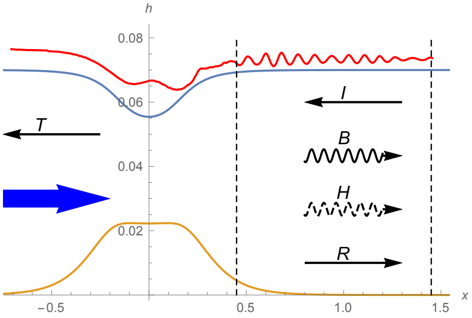

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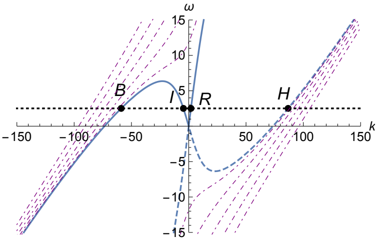
Dispersion relation and power spectrum

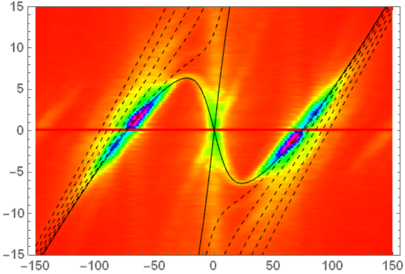


Dispersion relation (ω vs k)

$$\Omega^2 = (\omega - vk)^2 = gk\tanh(kh)$$

At fixed ω, there are 4 roots k^a_ω,
I, R are hydro: $k \propto \omega$
B and H are dispersive: $k \propto 1/h$
the H-root has $\omega\Omega < 0$, $\rightarrow$ a NEW.

In dashed-dotted, transverse modes



Obs. power spectrum of noise:

$$P(\omega, k) = \frac{\langle\langle\, |\delta h(\omega, k)|^2 \,\rangle\rangle}{|gk\tanh(kh)|^{1/2}},$$

Typical amplitude: 0.1 mm, $\delta h/h \sim 10^{-3}$.
Ensemble average over 80 realizations,
NEW are present in the spectrum.

I. Linear wave equation

- **Hypotheses:**
  - inviscid, incompressible, ideal fluid,
  - **2D** irrotational flow,
  - gravity is the only external force, neglect capillary effects.

- **Non-linear equations:**
  - $\vec{\nabla} \times \vec{v} = 0 \to \vec{v} = \vec{\nabla}\phi$;
  - continuity equation: $\Delta_{2D}\,\phi = 0$;
  - unpenetrable bottom: $(v_y - v_x \partial_x y_b)_{y=y_b} = 0$;
  - free surface: $(v_y - v_x \partial_x y_s)_{y=y_s} = 0$;
  - Bernouilli equation (continuity of pressure):

$$\frac{(\vec{v})^2}{2} + gy = cst. \text{ at } y = y_s(x) = h(x).$$

- **Linear perturbations:** $\phi(t,x,y) = \phi_0(x,y) + \delta\phi(t,x,y)$.

I. Wave equation

One finds that linear surface waves (**approxim.**) obey Unruh 2012

$$[(\partial_t + \partial_x v)(\partial_t + v\partial_x) - ig\partial_x \tanh(-ih(x)\partial_x)]\, \delta\phi(t,x) = 0, \quad (1)$$

- a **1+1D PDE of infinite order**,
- $v(x)$ is the horizontal component of the bckrd flow velocity,
- $h(x) = y_s(x)$, the background flow height.

The $1+1D$ dispersion relation is thus

$$(\omega - vk)^2 = gk \tanh(hk). \quad (2)$$

$\delta\phi$ is related to the **observable:** the linear variation of $h(x)$ by

$$\delta h(t,x) = -\frac{1}{g}(\partial_t + v\partial_x)\, \delta\phi. \quad (3)$$

Very similar eqs apply to density perturbations in BEC and gazes. (These eqs. have an Hamiltonian structure)

Quartic dispersion relation

- In stationary flows, work with (complex) stationary waves $e^{-i\omega t}\phi_\omega(x)$ with fixed lab. frequency ω.
- expand to 3rd order in $h\partial_x$:

$$\left[(-i\omega + \partial_x v)(-i\omega + v\partial_x) - g\partial_x h\partial_x - \frac{g}{3}\partial_x (h\partial_x)^3\right]\phi_\omega = 0. \quad (4)$$

preserving the ordering of $h(x)$ and ∂_x.

- The assoc. quartic dispersion relation is

$$(\omega - v\,k_\omega)^2 = c^2\,k_\omega^2\left(1 - \frac{h^2\,k_\omega^2}{3}\right), \quad (5)$$

$\mathbf{c^2} = \mathbf{gh}(\mathbf{x})$: the (local) group (velocity)^2 for low k_ω waves in fluide frame.

$\mathbf{h}(\mathbf{x})$ gives the x-dep. dispersive length (the "Planck" length)

Hydrodynamics, and black (white) hole metric

- In the hydrodynamical approximation, one neglects $(hk)^2 \ll 1$.
  Using, $c^2(x) = gh(x)$, the wave eq.

$$\left[(-i\omega + \partial_x v)(-i\omega + v\partial_x) - g\partial_x h(x)\partial_x - \frac{g}{3}\partial_x \left(h(x)\partial_x\right)^3\right] \phi_\omega = 0$$

  is a (*dim. reduced*) Klein-Gordon in a 2D space-time metric

$$ds^2 = -c(x)^2 dt^2 + (dx - v(x)dt)^2,$$

- There is a Killing horiz. **when** $v(x)$ **crosses** $c(x)$.
- In fact, the wave-energy (the hamiltonian) is **no longer positive def**.
  $\rightarrow$ the spectrum of statio modes (generically) contains **NEWs**.
- if v increases (*decreases*) along v, one gets a black (*white*) horizon,

  i.e., a decrease (*increase*) of k_ω for **counter-prop. waves**

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II. Background flow profiles

- **In experim.**, the bckd. flow is fixed by the **obstacle:** $h_B(x)$.
- **mathem.**, fully described by the **water depth** $h(x)$, since

$$v(x) = J/h(x), \quad c(x) = \sqrt{g\,h(x)}. \tag{6}$$

- **Monotonic flows** can be parameterized by

$$h(x) = h_0 + D \tanh\left(\frac{\sigma x}{D}\right). \tag{7}$$

- **Non-monotonic flows** by

$$h_{\mathrm{non-m}}(x) = h_0 + D \tanh(\frac{\sigma_1}{D}(x+L)) \tanh(\frac{\sigma_2}{D}(x-L)), \tag{8}$$

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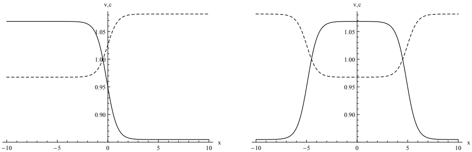
Trans-critical versus sub-critical flows

- the trans-critical character fixed by **"Froude number"** $F \equiv v/c$
- When $\mathbf{F}_{\max} > \mathbf{1}$: the flow is **transcritical**.
- A *"Killing horizon"* (exactly) corresponds to $v = c$, i.e., $F = 1$.
- The **surface gravity** $\kappa_G = |\partial_x (c - v)|_{v=c}$ is (exactly)

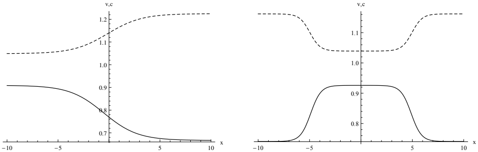
$$\kappa_G = |\partial_x F|_{F=1} \propto \sigma. \tag{9}$$

4 types of WH flows

velocity $v(x) > 0$ (flow to the right) (plain), and speed $c(x)$ (dashed)



trans-critical monot., and non-monot. flows.



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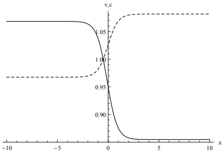

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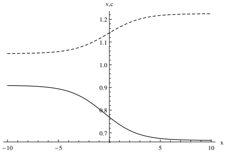

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III. Mode analysis

the scattering coeffs. are defined in the asymptotic (right) sub-critical region.

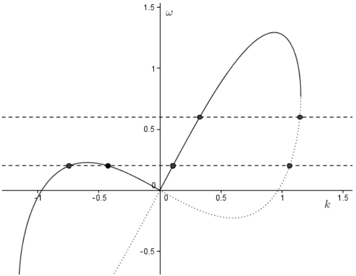
The 3 + 1 stationary modes (ABM)

In a **sub-critical flow**, the **3 + 1 stationary modes** are

- $\phi_\omega^{\rightarrow,d}$ is dispersive and right-moving in the lab frame; the blue-shifted mode
- $\phi_\omega^{\leftarrow}$ is hydrodynamic, and left-moving; the incoming mode
- $\phi_\omega^{\rightarrow}$ is hydrodynamic, and right-moving; the reflected mode
- $(\phi_{-\omega}^{\rightarrow,d})^*$ is dispersive, and right-moving. the "created" mode

NB.1 The last one (the NEW) has a **negative (Klein-Gordon) norm.**
(*the corresponding root lives on the* **negative** $\Omega \doteq \omega - vk$ *branch.*)

NB.2. There is a crit. freq. $\omega_{\max}$ above which the first 2 roots no longer exist.



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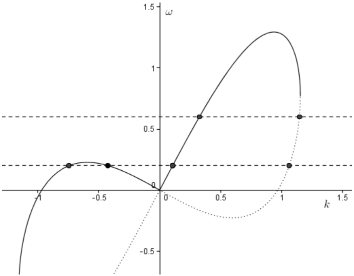


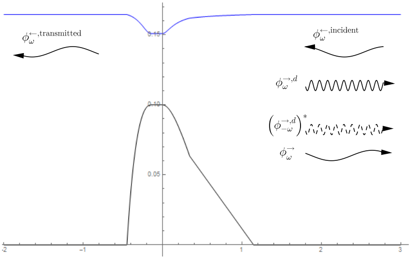
Asymptotically sub-critical flows

4-mode mixing: (below $\omega_{\max}$)

$$\phi_\omega^{\leftarrow,in} \to \alpha_\omega\, \phi_\omega^{\to,d,out} + \beta_\omega\, (\phi_{-\omega}^{\to,d,out})^* + A_\omega\, \phi_\omega^{\to,out} + \tilde{A}_\omega\, \phi_\omega^{\leftarrow,out}, \qquad (10)$$

and "unitarity" (i.e., conservation of the norm) gives

$$|\alpha_\omega|^2 - |\beta_\omega|^2 + |A_\omega|^2 + \left|\tilde{A}_\omega\right|^2 = 1. \qquad (11)$$



Michel-RP '14, precursors: RP-Finazzi '10, Scott R. review '12, Finazzi-Carusotto '12

The 4×4 S-matrix

Considering the four incoming modes, one has

$$\begin{pmatrix} \phi_\omega^{\leftarrow,\mathrm{in}} \\ \phi_\omega^{\rightarrow,d,\mathrm{in}} \\ \left(\phi_{-\omega}^{\rightarrow,d,\mathrm{in}}\right)^* \\ \phi_\omega^{\rightarrow,\mathrm{in}} \end{pmatrix} = \begin{pmatrix} \tilde{A}_\omega & \alpha_\omega & \beta_\omega & A_\omega^{(v)} \\ \bar{\alpha}_\omega & A_\omega & B_\omega & \alpha_\omega^{(v)} \\ \bar{\beta}_\omega & \bar{B}_\omega & \bar{A}_\omega & \beta_\omega^{(v)} \\ \bar{A}_\omega^{(v)} & \bar{\alpha}_\omega^{(v)} & \bar{\beta}_\omega^{(v)} & A_\omega^{(vv)} \end{pmatrix} \begin{pmatrix} \phi_\omega^{\leftarrow,\mathrm{out}} \\ \phi_\omega^{\rightarrow,d,\mathrm{out}} \\ \left(\phi_{-\omega}^{\rightarrow,d,\mathrm{out}}\right)^* \\ \phi_\omega^{\rightarrow,\mathrm{out}} \end{pmatrix}. \quad (12)$$

NB1. The (v)-mode $\phi_\omega^{\rightarrow,\mathrm{out}}$ is co-propagating and plays no signif. role.

NB2. In trans-critical monotonous flows,
it reduces to a 3×3 because there is no transmitted mode $\phi_\omega^{\leftarrow,\mathrm{out}}$.

NB3. In trans-critical non-monotonous which are asympt. sub-crit. flows,
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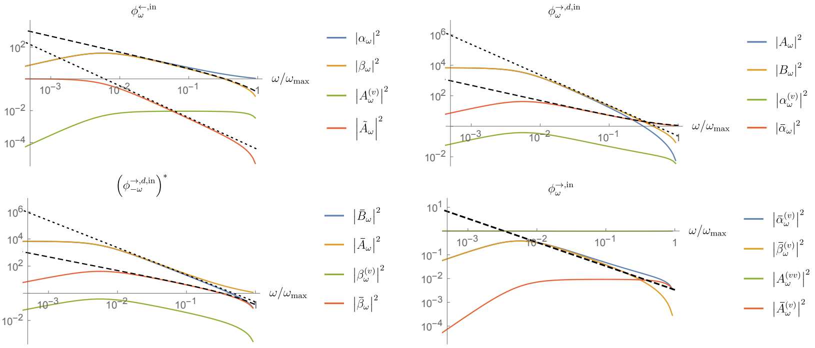

$F_{\max} = 1.4$, $L = h_{\mathrm{as}}$ (short obs.), $\sigma_{R/L} h_{\mathrm{as}} \sim 2$ (reasonable slopes).

1. Some norms are $\gg 100 \to$ **large amplification ! - quite unusual !**
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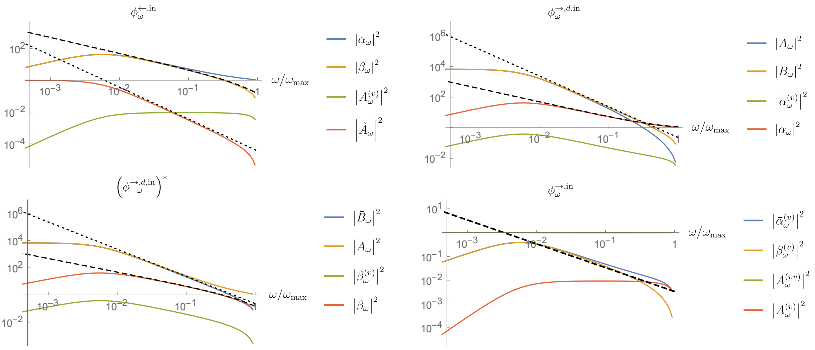

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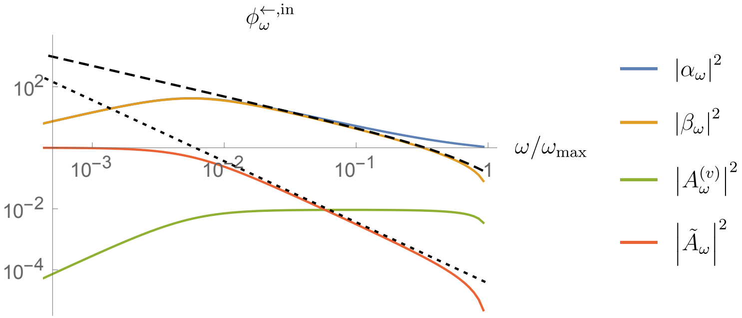
Link with Hawking radiation

$$\phi_\omega^{\leftarrow,\mathrm{in}} = \alpha_\omega \phi_\omega^{\rightarrow,d,\mathrm{out}} + \beta_\omega \left(\phi_{-\omega}^{\rightarrow,d,\mathrm{out}}\right)^* + A_\omega^{(v)} \phi_\omega^{\rightarrow,\mathrm{out}} + \tilde{A}_\omega \phi_\omega^{\leftarrow,\mathrm{out}}, \qquad (13)$$



- The dashed line is the Planck spectrum at the Hawk. temp. $T_H = \kappa/2\pi$.
- In a large frequency domain, excellent agreement. In part. $|\beta_\omega|^2 \sim T_H/\omega$.

- for low freq., $|\beta_\omega|^2 \sim |\alpha_\omega|^2 \sim \omega/\sigma_\beta$ for $\omega \to 0$.

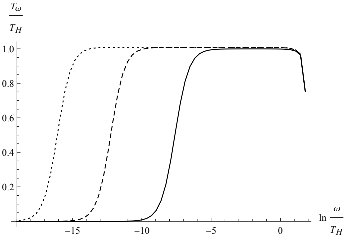
This is due to **transmission** across the obstacle: The transm. coefficient (in red) reaches 1: total transm.

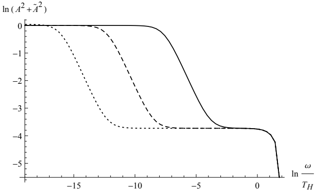
Verification

- To study the Hawkingness, plot of the effective temp. T_ω

$$|\beta_\omega|^2 \doteq (e^{\omega/T_\omega} - 1)^{-1}, \qquad (14)$$

for three different $L = 5$ (solid), 7 (dashed), and 10 (dotted)





- the constant value of T_ω closely agrees with $T_H = \kappa/2\pi$.
- except at ultra-low freq., **no significant** change at intermediate freq.
- the critical ultra-low freq. $\omega_c \propto e^{-2\, k^{\text{dec}}_{\omega=0}\, L} \ll T_H$ is not relevant.

- **Lesson: in transonic asympt. smooth flows,** **$|\beta_\omega^{\text{surface waves}}|^2$ closely agrees with the Hawking prediction.**

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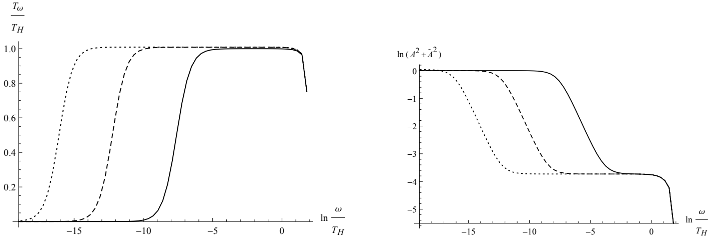

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  $|\beta_\omega^{\rm surface\ waves}|^2$ **closely agrees with the Hawking prediction.**

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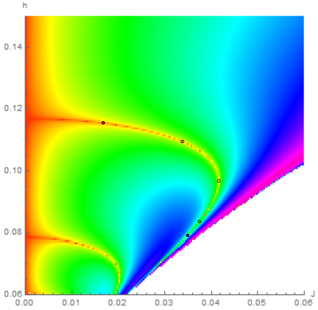
It is unclear if such flows could be realized.

Main reason:
"wave breaking of the undulation".

The zero freq. modulation on the downstream side becomes highly **non-linear**.

Plot: its amplitude versus h and J.



Strategy:

- conceive obstacles minimizing the undulation amplitude. (work in progress)
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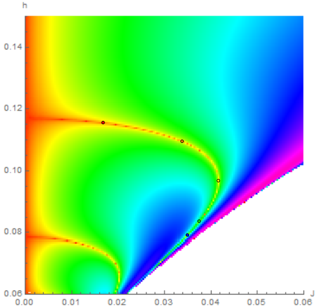

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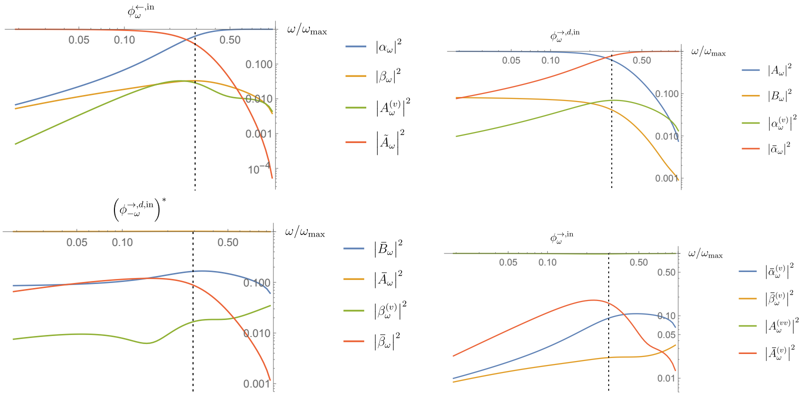


$F_{\max} = 0.8$, $L = 2h_{\mathrm{as}}$ (short obs.), $\sigma_{R/L} h_{\mathrm{as}} \sim 2$ (reasonable slopes).

Although $|\beta_\omega|^2 \neq 0$, there is $\rightarrow$ **no large amplification** .

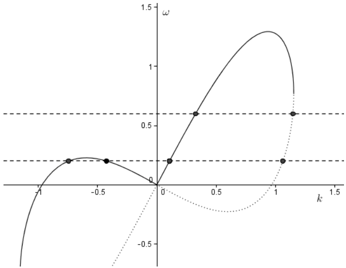
Because there is a critical freq. $\omega_{\min}$ below which **transmission** dominates.

sub-critical flows, the critical freq. $\omega_{\min}$

$F_{\max} < 1$ defines the **critical freq.**

$$\omega_{\min} \sim \frac{c}{3h}(F_{\max} - 1)^{3/2}, \quad \text{for } F_{\max} \to 1,$$

given by the double root of the disp. rel. for $F = F_{\max}$

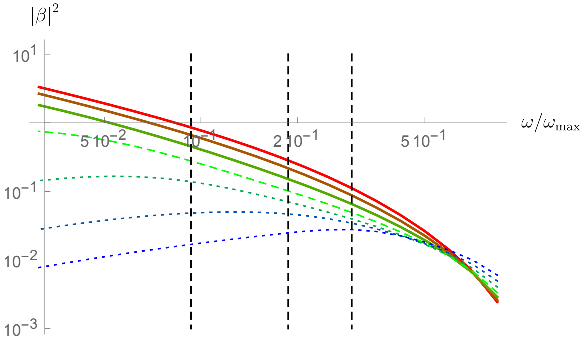


- for $\omega > \omega_{\min}$, **"wave blocking"**:
  there is a turning point (WKB) as in transcritical flows, hence, **little transmission :** $|\tilde{A}_\omega| \ll 1$.
- for $\omega < \omega_{\min}$, **"transmission":**
  there is no turning point, hence, **large transmission** $|\tilde{A}_\omega| \sim 1$, and little wave blocking and thus **little amplification** $|\beta_\omega|^2 \ll 1$**,**

In brief, because of transmission, sub-critical flows are less unstable.

Study of the properties when reducing $F_{\max}$

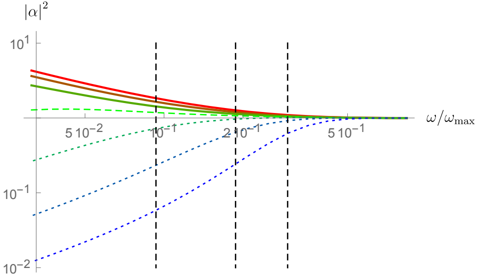
The 'evolution' of $|\beta_\omega|^2$ when reducing $F_{\max}$

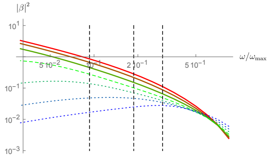


In a log scale, $|\beta_\omega|^2$ for a fixed F_{as} and 7 values of $F_{\max}$ from 1.2 to 0.8
The green dashed curve separates the 3 trans and the 3 subcritical flows.

Lesson: there is a smooth transition from trans- to sub-. which describes the reduction of the mode amplification.

The 'evolution' of $|\alpha_\omega|^2$ when reducing $F_{\max}$





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Lesson: In sub-critical flows, for $\omega < \omega_{\min}$,

$$|\beta_\omega|^2 \sim |\alpha_\omega|^2 \sim \omega,$$

Remember in Hawking case: $|\alpha_\omega|^2 = 1 + |\beta_\omega|^2$.

Remember that the Vancouver group used $|\beta_\omega|^2/|\alpha_\omega|^2$ to check "thermality".

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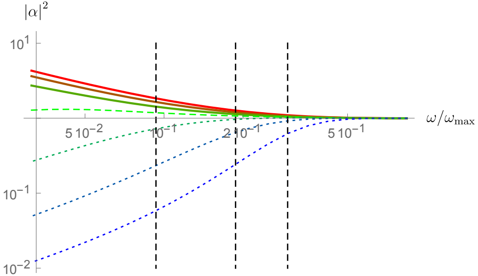

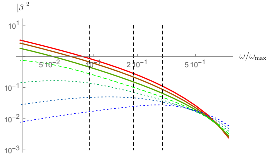

In a log scale, $|\alpha_\omega|^2$ for a fixed $F_{\rm as}$ and 7 values of $F_{\max}$ from 1.2 to 0.8
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IV. Interpreting the spectra

- Since the Hawking spectrum is (ex.) Planckian, $|\beta_\omega|^2 = 1/(e^{\omega/T_H} - 1)$ deviations are well characterized by an effective temperature $T_\omega^{\rm eff}$.

- Two temperatures have been used:

$$\ln \frac{|\beta_\omega|^2}{1 + |\beta_\omega|^2} = -\frac{\omega}{T_\omega^{\rm eff}}. \quad (15)$$

  Constancy of $T_\omega^{\rm eff}$ is equivalent to $|\beta_\omega|^2$ following the Planck law.

- The second one is defined by

$$\ln \left|\frac{\beta_\omega}{\alpha_\omega}\right|^2 = -\frac{\omega}{T_\omega^V}. \quad (16)$$

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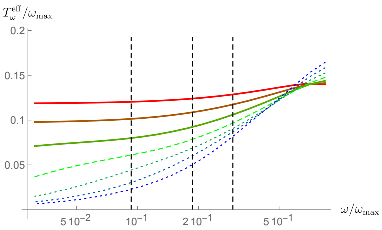
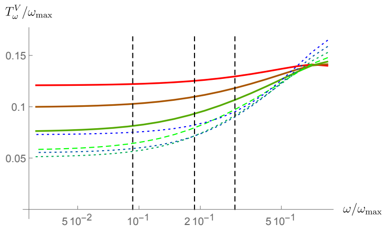




$T_\omega^{\rm eff}$ (left) and T_ω^V (right) for a fixed $F_{\rm as}$ and 7 values of $F_{\rm max}$ from 1.2 to 0.8

- For the 3 transcritical flows, they agree and are near constant.
- For the 3 subcritical flows, $T_\omega^{\rm eff}$ monoton. decreases with $F_{\rm as}$ (and ω).
- Instead, for low ω, T_ω^V **increases** when $F_{\rm as}$ decreases,

  This is because $|\alpha_\omega|^2$ decreases faster than $|\beta_\omega|^2$.
- **In brief, it is clear that the spectrum is no longer approx. Planckian,**
  it is also rather clear that several parameters are relevant.

V. Subcritical flows: the three regimes.

Because of transmission for $\omega < \omega_{\rm min}$,
the spectrum in subcritical flows splits into **3 separate** regimes:

- I. the simplest, most robust one is the **transition** in a narrow band centered on $\omega_{\rm min}$, where $|\tilde{A}_\omega|^2$ goes from ~ 1 to ~ 0.
- II. a **low frequency** regime where

$$|\beta_\omega|^2 \sim |\alpha_\omega|^2 \sim \omega/\sigma_\beta,$$

  hence **fully governed** by the freq. σ_β.
- III. a **high freq. regime**, where there is blocking, as in trans- flows. In this regime, one could expect to recover the Hawking prediction. In general, however, this is not the case.

NB. These properties should be observed/validated in future experiments.

I. The transitional regime around $\omega_{\rm min}$

nothing special to notice about **transmission**

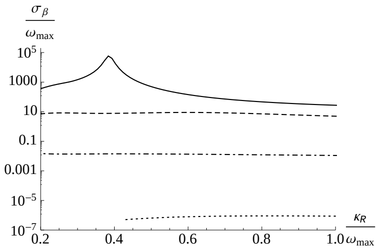
- as expected, $|\widetilde{A}_{\omega=\omega_{\rm min}}|^2 \sim 0.5$ for not too long obstacles.
- as expected, the slope

$$S \equiv -\left. \frac{d|\widetilde{A}_\omega|^2}{d\,(\ln \omega)} \right|_{\omega_{\rm min}} = -\omega_{\rm min} \left. \frac{d|\widetilde{A}_\omega|^2}{d\omega} \right|_{\omega_{\rm min}} . \qquad (17)$$

increases when increasing the length $2L$ of the obstacle: there is a sharper transition for longer obstacles.

Study of the freq. σ_β entering in $|\beta_\omega|^2 \sim |\alpha_\omega|^2 \sim \omega/\sigma_\beta$.

The most relevant parameters are $F_{\rm max}$, and the length $2L$.



4 values of $F_{\rm max}$: 0.6 (solid), 0.8 (dashed), 1 (dot-dashed) and 1.2 (dotted).

NB. the behavior in trans-crit. flows can be understood from **transmission**:

$$\sigma_\beta \sim \exp\{-2k^{\rm dec}_{\omega=0}(F_{\rm max}) \times (2L)\}$$

$k^{\rm dec}_{\omega=0}(F_{\rm max})$ is the zero-freq. imaginary wave vector
in the trans-critical flow evaluated on top of the obstacle.

II. The low frequency regime, $\omega < \omega_{\rm min}$

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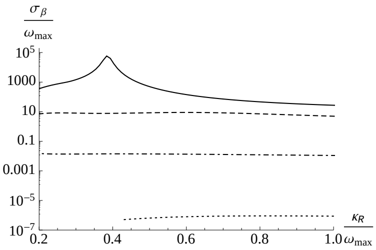

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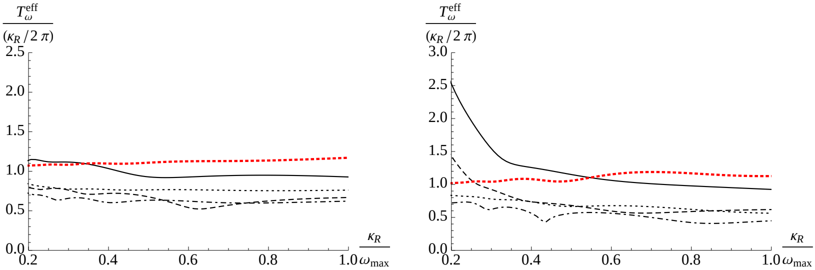
III. The high frequency regime

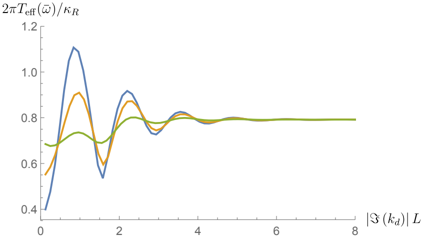


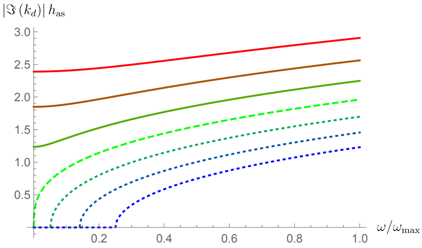
Effective temperature $T_\omega^{\rm eff}$ at the midpoint $(\omega_{\rm min} + \omega_{\rm max})/2$ of the high-freq. regime.

Four $F_{\rm max}$: 0.6 (solid), 0.8 (dashed), 1.0 (dot-dashed) and 1.2 (dotted).
On the left: $\kappa_L/\omega_{\rm max} = 0.25$ (top), $\kappa_L/\omega_{\rm max} = 0.75$ (bottom).

For sub- flows, is clear that κ_L matters when it is larger than κ_R, for the present obstacle with $2L/h_{\rm as} = 2.5$.

III. Explanation: residual trasmission





Left:

$2\pi T_{\rm eff}/\kappa_R$ at $\bar\omega = (\omega_{\rm min} + \omega_{\rm max})/2$ as a function of L for three values of κ_L.

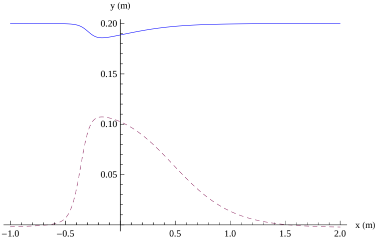
The amplitude of the oscillations increases with κ_L,
while they are exponentially damped for increasing $|\Im(k^d_\omega)|2L$.

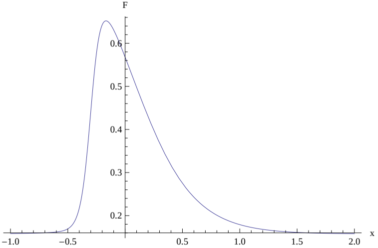
Right: $|\Im(k^d_\omega(0))|h_{\rm as}$, imaginary part of the decaying wavevector as fct of ω, for 7 values of $F_{\rm max}$ from 1.2 to 0.8.

The significant decrease of $|\Im(k^d_\omega(0))|h_{\rm as}$ with $F_{\rm max} < 1$ explains why,
for sub-crit flows, the spectrum of reflected modes on the Right side is affected by κ_L.

Vancouver experiment.

Vancouver experiment: I. Background flow





- On the left, the free surface (plain), and the obstacle (dashed).
- On the right, $F(x) = v(x)/c(x)$. The maximum $F_{\rm max} \simeq 0.7$, significantly less than 1, hence no Kil. horizon, no white hole.

- yet, they report observation of
  - wave blocking, **as if** no transmission, and
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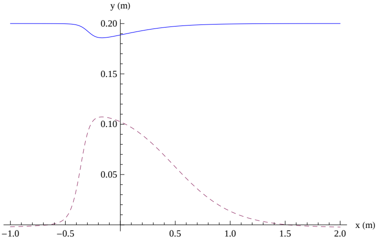

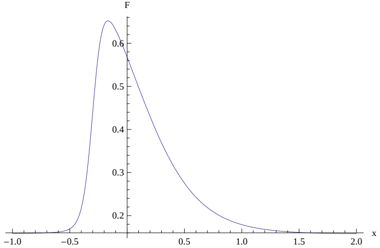

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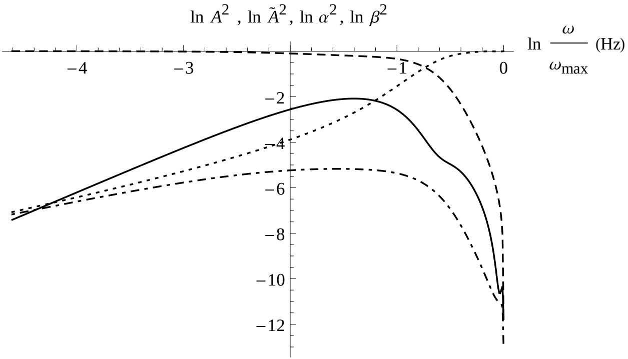
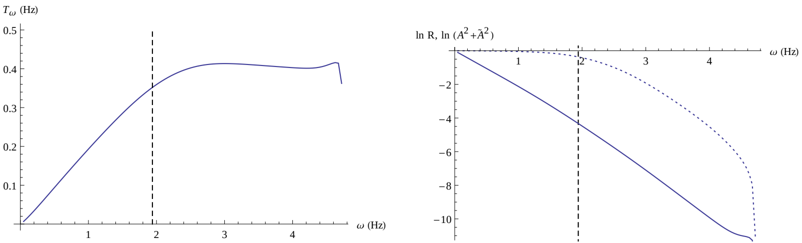
Vanc. exper. II. Numerically comp. scatt. coeffs



Log. of $|\alpha_\omega|^2$ (dotted), $|\beta_\omega|^2$ (dot-dashed), $|\tilde{A}_\omega|^2$ (dashed), and $|A_\omega|^2$ (solid), as functions of $\ln \omega/\omega_{\rm max}$ **numer. computed with quartic DR**

- NB. $\omega_{\rm max,\,4} \simeq 5Hz$, $\omega_{\rm min,\,4} \simeq 2Hz$, where ", 4" means "computed with quartic DR" .
- for $\omega < \omega_{\rm min,\,4}$, there is a severe drop of $|\alpha_\omega|^2$ below 1,
- also $|\beta_\omega|^2 \lesssim e^{-5} \ll 1$ for all ω.

Vanc. exper. III. Effective temperatures.



- **Left**, effective temperature T_ω in Hz.
  It vanishes for $\omega \to 0$. Because $|\beta_\omega|^2 \to 0$, (not reported by the Vanc. team).

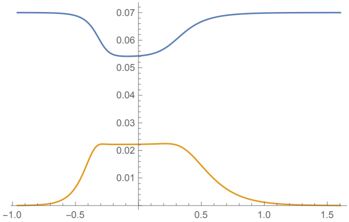
- **Right**, solid, $\ln R_\omega \equiv \ln |\beta_\omega|^2/|\alpha_\omega|^2$.
  - Essentially **linear** in ω, **as if** a thermal spectrum. (Vert. line $\omega = \omega_{\text{min},\,4}$)
  - Observed in Vancouver (slope in agreement of 30%).
  - Used by them as a criterion of "thermality".

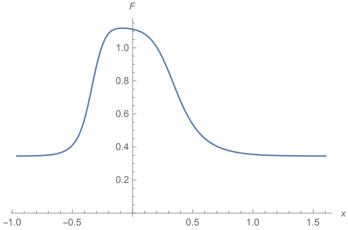
can one conceive improved experiments?

I. Lowering the amplitude of the undulation

- The V. team could not work with $F_{\max} > 0.7$ because of the undulation

- In principle, its amplitude can vanish:





Left: Free surface (blue), obstacle (brown);
Right: $F(x)$, flow **trans-critical** with $F_{\max} \simeq 1.12$.
obtained by **solving the non-linear hydro. eq.** **Unruh-2012, and FM-RP 2014**.

NB. the amplitude of the *undulation* vanishes.

The procedure to design obstacles, Unruh 2012, FM-RP 2014

- I. exploit 2D irrotational flow, i.e., use
  - Bernouilli eq.
  - velocity potential ϕ and streamline ψ as coordinates "x, y"
  - 2D Laplace eq. , so that $\Phi(z)$ holomorphic,
    where $\Phi = \phi + i\psi$, $z = x + iy$.
- II. choose the **free surface** arbitrarily : $y = y_s(\phi), \psi = \psi_s$
- III. Solve:
  - Use Bern. to get $x_s(\phi)$: 1st ODE,
  - use holomor. to get $x_b(\phi)$, $y_b(\phi)$: **the shape of the bottom**
    from $Z(\Phi)$, holom., evaluated at $\psi = 0$,
    from $Z(\phi + i\psi_s) = x_s(\phi) + iy_s(\phi)$.

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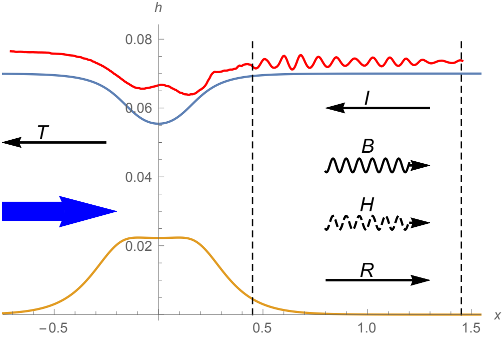
The Orsay-2 obstacle



In brown the 'designed' obstacle, in red the observed (time-average) surface.
The vertical lines show the region used to analyzed the mode content.

$F_{\rm max} = 0.83 \pm 0.03$ vs $F_{\rm max}^{\rm Vancouver} = 0.67 \pm 0.02$
More importantly, peak-to-peak amplitude of the undulation 5.0 mm.

A new obstacle *O3b* is currently being tested.

II. Extracting the scattering coefficients

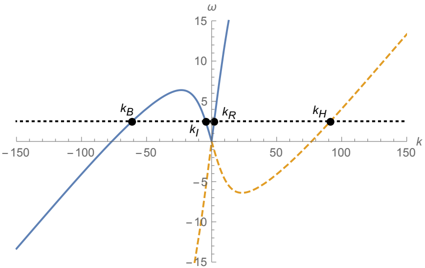
- in Vancouver, only $|\beta_\omega/\alpha_\omega|^2$ was measured.
- The noise degrades the measurements of the free surface. Maximal resolution: 0.1mm, close to the typical noise amplitude.
- To lower its impact, we use the constructive interferences between the various waves produced by the wave maker, i.e.,
- we study the (norm of the) two-point correlation function

$$G_2(\omega; k, k') \equiv \frac{\left|\left\langle \delta\tilde{h}(\omega, k)\, \delta\tilde{h}(\omega, k')^* \right\rangle\right|}{S_k S_{k'}} . \tag{18}$$

in the $k - k'$ plane, S_k is the "structure factor".

- this is quite similar to what is done in BEC, except that we can work at fixed ω.

The dispersion relation in the $k - k'$ plane

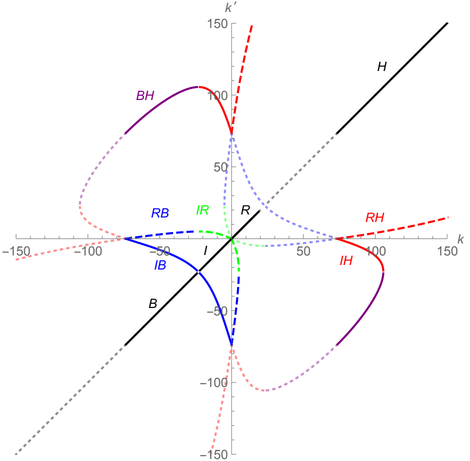


Top: dispersion relation ω vs k.

Right:
k_ω^a vs k_ω^b for two k's with the same ω.

the scattering induces correl. only among k's with the same ω, since flow is statio.

Hence $G_2^{\rm observed}(\omega; k, k')$ should $\neq 0$ only along these lines.



In solid $\omega > 0$, in pale $\omega < 0$.
Auto-correlations are along the diagonal.

Extracting the scattering coefficients

$$|\beta_\omega| = \left|\frac{G_2(\omega, k_I, k_H)}{G_2(\omega, k_I, k_I)}\right| \times \left|\frac{\partial_\omega k_I}{\partial_\omega k_H}\right|^{1/2},$$

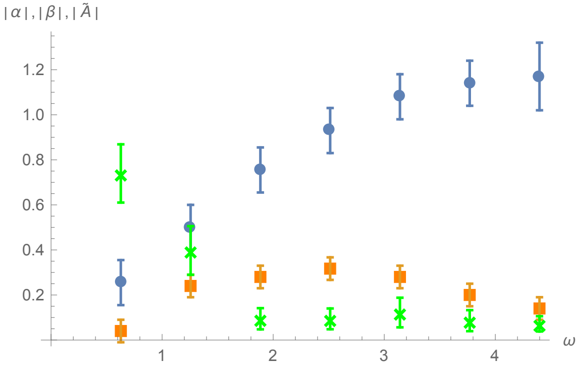
where k_I is the wv of the Incoming mode, and k_H that of the NEW.

Similarly

$$|\alpha_\omega| = \left|\frac{G_2(\omega, k_I, k_B)}{G_2(\omega, k_I, k_I)}\right| \times \left|\frac{\partial_\omega k_I}{\partial_\omega k_B}\right|^{1/2}$$

for the 'Blue shifted' mode with wv k_B.

The measured scattering coefficients



The norm of the measured scattering coefficients.
$|\alpha_\omega|$ in blue, $|\beta_\omega|$ in orange, $|\tilde{A}_\omega|$ (transmission) in green.

The crossover near $\omega_{\rm min} = 0.8$ between $|\alpha_\omega|$ and $|\tilde{A}_\omega|$ clearly visible.

Unitarity: $1 = |\alpha_\omega|^2 - |\beta_\omega|^2 + |A_\omega|^2 + |\tilde{A}_\omega|^2$ **obeyed within error bars.**

Comparison with numerical simulations

- In brief, the observed value of $|\beta_\omega|$ is about
  100 times larger than that numerically computed.

- this is (most probably) due to the resonant scattering on the undulation: although its amplit. $\sim$ 0.25 mm $\ll$ 40 mm of δh due to the obs.
  resonant anomalous Bragg scattering. (work in progress)

- the first experimental task would be to obtain a stable high $F_{\max}$ flow with an undulation whose amplitude significantly less than 0.25 mm.

Conclusions

- for **smooth and suff. trans-critical flows,** $F_{\max} > 1.1$,
  - Hawking's spectrum is found in a wide domain of frequency.
  - at the predicted temperature $T_{\text{eff}}(\omega) = \kappa/2\pi$.

- for **sub-critical flows,** $F_{\max} < 1$,
  - the **transmission coef.** $\tilde{A}_\omega \to 1$ for $\omega < \omega_{\min}$,
  - α_ω **and** β_ω **both** $\propto \omega$ **for** $\omega \to 0$,
  - even for $\omega > \omega_{\min}$, $T_{\text{eff}}(\omega)$ is generically non constant,
    (because of **residual** transmission across the obstacle).

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