



Detection of Gravitational Waves



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on behalf of the Virgo Naples group and on behalf of the LIGO Scientific and VIRGO collaborations





Introduction to Gravitational Waves



time ('ripples' in space-time)

Predicted by Einstein 100 years ago; confirmation by Hulse/Taylor/Weisberg

Emitted from accelerating mass distributions Sourced by the time-dependence of the quadrupole mass moment Practically, need massive objects at speeds approaching the speed of light

GWs carry *direct* information about the relativistic motion of bulk matter



Supernovae



Rotating neutron stars

Hunting the GW signals

Coalescent Binary Sytem



GW stochastic background



A Golden GW Signal



Emission during the inspiral phase $h_{+}(t) = A_{\rm GW}(t) \left(1 + \cos^{2} \iota\right) \cos \phi_{\rm GW}(t)$ $h_{\times}(t) = -2A_{\rm GW}(t) \cos \iota \sin \phi_{\rm GW}(t)$

U → the inclination angle between the direction of the detector as seen from the binary's center-of-mass, and the normal to the orbital plane

During the inspiral, if the phase ϕ_{GW} is computed using PN expansion, at the leading order the phase evolution depends on the chirp mass

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G} \left[\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right]^{3/5}$$

Addition parameters ($q = [m_1/m_2]$ and **L**), enter at the following orders

When the two compact objects are closer, PN fails and NR is used

Source parameter extraction - I

- From the inspiral, we measure the chirp mass M with good accuracy
- During the inspiral, the in-plane components of the spins cause the binary's orbital plane to precess around the the total angular momentum of the binary.
- <u>Depending on the orientation of the binary</u>, precession effects can produce an amplitude and phase modulation of the signal at the detector

Source parameter extraction - II

- r > 10 G M_{tot}/c² → Inspiral phase, PN expansion
 Chirp mass measurement
- $r < 10 \text{ G M}_{tot}/c^2 \rightarrow \text{Merger and ringdown, NR wave description}$
 - Measurement of the final BH mass and Spin
 - Amount of energy emitted similar to the entire inspiral phase

Cosmological corrections

- If the source is at a cosmological distance with redshift *z*, then
 - distance \rightarrow luminosity distance D_L ,
 - Frequency is the observed one , f, redshifted by (1+z) to the respect of the source frequency,
 - Each mass $m \rightarrow (1 + z) m$.

The detector



The detection principle



A typical sensitivity curve



Location in the sky

• GW laser interferometers are not pointing telescopes,

 Sky location can be reconstructed through the time of arrival of GW radiation at the different detector sites, as well as the relative amplitude and phase of the GWs in different detectors.



- The radiation emitted in a CBC along the orbital axis → circularly polarized
- The radiation emitted in a CBC on the equatorial plane → linearly polarized
- Observing with more than one detector we can infer both the location in the sky and the degree of elliptical polarization





Credit: S. Fairhurst



The 2007 GW network





H1- Hanford – Washington state

Virgo – Cascina (Pisa) – EGO site



GEO600 – Hannover - Germany



L1- Livingston – Louisiana state

A bit of history

- The LIGO project was approved in 1992 and inaugurated in 1999. Built at a cost of almost 3x10⁸ \$, LIGO was the largest single enterprise ever undertaken by the foundation. It started the operation in 2002.
- VIRGO was formally proposed in 1989 and approved in 1993. The construction was divided in two step: it started in 1996 and then completed in 2003. The first science run is date 2007. The total investment done by CNRS and INFN was almost 8 x 10⁷ \$.
- **GEO600** was proposed in 1994. Since September 1995 this British-German GW detector was under construction. The first science run was performed in 2002. In 2013 Squeezing light was used over one complete year!
- First attempt to exchange data and mix the data analysis groups started in 2004. The formal MoU of data sharing and common analysis among GEO-LIGO-VIRGO was signed in 2007.

GW Science From First Generation

THE ASTROPHYSICAL JOURNAL, 715:1438–1452, 2010 June 1 © 2010. The American Astronomical Society. All rights reserved. Printed in the U.S.A. doi:10.1088/0004-637X/715/2/1438

SEARCH FOR GRAVITATIONAL-WAVE BURSTS ASSOCIATED WITH GAMMA-RAY BURSTS USING DATA FROM LIGO SCIENCE RUN 5 AND VIRGO SCIENCE RUN 1

PHYSICAL REVIEW D 82, 102001 (2010)

Search for gravitational waves from compact binary coalescence in LIGO and Virgo data from S5 and VSR1

PHYSICAL REVIEW D 81, 102001 (2010)

All-sky search for gravitational-wave bursts in the first joint LIGO-GEO-Virgo run

THE ASTROPHYSICAL JOURNAL, 715:1453-1461, 2010 June 1 © 2010. The American Astronomical Society. All rights reserved. Printed in the U.S.A. doi:10.1088/0004-637X/715/2/1453

SEARCH FOR GRAVITATIONAL-WAVE INSPIRAL SIGNALS ASSOCIATED WITH SHORT GAMMA-RAY BURSTS DURING LIGO'S FIFTH AND VIRGO'S FIRST SCIENCE RUN

nature

Beating the spin-down limit on gravitational wave emission from

LETTERS

the Vela pulsar

arXiv:1104.2712v2 [astro-ph.HE] 15 Apr 2011

An upper limit on the stochastic gravitational-wave background of cosmological origin

The LIGO Scientific Collaboration* & The Virgo Collaboration*

CW and stochastic signal searches

Strain Sensitivity

10

10

100

10-2

Gravitational-wave Frequency (Hz)

LIGO-Vir

Frequency (Hz)

 For Crab and Vela pulsars we are below the "spin-down limit": constrain the fraction of spin-down energy due to GW J0534+2200

[Aasi et al. ApJ 2014]

Pulsar $h_{95\%}$ ε J0534+22001.8(1.6)x10^{-25}9.7(8.6)x10^{-5}J0835-45101.1(1.0)x10^{-24}6.0(5.5)x10^{-4}

• Upper Limits on the Stochastic Gravitational-Wave Background [PRL 113, 231101 (2014)]

				Strings
Frequency (Hz)	$f_{\rm ref}$ (Hz)	α	Ω_{lpha}	Limit - AdvDet
41.5-169.25		0	$(-1.8 \pm 4.3) \times 10^{-6}$	10 12 BBH BNS
170-600		0	$(9.6 \pm 4.3) \times 10^{-5}$	Stiff
600-1000	900	3	0.026 ± 0.052	10 ⁻¹⁴ Axion Slow-Roll Inflat
1000-1726	1300	3	-0.077 ± 0.53	10^{-16} 10^{-12} 10^{-12} 10^{-9} 10^{-6} 10^{-3} 10^{9} 10^{3} 10^{6}

All-Sky Burst Search



250

200

150

100

50

٥

All-sky, all-times search for signals which do not assume specific morphology (GW 'bursts') is least constrained transient search open to many source models and unexpected signals



IMBH Sensitivity Range in Mpc

Intermediate Mass Black Hole search uses same algorithm to target black hole binaries at larger masses than CBC searches



The GW network of the Advanced detectors



Advanced detectors

- Upgrade of the LIGO
- LIGO cost: \$205M (NSF) and \$16M in hardware from partners in Germany, UK, and Australia
- aLIGO approved in 2008, inauguration May 2015
- First observing run O1 from mid-September 2015 to mid-January 2016.
- Upgrade of Virgo
- aVirgo cost: 23 M from CNRS, INFN and NIKHEF
- aVirgo approved in 2011 and project started in 2012
- Installation to be completed in the first half of 2016

Local Superclusters



Ultimately 10x more sensitive \rightarrow 1000x more volume probed



First sensitivity target achieved already !



Compact Coalescing Binaries

Detection perspectives with advanced detectors Phys. Rev D85 (2012) 082002





LIGO upgrade concluded

The Advanced LIGO dedication ceremony was held at Hanford on May 19, 2015

H1- Hanford – Washington state

VIRGO will end the upgrade in 2016



L1- Livingston – Louisiana state

Hardware highlights

From the first generation to the second one



Active Seismic Isolation for in-vacuum Optical Tables



Credits to D. Hoak



Four-stage monolithic suspensions larger mirrors



Credits to D. Hoak

10x more laser power —> Reduced shot noise

Larger test masses —> Larger beam size —> Coating thermal noise coupling is reduced —> Less sensitive to radiation pressure noise from increased power

Arm Length Stabilization



Virgo pioneered the low frequency strategy



Super Attenuator Chain



LIGO and Virgo in the past



From LIGO to aLIGO: Sensitivity improvements



Data Analysis: Extracting the signal from the noise
How the signal might look like



Coherent Wave Burst (cWB) algorithm: minimal assumption for searching transients

- Require coherent signals in multiple detectors, using direction-dependent antenna response
- Look for excess power in time-frequency space using wavelet decomposition
 The event are ranked using a variable quoting
 the SNR of the cohorent signal in the

the SNR of the coherent signal in the network

$$\eta_c = \sqrt{\frac{2E_c}{(1 + E_n/E_c)}}$$

 $E_c \rightarrow$ Normalized coherent energy between the two detectors

 $E_n \rightarrow$ normalized noise energy derived by subtracting the reconstructed signal from the data

$$E_c = \sum_{i \in C} \sum_{n \neq m} w_n[i] P_{nm}[i] w_m[i]$$

Particularly useful also for low-latency transient search → trigger Multimessenger follow up: event reported 3 minutes after the data were collected

Matched filter for CBC search



Search use waveform predictions to perform matched filtering

- EOBNR → The effective-one-body (EOB) formalism combines perturbative results from the weak-field PN approximation with strong-field effects from the test-particle limit.
- IMR-Phenomen → It is based on extending frequency-domain PN expressions and hybridizing PN and EOB with NR waveforms.



Background estimation





The first run of the advanced detectors



Hanford (H1) and Livingston (L1) in action

The detector was in a rather stable conditions from the beginning of September 2015 and took data until January 2016

Here we present just the analysis of the data taken

in the period from September 12 to October 20, 2015

16 days of coincident data taking !!

16 days of data produced by 2 interferometers:

Monitor of environmental disturbances and interferometer's operating point control

- Each observatory site is equipped with an array of sensors:
 - seismometers, accelerometers, microphones,
 - magnetometers, radio receivers, weather sensors,
 - AC-power line monitors,
 - a cosmic-ray detector
- 10⁵ channels record the interferometer's operating point
- Data collection is synchronized to GPS time to better than 10 $\mu s.$
 - Timing accuracy verified with an atomic clock and a secondary GPS receiver at each observatory site.

The event

• On September 14, 2015 at 09:50:45 UTC the LIGO Hanford, WA, and Livingston, LA, observatories detected a coincident signal.

• The event was flagged as GW150914

 Exhaustive investigations of instrumental and environmental disturbances were performed, giving *no evidence* that GW150914 is an instrumental artifact







Estimated GW Strain Amplitude: GW150914

Full bandwidth waveforms without filtering. Numerical relativity models of black hole horizons during coalescence

Effective black hole separation in units of Schwarzschild radius ($R_s=2GM/c^2$); and effective relative velocities given by post-Newtonian parameter v/c = ($GM\pi f/c^3$)^{1/3}



Assessing the statistical significance of the event

Generic transient search: minimum bias

cWB version online: event detected with 3 minutes of latency

cWB version off-line: data reanalyzed to assess the statistical significance

Events classified in 3 different classes (\rightarrow trial factor):

- C1 class → events with time-frequency morphology of known populations of noise transients: excluded;
- C3 class → events with frequency that increases with time;
- C2 class → all remaining events.

Background evaluation \rightarrow Based on the time shift method: Number of shift produced an equivalent to 67400 years



Binary Coalescence search

- Search for GW emission by binary system: total mass range $1-99 \text{ M}_{\odot}$
- > > 4 M_{\odot} Model based on PN, BH perturbation theory and NR
- \geq ~ 2.5 x 10⁵ wave forms used to cover the parameter space
- > SNR of the Matched filter computed as function of time $\rho(t)$ and identify maxima and calculate χ^2 to test consistency with the matched template, then apply detector coincidence within 15 ms
- ➢ Background computed by shifting 10⁷ times equivalent to 680.000 years

 \blacktriangleright Combined SNR = 23.6 , FAR = 1/203,000 years \rightarrow 5.1 σ



The Parameter Estimation

Parameter Estimation: probability density functions (PDFs) of masses at the source frame



Source parameters for GW150914

Primary black hole mass	$36^{+5}_{-4}{ m M}_{\odot}$
Secondary black hole mass	$29^{+4}_{-4}{ m M}_{\odot}$
Final black hole mass	$62^{+4}_{-4}{ m M}_{\odot}$
Final black hole spin	$0.67\substack{+0.05\\-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift, z	$0.09\substack{+0.03\\-0.04}$

Estimated source parameters from GW150914. We report median values with 90% credible intervals that include statistical errors from averaging the results of different waveform models. Masses are given in the source frame: to convert in the detector frame multiply by (1+z). The source redshift assumes standard cosmology: $D_L \rightarrow z$ assuming Λ CDM with H₀ = 67.9 km s⁻¹ Mpc⁻¹ and Ω_m =0.306

Energy emitted and Angular momentum

- Energy emitted in GW
- GW luminosity peak

+ 0.5

$$3.6 - 0.4 \times 10^{56} \text{ erg/s}$$

 $200^{+30}_{-20} \text{ M}_{\odot} \text{ c}^{2}/\text{s}$

 $3^{+0.5}$ M_o c^2

Values consistent with the GR predictions

2%-3% of Newtonian energy in the inspirable phase ending at ($r \approx 5 GM/c^2$) + Num. Rel. prediction for the final phase 2%-3%

$${\cal F} \sim {c^3 |\dot{h}|^2 \over 32 \pi G} \sim ~~10^{-2}~{
m W}~{
m m}^{-2}$$

• Angular momentum

in the final phase of the inspiral, the total angular moment **L** dominated by the orbital one

Spin of the final BH $S = L - L_{emit. GW}$

	EOBNR	IMRPhenom	Overall
Detector-frame total mass M/M_{\odot}	$70.3^{+5.3}_{-4.8}$	$70.7^{+3.8}_{-4.0}$	$70.5^{+4.6\pm0.9}_{-4.5\pm1.0}$
Detector-frame chirp mass \mathcal{M}/M_{\odot}	$30.2^{+2.5}_{-1.9}$	$30.5^{+1.7}_{-1.8}$	$30.3^{+2.1\pm0.4}_{-1.9\pm0.4}$
Detector-frame primary mass m_1/M_{\odot}	$39.4^{+5.5}_{-4.9}$	$38.3^{+5.5}_{-3.5}$	$38.8^{+5.6\pm0.9}_{-4.1\pm0.3}$
Detector-frame secondary mass m_2/M_{\odot}	$30.9^{+4.8}_{-4.4}$	$32.2^{+3.6}_{-5.0}$	$31.6^{+4.2\pm0.1}_{-4.9\pm0.6}$
Detector-frame final mass $M_{\rm f}/{\rm M}_{\odot}$	$67.1^{+4.6}_{-4.4}$	$67.4^{+3.4}_{-3.6}$	$67.3^{+4.1\pm0.8}_{-4.0\pm0.9}$
Source-frame total mass $M^{\rm source}/M_{\odot}$	$65.0^{+5.0}_{-4.4}$	$64.6^{+4.1}_{-3.5}$	$64.8^{+4.6\pm1.0}_{-3.9\pm0.5}$
Source-frame chirp mass $\mathcal{M}^{\rm source}/M_{\odot}$	$27.9^{+2.3}_{-1.8}$	$27.9^{+1.8}_{-1.6}$	$27.9^{+2.1\pm0.4}_{-1.7\pm0.2}$
Source-frame primary mass $m_1^{\rm source}/{ m M}_{\odot}$	$36.3^{+5.3}_{-4.5}$	$35.1^{+5.2}_{-3.3}$	$35.7^{+5.4\pm1.1}_{-3.8\pm0.0}$
Source-frame secondary mass $m_2^{ m source}/{ m M}_{\odot}$	$28.6^{+4.4}_{-4.2}$	$29.5^{+3.3}_{-4.5}$	$29.1^{+3.8\pm0.2}_{-4.4\pm0.5}$
Source-fame final mass $M_{\rm f}^{\rm source}/{ m M}_{\odot}$	$62.0^{+4.4}_{-4.0}$	$61.6^{+3.7}_{-3.1}$	$61.8^{+4.2\pm0.9}_{-3.5\pm0.4}$
Mass ratio q	$0.79\substack{+0.18 \\ -0.19}$	$0.84\substack{+0.14\\-0.21}$	$0.82^{+0.16\pm0.01}_{-0.21\pm0.03}$
Effective inspiral spin parameter $\chi_{\rm eff}$	$-0.09\substack{+0.19\\-0.17}$	$-0.03\substack{+0.14 \\ -0.15}$	$-0.06^{+0.17\pm0.01}_{-0.18\pm0.07}$
Dimensionless primary spin magnitude a_1	$0.32^{+0.45}_{-0.28}$	$0.31\substack{+0.51\\-0.27}$	$0.31^{+0.48\pm0.04}_{-0.28\pm0.01}$
Dimensionless secondary spin magnitude a_2	$0.57_{-0.51}^{+0.40}$	$0.39_{-0.34}^{+0.50}$	$0.46^{+0.48\pm0.07}_{-0.42\pm0.01}$
Final spin $a_{\rm f}$	$0.67\substack{+0.06\\-0.08}$	$0.67\substack{+0.05 \\ -0.05}$	$0.67^{+0.05\pm0.00}_{-0.07\pm0.03}$
Luminosity distance $D_{\rm L}/{ m Mpc}$	390^{+170}_{-180}	440^{+140}_{-180}	$410^{+160\pm20}_{-180\pm40}$
Source redshift z	$0.083^{+0.033}_{-0.036}$	$0.093\substack{+0.028\\-0.036}$	$0.088^{+0.031\pm0.004}_{-0.038\pm0.009}$
Upper bound on primary spin magnitude a_1	0.65	0.71	0.69 ± 0.05
Upper bound on secondary spin magnitude a_2	0.93	0.81	0.88 ± 0.10
Lower bound on mass ratio q	0.64	0.67	0.65 ± 0.03
Log Bayes factor $\ln \mathcal{B}_{s/n}$	288.7 ± 0.2	290.1 ± 0.2	

Few Companion Results

Astrophysical implications (accepted on ApJL)

- GW150914 demonstrates the existence of stellar-mass black holes of M > 25M_☉,
 binary black holes can form in nature and merge within a Hubble time
- Binary black holes have been predicted to form both in isolated binaries and in dense environments by dynamical interactions .
- Formation of such massive black holes from stellar evolution
 - weak massive-star winds, and stellar environments with metallicity lower than 1/2 the solar value

Event rates evaluation \rightarrow 2–400 Gpc⁻³ yr⁻¹

- Considering only GW150914 event and assuming that the BBH merger rate is constant in the comoving frame, we infer a 90% credible range of 2–53 Gpc⁻³ yr⁻¹ (in the comoving frame)
- Incorporating all triggers that pass the search threshold (independent of their astrophysical origin) and several other models, we obtain rate estimates ranging from 6–400 Gpc⁻³ yr⁻¹

Stochastic back. of astrophysical origin

- Superposition of unresolved GW emission of binary black hole sources at larger distances.
- Prediction Ω_{GW} (~ f = 25 Hz) $1.1^{+2.7}_{-0.9}$ x 10 ⁻⁹ with 90% confidence.
- Potentially measurable by the Advanced LIGO/at their projected final sensitivity

Testing General Relativity

- The phase of the gravitational waveform during the inspiral can be ex- pressed as a power-series in f^{1/3}.
- The coefficients of this expansion can be computed in general relativity

-GR test : allow the coefficients to deviate from the nominal GR value

Compton Wave-length of the Graviton

C. M. Will, Phys. Rev. D 57, 2061 (1998).

• We assume a modified dispersion relation for gravitational waves

 $(v_g/c)^2 = 1 - \{h c / (\lambda_g E)\}^2$

• In the massive graviton theory an extra phase term is added to the CBC evolution (formally a 1PN order term) [C.Will PRD, 57, 2061 (1998)]

$$\phi_{MG}(f) = -(\pi D c) / [\lambda_g^2 (1+z) f]$$

• Our constrain on the 1PN terms permit to derive a down limit for the Compton wavelength of the graviton

$$\lambda_g = \mathbf{h} / (m_g c) > 10^{13} \text{ km}$$

- It corresponds to a limit $m_g < 1.2 \times 10^{-22} \text{ eV/c}^2$.
 - limit better than that set by Solar System observations
 - thousand time better of the binary pulsar bounds
 - worse than bounds from dynamics of galaxy clusters and weak lensing observations (model- dependent bounds)

The e.m. and v follow up



GW – HEN Coincidence search

• Search for coincident neutrino candidates to GW150914 within the data recorded by the IceCube and Antares neutrino detectors.

Time coincidence window ±500 s

- Number of neutrino candidates detected by IceCube and Antares were 3 and 0, respectively
- None of the neutrino candidates were directionally coincident with GW150914.



E.M.-messenger searches

About 74 MoUs: 19 countries, 150 instruments, covering the full spectrum from radio to very high-energy gamma rays 62 groups ready to observe at the time of the O1 start-up 20 group observed the sky once the trigger has been released



An article is in preparation: we summarize the em follow up effort

The Event – What LVC sent to the astronomers

16 Sept 05:39 UTC notification about the trigger identified by the online Burst analysis during ER8 (GCN 18330)



Event time 2015-09-14 09:50:45 UTC FAR 1.178e-08 Hz 1/2.7 yr

The 50% credible region spans about 200 deg² and the 90% region about 750 deg²

O3 Oct 2015 update → waveform reconstruction appears consistent with expectations for a binary black hole coalescence (GCN 18388)

▶ 12 Jan 2016 update → offline calibration and re-analysis FAR < 1/100 yr (GCN 18851)
 ▶ 13 Jan update → Refined localizations from CBC parameter estimation (GCN 18858)



A glance to What's Next

February 17.th 2016 Indian Government approval of Ligo-India Proposal



Cabinet grants 'in-principle' approval to the LIGO-India mega science proposal

The Union Cabinet chaired by the Prime Minister Shri Narendra Modi has given its 'in principle' approval to the LIGO-India mega science proposal for research on gravitational waves. The proposal, known as LIGO-India project (Laser Interferometer Gravitational-wave Observatory in India) is piloted by Department of Atomic Energy and Department of Science and Technology (DST). The approval coincides with the historic detection of gravitational waves a few days ago that opened up of a new window on the universe to unravel some of its greatest mysteries.

The LIGO-India project will establish a state-of-the-art gravitational wave observatory in India in collaboration with the LIGO Laboratory in the U.S. run by Caltech and MIT.

The project will bring unprecedented opportunities for scientists and engineers to dig deeper into the realm of gravitational wave and take global leadership in this new astronomical frontier.

LIGO-India will also bring considerable opportunities in cutting edge technology for the Indian industry which will be engaged in the construction of eight kilometre long beam tube at ultra-high vacuum on a levelled terrain.

The project will motivate Indian students and young scientists to explore newer frontiers of knowledge, and will add further impetus to scientific research in the country.

 6. PM inaugurates Centenary Celebrations of Gaudiya Mission and Math
 7. Excerpts of PM's Address at the launch of Shyama Prasad Mukherji Rurban Mission
 8. Text of PM's speech at the Foundation Stone laving accommon of Bradhan Mantri Awas Yo

2. Prime Minister of Nepal Calls on President

4. President of India's Message on the eve of

Independence Day of St. Lucia

3. President's Greetings on the eve of Birthday of

- laying ceremony of Pradhan Mantri Awas Yojana in Chhattisgarh
- 9. PM launches National Rurban Mission

10. PM in Naya Raipur

President's Secretariat

Guru Ravidasji

Prime Minister's Office



The detection of gravitationalwaves strongly enforces the proposal for future space-based detectors like LISA



Artistic view of Lisa Detector
Conclusions







Conclusion



844 Shing in phychilecommunications was very 20 Sociation (01)

Die Feldgleichungen der Gravitation. Von A. Exstern.

In vert vor kunnen sochierenen Stittelingen beite bil gereigt, wir nur an Föligheitungen die komptenen gebergen benn, die den Porsim allgemeinen Richteftlich songehreiten, die die in Bereiten Fassung belichtigen Spleitunitenen der Bannzeiterentigen deres zusamgebeitetigen Spleitunitenen der Bannzeiterentigen gegentber katernalisien.

Der Entwickungsamg wert deht folgender. Zustricht fach ich inferdangen, welche die Narrowsen Thomie en Nilweing extinitie und beitoligen subsituatione eine die Internationen eingestählt bei weiter waren. Ellernet fand inn, daß diesen Gleichungen elligendu keuristen eingenehmt. Die der Schult die Europterteisen der Sotrick- ererkbendet. Die Kondustrangssom ver dam nich der eintreteise Rept im speckolisiene daß 1-g is i gestauft weit under die Gleichungen der Theorie eine einberen Vereinflichung erfähren, Juste nurde der nie erreitun, die Legatiese sugeführt verden, Juste andere des fahren versiehen die Legatiese sugeführt verden, Just der Steine des Intergenweis der Hatter verdenzeiten.

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Published by American Physical Society⁷⁰

Volume 116, Number 6

<u>We detected Gravitational Waves</u> <u>and really opened a new way</u> of observing the universe



Extra slides

Waveform zoology



Waveform template

- Model complete (include also spin-orbit (1.5 PN) and spin-spin (2PN) effects
- Waveforms stop at ISCO.

$$h(f) = Cf^{-7/6}e^{-i\Psi(f)}$$

$$\begin{split} \Psi(f;M,\eta) &= 2\pi f t_C - 2\phi_C - \pi/4 \\ &+ \pi \left[\frac{38\,645}{756} - \frac{65}{9}\eta \right] \left[1 + 3\ln\left(\frac{v}{v_0}\right) \right] + \left\{ \frac{11\,583\,231\,236\,531}{4\,694\,215\,680} - \frac{640}{3}\pi^2 - \frac{6\,848}{21}\left(\gamma + \ln(4\,v)\right) \right\} \\ &+ \left(-\frac{15\,335\,597\,827}{3\,048\,192} + \frac{2\,255}{12}\pi^2 \right)\eta + \frac{76\,055}{1\,728}\eta^2 - \frac{127\,825}{1\,296}\eta^3 \right\} v^6 \qquad v = (\pi M f)^{1/3} \\ &+ \pi \left[\frac{77\,096\,675}{254\,016} + \frac{378\,515}{1\,512}\eta - \frac{74\,045}{756}\eta^2 \right] v^7 \bigg\}, \end{split}$$