

Strangeness, Gravitational Waves and Neutron Stars

Frascati, June 12, 2016

**Quark Deconfinement in
Neutron Stars
and
Astrophysical implications**

Ignazio Bombaci

Dipartimento di Fisica “E. Fermi”, Università di Pisa

Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Pisa

European Gravitational Observatory (EGO), Cascina

CONVERSION OF NEUTRON STARS TO STRANGE STARS AS THE CENTRAL ENGINE OF GAMMA-RAY BURSTS

IGNAZIO BOMBACI¹ AND BHASKAR DATTA^{2,3}

Received 1999 August 20; accepted 1999 December 17; published 2000 January 21

ABSTRACT

We study the conversion of a neutron star to a strange star as a possible energy source for gamma-ray bursts. We use different recent models for the equation of state of neutron star matter and strange quark matter. We show that the total amount of energy liberated in the conversion is in the range of $(1\text{--}4) \times 10^{51}$ ergs (1 order of magnitude larger than previous estimates) and is in agreement with the energy required to power gamma-ray burst sources at cosmological distances.

GAMMA-RAY BURSTS FROM DELAYED COLLAPSE OF NEUTRON STARS TO QUARK MATTER STARS

Z. BEREZHIANI,¹ I. BOMBACI,² A. DRAGO,³ F. FRONTERA,^{3,4} AND A. LAVAGNO⁵

Received 2002 September 12; accepted 2002 December 2

ABSTRACT

We propose a model to explain how a gamma-ray burst can take place days or years after a supernova explosion. Our model is based on the conversion of a pure hadronic star (neutron star) into a star made at least in part of deconfined quark matter. The conversion process can be delayed if the surface tension at the interface between hadronic and deconfined quark matter phases is taken into account. The nucleation time (i.e., the time to form a critical-size drop of quark matter) can be extremely long if the mass of the star is small. Via mass accretion the nucleation time can be dramatically reduced and the star is finally converted into the stable configuration. A huge amount of energy, on the order of $10^{52}\text{--}10^{53}$ ergs, is released during the conversion process and can produce a powerful gamma-ray burst. The delay between the supernova explosion generating the metastable neutron star and the new collapse can explain the delay inferred in GRB

References

- I. Bombaci, B. Datta, *Astrophys. Jout. Lett.* **530** (2000) L69**
- Z. Berezhiani, I. Bombaci, A. Drago, F. Frontera, A. Lavagno, *Nucl. Phys. B, P.S.* **113** (2002) 268**
- Z. Berezhiani, I. Bombaci, A. Drago, F. Frontera, A. Lavagno, *Astrophys. Jour.* **586** (2003) 1250**
- I. Bombaci, I. Parenti, I. Vidaña, *Astrophys. Jour.* **614** (2004) 314**
- A. Drago, A. Lavagno, G. Pagliara, *Eur. Phys. Jour. A* **19** (2004) 197**
- A. Drago, A. Lavagno, I. Parenti, *Astrophys. Jor.* **659** (2007) 1519**
- I. Bombaci, G. Lugones, I. Vidaña, *Astron. & Astrophys.* **462** (2007) 1017**
- I. Bombaci, P.K. Panda, C. Providencia, I. Vidaña, *Phys. Rev. D* **77** (2008) 083002**
- I. Bombaci, D. Logoteta, P.K. Panda, C. Providencia, I. Vidaña, *Phys. Lett. B* **680** (2009) 448**
- I. Bombaci, D. Logoteta, C. Providencia, I. Vidaña, *Astr. and Astrophys.* **528** (2011) A71**
- A. Drago, A. Lavagno, G. Pagliara, *Phys. Rev. D* **89** (2014) 043014**
- I. Bombaci, D. Logoteta, I. Vidaña, C. Providencia, *EPJ A* **52** (2016) 58**
- A. Drago et al, *EPJ A* **52** (2016) 40; EPJ A **52** (2016) 41**
- A. Drago, A. Lavagno, B. Metzger, G. Pagliara, *Phys. Rev. D* **93** (2016) 103001**

Neutron Stars: bulk properties

Mass	$M \sim 1.5 M_{\odot}$
Radius	$R \sim 10 \text{ km}$
Centr. Density	$\rho_c = (4 - 8) \rho_0$
Compactness	$R/R_g \sim 2 - 4$
Baryon number	$A \sim 10^{57}$
Binding energy	$B \sim 10^{53} \text{ erg}$ $B/A \sim 100 \text{ MeV}$ $B/(Mc^2) \sim 10\%$

Stellar structure:
General Relativity

Giant “atomic nucleus”
bound by gravity

$$M_{\odot} = 1.989 \times 10^{33} \text{ g} \quad R_{\odot} = 6.96 \times 10^5 \text{ km}$$

$$\rho_0 = 2.8 \times 10^{14} \text{ g/cm}^3 \text{ (nuclear saturation density)}$$

$$R_g \odot = 2.95 \text{ km}$$

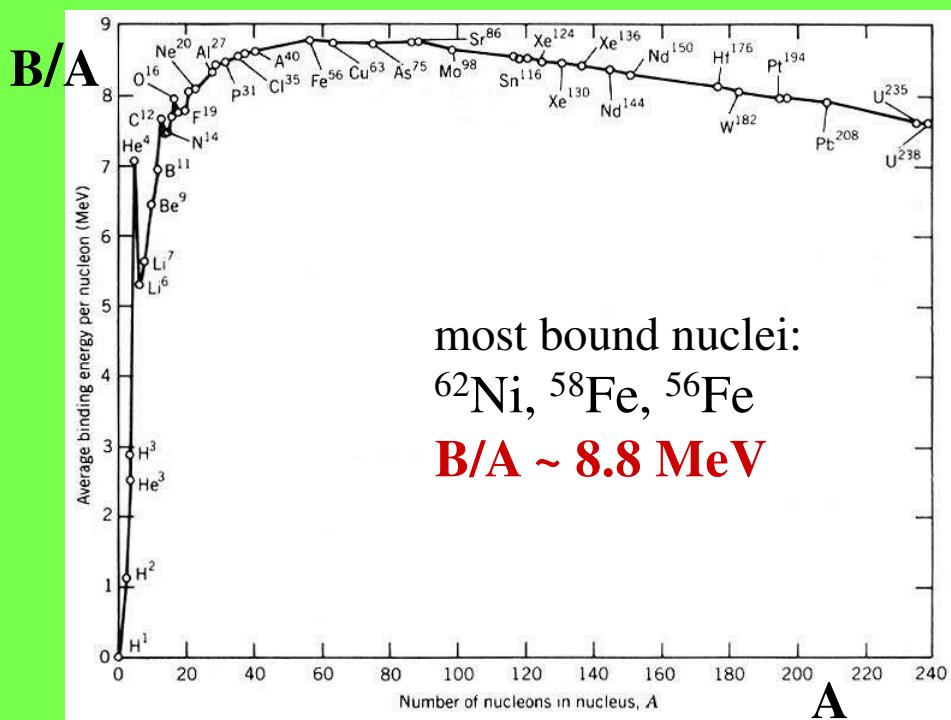
$$R_g \equiv 2GM/c^2 \text{ (Schwarzschild radius)}$$

Atomic Nuclei: bulk properties

Mass number $A = 1 - 238$ (natural stable isotopes)

Radius $R = r_0 A^{1/3} \sim (2 - 10) \text{ fm}$

Density $\rho \sim \rho_0 = 2.8 \times 10^{14} \text{ g/cm}^3$



$B/(Mc^2)$
 $\sim (0.1-1)\%$

bound by
nuclear
interactions

Relativistic equations for stellar structure

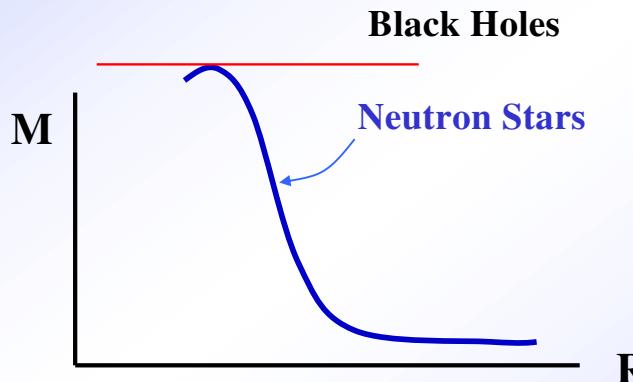
Tolman – Oppenheimer – Volkov equations (TOV)

$$\frac{dP}{dr} = -G \frac{m(r)\rho(r)}{r^2} \left(1 + \frac{P(r)}{c^2 \rho(r)} \right) \left(1 + 4\pi \frac{r^3 P(r)}{m(r) c^2} \right) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1}$$

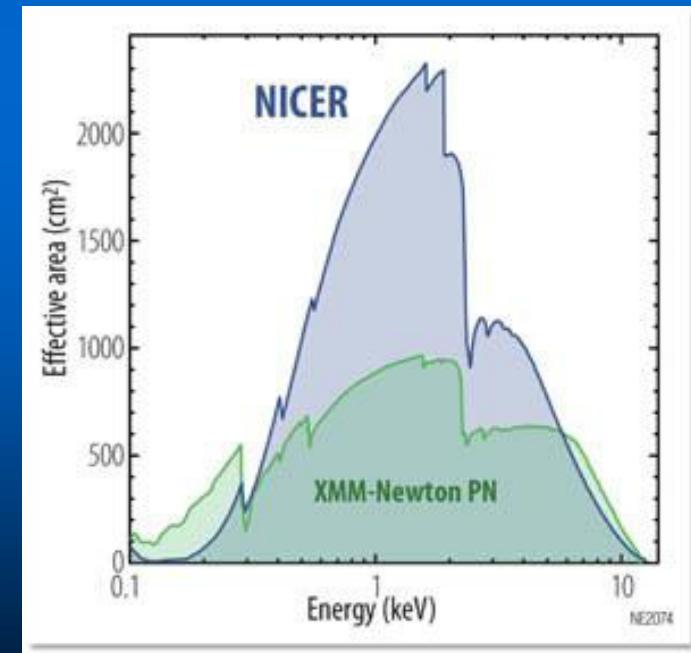
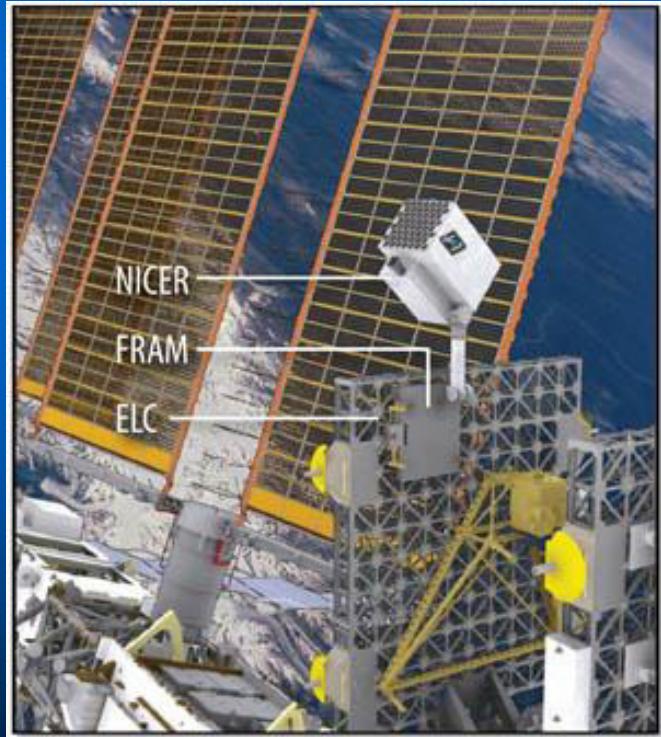
$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

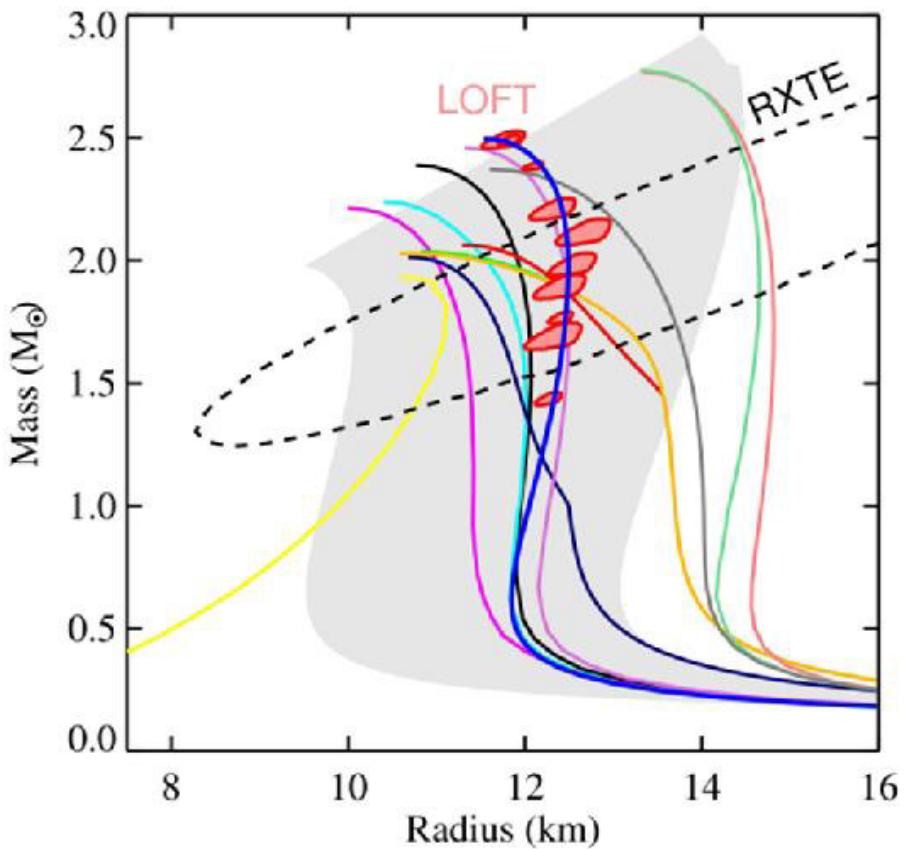
$$\frac{d\Phi}{dr} = -\frac{1}{\rho(r)c^2} \frac{dP}{dr} \left(1 + \frac{P(r)}{\rho(r)c^2} \right)^{-1}$$

One needs the
**equation of state (EOS) of
dense matter, $P = P(\rho)$,**
up to very high densities



$M_{\max}(\text{EOS}) \geq \text{all measured neutron star masses}$





NICER and LOFT
accurate measurements of
the mass and radius of
several neutron stars



Determination of the cold ($T=0$)
dense matter EOS

$$P = P(\rho)$$

Relativistic inverse stellar structure problem:
L. Lindblom, ApJ 398 (1992) 569
F. Özel, D. Psaltis, Phys. Rev. D 80 (2009) 103003

Neutron star physics in a nutshell

1) **Gravity** compresses matter at very high density

2) **Pauli principle**

Stellar constituents are different species of **identical fermions** (n , p , ..., e^- , μ^-)
→ antisymmetric wave function for particle exchange → Pauli principle

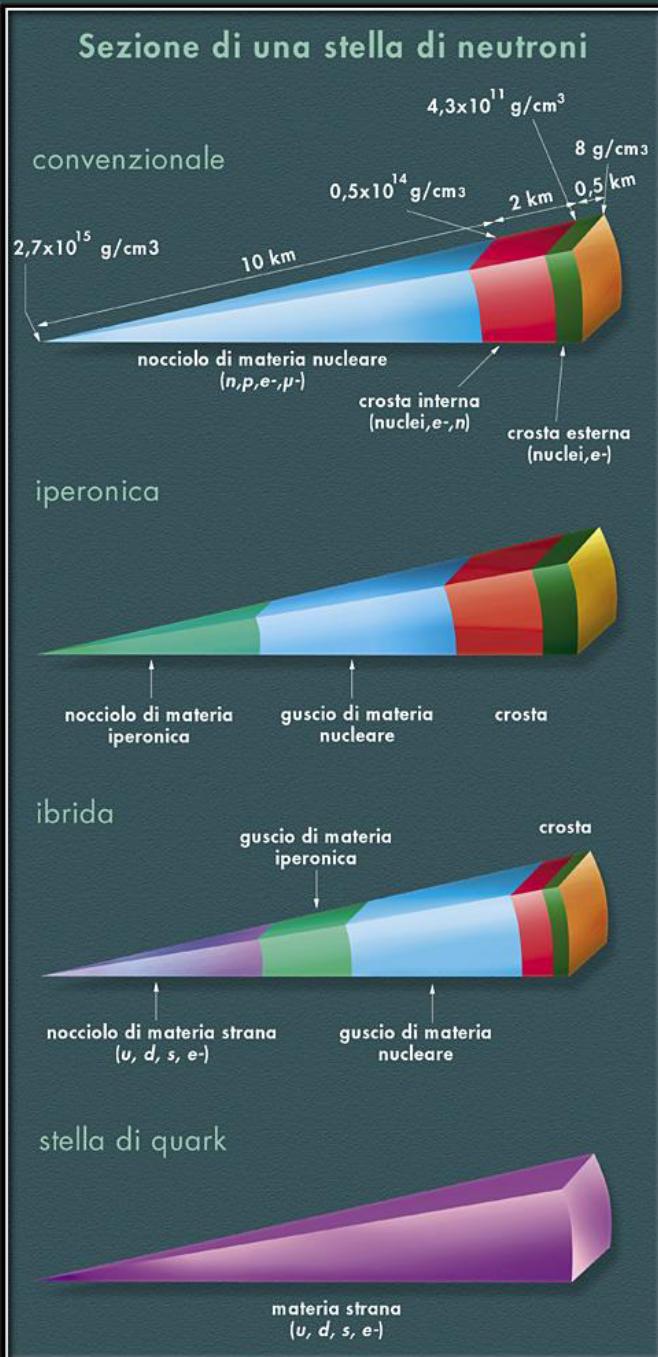
Chemical potentials $\mu_n, \mu_p, \dots, \mu_e$ rapidly increasing functions of density

3) **Weak interactions** change the isospin and strangeness content of dense matter to minimize energy

Cold catalyzed matter (Harrison, Wakano, Wheeler, 1958)

The ground state (minimum energy per baryon) of a system of **hadrons** and **leptons** with respect to their mutual **strong** and **weak interactions** at a given total baryon density n and temperature $T = 0$.

“Neutron Stars”



Nucleon Stars

Hyperon Stars

Hybrid Stars

Strange Stars

Hadronic Stars

Quark Stars

Neutron Stars in the QCD phase diagram

Lattice QCD at $\mu_b=0$ and finite T

- The transition to Quark Gluon Plasma is a crossover

Aoki et al., Nature, 443 (2006) 675

- Deconfinement transition temperature T_c

HotQCD Collaboration

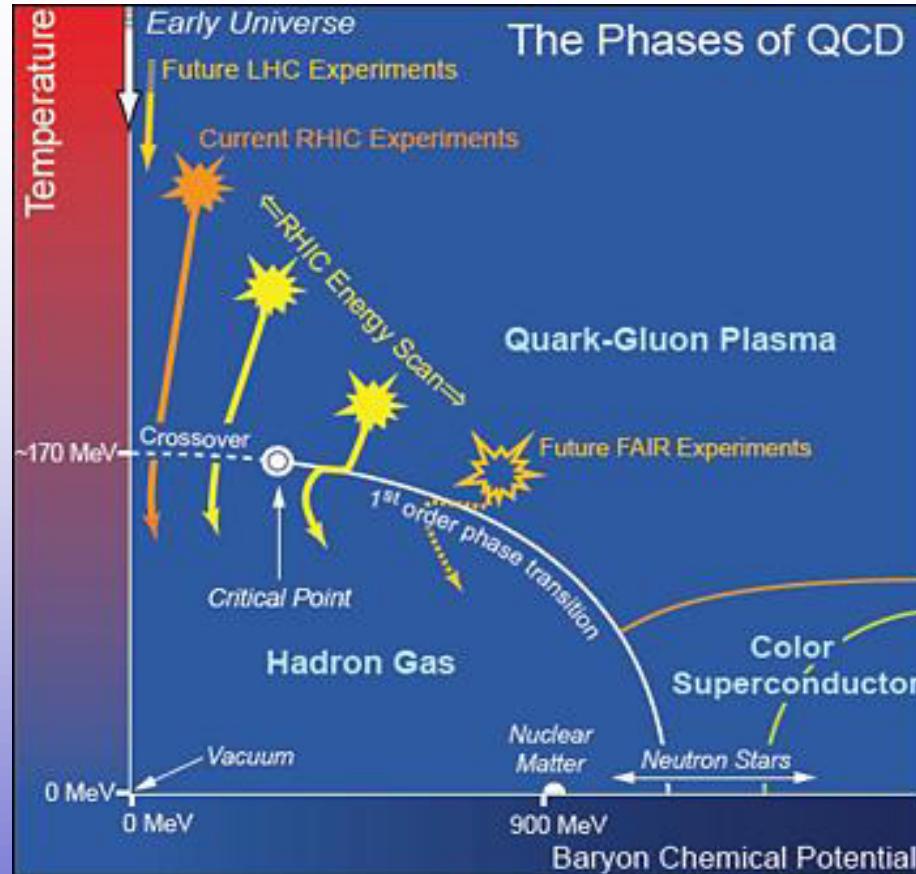
$$T_c = 154 \pm 9 \text{ MeV}$$

Bazarov et al., Phys.Rev. D85 (2012)
054503

Wuppertal-Budapest Collab.

$$T_c = 147 \pm 5 \text{ MeV}$$

Borsanyi et al., J.H.E.P. 09 (2010) 073



Neutron Stars: high μ_b and low T

Lattice QCD calculations are presently not possible

Quark deconfinement transition expected of the first order
Z. Fodor, S.D. Katz, Prog. Theor Suppl. 153 (2004) 86

“A link between lattice QCD and measured neutron star masses”

I. Bombaci, D. Logoteta, Mont. Not. Royal Astron. Soc. 433 (2013) L79

1st order phase transitions are triggered by the **nucleation** of a **critical size drop** of the **new (stable) phase** in a **metastable mother phase**

Virtual drops of the stable phase are created by small localized **fluctuations** in the state variables of the **metastable phase**



A common event in nature, e.g.:

- **fog or dew formation in supersaturated vapor**
- **ice formation in supercooled water**

Pure and distilled water at standard pressure (100 kPa) can be supercooled down to a temperature of -48.3 °C. In the tempearture range (-48.3 — 0) °C, water is in a metastable phase and ice cristals will form via a nucleation process.

Phase equilibrium

Gibbs' criterion
for phase equilibrium

$$\mu_H = \mu_Q \equiv \mu_0$$

$$T_H = T_Q \equiv T$$

$$P(\mu_H) = P(\mu_Q) \equiv P(\mu_0) \equiv P_0$$

μ_j = Gibbs' energy per baryon (j-phase average chemical pot.) $j = H, Q$

$$\mu_H = \frac{\varepsilon_H + P_H - s_H T}{n_{b,H}}$$

$$\mu_Q = \frac{\varepsilon_Q + P_Q - s_Q T}{n_{b,Q}}$$

ε_j = energy density, P_j = pressure, s_j = entropy density (including leptonic contributions)

Phase equilibrium

Gibbs' criterion
for phase equilibrium

$$\mu_H = \mu_Q \equiv \mu_0$$

$$T_H = T_Q \equiv 0$$

$$P(\mu_H) = P(\mu_Q) \equiv P(\mu_0) \equiv P_0$$

Cold deleptonized
Neutron Stars
 $t \gg t_{\text{cool}} \sim \text{a few } 10^2 \text{ s}$
cooling time

μ_j = Gibbs' energy per baryon (j-phase average chemical pot.) $j = H, Q$

$$\mu_H = \frac{\varepsilon_H + P_H - s_H \cancel{T}}{n_{b,H}}$$

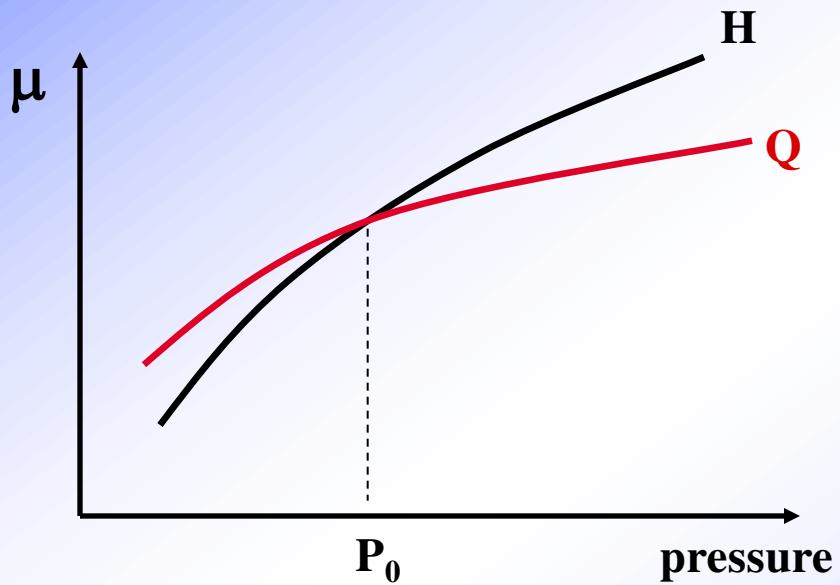
$$\mu_Q = \frac{\varepsilon_Q + P_Q - s_Q \cancel{T}}{n_{b,Q}}$$

ε_j = energy density, P_j = pressure, s_j = entropy density (including leptonic contributions)

In Hadronic Star cores when $P(r = 0) > P_0$

Hadronic matter phase is metastable

stable Quark matter phase
is formed by a nucleation process



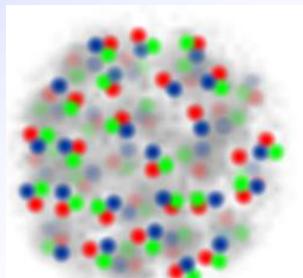
- Oscillation time of a virtual drop in the potential energy well:

$$v_0^{-1} \approx \tau_{\text{strong}} \approx 10^{-23} \text{ sec.} \ll \tau_{\text{weak}}$$

**Quark-flavor must be conserved
in the early stage of deconfinement**

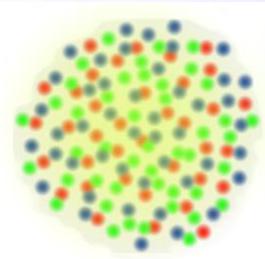
Q*-phase: flavor content is equal to that of β -stable HM at the same pressure and temperature

- Q-phase:** β -stable SQM.
Soon afterwards a critical-size drop of QM is formed,
the weak interaction re-establish β -equilibrium



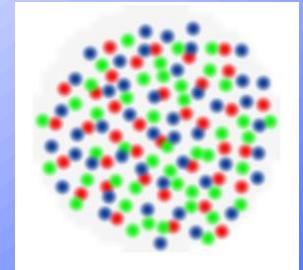
Hadron–phase

$$\tau_s \sim 10^{-23} \text{ s}$$

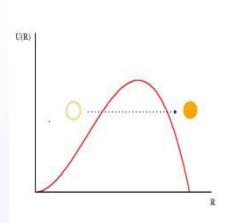


Q*-phase
Non- β -stable
quark-matter drop

$$\tau_w \sim 10^{-8} \text{ s}$$



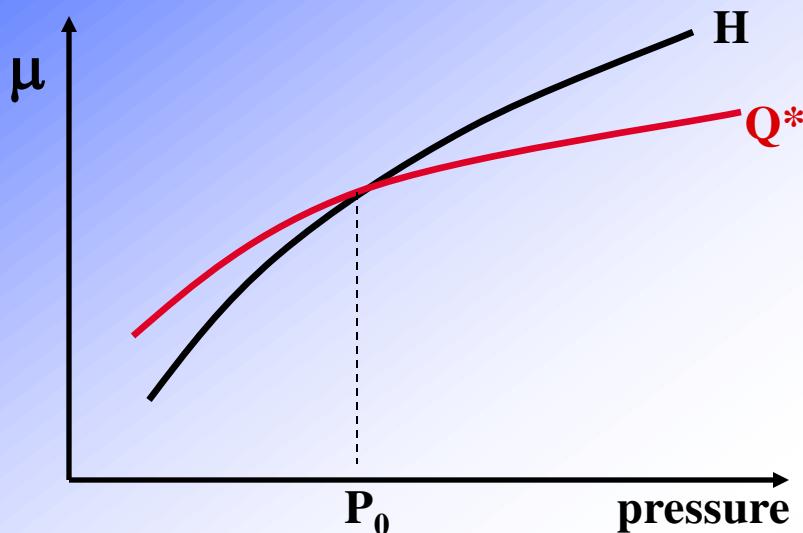
Q-phase



First drop of
 Q^* -matter



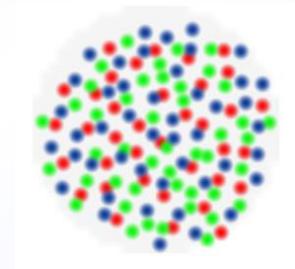
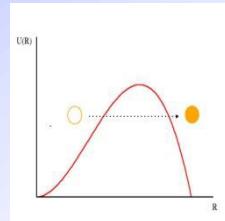
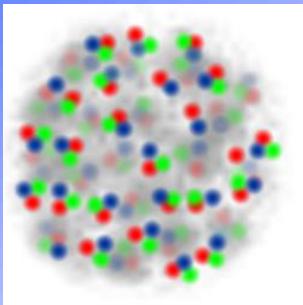
stellar conversion process $HS \rightarrow QS$



Hadronic Stars with $P_{\text{centr}} > P_0$
are metastable to the conversion
to Quark Stars

The mean lifetime of the metastable Hadronic Star configuration is related to nucleation time of the first drop of Q^* -phase

Direct nucleation of a β -stable quark matter drop



The direct nucleation of the β -stable quark drop (Q-phase drop) is a ***high order weak process***

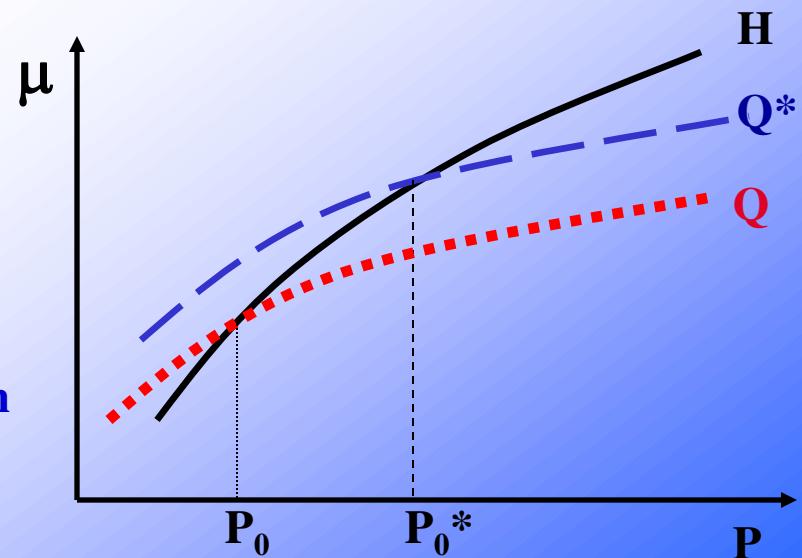
suppressed by a factor $\sim G_F^{2N/3}$

For a critical size drop

$$N = 100 - 1000. \quad \rightarrow$$

$$\rightarrow \tau_Q \gg T_{\text{univ}}$$

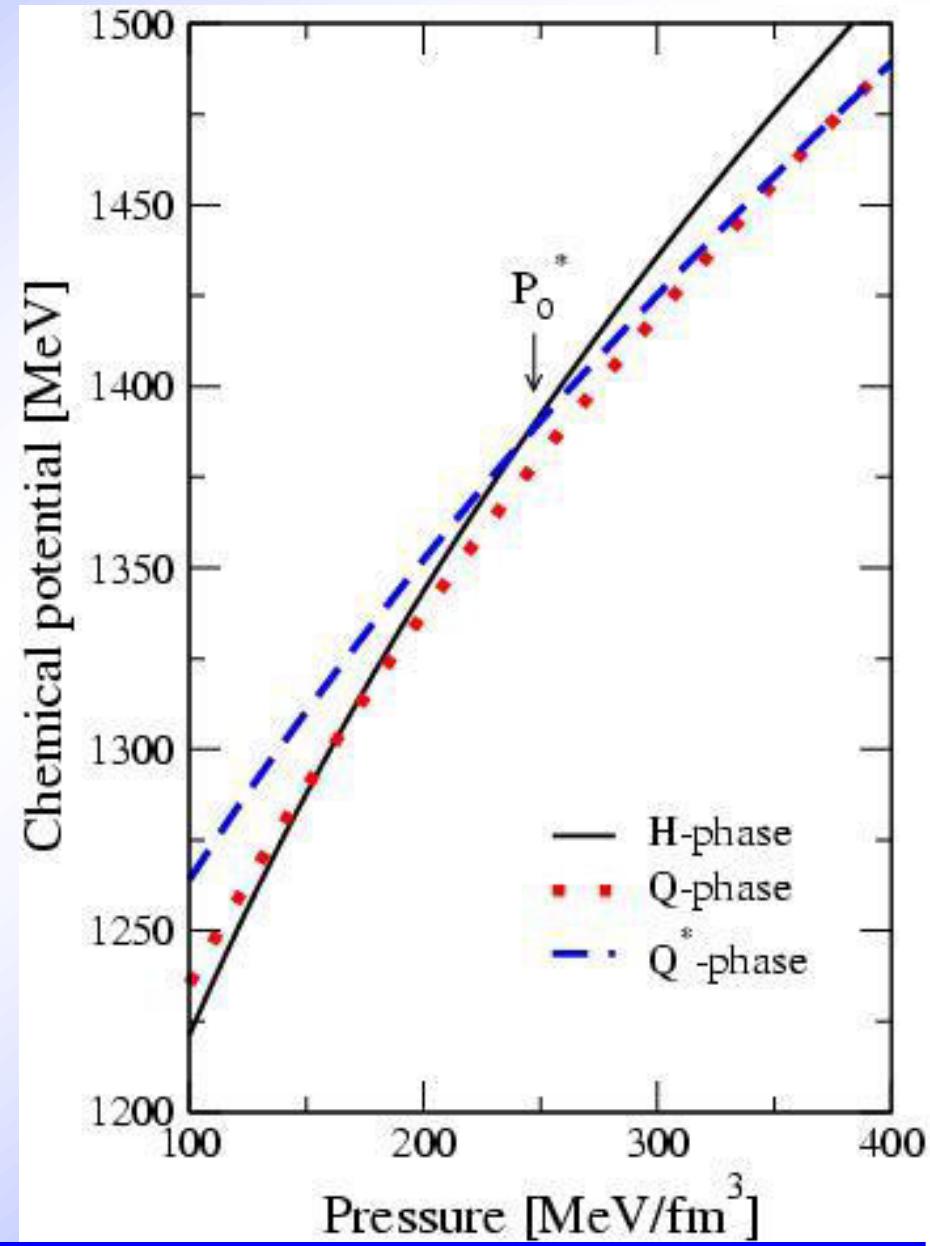
β -stable
quark-matter drop
("normal" or CFL)



This is the same reason that impedes that an iron nucleus converts to a drop of SQM, even in the case the Bodmer-Witten hypothesis is fulfilled.

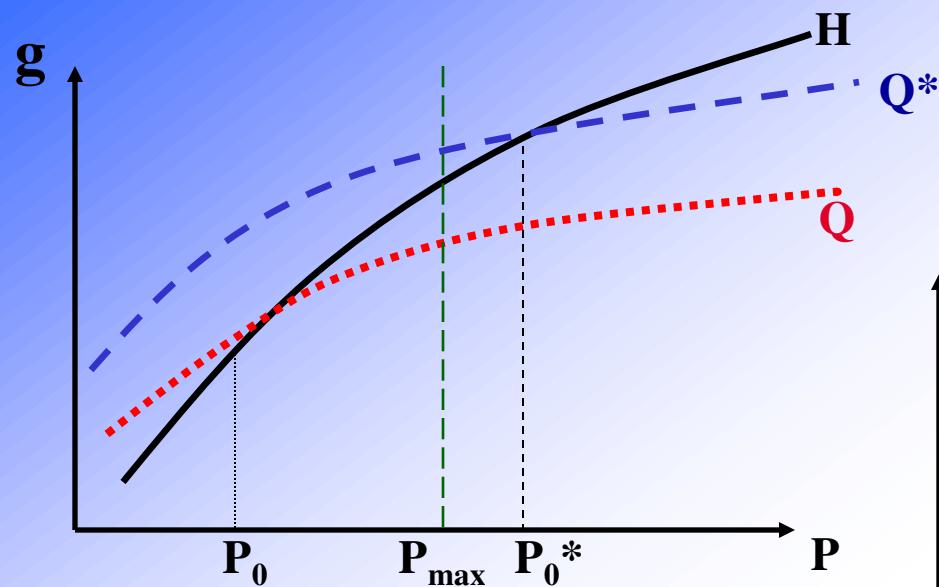
**Flavor conservation in the
intermediate transitory
 Q^* -phase**

**inhibits quark matter nucleation
in pure hadronic stars**

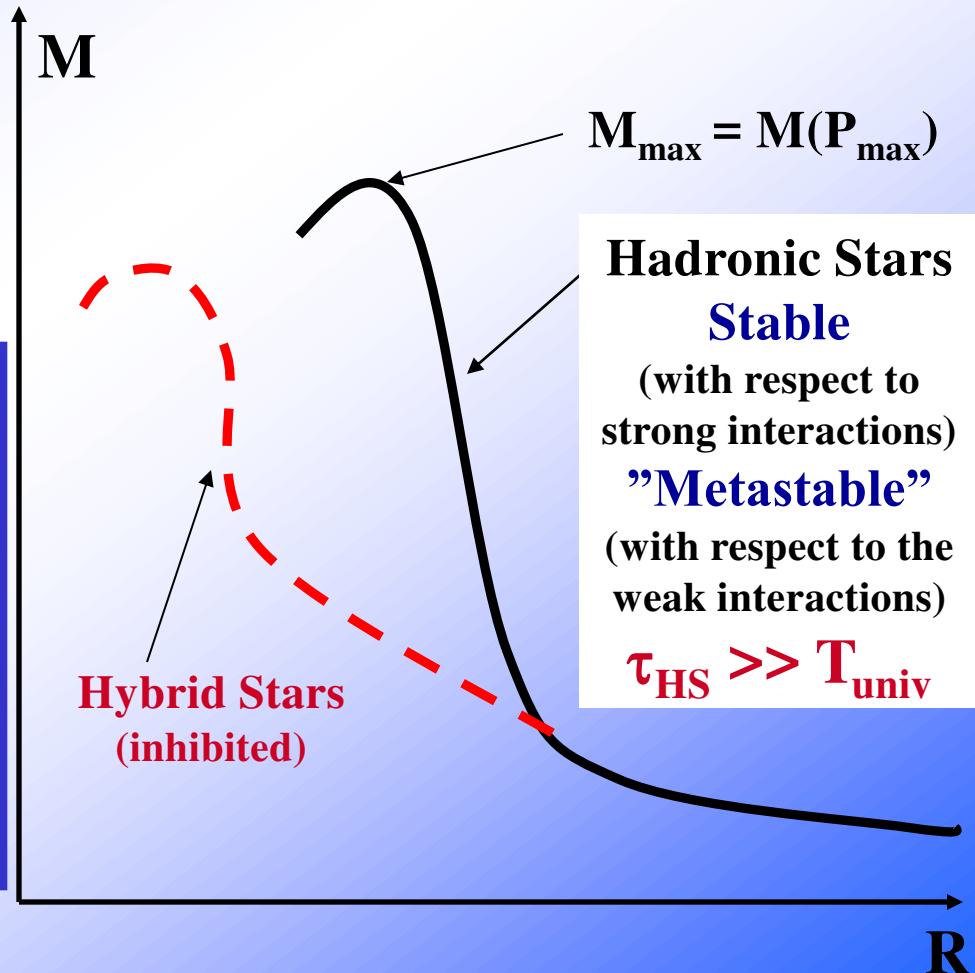


GM3 + MIT bag model ($B=152.45$ MeV/fm³, $m_s=150$ MeV)

A consequence of flavor conservation in the intermediate transitory Q^* -phase



In some case (depending on the EOS model parameters) **the formation of a QM-core in a compact star could be inhibited by flavor conservation in the intermediate Q^* -phase, even in the cases where the β -stable Q -phase has a lower Gibbs' energy per baryon than the hadronic-phase.**



Fluctuations of the number N of particles in a fixed volume V

**Root-mean-square fluctuation
(dispersion)**

$$\langle (\Delta N)^2 \rangle^{1/2} = \sqrt{N}$$

Relative fluctuation

$$\delta N = \frac{\langle (\Delta N)^2 \rangle^{1/2}}{N} = \frac{1}{\sqrt{N}}$$

Landau, Lifšits, Statistical Mechanics

$$R \approx 10 \text{ fm}, \quad n_B \approx 3n_0 \approx 0.48 \text{ fm}^{-3}$$

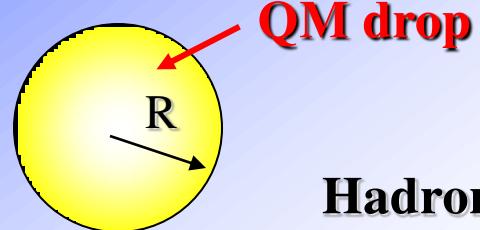
$$N = \frac{4}{3} \pi R^3 n_B \approx 2 \times 10^3,$$

$$\sqrt{N} \approx 45 \quad \delta N \approx 0.022$$

Quantum nucleation theory

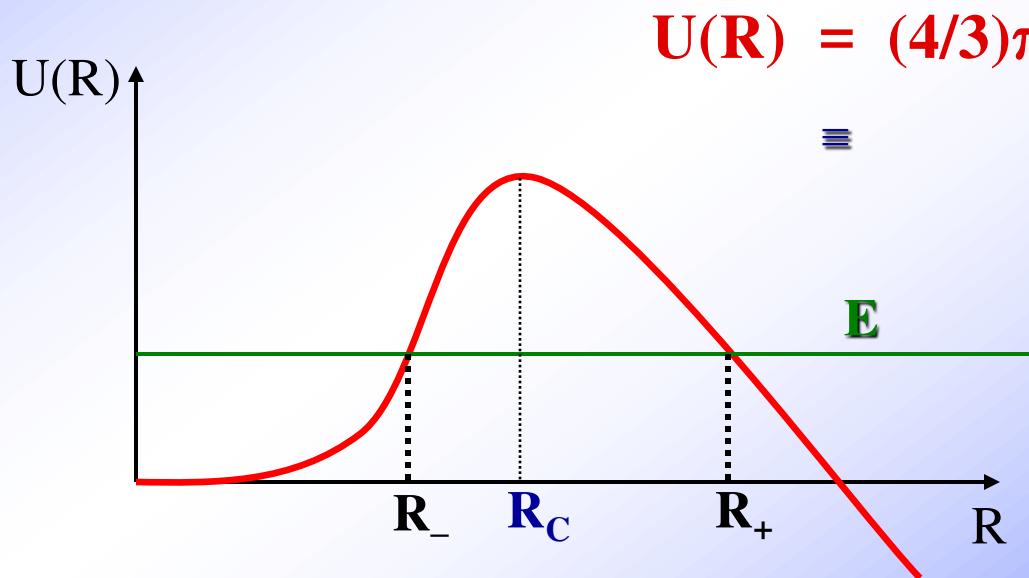
I.M. Lifshitz and Y. Kagan, 1972; K. Iida and K. Sato, 1998

Quantum fluctuation of a virtual drop of QM in HM



$$L = \frac{1}{2} M(R) \dot{R}^2 - U(R)$$

$$M(R) = 4\pi \rho_H (1 - n_{Q^*}/n_H)^2 R^3$$



$$U(R) = (4/3)\pi R^3 n_{Q^*} (\mu_{Q^*} - \mu_H) + 4\pi\sigma R^2$$
$$\equiv a_v R^3 + a_s R^2$$

As $R > R_C$ the drop grows with no limitation.

$R_C \equiv$ radius of the critical size drop

Probability of tunneling

Oscillation frequency of the virtual drop inside the potential well

$$v_0 = (dI/dE)^{-1} \quad \text{for } E = E_0$$

$$I(E_0) = \frac{2}{3} \pi \hbar$$

$$I(E) = 2 \int_0^{R_-} dR \sqrt{2M(R)[E - U(R)]}$$

Action of the zero point oscillations

Penetrability of the potential barrier

$$p_0 = \exp \left[-\frac{A(E_0)}{\hbar} \right]$$

$$A(E) = 2 \int_{R_-}^{R_+} dR \sqrt{2M(R)[U(R) - E]}$$

Action under the potential barrier

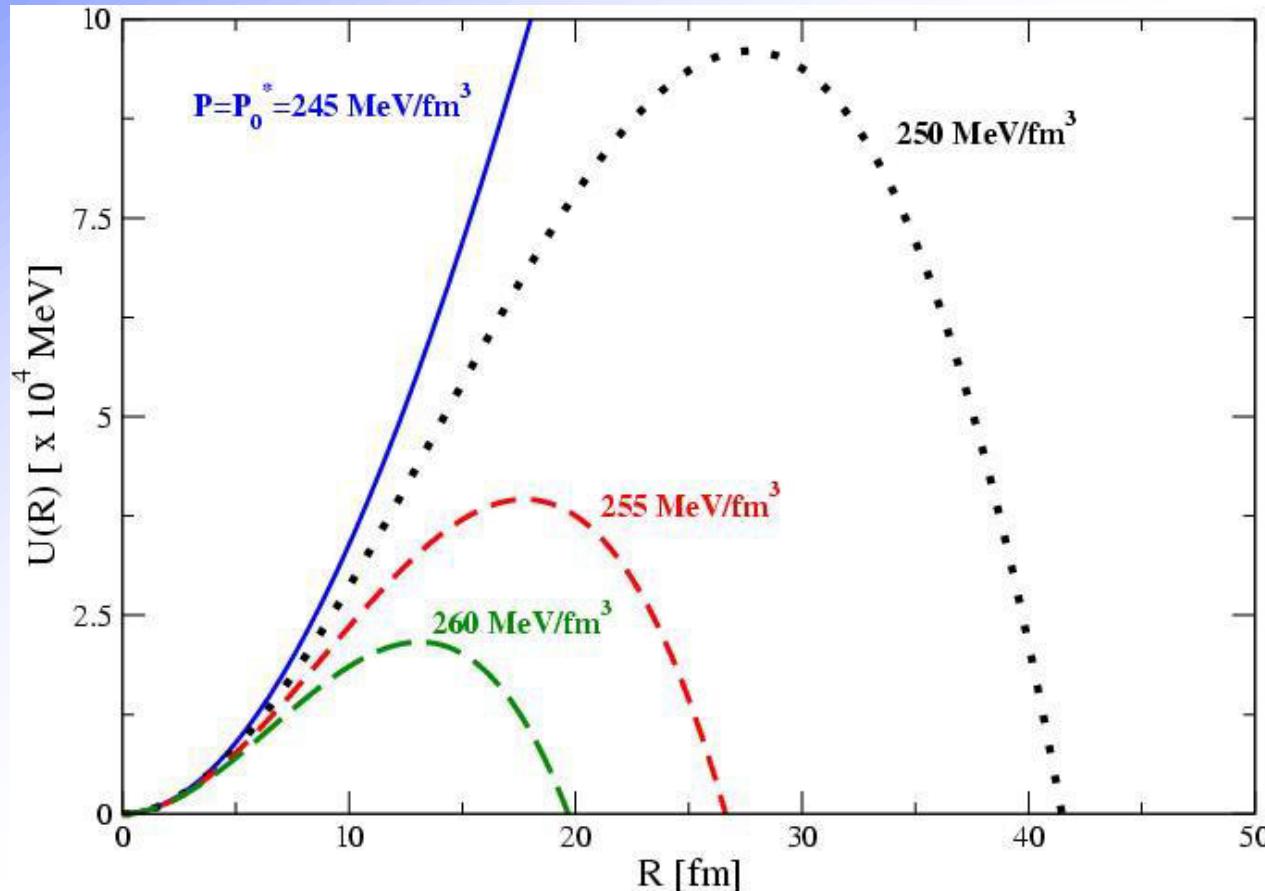
Quantum nucleation time

$$\tau_q = (v_0 p_0 N_c)^{-1}$$

$N_c \sim 10^{48}$
numb. of nucleation centers in the star core

Potential energy barrier between the H-phase and the Q*-phase

$$U(R,P) = (4/3)\pi n_{Q^*} (\mu_{Q^*} - \mu_H) R^3 + 4\pi\sigma R^2 \\ = a_v(P) R^3 + a_s R^2$$

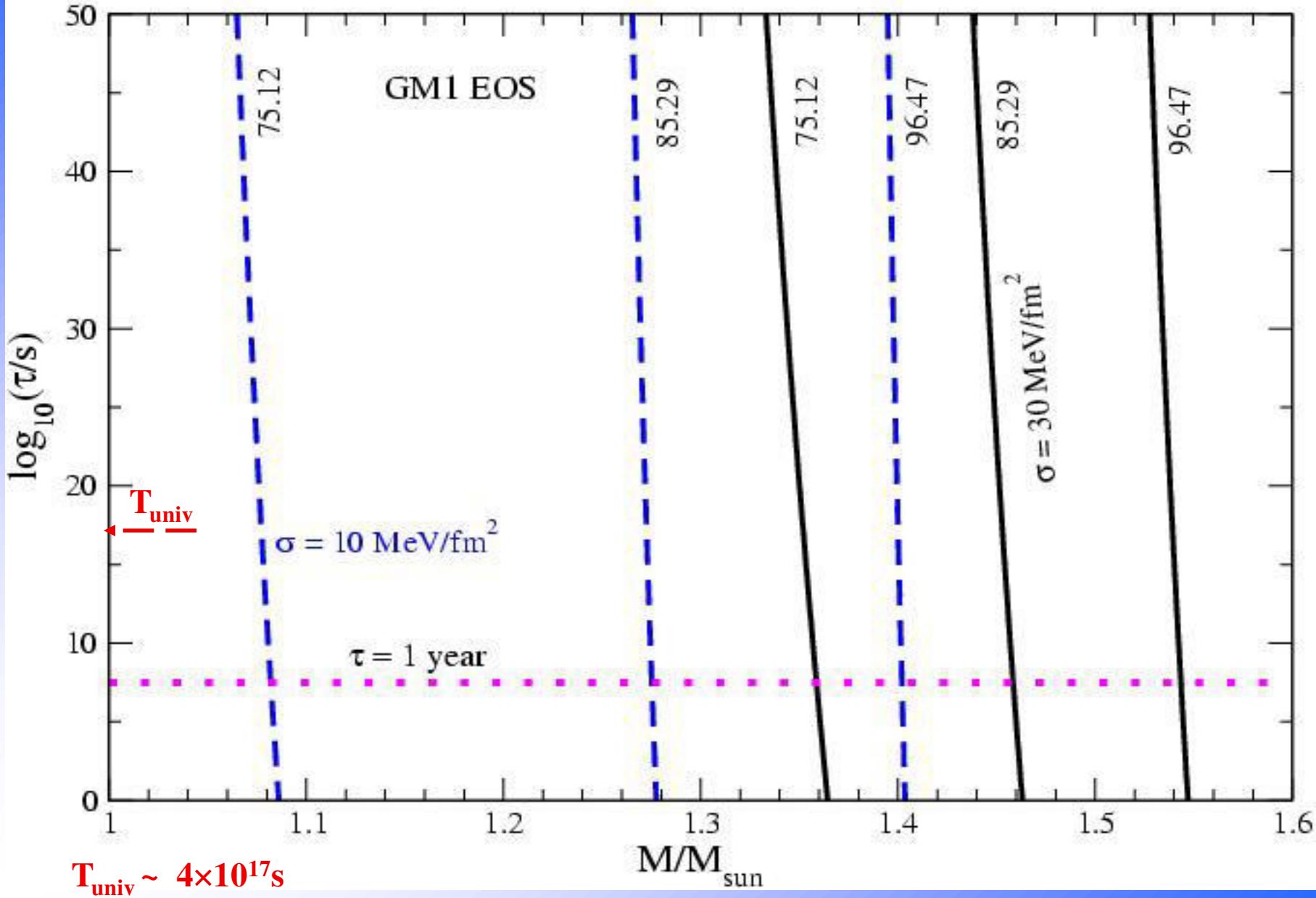


$T = 0$

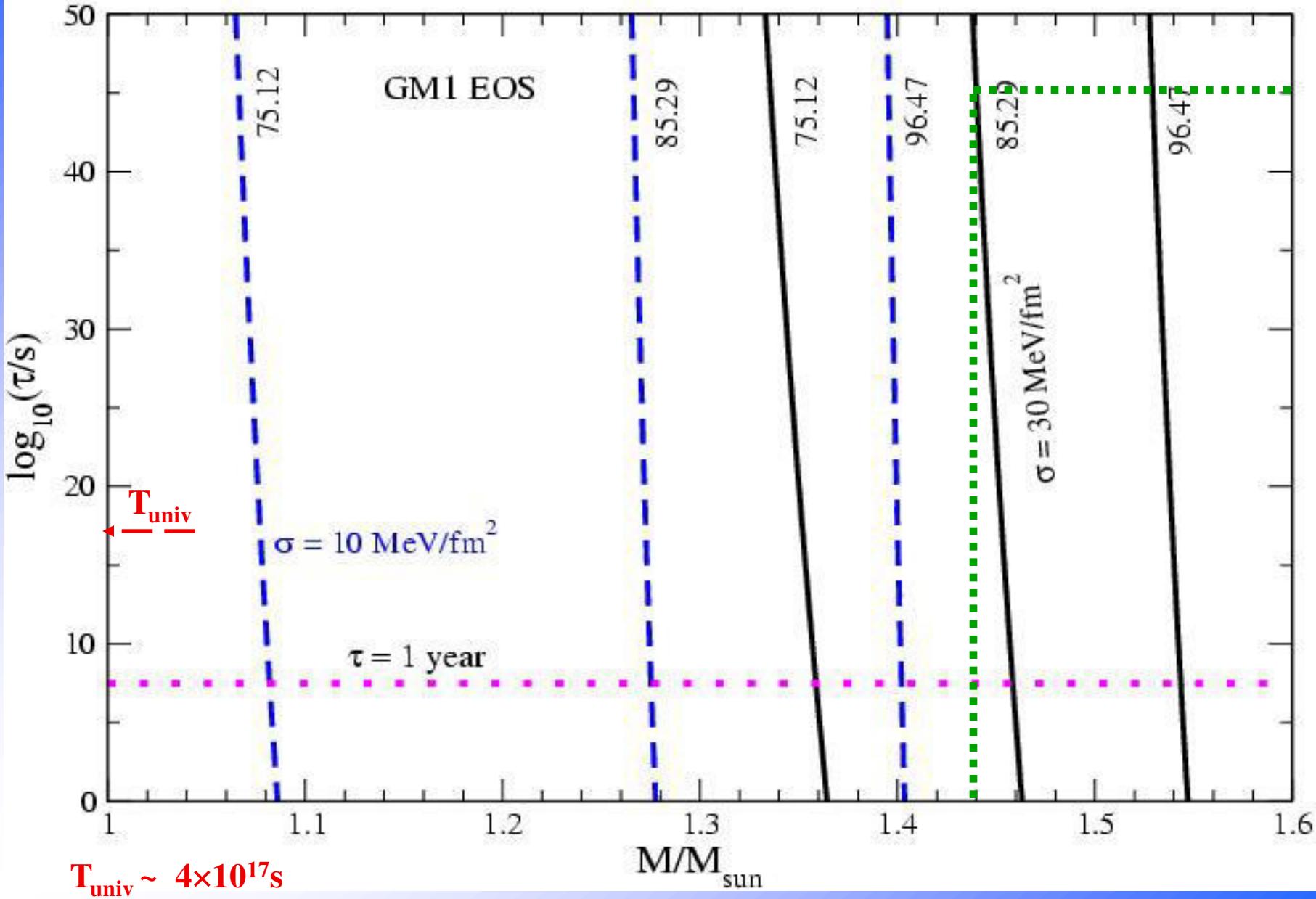
GM3 + MIT bag model
($B = 152.45 \text{ MeV/fm}^3$,
 $m_s = 150 \text{ MeV}$)

$\sigma = 30 \text{ MeV/fm}^2$

Hadronic Star mean-life time



Hadronic Star mean-life time



The critical mass of metastable Hadronic Stars

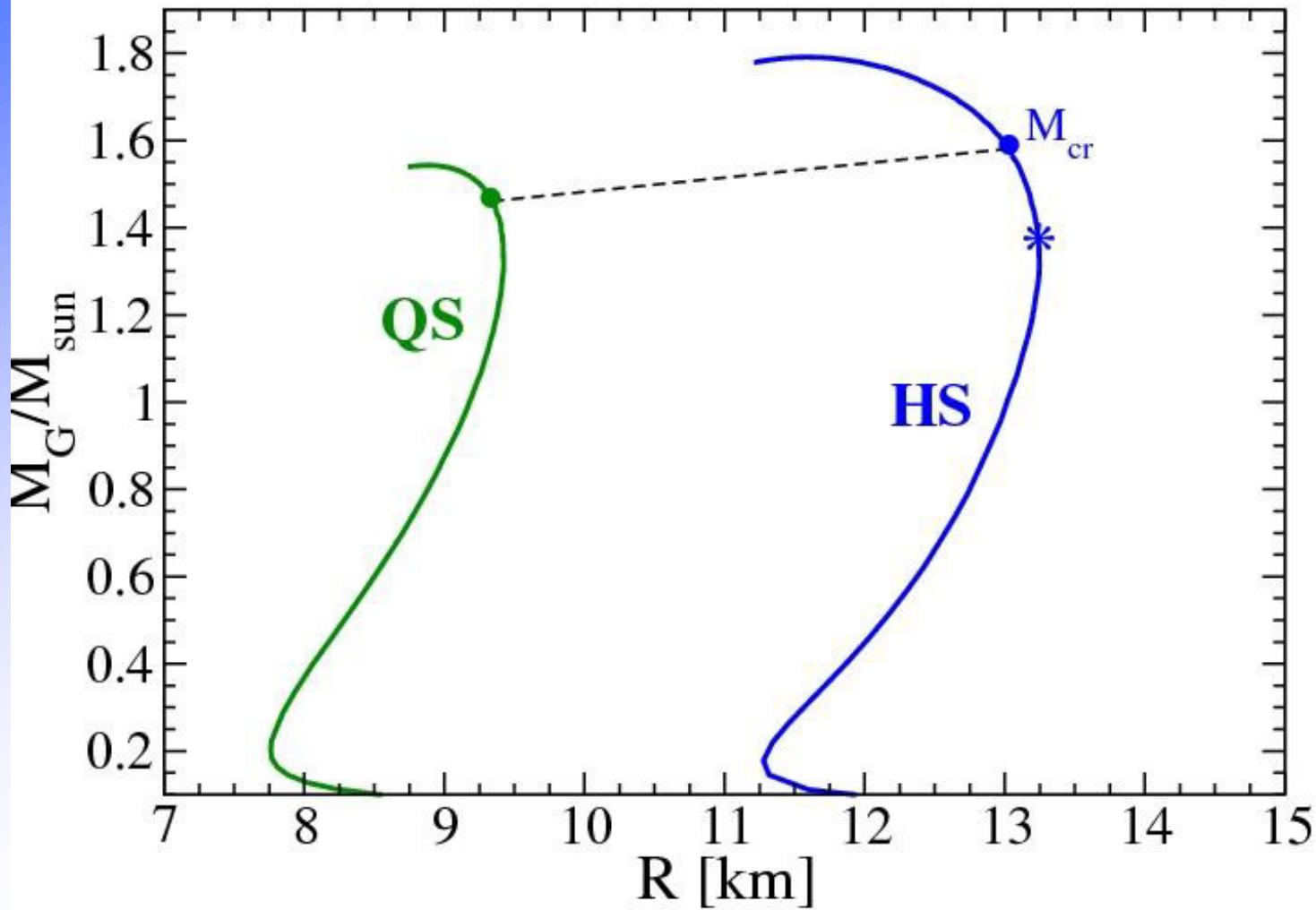
Def.: $M_{cr} \equiv M_{HS}(\tau = 1 \text{ yr})$

- HS with $M_{thr} < M_{HS} < M_{cr}$ are metastable with $\tau = 1 \text{ yr} \div \infty$
- HS with $M_{HS} > M_{cr}$ are very unlikely to be observed



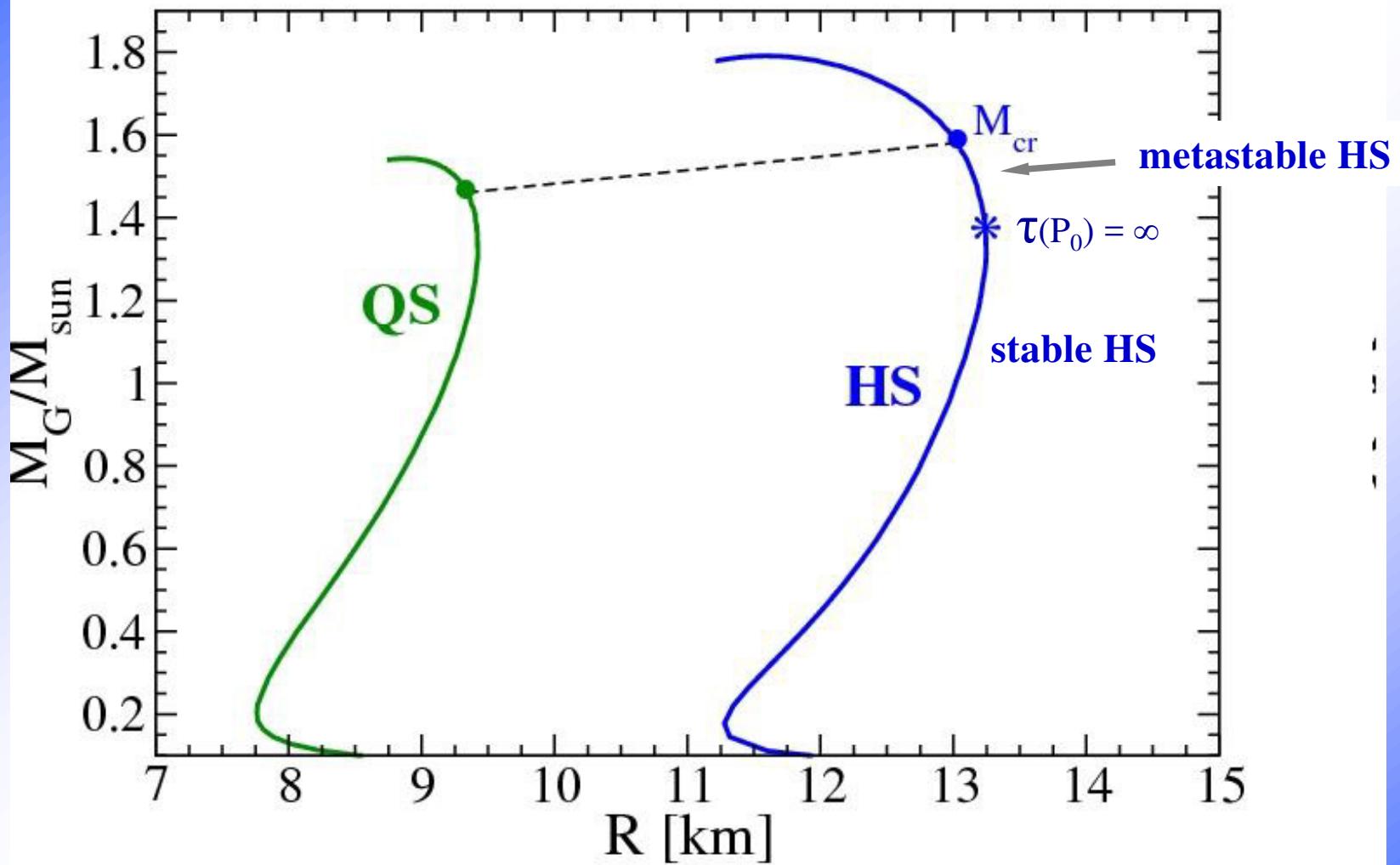
The critical mass M_{cr} plays the role of an effective maximum mass for the hadronic branch of compact stars

The two families of Compact Stars



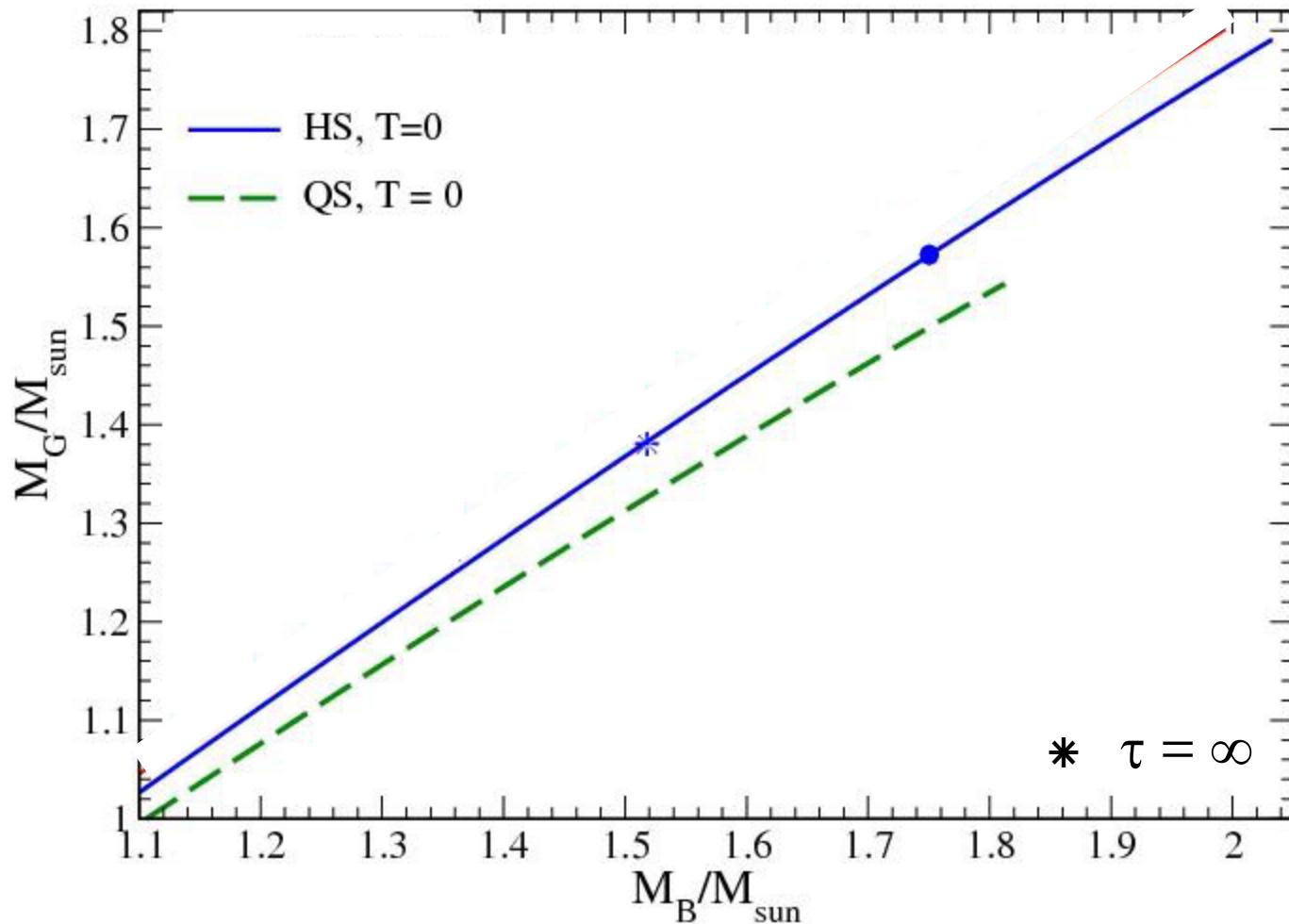
GM1($X\sigma = 0.6$) + MIT bag model ($B=85 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

The two families of Compact Stars



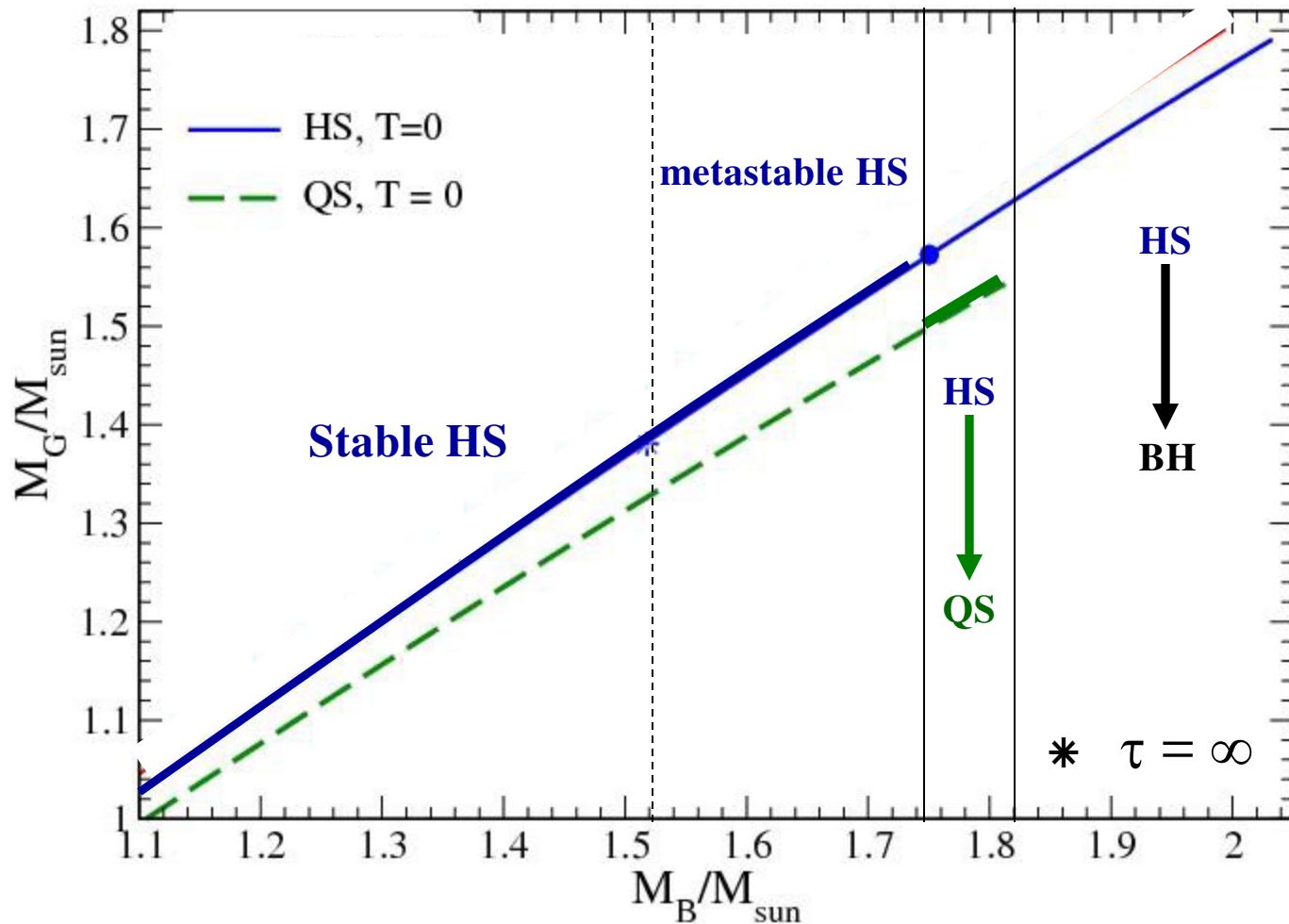
GM1($X\sigma = 0.6$) + MIT bag model ($B=85 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

Evolution of a cold HS



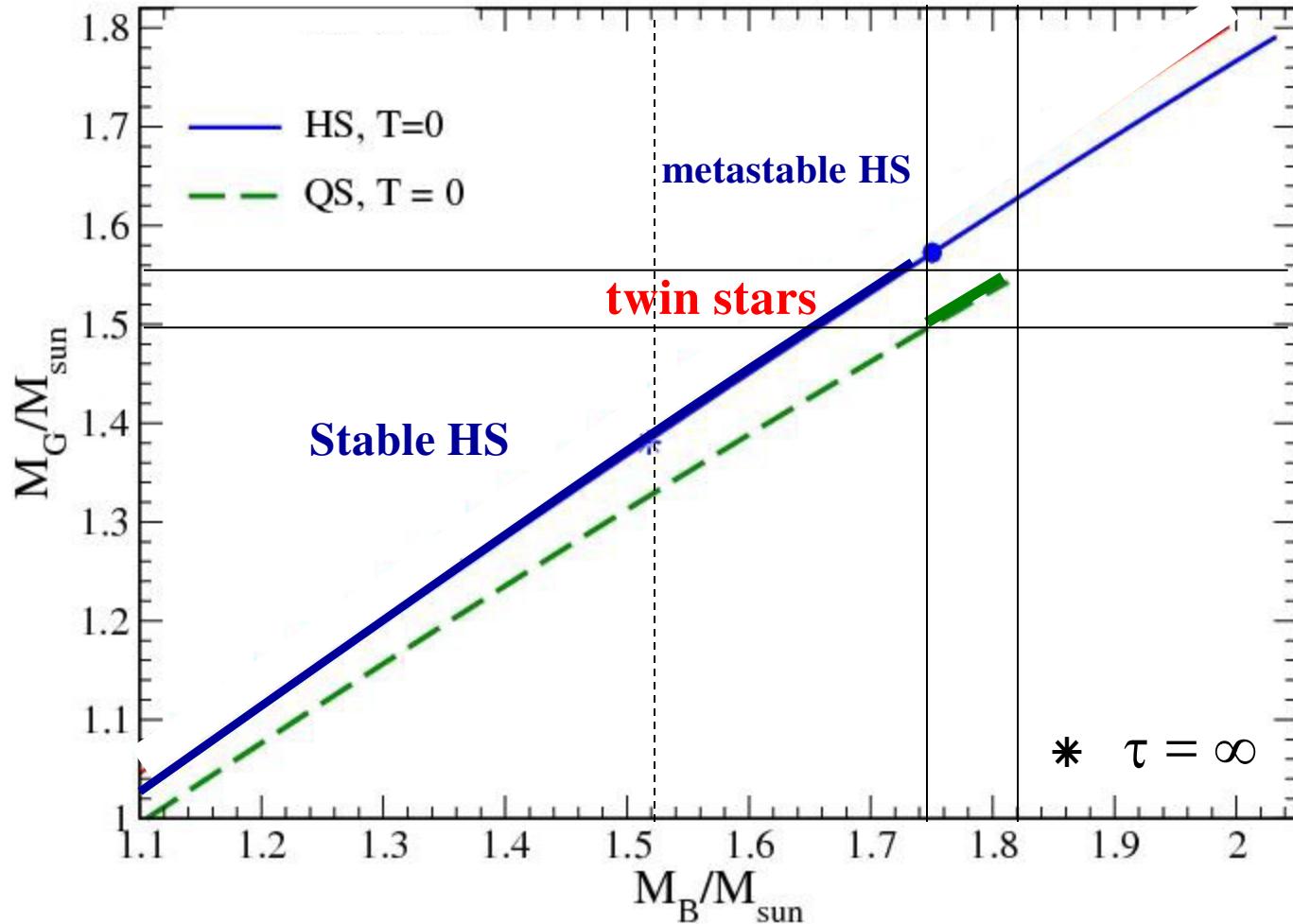
GM1($X\sigma = 0.6$) + MIT bag model ($B=85 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

Evolution of a cold HS



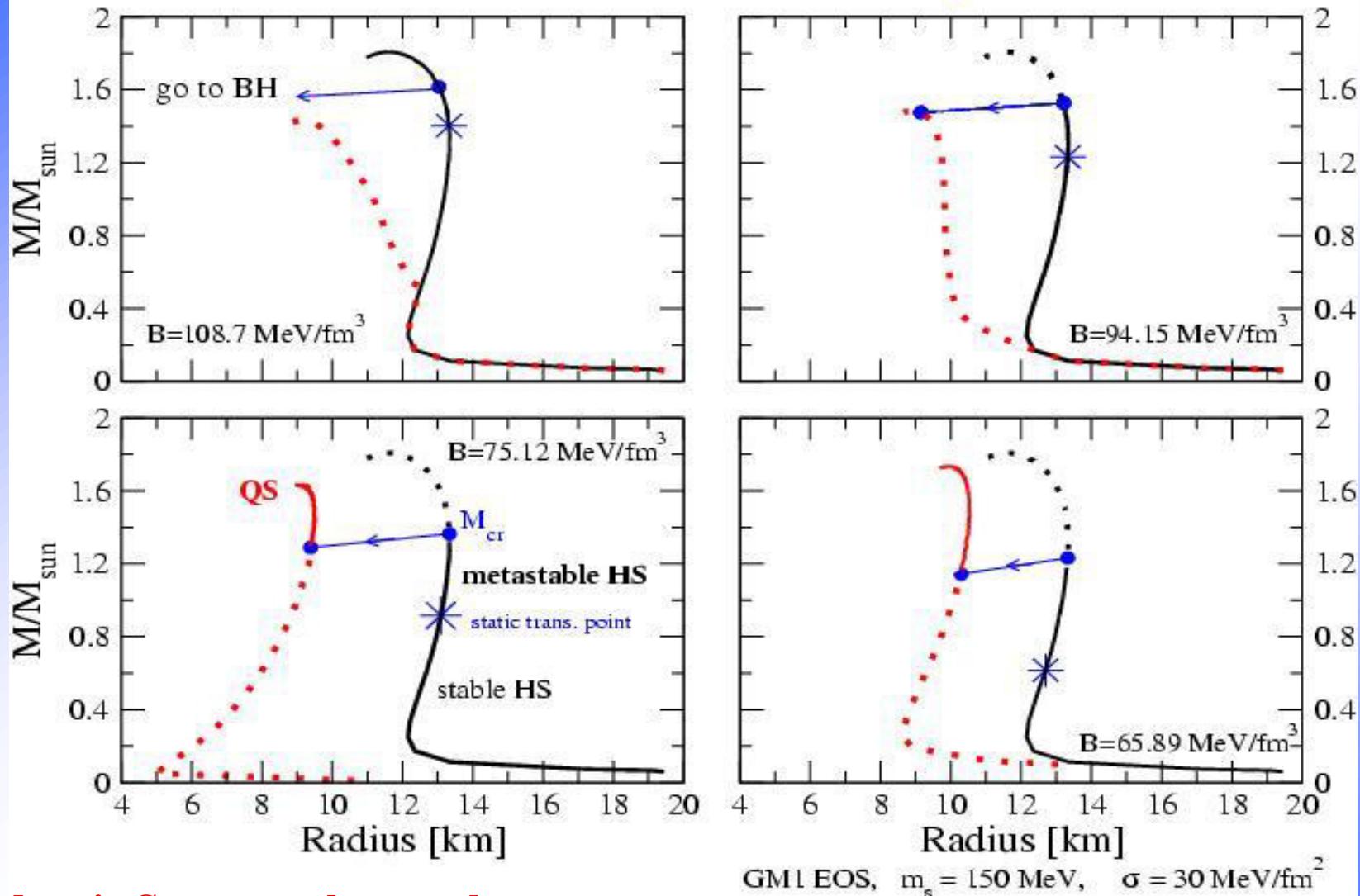
GM1($X\sigma = 0.6$) + MIT bag model ($B=85$ MeV/fm 3 , $m_s=150$ MeV)

Evolution of a cold HS



GM1($X\sigma = 0.6$) + MIT bag model ($B=85 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

The two families of Compact Stars



Hadronic Stars: nucleons + hyperons

Two “heavy” Neutron Stars

PSR J1614–2230

$M_{NS} = 1.97 \pm 0.04 M_{\odot}$

NS – WD binary system (He WD)

$M_{WD} = 0.5 M_{\odot}$ (companion mass)

$P_b = 8.69$ hr (orbital period) $P = 3.15$ ms (PSR spin period)

$i = 89.17^\circ \pm 0.02^\circ$ (inclination angle)

P. Demorest et al., Nature 467 (2010) 1081

PSR J0348+0432

$M_{NS} = 2.01 \pm 0.04 M_{\odot}$

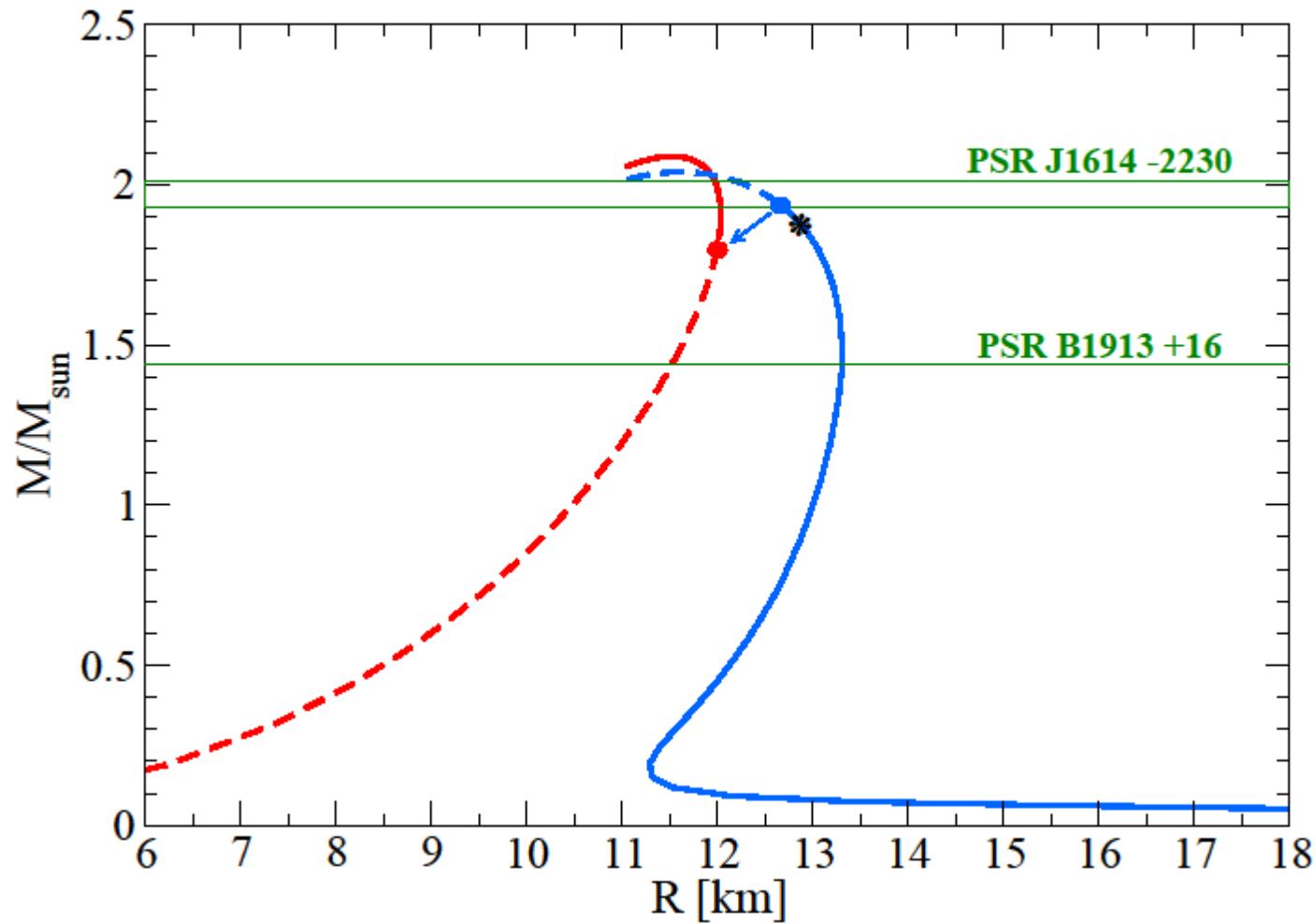
NS – WD binary system

$M_{WD} = 0.172 \pm 0.003 M_{\odot}$ (companion mass)

$P_b = 2.46$ hr (orbital period) $P = 39.12$ ms (PSR spin period)

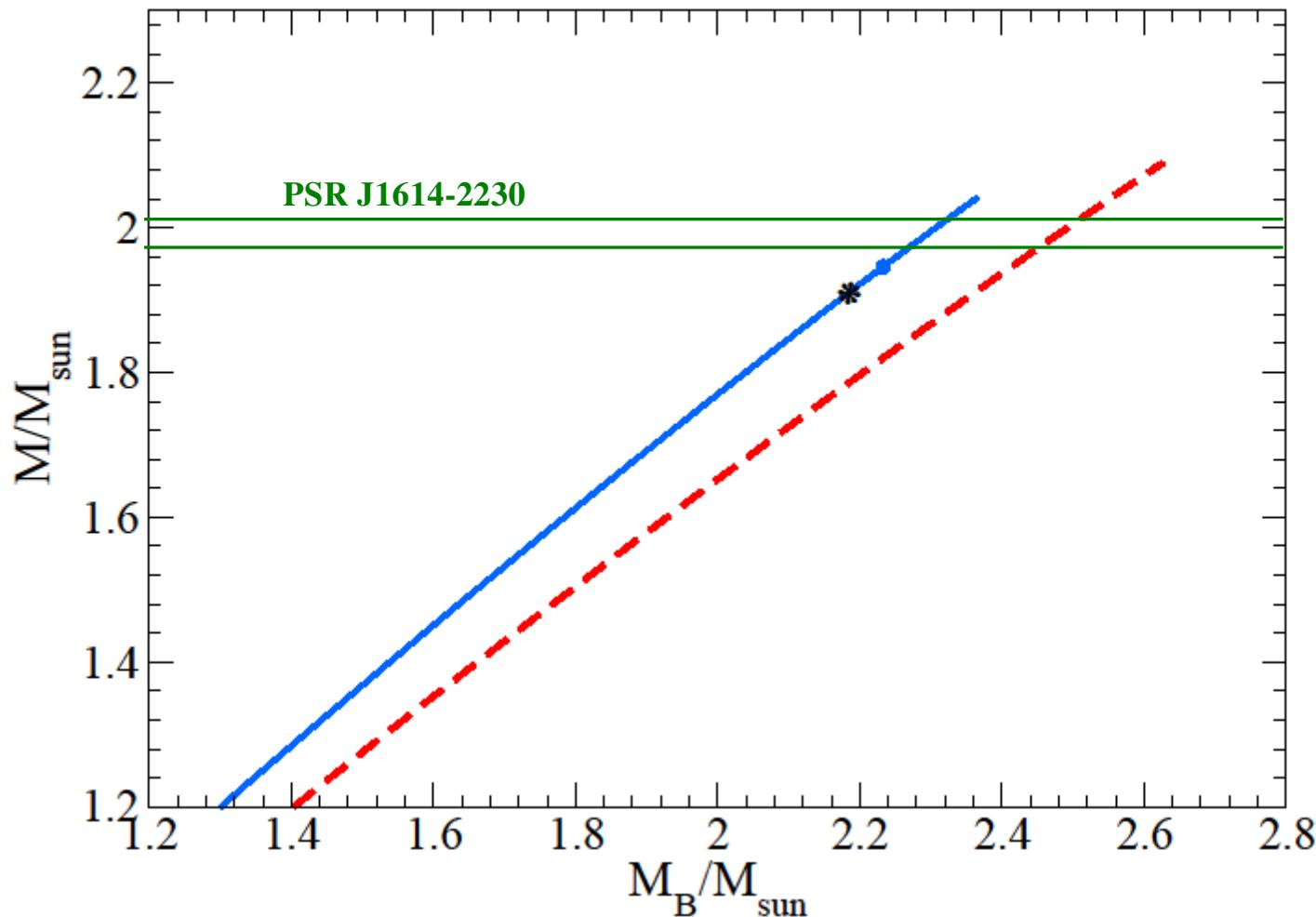
$i = 40.2^\circ \pm 0.6^\circ$ (inclination angle)

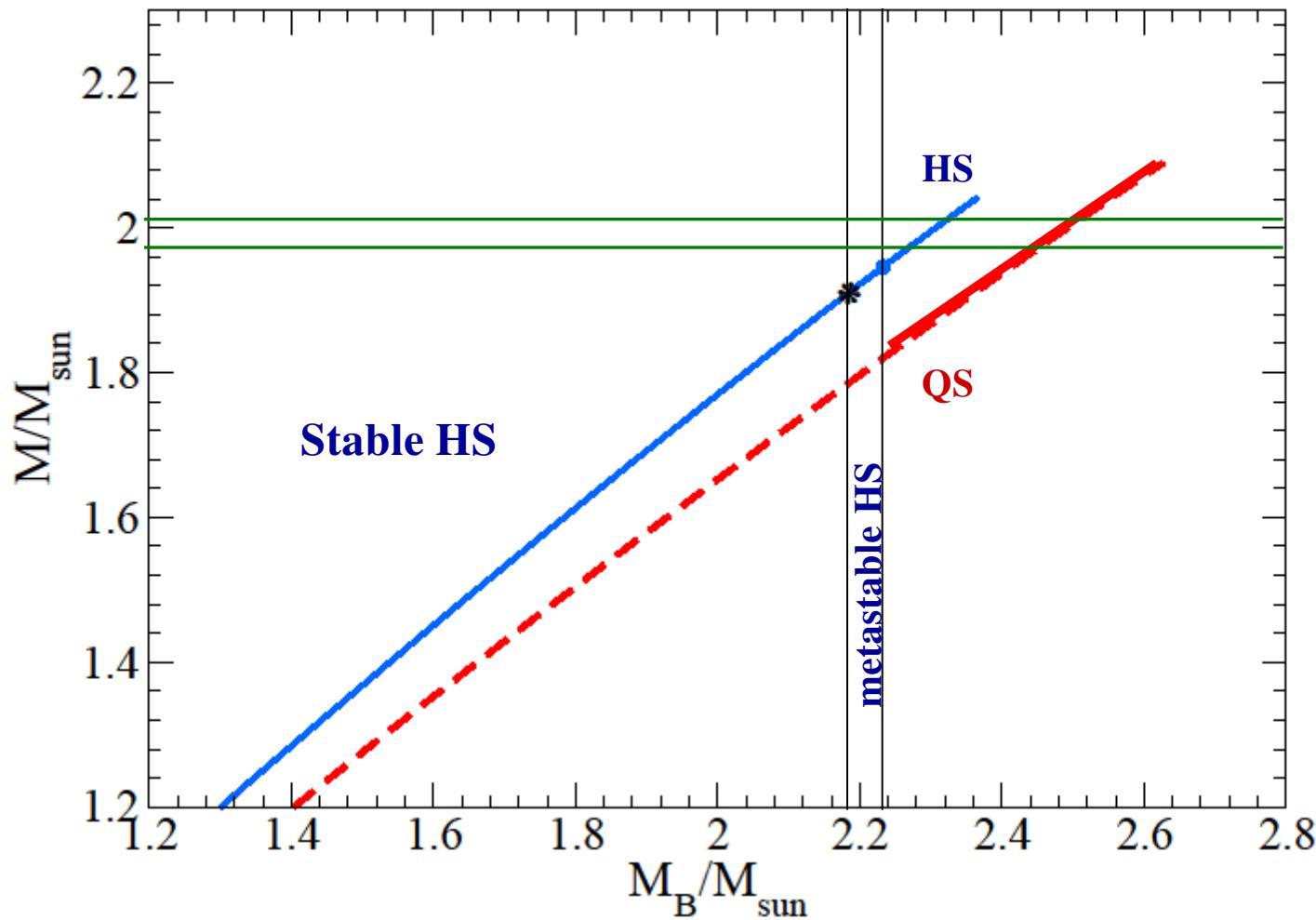
Antoniadis et al., Science 340 (2013) 448



Bombaci, et al. EPJ A 52 (2016) 58

SQM EOS: Alford et al. *Astrophys. J.* 629 (2005); Fraga et al., *Phys. Rev. D* 63 (2001)





The limiting mass of compact stars

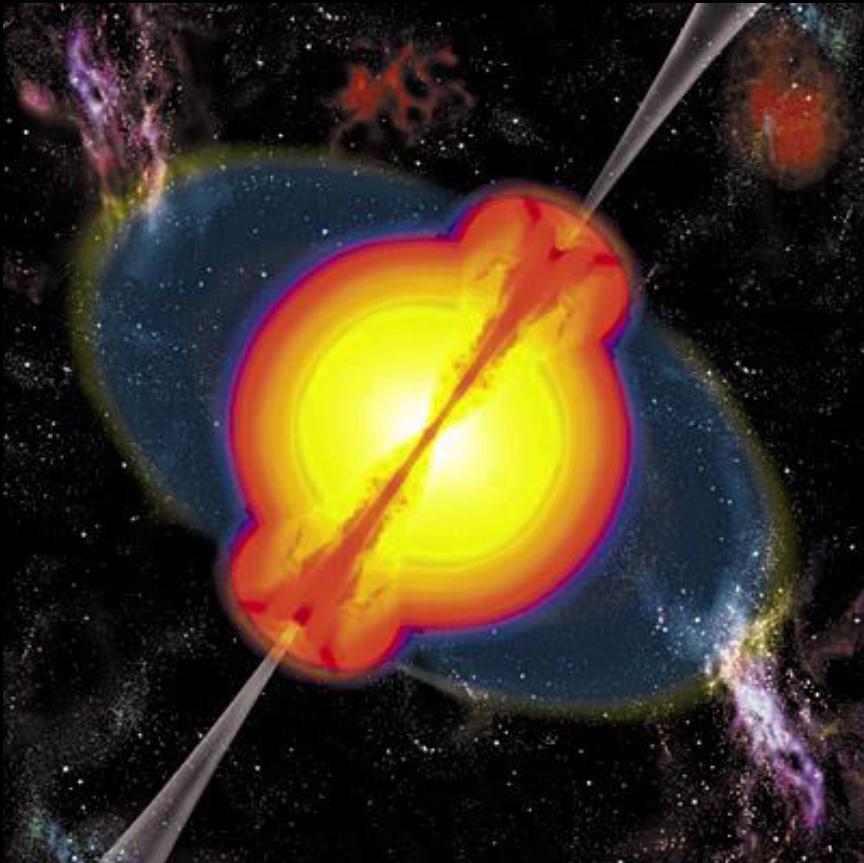
The metastability of HS and the existence of two families of compact stars, demands an **extension of the concept of maximum mass** of a “neutron star” with respect to the *classical* one introduced by **Oppenheimer & Volkoff**

Hadronic Stars with a “short” *mean-life time* are very unlikely to be observed

A new **operational** definition of NS limiting mass

- If $\tau(M_{HS,max}) \sim T_{univ}$ or $\tau(M_{HS,max}) \gg T_{univ}$
 $M_{lim} = M_{HS,max}$
- If $M_{cr} < M_{HS,max}$ i.e. $\tau(M_{HS,max}) < 1 \text{ yr}$
 $M_{lim} = \max[M_{cr}, M_{QS,max}]$
- If $1 \text{ yr} < \tau(M_{HS,max}) < T_{univ}$
 $M_{lim} = \max[M_{HS,max}, M_{QS,max}]$

Total energy released in the stellar conversion

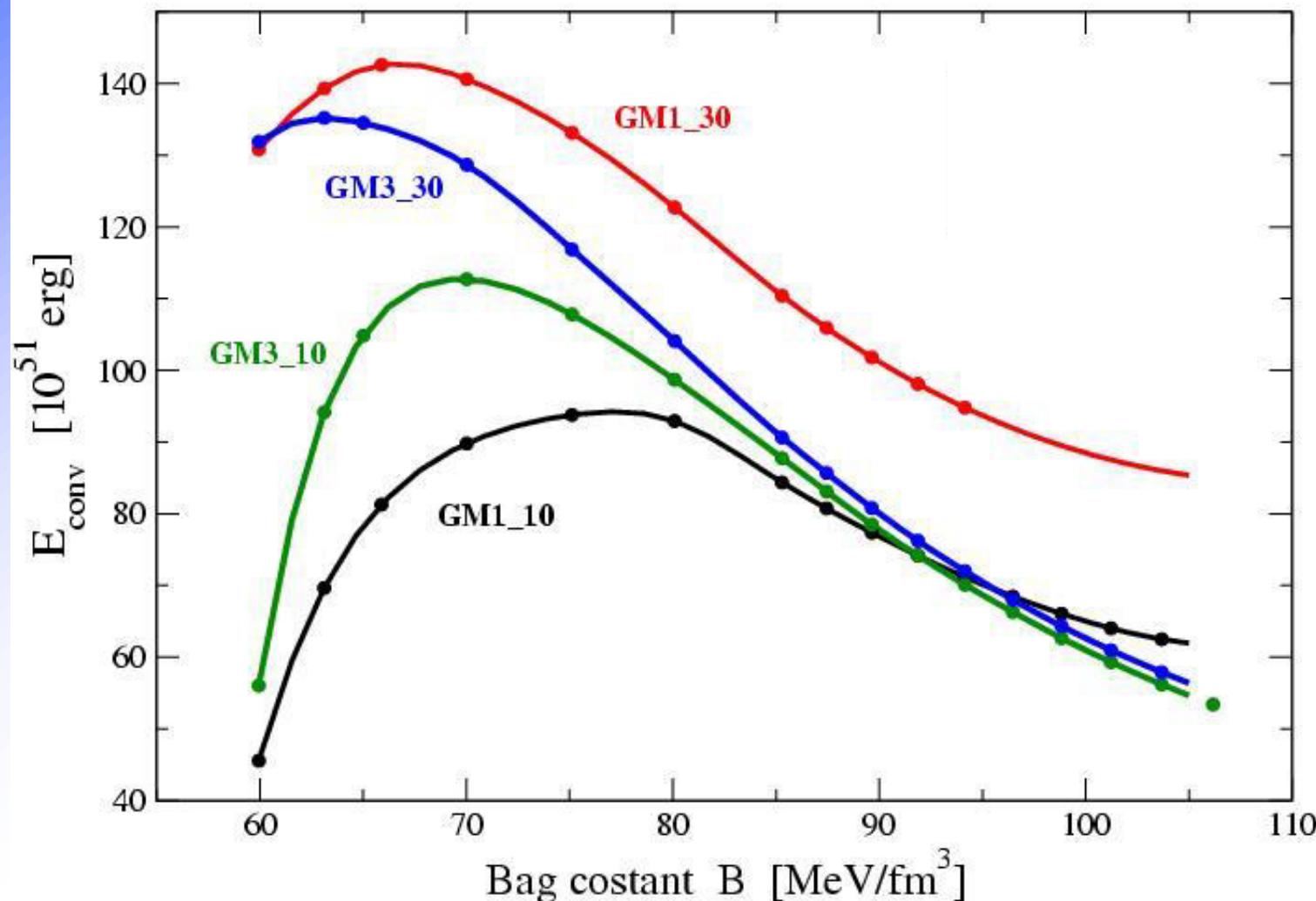


Assuming that the **stellar baryonic mass is conserved** during the stellar conversion, the total energy released in the process is :

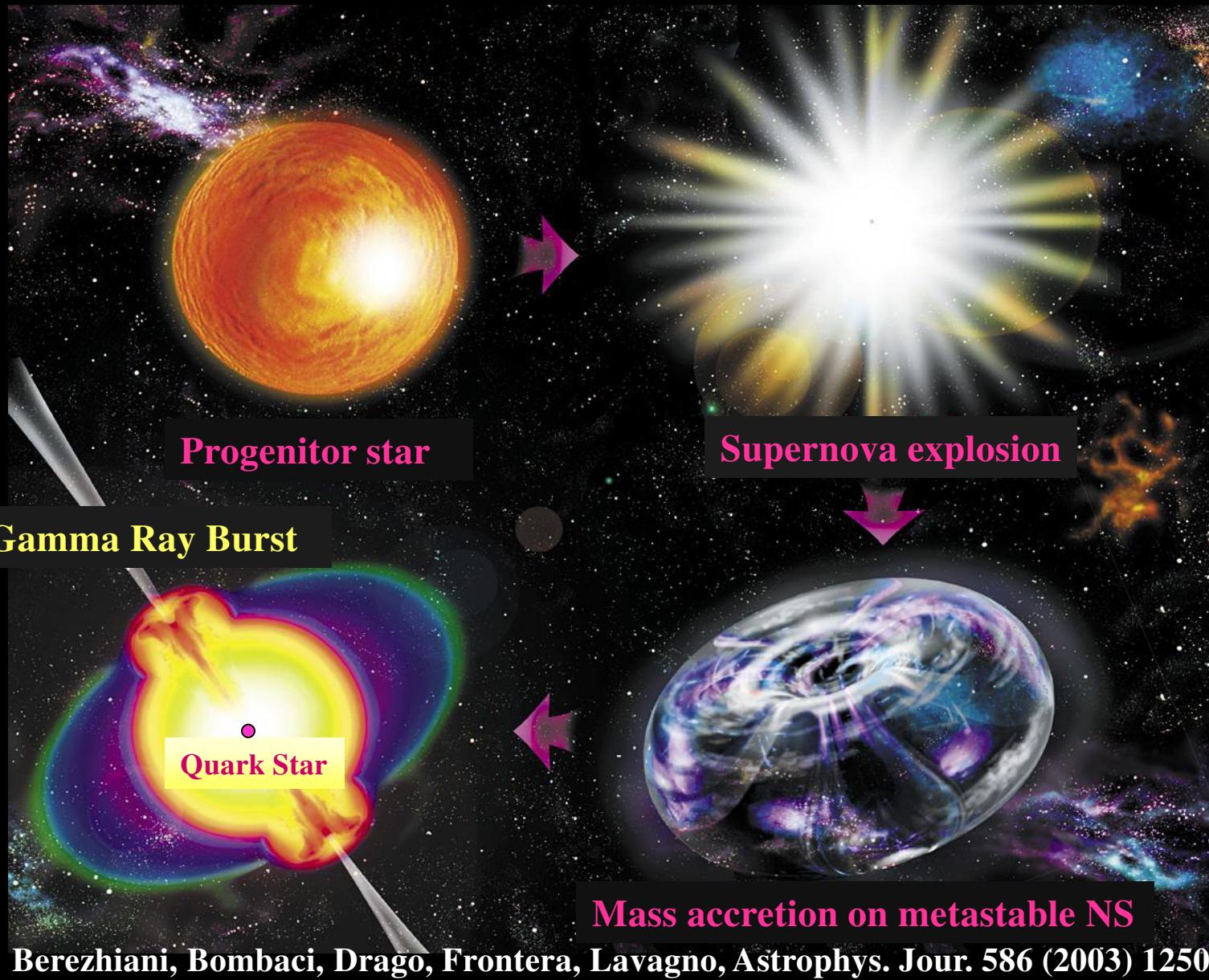
$$E_{\text{conv}} = M_{\text{cr}} - M_{\text{QS}}(M^b_{\text{cr}})$$

I. Bombaci, & Bhaskar Datta,
Astrophys. J. Lett. 530 (2000) L69

Total energy released in the stellar conversion



Supernova-GRB connection: Hadronic Star → Quark Star conversion model



Metastability of cold and deleptonized HS

**Quark deconfinement phase transition in
Cold ($T = 0$) and neutrino-free HS**

**Formation of the first drop of QM:
Quantum nucleation process**

- Z. Berezhiani, Bombaci, Drago, Frontera, Lavagno, *Astrophys. Jour.* 586 (2003) 1250
- I. Bombaci, I. Parenti, I. Vidaña, *Astrophys. Jour.* 614 (2004) 314
- I. Bombaci, G. Lugones, I. Vidaña, *Astron. & Astrophys.* 462 (2007) 1017
- I. Bombaci, P.K. Panda, C. Providencia, I. Vidaña, *Phys. Rev. D* 77 (2008) 083002

Proto-Neutron Stars (new born NS)

Hot lepton-rich PNS

$$2 \text{ s} < t < t_v \sim 10 - 20 \text{ s}$$

Trapped neutrinos

Hot matter

$t = 0$ (stellar core bounce),

$$T_c = 10 - 30 \text{ MeV}$$

t_v = neutrino trapping time

Hot neutrino-free PNS

$$t_v < t < t_{\text{cool}} \sim \text{a few } 10^2 \text{ s}$$

Neutrinos free matter

Hot matter (isoentropic core)

$$S/N_b \sim \text{const} = 1 - 2 k_B$$

Proto-Neutron Stars (new born NS)

Hot lepton-rich PNS

$$2 \text{ s} < t < t_v \sim 10 - 20 \text{ s}$$

Trapped neutrinos

Hot matter

$t = 0$ (stellar core bounce),

$$T_c = 10 - 30 \text{ MeV}$$

t_v = neutrino trapping time

Hot neutrino-free PNS

$$t_v < t < t_{\text{cool}} \sim \text{a few } 10^2 \text{ s}$$

Neutrinos free matter

Hot matter (isoentropic core)

$$S/N_b \sim \text{const} = 1 - 2 k_B$$



Cold deleptonized Neutron Stars

$t > t_{\text{cool}} \sim \text{a few } 10^2 \text{ s}$ cooling time

Neutrinos free matter

Cold matter ($T \ll 1 \text{ MeV}$)

Burrows, Lattimer, ApJ 307 (1986) 178; Prakash et al, Phys. Rep. 280 (1997) 1;
Pons et al. ApJ 513 (1999) 780

Proto-Neutron Stars (new born NS)

Hot lepton-rich PNS

$$2 \text{ s} < t < t_\nu \sim 10 - 20 \text{ s}$$

Trapped neutrinos

Hot matter

$t = 0$ (stellar core bounce),

$$T_c = 10 - 30 \text{ MeV}$$

t_ν = neutrino trapping time

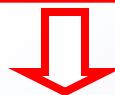
Hot neutrino-free PNS

$$t_\nu < t < t_{\text{cool}} \sim \text{a few } 10^2 \text{ s}$$

Neutrinos free matter

Hot matter (isoentropic core)

$$S/N_b \sim \text{const} = 1 - 2 k_B$$



In a **proto-neutron star** the **quark deconfinement phase transition** will be likely triggered by a **thermal nucleation process**

Thermal nucleation of QM in hot and dense hadronic matter

earlier studies:

J.E. Horvath, O.G. Benvenuto, H. Vucetich, Phys. Rev. D 45 (1992) 3865

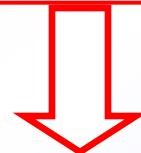
J.E. Horvath, Phys. Rev. D 49 (1994) 5590

M.L. Olesen, J. Madsen, Phys. Rev. D 49 (1994) 2698

H. Heiselberg, arXive:hep-ph/9501374

Thermal nucleation of a critical size drop of QM

for $T > 2 - 3 \text{ MeV}$



**hadronic stars converted to quark stars
within the first seconds after birth**

Based on an estimate of the thermal nucleation rate based on “typical” values of the central stellar properties (T_c , P_c ,)



To establish if a
new born hadronic star
(proto-hadronic star)
could survive the early
stages of its evolution
without “decaying” to a
quark star

Bombaci, Logoteta, Panda, Providencia, Vidaña, Phys. Lett. B 680 (2009) 448
Bombaci, Logoteta, Providencia, Vidaña, Astr. and Astrophys. 528 (2011) A71
Logoteta, Bombaci, Providencia, Vidaña, Phys. Rev. D 85 (2012) 023003

Phase equilibrium

Gibbs' criterion
for phase equilibrium

$$\mu_H = \mu_Q \equiv \mu_0$$

$$T_H = T_Q \equiv T$$

$$P(\mu_H) = P(\mu_Q) \equiv P(\mu_0) \equiv P_0$$

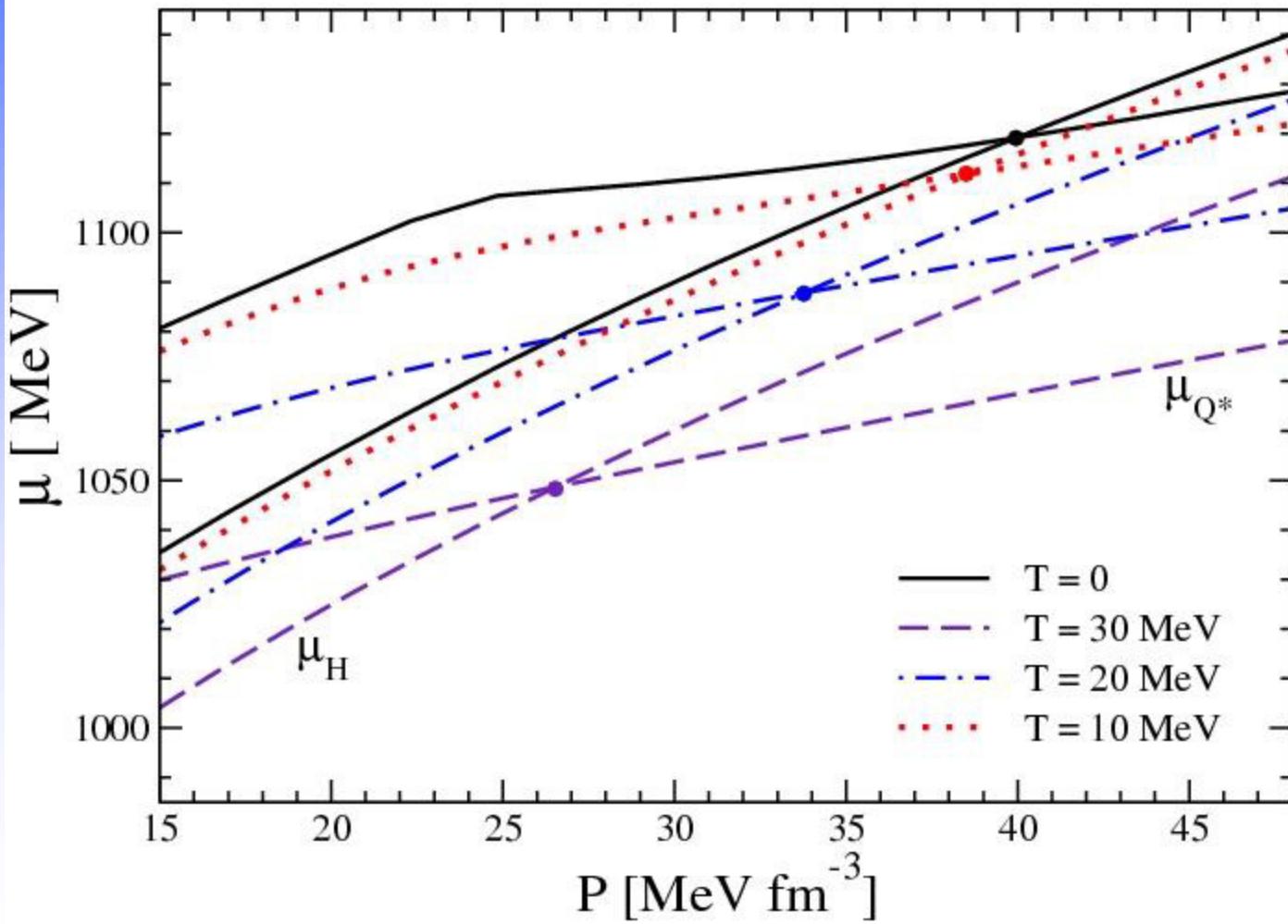
μ_j = Gibbs' energy per baryon (j-phase average chemical pot.) $j = H, Q$

$$\mu_H = \frac{\varepsilon_H + P_H - s_H T}{n_{b,H}}$$

$$\mu_Q = \frac{\varepsilon_Q + P_Q - s_Q T}{n_{b,Q}}$$

ε_j = energy density, P_j = pressure, s_j = entropy density (including leptonic contributions)

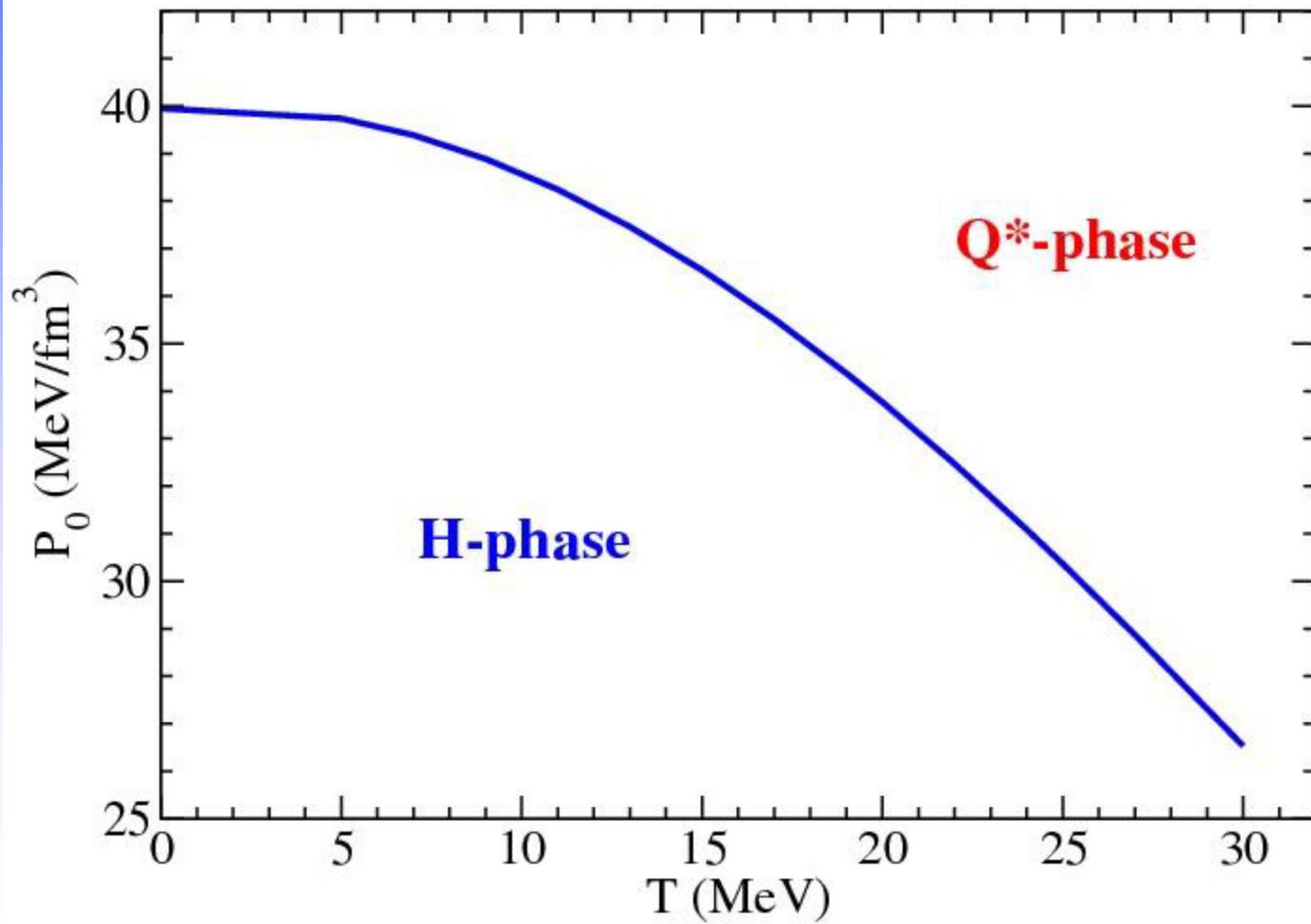
Gibbs' energies per baryon



ν -free matter

GM1 + MIT bag model ($B=85$ MeV/fm 3 , $m_s=150$ MeV)

Phase equilibrium curve



ν -free matter

GM1 + MIT bag model ($B=85$ MeV/fm 3 , $m_s=150$ MeV)

Specific latent heat Q

T (MeV)	Q (MeV)	n_{Q^*} (fm $^{-3}$)	n_H (fm $^{-3}$)	P_0 (MeV)
0	0.0	0.453	0.366	39.95
5	0.56	0.451	0.364	39.74
10	2.40	0.447	0.358	38.58
15	5.71	0.439	0.348	36.55
20	10.60	0.428	0.334	33.77
25	17.17	0.414	0.316	30.36
30	25.44	0.398	0.294	26.53

GM1 + MIT bag model (B=85 MeV/fm 3 , m_s=150 MeV)

ν -free matter

$$Q = \tilde{W}_{Q^*} - \tilde{W}_H = T(\tilde{S}_{Q^*} - \tilde{S}_H)$$

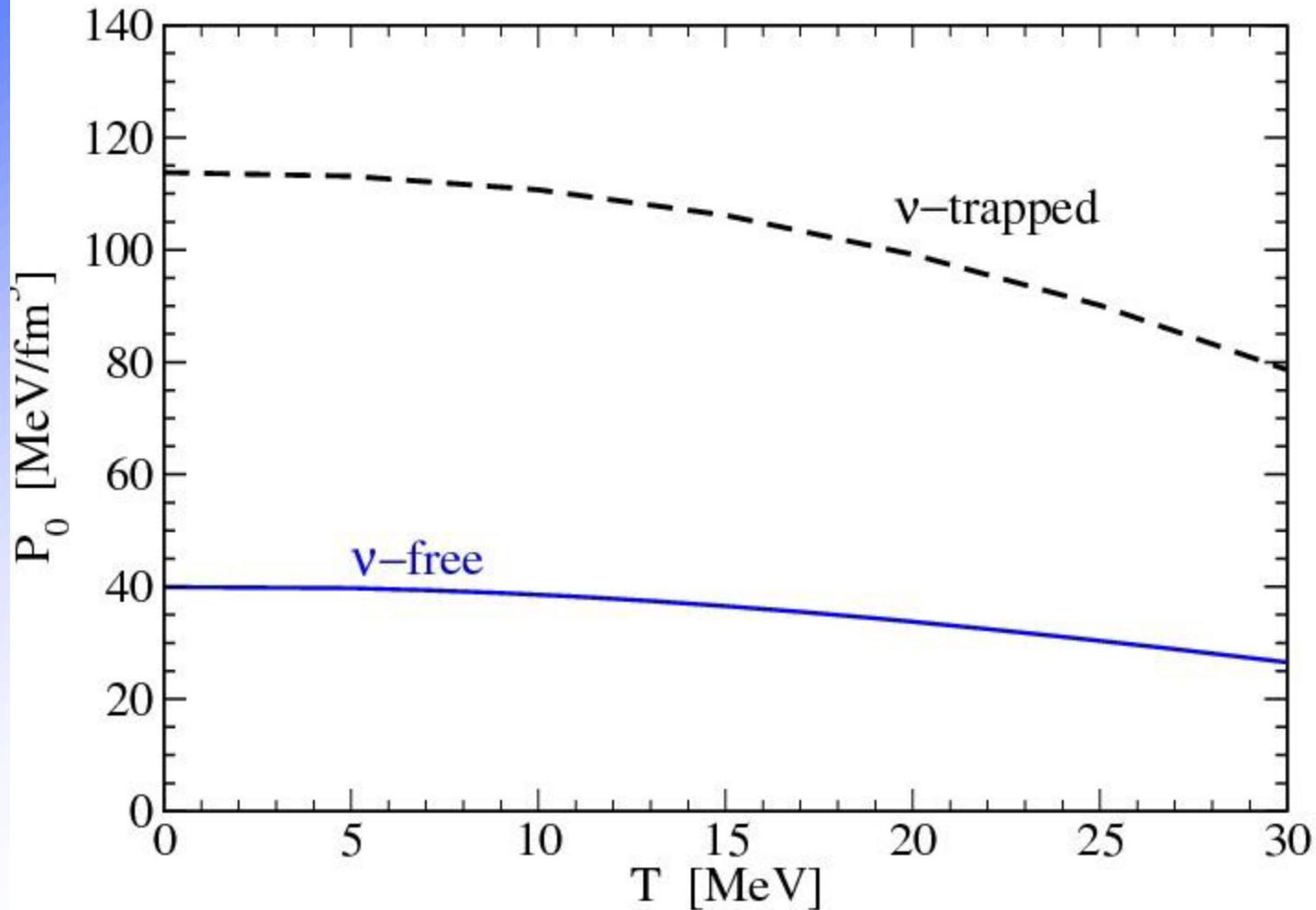
\tilde{W}_{Q^*} , \tilde{W}_H enthalpy per baryon

\tilde{S}_{Q^*} , \tilde{S}_H entropy per baryon

Clapeyron – Clausius equation

$$\frac{dP_0}{dT} = -\frac{n_H n_{Q^*}}{n_{Q^*} - n_H} \frac{Q}{T}$$

Phase equilibrium curve

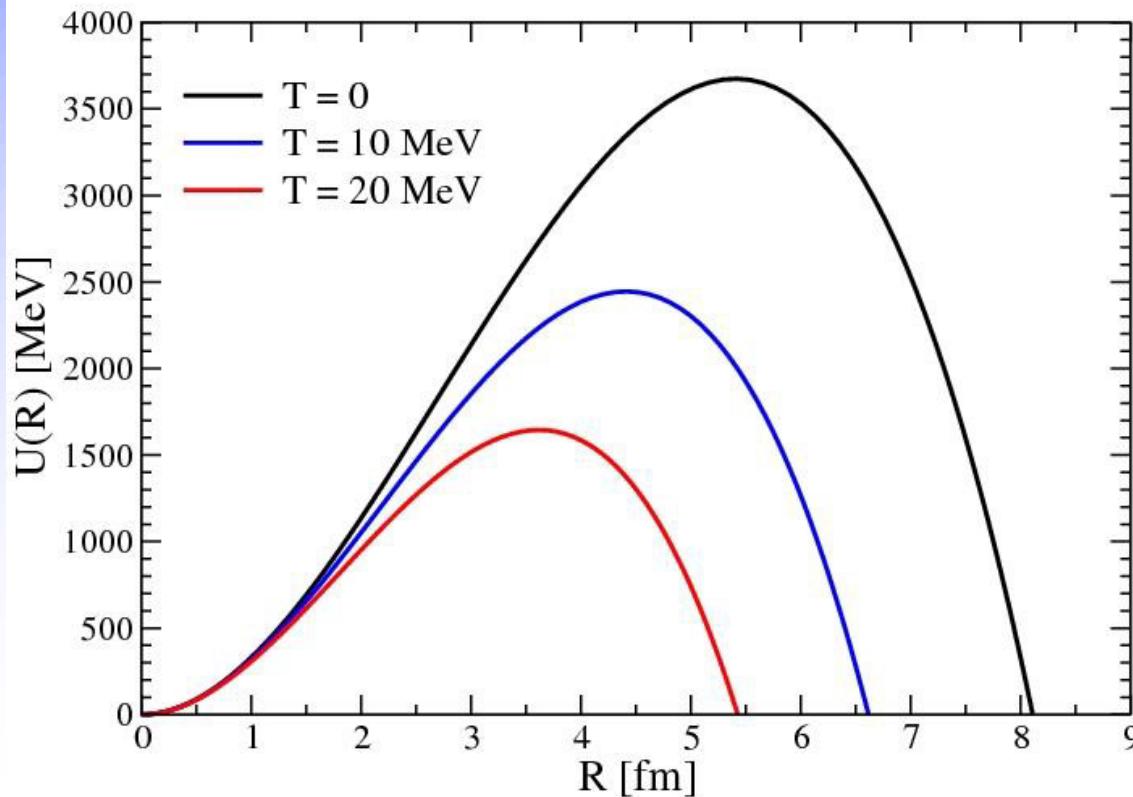


GM1 + MIT bag model ($B=85$ MeV/fm 3 , $m_s=150$ MeV)

Potential energy barrier between the H-phase and the Q*-phase

$$\begin{aligned} U(R, P, T) &= (4/3)\pi n_{Q^*} (\mu_{Q^*} - \mu_H) R^3 + 4\pi\sigma R^2 \\ &= a_v(P, T) R^3 + a_s R^2 \end{aligned}$$

We assume
 σ temp.
independent



$$P = 57 \text{ MeV/fm}^3$$

GM1 + MIT bag model
($B=85 \text{ MeV/fm}^3$,
 $m_s=150 \text{ MeV}$)

$$\sigma = 30 \text{ MeV/fm}^2$$

Thermal nucleation theory

J.S. Langer, Phys. Rev. Lett. 21 (1968) 973; Ann. Phys. (N.Y.) 54 (1969) 258

J.S. Langer, L.A. Turski, Phys. Rev. A8 (1973) 3230

L.A.

Turski, J.S. Langer, Phys. Rev. A22 (1980) 2189

Thermal
nucleation rate

$$I = \frac{\kappa}{2\pi} \Omega_0 \exp\left(-\frac{U(R_c, T)}{T}\right)$$

$\kappa \equiv$ dynamical prefactor (related to the growth rate of the drop radius R near the critical radius R_c)

$\Omega_0 \equiv$ statistical prefactor (measures the phase-space volume of the saddle point region around R_c)

$U(R_c, T) \equiv$ activation energy

Statistical prefactor

L. Csernai, J.I. Kapusta,
Phys. Rev. D 46 (1992)

$$\Omega_0 = \frac{2}{3\sqrt{3}} \left(\frac{\sigma}{T} \right)^{3/2} \left(\frac{R}{\xi_Q} \right)^4$$

ξ_Q = quark correlation length

$\xi_Q = 0.7$ fm (CK1992)

dynamical prefactor

$$\kappa = \frac{2\sigma}{R_C^3 (\Delta w)^2} \left[\lambda T + 2 \left(\frac{4}{3} \eta + \zeta \right) \right]$$

R. Venugopalan, A.P. Vischer, Phys. Rev. E 49 (1994) 5849

L. Csernai, J.I. Kapusta, E. Osnes, Phys. Rev. D 67 (2003) 045003

$\Delta w = w_{Q^*} - w_H$ enthalpy density difference

λ = thermal conductivity, η = shear viscosity, ζ = bulk viscosity

According to P. Danielewicz, Phys. Lett. B 146 (1984) 168, we assume

$$\lambda = 0, \quad \zeta = 0$$

$$\eta = \frac{7.6 \times 10^{26}}{(T/\text{MeV})^2} \left(\frac{n_H}{n_0} \right)^2 \frac{\text{MeV}}{\text{fm}^2 \text{ s}}$$

$$n_0 = 0.16 \text{ fm}^{-3}$$

**Thermal
nucleation time**

$$\tau_{\text{th}} = (V_{\text{nuc}} I)^{-1}$$

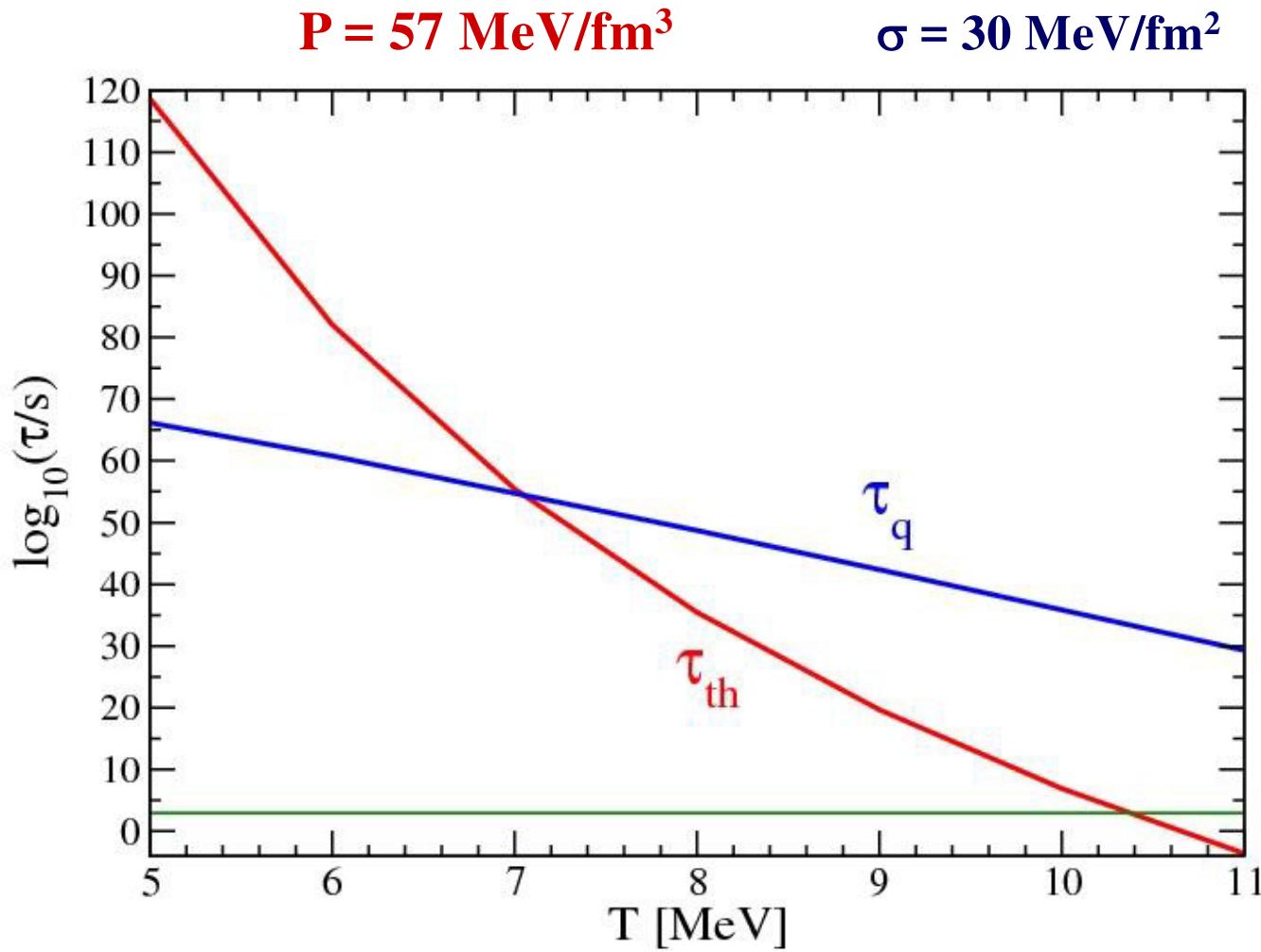
$$V_{\text{nuc}} = \frac{4}{3}\pi R_{\text{nuc}}^3$$

volume of the innermost stellar region
where $P(r) \cong P(r=0)$. $R_{\text{nuc}} \sim 100 \text{ m}$

Recent papers on transport
properties of neutron matter
and beta-stable NM:

- O. Benhar, M. Valli, Phys. Rev. Lett. 99 (2007) 232501
- O. Benhar, A. Polls, M. Valli, I. Vidaña, Phys. Rev. C 81 (2010) 024305
- O. Benhar, A. Carbone, arXiv:0912.0129
- H.F. Zhang, U. Lombardo, W. Zuo, Phys. Rev. C 82 (2010) 015805

Thermal and quantum nucleation time



GM1 + MIT bag model ($B=85 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

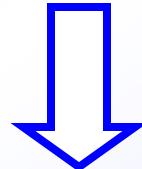
proto-Hadronic Star

Hot neutrino-free PHS

$$t_{\nu} < t < t_{\text{cool}} \sim \text{a few } 10^2 \text{ s}$$

$$S/N_b \sim \text{const} = 1 - 2 k_B$$

$$\tau = \min(\tau_q, \tau_{\text{th}}) \gg t_{\text{cool}}$$

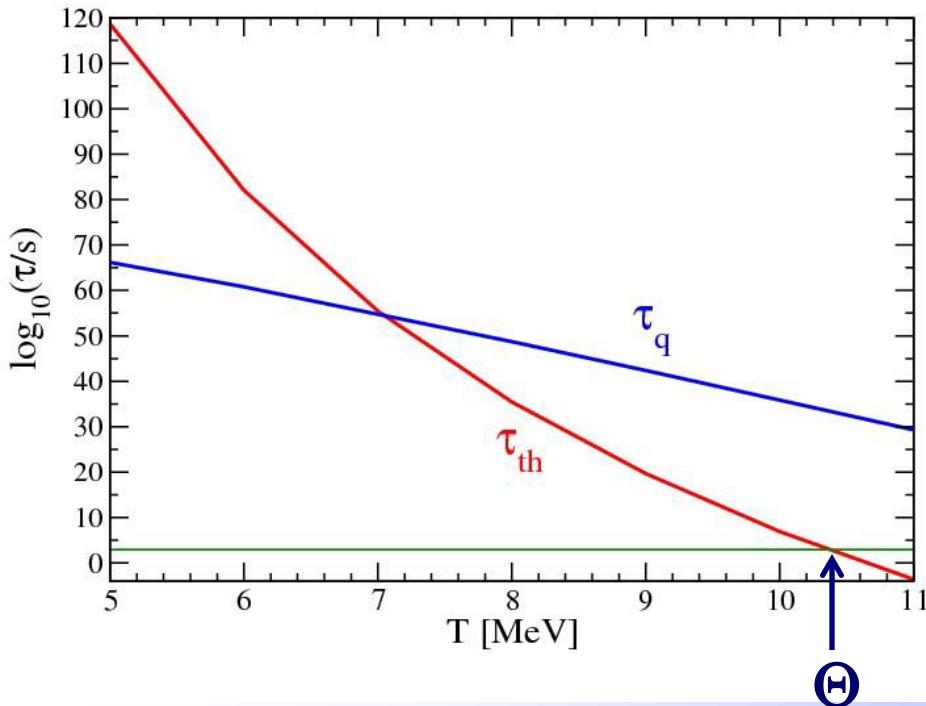


**Q*-matter nucleation will not likely occur in the PHS.
This star will evolve to a cold Hadronic Star**

Limiting conversion temperature Θ for the proto-Hadronic Star

$$\Theta \equiv T_c (\tau = 10^3 \text{ s})$$

$P = 57 \text{ MeV/fm}^3$ $\sigma = 30 \text{ MeV/fm}^2$



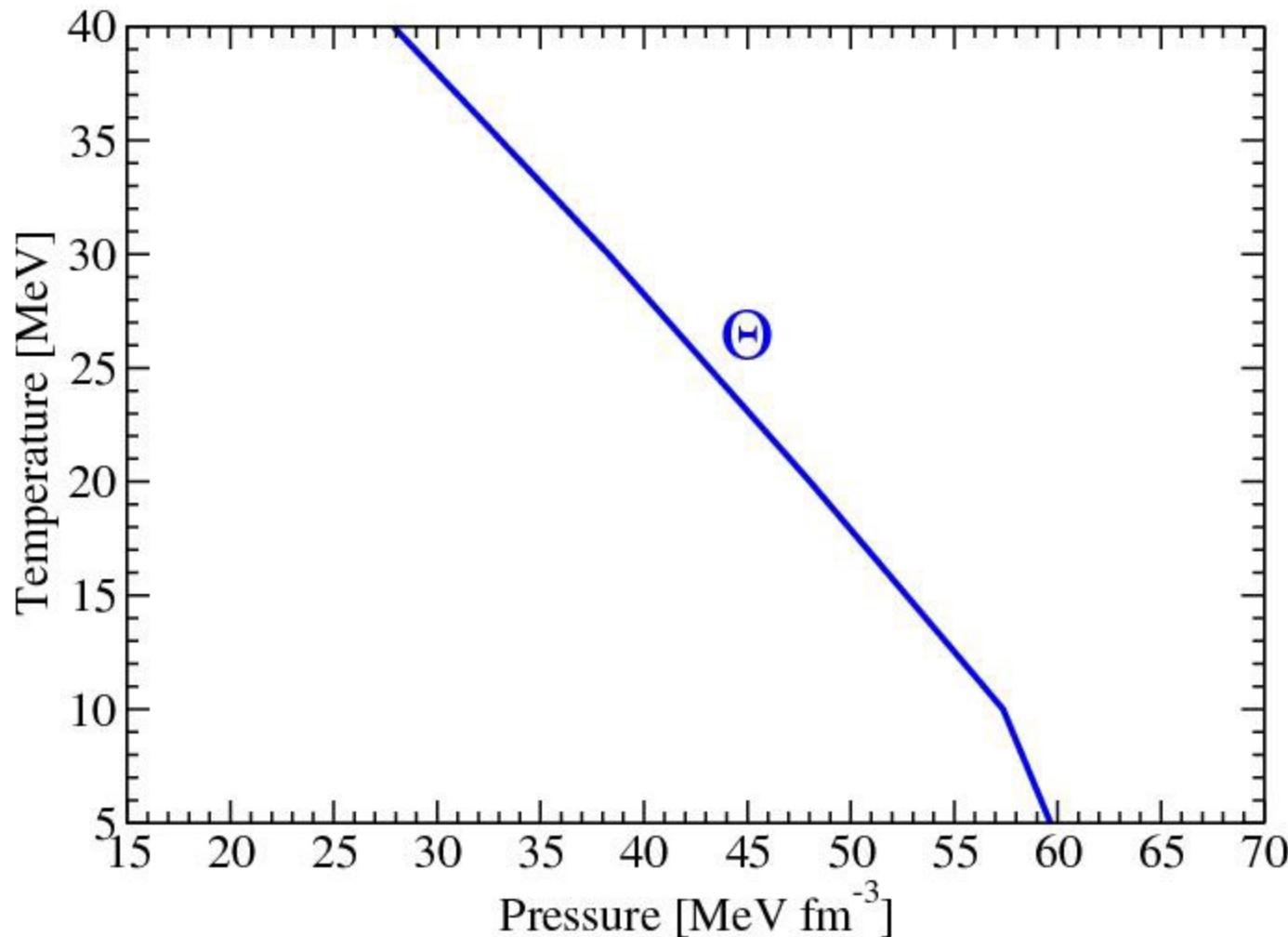
$$\Theta = \Theta(P_c)$$

↓

$\Theta = \Theta(M_G)$

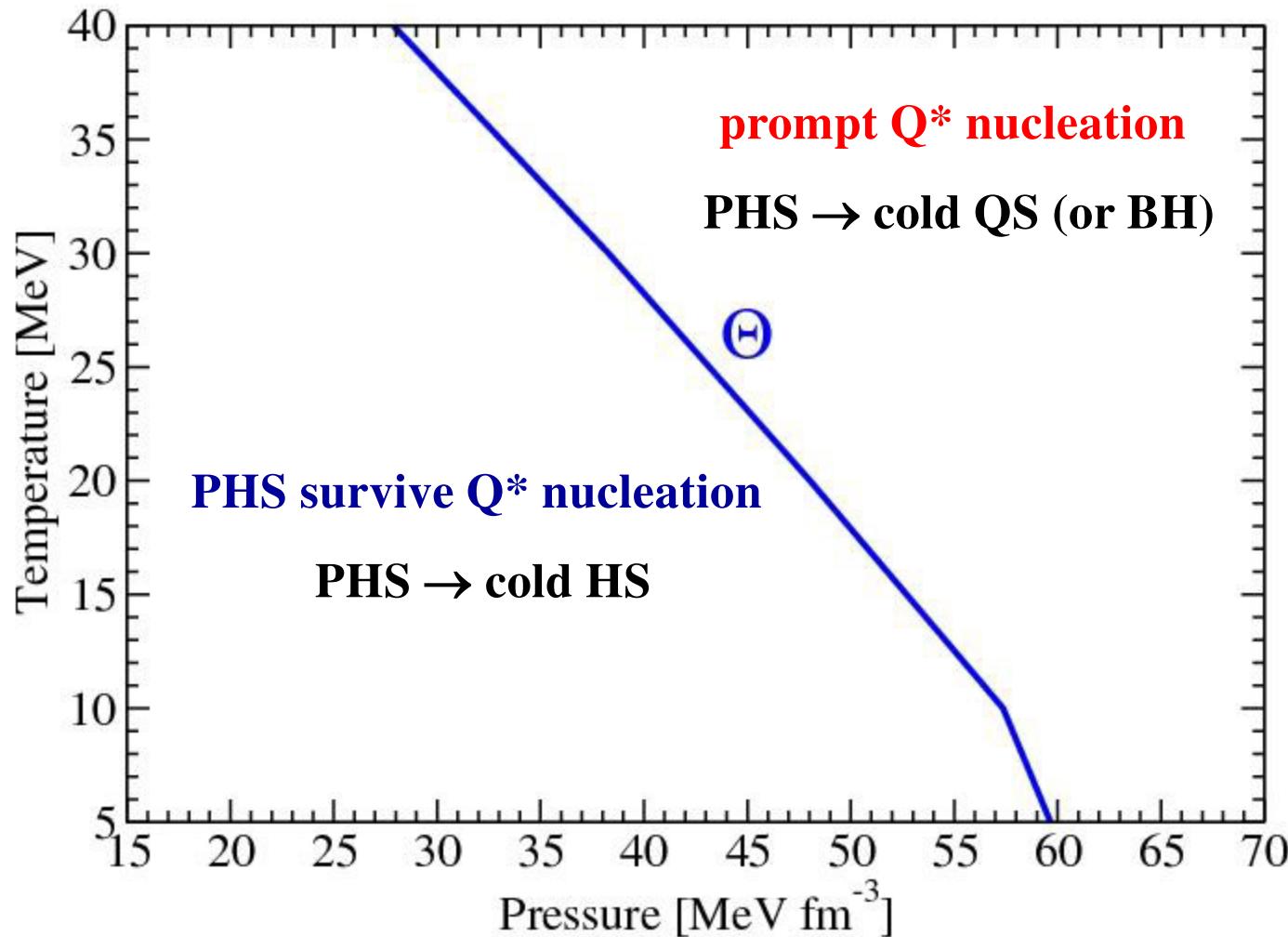
TOV eq.

The limiting conversion temperature of proto-HS



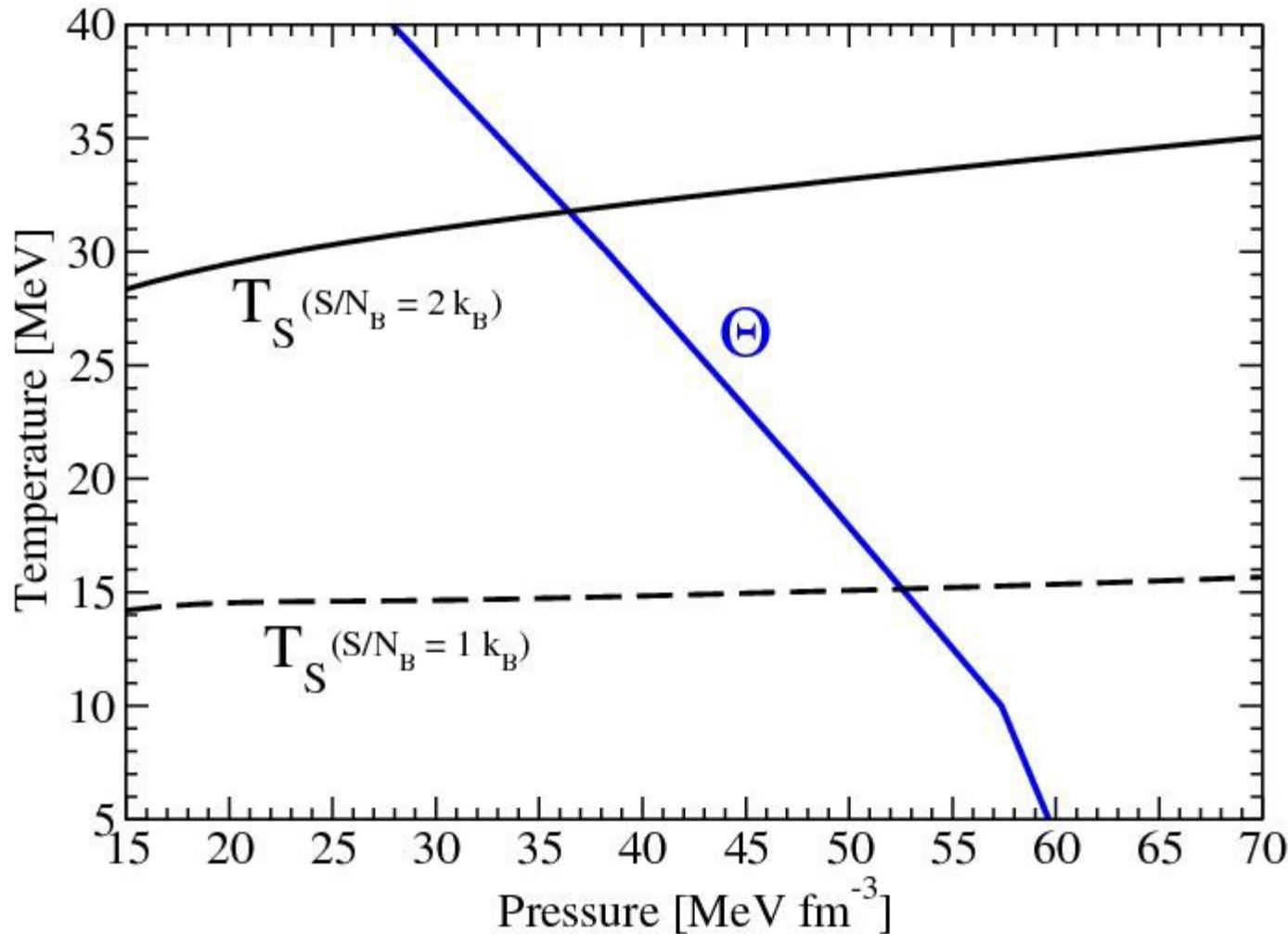
GM1 + MIT bag model ($B=85$ MeV/fm³, $m_s=150$ MeV)

The limiting conversion temperature of proto-HS



GM1 + MIT bag model ($B=85$ MeV/fm³, $m_s=150$ MeV)

The limiting conversion temperature of proto-HS



GM1 + MIT bag model ($B=85$ MeV/fm³, $m_s=150$ MeV)

Critical mass configuration for the proto-Hadronic stellar sequence

$$M_{cr} = M(P_S, \Theta_S)$$

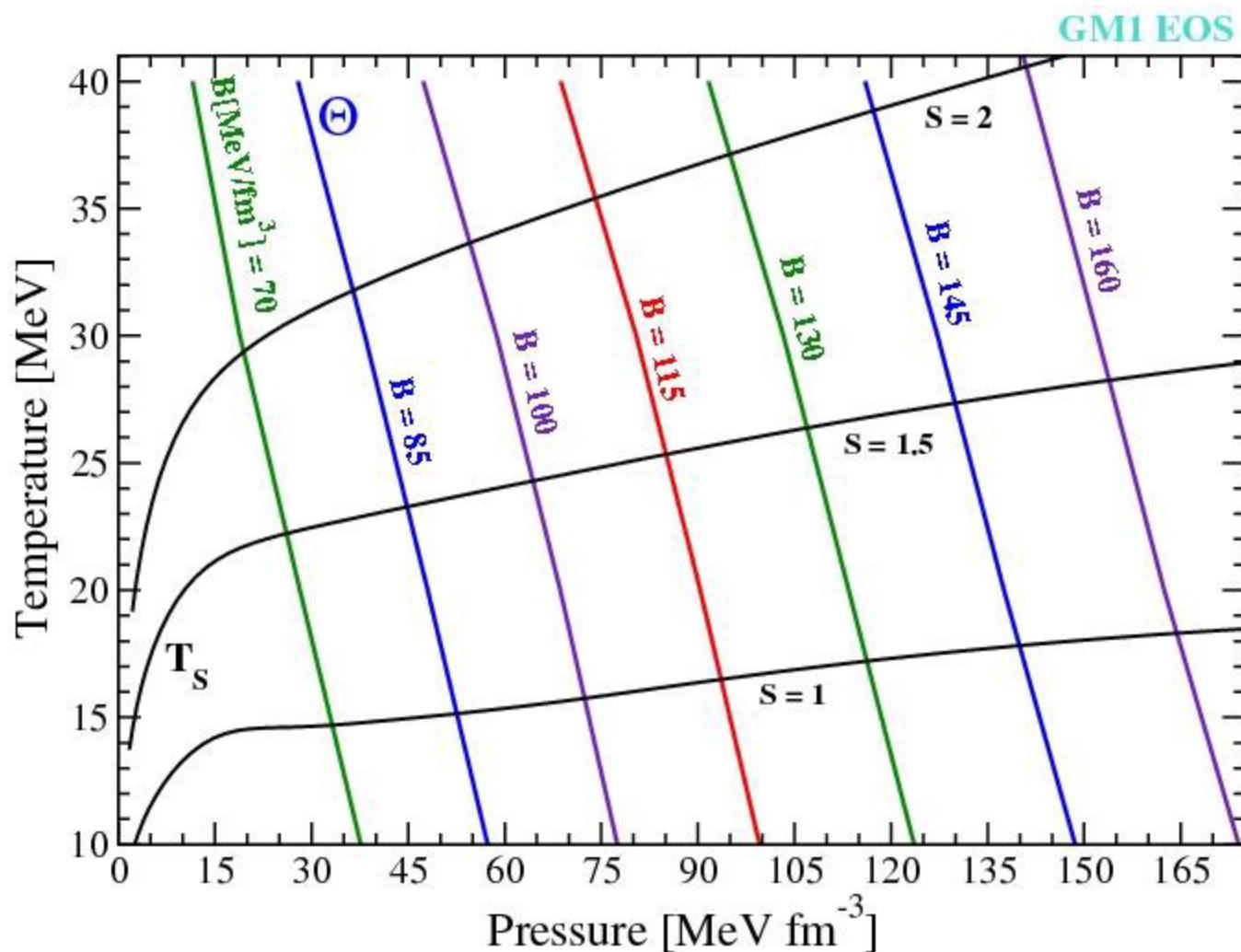
Gravitational mass

$$M_{B,cr} = M(P_S, \Theta_S)$$

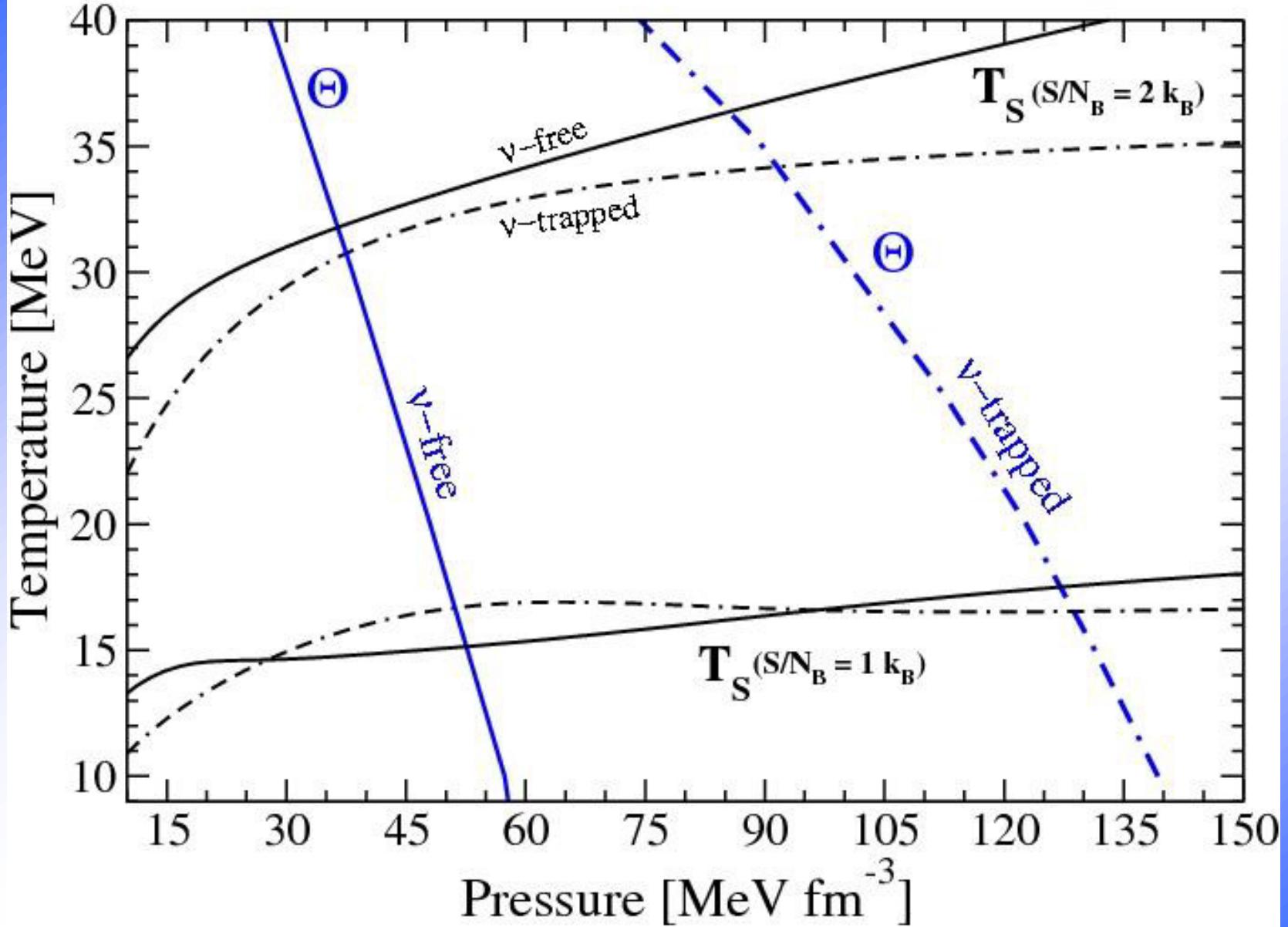
Baryonic mass

S/N_B (k_B)	M_{cr} (M_\odot)	$M_{B,cr}$ (M_\odot)	\mathcal{M} (M_\odot)
0.0	1.573	1.752	1.573
1.0	1.494	1.643	1.485
2.0	1.390	1.492	1.361

\mathcal{M} = grav. mass
of the **cold HS**
with the same
baryonic mass

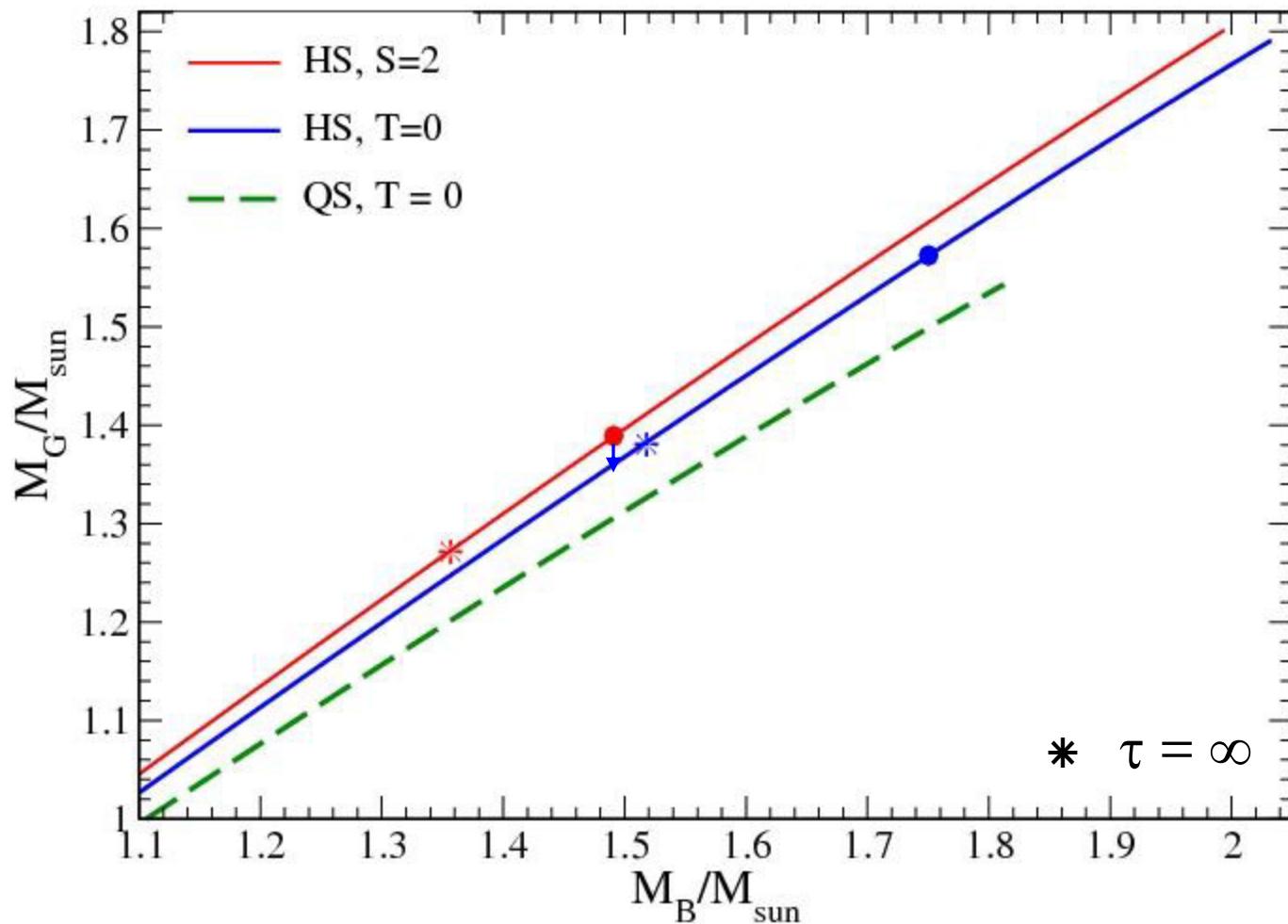


The limiting conversion temperature of proto-HS



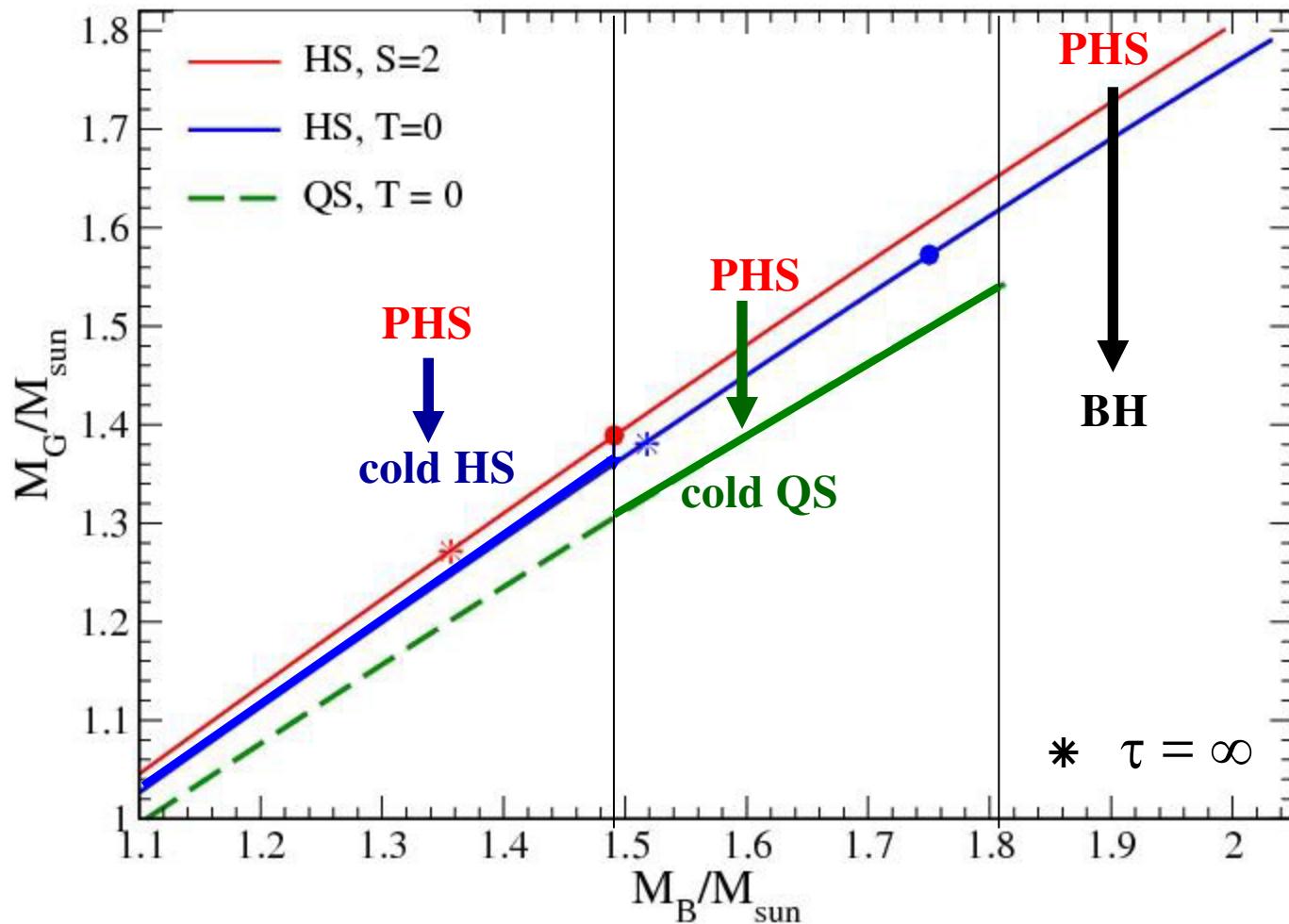
GM1 + MIT bag model ($B=85$ MeV/fm³, $m_s=150$ MeV)

Evolution of a proto-HS



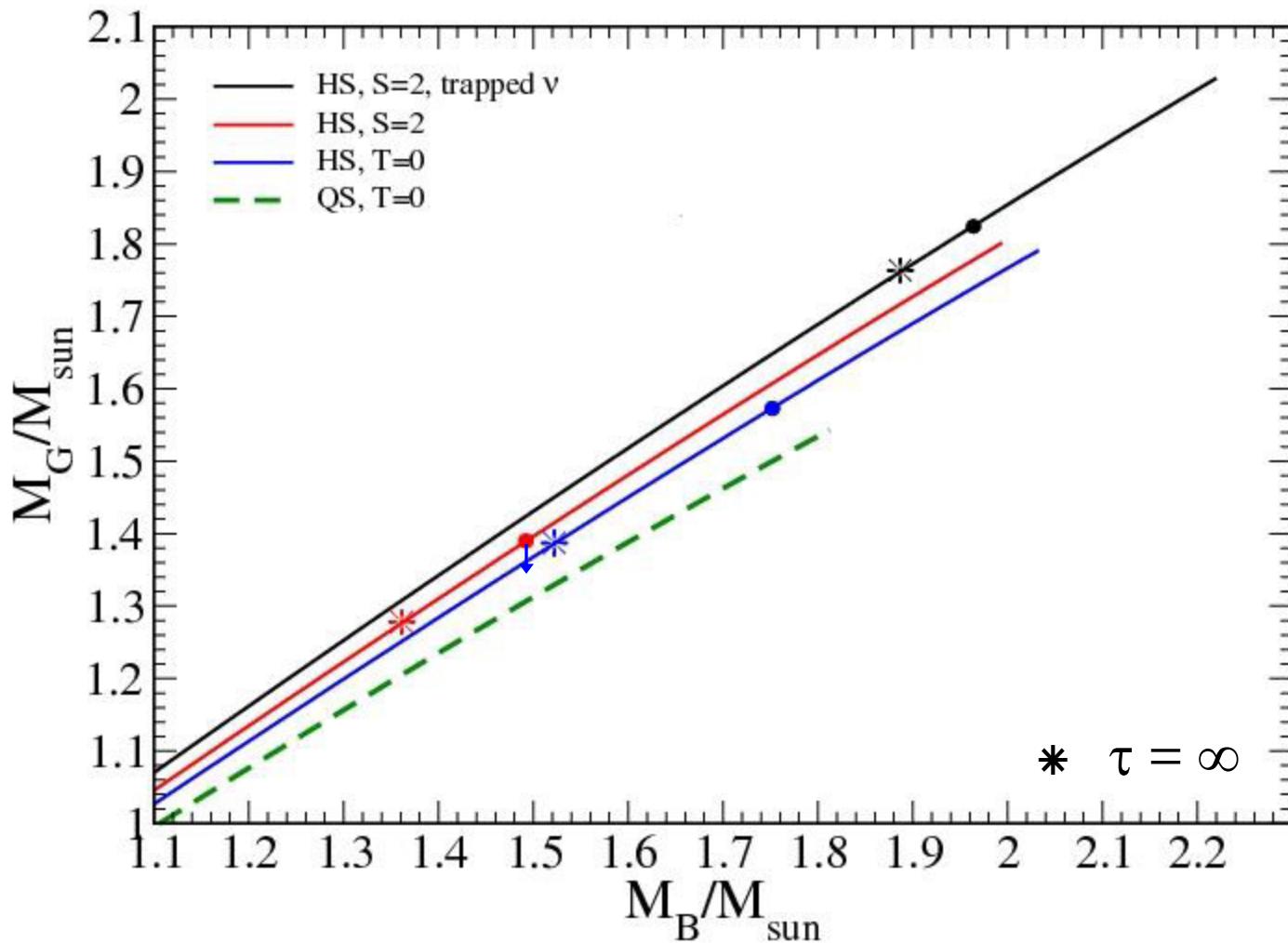
GM1 + MIT bag model ($B=85 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

Evolution of a proto-HS



GM1 + MIT bag model ($B=85 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

Evolution of a proto-HS: effect of ν -trapping



GM1 + MIT bag model ($B=85$ MeV/fm 3 , $m_s=150$ MeV)

Conclusions

A transition to a **quark-deconfined phase** of dense matter is expected in “massive” Neutron Stars.

If this phase transition is of the first order then:

“massive” **Hadronic Stars** are **metastable**
to the conversion to **Quark Stars** (hybrid or strange stars)

There exist in nature **two different branches of compact stars**

stellar conversion HS \rightarrow QS
 $E_{\text{conv}} \sim 10^{53}$ erg (possible energy source for some GRBs)

**extension of the concept of limiting mass of compact stars
with respect to the *classical* one given by
Oppenheimer and Volkoff**

We have studied **quark deconfinement phase transition** in
hot β -stable ν -free and **ν -trapped hadronic matter**

We have introduced the new concepts of
limiting conversion temperature Θ and **critical mass M_{cr}**
for proto-hadronic stars

Proto-Hadronic Stars with **$M < M_{cr}$** could survive the early stages of their evolution without decaying to a Quark Star, thus evolving to a **cold Hadronic Star**

A prompt formation of a critical size drop of QM take place when **$M > M_{cr}$** . These proto-Hadronic Stars evolve to **cold Quark Stars** or collapse to a **Black Hole**