COLD ASYMMETRIC QUARK MATTER

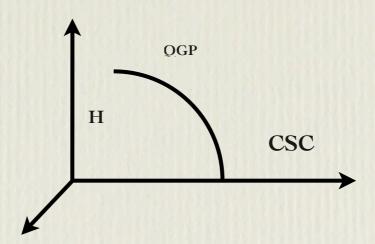
Massimo Mannarelli INFN-LNGS

massimo@lngs.infn.it

A. Mammarella and M.M. Phys.Rev. D92 (2015) 8, 085025 S. Carignano, A. Mammarella and M.M. Phys.Rev. D93 (2016) no.5, 051503

OUTLINE

General motivations

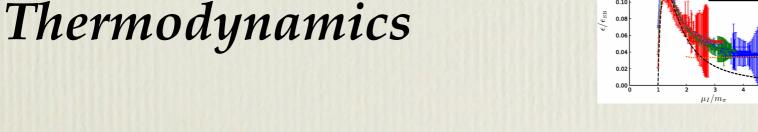


Chiral perturbation theory

$$\mathcal{L} = \frac{F_0^2}{4} \text{Tr}(D_{\nu} \Sigma D^{\nu} \Sigma^{\dagger}) + \frac{F_0^2}{4} \text{Tr}(X \Sigma^{\dagger} + \Sigma X^{\dagger})$$

Condensed phases

Thermodynamics



Conclusions

GENERAL MOTIVATIONS

- Heavy nuclei have more neutrons than protons
- Neutron stars have much more neutrons than protons
- At sufficiently large densities, hyperons should appear

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Goal: describing nuclear matter in isospin/strangeness rich environment

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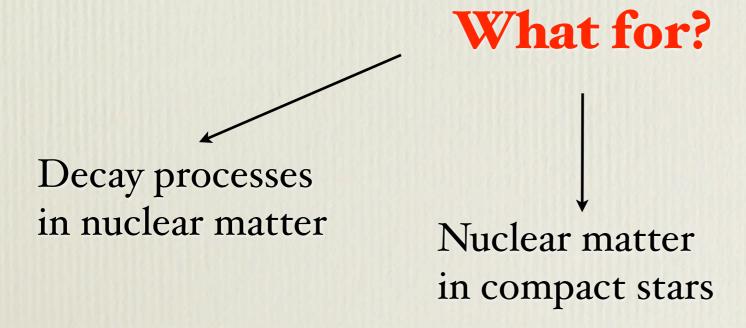
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Decay processes in nuclear matter

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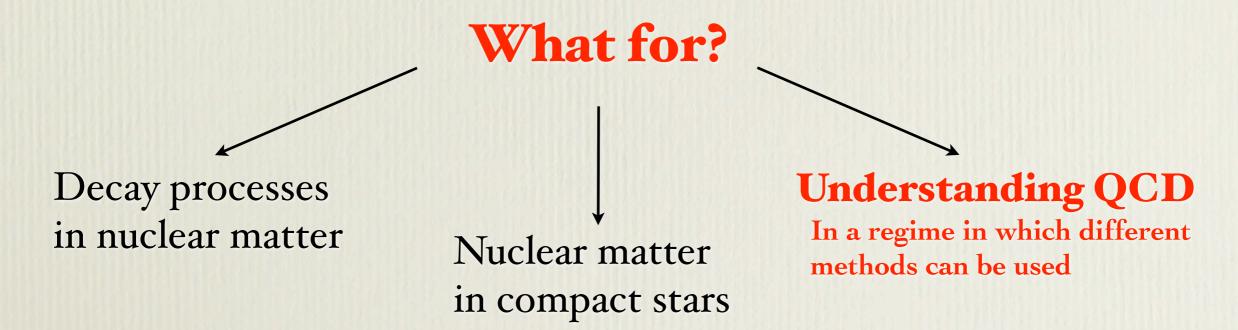
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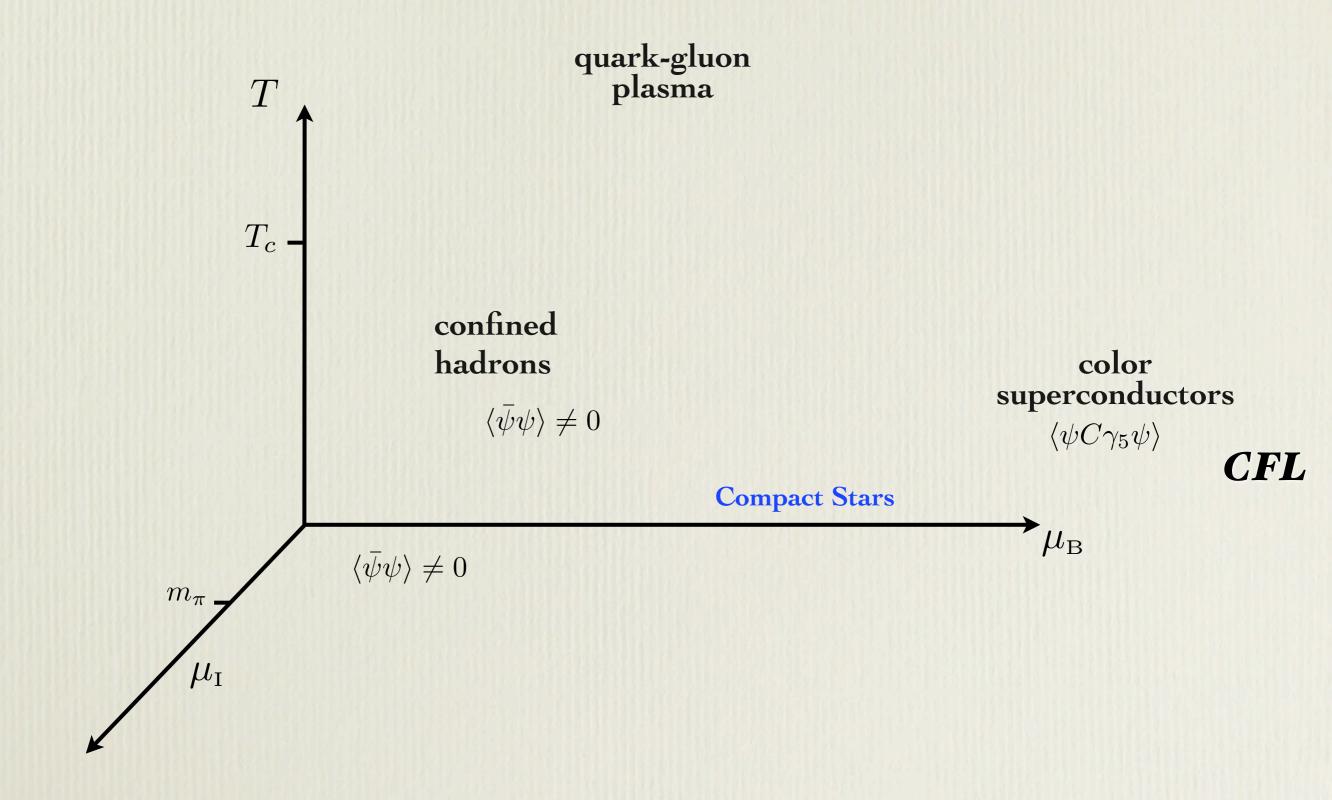


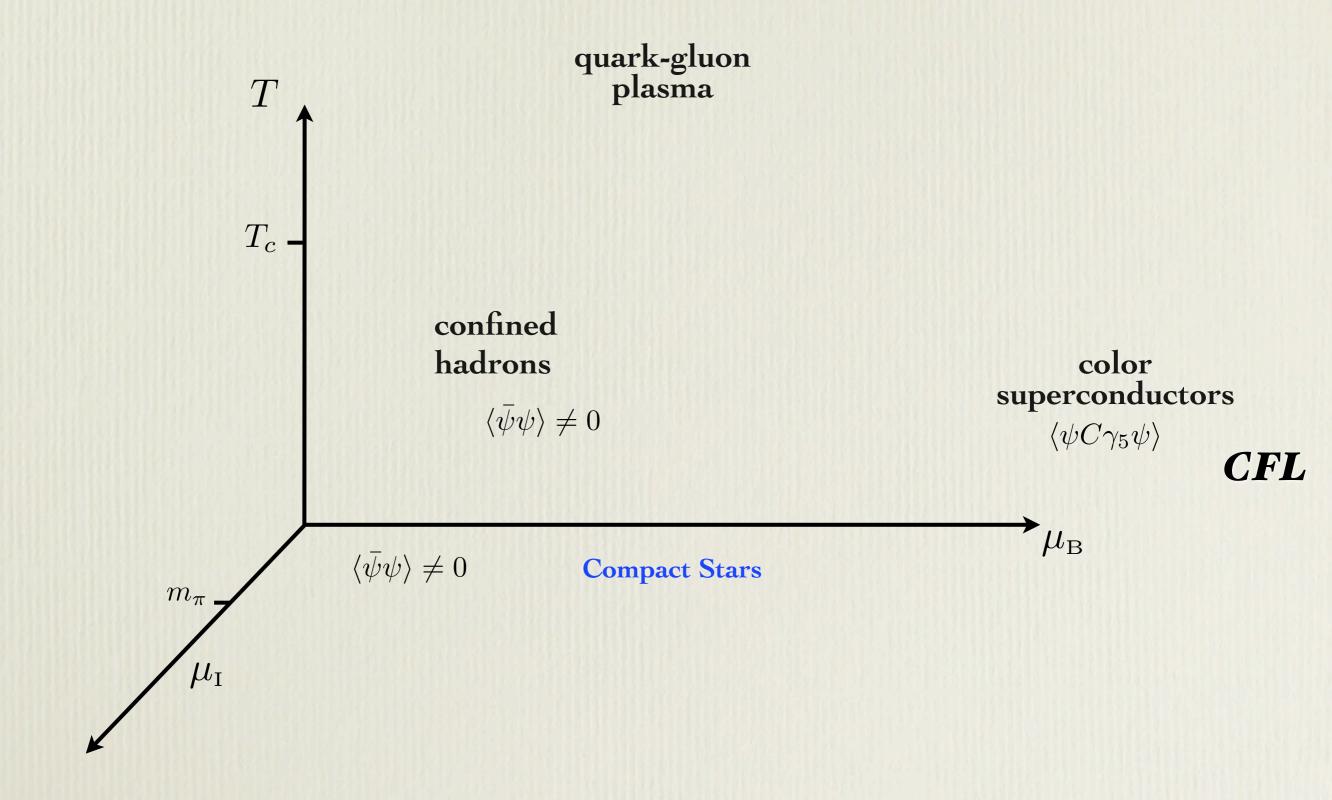
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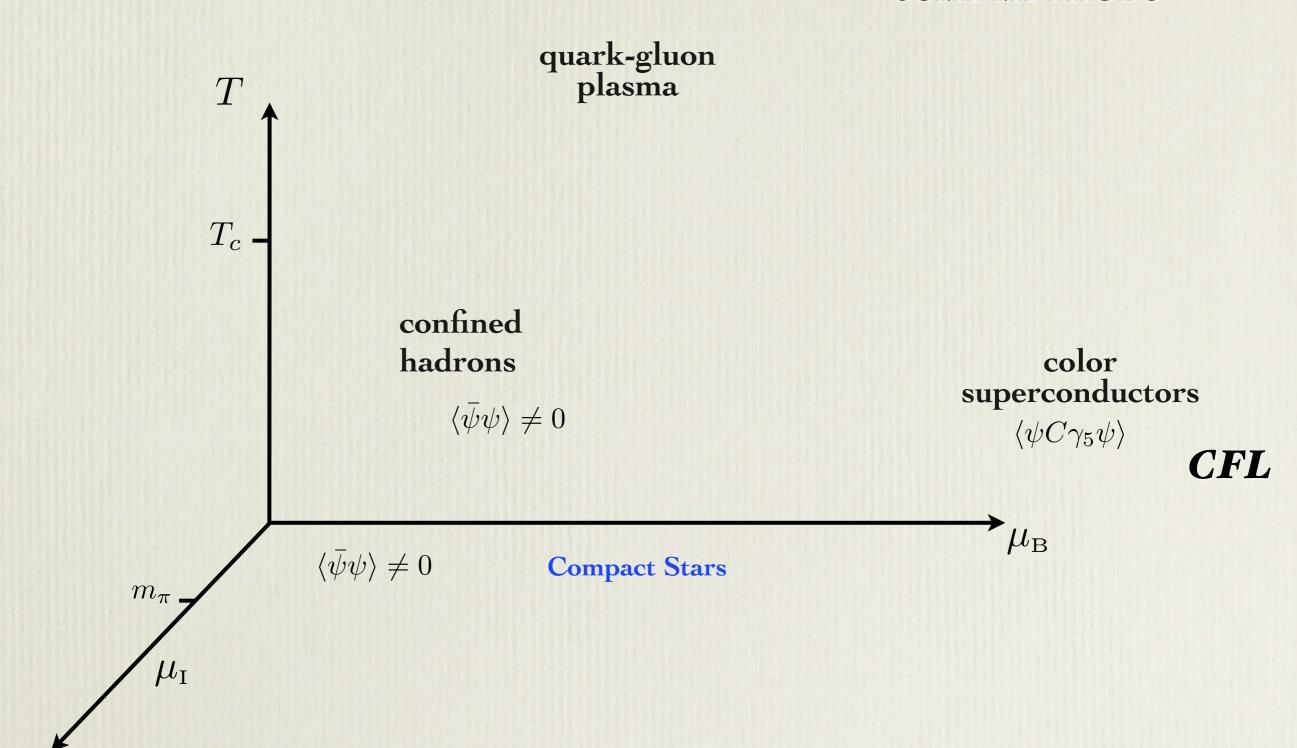
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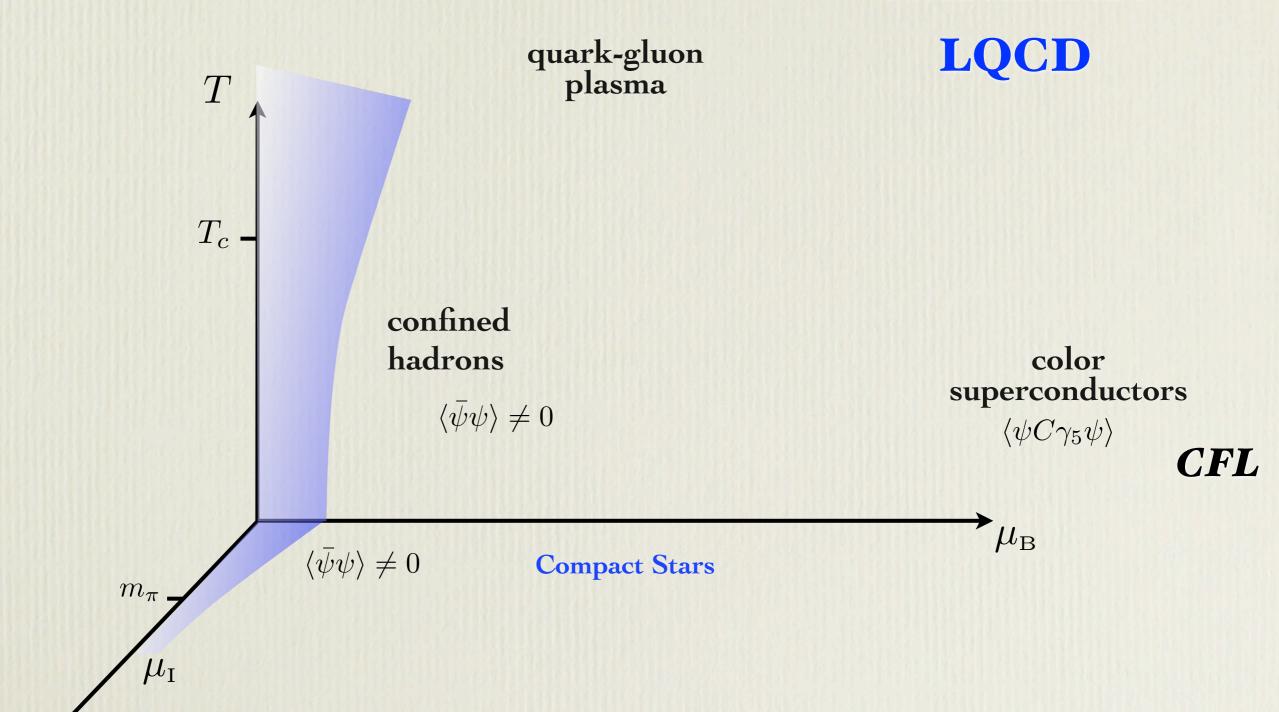


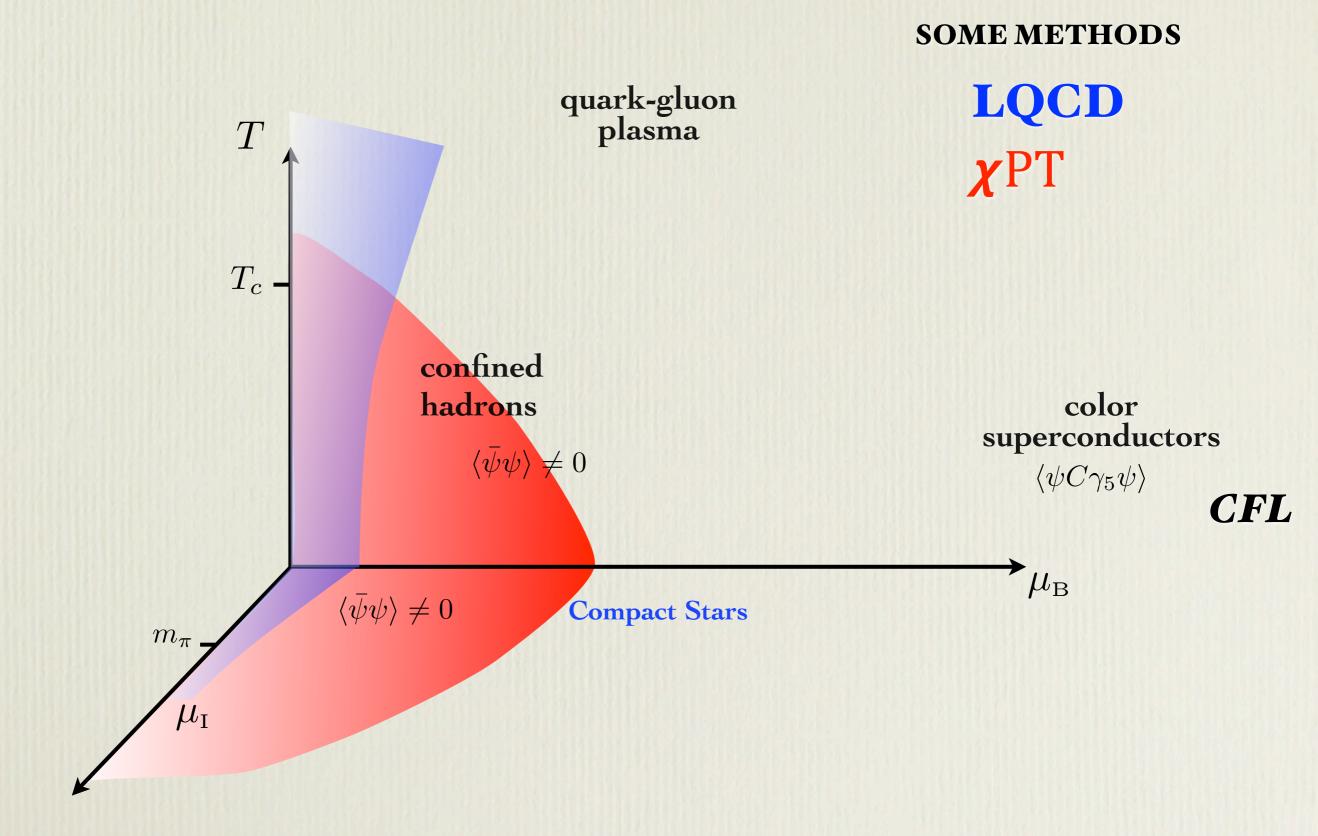


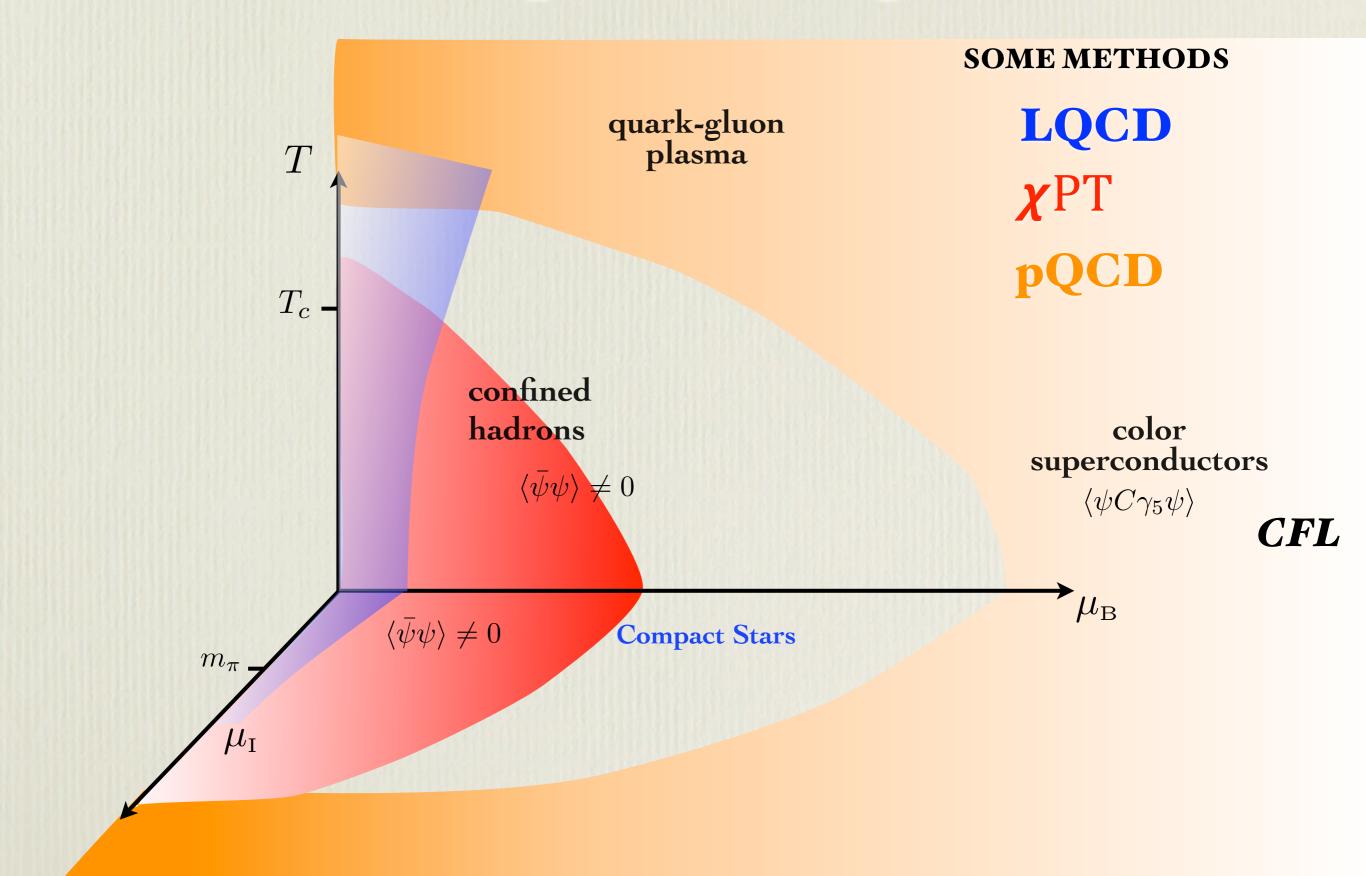
SOME METHODS

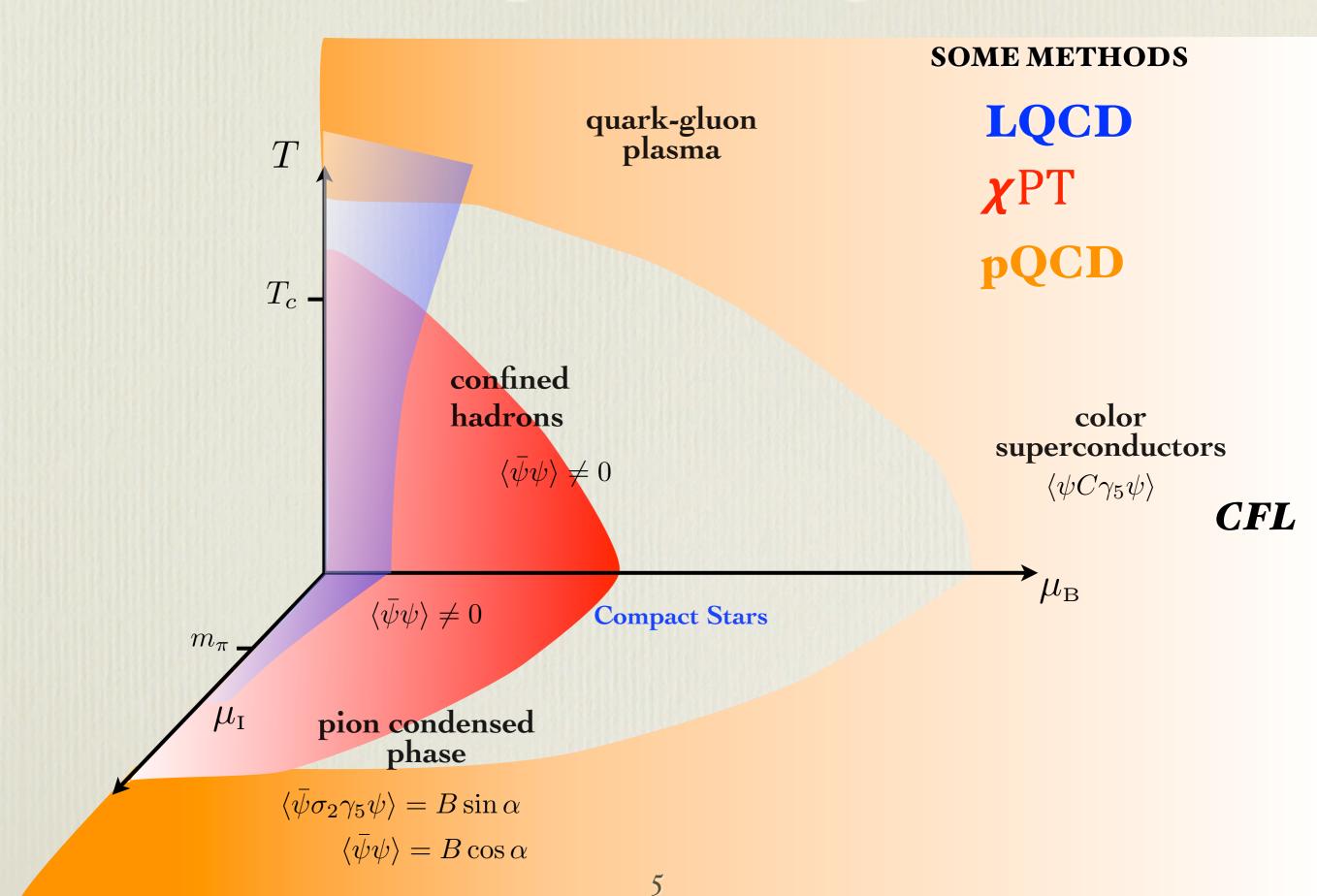


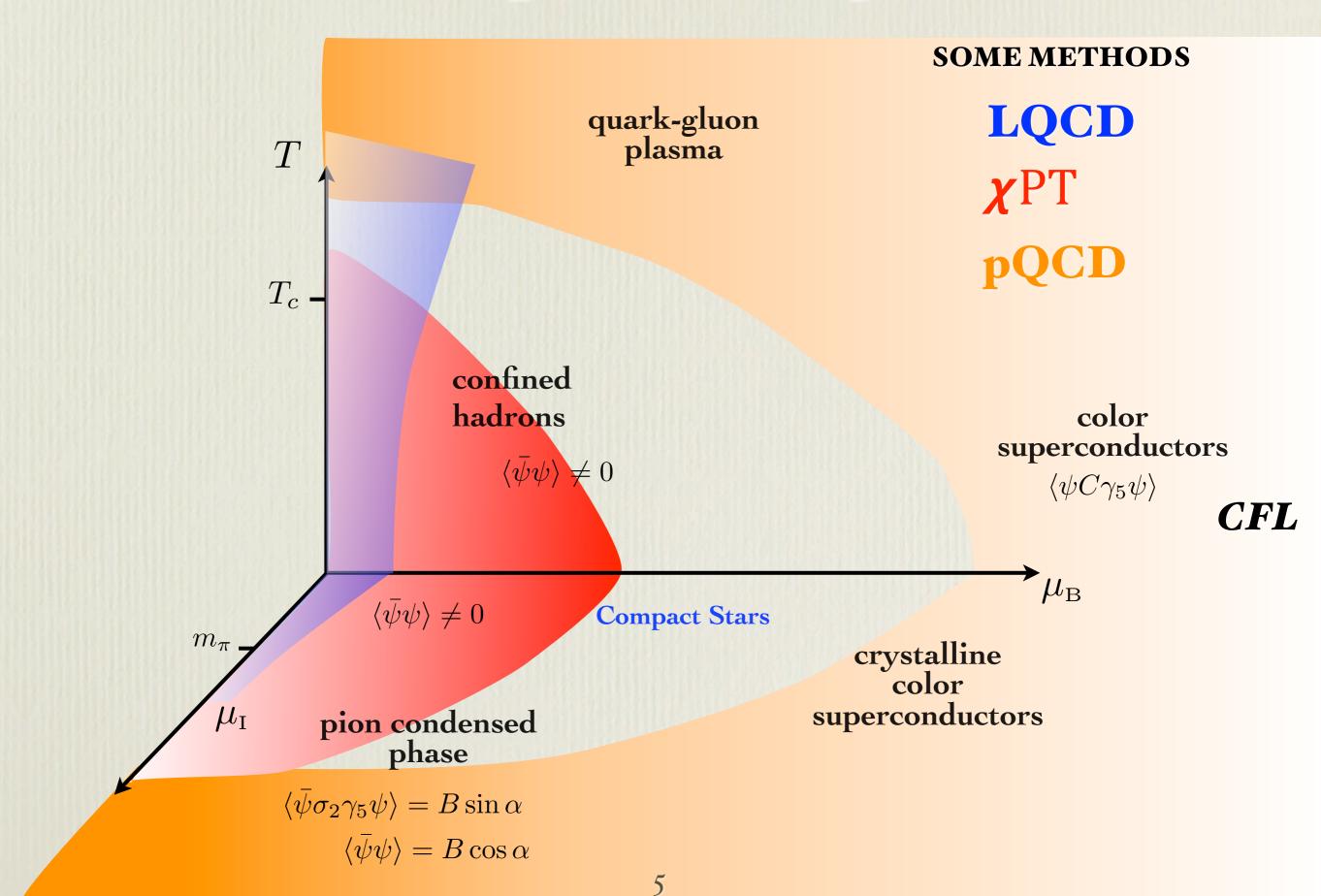


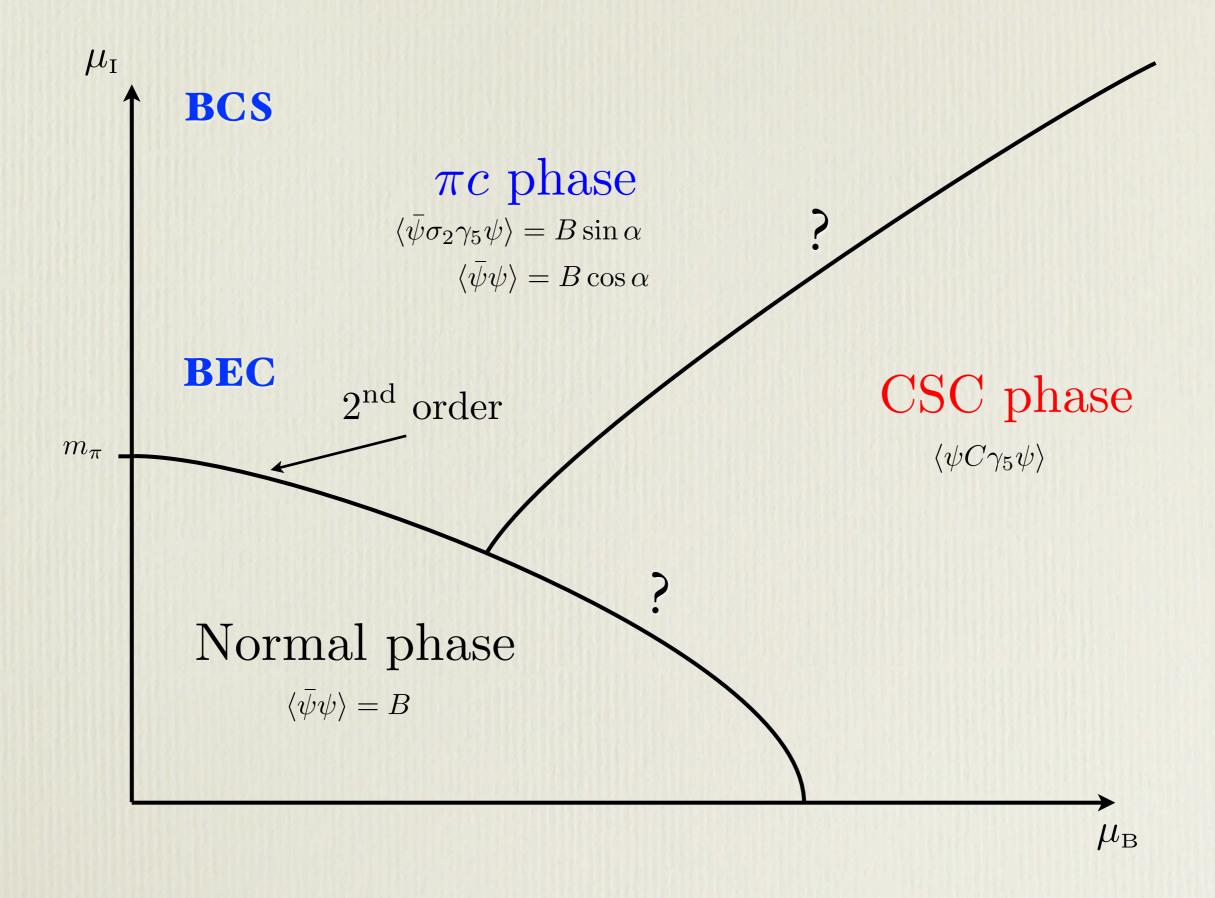












Chiral perturbation theory (xPT)

Setting

At non-asymptotic energy scales QCD is a nonperturbative theory

χPT is a low energy effective field theory of QCD: A realization of hadronic matter at soft energy scales

$$p \ll \Lambda \sim 1 \, \mathrm{GeV}$$

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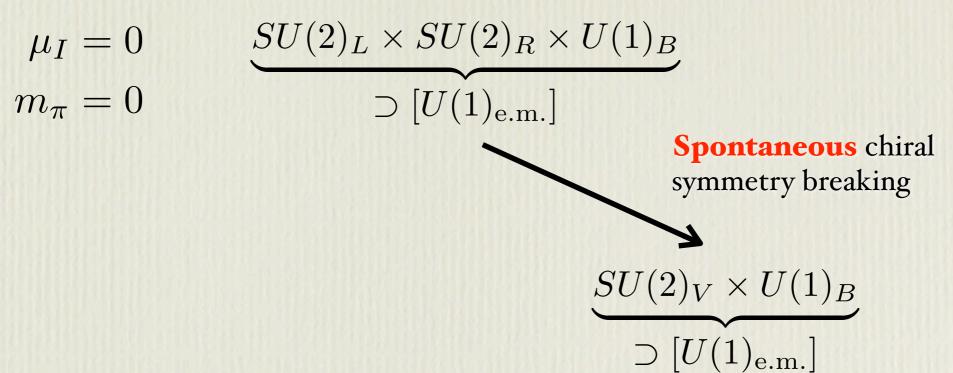
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In the following we do not include baryons and vector mesons

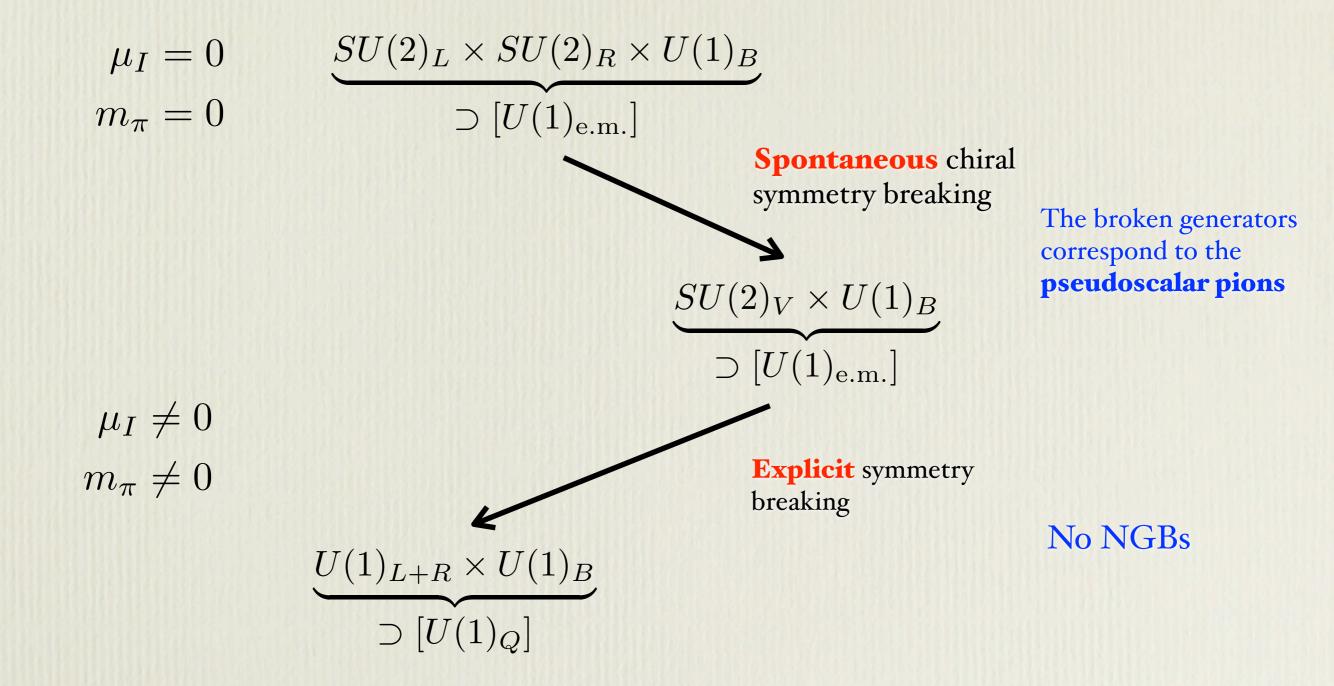
$$|\mu_B| \lesssim 940 \text{ MeV}$$
 $|\mu_I| \lesssim 770 \text{ MeV}$

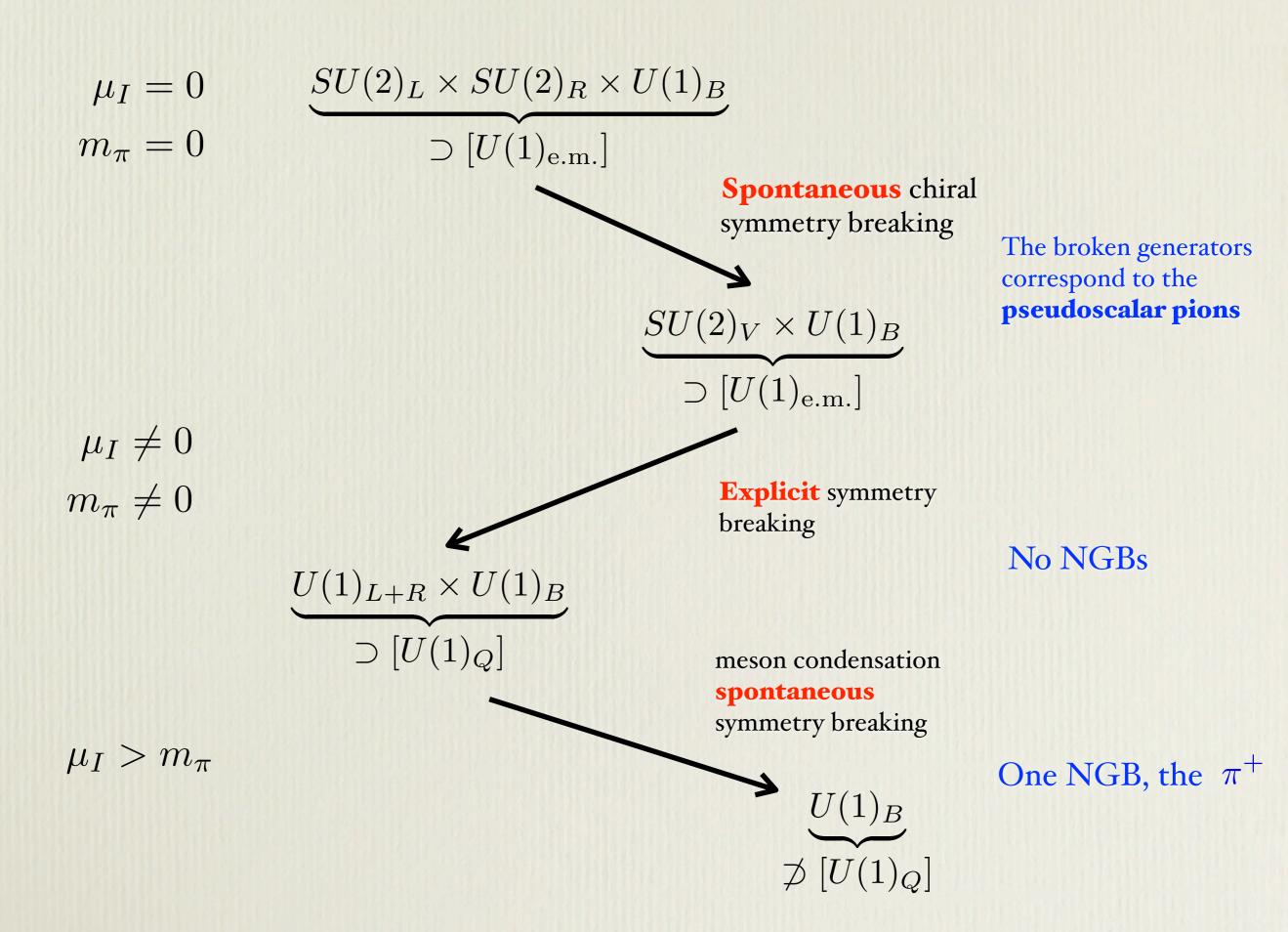
$$\mu_I = 0$$
 $\underbrace{SU(2)_L \times SU(2)_R \times U(1)_B}_{D_{e.m.}}$ $\mathcal{S}U(1)_{e.m.}$

$$\mu_I=0$$
 $SU(2)_L imes SU(2)_R imes U(1)_B$ $\supset [U(1)_{\mathrm{e.m.}}]$ Spontaneous chiral symmetry breaking $SU(2)_V imes U(1)_B$ $\supset [U(1)_{\mathrm{e.m.}}]$



The broken generators correspond to the **pseudoscalar pions**





Leading order Lagrangian

The $\mathcal{O}(p^2)$ Lorentz invariant Lagrangian density for pseudoscalar mesons

$$\mathcal{L} = \frac{F_0^2}{4} \text{Tr}(D_{\nu} \Sigma D^{\nu} \Sigma^{\dagger}) + \frac{F_0^2 m_{\pi}^2}{4} \text{Tr}(\Sigma^{\dagger} + \Sigma^{\dagger})$$

We have introduced the covariant derivative to take into account in-medium propagation

$$D_{\mu}\Sigma = \partial_{\mu}\Sigma - \frac{i}{2}[v_{\mu}, \Sigma]$$

Gasser and Leutwyler, Annals Phys. 158, 142 (1984)

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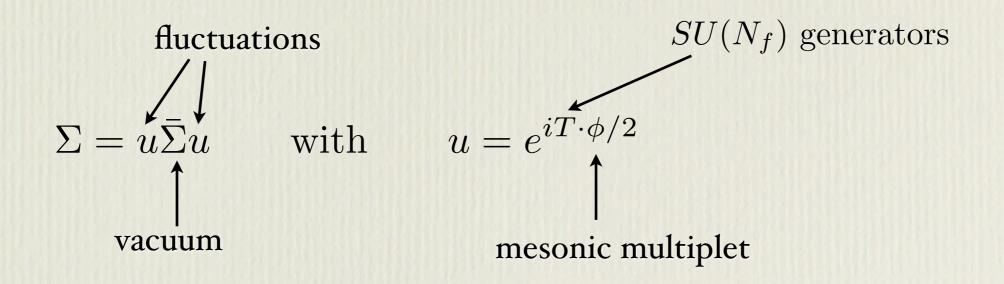
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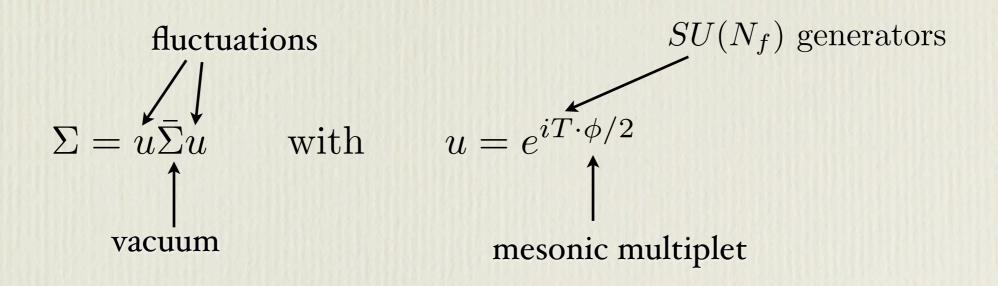
Formally preserving the Lorentz invariance

$$v^{\mu}=-2eQA^{\mu}-2\mu\delta^{\mu0}$$
 irrelevant for mesons
$$\mu=\mathrm{diag}(\mu_u,\mu_d)=\frac{\mu_B}{3}+\frac{\mu_I\sigma_3}{2}$$

Expanding (Here $N_f = 2$)



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• The simplest choice is of course $\bar{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$ar{\Sigma} = \left(egin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}
ight)$$

Corresponding to the standard nonlinear realization of χ PT

$$\Sigma = e^{iT \cdot \phi}$$

Mass splitting

proportional to the isospin charge

$$m_{\pi^0} = m_{\pi}$$
 $m_{\pi^-} = m_{\pi} + \mu_I$ $m_{\pi^+} = m_{\pi} - \mu_I$

what happens for $\mu_I > m_{\pi}$?

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A vanishing meson mass may imply the onset of an instability

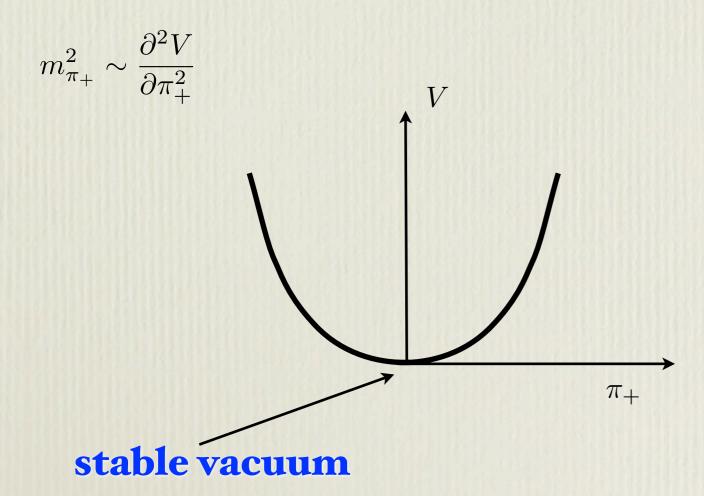
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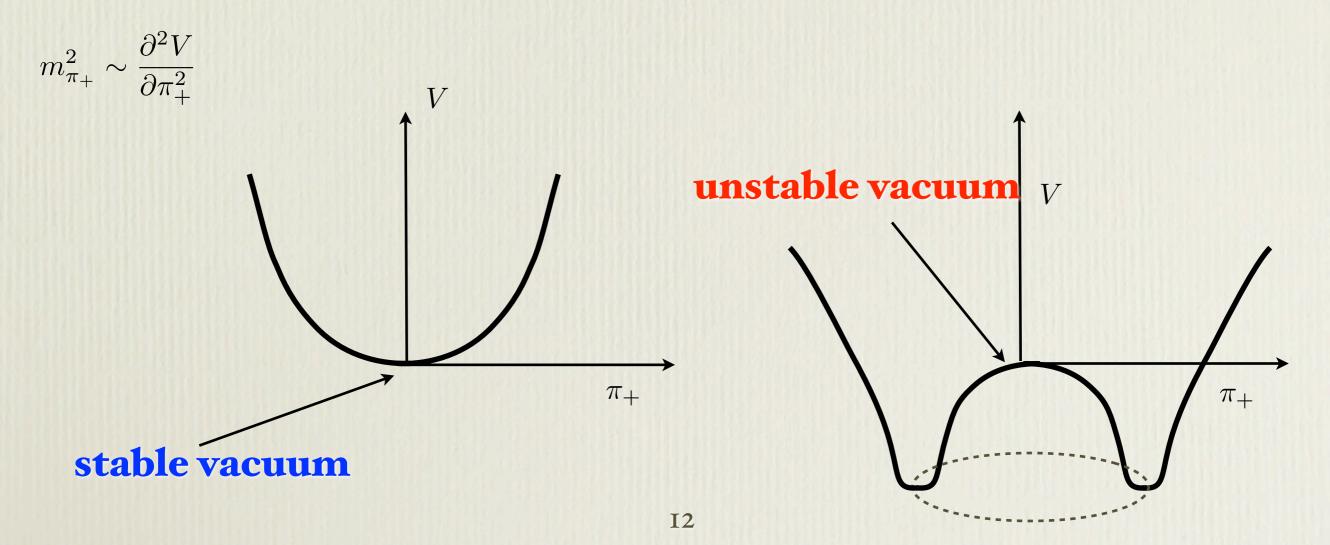
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Increasing the isospin charge

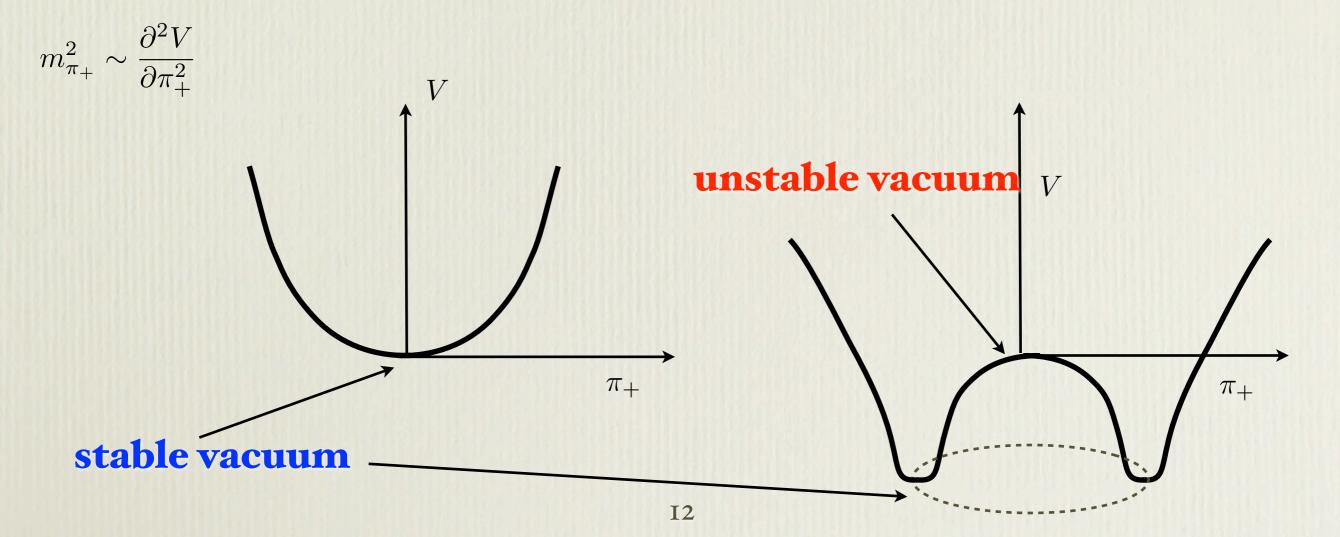
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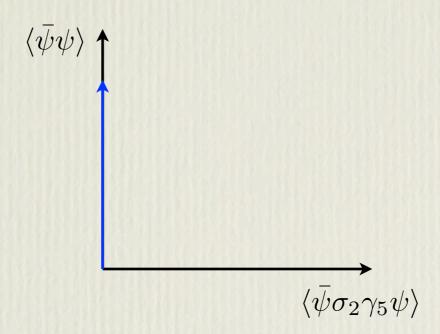
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$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$$

$$\mu_I = 0$$

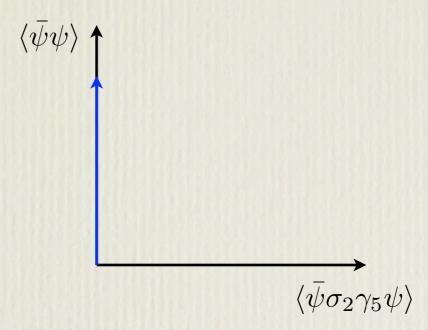


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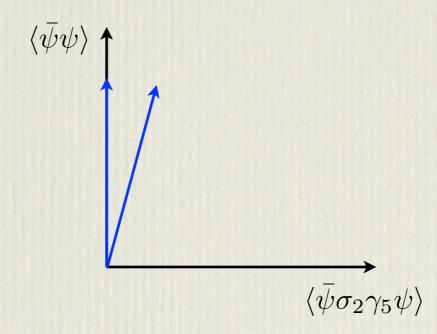
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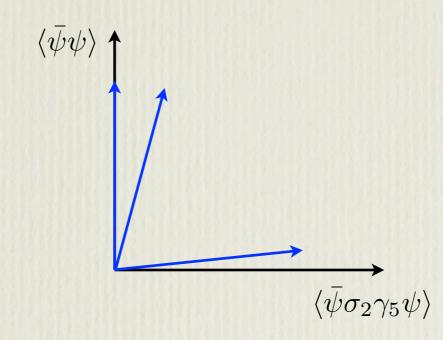
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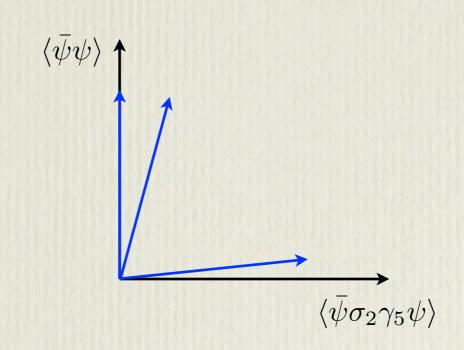
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Lagrangian at the vev

$$\mathcal{L}_0(\alpha, \mu_I, n_3) = F_0^2 m_\pi^2 \cos \alpha + \frac{F_0^2}{2} \mu_I^2 \sin^2 \alpha (1 - n_3^2)$$

for
$$\mu_I < m_{\pi}$$

for $\mu_I > m_{\pi}$

$$\cos \alpha = 1$$

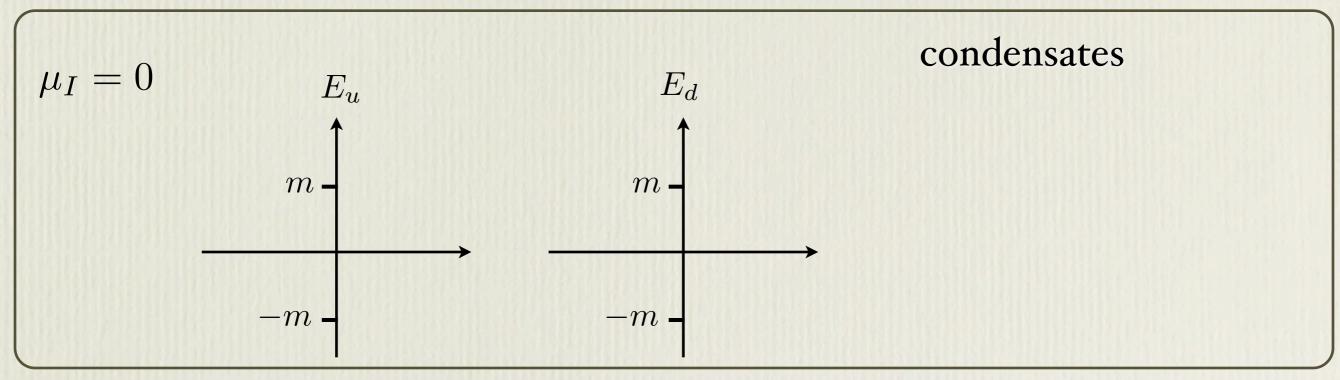
$$\cos \alpha_{\pi} = m_{\pi}^2/\mu_I^2$$

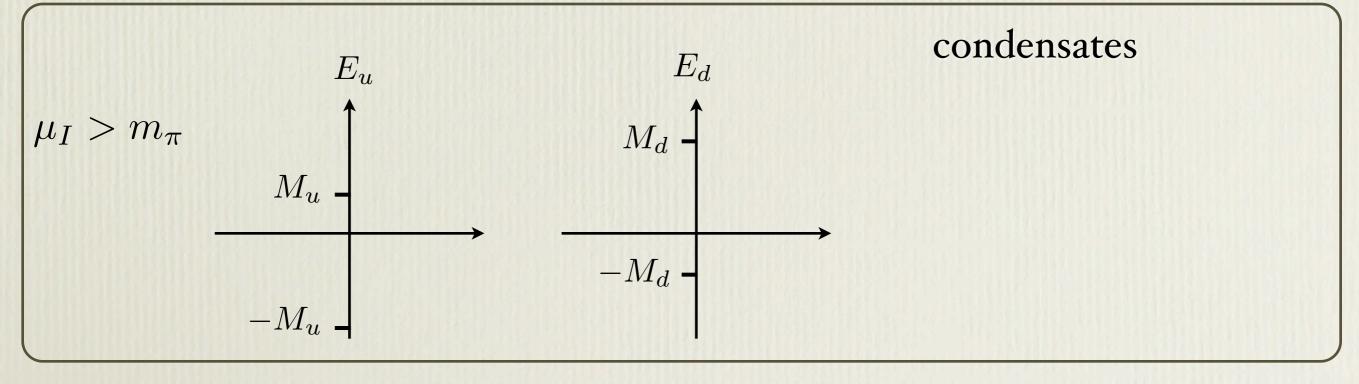
$$\mathcal{L}_0$$
 independent of \boldsymbol{n}

$$n_3 = 0$$
 residual $O(2)$ symmetry

$$M_u = -\frac{\mu_I}{2} \pm m$$

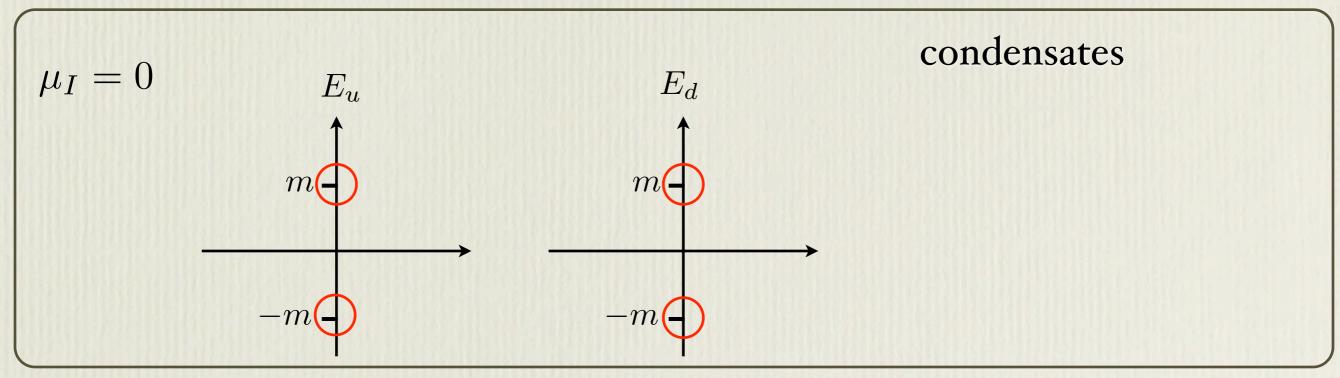
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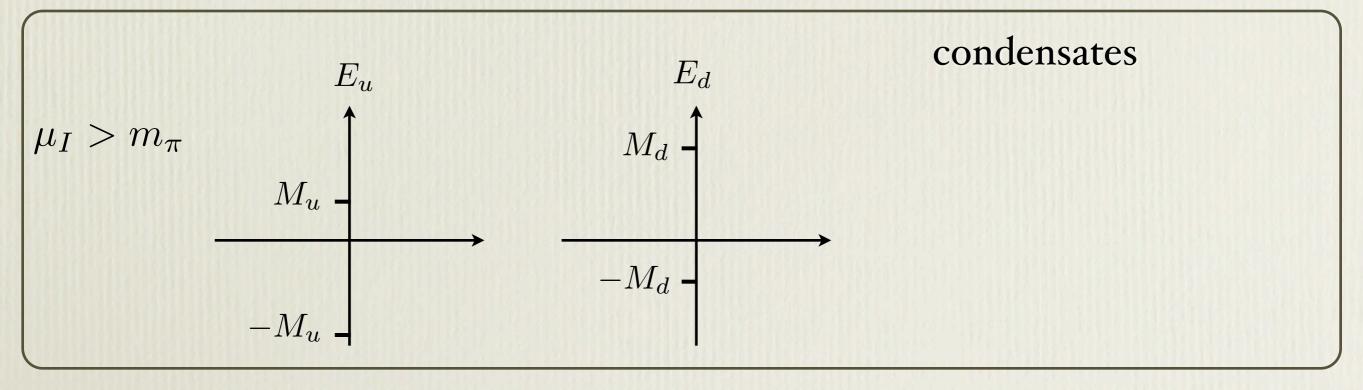




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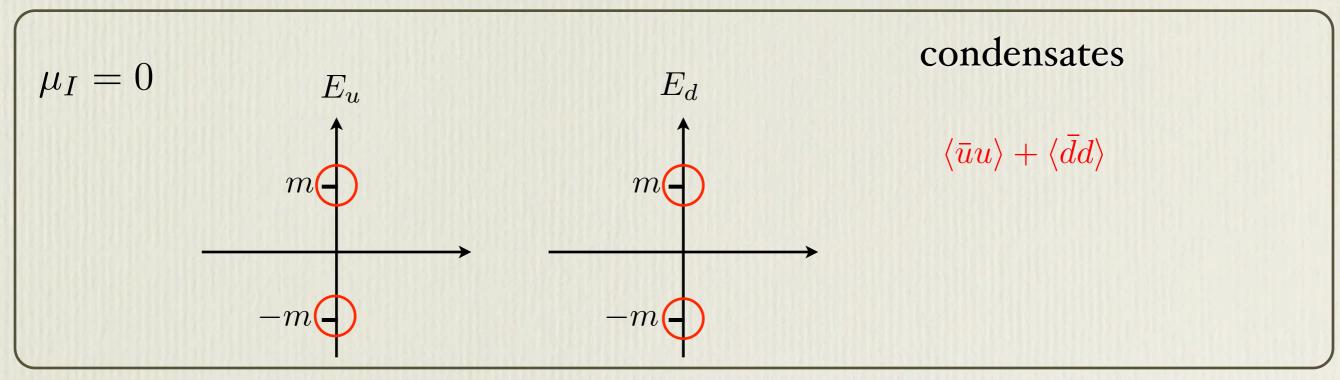
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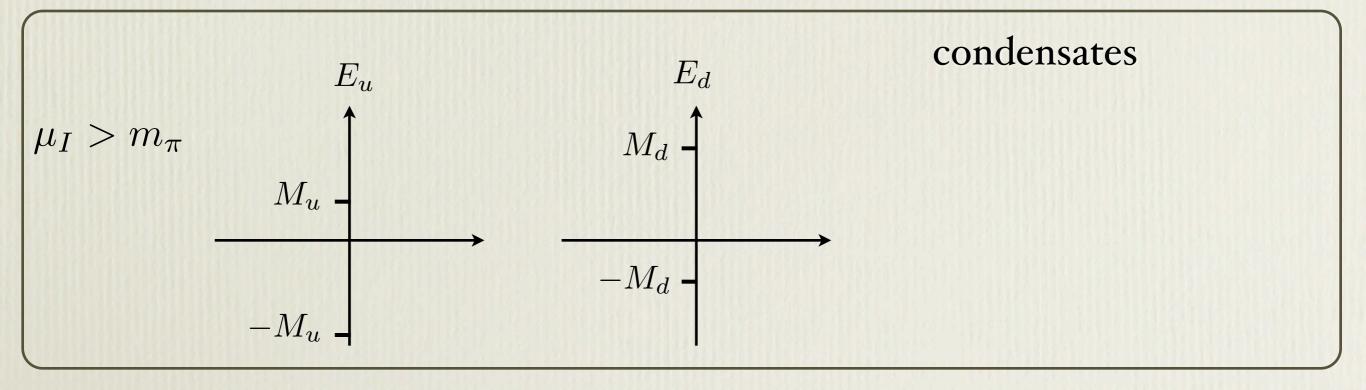




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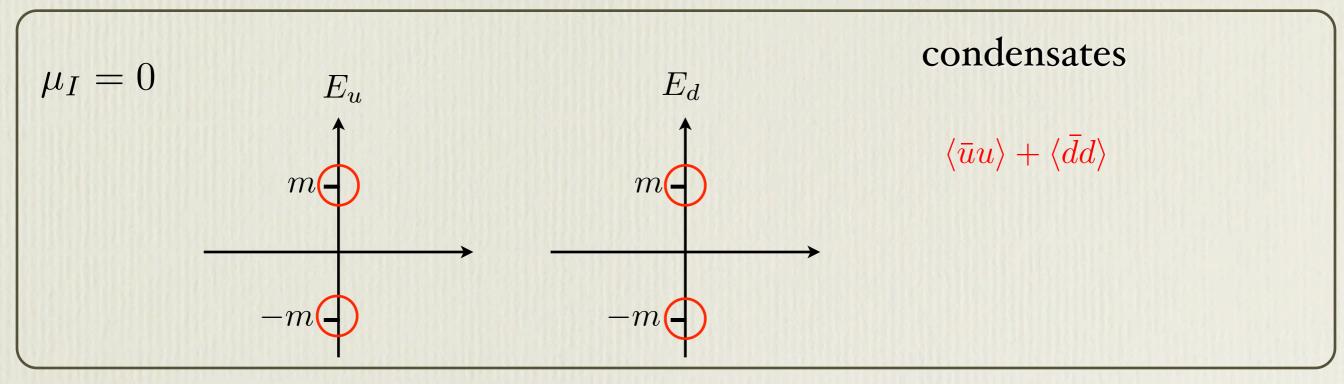
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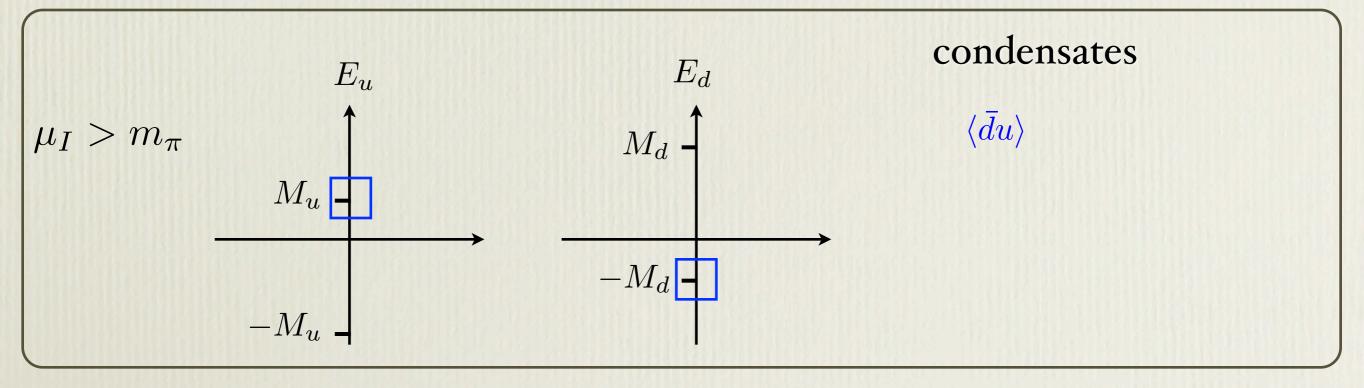




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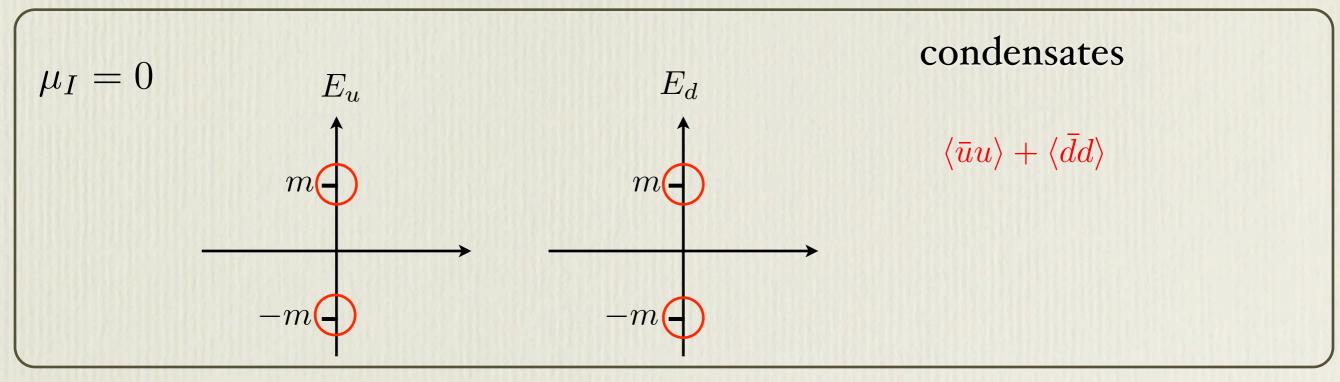
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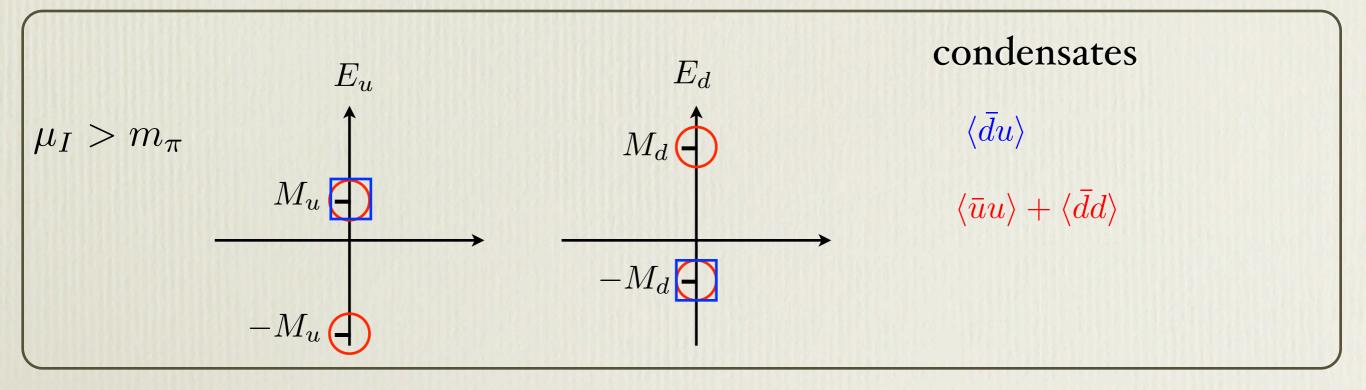




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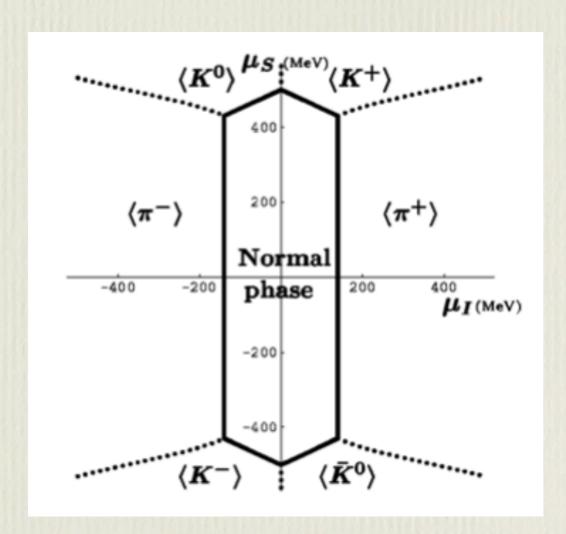
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Condensed phases

Phase diagram



solid line: second order

dotted line: first order

Kogut and Toublan PhysRevD.64.034007

In the condensed phases, a superfluid of charged bosons: a superconductor!

$$M_D^2 = M_M^2 = F_0^2 e^2 (\sin \alpha)^2$$

A. Mammarella and M.M. Phys.Rev. D92 (2015) 8, 085025

Mixing and mass splitting

In the condensed phases mesons mix and have nontrivial mass splitting

$$\begin{pmatrix} \tilde{\pi}_{+} \\ \tilde{\pi}_{-} \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \pi_{+} \\ \pi_{-} \end{pmatrix}$$

mass eigenstates charge eigenstates

$$U_{ij} \equiv U_{ij}(m_{\pi}, \mu_I, \mathbf{E})$$

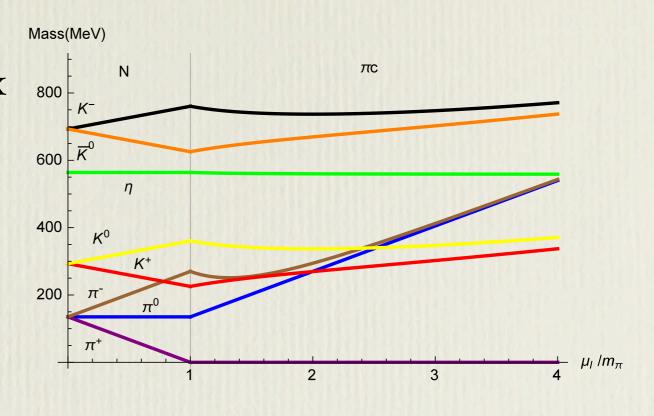
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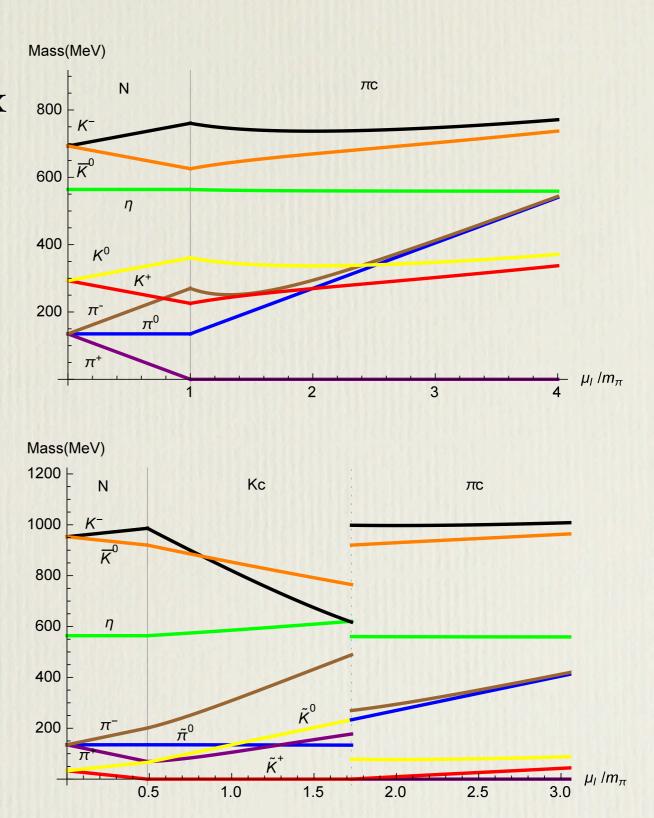
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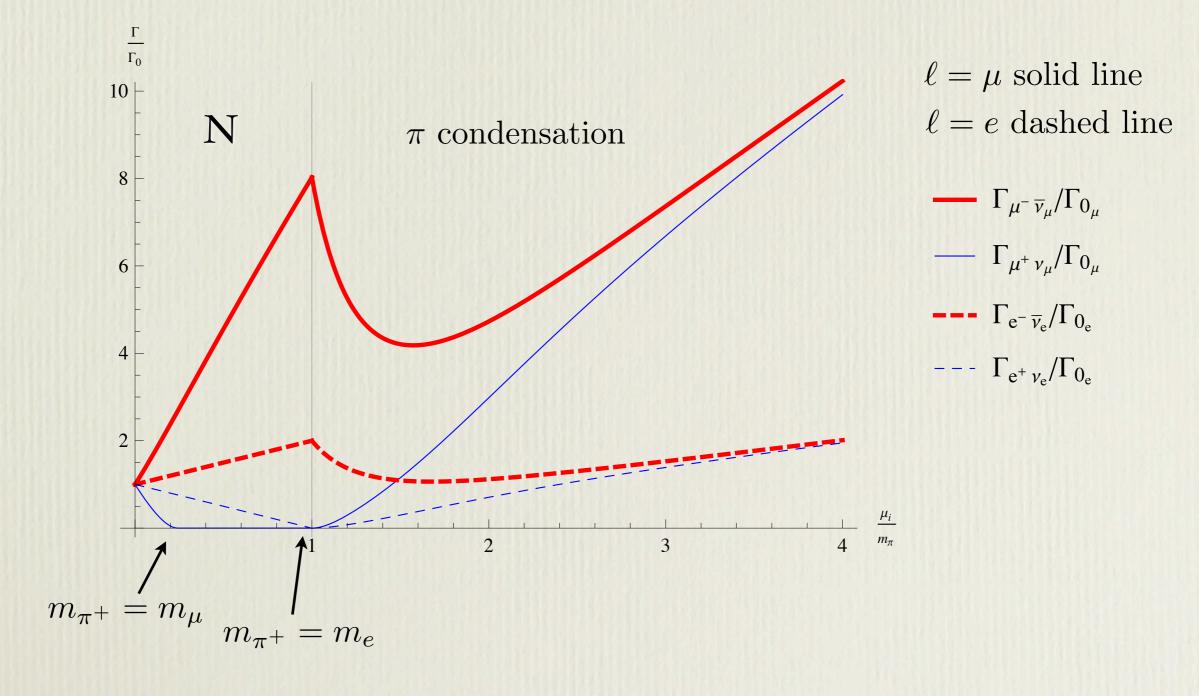
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Leptonic decays

Processes
$$\tilde{\pi}_- \to \ell^{\pm} \nu_{\ell}$$
 and $\tilde{\pi}_+ \to \ell^{\pm} \nu_{\ell}$



"Thermodynamics"

Leading order results

Analytic expressions of the pressure, number density and

for
$$\mu_I > m_{\pi}$$

$$p_{\text{LO}}^{\pi c} = \frac{f_{\pi}^2 \mu_I^2}{2} \left(1 - \frac{m_{\pi}^2}{\mu_I^2} \right)^2$$

$$n_{I,\text{LO}}^{\pi C} = \frac{\partial p_{\text{LO}}^{\pi C}}{\partial \mu_I} = f_{\pi}^2 \mu_I \left(1 - \frac{m_{\pi}^4}{\mu_I^4} \right)$$

$$\epsilon_{\text{LO}}^{\pi c} = \frac{f_{\pi}^2 \mu_I^2}{2} \left(1 + 2 \frac{m_{\pi}^2}{\mu_I^2} - 3 \frac{m_{\pi}^4}{\mu_I^4} \right)$$

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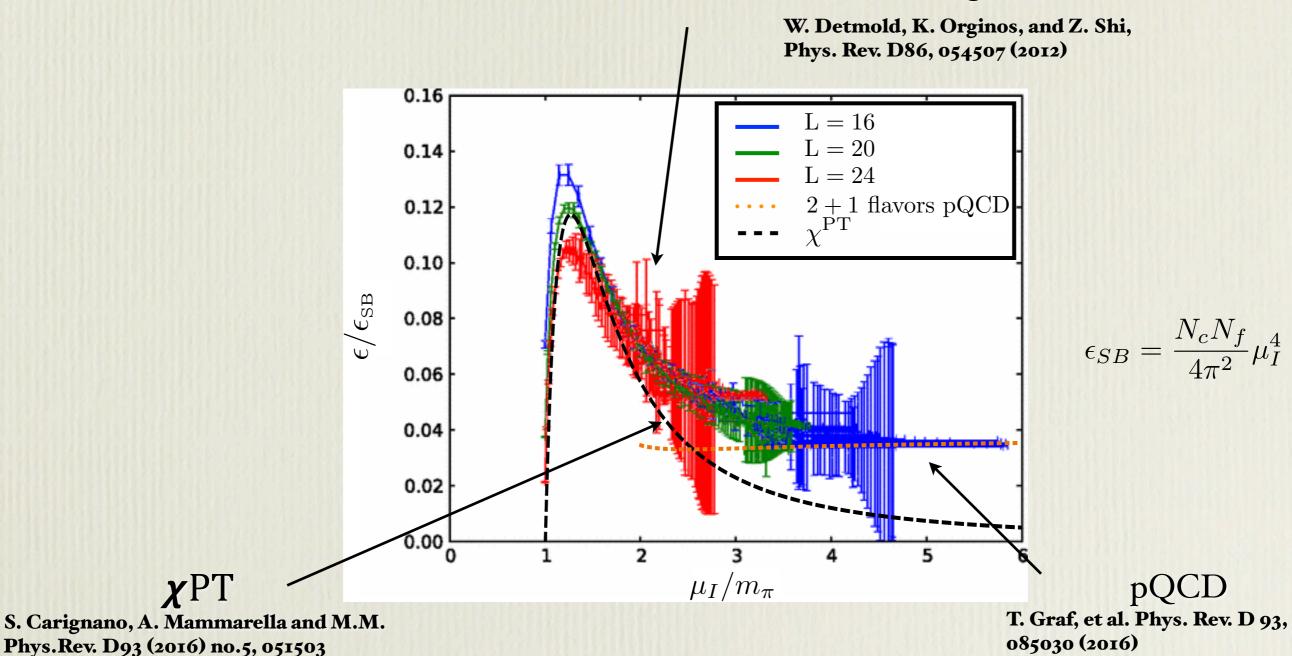
Equation of State

$$\epsilon_{\text{LO}}^{\pi c}(p) = 2\sqrt{p(2f_{\pi}^{2}m_{\pi}^{2} + p)} - p$$

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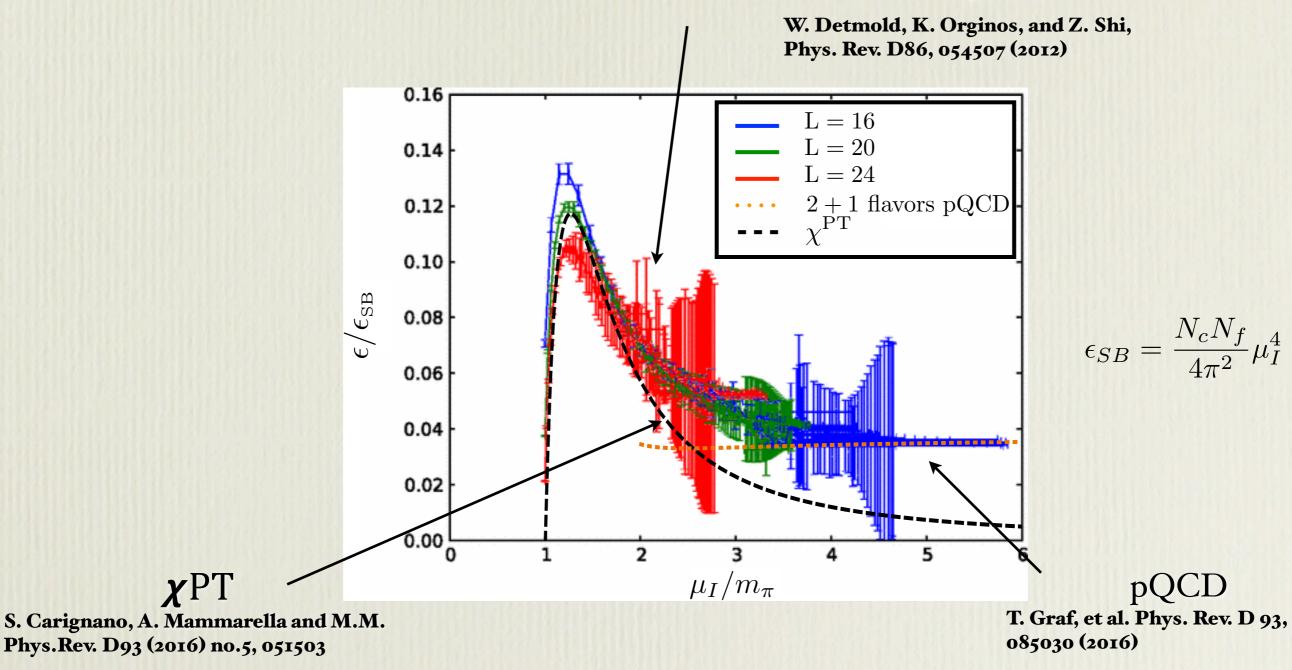
Comparison with different methods

Microcanonical lattice QCD simulations



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Leading order χ PT correctly reproduces the peak structure

$$\mu_{I,\text{LQCD}}^{\text{peak}} = \{1.20, 1.25, 1.275\} m_{\pi}$$

$$\mu_{I,\chi\text{PT}}^{\text{peak}} = (\sqrt{13} - 2)^{1/2} m_{\pi} \simeq 1.276 m_{\pi}$$

Origin of the peak

"The system for μ_I < 1.3 m_{π} , can be identified as a **pion gas**. When $\mu_I \sim \mu^I_{peak}$, pions start to condense and the system resides in the BEC state. The plateau beginning to form beyond $\mu_I \approx 3 m_{\pi}$, may indicate a crossover from the BEC to BCS state, however higher precision and larger μ_I is required to make a definite statement."

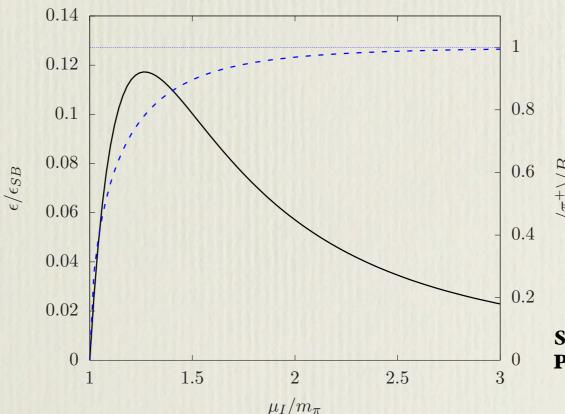
W. Detmold, K. Orginos, and Z. Shi, Phys. Rev. D86, 054507 (2012)

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In χ PT we have access to the pion condensate. At the peak it is close to its maximum value. The peak seems to be related with the saturation of the condensate.



S. Carignano, A. Mammarella and M.M. Phys.Rev. D93 (2016) no.5, 051503

BEC-BCS crossover

The **BEC** and the **BCS** are not separated by a phase transition

BEC - tightly bound pairs (dimers)

BCS - correlated (far away) fermion pairs

 χ PT naturally captures the BEC side. Because χ PT deals with bosons Can χ PT capture the BCS side?

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What about the ground state properties?

Conformal limit

In fermionic systems the crossover region is characterised by a divergent s-wave scattering length.

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However in χ PT at $\mu_I = \sqrt{3}m_\pi$ $\epsilon = 3p \qquad \qquad \text{No trace anomaly}$ $\frac{\partial^2 \epsilon}{\partial p^2} = 0 \qquad \text{Change from two different regimes}$ $c_s = \sqrt{\frac{2}{3}} \qquad \qquad \text{Correct value for a system of maximally stressed two noninteracting}$ Fermi fluids

Conclusions

- The realistic conditions in heavy nuclei and in compact stars require a nonvanishing isospin chemical potential
- If isospin is broken nontrivial mass dependence
- In the condensed phase there is mixing and mesons have nontrivial masses and decay patterns
- LO χ PT seems to lead to the correct EoS below the ϱ mass
- Need of more refined lattice QCD simulations
- We can easily do kaons

New Call for GSSI PhD Applications 2016/17 now open

Print

The Gran Sasso Science Institute (GSSI), founded in 2012 in L'Aquila (Italy) as Center for Advanced Studies of the National Institute for Nuclear Physics (INFN) and then established in March 2016 as a new university providing post-graduate education, offers 41 PhD fellowships for the academic year 2016/17.

The GSSI invites applications for 10 fellowships in "Astroparticle Physics", 11 in "Mathematics in Natural, Social and Life Sciences", one of which funded by the Italian Institute of Technology (IIT), 10 in "Computer Science" and 10 in "Urban Studies and Regional Science". The official language for all PhD courses is English.



The fellowships are awarded for three years and their yearly amount is € 16.159,91 gross. All PhD students have free accommodation at the GSSI facilities and use of the canteen.

The application must be submitted through the online form available at www.gssi.it/phd/ by 1st September 2016 at 18.00 (Italian time zone).

For more information, please consult the Call for Applications at <u>www.gssi.it/phd/</u> or write an email to <u>info@gssi.infn.it</u> or call +39 0862 4280262.

BASIC LITERATURE

- Shapiro and Teukolsky "Black holes, white dwarfs, and neutron stars"
- Son and Stephanov, ``QCD at finite isospin density'' PRL 86, 592 (2001)
- Kogut and Toublan, ``QCD at small nonzero quark chemical potentials" PRD 64, 34007, (2001)
- Barducci, Casalbuoni, Pettini and Ravagli, "Pion and kaon condensation in a 3-flavor NJL model", PRD 71, 016011, (2005).

backup

Superfluid vs Superconductors

Definitions

Superfluid: frictionless fluid with $v = \nabla \phi \Rightarrow \nabla \times v = 0$ (irrotational or quantized vorticity)

Superconductor: "screening" of magnetic fields: Meissner effect (almost perfect diamagnet)

Superfluid

Broken global symmetry

Goldstone theorem



"Easy" transport of the quantum numbers of the broken group

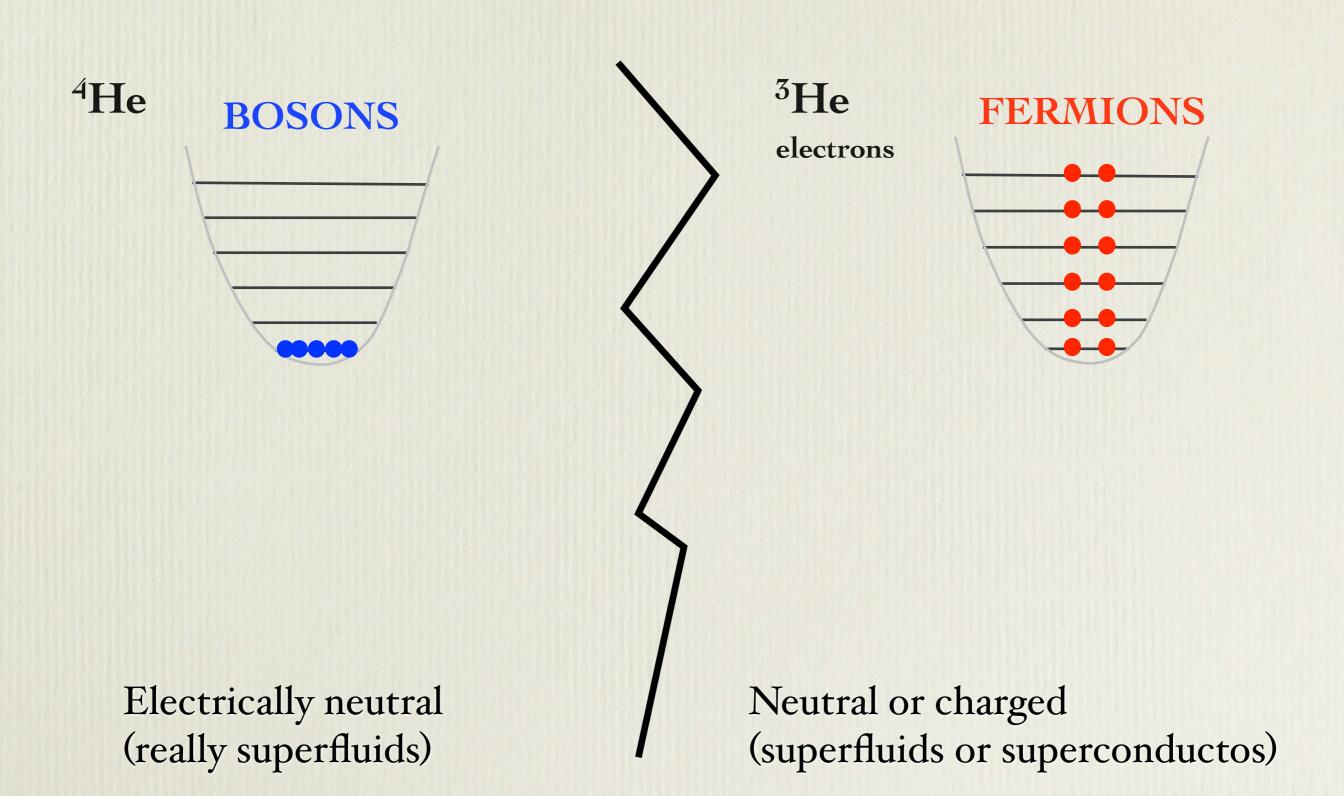
Superconductor

"Broken gauge symmetry"

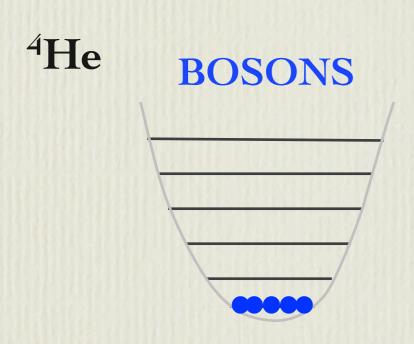
Higgs mechanism ———

Gauge fields with mass, M, penetrate for a length $\lambda \propto 1/M$

Fermionic and bosonic superfluids at T=0



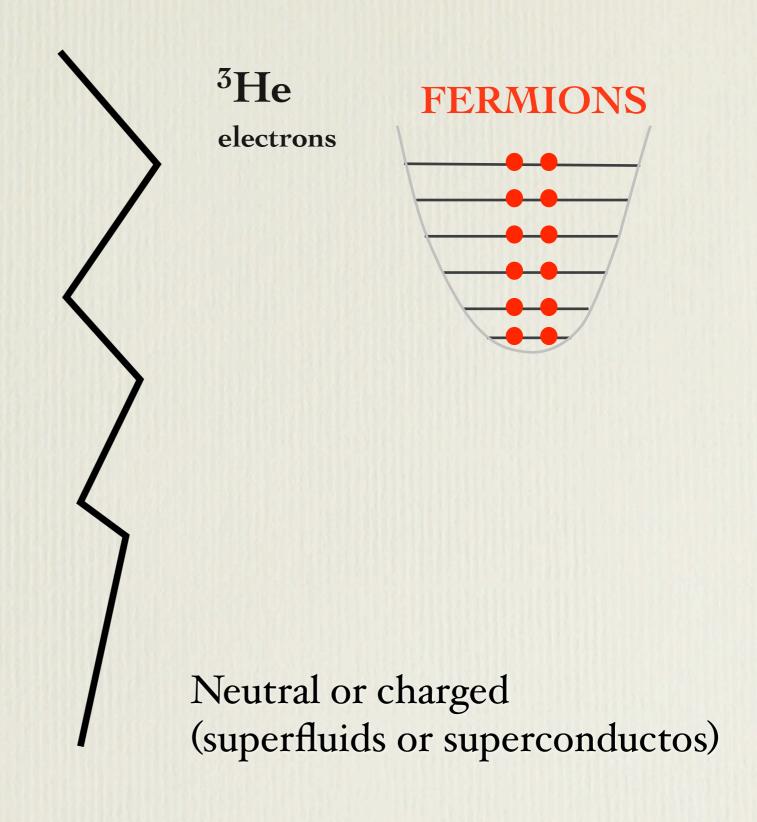
Fermionic and bosonic superfluids at T=0



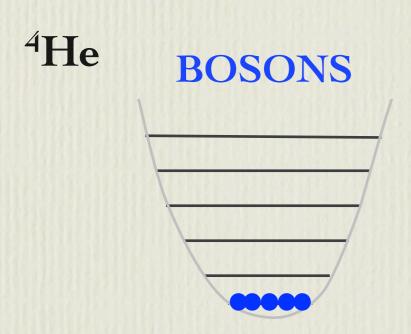
Bosons "like" to move together, no dissipation

 4 He becomes superfluid at $T_c \approx 2.17$ K, Kapitsa et al (1938)

Electrically neutral (really superfluids)



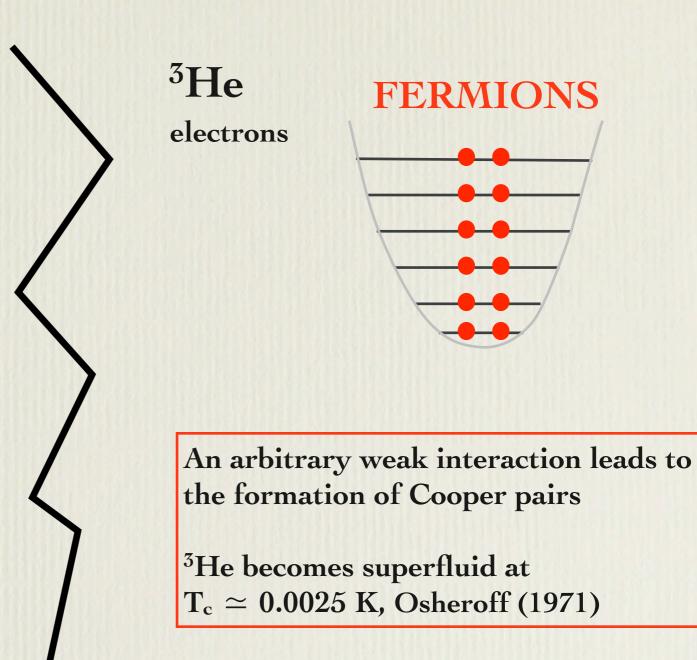
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Electrically neutral (really superfluids)



Neutral or charged (superfluids or superconductos)