Nucleon form factors,

generalized parton distributions, parton angular momentum

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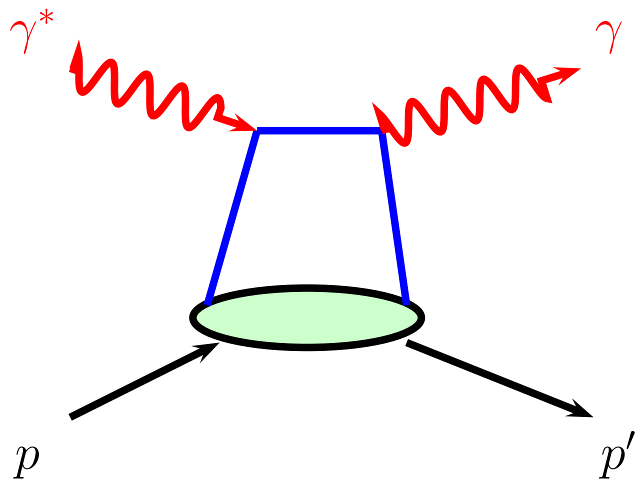
- **Introduction**
- **GPD analysis of form factors**
- **Physical interpretation**
- **Applications: WACS, DVCS, DVMP**
- **Parton angular momenta**
- **Summary**

(mainly) based on M. Diehl and P. K. arXiv:1302.4604

Interest in GPDs



handbag factorization

common picture for DVCS and DVMP in

gen. Bjorken regime (large Q^2 and W, fixed x_B)

GPDs are universal

- seems to work successfully for small $-t$

  GPDs extracted from DVMP (Goloskokov-K)

  also fit DVCS   K.-Moutarde-Sabatie(12)

  combined fit to HERA DVCS and DVMP data:

  Meskauskas-Müller(11)

$$\text{GPDs} \sim \langle p'|\overline{\Psi}(-z/2)\Gamma\Psi(z/2)|p\rangle$$

$$\Gamma = \gamma^+,\, \gamma^+\gamma_5,\, \sigma^{+j}$$

- GPDs allow for a determination of parton angular momenta
- localization of partons in impact parameter plane

  requires Fourier transform of GPDs with respect to $\Delta = p' - p$ ($\Delta^2 = t$)

  need GPDs at large $-t$
- GPDs at large $-t$ control wide-angle Compton scattering (WACS)

Sum rules for nucleon form factors

GPDs respect sum rules (hold for all $\xi = \frac{(p-p')^+}{(p+p')^+}$, work in frame with $\xi = 0$)

$$F_1^q(t) = \int_0^1 dx H_v^q(x,t) \qquad F_2^q(t) = \int_0^1 dx E_v^q(x,t)$$

$$K_v^q(x,t) = K^q(x,0,t) + K^q(-x,0,t) \qquad K^{\bar{q}} = -K^q(-x,0,t) \qquad (K^q = H^q, E^q)$$

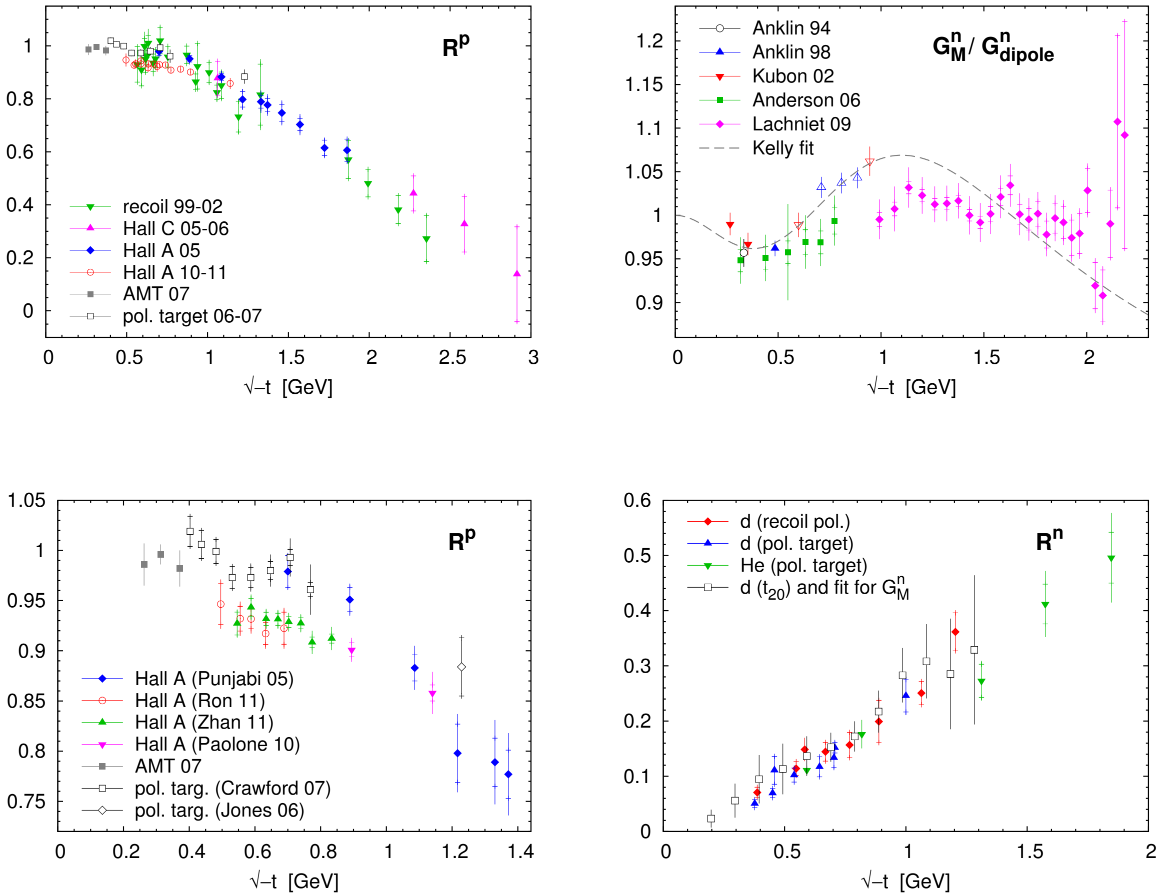
only difference $K^q - K^{\bar{q}}$ contribute to elm form factors (C-invariance)

$$F_i^p = e_u F_i^u + e_d F_i^d + e_s F_i^s \qquad F_i^n = e_u F_i^d + e_d F_i^u + e_s F_i^s \qquad i = 1,2$$

plenty of data on $G_M^p, G_M^n, G_E^p, G_E^n$ ($\Rightarrow F_1^p, F_1^n, F_2^p, F_2^n$) are available
used to extract the GPDs H_v^q and E_v^q with help of a suitable parametrization

previous work: DFJK hep-ph/0408173 and Guidal et al hep-ph/0410251
update of DFJK(04): Diehl-K(13)
use new form factor data: extend to larger $-t$ (G_E^p, G_M^n, G_E^n)
new accurate data at low $-t$
(occasionally in conflict with old data)

The form factor data



$$R_i = \mu_i G_E^i / G_M^i$$

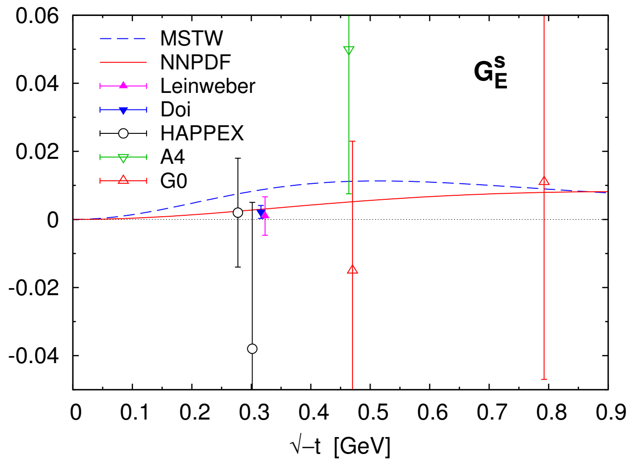
R_i and G_M^i have uncorrelated errors

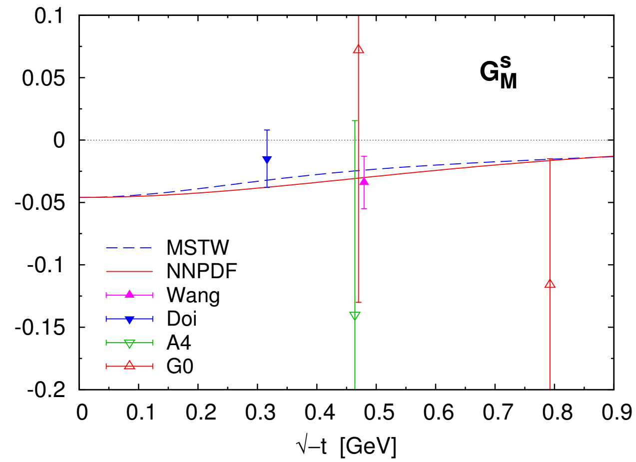
G_M^n data: MAMI, PSI data only used partially

R^p data: polarized target data not used Jones(06), Crawford(07)

R^n (measured) eventually reconstructed by us from published G_E^n

Strangeness





estimate of F_1^s: $H_v^s(x,t) = [s(x) - \bar{s}(x)] \exp\left[t\alpha_s'(1-x)\log 1/x\right]$

(analogous to $H_v^{u(d)}$) $\quad \alpha_s' = 0.95\,\mathrm{GeV}^2 \quad s - \bar{s}$ from MSTW(08), NNPDF(12)

estimate of F_2^s: VMD $\quad F_2^s = \mu_s \sum_{i=1}^{3} \frac{a_i}{m_i^2 - t} \quad \xrightarrow{-t\to\infty} 1/t^3$ (dim. counting)

three poles $\Phi(1020)$, $\Phi(1680)$, $\Phi(2170)$ $\quad$ PDG(12) $\Longrightarrow$

$a_i = m_1^2 m_2^2 m_3^2 \Big[\prod_{j\neq i}(m_i^2 - m_j^2)\Big]^{-1} \qquad \mu_s = -0.046$ Leineweber(03)

Results on $G^s_{M(E)}$ consistent with experiment and lattice

strangeness FFs are smaller than errors on $F_i^{u(d)}$ $\Rightarrow$ neglected in analysis

Parametrization of the GPDs

ANSATZ:
$$H_v^q(x,t) = q_v(x)\exp\left[t f_q(x)\right]$$
$$E_v^q(x,t) = e_v^q(x)\exp\left[t g_q(x)\right]$$
$$f_q = \left[\alpha_q' \log(1/x) + B_q\right](1-x)^3 + A_q\, x(1-x)^2$$
$$g_q = \left[\alpha_q' \log(1/x) + D_q\right](1-x)^3 + C_q\, x(1-x)^2$$

$q_v(x)$ PDFs from ABM (Alekhin et al (12) (CT, MSTW, NNPDF ... probed as well)

$e_v^q = \kappa_q N_q x^{-\alpha_q}(1-x)^{\beta_q}(1+\gamma_q\sqrt{x})$ $\qquad\qquad$ $\kappa_q = \int dx e_v^q(x)$

Motivation:

small x: Regge-like term dominates - $K_v^q \sim x^{-\alpha_q - t\alpha_q'}$

large x: last term dominates

$$\text{Fourier transform}: \qquad k_v^q(x,b^2) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2}\,\mathrm{e}^{-i\mathbf{b}\mathbf{\Delta}}\,K_v^q(x,t=\Delta^2)$$

Burkardt(02): $q_v(x,b^2)$ is difference of densities for quarks and antiquarks at given x and transverse distance b from the proton's momentum center

average impact parameter associated with q_v: $\langle b^2\rangle_x^q = 4f_q(x)$

factor $(1-x)^2$ leads to $\langle b^2\rangle_x^q \sim (1-x)^2$ in the limit $x\to 1$

guarantees finite size of proton

The fits

not all parameters can be varied independently (but more as in DFJK(04))
otherwise large uncertainties of parameters and violations of positivity bound:

$$\frac{\left[e_v^q(x)\right]^2}{8m^2} \leq \exp(1)\left[\frac{g_q(x)}{f_q(x)}\right]^3\left[f_q(x)-g_q(x)\right]\left\{\left[q_v(x)\right]^2-\left[\Delta q_v(x)\right]^2\right\}$$

we fix: $\alpha_d = \alpha_u = \alpha$ $\qquad \alpha'_u - \alpha'_d = 0.1\,\mathrm{GeV}^2$ $\qquad \gamma_u = 4$ $\qquad \gamma_d = 0$

β_q: various fits; look for solution with minimal χ^2 not violating positivity

default fit (ABM1): PDFs from ABM(12) at scale $\mu = 2\,\mathrm{GeV}$

q	u	d
A_q	1.264 ± 0.050	4.198 ± 0.231
B_q	0.545 ± 0.062	0.206 ± 0.073
C_q	1.187 ± 0.087	3.106 ± 0.249
D_q	0.333 ± 0.065	-0.635 ± 0.076

$\beta_u = 4.65 \qquad \beta_d = 5.25$

$\alpha = 0.603 \pm 0.020$

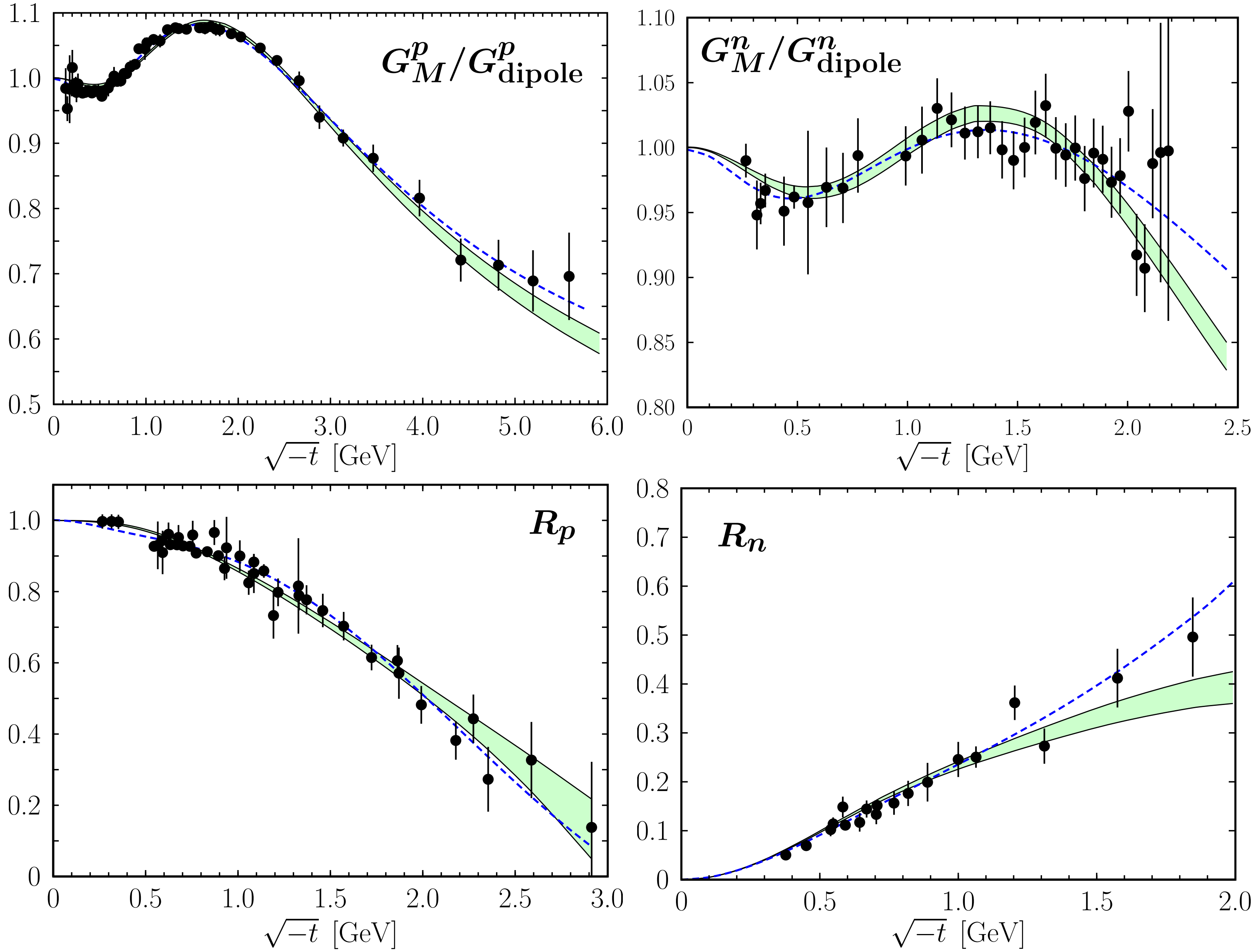
$\alpha'_d = (0.861 \pm 0.026)\,\mathrm{GeV}^2$

$\chi^2 \approx 220$ for 178 data points

All quantities in units GeV^{-2}

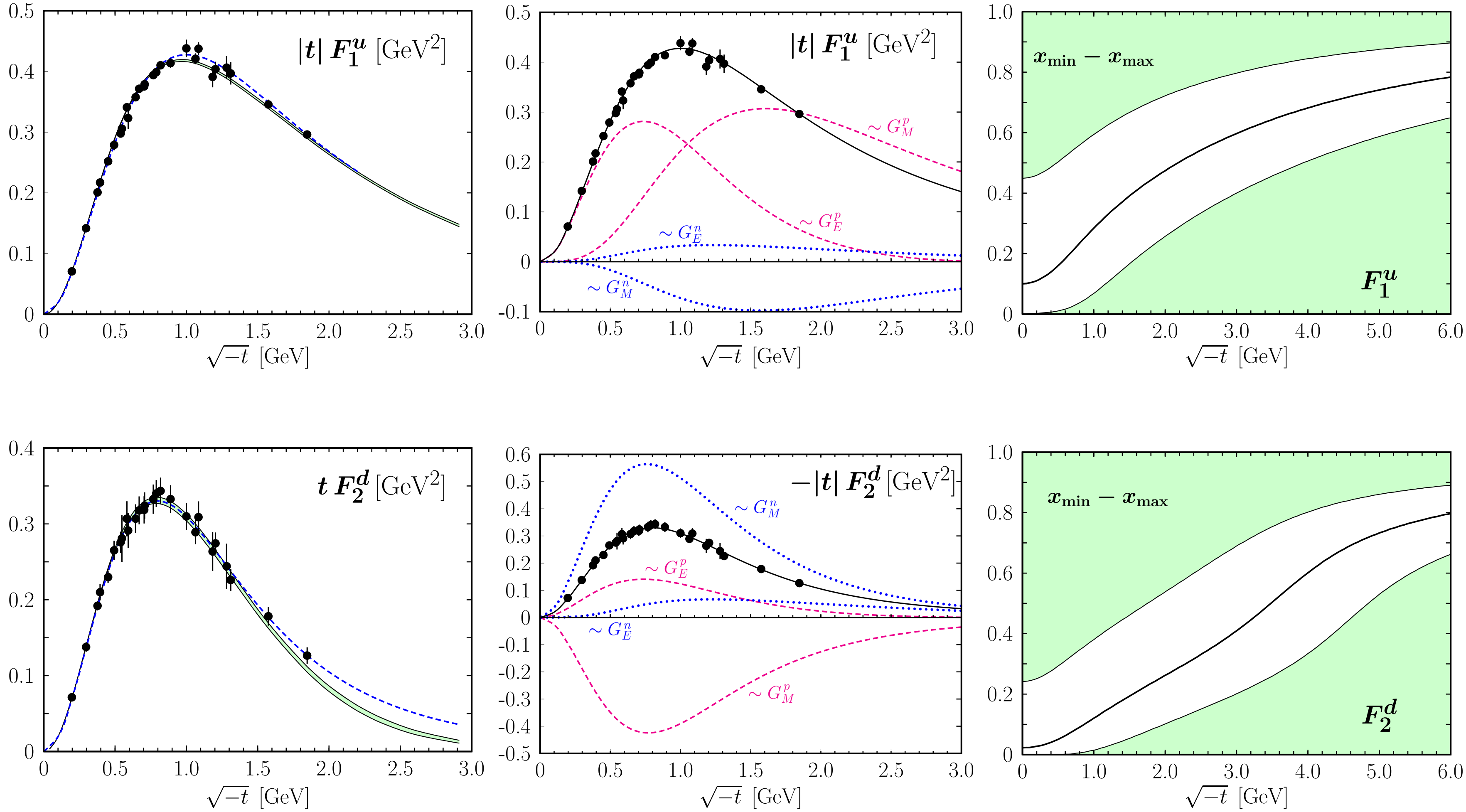
variations of fit: other PDFs, ABMx: $\gamma_d = \gamma_u = 4$, with strangeness,
alternative data (R^p or G_M^p Arrigton(05),instead (07))

Sachs form factors



default fit: green bands

The flavor form factors



fit to the flavor FFs
(interpolated FF data)
see also (Cates et al(11))

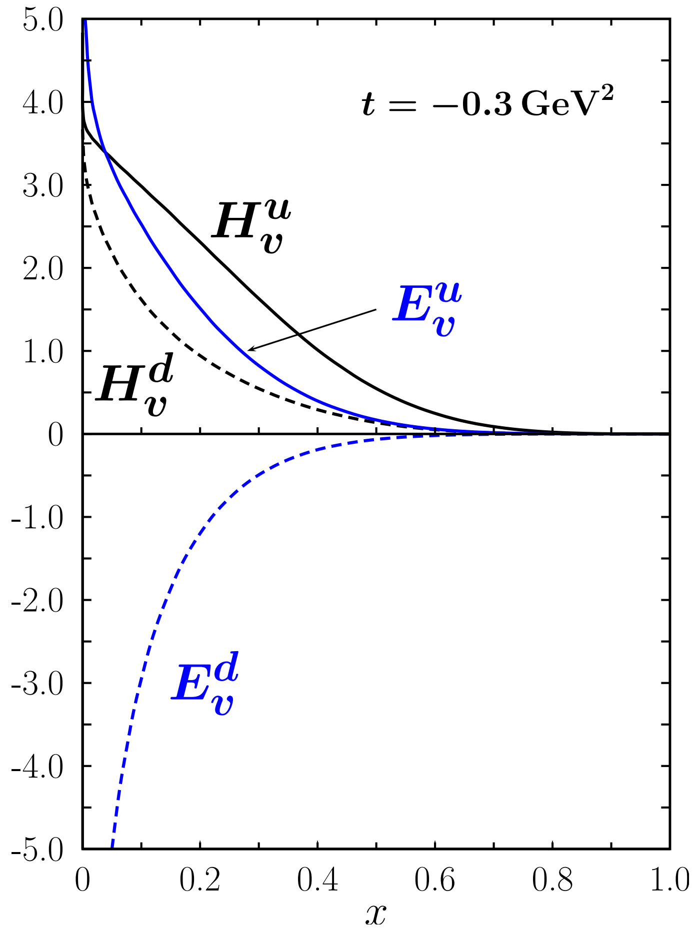
decomposition into
Sachs FFs

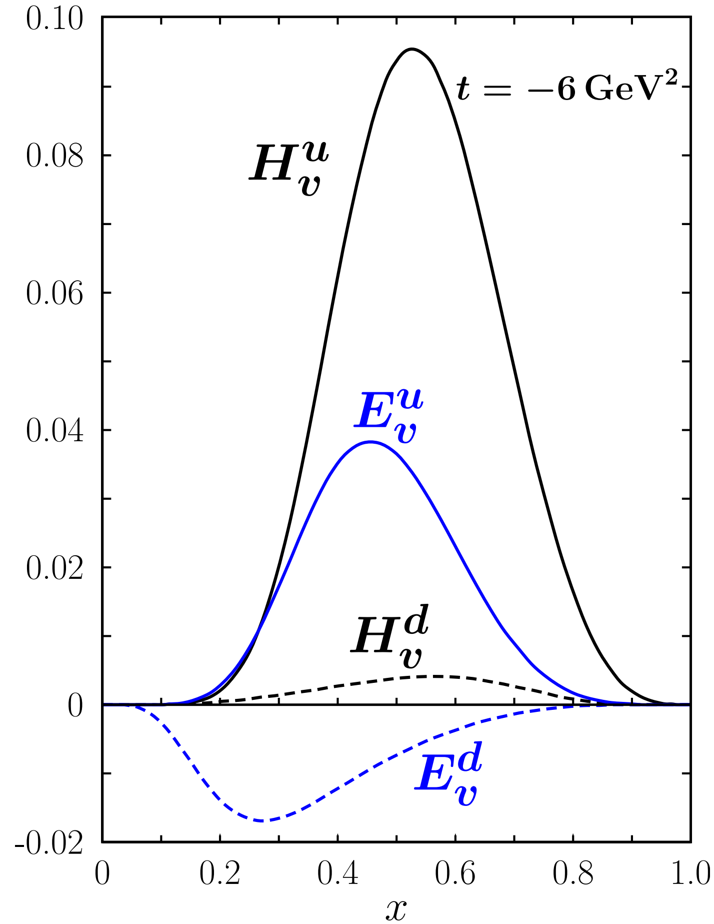
$$\int_{x_{max}(0)}^{1(x_{min})} dx K_v^q(x,t) = 5\% F_i^q(t)$$

$x \leftrightarrow t$ correlation

green regions - GPDs not determined

The GPDs H and E ($\mu = 2\,\text{GeV}$)





$K_v^q \sim x^{-\alpha_q - t\alpha'_q}$ at small x

singular (zero) at small (large) $-t$

makes $x \leftrightarrow t$ correlation obvious

$K_v^q \sim (1-x)^{\beta_q}$ at large x

pronounced peak

position moves to larger x and becomes narrower with increasing $-t$

Large$-t$ behavior of flavor form factors

at large t: dominance of narrow region of large x:

$q_v \sim (1-x)^{\beta_q}$, $f_q \sim A_q(1-x)^2$ (analogously for F_2^q)

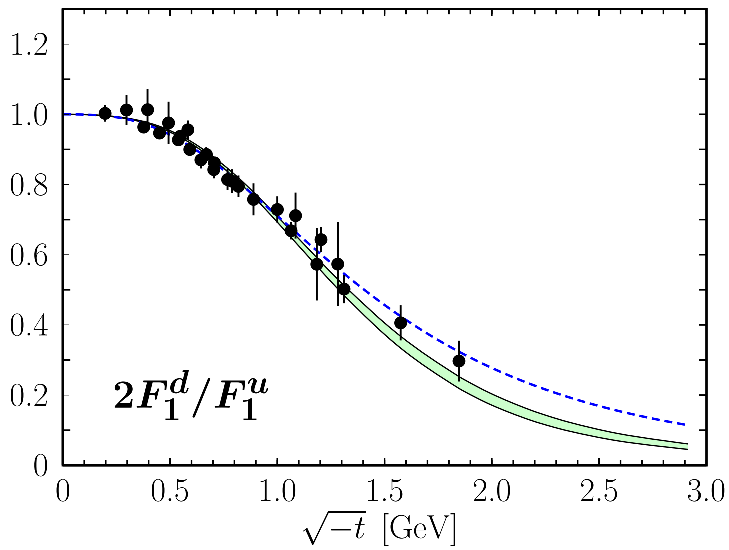
Saddle point method provides $1 - x_s = \left(\frac{2}{\beta_q} A_q |t|\right)^{-1/2}$ $F_1^q \sim |t|^{-(1+\beta_q)/2}$

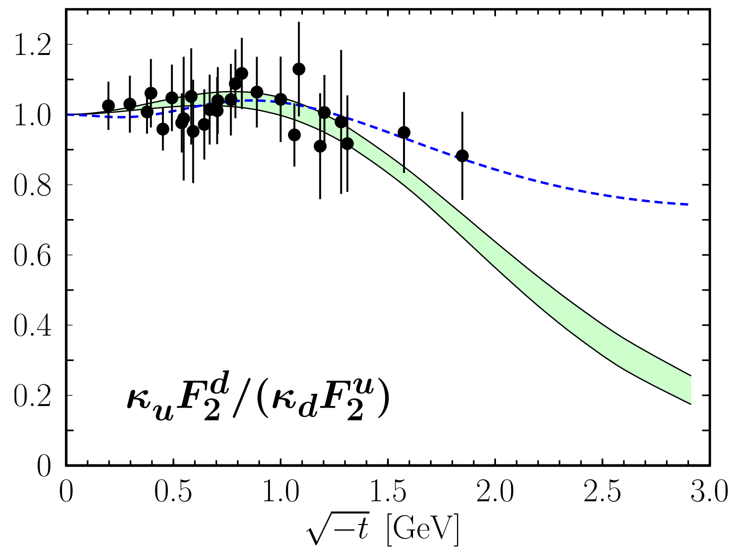
derivation of power law requires that x_s lies in sensitive x-region

active parton carries most of proton momentum while the spectators are soft

region of Feynman mechanism (see also Jain-Ralston (14))

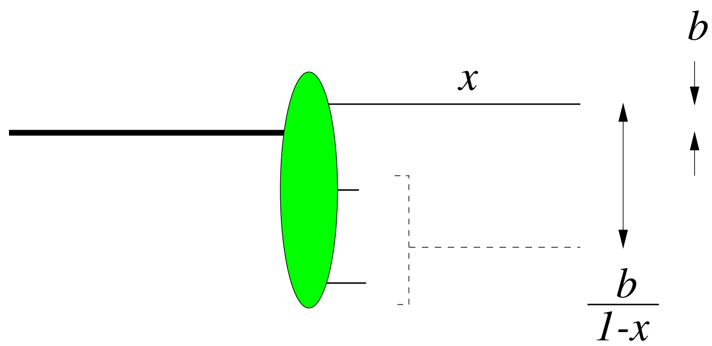
sheds doubts on usefullness of 'scaling' tests





ABM PDFs: $\beta_u \simeq 3.4$, $\beta_d \simeq 5$, e_v^q: $\beta_u = 4.65$, $\beta_d = 5.25$

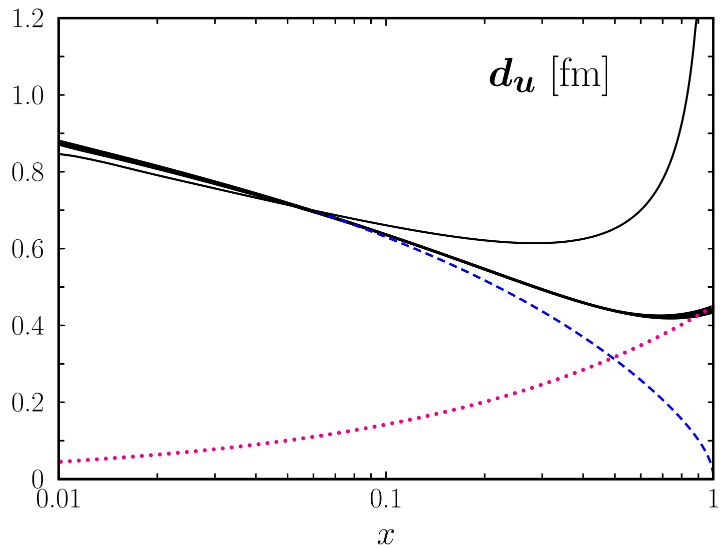
Impact parameter distributions



consider the FT of K_v^q

$\mathbf{b}$ transverse distance between struck quark and proton's center of momentum: $\sum_i x_i \mathbf{b_i} = 0$ (chosen, $\sum_i x_i = 1$)

$\mathbf{b}/(1-x)$ relative distance between struck quark and cluster of spectators



average distance:

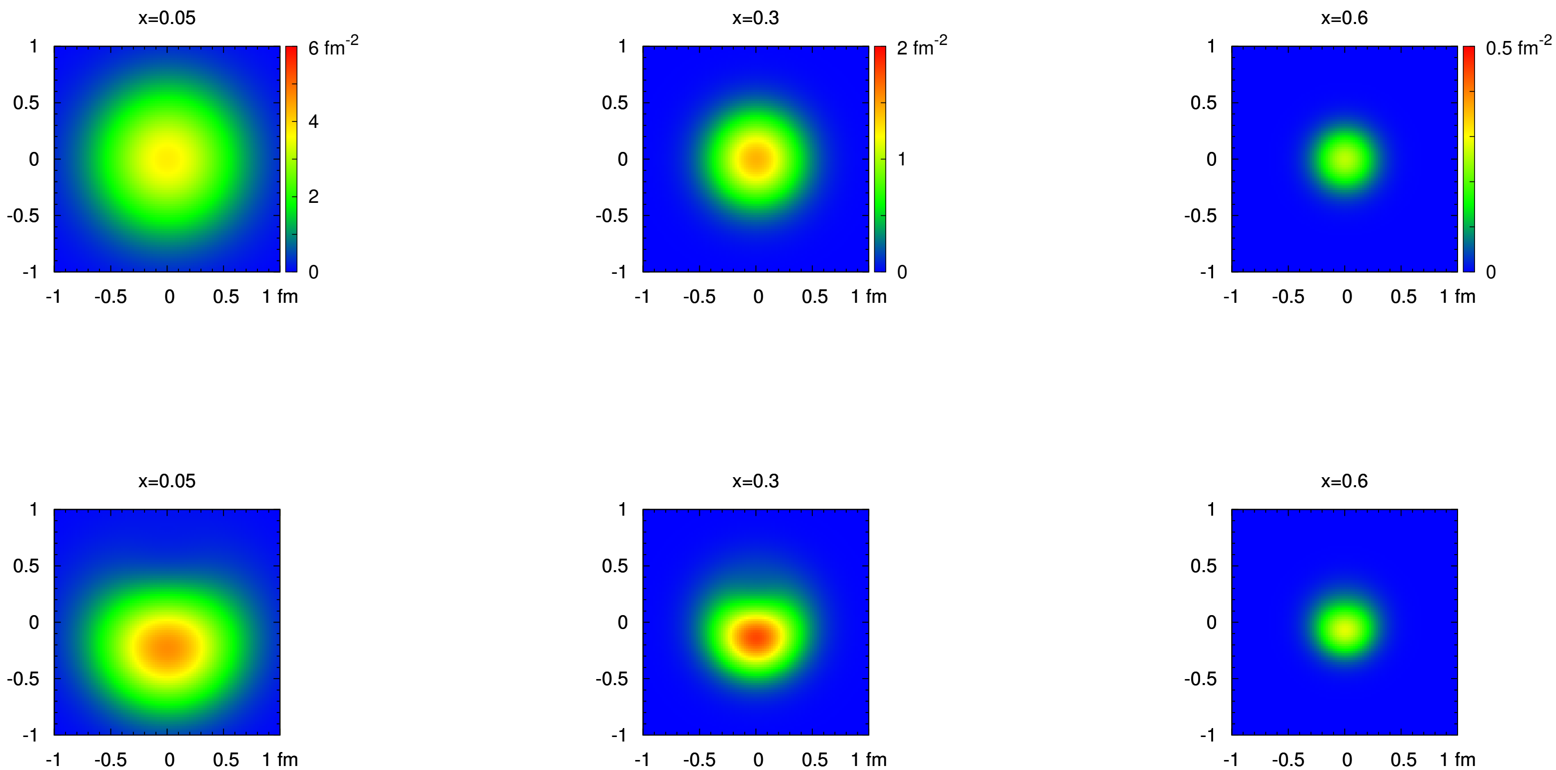
$$d_q = \frac{\sqrt{\langle b^2 \rangle_x^q}}{1-x} \quad = 2\frac{\sqrt{f_q(x)}}{1-x}$$

d_q provides estimate of size of hadron

Regge-type term; A-term; full profile fct

Frequently applied Regge parametrization $f_R = \alpha_R' \ln 1/x + B_R$ can only be used at small x (small $-t$) unphysical at large x (large $-t$) (confinement)

Tomography of d_v quarks



$$q_v^X(x,\mathbf{b}) = q_v(x,\mathbf{b}) - \frac{b^y}{m}\frac{\partial}{\partial \mathbf{b}^2} e_v^q(x,\mathbf{b})$$

u-quark density shifted upwards

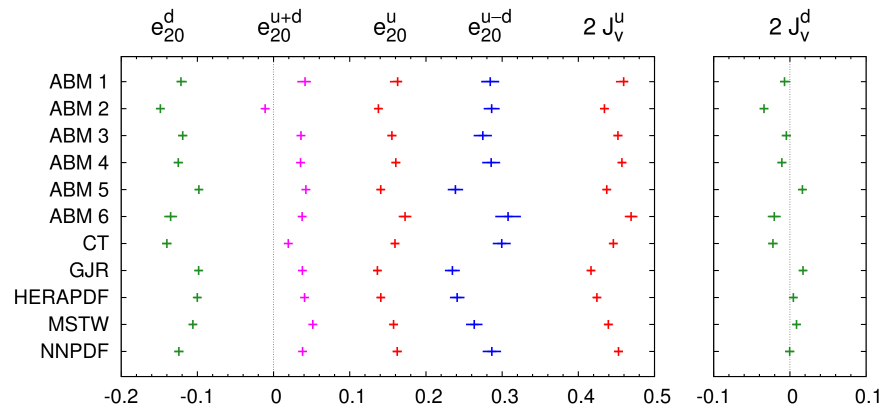
Burkardt(02), Jaffe(96)

Ji's sum rule

Ji(96) $2J_v^q = h_{20}^q + e_{20}^q$ $h_{20}^q(\xi = t = 0)$ known from PDFs

sum of 2nd moments at $t = 0$ gives two times the angular momentum carried by quarks of flavor q minus corresponding antiquark contribution

fits	$2J_v^u$	$2J_v^d$
ABM 1	$0.460^{+0.006}_{-0.010}$	$-0.007^{+0.008}_{-0.006}$
all fits	$0.460^{+0.018}_{-0.048}$	$-0.007^{+0.021}_{-0.033}$
ABM 0	$0.560^{+0.009}_{-0.010}$	$-0.019^{+0.009}_{-0.009}$



(ABM 0: $\mu = 1\,\mathrm{GeV}$)

orbital angular momentum: subtract $\tilde{h}_{10}^v(0)$ from polarized PDFs DSSV(09)

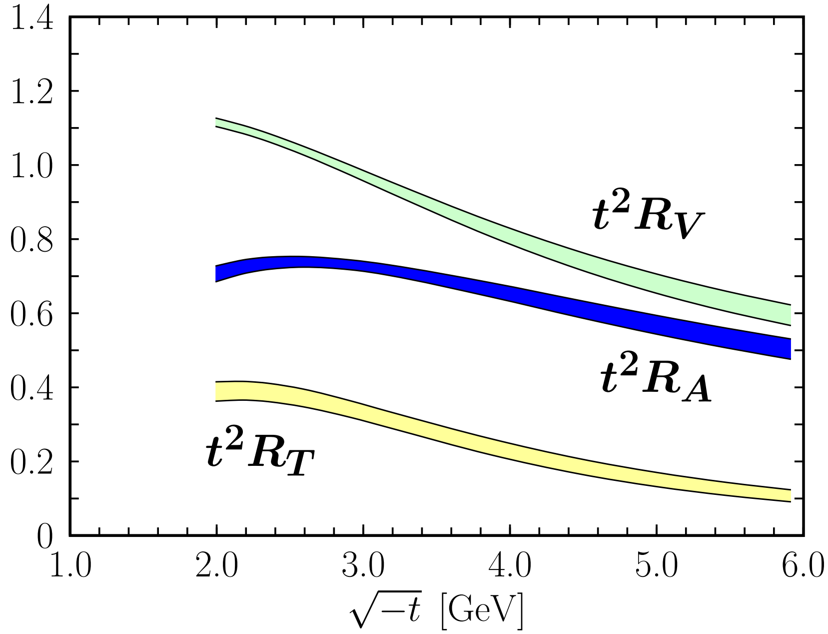
$$L_v^u = -0.141^{+0.025}_{-0.033} \qquad L_v^d = 0.114^{+0.034}_{-0.035}$$

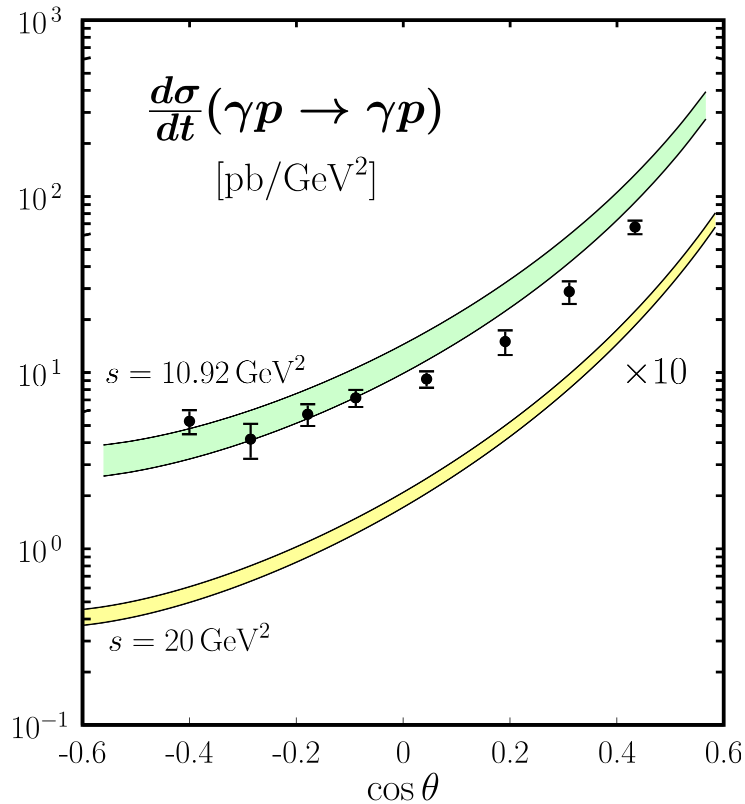
differs from DFJK4 - Δq_v from BB(02) and DSSV(09) differ

errors likely underestimated, systematic study of all pol. PDFs not done

The Compton cross section

$s,\ -t,\ -u \gg m^2$





$R_V = \sum_q e_q^2 \int_0^1 \frac{dx}{x} H^{q_v}(x,t)$

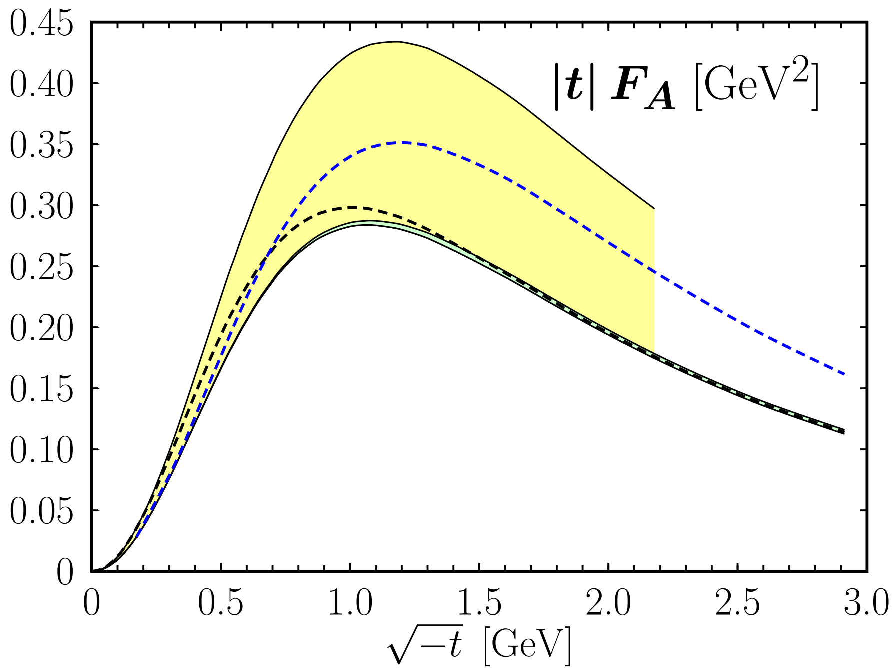
$$\frac{d\sigma}{dt} = \frac{\pi \alpha_{\rm em}^2}{s^2} \frac{(s-u)^2}{-us} \left[R_V^2(t) - \frac{t}{4m^2} R_T^2(t) + \frac{t^2}{(s-u)^2} R_A^2(t) \right] + \mathcal{O}(\alpha_s)$$

data: JLab E99-114 $\quad (\cos\theta = \pm 0.6\text{: } t(u) = -1.85(-1.76)\,\mathrm{GeV}^2)$

parameter free prediction

Radyushkin hep-ph/9803316; DFJK hep-ph/9811253; Huang-K-Morii hep-ph/0110208

The axial form factor



sum rule:

$$F_A(t) = \int_0^1 dx \Big[\widetilde{H}_v^u - \widetilde{H}_v^d \Big] + 2 \int_0^1 dx \Big[\widetilde{H}^{\bar{u}} - \widetilde{H}^{\bar{d}} \Big]$$

no new data; old data (covering a fairly large range of t): Kitagaki et al (83)
in form of dipole parametrization (yellow band)

no attempt to analyse it like H, E Δq from DSSV(09)

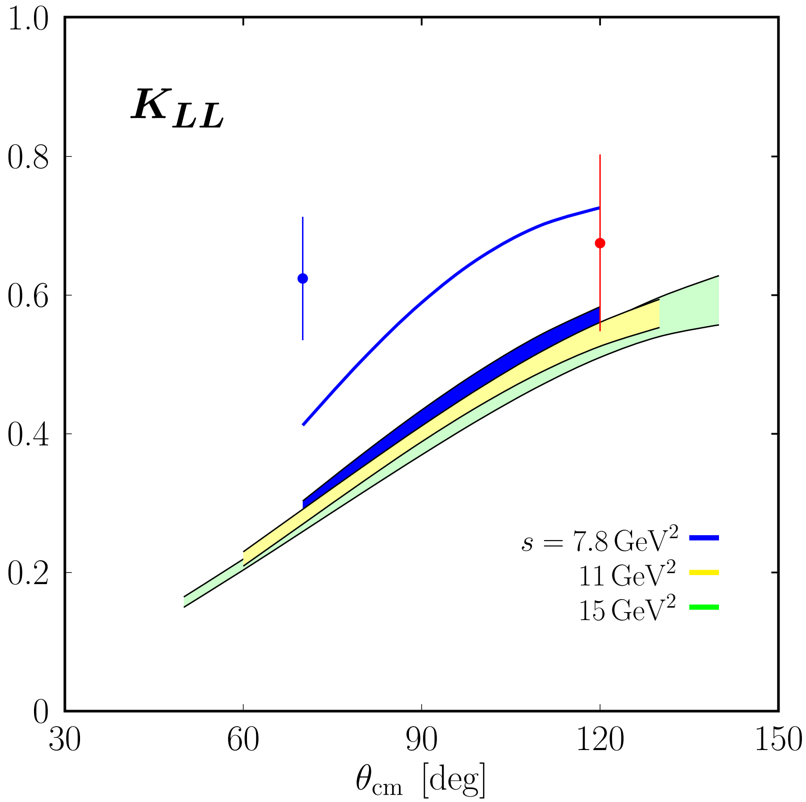
use $\widetilde{H}^q = \Delta q(x) \exp\left[t \tilde{f}_q(x,t)\right]$ assume $\tilde{f}_q = f_q$ (green band)

density interpretation of $q_v(x,b^2) \pm \Delta q_v(x,b^2)$ implies $\tilde{f}_q \leq f_q$

$\tilde{f}_q < f_q$ increases F_A: 50% increase of F_A possible (blue dashed line)

black dashed line: with an estimate of sea contribution

Spin correlations in WACS



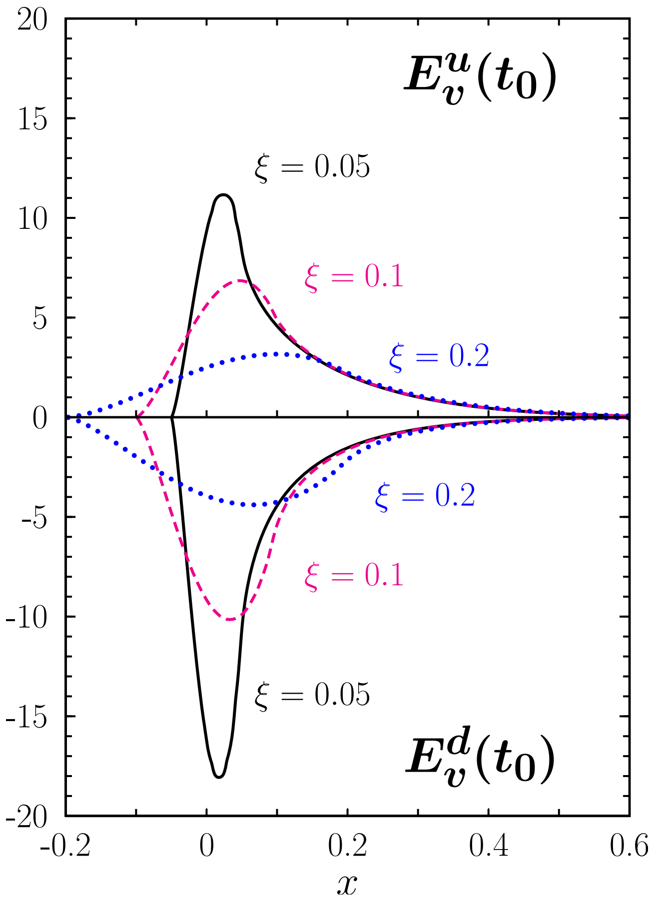
$$K_{LL} \simeq A_{LL} \simeq K_{LL}^{KN}\frac{R_A}{R_V} \times \left[1 - \frac{t^2}{2(s^2+u^2)}(1 - \frac{R_A^2}{R_V^2})\right]^{-1}$$

Klein-Nishina for massless quarks:
$K_{LL}^{KN} = (s^2 - u^2)/(s^2 + u^2)$

data: E07-002 $(s = 7.8\,\mathrm{GeV}^2 \quad t = -2.1\,\mathrm{GeV}^2)$
E99-114 $(s = 6.9\,\mathrm{GeV}^2 \quad u = -1.04\,\mathrm{GeV}^2)$
proton mass is not really negligible at available kinematics

R_A not well determined? see example

Skewness dependence, DVCS and DVMP



Radyushkin-Musatov(2000)
double distribution ansatz: (no D-term)

$$K_v^q(x,\xi,t) = \int_0^1 d\rho \int_{\rho-1}^{1-\rho} d\eta \delta(\rho+\xi\eta-x) \times K_v^q(\rho,0,t) w(\rho,\eta) \quad (1)$$

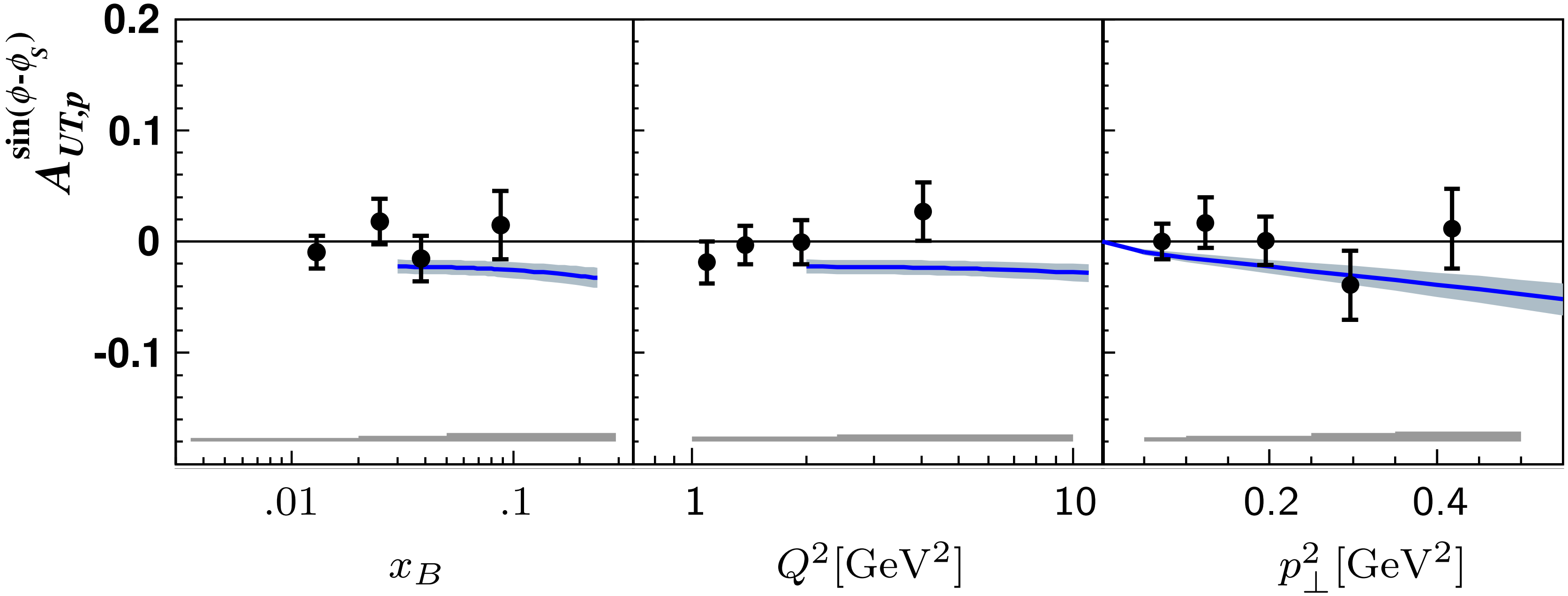
frequently used:

$$w = \frac{3}{4}\frac{(1-\rho)^2-\eta^2}{(1-\rho)^3}$$

minimum value of $-t$:
$-t_0 = 4m^2\xi^2/(1-\xi^2)$

frequently applied for parametrizations of GPDs
succesful applications to DVCS and DVMP, in particular at small ξ
analysis of ρ^0 and ϕ cross section data fixes well H for gluon and sea quarks for given valence-quark H

Probing E



ρ^0 production: COMPASS(12) $W = 8.1\,\mathrm{GeV}$ $\qquad A_{UT}^{\sin(\phi-\phi_s)} \sim \mathrm{Im}[\mathcal{E}^*\mathcal{H}]$

Goloskokov-K.(08) $\qquad$ ($\mathcal{K}$ convolution)

sum rule for 2nd moments of E at $\xi = t = 0$: $\quad$ Diehl-Kugler(07)

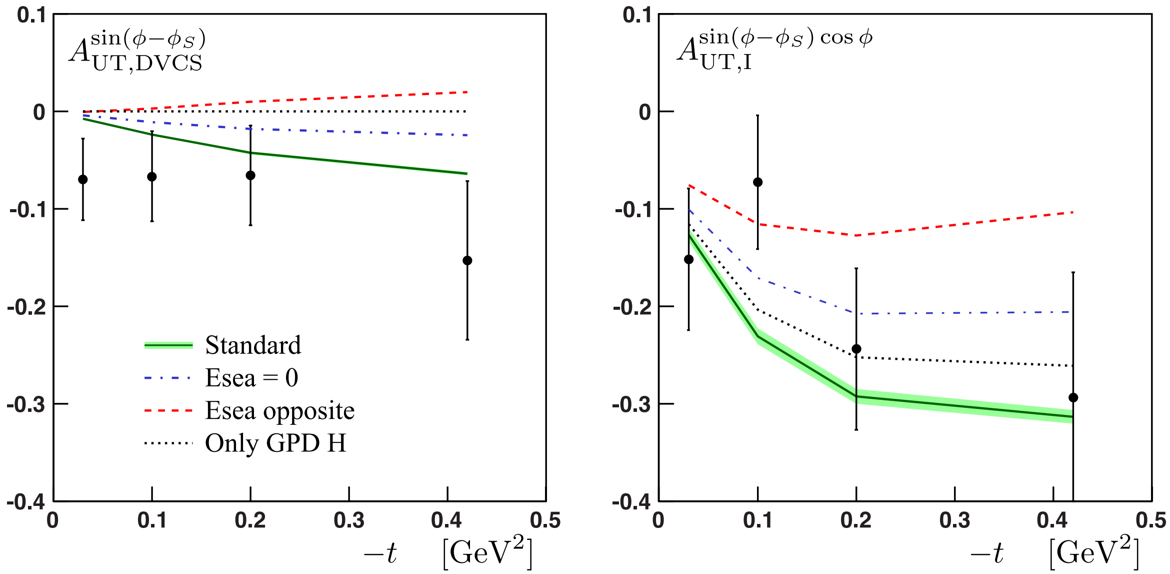
$e_{20}^g = -\sum e_{20}^{q_v} - 2\sum e_{20}^{\bar{q}} \qquad e_{20}^{u_v} + e_{20}^{d_v}$ very small

2nd moments of gluon and sea cancel almost completely

no nodes: other moments too

(ϕ azimuthal angle between lepton and hadron plane, ϕ_s orientation of target spin vector)

Probing E cont.



DVCS data: HERMES(08) $Q^2 \simeq 2.5\,\mathrm{GeV}^2$ K.-Moutarde-Sabatie(13)

LO: no contrib. from gluons no cancellation, we see $E^{\rm sea}$

restriction of $|E^s|$ by positivity bound (flavor symm. sea assumed)

$$\frac{\left[e^s(x)\right]^2}{8m^2} \leq \exp(1)\left[\frac{g_s(x)}{f_s(x)}\right]^3 \left[f_s(x) - g_s(x)\right]\left\{\left[s(x)\right]^2 - \left[\Delta s(x)\right]^2\right\}$$

negative $E^{\rm sea}$ favored in both cases

Angular momenta of partons

Ji's sum rule for 2nd moments of H and E

$$J^a = \frac{1}{2}\Big[q^a_{20} + e^a_{20}\Big] \qquad J^g = \frac{1}{2}\Big[g_{20} + e^g_{20}\Big] \qquad (\xi = t = 0)$$

q^a_{20}, g_{20} from ABM11 (NLO) PDFs $(a = u, d, s, \bar{u}, \bar{d}, \bar{s})$

$e^{a_v}_{20}$ from form factor analysis Diehl-K. (13):

$e^s_{20} \simeq 0 \ldots - 0.026$ from analysis of A_{UT} in DVCS and pos. bound

e^g_{20} from sum rule for e_{20} $(e^g_{20} \simeq -6e^s_{20})$

(Goloskokov-K (09), K. 1410.4450)

$$\begin{aligned} J^{u+\bar{u}} &= 0.261 \ldots 0.235\,; \\ J^{d+\bar{d}} &= 0.035 \ldots 0.009\,; \\ J^{s+\bar{s}} &= 0.017 \ldots - 0.009\,; \\ J^g &= 0.187 \ldots 0.265\,; \end{aligned}$$

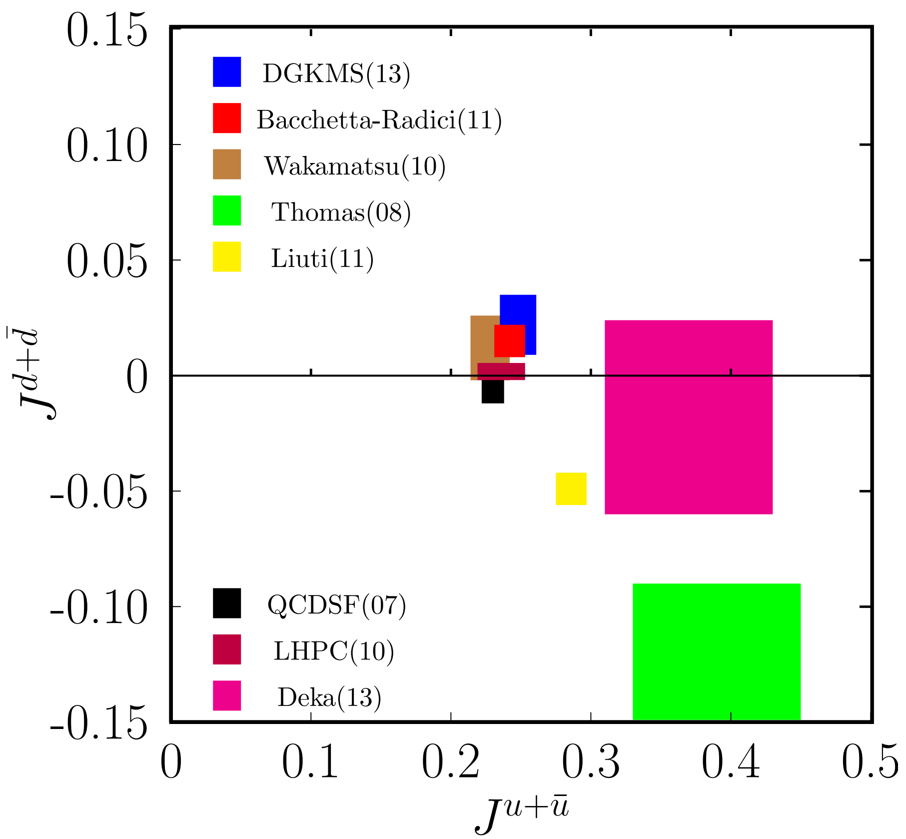
J^i quoted at scale $2\,\mathrm{GeV}$

$\sum J^i = 1/2$

(spin of the proton)

need better determ. of E^{sea} (smaller errors of A_{UT} in DVCS)

Comparison with other results



Deka et al (13): $J^g = 0.14 \pm 0.04$

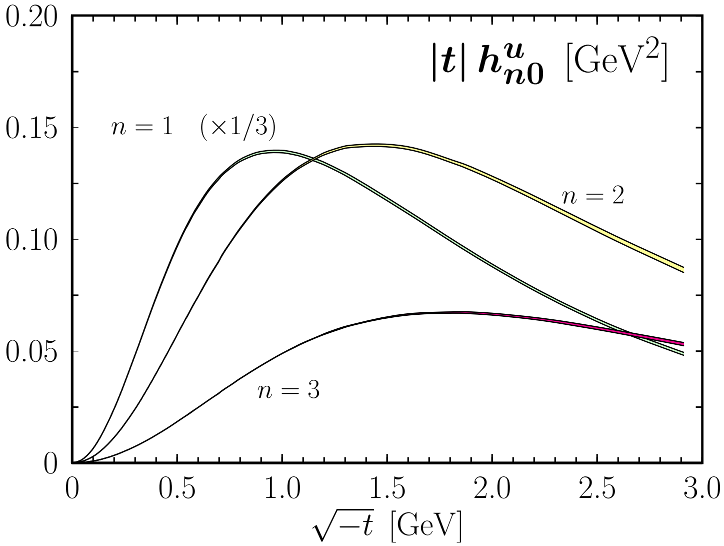
our value: $J^g = 0.187 \ldots 0.265$

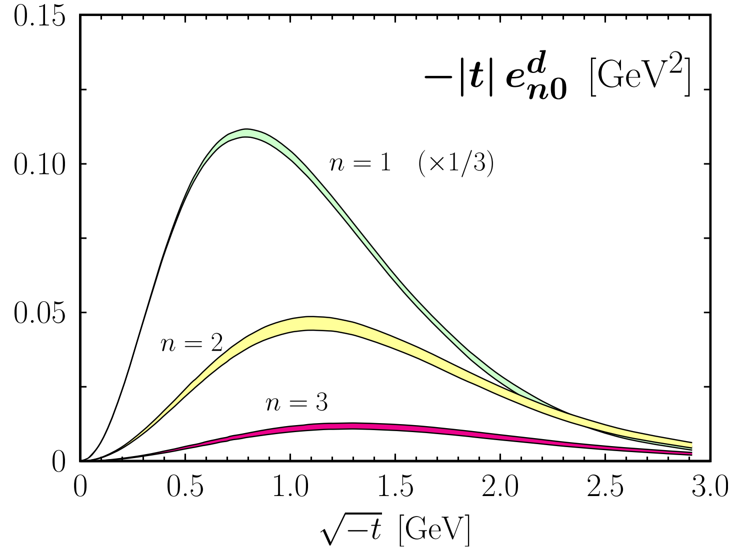
Summary

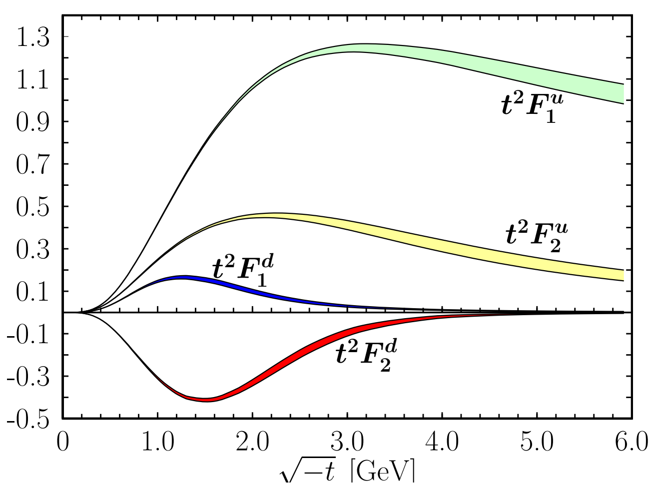
- On the basis of physically motivated parameterizations information on H and E for valence quarks and at $\xi = 0$ has been extracted from data on nucleon elm. form factors
- Parameterization is not unique but results are theoretically consistent and imply the physics of the Feynman mechanism at large t
- Analysis can be improved with large $-t$ form factor data from Jlab12
- $J_v^u = 0.230^{+0.009}_{-0.024}$, $L_v^u = -0.141^{+0.025}_{-0.033}$, $J_v^d = -0.004^{+0.010}_{-0.016}$, $L_v^d = 0.114^{+0.034}_{-0.035}$ and J for all quarks and gluons
- Polarized and unpolarized WACS can be predicted now and found to be in reasonable agreement with experiment (RA,m!)
- The zero skewness GPDs in combination with the double distribution ansatz leads to GPDs (and estimates of K for gluons and sea quarks) which allow to compute DVCS and DVMP in fair agreement with experiment

Backup

Mellin moments







$$k^q_{n0}(t) = \int_0^1 dx x^{n-1} K^q_v(x,t)$$

strong decrease with index n

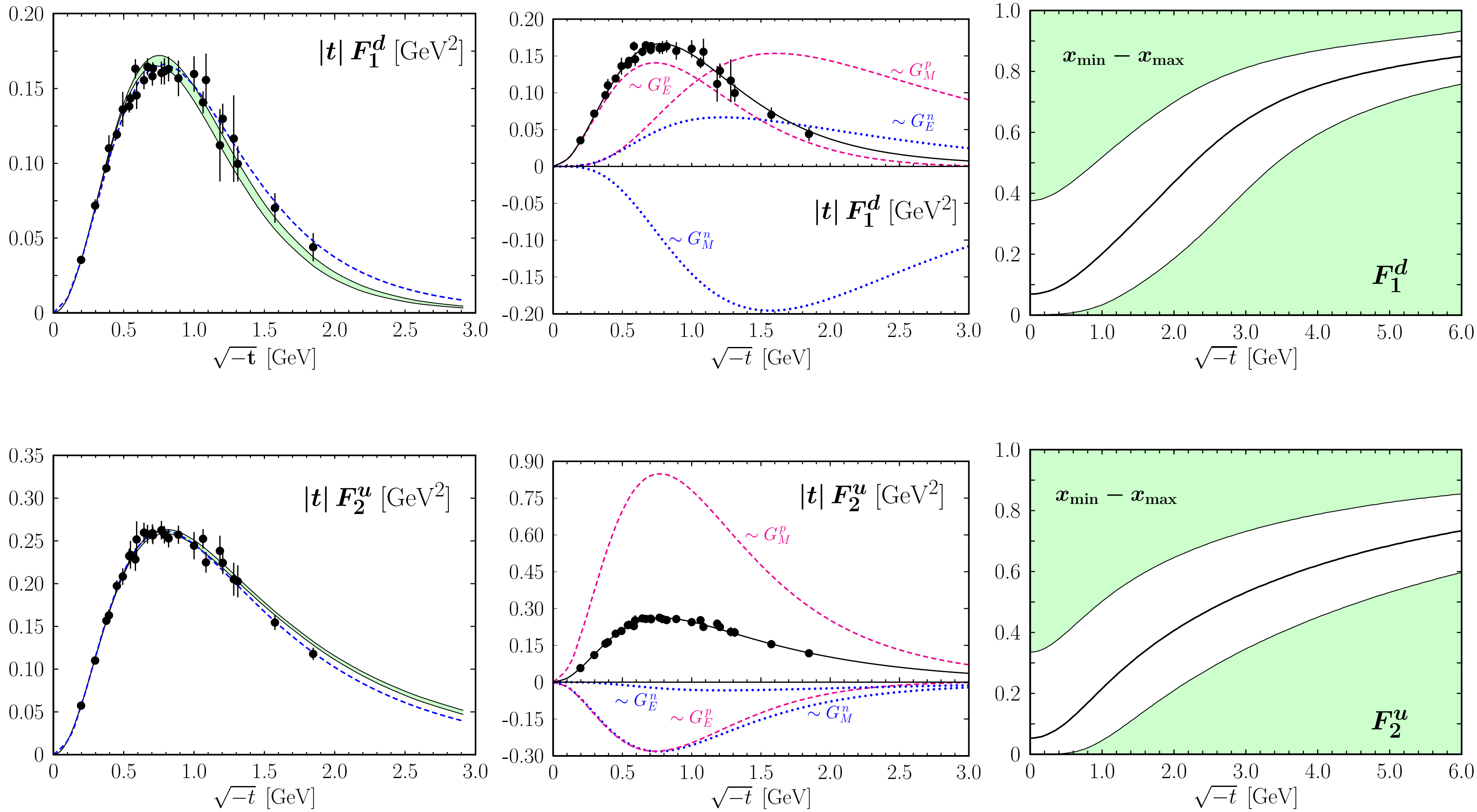
(because of strong decrease of GPDs with x)

decrease of moments with $-t$ becomes slower as n increases

u moments are larger than d ones at large $-t$

explains why F_i^n negative

The flavor form factors



fit to the flavor FFs (interpolated FF data)

decomposition into Sachs FFs

$$\int_{x_{max}(0)}^{1(x_{min})} dx K_v^q(x,t) = 5\% F_i^q(t)$$

$x \leftrightarrow t$ correlation

green bands - GPDs not determined