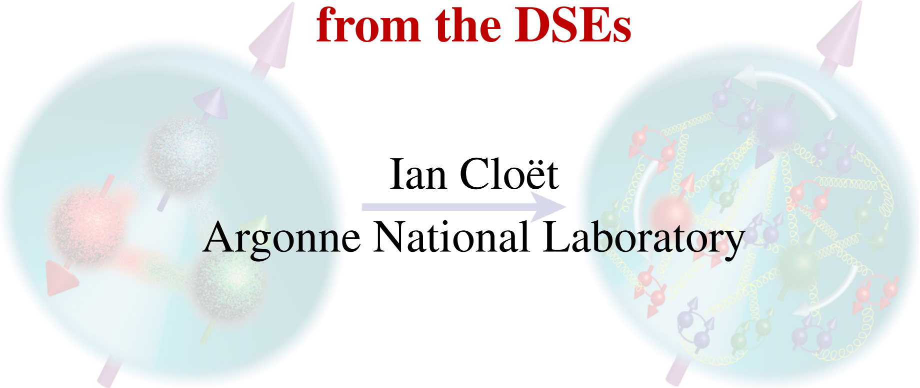
Nucleon Transverse Structure from the DSEs



Ian Cloët

Argonne National Laboratory

Probing transverse nucleon structure at high momentum transfer

ECT*, 18–22 April 2016

U.S. DEPARTMENT OF ENERGY | Office of Science

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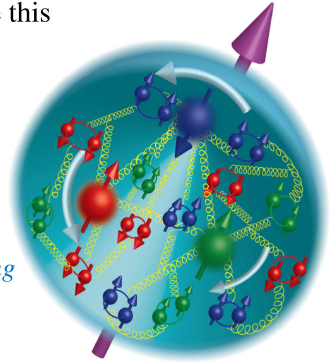
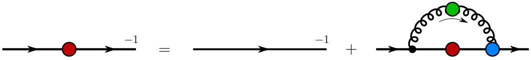
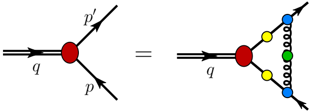
QCD: The Unifying Challenge

- Understanding QCD means to chart and compute this distribution of matter and energy within hadrons and nuclei – together with the complementary process of fragmentation functions
  - *a priori* have no idea what QCD can produce – but gives raise to ∼98% of mass in the visible universe
  - must understand the emergent phenomena of *confinement* and *dynamical chiral symmetry breaking*
  - *best promise for progress is a strong interplay between experiment and theory*



- A key pathway is provided by new data on nucleon elastic form factors, TMDs, etc $\Longrightarrow$ diquarks, OAM, etc
- *In the DSEs an understanding of QCD is gained by exposing the properties and behaviour of its dressed propagators, dressed vertices and interaction kernels*





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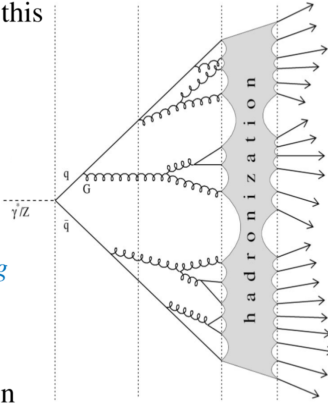

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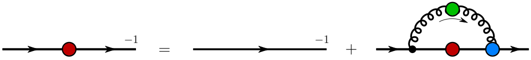
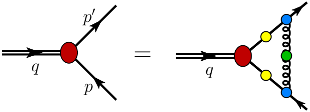


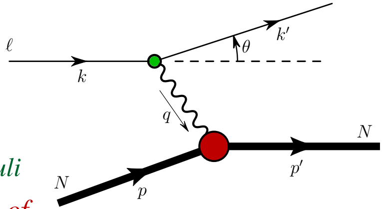


Nucleon Electromagnetic Form Factors

- Nucleon electromagnetic current

$$\langle J^\mu \rangle = \bar{u}(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\,M} F_2(Q^2) \right] u(p)$$

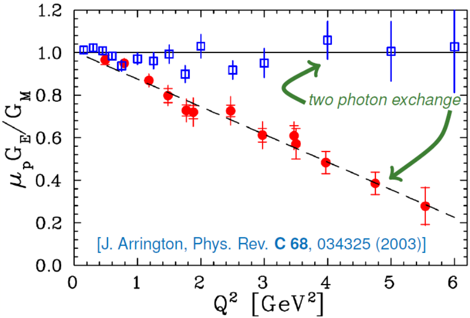
Dirac (F_1) *Pauli* (F_2)



- *Provide vital information on the distribution of charge and magnetization within hadrons and nuclei*
  - form factors also directly probe confinement at all energy scales
- Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:
  - proton radius puzzle
  - $\mu_p\, G_{Ep}/G_{Mp}$ ratio and a possible zero-crossing
  - flavour decomposition and evidence for diquark correlations
  - meson-cloud effects
  - seeking verification of perturbative QCD scaling predictions & scaling violations

Nucleon Sachs Form Factors

- Experiment gives Sachs form factors: $G_E = F_1 - \frac{Q^2}{4\,M^2} F_2 \quad G_M = F_1 + F_2$
- Until the late 90s Rosenbluth separation experiments found that the $\mu_p\, G_{Ep}/G_{Mp}$ ratio was flat
- Polarization transfer experiments completely altered our picture of nucleon structure
  - distribution of charge and magnetization are not the same



- Proton charge radius puzzle $[7\sigma]$

$$\left\langle r_E^2 \right\rangle = -6\, \frac{\partial}{\partial Q^2}\, G_E(Q^2)\Big|_{Q^2=0}$$

$$r_{Ep} = 0.84087 \pm 0.00039 \text{ fm}$$

muonic hydrogen [Pohl *et al.* (2010)]

$$r_{Ep} = 0.8775 \pm 0.0051 \text{ fm}$$

CODATA: $e\,p$ + e-hydrogen

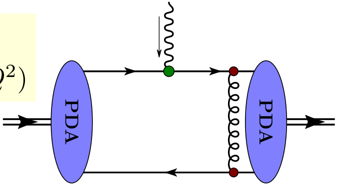
- one of the most interesting puzzles in hadron physics
- so far defies explanation

Form Factors in Conformal Limit ($Q^2 \to \infty$)

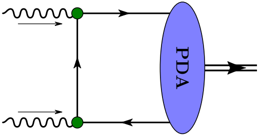
- At asymptotic energies hadron form factors factorize into *parton distribution amplitudes* & a hard scattering kernel [Farrar, Jackson; Lepage, Brodsky]
  - only the valence Fock state ($\bar{q}q$ or qqq) can contribute as $Q^2 \to \infty$
  - both confinement and asymptotic freedom in QCD are important in this limit

- Most is known about $\bar{q}q$ bound states, e.g., for the pion:

$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2)$$



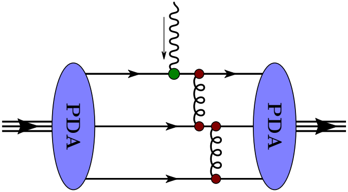
$$Q^2\, F_{\gamma^*\gamma\pi}(Q^2) \to 2\, f_\pi$$



- For the nucleon, normalization is not known

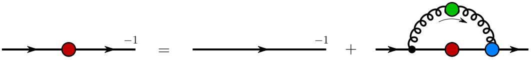
$$G_{E,M}(Q^2 \to \infty) \propto \alpha_s^2(Q^2)/Q^4$$

  - orbital angular momentum effects approach



- *Gluons play a critical role – formalism must reflex this!*

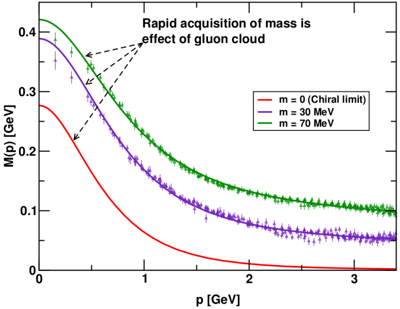
- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability $\Longrightarrow$ must implement a symmetry preserving truncation
- The most important DSE is QCD's gap equation $\Longrightarrow$ quark propagator



  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i\not p + M(p^2)}$$

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



- $S(p)$ has correct perturbative limit
- mass function, $M(p^2)$, exhibits dynamical mass generation
- complex conjugate poles
  - no real mass shell $\Longrightarrow$ confinement

Nucleon Structure

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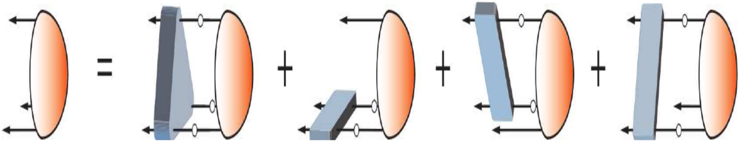
- A robust description of the nucleon as a bound state of 3 dressed-quarks can only be obtained within an approach that respects Poincaré covariance
- Such a framework is provided by the Poincaré covariant Faddeev equation

image from Gernot Eichmann



  - sums all possible interactions between three dressed-quarks
  - much of 3-body interaction can be absorbed into effecive 2-body interactions
  - *Faddeev eq. has solutions at discrete values of* $p^2\ (= M^2) \implies$ *baryon spectrum*
- A *prediction* of these approaches is that owing to DCSB in QCD – strong diquark correlations exist within baryons
  - any interaction that describes colour-singlet mesons also generates *non-pointlike* diquark correlations in the colour-$\bar{3}$ channel
  - where *scalar* (0^+) *& axial-vector* (1^+) *diquarks* most important for the nucleon

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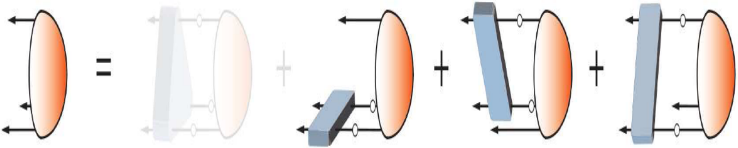

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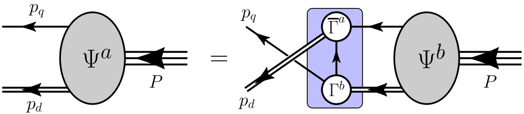
Diquarks

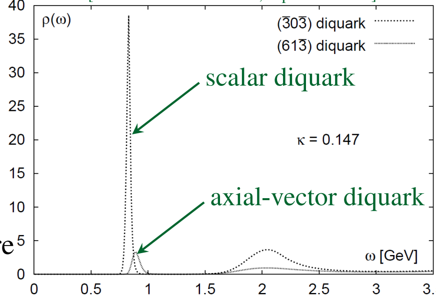
- Diquarks are dynamically generated correlations between quarks inside baryons



  - typically diquark sizes are similar to analogous mesons: $r_{0^+} \sim r_\pi$, $r_{1^+} \sim r_\rho$

- These dynamic qq correlations are not the static diquarks of old
  - all quarks participate in all diquark correlations
  - in a given baryon the Faddeev equation predicts a probability for each diquark cluster
  - for the nucleon: scalar $(0^+) \sim 70\%$
    axial-vector $(1^+) \sim 30\%$

[I. Wetzorke and F. Karsch, hep-lat/0008008]



- *Faddeev equation spectrum has significant overlap with constituent quark model and limited relation to Lichtenberg's quark+diquark model*

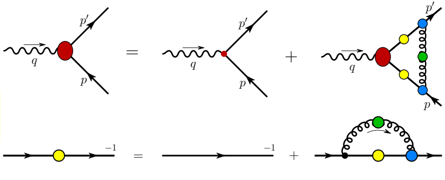
- Mounting evidence from hadron structure (e.g. PDFs, form factors) and lattice

Nucleon EM Form Factors from DSEs

- A robust description of form factors is only possible if *electromagnetic gauge invariance* is respected; equivalently all relevant *Ward-Takahashi identities* (WTIs) must be satisfied

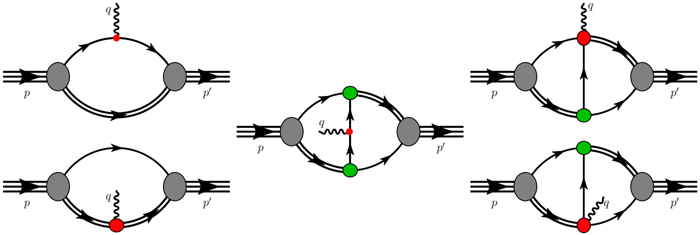
- For quark-photon vertex WTI implies:

$$q_\mu \, \Gamma^\mu_{\gamma qq}(p', p) = \hat{Q}_q \left[S_q^{-1}(p') - S_q^{-1}(p) \right]$$



  - transverse structure unconstrained

- Diagrams needed for a gauge invariant nucleon EM current in (our) DSEs



- Feedback with experiment can shed light on elements of QCD via DSEs

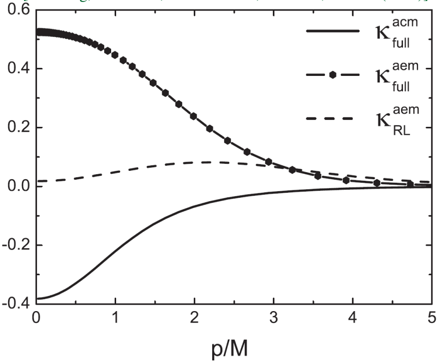
Beyond Rainbow Ladder Truncation

- Include *"anomalous chromomagnetic"* term in quark-gluon vertex

$$\frac{1}{4\pi} g^2 D_{\mu\nu}(\ell)\,\Gamma_\nu(p',p) \;\rightarrow\; \alpha_{\text{eff}}(\ell)\, D^{\text{free}}_{\mu\nu}(\ell)\,[\gamma_\nu + i\sigma^{\mu\nu} q_\nu\, \tau_5(p',p) + \ldots]$$

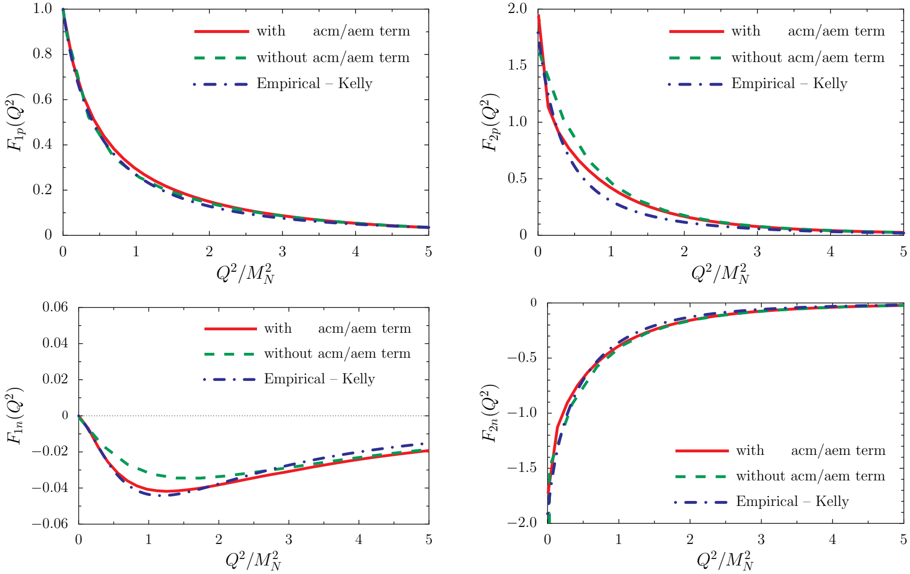
- In chiral limit *anomalous chromomagnetic* term can only appear through DCSB – not chirally symmetric and flips quark helicity
- EM properties of a spin-$\frac{1}{2}$ point particle are characterized by two quantities:
  - charge: e & magnetic moment: μ
- Expect strong gluon dressing to produce non-trivial electromagnetic structure for a dressed quark
  - recall dressing produces – from massless quark – a $M \sim 400\,\text{MeV}$ dressed quark
- Large anomalous chromomagnetic moment in the quark-gluon vertex – *produces a large quark anomalous electromagnetic moment*
  - *dressed quarks are not point particles!*

[L. Chang, Y. -X. Liu, C. D. Roberts, PRL **106**, 072001 (2011)]



Nucleon Dirac & Pauli form factors

[ICC, G. Eichmann, B. El-Bennich, T. Klahn and C. D. Roberts,, Few Body Syst. **46**, 1 (2009)]



- quark aem term has important influence on Pauli form factors at low Q^2

Proton G_E/G_M *Ratio*

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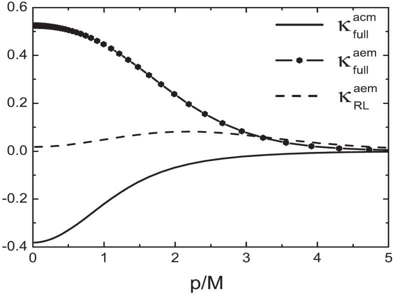
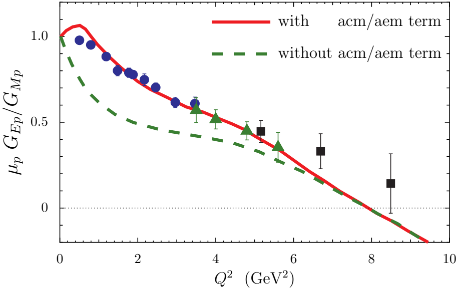
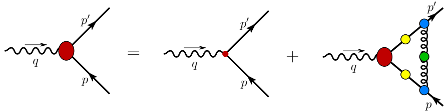
[L. Chang, Y. -X. Liu, C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011)]



[I. C. Cloët, C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014)]



- Quark anomalous magnetic moment required for good agreement with data
  - important for low to moderate Q^2
  - power law suppressed at large Q^2



- Illustrates how feedback with EM *form factor measurements* can help constrain the *quark–photon vertex* and therefore the *quark–gluon vertex* within the DSE framework
  - knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs $\Leftrightarrow$ confinement

Neutron G_E/G_M Ratio

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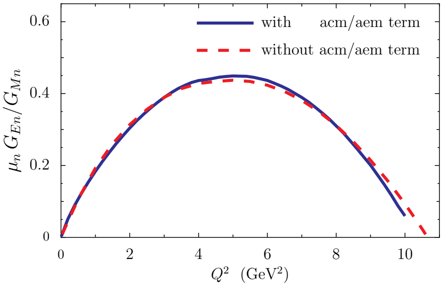
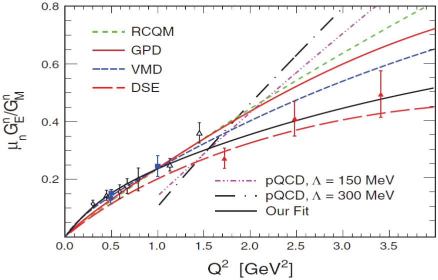
[ICC, C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014)]



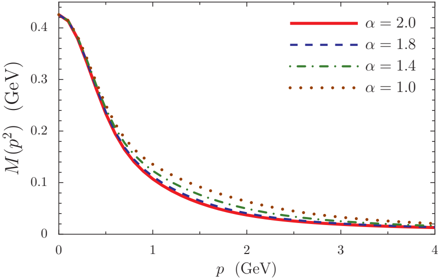
[S. Riordan *et al*, Phys. Rev. Lett. **105**, 262302 (2010)]

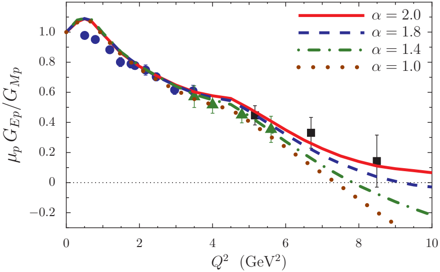


- Quark anomalous chromomagnetic moment – which drives the large anomalous electromagnetic moment – has only a minor impact on neutron Sachs form factor ratio
- Predict a zero-crossing in G_{En}/G_{Mn} at $Q^2 \sim 11\,\text{GeV}^2$
- Turn over in G_{En}/G_{Mn} will be tested at Jefferson Lab
- DSE *predictions* were confirmed on domain $1.5 \lesssim Q^2 \lesssim 3.5\,\text{GeV}^2$

Proton G_E form factor and DCSB

[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]





- Find that slight changes in $M(p^2)$ on the domain $1 \lesssim p \lesssim 3\,\text{GeV}$ have a striking effect on the G_E/G_M proton form factor ratio
  - *strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD*
- Zero in $G_E = F_1 - \frac{Q^2}{4\,M_N^2}\,F_2$ largely determined by evolution of $Q^2\,F_2$
  - F_2 is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment – *vanishes in perturbative limit*
  - the quicker the perturbative regime is reached the quicker $F_2 \to 0$

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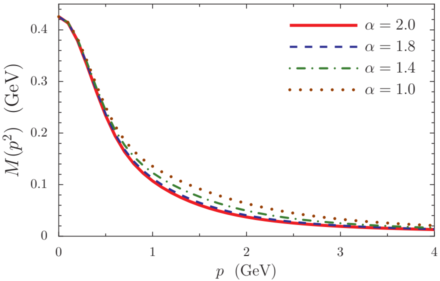
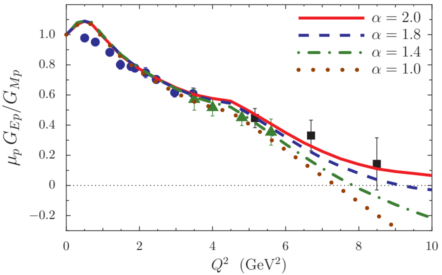
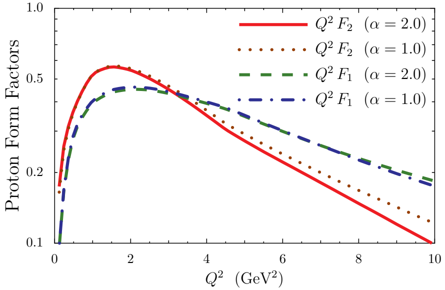
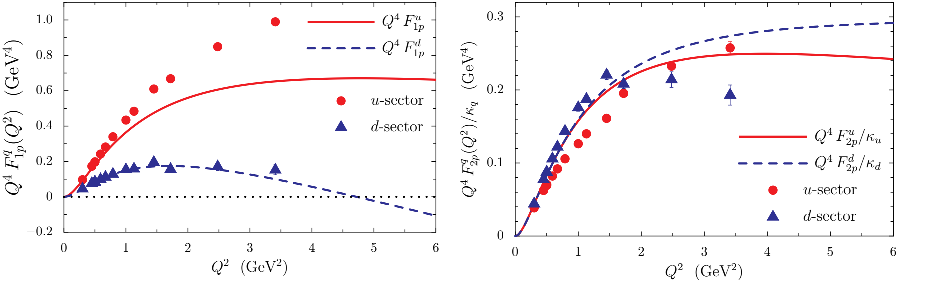




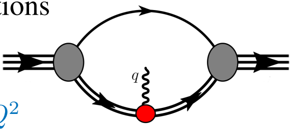


- Recall: $G_E = F_1 - \frac{Q^2}{4 M_N^2} F_2$
- Only G_E is senitive to these small changes in the mass function
- *Accurate determination of zero crossing would put important contraints on quark-gluon dynamics within DSE framework*

[I. C. Cloët, W. Bentz, A. W. Thomas, Phys. Rev. C **90**, 045202 (2014)]



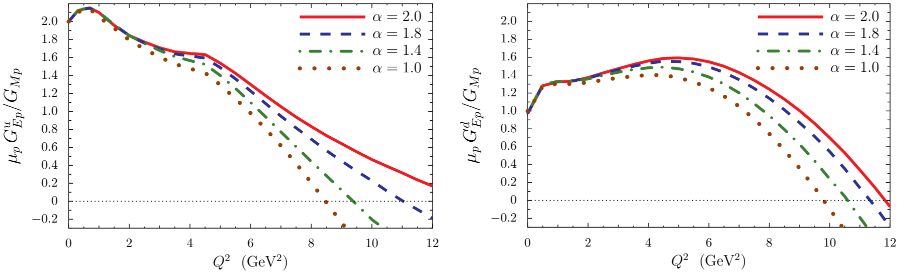
- Prima facie, these experimental results are remarkable
  - u and d quark sector form factors have very different scaling behaviour
- However, when viewed in context of diquark correlations results are straightforward to understand
  - in proton (uud) the d quark is "bound" inside a scalar diquark $[ud]$ 70% of the time; $u[ud]$ diquark $\implies 1/Q^2$



- Zero in F_{1p}^d a result of interference between scalar and axial-vector diquarks
  - location of zero indicates relative strengths – correlated with d/u ratio as $x \to 1$

Flavour separated Sachs ratio

[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]

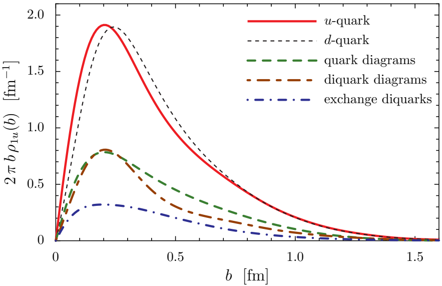


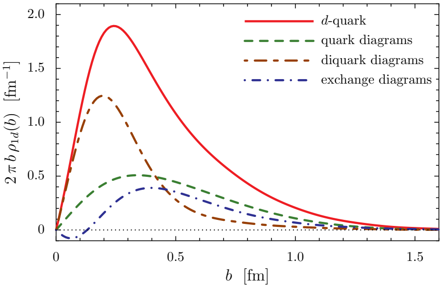
- Flavour sector form factors defined by:

$$f(Q^2) = e_u\, f_u(Q^2) + e_d\, f_d(Q^2)$$

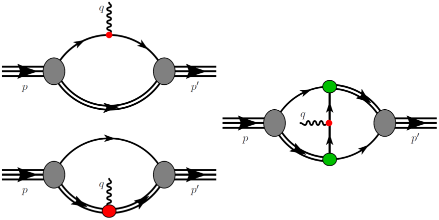
- Effect driven largely by the u-quark sector
- The singly represented d-quark is about 80% of the time inside a diquark
- The d-quark also becomes parton-like more quickly as α increases but it is hidden from view because of the diquark correlations

Proton Transverse Charge Densities





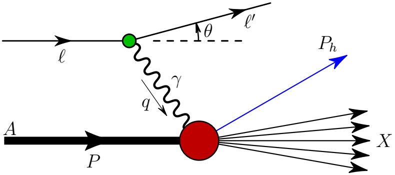
- Sum proportional to proton and difference to neutron transverse charge densities
- d-quarks sit at larger b than u-quarks
  - primarily from scalar-scalar diquark exchange type diagrams with exchanged d-quark



- About 10 different diagrammatic contributions – many subtle effects give rise to these densities

Probing Transverse Momentum with SIDIS

leading twist		quark polarization: unpolarized [U]	longitudinal [L]	transverse [T]
nucleon polarization	U	$f_1 =$ unpolarized		$h_1^\perp =$ − Boer-Mulders
	L		$g_1 =$ − helicity	$h_{1L}^\perp =$ − worm gear 1
	T	$f_{1T}^\perp =$ − Sivers	$g_{1T}^\perp =$ − worm gear 2	$h_{1T} =$ − transversity $h_{1T}^\perp =$ − pretzelosity

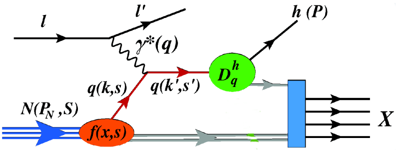


- The new frontier in hadron physics is the 3D imaging of the quarks & gluons
- SIDIS cross-section on nucleon has 18 structure functions – factorize as:

$$F(x, z, P_{h\perp}^2, Q^2) \propto \sum f^q(x, k_T^2) \otimes D_q^h(z, p_T^2) \otimes H(Q^2)$$

  - reveals correlations between parton transverse momentum, its spin & nucleon spin
- *Parametrization of these functions is not sufficient – must calculate in a framework with a well defined connection to QCD*
- Fragmentation functions are particularly challenging & therefore interesting

leading twist		quark polarization		
		unpolarized [U]	longitudinal [L]	transverse [T]
nucleon polarization	U	$f_1 =$ unpolarized		$h_1^\perp =$ − Boer-Mulders
	L		$g_1 =$ − helicity	$h_{1L}^\perp =$ − worm gear 1
	T	$f_{1T}^\perp =$ − Sivers	$g_{1T}^\perp =$ − worm gear 2	$h_{1T} =$ − transversity $h_{1T}^\perp =$ − pretzelosity



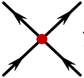
- The new frontier in hadron physics is the 3D imaging of the quarks & gluons
- SIDIS cross-section on nucleon has 18 structure functions – factorize as:

$$F(x, z, P_{h\perp}^2, Q^2) \propto \sum f^q(x, k_T^2) \otimes D_q^h(z, p_T^2) \otimes H(Q^2)$$

  - reveals correlations between parton transverse momentum, its spin & nucleon spin
- *Parametrization of these functions is not sufficient – must calculate in a framework with a well defined connection to QCD*
- Fragmentation functions are particularly challenging & therefore interesting

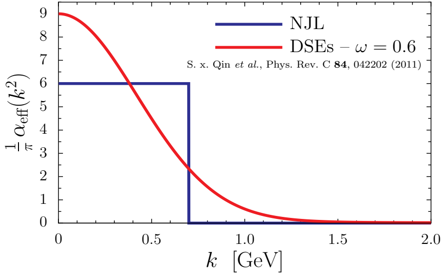
Continuum QCD

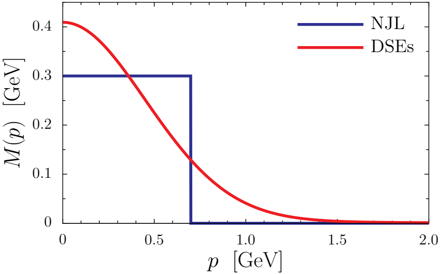
"integrate out gluons" →



$\frac{1}{m_G^2}\ \Theta(\Lambda^2 - k^2)$

- this is just a modern interpretation of the Nambu–Jona-Lasinio (NJL) model
- model is a Lagrangian based covariant QFT which exhibits dynamical chiral symmetry breaking & it elements can be QCD motivated via the DSEs

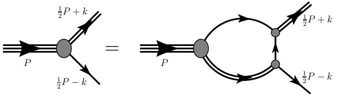




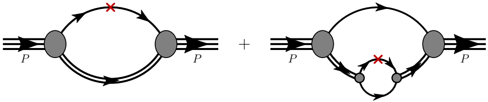
- The NJL model is very successful - provides a good description of numerous hadron properties: form factors, PDFs, in-medium properties, etc
  - however the NJL model has no direct link to QCD
  - in general NJL has no confinement – but can be implemented with proper-time RS

Nucleon quark distributions

- Nucleon = quark+diquark

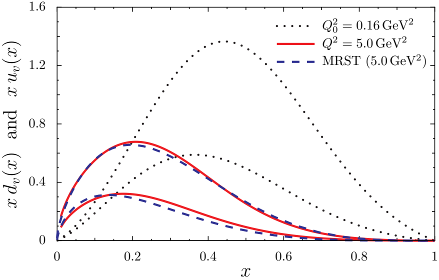


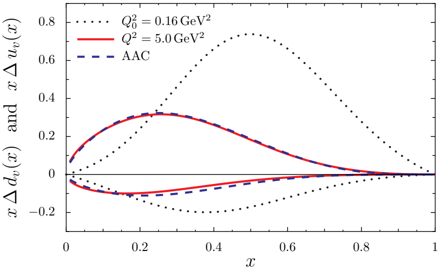
- PDFs given by Feynman diagrams: $\langle\gamma^+\rangle$



- Covariant, correct support; satisfies sum rules, Soffer bound & positivity

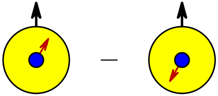
$$\langle q(x) - \bar{q}(x)\rangle = N_q, \quad \langle x\,u(x) + x\,d(x) + \ldots\rangle = 1, \quad |\Delta q(x)|\,,\ |\Delta_T q(x)| \leqslant q(x)$$





[ICC, W. Bentz and A. W. Thomas, Phys. Lett. B **621**, 246 (2005)]

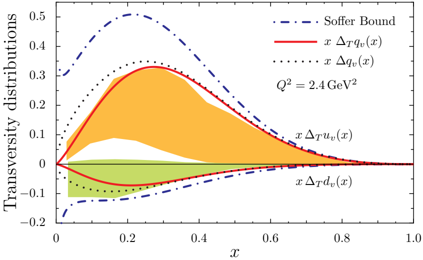
Nucleon transversity quark distributions

$\Delta_T q(x) =$ 

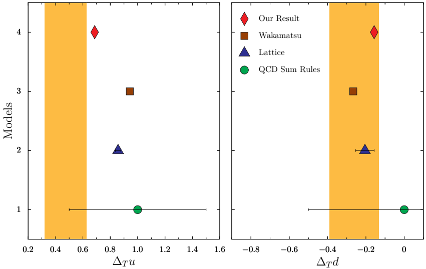
- Sum rule gives tensor charge

$$g_T = \int dx\, [\Delta_T u(x) - \Delta_T d(x)]$$

- quarks in eigenstates of $\gamma^\perp \gamma_5$
- Non-relativistically: $\Delta_T q(x) = \Delta q(x)$ – a measure of relativistic effects
- Helicity conservation: no mixing bet'n $\Delta_T q$ & $\Delta_T g$: $J \leqslant \frac{1}{2} \Rightarrow \Delta_T g(x) = 0$
- Therefore for the nucleon $\Delta_T q(x)$ is valence quark dominated
- At model scale we find: $g_T = 1.28$ *compare* $g_A = 1.267$ (input)



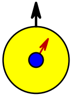
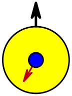
[ICC, W. Bentz and A. W. Thomas, Phys. Lett. B **659**, 214 (2008)]



[M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)]

Nucleon transversity quark distributions

Argonne NATIONAL LABORATORY

$\Delta_T q(x) =$  $-$ 

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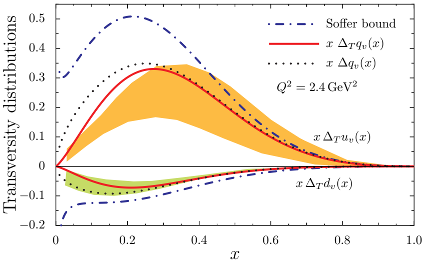
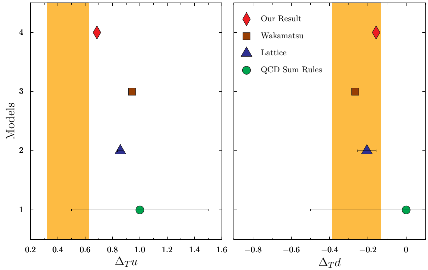
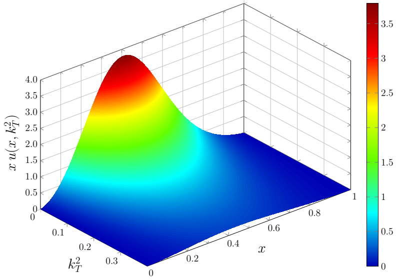


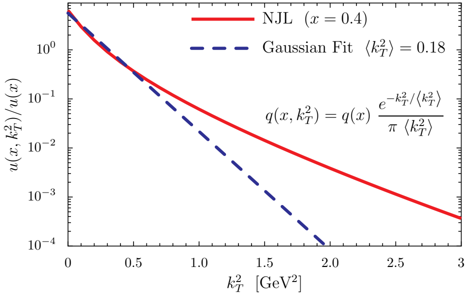
[ICC, W. Bentz and A. W. Thomas, Phys. Lett. B **659**, 214 (2008)]



[M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)]

Transverse Momentum Dependent PDFs





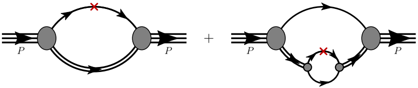
- So far only considered the simplest spin-averaged TMDs – $q(x, k_T^2)$
- Rigorously included transverse momentum of diquark correlations in TMDs

$$q_{D/N}(x,k_T^2) = \int_0^1 dy \int_0^1 dz \int d^2\vec{q}_\perp \int d^2\vec{\ell}_\perp \; \delta(x-yz)\;\delta(\vec{\ell}_\perp - \vec{k}_\perp - z\vec{q}_\perp)\; f_{D/N}(y,\vec{q}_\perp)\; f_{q/D}(z,\vec{\ell}_\perp)$$

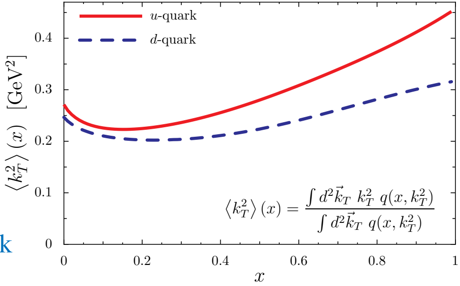
- Scalar diquark correlations greatly increase $\langle k_T^2\rangle$

$$\langle k_T^2\rangle_u^{Q^2=Q_0^2} = 0.43\,\text{GeV}^2 \qquad \langle k_T^2\rangle = 0.31\,\text{GeV}^2\ \text{[HERMES]},\quad 0.41\,\text{GeV}^2\ \text{[EMC]}$$

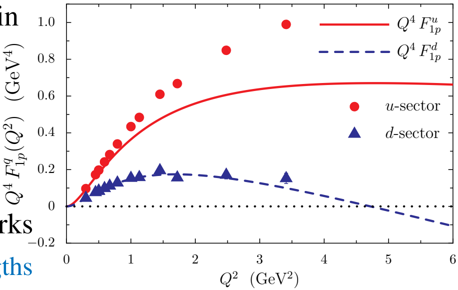
Flavour Dependence & Diquarks



- Scalar diquark correlations give sizable flavour dependence in $\langle k_T^2 \rangle$
  - 70% of proton (uud) WF contains a scalar diquark $[ud]$; $M_s \simeq 650\,\text{MeV}$, with $M \simeq 400\,\text{MeV}$ difficult for d-quark to be at large x



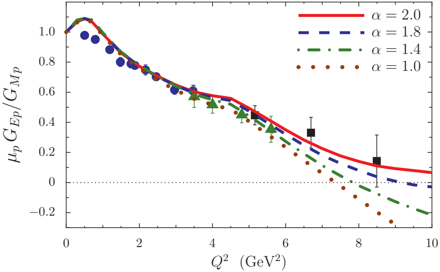
- Scalar diquark correlations also explain the very different scaling behaviour of the quark sector form factors
  - $u[ud]$ diquark $\Longrightarrow$ extra $1/Q^2$ for d
- Zero in F_{1p}^d a result of interference between scalar and axial-vector diquarks
  - location of zero indicates relative strengths – correlated with d/u ratio as $x \to 1$



[ICC, Bentz, Thomas, PRC **90**, 045202 (2014)]

Conclusion

- Using the DSEs we find that DCSB drives numerous effects in QCD, e.g., hadron masses, confinement and many aspects of hadron structure
  - e.g. location of zero's in form factors – G_{Ep}, F_{1p}^{d}, etc – provide tight constraints on QCD dynamics
  - predict zero in G_{En}/G_{Mn} independent rate of change of DCSB with scale



- Important progress toward nucleon TMD results
  - have rigorously included transverse momentum dependence of scalar and axial-vector diquark correlations
  - results in a dramatic increase in $\langle k_T^2 \rangle$ and a significant flavour dependence of the TMDs