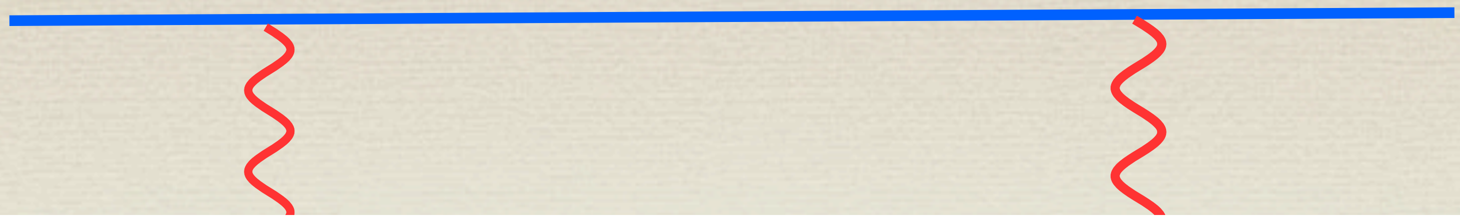


Two Photon Exchange Contribution at large Q^2 : QCD factorization framework

Nikolay Kivel

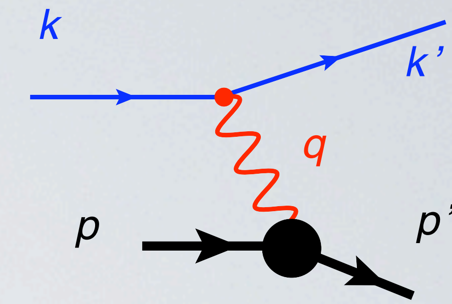
HELMHOLTZ Institut Mainz

"Probing Transverse Nucleon Structure at Higher Momentum Transfer", Trento, April 18-24

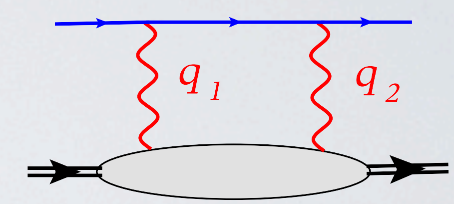
TPE corrections

reduced cross section

$$\sigma_R^{1\gamma} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$



$$\sigma_R^{\rm exp} = \sigma_R^{1\gamma}(1 + \delta_{\rm el,\ virt} + \delta_{\rm brems})$$



Rad. Corr. Tsai 1961, Mo, Tsi 1969

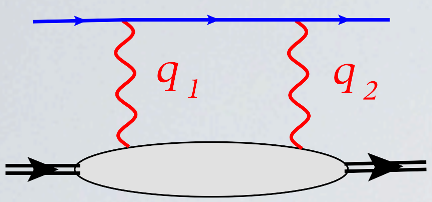
$$\sigma_R^{\rm exp} = \sigma_R^{1\gamma,\rm MT}(1 + \delta_{\rm virt}^{\rm MT} + \delta_{\rm brems}^{\rm MT})$$

Different calc of the TPE contribution gives

$$\delta_{el,\rm virt} - \delta_{el,\rm virt}^{\rm MT} = \delta_{2\gamma} - \delta_{2\gamma}^{\rm MT}$$

$$\sigma_R^{1\gamma,\rm MT} = \sigma_R^{1\gamma}(1 + \delta_{2\gamma} - \delta_{2\gamma}^{\rm MT})$$

TPE corrections



$$= \frac{e^2}{Q^2}\bar{u}(k')\gamma^\mu u(k)\bar{N}(p')\left[\gamma^\mu \underline{\delta\tilde{G}_M^{2\gamma}} - \frac{(p+p')^\mu}{2m}\underline{\delta\tilde{F}_2^{2\gamma}} + \frac{(p+p')^\mu}{2m^2}\not{k}\ \underline{\tilde{F}_3}\right]N(p)$$

hadronic part

Guichon, Vnaderhaeghen 2003

$$\sigma_R^{1\gamma,\mathrm{MT}} = G_M^2 + \frac{\varepsilon}{\tau}G_E^2 + 2G_M \mathrm{Re}\left[\delta\tilde{G}_M^{2\gamma} + \varepsilon\frac{\nu}{m^2}\tilde{F}_3 - G_M\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}}\right]$$

← TPE corr ↓

$$+2\frac{\varepsilon}{\tau}G_E \mathrm{Re}\left[\delta\tilde{G}_E^{2\gamma} + \frac{\nu}{m^2}\tilde{F}_3 - G_E\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}}\right]$$

$$\delta\tilde{G}_E^{2\gamma} = \delta\tilde{G}_M^{2\gamma} - (1+\tau)\delta\tilde{F}_2^{2\gamma}$$

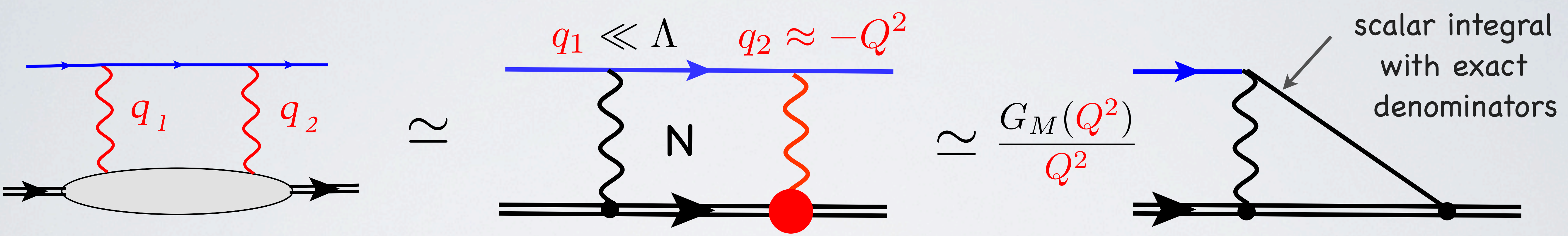
Tsai 1961, Mo, Tsi 1969

$$\mathrm{Re}\,\delta_{2\gamma}^{\mathrm{MT}} = \frac{\alpha}{\pi}\left\{\ln\frac{\lambda^2}{s-m^2}\ln\left|\frac{s-m^2}{u-m^2}\right| - \frac{1}{2}\ln^2\frac{s-m^2}{s} - \mathrm{Li}_2\left(\frac{s-m^2}{s}\right) - (s\leftrightarrow u)\right\}$$

TPE at large-Q^2: problems

- Tsai 1961, Mo, Tsi 1969

both diagrams calculated using only nucleon intermediate state and using $q_1=0$ or $q_2=0$ in both numerator and denominator of the integral reducing it to a vertex integral. This yields correct IR-divergent terms (soft approximation).

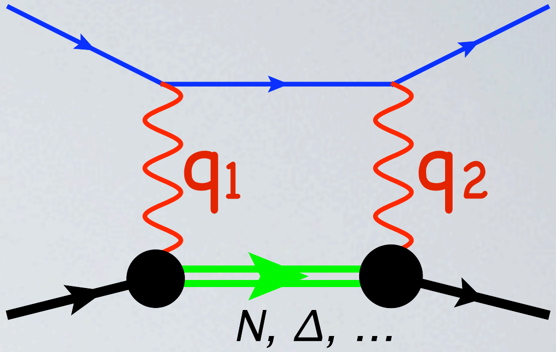


large Log at large energy

$$\delta_{2\gamma}^{\rm MT} \sim G_M(Q^2) \ln\left|\frac{s}{u}\right| \ln\frac{\lambda^2}{s} + \dots$$

- MT contribution includes the region where all lines are highly virtual and therefore can not be applied at large values of energy s
- separation of the short and large distances must be done consistently in order to avoid a double counting

TPE corrections in hadronic approach



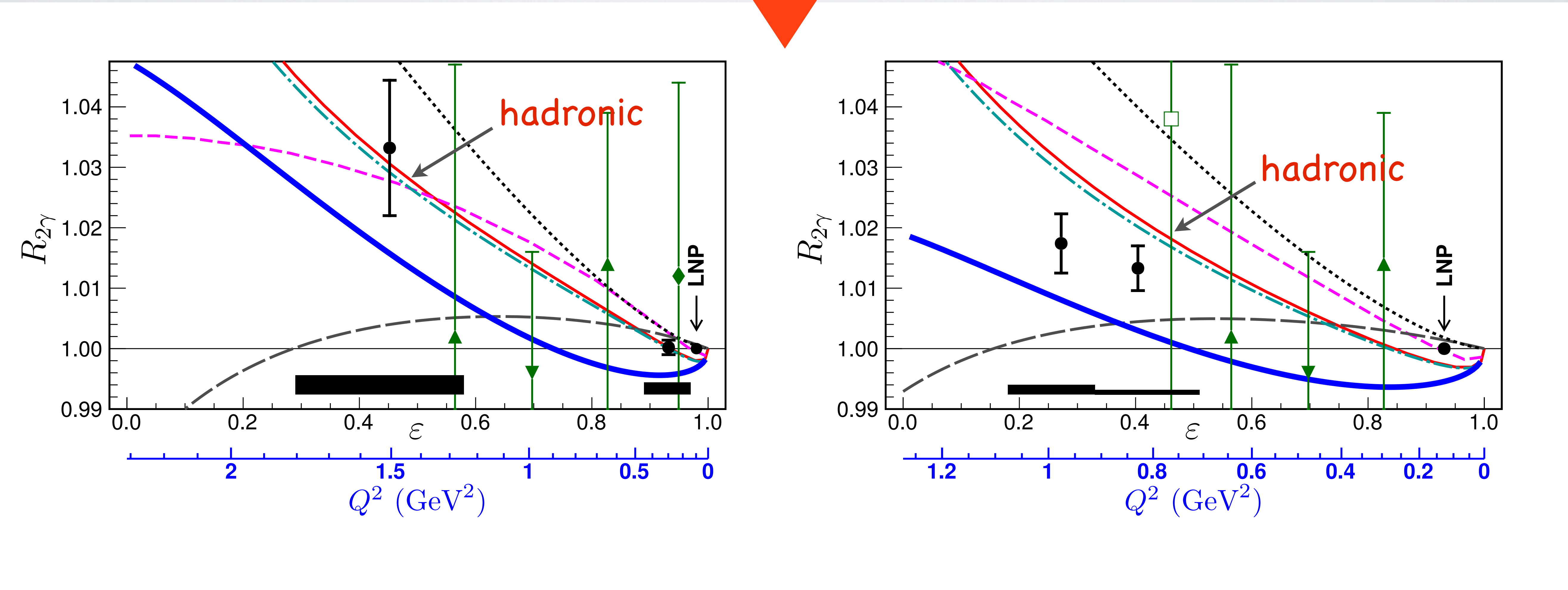
- Maximon, Tjon 2000 same as above but making above approximations only in the numerator (calc. with 4-point function) and using on-shell form factors in the loop integrals
- Kondratyuk, Blunden, Melnitchouk, Tjon 2003, 2005, 2007; further improvement by keeping the full numerator + higher resonance states
- Borisyuk, Kobushkin 2006, 2007, 2008, 2012, 2014 dispersion relations elastic & Δ, $N+\pi$, results are in agreement (qualitatively)
- Tomalak, Vanderhaeghen 2014 dispersion relations, only the elastic term
- Zhou, Yang 2014 hadronic (recalculation): elastic & Δ
- Bystritsky, Kuraev, Tomasi-Gustafsson 2006 hadronic + assumption about that dominant region: $q_1 \approx q_2 \approx q/2$(?), TPE effect is very small

TPE corrections in hadronic approach

TPE can resolve the discrepancy at small and intermediate values of the momentum transfer $Q^2 < 2.5 GeV^2$

$$R_{2\gamma} = \frac{\sigma(e^+p)}{\sigma(e^-p)}$$

VEPP-3 Rachek et al, 2015

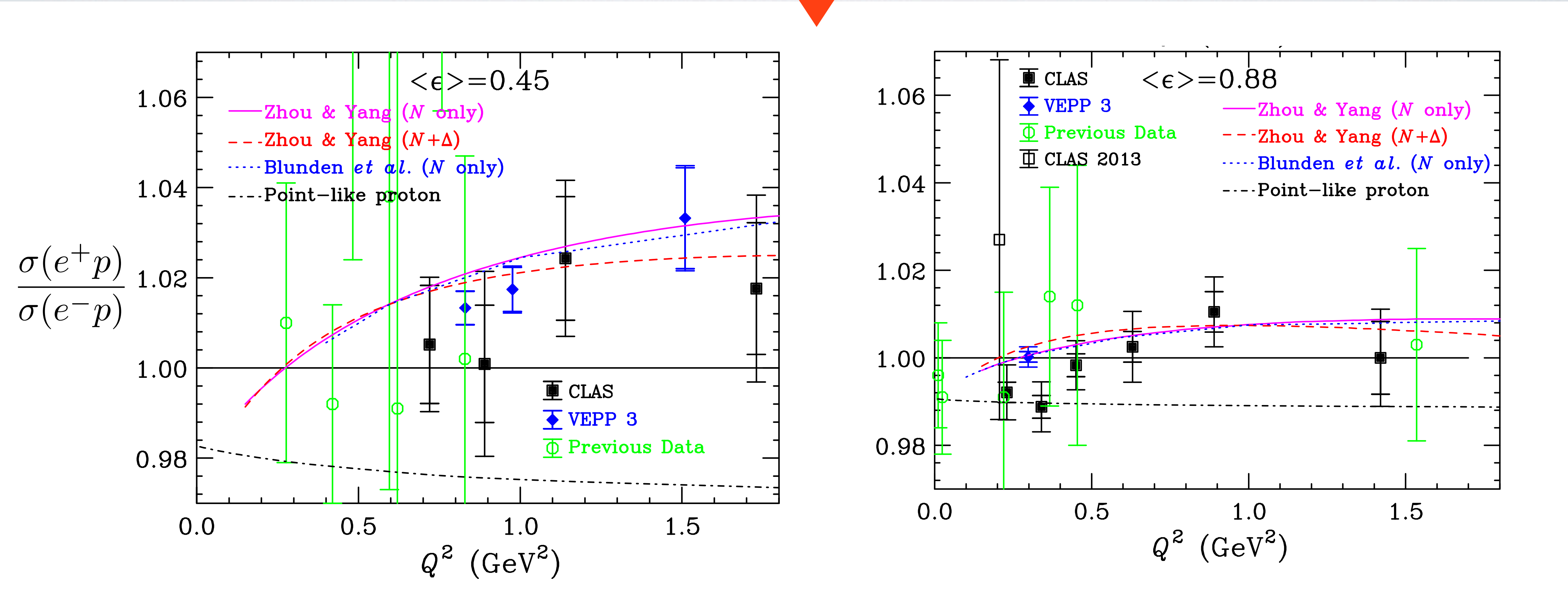


TPE corrections in hadronic approach

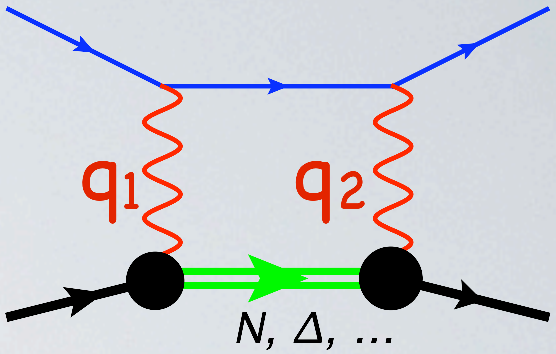
TPE can resolve the discrepancy at small and intermediate values of the momentum transfer $Q^2 < 2.5 GeV^2$

$$R_{2\gamma} = \frac{\sigma(e^+p)}{\sigma(e^-p)}$$

Class: Rimal et all, 2016 hep-ex 1603.00315



TPE corrections in hadronic approach



works well at low to moderate(?) values of Q^2

ambiguities: on-shell FFs, models for excited states FFs

$\Rightarrow$ more and more parameters,
less and less reliable at high Q^2

- The region with the highly virtual photons involves short distance interactions and therefore must be described in terms of QCD degrees of freedom: quarks and gluons

$\Rightarrow$ systematic description can be developed using factorization approach

TPE at large-Q^2: hard spectator approach

Borisyuk, Kobushkin, 2008

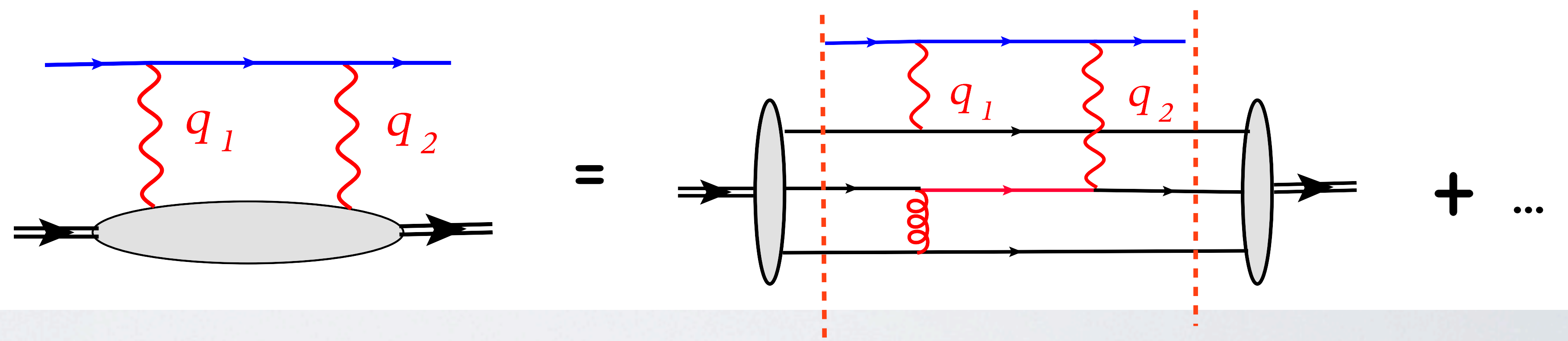
NK, Vanderhaeghen, 2009

both photons are highly virtual

$$q_1^2 \sim q_2^2 \sim q^2 = (p' - p)^2 \equiv -Q^2$$

applicable only for large-angle scattering

$$s \sim -t \sim -u \gg \Lambda^2$$



$$\varphi_N(x_i) * T_H(x, x', \varepsilon, Q^2) * \varphi_N(x_i')$$

scaling behavior $Q^2 \to \infty$

Form Factors

$$G_M \sim \alpha_S^2(Q) \frac{\Lambda^4}{Q^4}$$

$$F_2 \sim \frac{\Lambda^6}{Q^6}$$

TPE amplitudes

$$\delta \tilde{G}_M^{2\gamma} \sim \frac{\nu}{m^2} \tilde{F}_3 \sim \alpha_{em} \alpha_S(Q^2) \frac{\Lambda^4}{Q^4}$$

$$\delta \tilde{F}_2^{2\gamma} \sim \alpha_{em} \frac{\Lambda^6}{Q^6}$$ helicity flip, subleading

TPE at large-Q^2: hard spectator approach

☛ such contribution is complementary to the where one photon is soft

Cross section

$$\sigma_R^{1\gamma,\mathrm{MT}} = \sigma_R^{1\gamma}\left(1 + \delta_{2\gamma}^{\mathrm{hard}}\right)$$

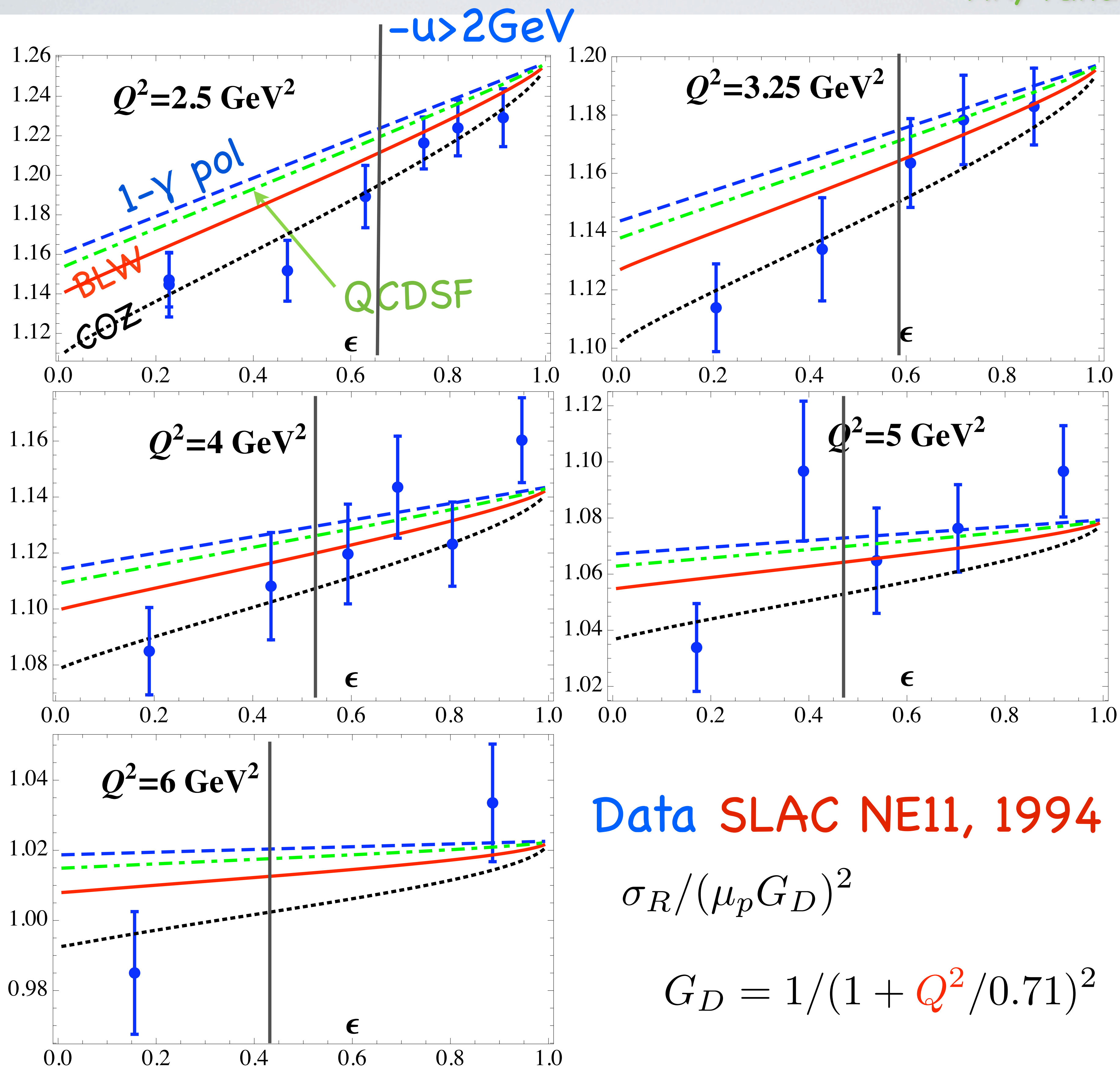
$$\sigma_R^{1\gamma,\mathrm{MT}} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2 G_M \mathrm{Re}\left[\delta \tilde{G}_M^{2\gamma} + \varepsilon \frac{\nu}{m^2} \tilde{F}_3\right]_{\mathrm{hard}} + 2 \frac{\varepsilon}{\tau} G_E \mathrm{Re}\left[\frac{\nu}{m^2} \tilde{F}_3\right]_{\mathrm{hard}}$$

← hard TPE corr

$$s \sim -t \sim -u \gg \Lambda^2$$

Reduced cross section: MT + hard TPE correction

NK, Vanderhaeghen, 2009



Data SLAC NE11, 1994

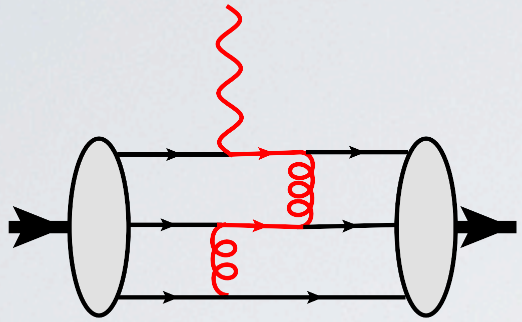
$$\sigma_R/(\mu_p G_D)^2$$

$$G_D = 1/(1 + Q^2/0.71)^2$$

TPE at large-Q^2: soft spectator contribution

em form factor F_1 — Chernyak, Zhitnitsky 1977; Brodsky, Lepage 1979

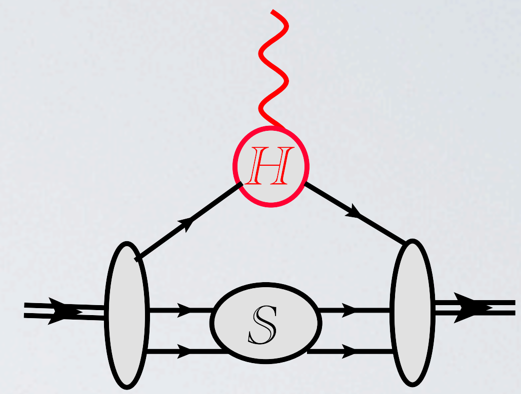
hard-spectator scattering



$\sim \frac{\Lambda^4}{Q^4}$

Duncan, Mueller 1980

soft-spectator scattering



$\sim \frac{\Lambda^4}{Q^4}$

- QCD factorization: soft spectator contribution is important at accessible values of Q^2
- Soft spectator scattering is especially important in processes with baryons

- Qualitatively agrees with observations of the large soft-overlap contribution in many phenomenological calculations:

LC wave functions
Isgur, Smith 1984

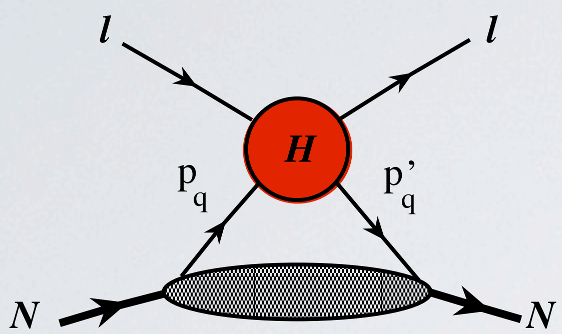
QCD sum rules
Nesterenko, Radyushkin 1983
Braun et al, 2002, 2006

GPD (handbag)-model
Radyushkin 1998
Kroll et al, 1998,2002,004,05
WA production/annihilation
$\gamma\gamma \leftrightarrow N\bar{N},\, N\pi\,,\pi\pi$

TPE at large-Q^2: partonic approach

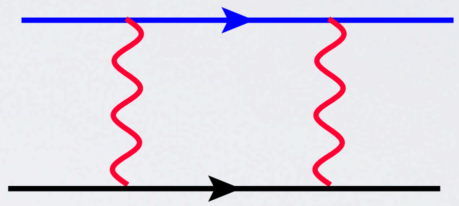
Partonic or GPD model (similar to "handbag" approach in WACS)

Afanasev, Brodsky, Chen, Carlson, Vanderhaeghen (2004, 2005)



large-t GPD

Hard partonic scattering



gives about 50% of the difference

☛ separation of the hard and soft photon contributions

☛ separation of hard and soft spectator contributions

Is it possible to formulate a more systematic approach in the framework of QCD inspired effective theory?

Soft spectator scattering in the SCET framework

description of the soft spectator contribution involves 3 different scales

1. Factorisation of hard modes: $p_h^2 \sim Q^2 \gg \Lambda^2$ (hard subprocess)

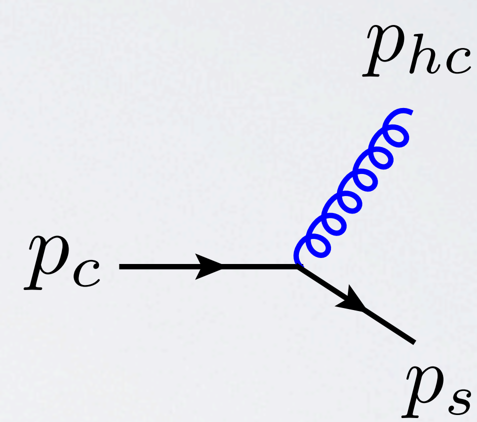
QCD → SCET-I $\quad F^{(s)}(Q^2, Q\Lambda, \Lambda^2) \simeq H(Q^2) * f(Q\Lambda, \Lambda^2)$

+ $\perp$ -

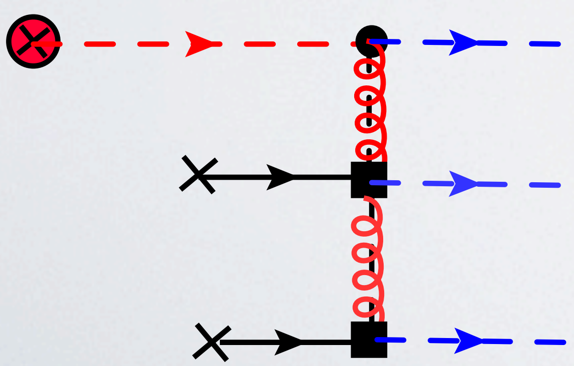
$p_{hc} \sim (Q, \sqrt{\Lambda Q}, \Lambda)$ hard-collinear

$p_c \sim (Q, \Lambda, \Lambda^2/Q)$ collinear

$p_s \sim (\Lambda, \Lambda, \Lambda)$ soft



2. Factorization of hard-collinear modes $p_{hc}^2 \sim Q\Lambda \gg m_N^2$

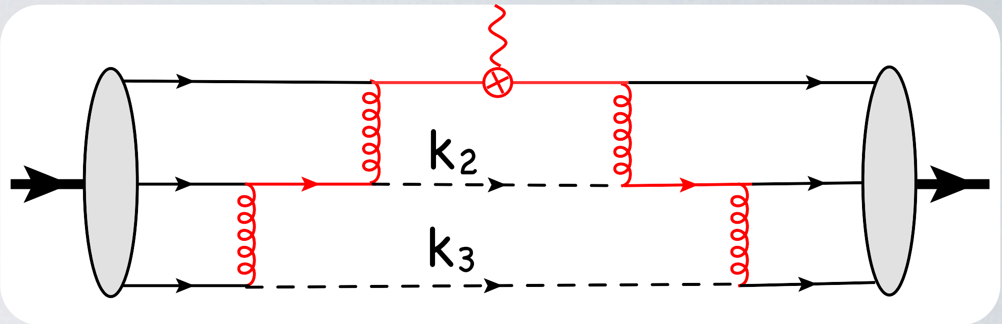


$$f(Q\Lambda, \Lambda^2) \simeq J_{hc}(Q\Lambda) * S[p_s] * \phi_N[p_c]$$

only after this we get a complete Q^2 behavior

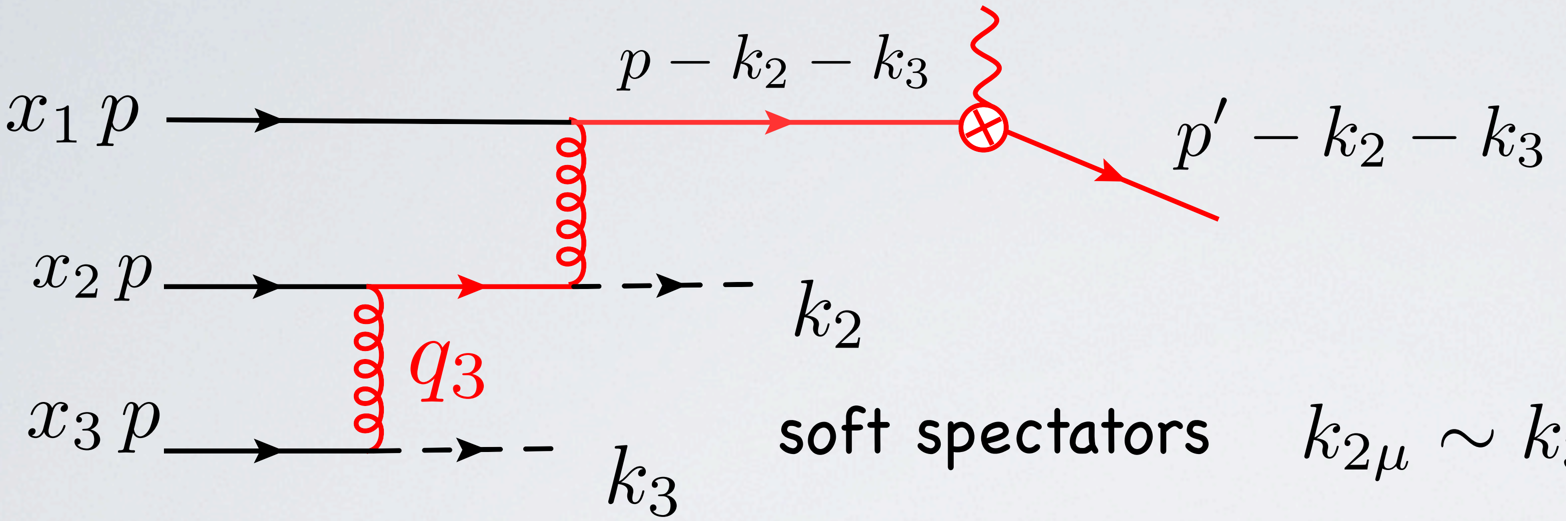
Soft spectator scattering at large Q^2

Duncan, Mueller 1980



$\sim \alpha_s^4(\Lambda Q)\ln[Q/\Lambda]/Q^4$

$$p \simeq \frac{1}{2}(Q,0,0,Q)$$



soft spectators $k_{2\mu} \sim k_{3\mu} \sim \Lambda$

$$q_3 = x_3 p - k_3 \qquad q_3^2 \sim (p\cdot k_3) \sim Q\Lambda \ll Q^2$$

all red lines can be described as hard-collinear

$$q_{hc} \sim (Q,\mathbf{0},\pm Q) + k \qquad q_{hc}^2 \sim Q\Lambda$$

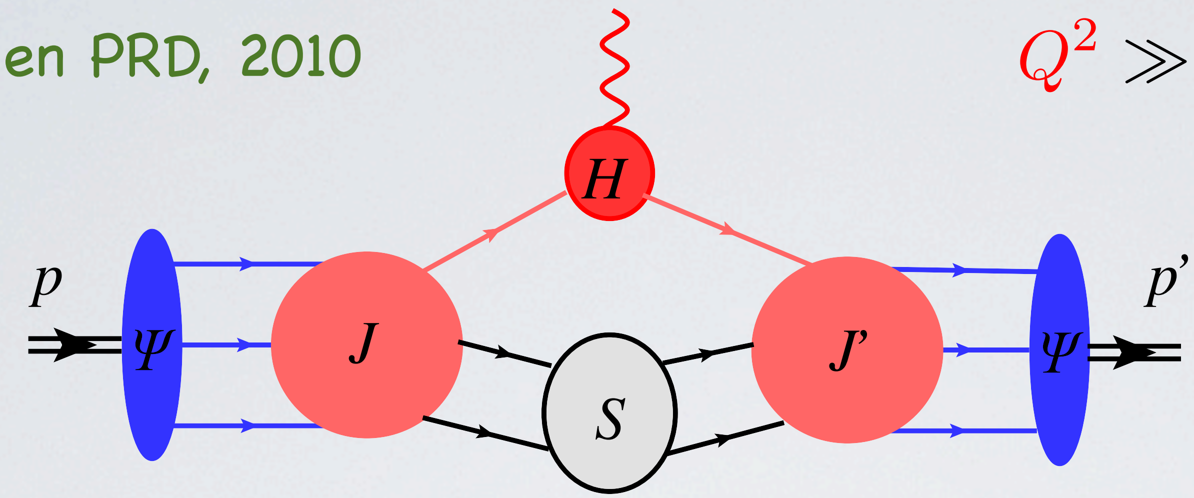
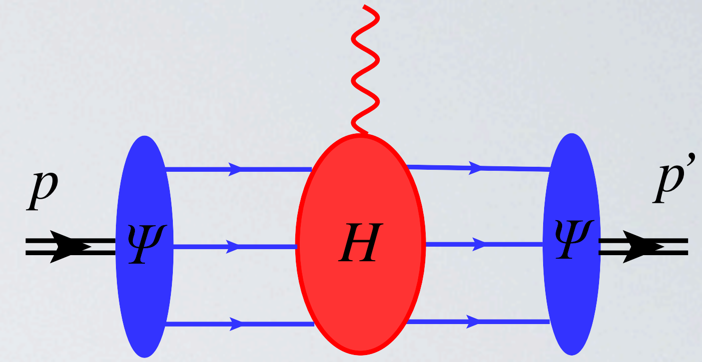
collinear soft

residual momenta $k_\mu \sim \mathcal{O}(\Lambda)$

Soft specator scattering at large Q^2 : SCET factorization

NK, Vanderhaeghen PRD, 2010

$$Q^2 \gg Q\Lambda \gg m_N^2$$

$F_1(Q) =$  + 

$$F_1^{(s)}(Q^2) \simeq C_A(Q^2, \mu_{hc})\, \boldsymbol{\Psi}_{out}(y_i, \mu) * \int_0^\infty d\omega_1 d\omega_2\; \mathbf{J}(y_i, \omega_{1,2}Q, \mu_{hc}, \mu)$$
$$\boldsymbol{\Psi}_{in}(x_i, \mu) * \int_0^\infty d\nu_1 d\nu_2\; \mathbf{J}(x_i, \nu_{1,2}Q, \mu_{hc}, \mu)\; \boldsymbol{S}(\omega_1, \omega_2, \nu_1, \nu_2; \mu)$$

$$\mu_{hc} \sim \sqrt{\Lambda Q}, \quad \mu \sim \omega_{1,2} \sim \nu_{1,2} \sim \Lambda$$

Soft correlation function:

$$\boldsymbol{S}(\omega_i, \nu_i; \mu) = \int \frac{d\eta_1}{2\pi} \int \frac{d\eta_2}{2\pi}\, e^{-i\eta_1\nu_1 - i\eta_2\nu_2} \int \frac{d\lambda_1}{2\pi} \int \frac{d\lambda_2}{2\pi} e^{i\lambda_1\omega_1 + i\lambda_2\omega_2} \langle 0|\, \mathbf{O}_S(\eta_i, \lambda_i)\, |0\rangle$$

$$\mathbf{O}_S(\eta_i, \lambda_i) = \varepsilon^{i'j'k'} \left[Y_n^\dagger(0)\right]^{i'l} \left[Y_n^\dagger q(\lambda_1 n)\right]^{j'} C\Gamma \left[Y_n^\dagger q(\lambda_2 n)\right]^{k'}$$
$$\times \varepsilon^{ijk} \left[S_{\bar n}(0)\right]^{li} \left[\bar q S_{\bar n}(\eta_1 \bar n)\right]^{j} \bar\Gamma C \left[\bar q S_{\bar n}(\eta_2 \bar n)\right]^{k}$$

QCD factorization at moderate values of Q²

☛ moderate values of Q^2 : $Q\Lambda \sim m_N^2$ hard-collinear scale is not large

$$Q^2 = 9 - 25\text{GeV}^2 \qquad \Lambda \simeq 0.3\text{GeV} \quad \Rightarrow \quad Q\Lambda \simeq 0.9 - 1.5\text{GeV}^2$$

the actual region for the planned experiments $Q^2 \gg Q\Lambda \lesssim m_N^2$

one can factorise in a systematic framework only the hard modes

the interactions of hard-collinear particles with soft spectators must be considered as nonperturbative

Advantages of the SCET framework

SCET matrix elements are defined by operators in EFT and universal

the hard spectator contributions are natural part of the total picture

if necessary the large logarithms can be resummed using RG evolution

Soft spectator scattering at large Q^2

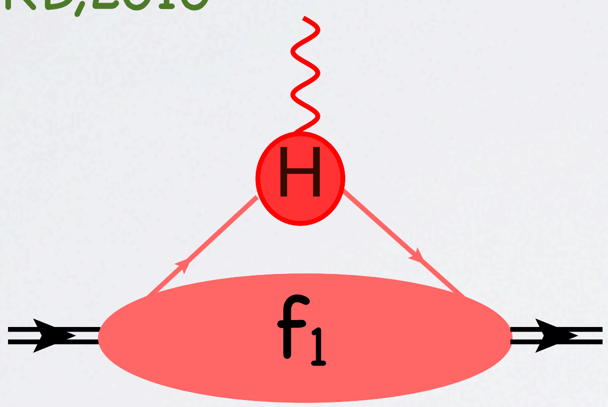
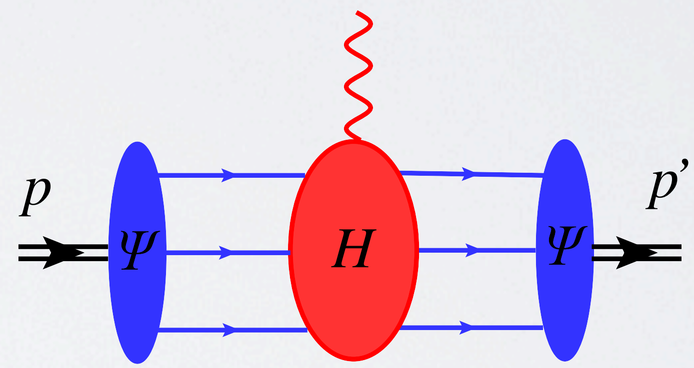
- moderate values of Q^2: $Q\Lambda \sim m_N^2$ hard-collinear scale is not large

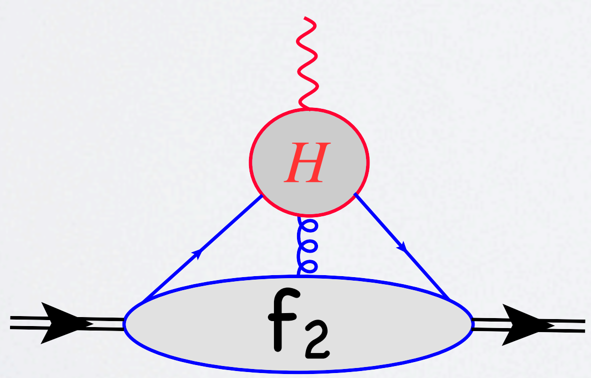
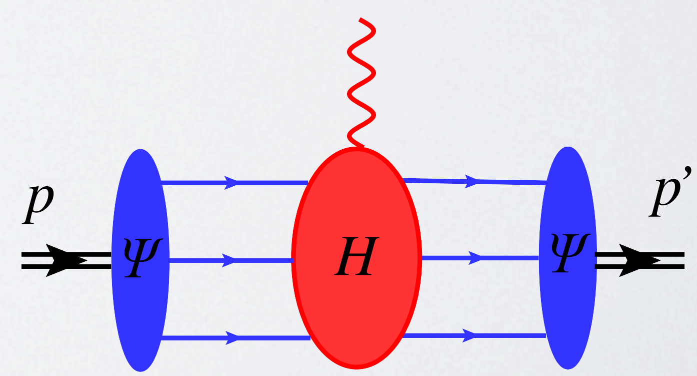
$$Q^2 = 9 - 25\text{GeV}^2$$
$$\Lambda \simeq 0.3\text{GeV}$$

$$\Rightarrow \quad Q\Lambda \simeq 0.9 - 1.5\text{GeV}^2$$

$$Q^2 \gg Q\Lambda \sim m_N^2$$

NK, Vanderhaeghen PRD,2010

$F_1(Q) =$  + 

$F_2(Q) =$  $+ \dfrac{4m_N^2}{Q^2}$ f$_1$ + 

TPE factorization within the SCET framework

Basic idea is to construct expansion with respect to large scale 1/Q

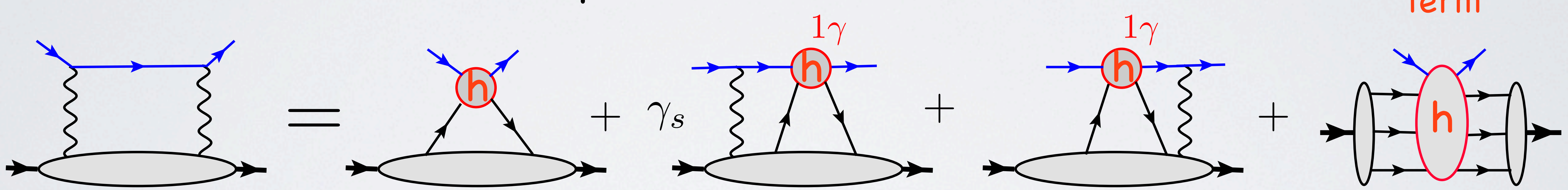
in the large-angle scattering domain $s \sim -t \sim -u \gg \Lambda^2$

large values of ε

NK, Vanderhaeghen 2012

soft spectator contributions

hard spectator term



2 photons are hard

one hard photon & one soft photon γ_s

$\phi_N(x_i)$

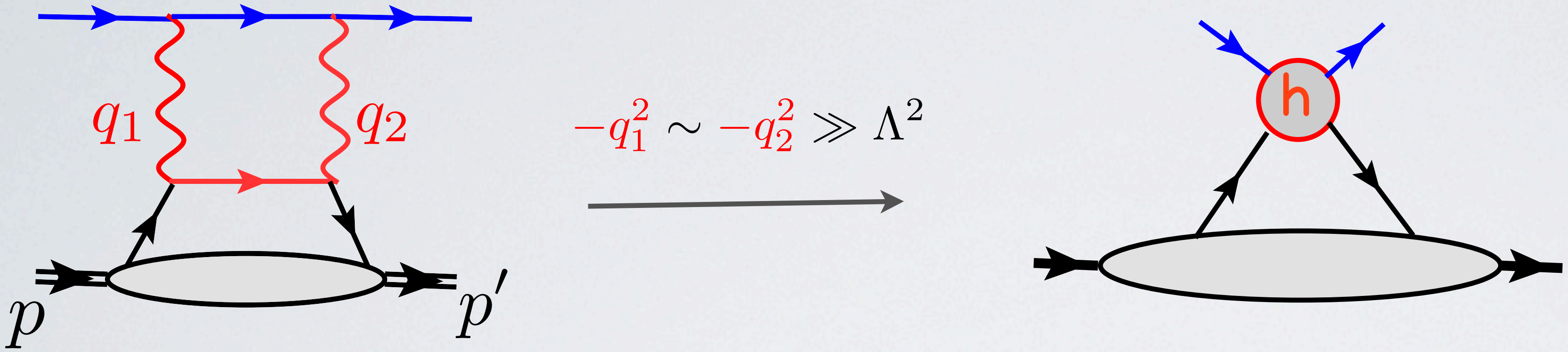
Nonperturbative input

two SCET matrix elements

nucleon distribution amplitude

TPE factorization at large Q²: the hard photon configuration

leading order QCD , next-to-leading order QED: 2-hard photon configuration



$\sim C(s,t)\,\mathcal{F}_1(t)$

SCET FF $\quad \langle p'|\bar{\chi}_n(0)\gamma_\perp \chi_{\bar{n}}(0) - \bar{\chi}_{\bar{n}}(0)\gamma_\perp \chi_n(0)|p\rangle_{SCET} = \bar{N}(p')\gamma_\perp N(p)\mathcal{F}_1(t)$

quark momenta $\quad p \simeq Q\dfrac{\bar{n}}{2} \qquad p' \simeq Q\dfrac{n}{2} \qquad n = (1,\vec{0},-1) \qquad \bar{n} = (1,\vec{0},1)$

quark “jets” $\quad \chi_{\bar{n}}(0) = \mathrm{P}\exp\left\{ig\int_{-\infty}^{0} ds\, n\cdot A_{hc}^{(\bar{n})}(sn)\right\}\frac{1}{4}\not{\bar{n}}\not{n}\,\psi_{hc}(0)$

How we can estimate SCET FF $\mathcal{F}_1$?

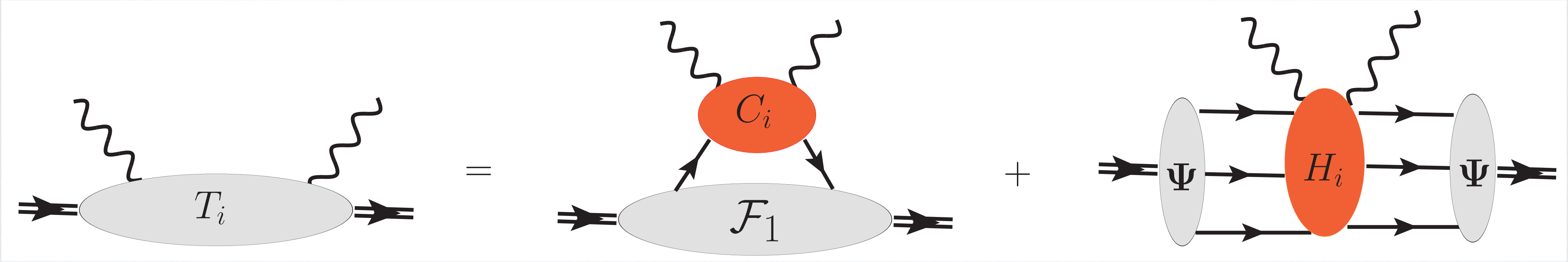
How we can estimate SCET FF $\mathcal{F}_1$?

☛ use universality and if possible constrain it from other process

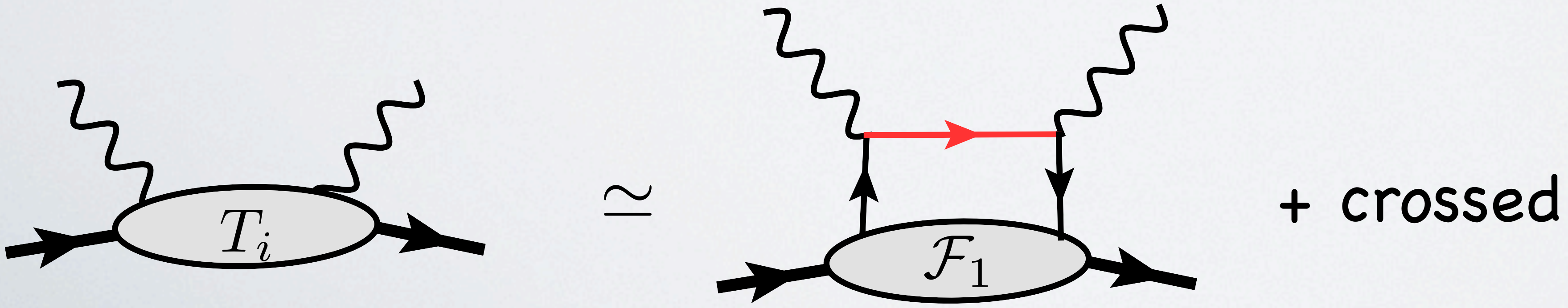
☛ Wide Angle Compton Scattering $s \sim -u \sim -t \gg \Lambda^2$

- Existing JLab data indicate that the soft spectator scattering strongly dominates over hard spectator contribution

- Factorization with the same SCET FF   NK, Vanderhaeghen 2012



Leading order contribution



$$T_i \simeq C_i(s,t)\mathcal{F}_1(t)$$

Wide Angle Compton Scattering in SCET

LO result $\mathcal{R} = \frac{T_2(s,t)}{C_2(s,t)} \simeq \frac{T_4(s,t)}{C_4(s,t)} \simeq \frac{T_6(s,t)}{C_6(s,t)} \simeq \mathcal{F}_1(t)$

the hard-spectator corrections are neglected

$$\frac{d\sigma}{dt} \simeq \frac{2\pi\alpha^2}{(s-m^2)^2}\left(\frac{s}{s+t}+\frac{s+t}{s}\right)|\mathcal{R}(t)|^2 \approx \frac{d\sigma^{\rm KN}}{dt}|\mathcal{R}(t)|^2$$

gives reliable description for

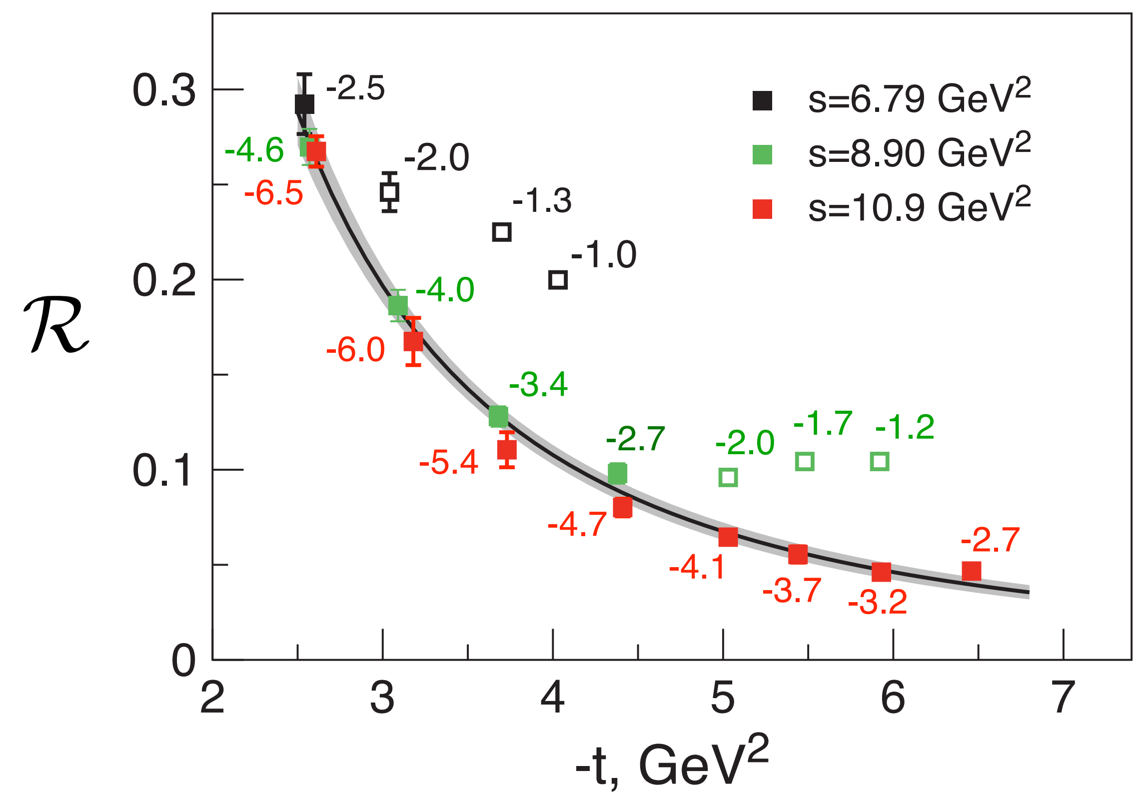
$|t|$ & $|u|>2.5$ GeV^2

empirical fit $\mathcal{F}_1(t) \approx \left(\frac{\Lambda^2}{-t}\right)^n$

$$n = 2.09 \pm 0.06$$

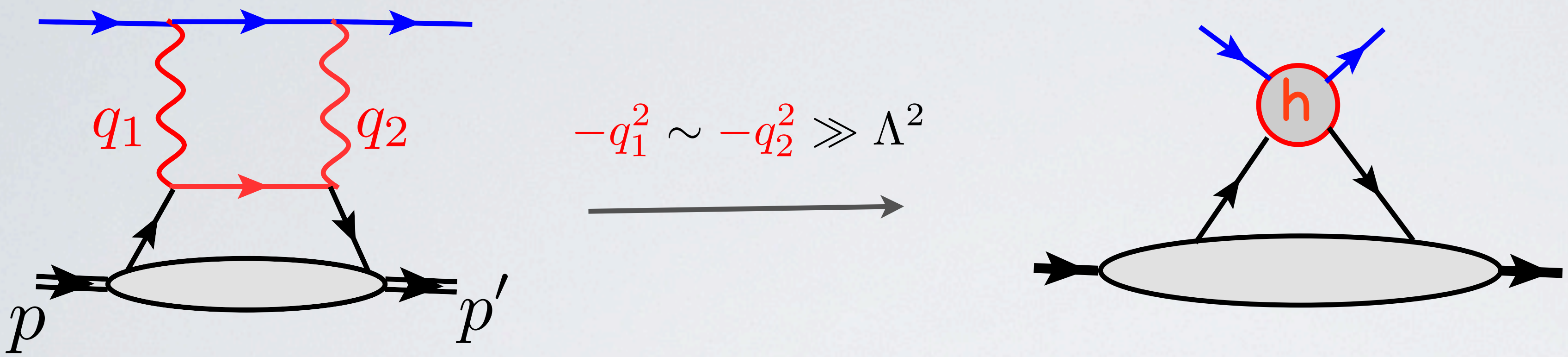
$$\Lambda = 1.17 \pm 0.01$$

used data: JLab, Hall A, 2007



TPE factorization at large Q²: the hard photon configuration

leading order QCD , next-to-leading order QED: 2-hard photon configuration



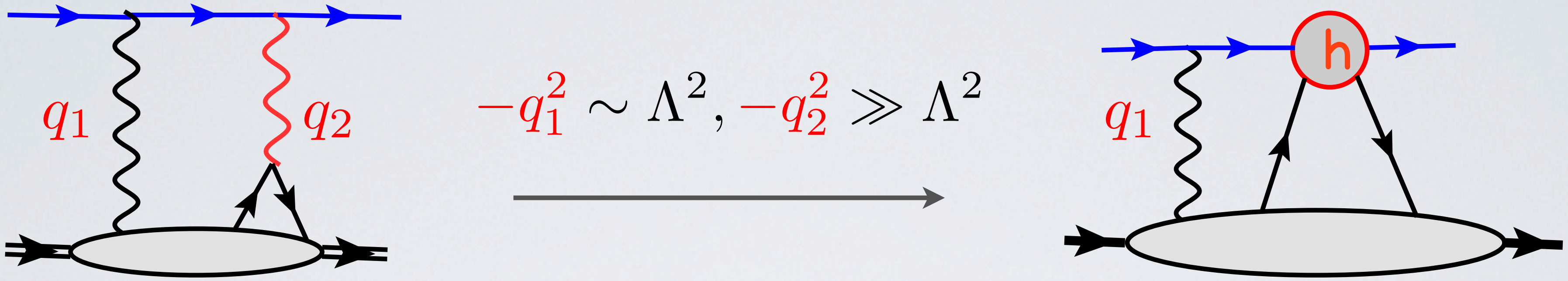
$$\delta \tilde{G}_M^{(s)}(\varepsilon, Q) = \frac{\alpha}{\pi} C_M(\varepsilon, \mu_F)\, \mathcal{F}_1(t) \qquad \tilde{F}_3^{(s)}(\varepsilon, Q) = \frac{\alpha}{\pi} C_3(\varepsilon)\, \mathcal{F}_1(t)$$

$$C_M(\varepsilon, \mu_F) = \ln\left|\frac{u}{s}\right| \ln\frac{s}{\mu_F^2} + \ldots$$

μ_F separates region where both γ are hard from region soft-γ & hard-γ

$$\mu_F \sim \Lambda \simeq 0.3 - 0.5\mathrm{GeV}$$

TPE at large-Q²: soft-hard configuration



$$\bar{u}(k')\gamma^{\mu}u(k)\frac{4\pi\alpha}{Q^2}\ \langle p'|(\bar{\chi}_n\gamma_{\perp\mu}\chi_{\bar{n}} + \bar{\chi}_{\bar{n}}\gamma_{\perp\mu}\chi_n)Y^{\dagger}_{k'}S_k|p\rangle_{SCET}$$

soft photons radiation is described by the WL's

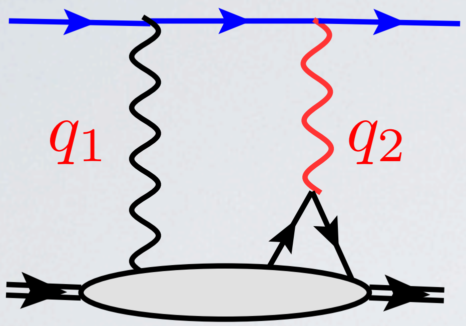
$$Y^{\dagger}_{k'}(0) = \mathrm{P}\exp\left\{-ie\int_0^{\infty} dt\ v\cdot B^{(s)}(tv)\right\} \qquad v = 2k'/Q$$

$$S_{k'}(0) = \mathrm{P}\exp\left\{-ie\int_{-\infty}^{0} dt\ \bar{v}\cdot B^{(s)}(t\bar{v})\right\} \qquad \bar{v} = 2k/Q$$

soft photon virtuality $q_1^2 \lesssim \Lambda^2$ includes IR QED singularity

Hadronic models for the SCET amplitudes

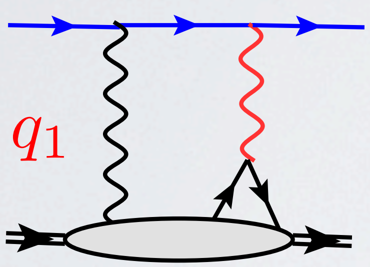
Hierarchy of the soft scales:



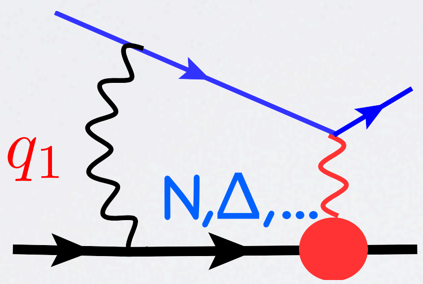
$q_1 \ll \Lambda$ ultrasoft photon interacts with the point-like proton

$q_1 \sim \Lambda$ resolution is related to the hard-collinear scale

assume that the dominant contribution arises from the ultrasoft region



$\simeq$



$$+\dots \qquad \sim \frac{G_M(Q^2)}{Q^2}\int d^D l \frac{1}{[l^2-\lambda^2][-2(lk)][2(lp)]}$$

minimalistic model: elastic contribution only

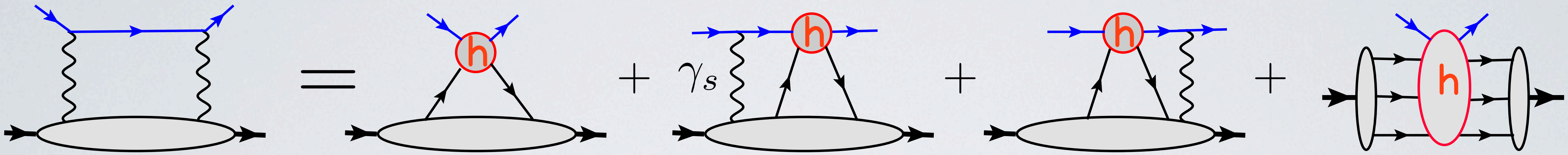
$$\langle p'|(\bar{\chi}_n\gamma_{\perp\mu}\chi_{\bar{n}} + \bar{\chi}_{\bar{n}}\gamma_{\perp\mu}\chi_n)Y^{\dagger}_{k'}S_k|p\rangle_{SCET} \simeq \bar{N}(p')\gamma_\perp N(p)G_M(Q)\ln\frac{\lambda^2}{\mu_F^2}\ln\left|\frac{s-m_N^2}{u-m_N^2}\right|$$

contributions of higher resonances are suppressed by 1/Q

QCD factorization of TPE amplitudes: summary

soft spectator contributions

hard spectator contributions



$$\delta\tilde{G}_M(\varepsilon, Q) = C_M(\varepsilon,\mu_0)\,\mathcal{F}_1(t) + g_1(\varepsilon, Q, \mu_0) + \mathbf{\Psi}_N * H_M * \mathbf{\Psi}_N$$

$$\frac{\nu}{m^2}\tilde{F}_3(\varepsilon, Q^2) = C_3(\varepsilon)\mathcal{F}_1(t) + \mathbf{\Psi}_N * H_3 * \mathbf{\Psi}_N$$

SCET FF $\mathcal{F}_1(t)$ estimated using WACS data $-t = 2.5 - 6.5\ \mathrm{GeV}^2$

minimalistic model $g_1(\varepsilon, Q) \simeq \frac{\alpha}{\pi} G_M(Q) \ln\frac{\lambda^2}{\mu_0^2} \ln\left|\frac{s - m_N^2}{u - m_N^2}\right|$ $\mu_0 \simeq 0.350 - 600\mathrm{MeV}$

Nucleon DA $\varphi_N(x_i) \simeq 120\, f_N x_1 x_2 x_3 \{1 + r_-(x_1 - x_2) + r_+(1 - 3x_3)\}$

Braun Lenz Wittmann 2006 $f_N = (5.0 \pm 0.5)\times 10^{-3}\mathrm{GeV}^2$ $r_- \simeq 1.37$ $r_+ \simeq 0.35$

Reduced cross section

$$\sigma_R^{1\gamma,\mathrm{MT}} = \sigma_R^{1\gamma}\left(1 + \delta_{2\gamma} - \delta_{2\gamma}^{\mathrm{MT}}\right)$$

$$\sigma_R^{1\gamma,\mathrm{MT}} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2 G_M \mathrm{Re}\left[\delta \tilde{G}_M^{2\gamma} + \varepsilon \frac{\nu}{m^2} \tilde{F}_3 - G_M \frac{1}{2} \delta_{2\gamma}^{\mathrm{MT}}\right]$$
$$+ 2 \frac{\varepsilon}{\tau} G_E \mathrm{Re}\left[\delta \tilde{G}_E^{2\gamma} + \frac{\nu}{m^2} \tilde{F}_3 - G_E \frac{1}{2} \delta_{2\gamma}^{\mathrm{MT}}\right]$$

IR QED soft singularity cancel

$$\delta \tilde{G}_E^{2\gamma} + \frac{\nu}{m^2} \tilde{F}_3 - G_E \frac{1}{2} \delta_{2\gamma}^{\mathrm{MT}} = -\frac{4m^2}{Q^2}\left[\delta \tilde{G}_M^{2\gamma} + \frac{\nu}{m^2} \tilde{F}_3 - G_M \frac{1}{2} \delta_{2\gamma}^{\mathrm{MT}}\right]$$

\+ contributions of subleading SCET operators

h-h

h-s

$$\delta \tilde{G}_M^{2\gamma} - G_M \frac{1}{2} \delta_{2\gamma}^{\mathrm{MT}} = \frac{\alpha}{\pi} \mathcal{F}(t) \ln\left|\frac{u}{s}\right| \ln \frac{s}{\mu_F^2} + \frac{\alpha}{\pi} G_M(Q^2) \ln\left|\frac{s-m^2}{u-m^2}\right| \ln \frac{\lambda^2}{\mu_F^2}$$
$$- \frac{\alpha}{\pi} G_M(Q^2) \ln\left|\frac{s-m^2}{u-m^2}\right| \ln \frac{\lambda^2}{s-m^2} + \cdots$$

Reduced cross section $\sigma_R^{1\gamma,\mathrm{MT}} = \sigma_R^{1\gamma}(1+\delta_{2\gamma}-\delta_{2\gamma}^{\mathrm{MT}})$

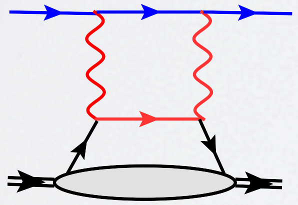
$$\sigma_R^{1\gamma,\mathrm{MT}} = G_M^2 + \frac{\varepsilon}{\tau}G_E^2 + 2G_M \mathrm{Re}\left[\delta\tilde{G}_M^{2\gamma} + \varepsilon\frac{\nu}{m^2}\tilde{F}_3 - G_M\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}}\right]$$

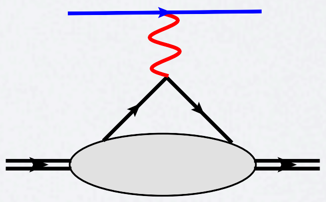
$$+2\frac{\varepsilon}{\tau}G_E \mathrm{Re}\left[\delta\tilde{G}_E^{2\gamma} + \frac{\nu}{m^2}\tilde{F}_3 - G_E\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}}\right]$$

h-h

h-s

$$\delta\tilde{G}_M^{2\gamma} - G_M\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}} = \frac{\alpha}{\pi}\mathcal{F}(t)\ln\left|\frac{u}{s}\right|\ln\frac{s}{\mu_F^2} \quad -\frac{\alpha}{\pi}G_M(Q^2)\ln\left|\frac{u-m^2}{s-m^2}\right|\ln\frac{s^2-m^2}{\mu_F^2} + \cdots$$





$$\mu_F \sim \Lambda \simeq 0.3 - 0.5\mathrm{GeV}$$

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$R = G_E/G_M \leq 0.25$ for $Q^2 \geq 2.5$ GeV²

subleading SCET contributions are relatively small at large Q², then

$$\delta\tilde{G}_E^{2\gamma} + \frac{\nu}{m^2}\tilde{F}_3 - G_E\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}} \approx -\frac{4m^2}{Q^2}\left[\delta\tilde{G}_M^{2\gamma} + \frac{\nu}{m^2}\tilde{F}_3 - G_M\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}}\right]$$

$$+2\frac{\varepsilon}{\tau}G_E \mathrm{Re}\left[\delta\tilde{G}_E^{2\gamma} + \frac{\nu}{m^2}\tilde{F}_3 - G_E\frac{1}{2}\delta_{2\gamma}^{\mathrm{MT}}\right] \sim \frac{\alpha}{\tau}\mathcal{O}(R) \ll \frac{\varepsilon}{\tau}G_E^2 \sim \mathcal{O}(R^2)$$

Reduced cross section

$$\sigma_R^{1\gamma,\mathrm{MT}} = \sigma_R^{1\gamma}\left(1 + \delta_{2\gamma} - \delta_{2\gamma}^{\mathrm{MT}}\right)$$

$$\sigma_R^{1\gamma,\mathrm{MT}} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2 G_M \mathrm{Re}\left[\delta \tilde{G}_M^{2\gamma} + \varepsilon \frac{\nu}{m^2} \tilde{F}_3 - G_M \frac{1}{2} \delta_{2\gamma}^{\mathrm{MT}}\right]$$

fix the ratio $R = G_E / G_M$ from pol. transfer data

Nucleon DA $\quad \varphi_N(x_i) \simeq 120\, f_N x_1 x_2 x_3 \{1 + r_-(x_1 - x_2) + r_+(1 - 3x_3)\}$

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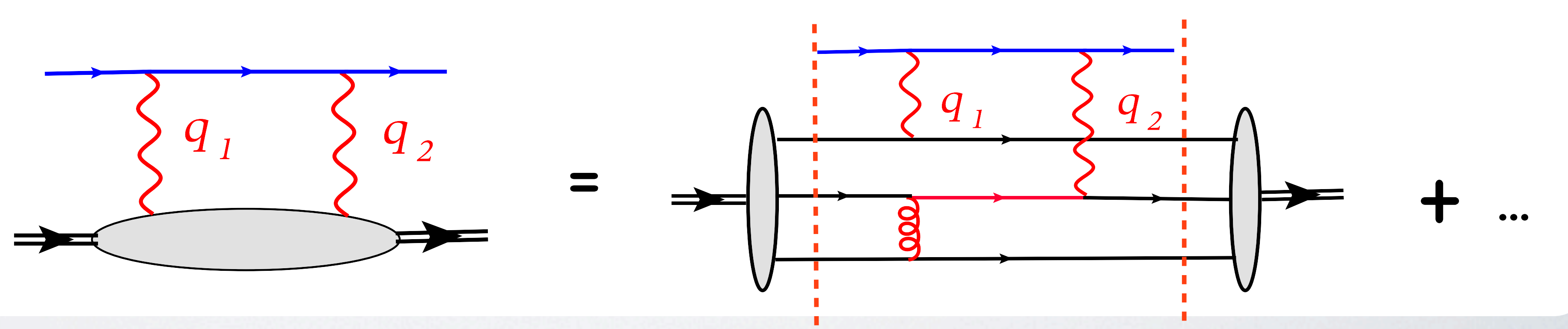

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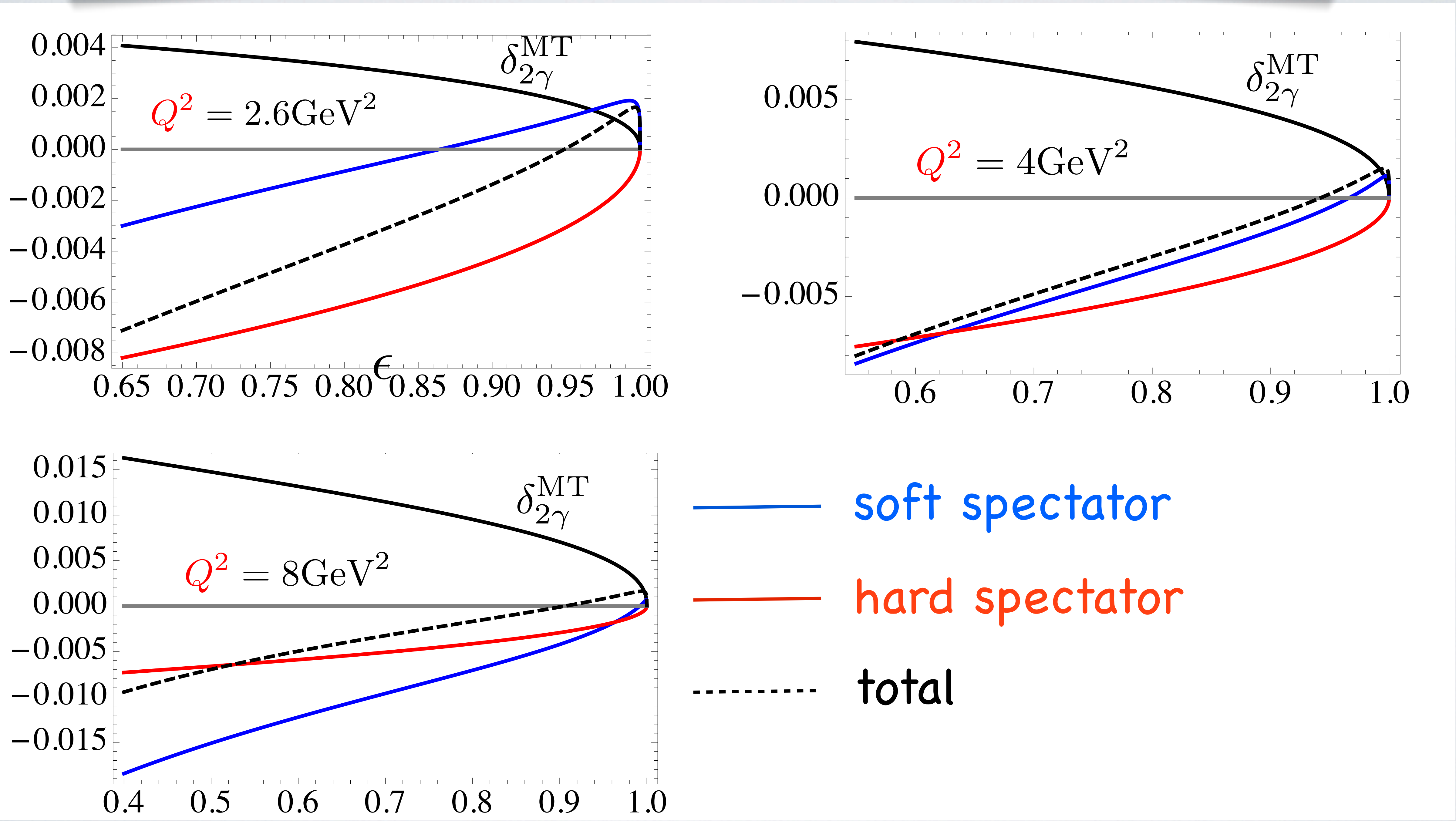
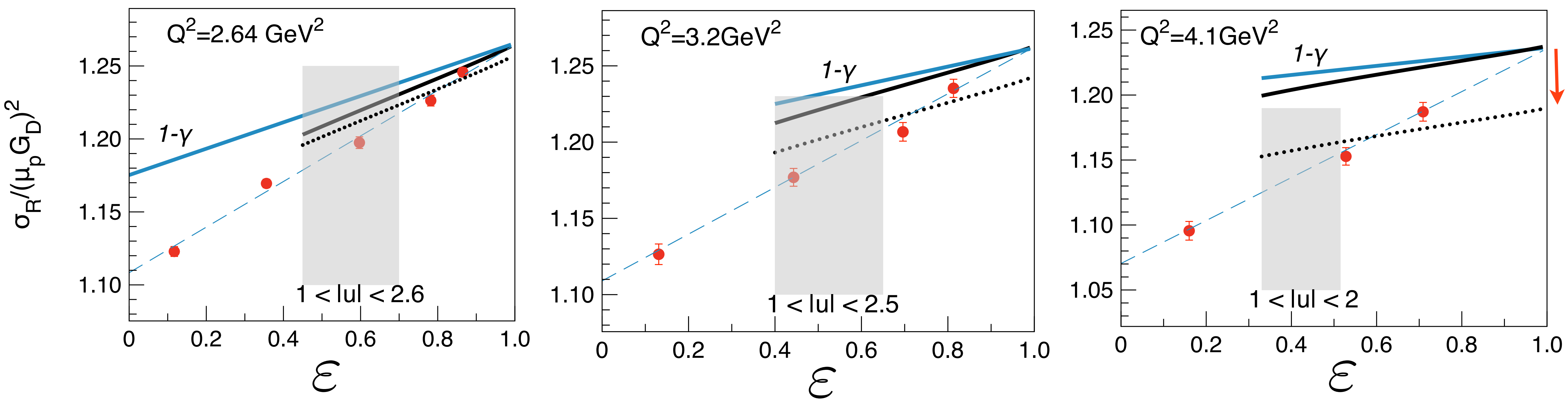


Reduced cross section: JLAB data

NK, Vanderhaeghen

$$\sigma_R(\varepsilon, Q) \simeq G_M^2 \left(1 + \frac{\varepsilon}{\tau} R^2\right) + 2G_M Re\left\{\underbrace{\delta(\varepsilon, Q) - \tfrac{1}{2}\delta_{2\gamma}^{\mathrm{MT}}(\varepsilon, Q)}_{\text{TPE correction}}\right\}$$

Data: JLab, Qattan et al, 2005



this fit:	$G_M = 0.1356$	$G_M = 0.1001$	$G_M = 0.0654$
Guttman	0.1360	0.1009	0.0667
Arrington	0.1352	0.0989	0.0647

Non-linear behavior of the $\sigma_R(\varepsilon, Q)$ at large fixed Q ?

Or the calculations of h-s underestimate this contribution?

TPE amplitudes

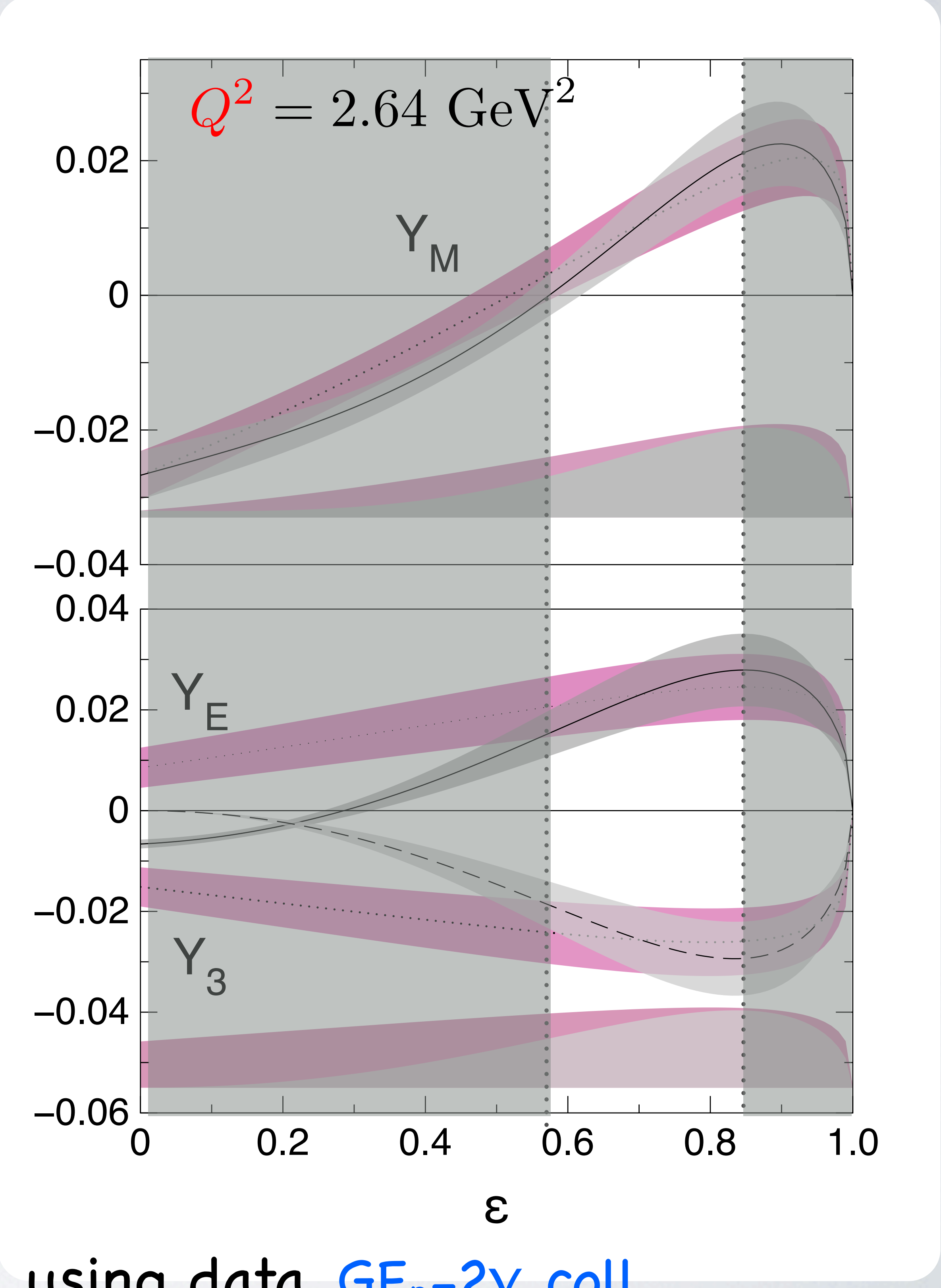
Guttman et all, 2011

$$Y_M(\varepsilon, Q) = Re\frac{\delta\tilde{G}_M(\varepsilon, Q)}{G_M(Q)}$$

$$Y_E(\varepsilon, Q) = Re\frac{\delta\tilde{G}_E(\varepsilon, Q)}{G_M(Q)}$$

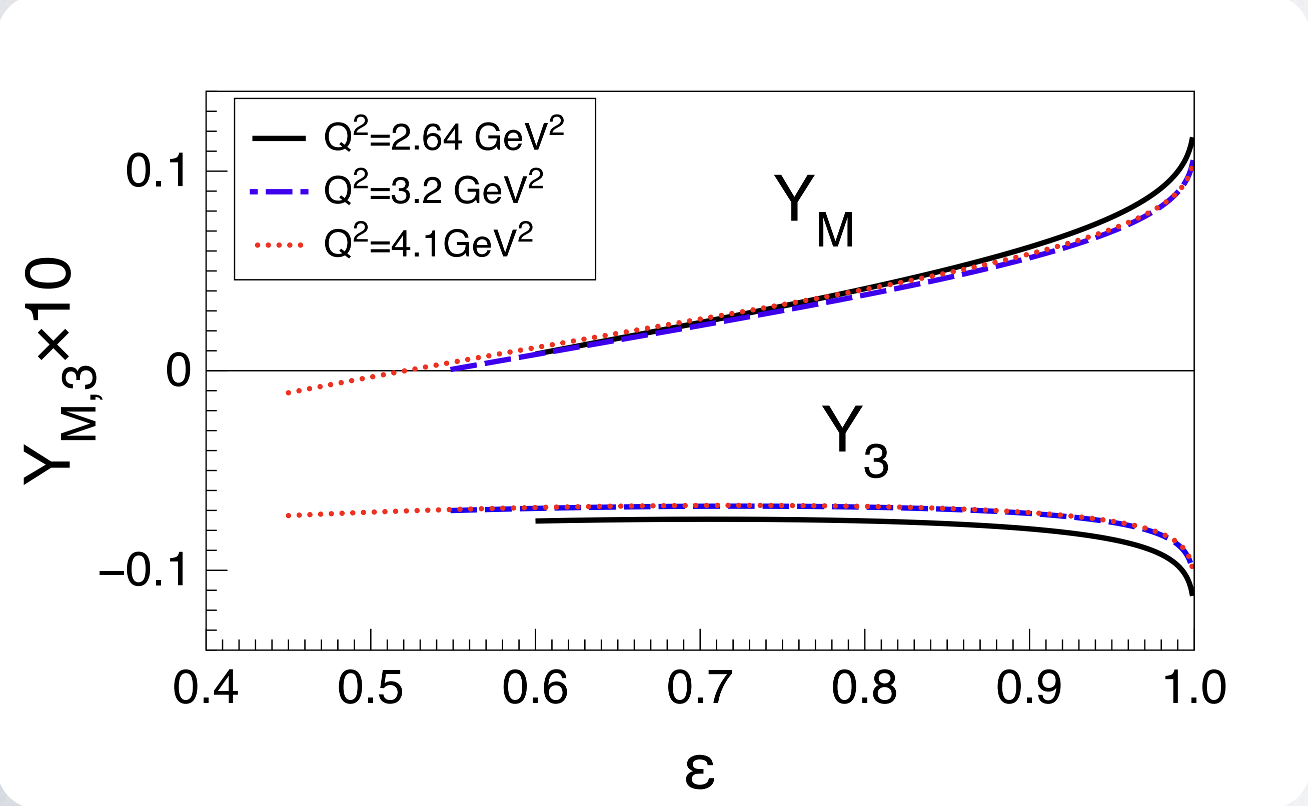
$$Y_3(\varepsilon, Q) = Re\frac{\tilde{F}_3(\varepsilon, Q)}{G_M(Q)}$$

Phenomenological analysis with linear behavior



using data GE_p-2γ coll.

Factorization + min.model h-s term



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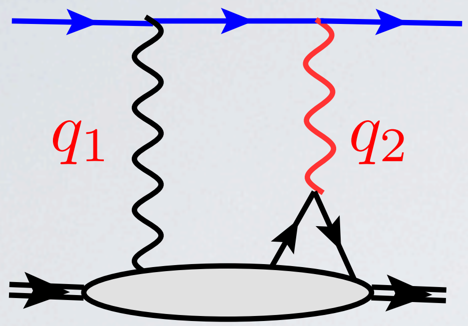

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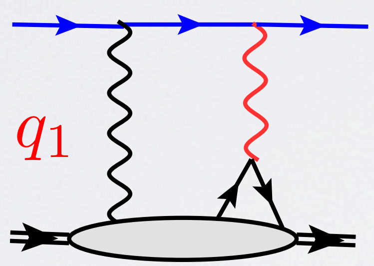
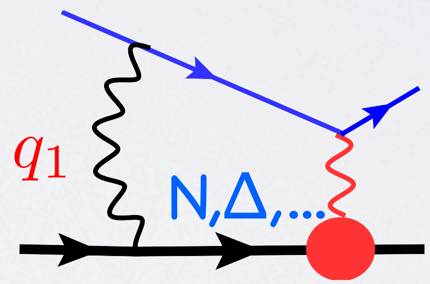
$q_1 \ll \Lambda$



$\simeq$



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contribution of higher resonances are suppressed by 1/Q

Conclusions

- We suggest QCD factorization approach for the kinematical region

  $s \sim -u \sim -t \gg \Lambda^2$ (relatively large ε)

- Hard and soft spectator contributions are included

- Contributions with 2 hard and hard-soft photons are taken into account consistently

- Data for WACS allows one to fix the contribution with hard photons & soft spectator scattering using the universality of SCET FFs

- Simple models used for SCET amplitudes describing the configuration when one of the photons is soft

- Obtained results give can not provide the full description of the discrepancy between Rosenbluth and polarised data

- The largest ambiguity is due to SCET amplitudes describing the configuration when one of the photons is soft. This contribution is model dependent