Nucleon GPDs from DVCS

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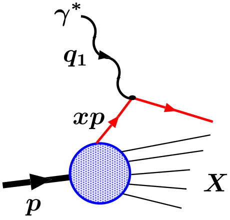
ECT* Workshop
Probing Transverse Nucleon Structure at High Momentum Transfer
Trento, Italy, 18–22 April 2016

Parton distribution functions

- Deep-inelastic scattering, $-q_1^2 \to \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \mathrm{const}$



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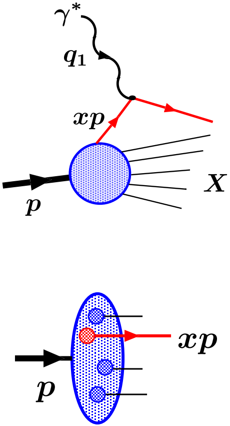

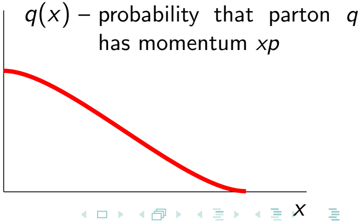

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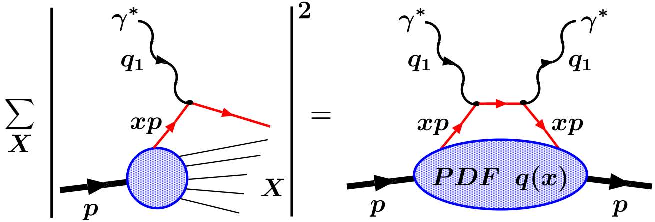
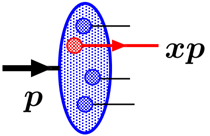
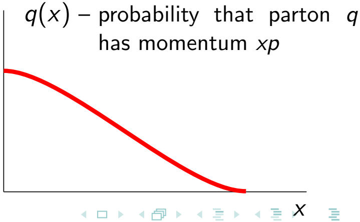
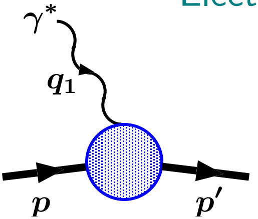






Electromagnetic form factors



$F_{1,2}(t = q_1^2)$

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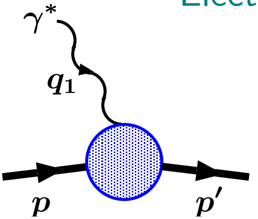
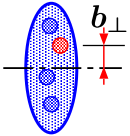
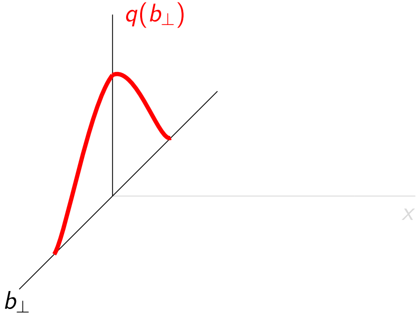


- Transversal density

$$q(b_\perp) = [\text{2-dim F.T.}] \otimes F_1(t = q_1^2)$$





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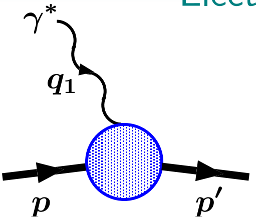

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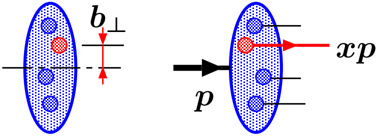

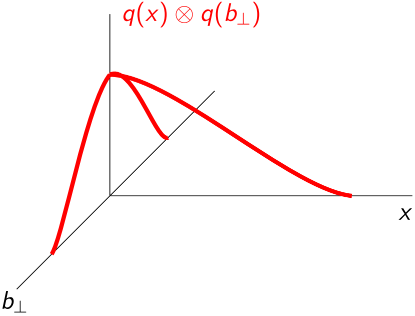

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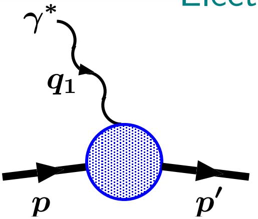

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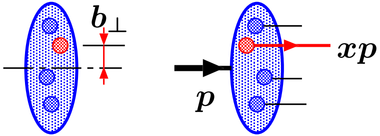

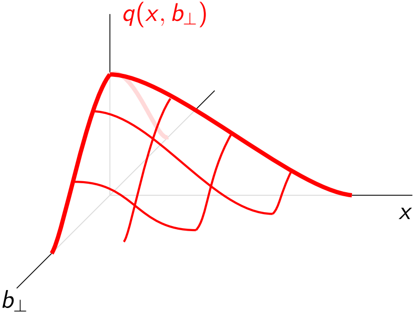

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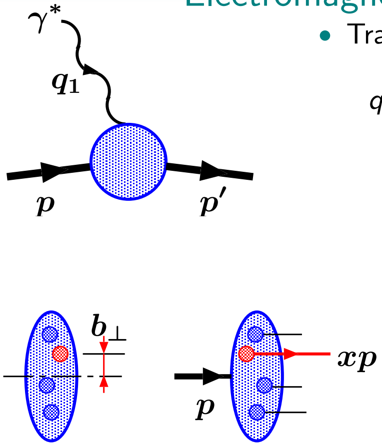

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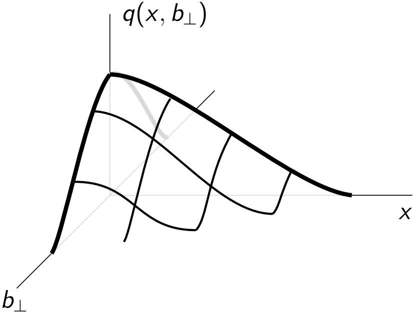


- "skewless" GPD: $H^q(x, 0, t = \Delta^2) = \int \mathrm{d}b_\perp \, e^{i\Delta \cdot b_\perp} q(x, b_\perp)$

  [Burkardt '00, Ralston, Pire '02]

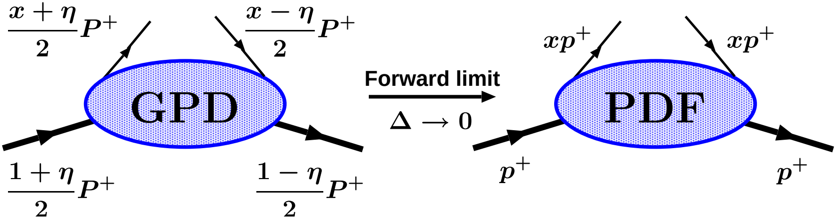
Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2 | \bar{q}(-z)\gamma^+ q(z) | P_1 \rangle \Big|_{z^+=0,\, \mathbf{z}_\perp=\mathbf{0}}$$

$$\widetilde{F}^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2 | \bar{q}(-z)\gamma^+\gamma_5 q(z) | P_1 \rangle \Big|_{z^+=0,\, \mathbf{z}_\perp=\mathbf{0}}$$

(and similarly for gluons F^g and $\widetilde{F}^g$).



$$P = P_1 + P_2\ ; \qquad t = \Delta^2 = (P_2 - P_1)^2\ ; \qquad \eta = -\frac{\Delta^+}{P^+}\ \text{(skewedness)}$$

Some properties of GPDs

- Decomposing into spin-non-flip and spin-flip part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

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- Forward limit ($\Delta \to 0$): $\Rightarrow$ GPD $\to$ PDF

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- Sum rules:

$$\int_{-1}^{1} dx \left\{ \begin{array}{c} H^q(x,\eta,t) \\ E^q(x,\eta,t) \end{array} \right. = \left\{ \begin{array}{ll} F_1^q(t) & \text{Dirac} \\ F_2^q(t) & \text{Pauli} \end{array} \right.$$

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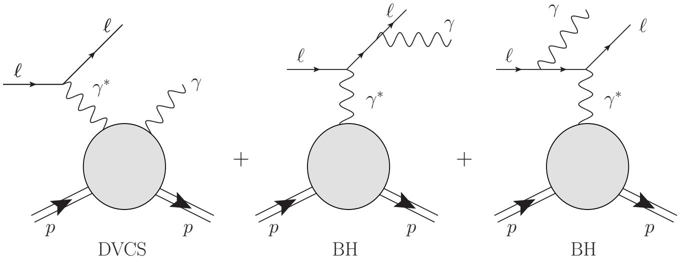
- "Ji's sum rule" (related to proton spin problem)

$$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x \Big[H^q(x,\eta,t) + E^q(x,\eta,t) \Big]_{t\to 0} \qquad \text{[Ji '96]}$$

DVCS

- Access to GPDs: Deeply virtual Compton scattering (DVCS) — "gold plated" process of exclusive physics
- DVCS is measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I} \; .$$

$$\mathcal{I} \quad \propto \quad \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\} ,$$

$$|\mathcal{T}_{\rm DVCS}|^2 \quad \propto \quad \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\} ,$$

- Choosing polarizations (and charges) we focus on particular harmonics:

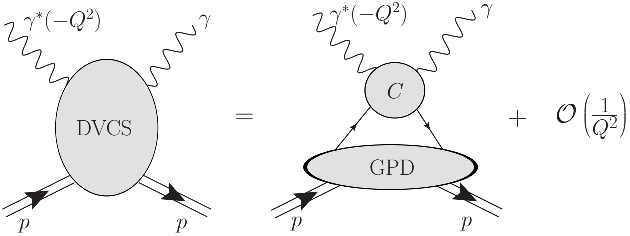
$$c^{\mathcal{I}}_{1,\rm unpol.} \propto \left[F_1 \, \mathfrak{Re}\, \mathcal{H} - \frac{t}{4M_p^2} F_2 \, \mathfrak{Re}\, \mathcal{E} + \frac{x_{\rm B}}{2 - x_{\rm B}} (F_1 + F_2) \, \mathfrak{Re}\, \widetilde{\mathcal{H}} \right]$$

[Belitsky, Müller et. al '01–'14]

- $\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2), \ldots$ — four Compton form factors (CFFs)

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



- Compton form factor is a convolution:

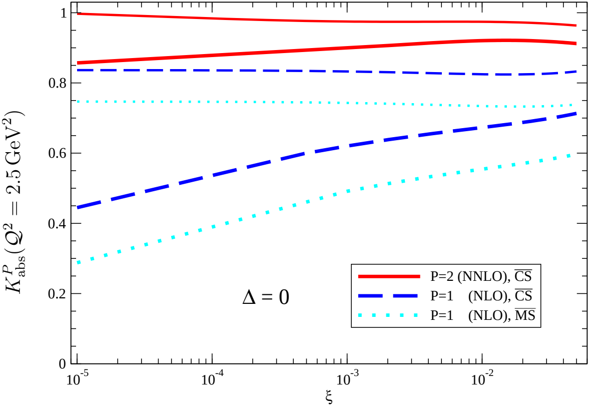
$$^{a}\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \frac{x_{\rm B}}{2-x_{\rm B}}, \frac{\mathcal{Q}^2}{\mathcal{Q}_0^2}) \; H^a(x, \frac{x_{\rm B}}{2-x_{\rm B}}, t, \mathcal{Q}_0^2)$$

$a = q, G$

- $H^a(x, \eta, t, \mathcal{Q}_0^2)$ — Generalized parton distribution (GPD)

(N)NLO corrections

- [K.K., Müller and Passek-K. '07]

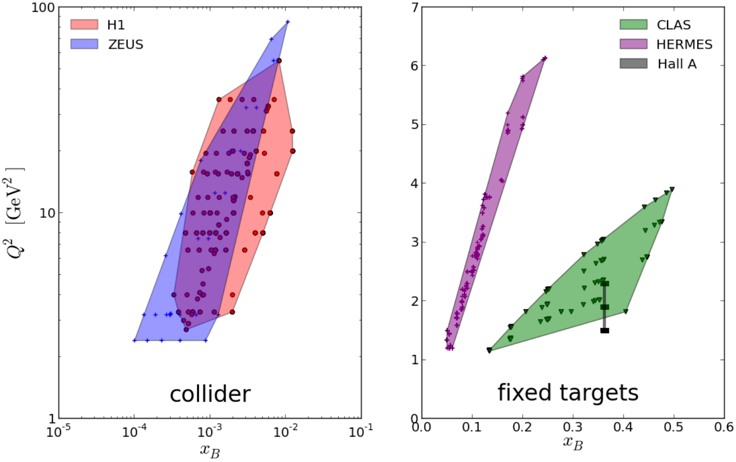


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)=\alpha_\Sigma(0) + 0.05$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0)=\alpha_\Sigma(0) - 0.02$

$$K^P_{\rm abs} \equiv \left| \frac{\mathcal{H}^{(P)}}{\mathcal{H}^{(P-1)}} \right|$$

Experimental coverage (1/2)



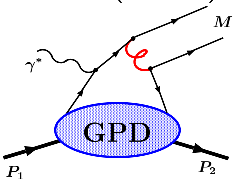
- Coming soon: COMPASS, JLab12, . . . EIC

Experimental coverage (2/2)

Collab.	Year	Observables	Kinematics x_B	Kinematics Q^2 [GeV^2]	Kinematics $\|t\|$ [GeV^2]	No. of points total	No. of points indep.
HERMES	2001	$A_{LU}^{\sin\phi}$	0.11	2.6	0.27	1	1
CLAS	2001	$A_{LU}^{\sin\phi}$	0.19	1.25	0.19	1	1
CLAS	2006	$A_{UL}^{\sin\phi}$	0.2–0.4	1.82	0.15–0.44	6	3
HERMES	2006	$A_{C}^{\cos\phi}$	0.08–0.12	2.0–3.7	0.03–0.42	4	4
Hall A	2006	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.36	1.5–2.3	0.17–0.33	4×24+12×24	4×24+12×24
CLAS	2007	$A_{LU}(\phi)$	0.11–0.58	1.0–4.8	0.09–1.8	62×12	62×12
HERMES	2008	$A_{C}^{\cos(0,1)\phi}$, $A_{UT,DVCS}^{\sin(\phi-\phi_S)}$, $A_{UT,I}^{\sin(\phi-\phi_S)\cos(0,1)\phi}$, $A_{UT,I}^{\cos(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12*+12+12 12+12 *12*	*4*+4+4 4+4 *4*
CLAS	2008	$A_{LU}(\phi)$	0.12–0.48	1.0–2.8	0.1–0.8	66	33
HERMES	2009	$A_{LU,I}^{\sin(1,2)\phi}$, $A_{LU,DVCS}^{\sin\phi}$, $A_{C}^{\cos(0,1,2,3)\phi}$	0.05–0.24	1.2–5.75	<0.7	18+18+18 18+18+*18*+*18*	6+6+6 6+6+*6*+*6*
HERMES	2010	$A_{UL}^{\sin(1,2,3)\phi}$, $A_{LL}^{\cos(0,1,2)\phi}$	0.03–0.35	1–10	<0.7	12+12+*12* 12+*12*+12	4+4+*4* 4+*4*+4
HERMES	2011	$A_{LT,I}^{\cos(\phi-\phi_S)\cos(0,1,2)\phi}$, $A_{LT,I}^{\sin(\phi-\phi_S)\sin(1,2)\phi}$, $A_{LT,BH+DVCS}^{\cos(\phi-\phi_S)\cos(0,1)\phi}$, $A_{LT,BH+DVCS}^{\sin(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12+12+12* *12+12* *12*+12 12	*4+4+4* *4+4* *4*+4 4
HERMES	2012	$A_{LU,I}^{\sin(1,2)\phi}$, $A_{LU,DVCS}^{\sin\phi}$, $A_{C}^{\cos(0,1,2,3)\phi}$	0.03–0.35	1–10	<0.7	18+*18*+*18* 18+18+*18*+*18*	6+*6*+6 6+6+*6*+*6*
CLAS	2015	$A_{LU}(\phi)$, $A_{UL}(\phi)$, $A_{LL}(\phi)$	0.17–0.47	1.3–3.5	0.1–1.4	166+166+166	166+166+166
CLAS	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.1–0.58	1–4.6	0.09–0.52	2640+2640	2640+2640
Hall A	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.33–0.40	1.5–2.6	0.17–0.37	480+600	240+360

Alternative processes for GPD access

- Deeply virtual meson production (DVMP) $\gamma^* p \to M p$.



  - Theory more "dirty" than for DVCS (second "soft" function appears: meson distribution amplitude)
  - Different mesons enable access to different flavours of GPDs

[P. Kroll's talk?]

- Wide-angle Compton scattering (WACS) [Tommorrow's talks]
  - WACS: proton momentum transfer t is large (unlike DVCS, where photon virtuality is large: $Q^2 \gg t$!)
  - data reasonably described by GPD models [Diehl, Kroll, '13]
- double DVCS $\gamma^* p \to \gamma^* p$, timelike DVCS, ...

Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*

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- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*
- When the dimensionality increases, the volume of the space increases so fast that the available data becomes sparse.
- Analogously, in contrast to *PDFs*(x), it is very difficult to perform truly model independent extraction of *GPDs*(x, η, t)
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of *CFFs*(x_B, t)
- (Dependence on additional variable, photon virtuality Q^2, is in principle known — given by evolution equations.)

Modelling sea quark and gluon GPDs

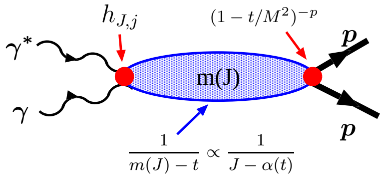
- Instead of considering momentum fraction dependence $H(x, \ldots)$
- ...it is convenient to make a transform into complementary space of conformal moments j:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^j\, C_j^{3/2}(x/\eta)\, H^q(x, \eta, \ldots)$$

- They are analogous to Mellin moments in DIS: $x^j \to C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials
- At LO easy multiplicative evolution (pQCD series behaviour and evolution of CFFs studied also to NNLO)

SO(3) partial wave expansion

- To model η-dependence of GPD's $H_j(\eta, t)$ consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$ and perform SO(3) partial wave expansion:



$$H_j(\eta, t) = \sum_{J=J_{\rm min}}^{j+1} h_{J,j} \frac{1}{J - \alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2}\right)^p} \eta^{j+1-J} d^J_{0,\nu}\left(\frac{1}{\eta}\right)$$

- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
  $\nu = 0, \pm 1$ — depending on hadron helicities
- Similar to "dual" parametrization [Polyakov, Shuvaev '02]
- We take leading waves $J = j+1, j-1, \cdots$ and expand for small η.

Modelling valence quark GPDs

- **Hybrid models** at LO
- Sea quarks and gluons modelled like just described (conformal moments + SO(3) partial wave expansion + $\mathcal{Q}^2$ evolution).
- Valence quarks model (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t) = n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- Fixed: n (from PDFs), $\alpha(t)$ (eff. Regge), p (counting rules)

$$\alpha^{\rm val}(t) = 0.43 + 0.85\,t/\mathrm{GeV}^2 \quad (\rho,\,\omega)$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations

$$\mathfrak{Re}\,\mathcal{H}(\xi, t) =$$
$$\frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) \mathfrak{Im}\,\mathcal{H}(\xi', t) - \frac{C}{\left(1 - \frac{t}{M_C{}^2}\right)^2}$$

- Typical set of free parameters:

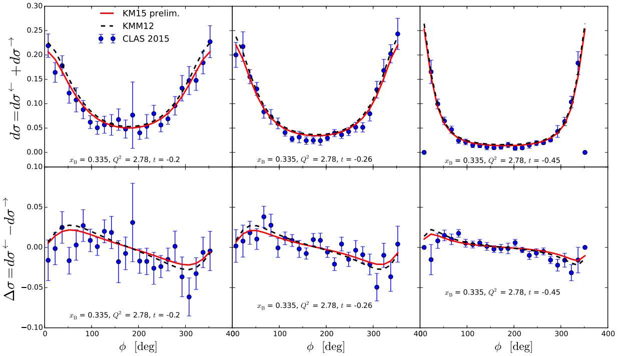
$M_0^{\rm sea}$, $s_{\rm sea}^{(2,4)}$, $s_G^{(2,4)}$	sea quarks and gluons H
$r^{\rm val}$, $M^{\rm val}$, $b^{\rm val}$	valence H
$\tilde{r}^{\rm val}$, $\tilde{M}^{\rm val}$, $\tilde{b}^{\rm val}$	valence $\widetilde{H}$
C, M_C	subtraction constant (H, E)
r_π, M_π	"pion pole" $\widetilde{E}$

Model	KM09a	KM09b	KM10	KM10a	KM10b	KMS11	KMM12	KM15
free params.	{3}+(3)+5	{3}+(3)+6	{3}+15	{3}+10	{3}+15	NNet	{3}+15	{3}+15
χ^2/d.o.f.	32.0/31	33.4/34	135.7/160	129.2/149	115.5/126	13.8/36	123.5/80	240./275
F_2	{85}	{85}	{85}	{85}	{85}		{85}	{85}
$\sigma_{\rm DVCS}$	(45)	(45)	51	51	45		11	11
$d\sigma_{\rm DVCS}/dt$	(56)	(56)	56	56	56		24	24
$A_{LU}^{\sin\phi}$	12+12	12+12	12	16	12+12		4	13
$A_{LU,I}^{\sin\phi}$			18	18		18	6	6
$A_{C}^{\cos 0\phi}$							6	6
$A_{C}^{\cos\phi}$	12	12	18	18	12	18	6	6
$\Delta\sigma^{\sin\phi,w}$			12				12	63
$\sigma^{\cos 0\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}/\sigma^{\cos 0\phi,w}$		4			4			
$A_{UL}^{\sin\phi}$							10	17
$A_{LL}^{\cos 0\phi}$							4	14
$A_{LL}^{\cos\phi}$								10
$A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}$							4	4

- [K.K., Müller, et al. '09–'15]
- These models are available at WWW (google for "gpd page")

2015 CLAS cross-sections (1/2)

- Restriction to kinematics where leading-order framework should be valid: $-t/Q^2 < 0.25$ with $Q^2 > 1.5\,\mathrm{GeV}^2$, means using 48 out of measured 110 $x_{\rm B}$–Q^2–t bins.



- $\chi^2/\mathrm{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$

ϕ-space vs. harmonics (1/2)

- ϕ-space figures and perfect χ^2 are not revealing the whole story
- Instead to $\sigma(\phi)$ it is favourable to work with harmonics like

$$\sigma^{\sin n\phi, w} \equiv \frac{1}{\pi} \int_{-\pi}^{\pi} dw \, \sin(n\phi)\, \sigma(\phi) \,,$$

with specialy weighted Fourier integral measure

$$dw \equiv \frac{2\pi \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}{\int_{-\pi}^{\pi} d\phi \, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} d\phi \,,$$

thus cancelling strongly oscillating factors $1/(\mathcal{P}_1(\phi)\mathcal{P}_2(\phi))$ in Bethe-Heitler and interference terms in $d\sigma$. Series of such weighted harmonic terms converges then faster with increasing n than normal Fourier series.

ϕ-space vs. harmonics (2/2)

- How many harmonics to extract?
- One approach:
  1. Fit harmonic expansion

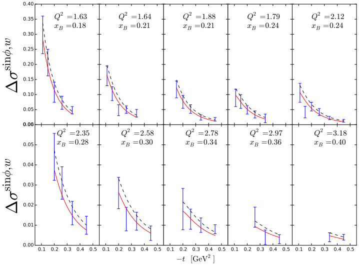
     $$\sigma(\phi) = c_0 + c_1 \cos\phi + \cdots + s_1 \sin\phi + \cdots$$

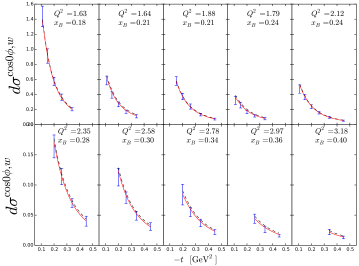
     to randomly chosen subset of data in a bin, and calculate χ^2 error for description of the rest of data (so called *cross-validation* procedure)
  2. Increase the number of harmonics until $\chi^2/\mathrm{d.o.f}$ starts to fall
- Highest extractable harmonics in 2015 cross-section data:

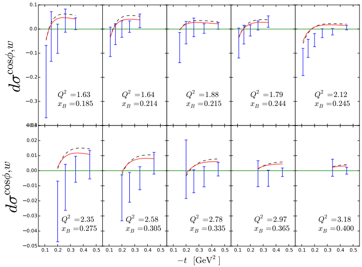
	CLAS		Hall A	
	sine	cosine	sine	cosine
$\Delta\sigma^w$	0.9 ± 0.4	0.1 ± 0.3	1.1 ± 0.3	0.1 ± 0.3
$d\sigma^w$	0.3 ± 0.6	0.7 ± 0.7	0.6 ± 0.8	1.5 ± 0.7

- (So $\Delta\sigma^w = s_1 \sin\phi$ and $d\sigma^w = c_0 + c_1 \cos\phi$ is enough.)

2015 CLAS cross-sections (2/2)



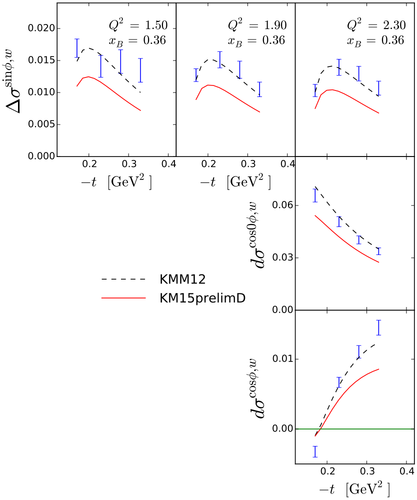


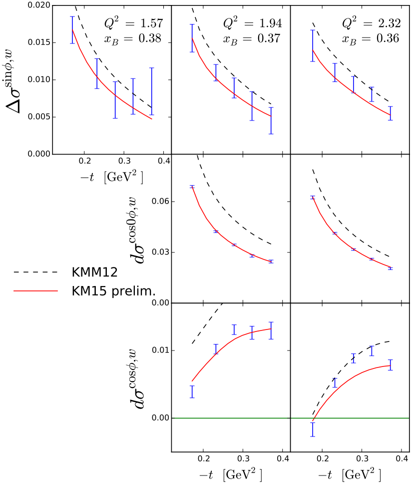


- $\chi^2/\mathrm{npts} = 62.2/48$ for $d\sigma^{\cos\phi,w}$

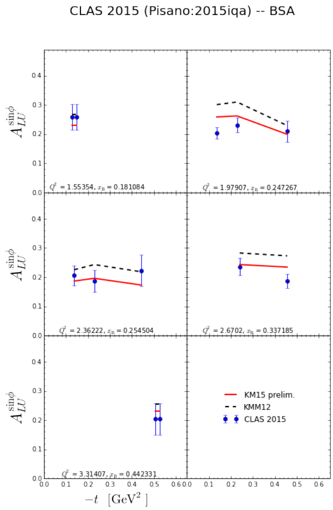
(O.K. but not so perfect as in ϕ-space)

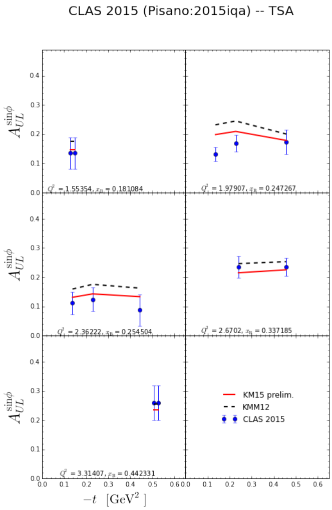
2006 vs 2015 Hall A cross-sections



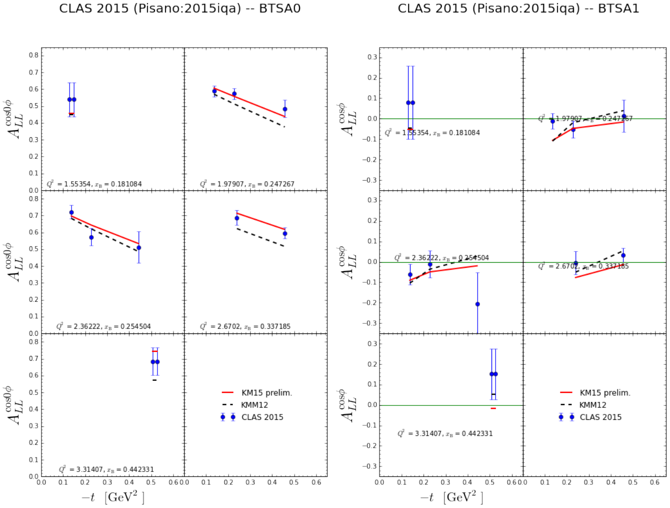


2015 CLAS asymmetries (1/2)

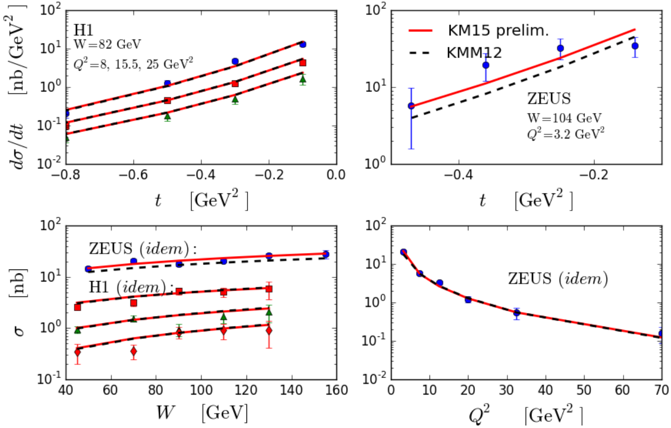




2015 CLAS asymmetries (2/2)



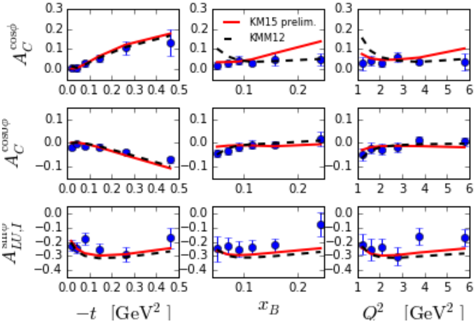
H1 (2007), ZEUS (2008)



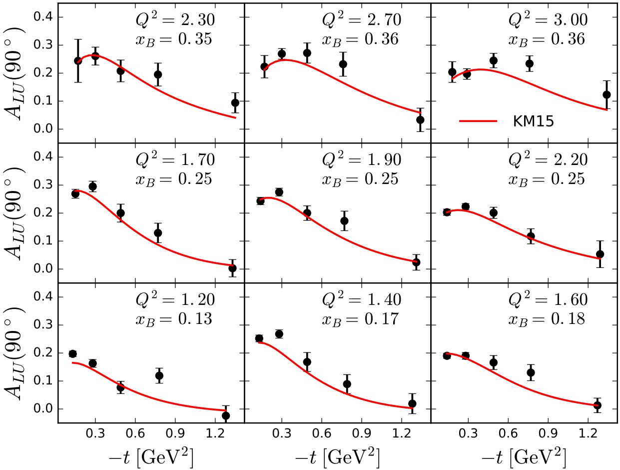
HERMES (2012)

$$BCA \equiv \frac{\mathrm{d}\sigma_{e^+} - \mathrm{d}\sigma_{e^-}}{\mathrm{d}\sigma_{e^+} + \mathrm{d}\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi} \cos\phi \sim \mathfrak{Re}\,\mathcal{H}$$

$$BSA \equiv \frac{\mathrm{d}\sigma_{e^\uparrow} - \mathrm{d}\sigma_{e^\downarrow}}{\mathrm{d}\sigma_{e^\uparrow} + \mathrm{d}\sigma_{e^\downarrow}} \sim A_{LU}^{\sin 1\phi} \sin\phi \sim \mathfrak{Im}\,\mathcal{H}$$

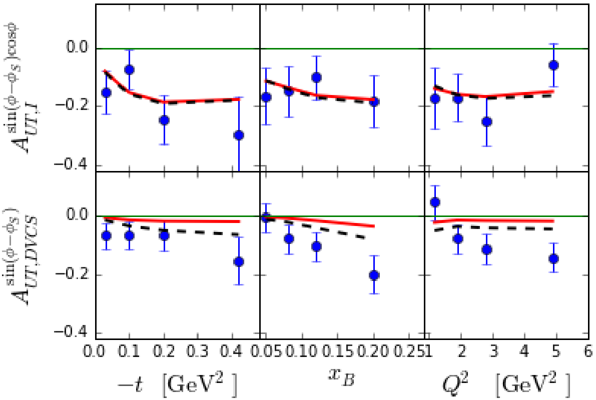


CLAS (2007)



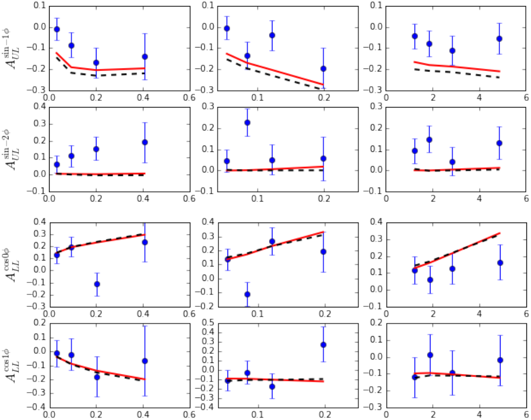
- Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.

Transversally polarized target — HERMES (2008)

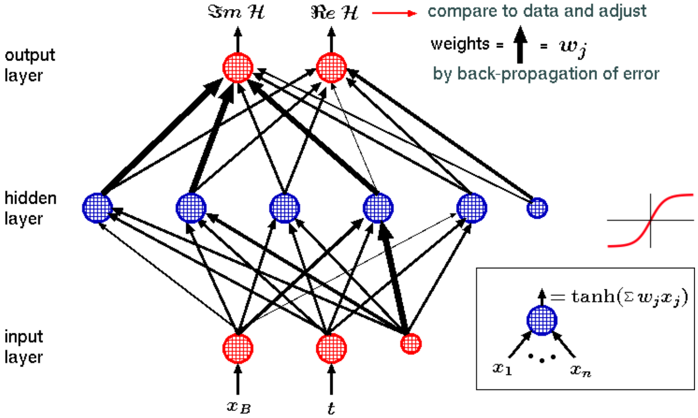


Longitudinally polarized target — HERMES (2010)

- Surprisingly large $\sin(2\phi)$ harmonic of A_{UL} cannot be described within this leading twist framework



Neural networks



- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias

Function fitting by a neural net

- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).

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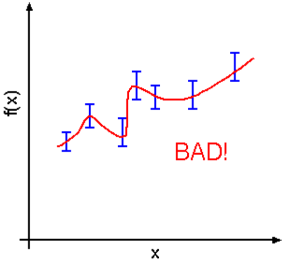

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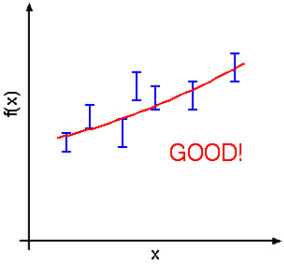

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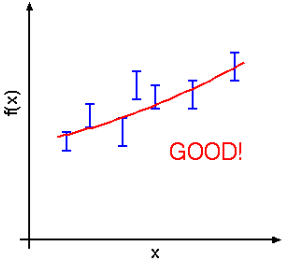
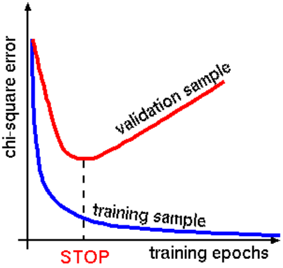
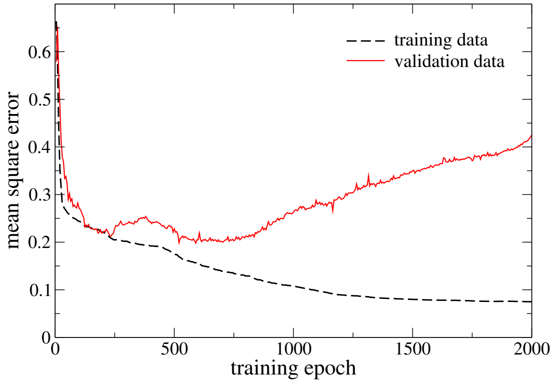
- With simple training of neural nets to data there is a danger of overfitting (a.k.a. overtraining)
- Solution: Divide data (randomly) into two sets: *training* sample and *validation sample*. Stop training when error of validation sample starts increasing.

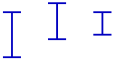




Example of a training with crossvalidation



Fitting by a set of neural networks



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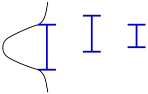

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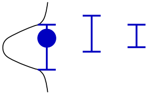

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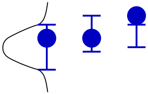

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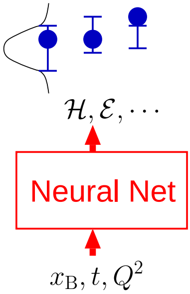

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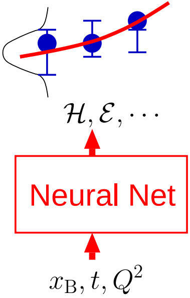

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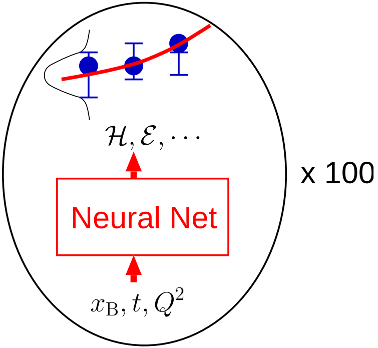

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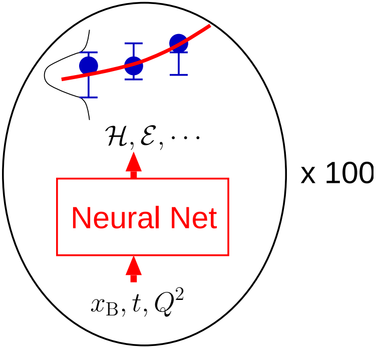
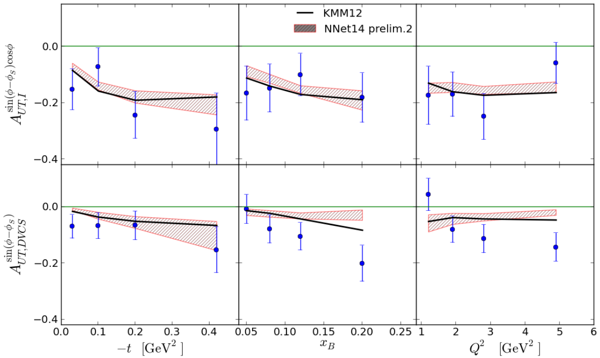


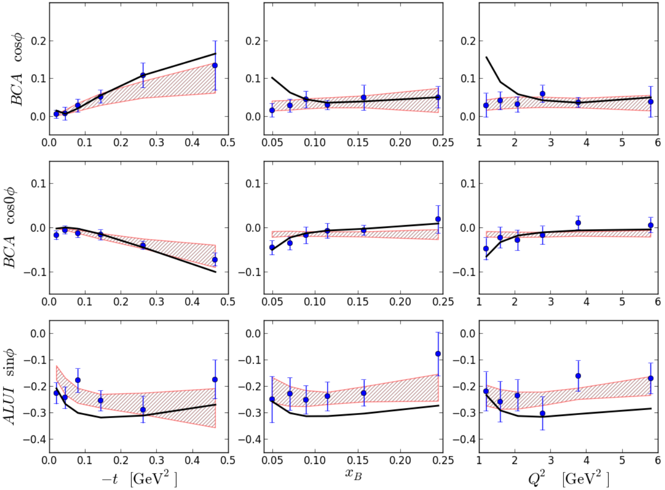
- Training networks on Monte Carlo replicated data preserves experimental uncertainties and their correlations [Giele et al. '01]
- Already successfully applied to PDF fitting by [NNPDF] group. Has even larger potential in GPD fitting with GPDs being less-known functions of more variables.

Preliminary neural Net HERMES fit

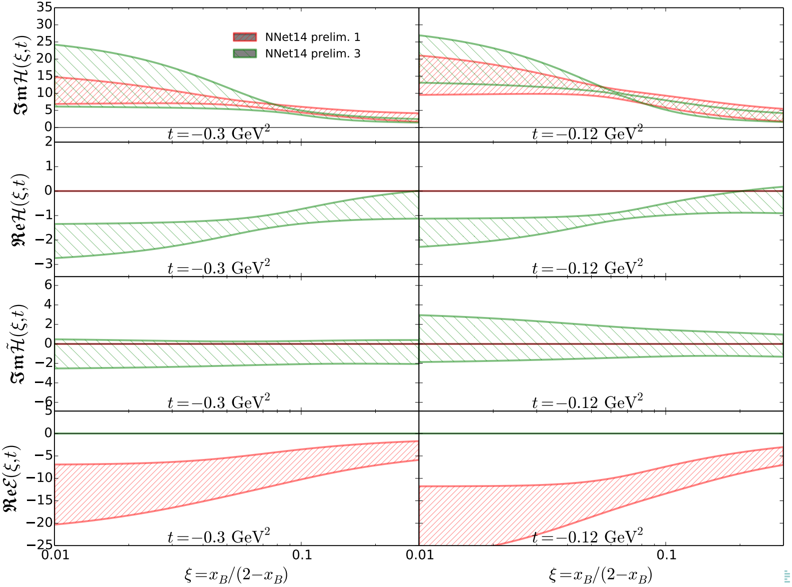
- Fit to all HERMES DVCS data with two types of neural nets
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{H},\ \mathfrak{Im}\,\tilde{\mathcal{H}})$: $\chi^2/n_{\rm pts} = 135.4/144$
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{E})$: $\chi^2/n_{\rm pts} = 120.2/144$



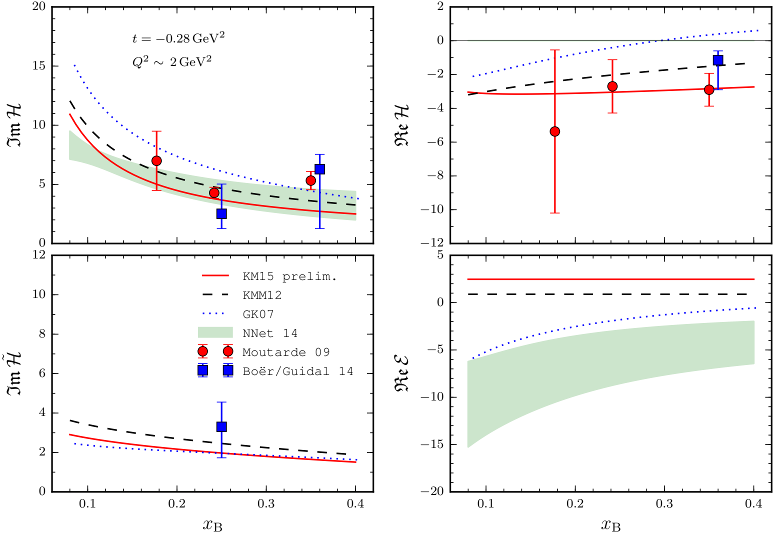
Neural Net HERMES fit - BSA/BCA



Neural Net HERMES fit - CFFs



Comparison of various approaches



Summary

- Global fits of all proton DVCS data using flexible hybrid models are in healthy shape
- Data clearly restrict $H(x, x, t)$, and to some extent $\tilde{H}$, but any information about E is very model-dependent
- New 2015 data relieve some old tensions
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The End