Form Factors and Their Interpretation

Gerald A. Miller, U. of Washington

Why study form factors?

- Confinement
- Confinement vs- chiral symmetry breaking for light quarks in nucleon
- Relation to Proton Radius- measurements using electrons and muons have different results: new physics? What is proton radius
- Dynamics -how does proton stick together at high momentum transfer

We need form factors

Outline

1. How not to and how to analyze electromagnetic form factors- transverse density

2. Model independent proton, neutron transverse charge density proton transverse magnetization density

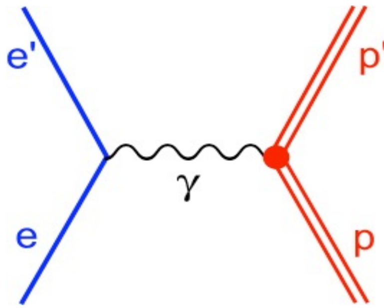
3. Connection with proton radius puzzle

4. Form factor dynamics and implications

5. Pion time-like data, transverse charge density, dynamics

Transverse Charge Densities.
Gerald A. Miller, arXiv:1002.0355 [nucl-th] ARNPS 60,1 2010

Electron-nucleon scattering



$J_\mu = \langle p' | \Gamma_\mu | p \rangle$

Nucleon vertex:

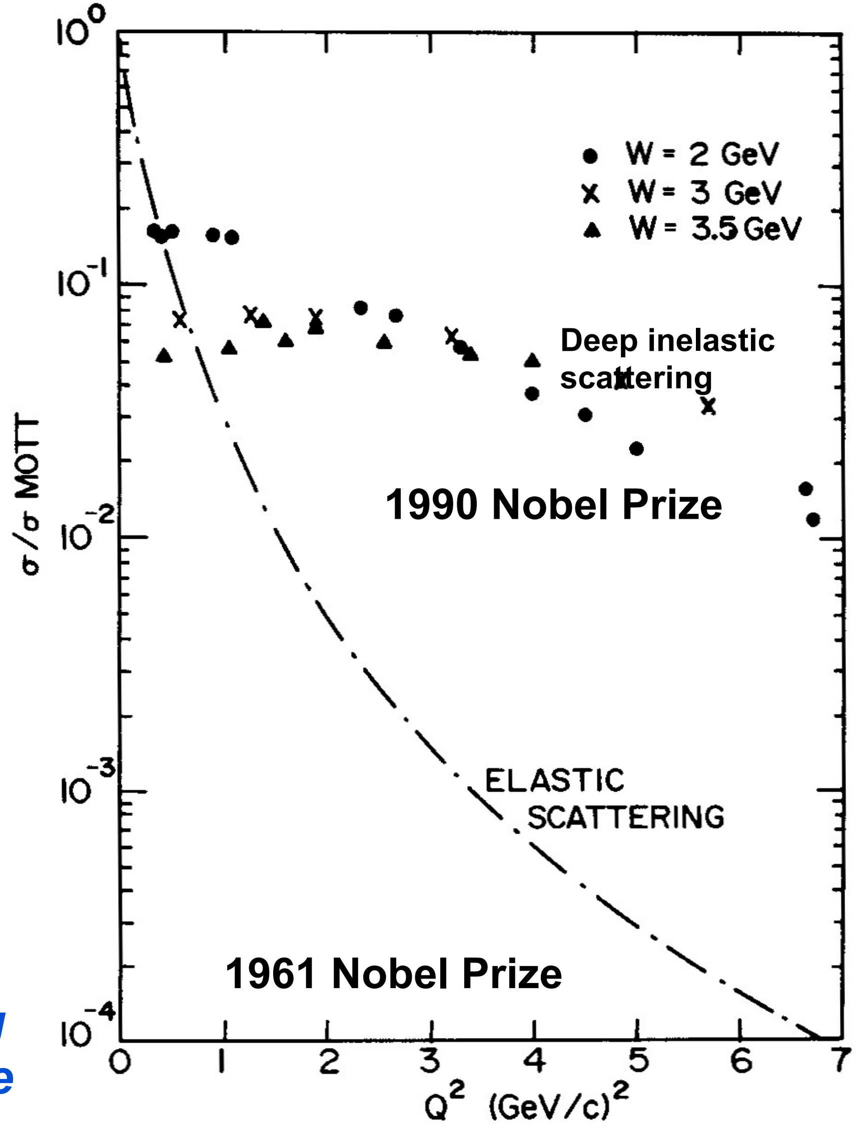
$$\Gamma_\mu(p',p) = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2M} F_2(Q^2)$$

Dirac **Pauli**

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(G_E^2 + \frac{\tau}{\epsilon} G_M^2\right) / (1+\tau)$$

Cross section for scattering from a point-like object

Form factors describing nucleon shape/structure



$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \; G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$$

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

WRONG

$$G_E(\vec{q}^{\,2}) = \int d^3r\rho(r)e^{i\vec{q}\cdot\vec{r}} \rightarrow \int d^3r\rho(r)(1-\vec{q}^{\,2}r^2/6+\cdots)$$

Coefficient of q^2 term is average of r^2, But what is r?

Correct non-relativistic:

wave function invariant under Galilean transformation

Neutron example Why FT of G_E is not a charge density

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$$

Big

$$\text{Low } Q^2: \quad G_E(Q^2) = -Q^2\left(\frac{R_1^2}{6} + \frac{\kappa_n}{4M^2}\right)$$

-

Non-zero neutron charge density caused by anomalous magnetic moment??????????

Problem- non relativistic doesn't work

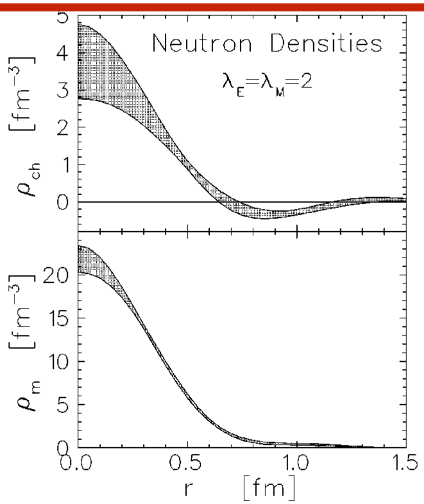
Need relativistic treatment

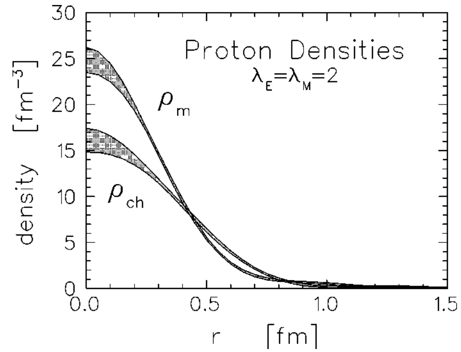
Relativistic : wave function is frame dependent, initial and final states differ

interpretation of Sachs FF is wrong

Final wave function is boosted from initial

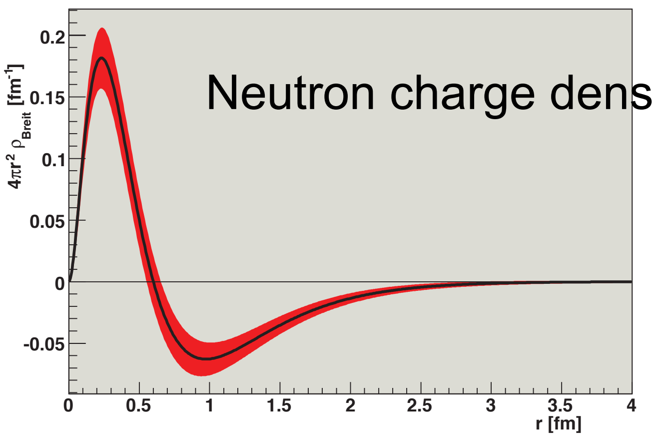
Relation between 3- dimensional and transverse densities- experimentalists love to 3 D F transform form factors

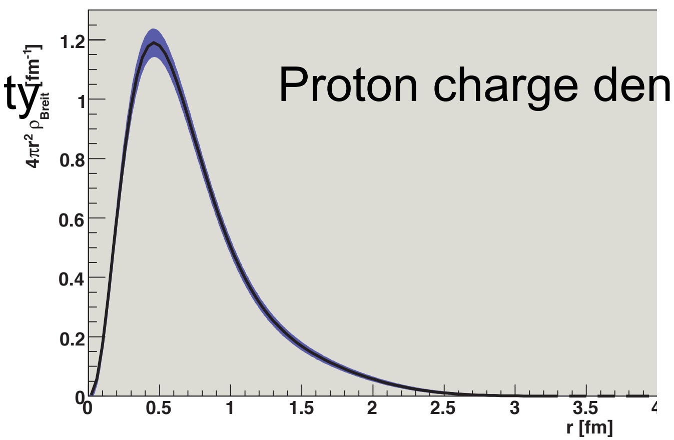




Kelly 2002

NSAC 2007

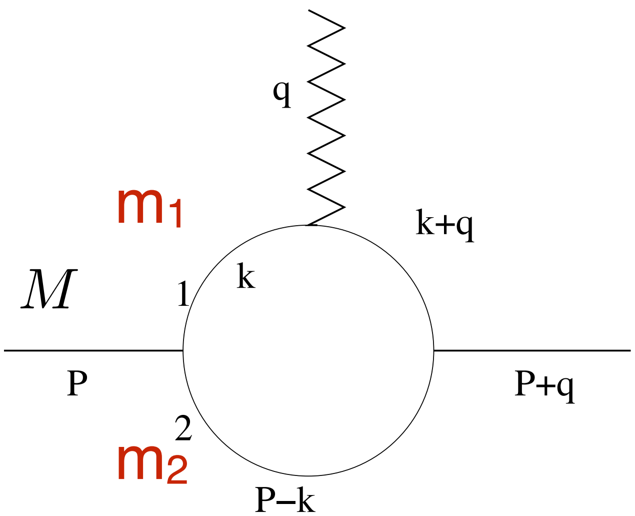




Sorry, not correct! No density interpretation of 3D FT of form factors but there is a way to make this kind of plot correctly.

Toy model GAM, Phys.Rev.C80:045210,2009.

- Scalar meson, mass M made of two scalars one neutral, $M=m_1+m_2-B$, $B>0$
- Exact covariant calculation of form factor



Direct evaluation of graph gives covariant, gauge invariant $F(Q^2)$, can be studied, vary masses look for |wave functions|^2

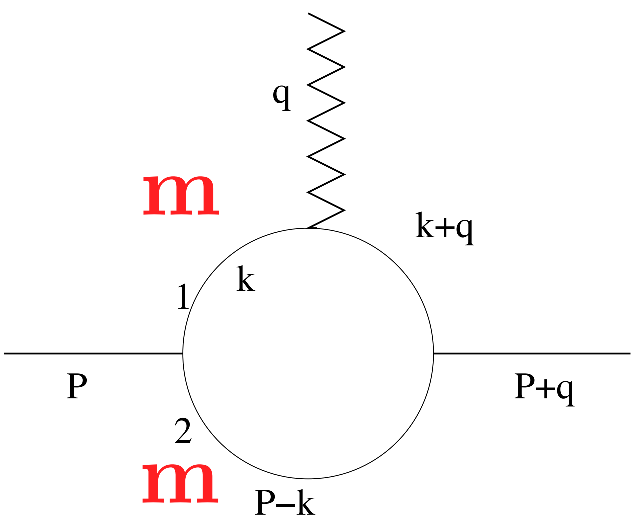
$$F(Q^2)(2P^\mu + q^\mu) = 4\frac{g^2}{16\pi^2}(2P^\mu + q^\mu)\int_0^1 dx \frac{x\,\mathrm{Tanh}^{-1}\left[\frac{\sqrt{Q^2}(1-x)}{4m^2-x(1-x)M^2+(1-x)^2Q^2}\right]}{\sqrt{Q^2}\sqrt{4m^2-x(1-x)M^2+(1-x)^2Q^2}}$$

Toy model $\mathbf{M = 2m - B,\ B > 0}$

- Infinite momentum frame, same result
- Integrate over minus-component, same result

$$F(Q^2) = \frac{g^2}{16\pi^2} \int d^2\kappa \int_0^1 \frac{dx}{x(1-x)} \psi^*(x, \boldsymbol{\kappa} + (1-x)\mathbf{q}) \psi(x, \boldsymbol{\kappa})$$

$$\psi(x, \boldsymbol{\kappa}) = g \left[M^2 - \frac{\boldsymbol{\kappa}^2 + m^2}{x(1-x)} \right]^{-1}$$



- **Non-relativistic limit, m/M >>B/M**
- **F(Q) IS 3Dim FT**
- **Leading order Chiral EFT**

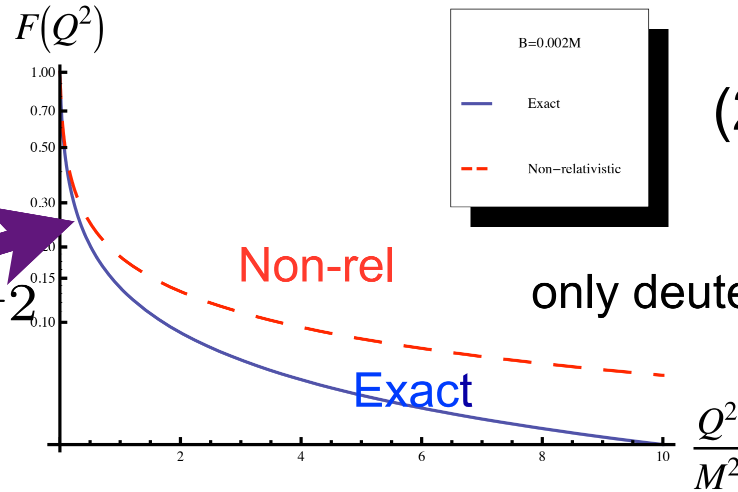
$$x \to \frac{m + \kappa^3}{M}, \psi \to \frac{g}{2m(-B - \frac{\kappa^2}{m})}$$

How small must B/M be for non-relativistic approximation to work?

Validity of non-relativistic (NR) approximation

validity of NR approx needed for form factor to be 3 D FT of density

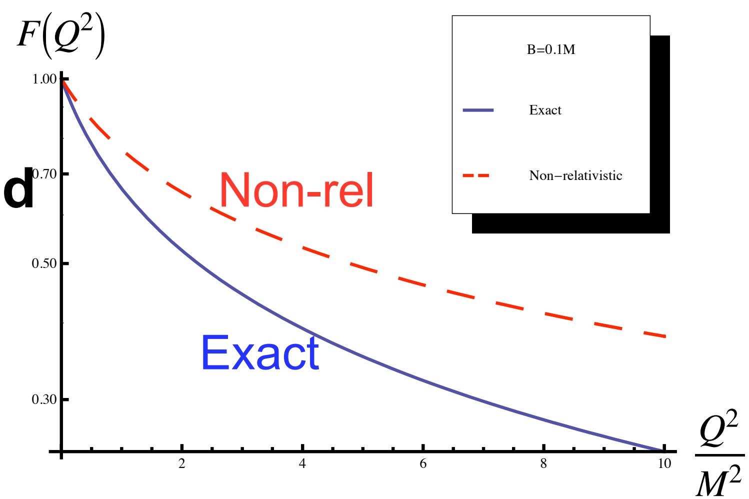
$Q^2 \leq 0.2M^2$



(2m-M)/M=0.002

only deuteron kinematics are non-rel

Relativity needed

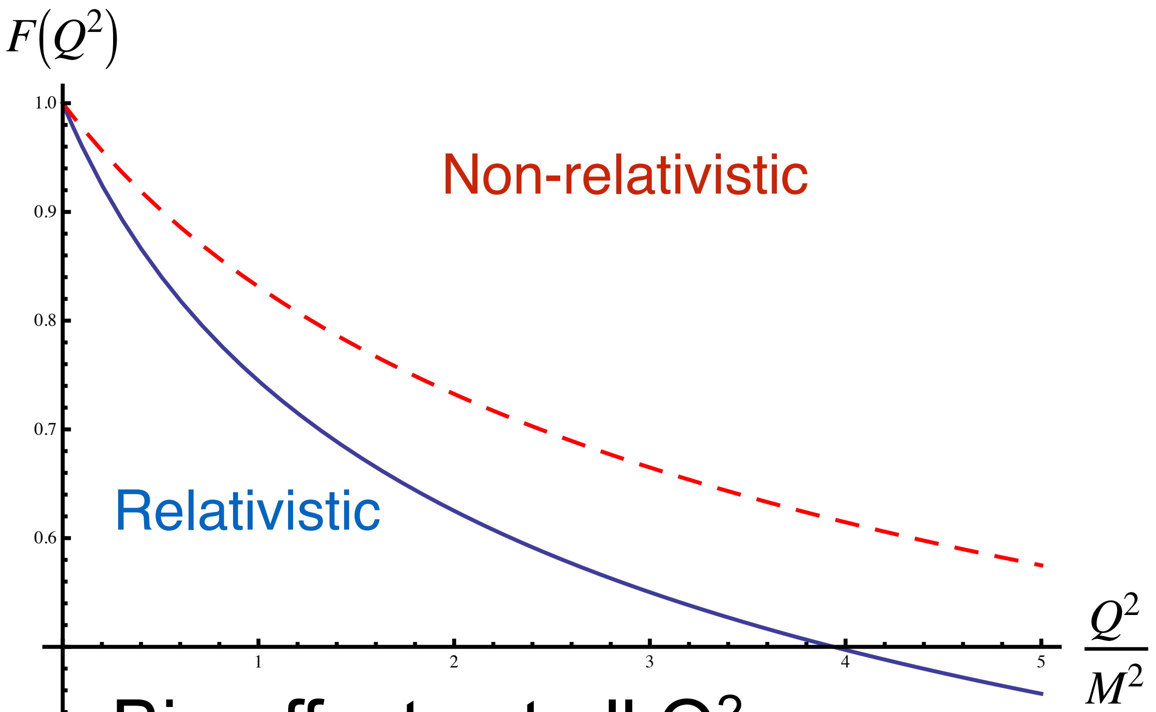


(2m-M)/M=0.1

Exact *vs* non-relativistic Form factors for the case $m_1 = m_2 = m$.

"quark-diquark nucleon"

$$m_2 = 2m_1, m = 400 \text{ MeV}, B = 260 \text{ MeV} = 0.276\ M$$



- Big effects at all Q^2
- Form factor is not 3 D FT of density
- Relativistic analysis of form factors is needed

Light front, Infinite momentum frame

$$\text{“}Time\text{”},\ x^+ = x^0 + x^3, \quad \text{“}Evolve\text{”},\ p^- = p^0 - p^3$$

$$\text{“}Space\text{”},\ x^- = x^0 - x^3, \quad \text{“}Momentum\text{”},\ p^+ (\text{Bjorken})$$

$$Transverse\ position,\ momentum \quad \mathbf{b}, \mathbf{p}$$

These variables are used in GPDs, TMDs, standard variables

transverse boosts in kinematic subgroup

$$\mathbf{k} \to \mathbf{k} - k^+ \mathbf{v} \qquad |\mathbf{R} = \mathbf{0}, \lambda\rangle = \int d^2p |\mathbf{p}, \lambda\rangle$$

$$\text{space} - \text{like}\ q^\mu, q^+ = 0,$$

$$\text{momentum transfer in transverse direction}$$

then density is 2 Dimensional Fourier Transform

Model independent transverse charge density

Light Front Charge Density Operator

$$\rho_\infty(x^-,\mathbf{b}) = \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

$$\boxed{\rho(b) \equiv \int dx^- \rho_\infty(x^-,\mathbf{b}) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)}$$

The true density is the 2 Dimensional FT of F_1!

Soper '77

$$\text{Density is } u-\bar{u},\ d-\bar{d}$$

What is charge density at the center of the neutron?

- **Neutron has no charge, but charge density need not vanish**
- **Is central density positive or negative?**

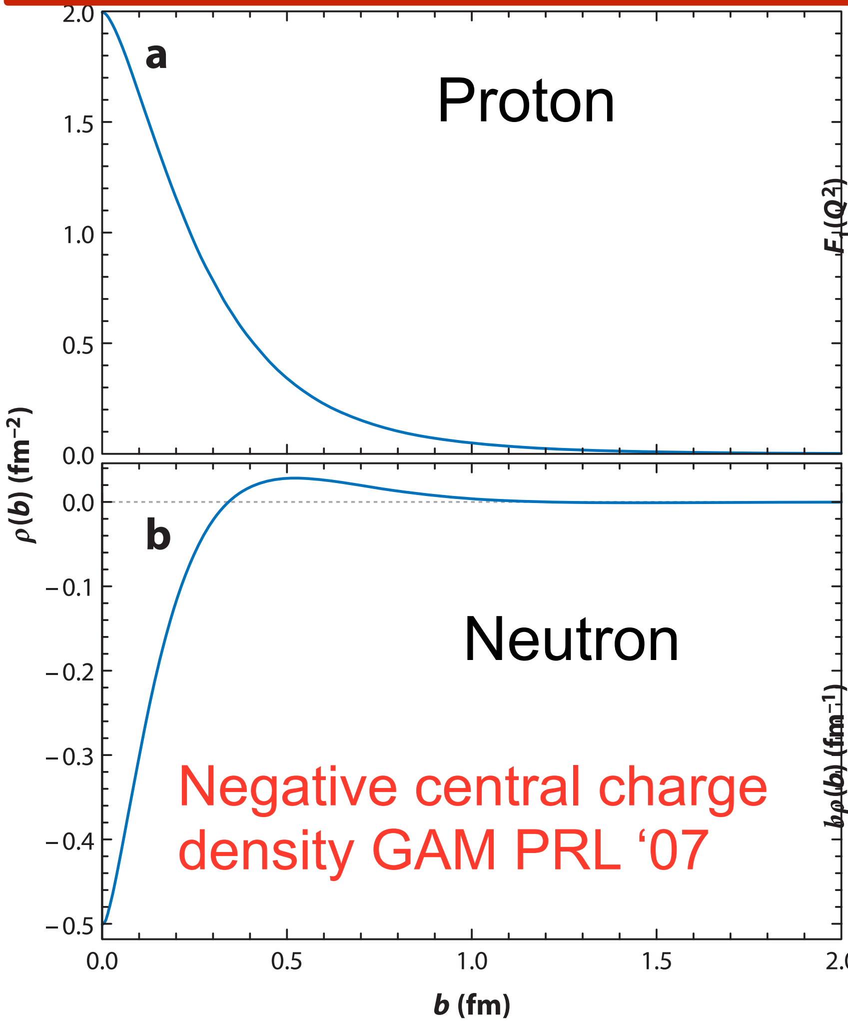
Fermi: n fluctuates to $p\pi^-$

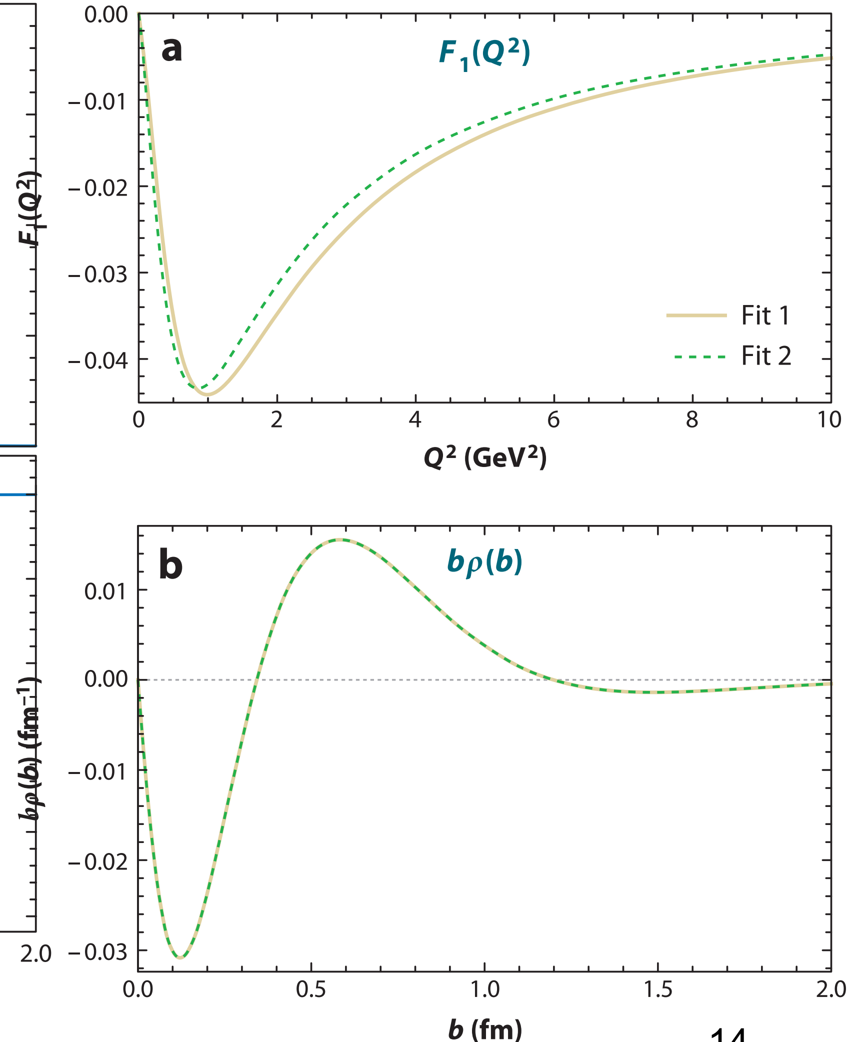
p at center, pion floats to edge

One gluon exchange favors dud

Real question- how does form factor relate to charge density?

Transverse charge densities from parameterizations (Alberico)





Why is central density of neutron negative ?

- d quarks dominate DIS from neutron at high x
- d quarks dominate at neutron center, or

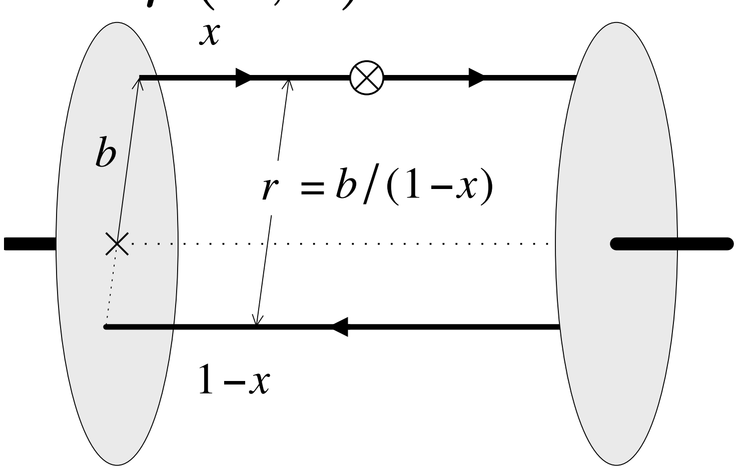
$$\pi^- = \bar{u}d$$

Quarks vs anti-quarks

Impact parameter dependent GPD Burkardt

Probability that quark at b from CTM has long momentum fraction x: $\rho(x,b)$

$$\rho(b) = \int dx\, \rho(x,b)$$



$$b = (1-x)r$$

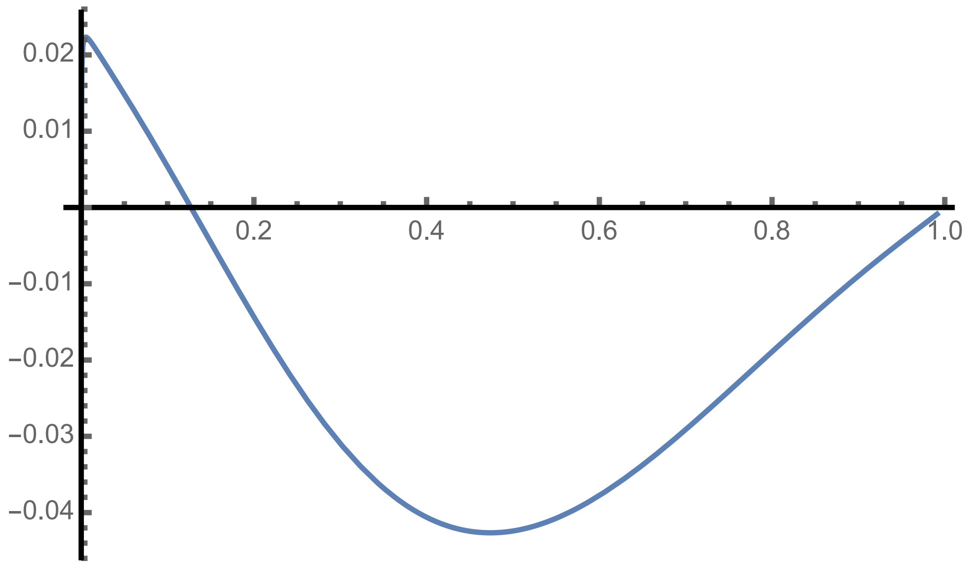
b is distance from center of momentum
r is relative distance
can get b=0 from x=1
if so r can be large - b=0 is not true center

Neutron interpretation ρ(x,b)

GAM, J. Arrington, PRC78,032201R '08

Using Kroll's GPD model

$$\rho_n(x, b=0)$$



$$b[\mathrm{GeV}^{-1}]$$

Negative density comes from high x, valence d quark effect

Caution- the GPDs are fit to form factors and DIS. Not enough DVCS data yet to independently get (x,b) dependence, other fits are possible.

Transverse Nucleon anomalous magnetization density

$$\vec{\mu}\cdot\vec{B} = \langle X| \int d^3r \frac{1}{2}(\vec{r}\times\vec{j})\cdot\vec{B}|X\rangle$$

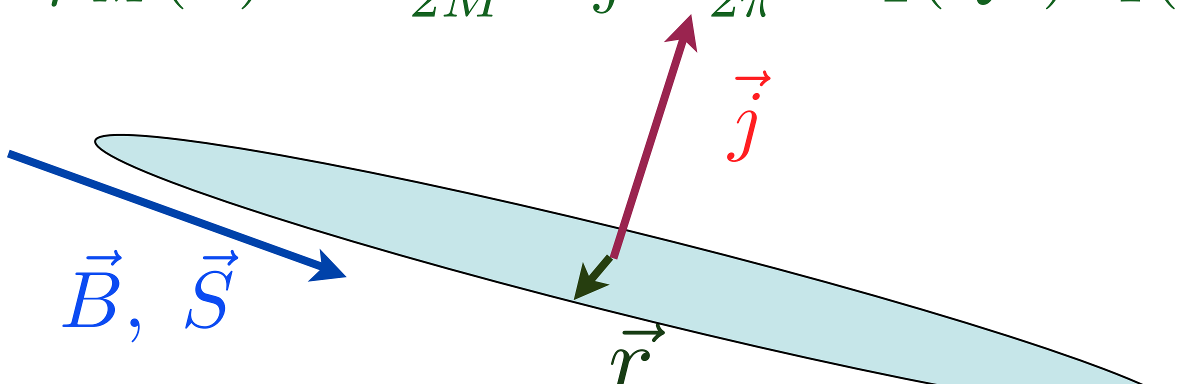
Transversely polarized target

$\vec{B}$ in x-direction

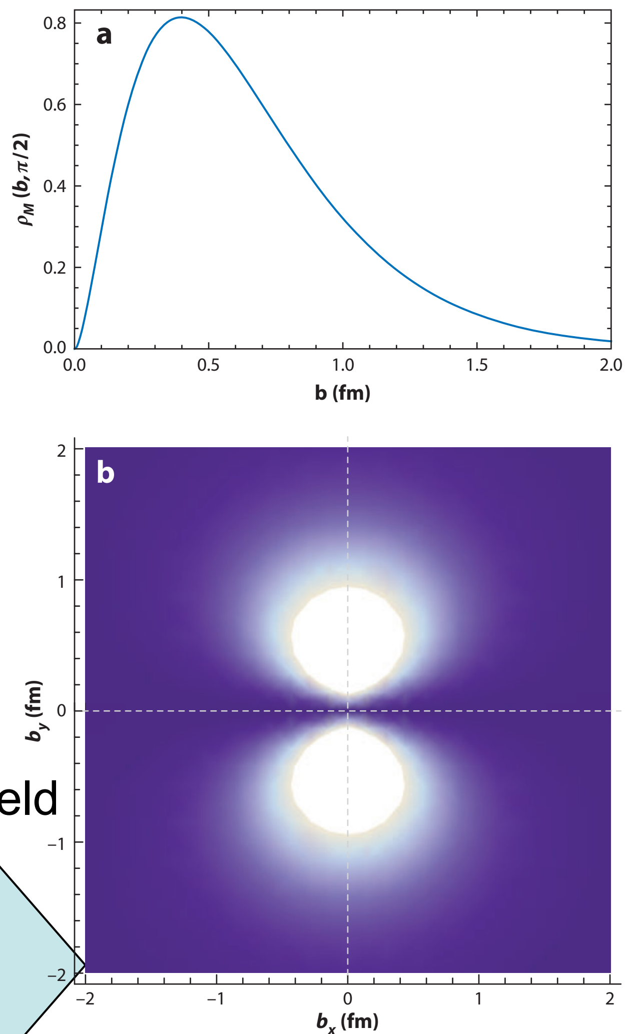
$1/2(\vec{r}\times\vec{j})$ in Infinite momentum frame

Magnetization density

$$\rho_M(\mathbf{b}) = \frac{\sin^2\phi}{2M} b \int \frac{Q^2 dQ}{2\pi} F_2(Q^2) J_1(Qb)$$



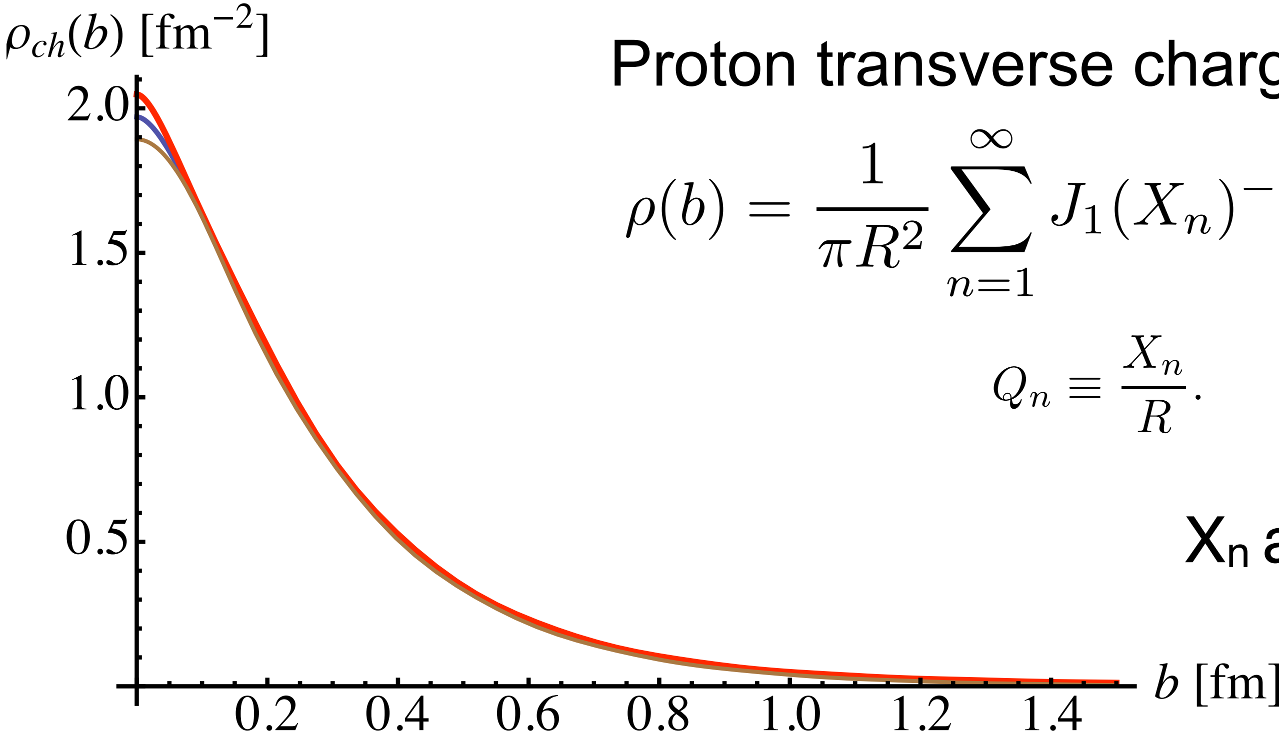
Transverse Nucleon anomalous magnetization density



Direction of magnetic field

How well are these known now?

- Analyze effect of experimental uncertainty and due to finite range of Q^2 - incompleteness



Proton transverse charge density

$$\rho(b) = \frac{1}{\pi R^2} \sum_{n=1}^{\infty} J_1(X_n)^{-2} F(Q_n^2) J_0(X_n \frac{b}{R}),$$

$$Q_n \equiv \frac{X_n}{R}.$$

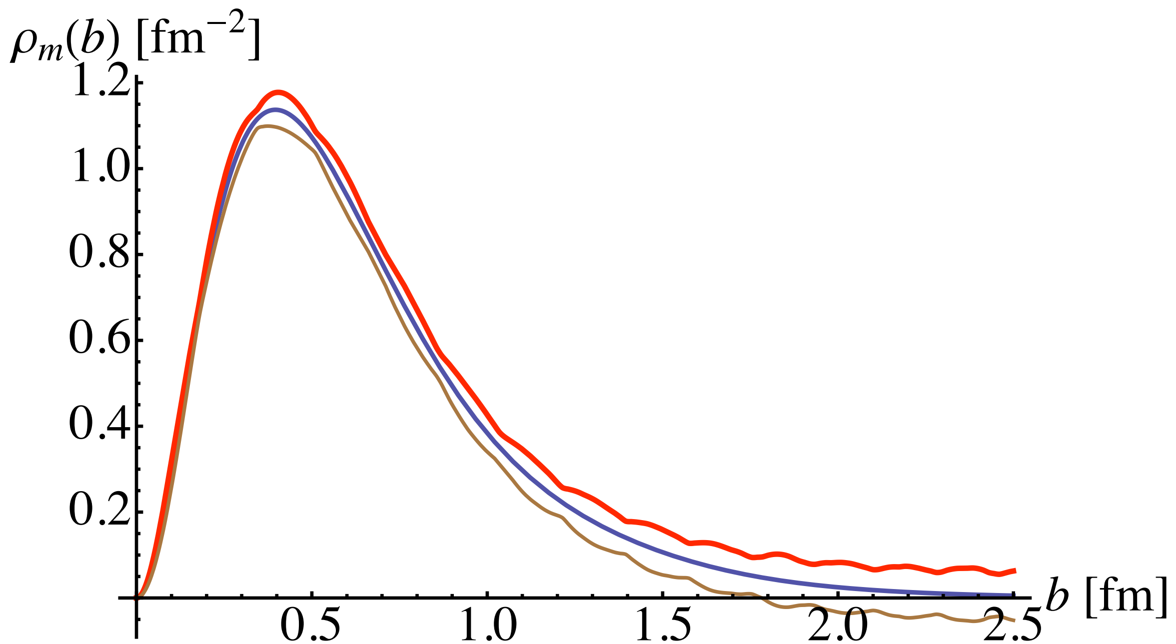
X_n are 0's of J_0

Venkat, Arrington, Miller, Zhan new analysis-

Phys. Rev. C **83**, 015203 (2011)

Proton anomalous magnetization density

$$\rho_m^{FRA} = \frac{1}{\pi R_2^2} \sum_{n=1}^{\infty} J_2^{-2}(X_{1,n}) b Q_{1,n} F_2(Q_{1,n}^2) J_1(Q_{1,n} b), \; Q_{1,n} \equiv \frac{X_{1,n}}{R_2}$$



Connection to proton radius puzzle: Why G_E seems like FT of density

Leading order interaction in atomic physics

Nucleon vertex is matrix element of Γ_0 between proton on-shell spinors

$V(r) = e^2 \int \frac{d^3q}{(2\pi)^3} \frac{G_E(|\vec{q}|^2)}{|\vec{q}|^2} e^{i\vec{q}\cdot\vec{r}}$ ($q^0 = 0$ Coulomb gauge)

$V(r) = \frac{e^2}{4\pi} \int \frac{d^3r'}{|\vec{r}-\vec{r}'|} (3\,\mathrm{Dim\,FT\,of\,G_E}(|\tilde{q}|^2))$

$3\,\mathrm{Dim\,FT\,of\,G_E}(|\tilde{q}|^2)$ acts as charge density for proton Coulomb interaction

In atomic physics $r_p^2 \equiv -6G'_E(Q^2 = 0)$

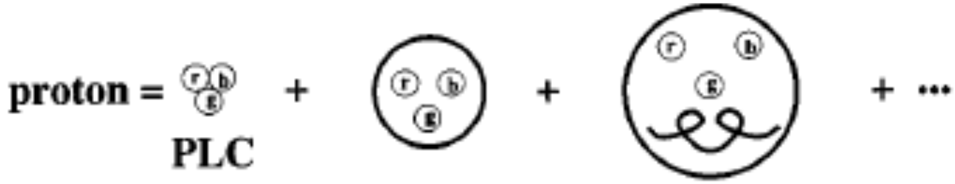
Extractions of r_p^2 using H atom differs from that of muonic H

Electron scattering experiments are challenged

Proton structure: issues needing more than form factors

- Proton is complicated object consisting of many Fock space configurations:

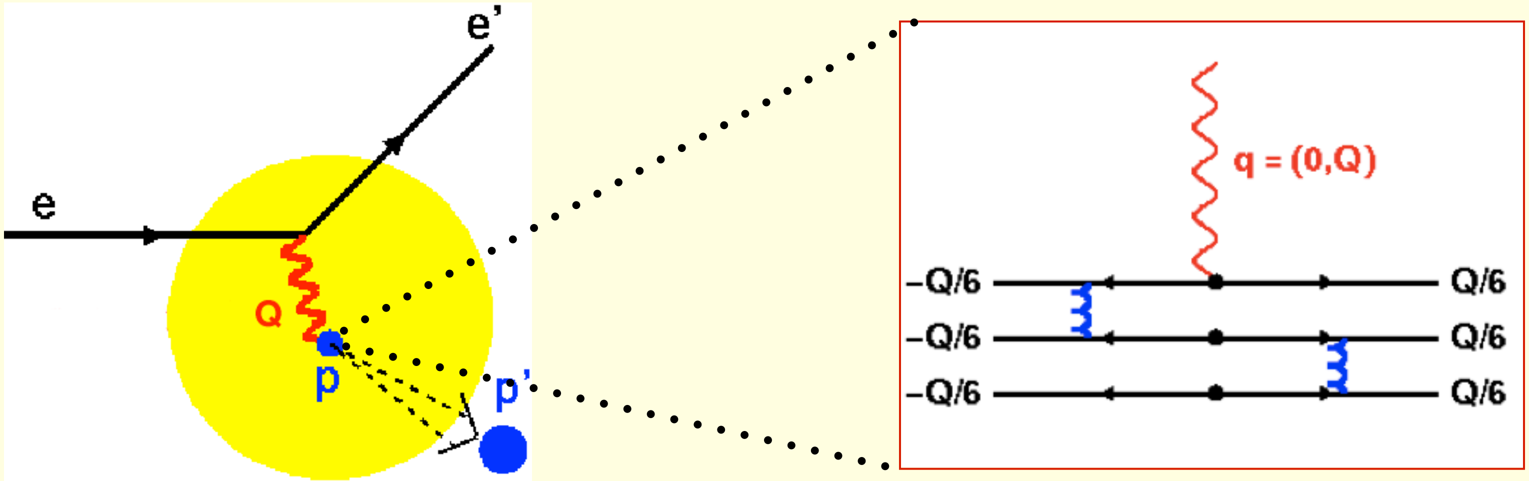
$$3q\,,4q\bar{q}\,,4q\bar{q}g\,,3qg\,,3q2g\;\cdots$$



- These configurations have different spatial extents
- PLC - point like configurations (small size)
- BLC -blob like configurations (large size)
- Hypothesis of pert. QCD - form factors at high momentum transfer are caused by PLC

Why PLC ?

Example: e-p scattering

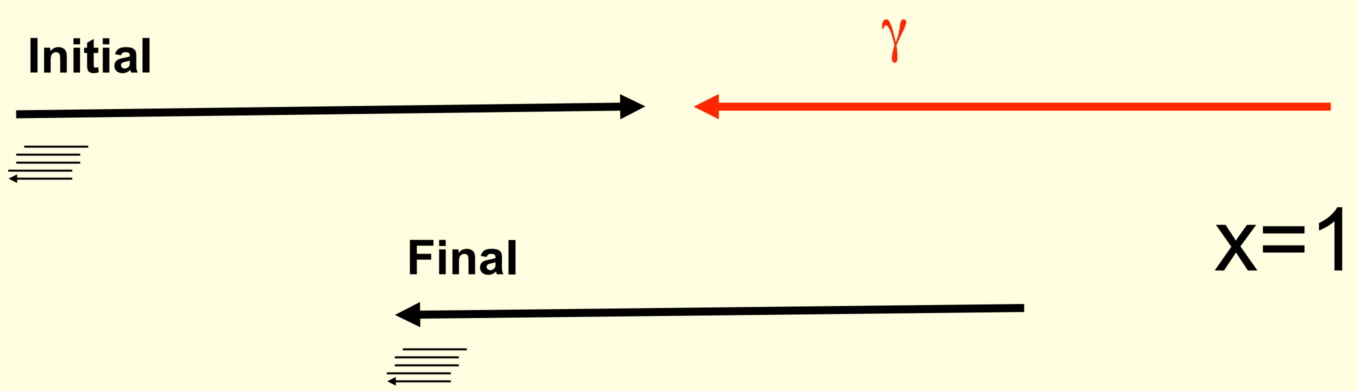


Momentum of exchanged gluon ~Q, separation ~1/Q

- At high enough Q an exclusive interaction occurs if the transverse size of the hadron is smaller than the equilibrium size.
- Perturbative reasoning-also non-perturbative

Why not PLC ?

e-p scattering **Feynman mechanism**



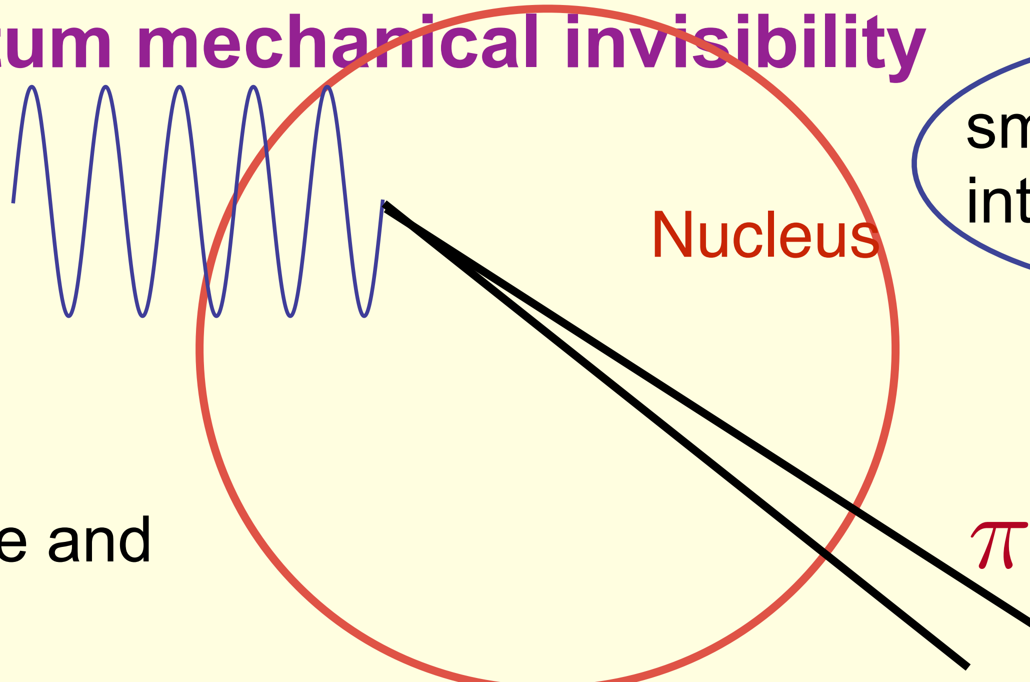
Transverse size not affected –no PLC

Interesting dynamical question about QCD –do PLC exist and participate?
Making PLC is squeezing- and is the interesting part

PLC vs Feynman mechanism

Implications of PLC vs Feynman

- Color transparency **High momentum transfer turns the proton into a color neutral Point Like Configuration- PLC**
- **These do not interact, not absorbed by nuclei, cast no shadow**
- **Quantum mechanical invisibility**



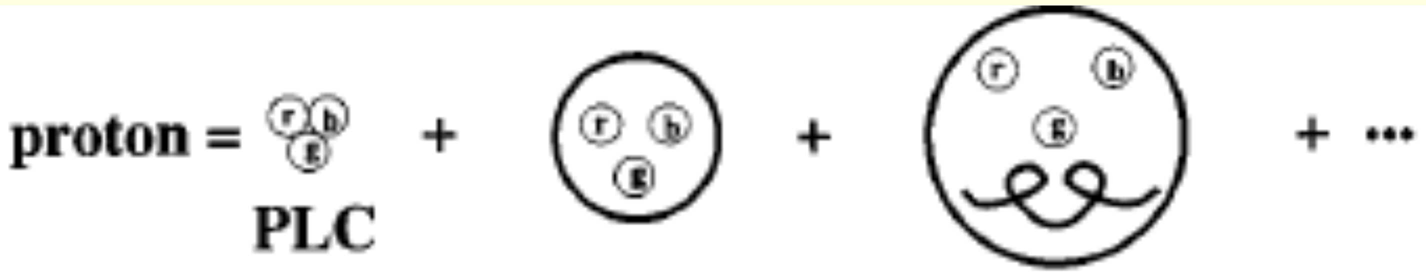
small size- limited interaction

$\pi,\ \rho,\ p$

Squeeze and freeze

Study in (e,e’p), (e,e’,Pi) etc on nuclei

Implications of PLC vs Feynman- EMC Effect Suppression of PLC in medium

free


place in medium:

normal size components attracted energy goes down

PLC does not interact- color screening

energy denominator increased, **PLC suppressed**

quarks in bound protons lose momentum in medium Studied in Nuclear DIS

Lattice FF calculations could be extended to tell PLC vs Feynman

Pion form factor

Determination of F_π via Pion Electroproduction

At low Q^2<0.3 GeV^2, the π^+ form factor can be measured **exactly** using high energy π^+ scattering from atomic electrons.

$\Rightarrow$ 300 GeV pions at CERN SPS. *[Amendolia et al., NP **B277**(1986)168]*

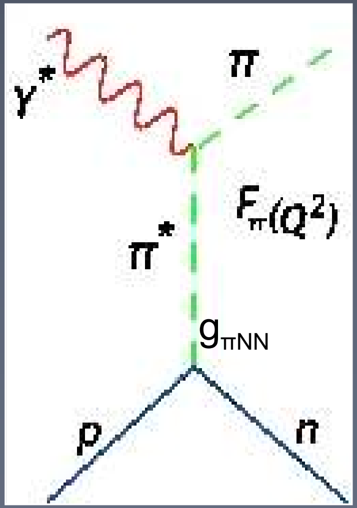
$\Rightarrow$ Provides an accurate measure of the π^+ charge radius.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

To access higher Q^2, one must employ the $p(e,e'\pi^+)n$ reaction.

- t-channel process dominates σ_L at small $-t$.
- In the Born term model:

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} \; g_{\pi NN}^2(t)\, F_\pi^2(Q^2,t)$$



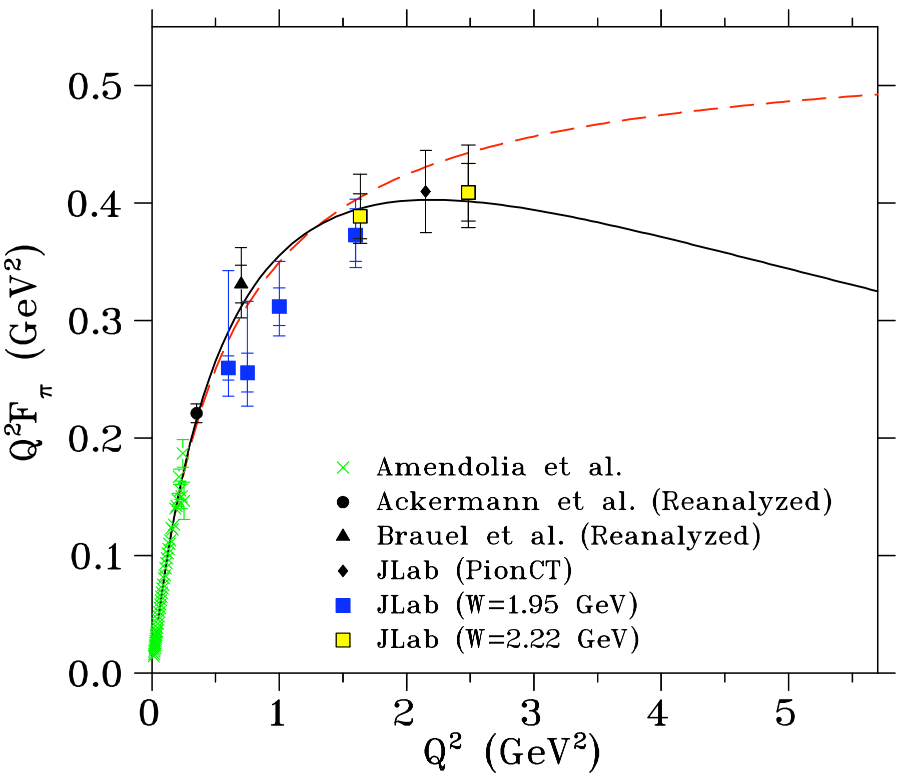
Pion Transverse Charge Density GAM Phys.Rev.C79:055204

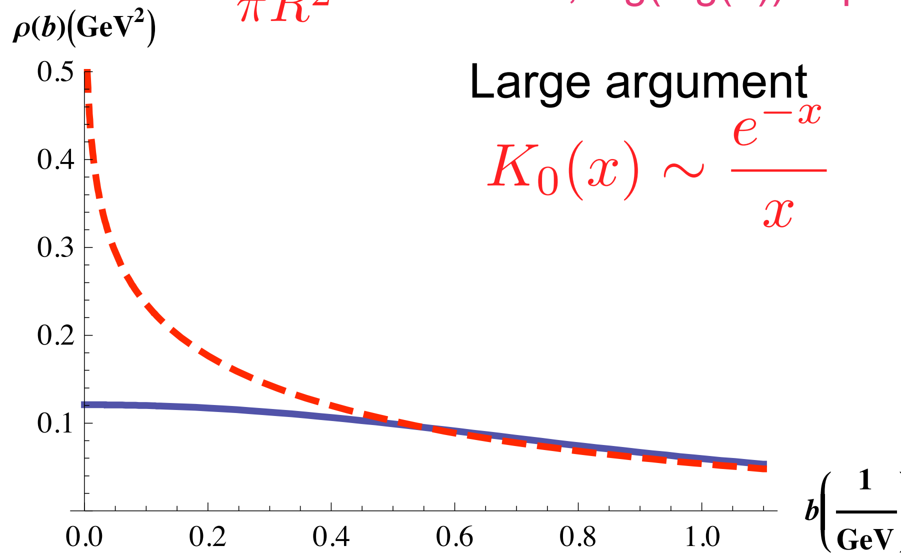
$$F_\pi(Q^2) = 1/(1 + R^2 Q^2/6),$$

$$\rho_\pi(b) = \frac{3K_0(\frac{\sqrt{6}b}{R})}{\pi R^2}$$

Singular - varies as log (b) small b, log(log(b)) in pQCD

Dashed- monopole fit, solid rel. cqm Huang





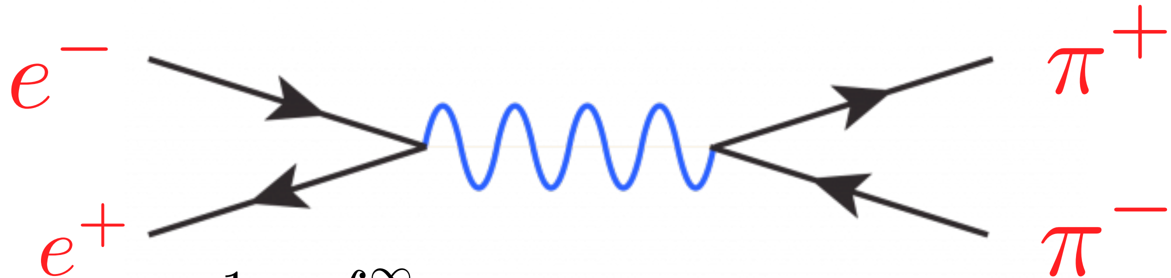
Large argument

$$K_0(x) \sim \frac{e^{-x}}{x}$$

Singular behavior at b=0 is not necessary to describe the current space-like form factor, but could a characterization tool

Pionic Transverse Density From Time-like and Space-Like Probes

Gerald A. Miller[1], Mark Strikman[2], Christian Weiss[3] Phys. Rev. D **83**, 013006 (2011)



$$\rho(b) = \frac{1}{(2\pi)} \int_0^\infty dQ Q J_0(Qb) F_\pi(Q^2)$$

$$F_\pi(t) = \frac{1}{\pi} \int_{4m_\pi^2}^\infty dt' \frac{Im F_\pi(t')}{t' - t + i\epsilon}.$$

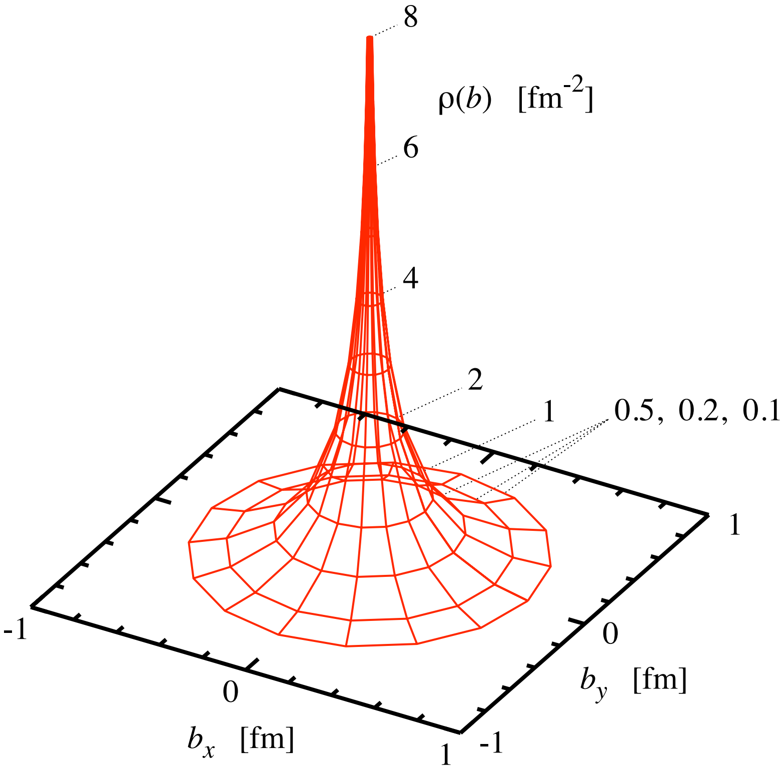
Dispersion relation uses time-like data

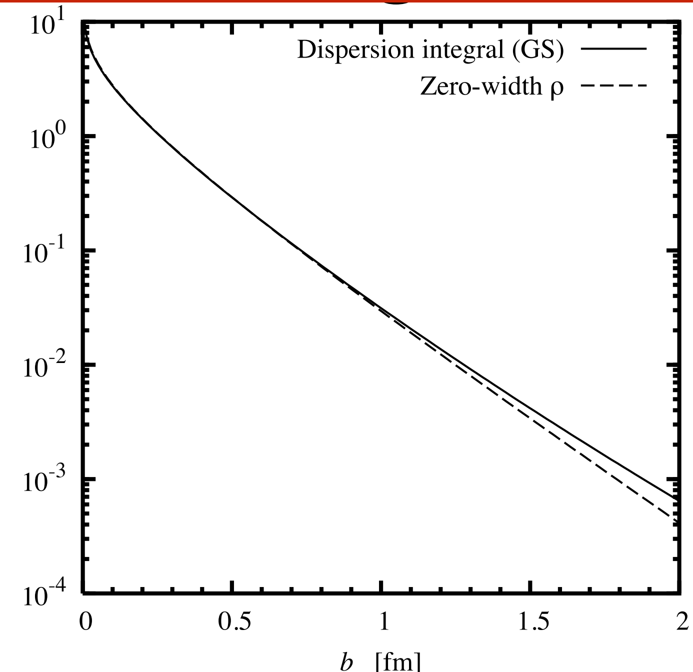
$$K_0(x) \sim \frac{e^{-x}}{x} \qquad \rho(b) = \frac{1}{2\pi} \int_{4m_\pi^2}^\infty dt K_0(\sqrt{t}b) \frac{Im F_\pi(t)}{\pi}.$$

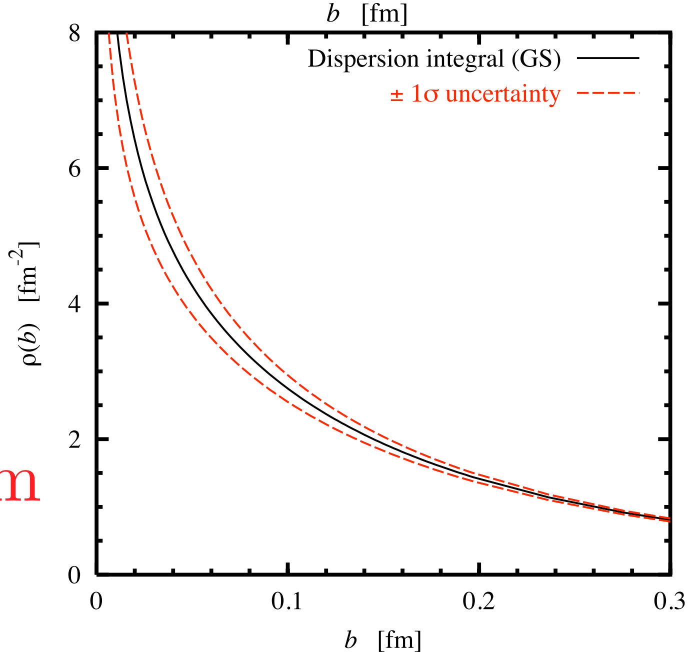
Low t dominates except for very small values of b

Model needed for high t: C. Bruch et al E. J Phys.C39, 41

Pion Transverse Charge Density







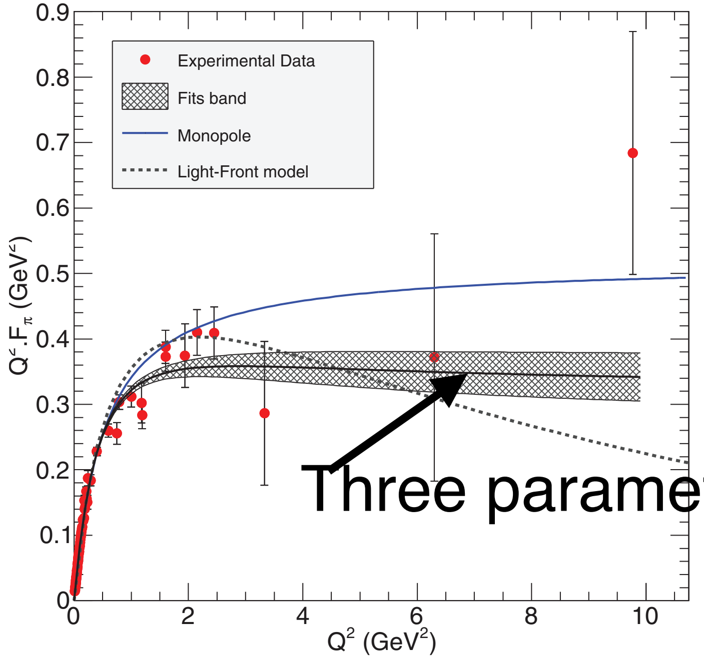
$\rho_\pi(b)$ is known for $b > 0.1$ fm

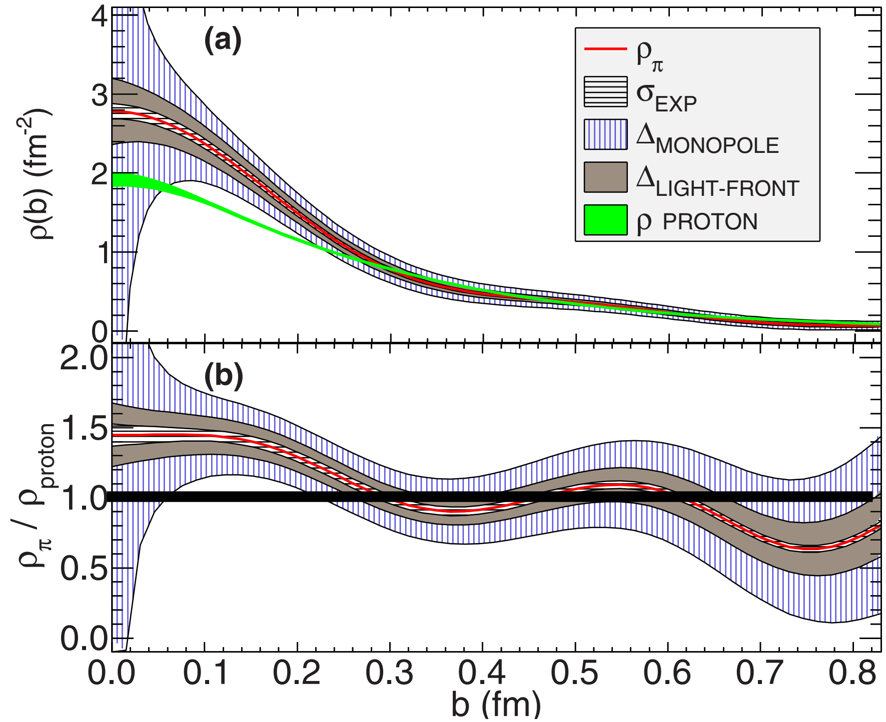
PHYSICAL REVIEW C **90**, 025211 (2014)

Pion transverse charge density and the edge of hadrons

Marco Carmignotto,[1,*] Tanja Horn,[1] and Gerald A. Miller[2]

data at t<0

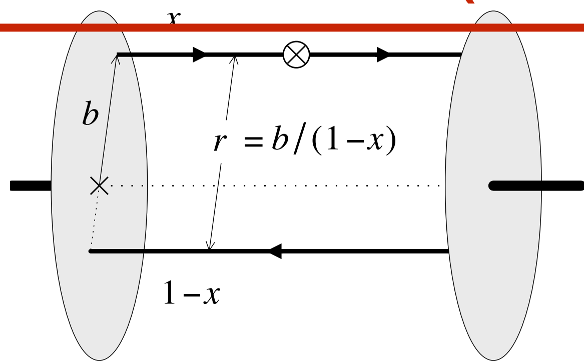




- Data is below the monopole form
- Uncertainy in pion transverse density dominated by incompleteness
- pion and proton have same transverse density $0.3 < b < 0.6$ fm

quark-diquark structure of proton?

b vs r : b=(1-x)r



r=q$\bar{\text{q}}$ transverse separation, b is distance from cm

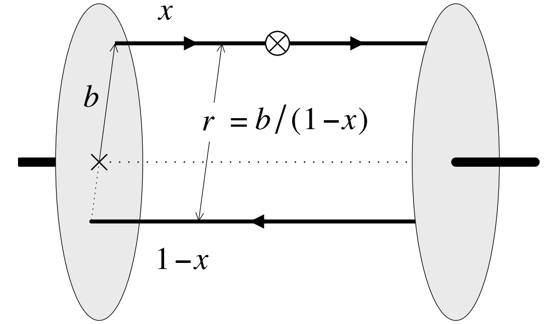
r-distribution $\rho_r(x,r) = (1-x)^2 \rho(x, b = (1-x)r)$

Singular in b is not necessarily singular in r
b distribution is accessible in elastic scattering, -dist is NOT
GPD is needed to tell- three models with same form factor below

A tale of two authors- three models (many more: Cloet, Eichmann)

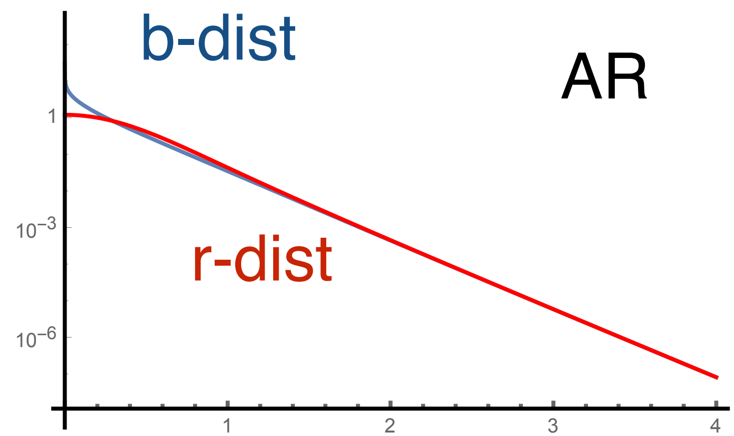
- Anatoly Radyushkin (AR) -ad hoc - quark dist .times function of Q^2
- Broniowski et al quark spectral function, confinement plus VMD
- Broniowski et al NJL- spontaneous symmetry breaking

b vs r : b=(1-x)r

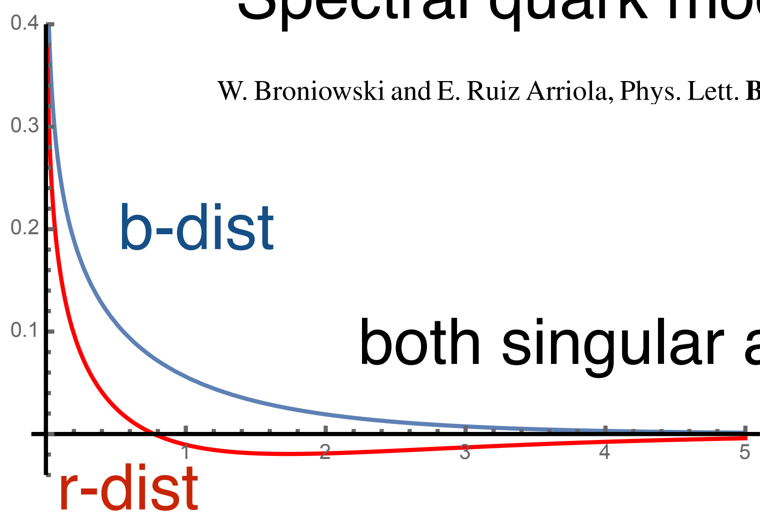


r=q $\overline{\text{q}}$ transverse separation,
b is distance from cm

r-distribution $\rho_r(x,r) = (1-x)^2 \rho(x, b=(1-x)r)$



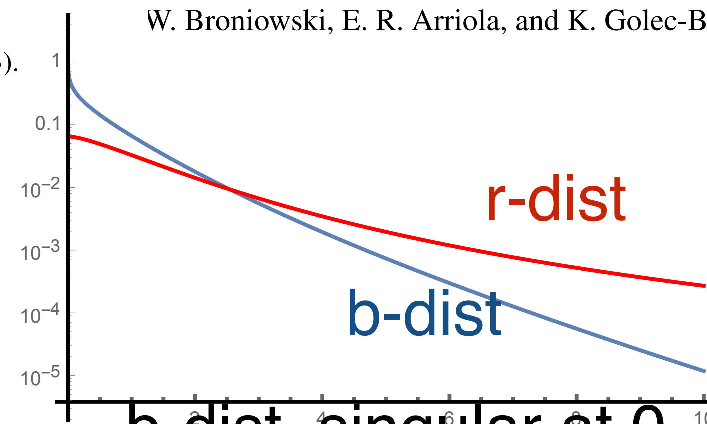
Spectral quark model



W. Broniowski and E. Ruiz Arriola, Phys. Lett. **B574**, 57 (2003).

NJL

W. Broniowski, E. R. Arriola, and K. Golec-Biernat,



b-dist singular at 0

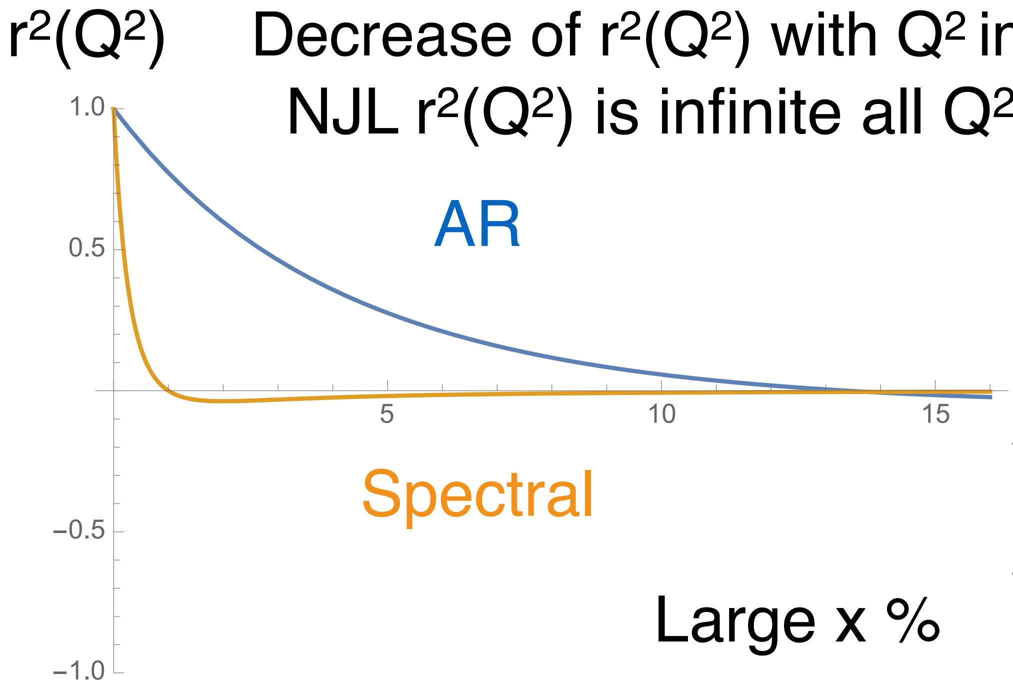
PLC existence: yes or no?

Frankurt, Miller Strikman Nucl. Phys. A555 (1993) 752

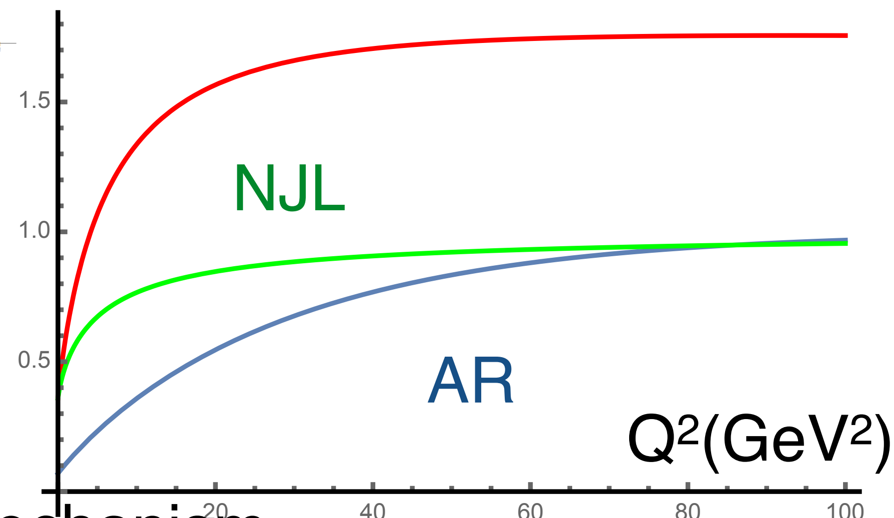
$$r^2(Q^2) = \frac{\int dx d^2b\, r^2\, e^{i\mathbf{Q}\cdot\mathbf{b}} \rho(x,b)}{\int dx d^2b\, r^2\, \rho(x,b)},\; r = b/(1-x)$$

Decrease of $r^2(Q^2)$ with Q^2 implies PLC exists

NJL $r^2(Q^2)$ is infinite all Q^2



Feynman mechanism?



All three models are Feynman mechanism

Summary of pion models

Model	Singular (b=0)	PLC	Feynman
AR	No	Yes	Yes
Spectral	Yes	YES (low Q)	Yes
NJL	No	NO	Yes

No one to one correspondence between b=0 sing., PLC
Need more models, ultimately lattice

Summary

- Much data exist, Jlab12 will improve data set, many experiments -talks here Form factors needed
- Charge density is not a 3 dimensional Fourier transform of G_E
- Interpret form factor as determining transverse charge and magnetization densities
- Nucleon transverse densities known now to high precision, neutron central t.density is negative caused by valence quarks
- Pion transverse density known fairly well, it is singular at origin
- Form factors do not tell all we want to know: Feynman vs PLC, b vs r, flavor composition, extension of lattice calcs (Syritsyn, Young, Portelli)