# Nucleon Form Factors in time-like and space-like regions

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Probing transverse nucleon structure at high momentum transfer
ECT* - European Center for Theoretical Studies in Nuclear Physics and Related Areas
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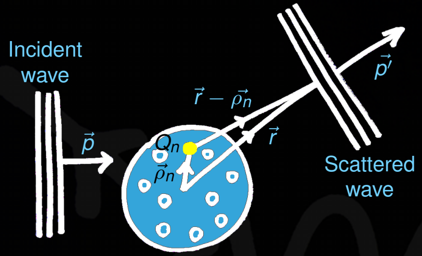
# AGENDA

- Nucleon Electromagnetic Form Factors
  - Definition and properties
- The space-like region
  - Proton radius
  - Rosenbluth versus Akhiezer-Rekalo
- The time-like region
  - Unphysical region
  - Threshold
  - An amazing effect
- The asymptotic region
- Analytic $R = \mu G_E / G_M$
- Conclusions

# SEMI-CLASSICAL DEFINITION




Amplitude of the scattered wave

$$\mathcal{A}_n(\vec{r}) = Q_0\, e^{i\,\vec{p}'\cdot\vec{r}}\, Q_n\, e^{i\,(\vec{p}-\vec{p}')\cdot\vec{\rho}_n}$$

$$\mathcal{A}(\vec{r}) = \sum_n \mathcal{A}_n(\vec{r}) = Q_0\, e^{i\,\vec{p}'\cdot\vec{r}} \sum_n Q_n\, e^{i\,\vec{q}\cdot\vec{\rho}_n}$$

Form factor

$$F(\vec{q}) = \frac{1}{Q_0} \langle\phi| \sum_n Q_n\, e^{i\,\vec{q}\cdot\vec{\rho}_n} |\phi\rangle$$

The **unpolarized** differential cross section on an extended object

$$\frac{d\sigma}{d\Omega} = \left|F(\vec{q})\right|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}}$$

Form factor **modulus** is obtained by comparing theory and experiment

In the Breit frame the Fourier transform of the form factor

$$\rho(\vec{r}) = \frac{Q_0}{(2\pi)^3} \int d^3\vec{q}\, F(\vec{q})\, e^{-i\,\vec{q}\cdot\vec{r}}$$

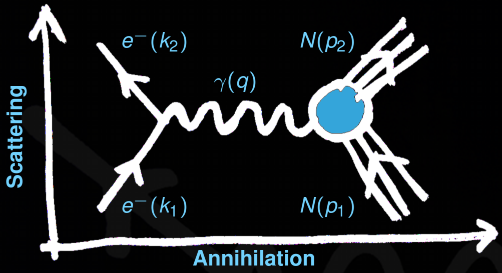
is the **charge spatial distribution.**
The form factor is Fourier transform of $\rho(\vec{r})$

$$F(\vec{q}) = \frac{1}{Q_0} \int d^3\vec{r}\, \rho(\vec{r})\, e^{i\,\vec{q}\cdot\vec{r}}$$

# NUCLEON ELECTROMAGNETIC FORM FACTORS

- Form factors characterize the internal structure of a hadron $\Rightarrow F_{\textbf{point-like}} = \text{constant}$.
- Elastic form factors contain information on the hadron ground state.
- In a parity and $T$-invariant theory, the electromagnetic structure of a particle of spin $S\hbar$ is defined by **$2S+1$ form factors**.
- Neutron and proton form factors are different.
- Playground for **theory** and **experiment**:
  - at low $q^2$ probe the size of the hadron;
  - at high $q^2$ test the QCD counting rule.

# DIRAC AND PAULI FORM FACTORS




- Scattering: $e^- N \to e^- N$
  **Space-like** kinematic region

$$q^2 = -2\omega_1\omega_2(1-\cos\theta_e) \leq 0$$

- Annihilation: $e^+e^- \leftrightarrow N\bar{N}$
  **Time-like** kinematic region

$$q^2 = 4\omega^2 > 0$$

Scattering amplitude in **Born** approximation

$$\mathcal{M} = \frac{1}{q^2}\left[e\,\bar{u}(k_2)\gamma_\mu u(k_1)\right]\underbrace{\left[e\,\overline{U}(p_2)\Gamma^\mu(p_1,p_2)U(p_1)\right]}_{\text{Nucleon EM 4-current: } J_N^\mu}$$

**From Lorenz and gauge invariance**

$$\Gamma^\mu(p_1,p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^N(q^2)$$

- Dirac FF: $F_1^N(q^2)$, $F_1^N(0) = \mathcal{Q}_N$
- Pauli FF: $F_2^N(q^2)$, $F_2^N(0) = \kappa_N$

$\mathcal{Q}_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

# Sachs Form Factors

**Breit frame**

No energy exchanged

$$p_1 = (E, -\vec{q}/2)$$
$$p_2 = (E, \vec{q}/2)$$
$$q = (0, \vec{q})$$

**Nucleon elecrtomagnetic four-current**

$$J_N^\mu = (J_N^0, \vec{J}_N) \quad \begin{cases} \rho_q = J_N^0 = e\left[F_1^N + \frac{q^2}{4M_N^2}F_2^N\right] \\ \vec{J}_N = e\,\overline{U}(p_2)\vec{\gamma}U(p_1)\left[F_1^N + F_2^N\right] \end{cases}$$

**Sachs Nucleon Form Factors**

$$G_M^N(q^2) = F_1^N(q^2) + F_2^N(q^2) \qquad G_E^N(q^2) = F_1^N(q^2) + \frac{q^2}{4M_N^2}F_2^N(q^2)$$

In the Breit frame represent the **Fourier transforms** of **charge** and **magnetic moment spatial distributions** of the nucleon

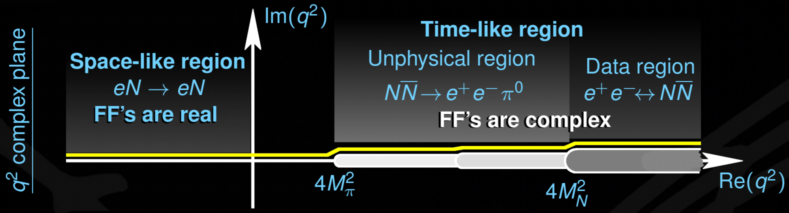
Normalization at $q^2 = 0$

- $G_E^N(0) = \mathcal{Q}_N$
- $G_M^N(0) = \mu_N$

$$\mu_N = \mathcal{Q}_N + \kappa_N$$

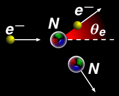
is the nucleon magnetic moment

# Cross sections and analyticity



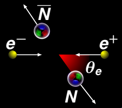

**Crossing: tot. helicity** $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

$$G_E(4M_N^2) = G_M(4M_N^2)$$




## Elastic scattering

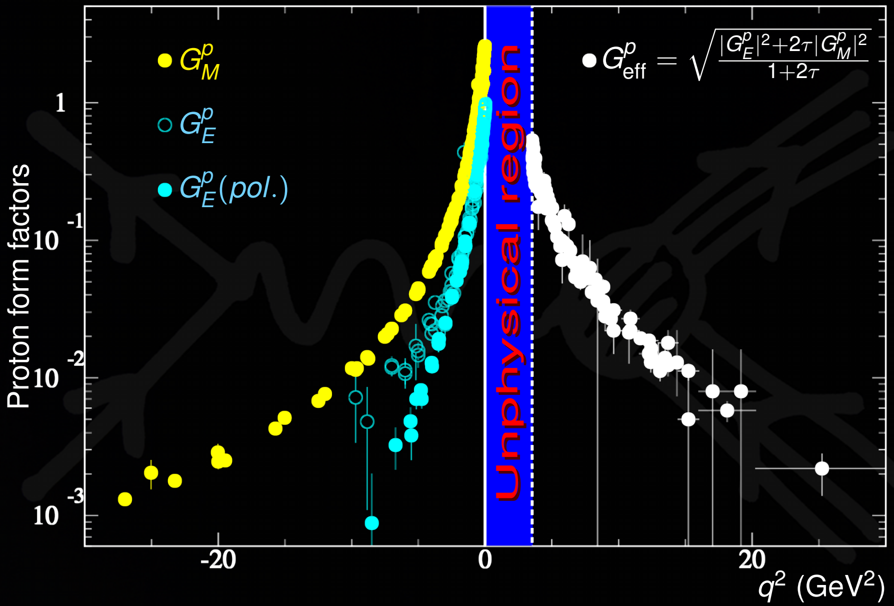
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2\frac{\theta_e}{2}}{4\omega_1^3 \sin^4\frac{\theta_e}{2}} \left[ G_E^2 - \tau \left(1 + 2(1-\tau)\tan^2\frac{\theta_e}{2}\right) G_M^2 \right] \frac{1}{1-\tau} \qquad \tau = \frac{q^2}{4M_N^2}$$



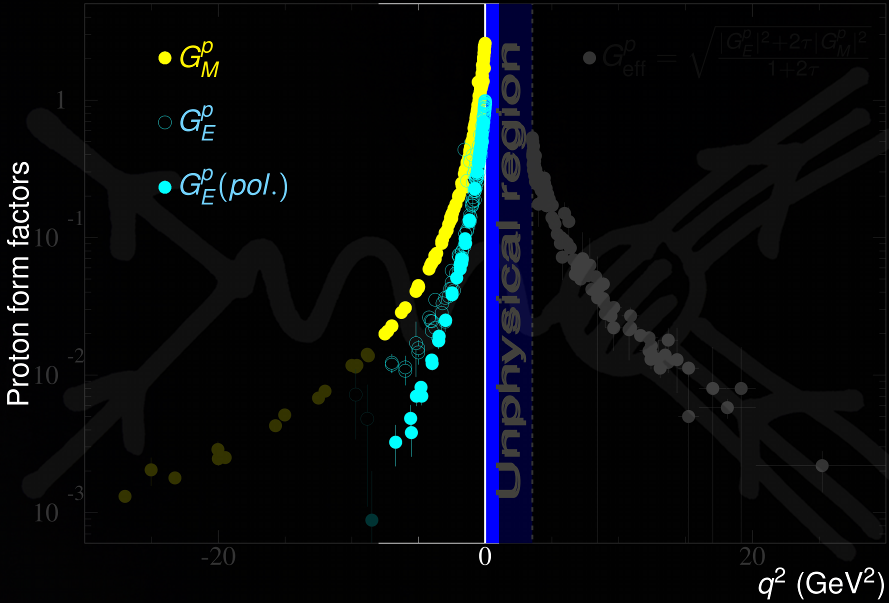

## Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2} \left[ (1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2 \right] \qquad \beta = \sqrt{1 - \frac{1}{\tau}}$$

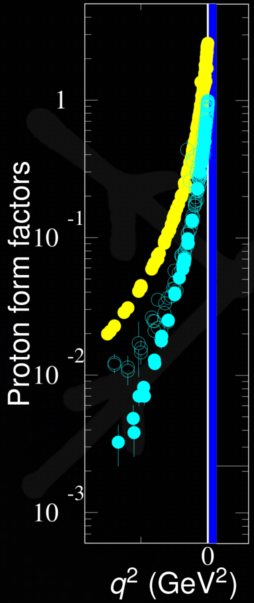
# THE PROTON RADIUS



# The proton radius



# The proton radius




$$G_E^p(q^2) = \int d^3\vec{r}\,\rho(r)\,e^{i\,\vec{q}\cdot\vec{r}} = 1 + \frac{1}{6}q^2\left\langle r_c^2\right\rangle + \mathcal{O}(q^4)$$

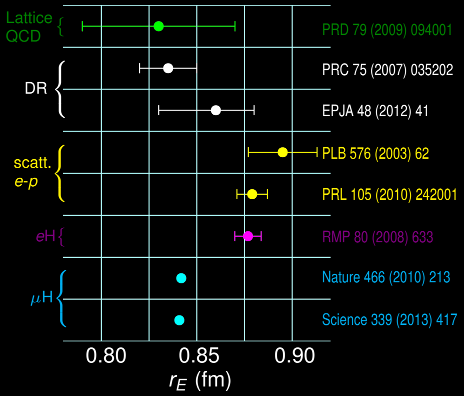
$\rho(r)$: normalized spherical charge density

The charge radius

$$r_E = \sqrt{\langle r_c^2\rangle} = \sqrt{4\pi\int_0^\infty r^4\rho(r)\,dr} = \sqrt{\frac{6}{G_E^p(0)}\left.\frac{dG_E^p}{dq^2}\right|_{q^2=0}}$$

| Charge density $\rho(r)$ | Form factor $G_E^p(q^2)$ | Charge radius $r_E$ | Comments |
|---|---|---|---|
| $\delta^3(r)$ | 1 | 0 | pointlike |
| $e^{-\lambda r}$ | $\lambda^4/(q^2+\lambda^2)^2$ | $2\sqrt{3}/\lambda$ | dipole |
| $e^{-\lambda r}/r$ | $\lambda^2/(q^2+\lambda^2)$ | $\sqrt{6}/\lambda$ | monopole |
| $e^{-\lambda r^2}/r^2$ | $e^{-q^2/(4\lambda^2)}$ | $1/\sqrt{2\lambda}$ | gaussian |

# THE PROTON RADIUS




Ongoing discussions. . .

- $q^2 \to 0^-$ extrapolation
- Radiative corrections
- Two-photon exchange
- Coulomb corrections

Analyticity via dispersion relations and QCD counting rules can give directly the proton radius. . .

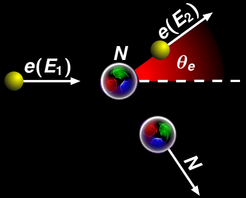
Logarithmic derivative of form factor at $q^2 = 0$ by means of dispersion relations for the logarithm

$$r_E^2 = \frac{12M_\pi^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\ln |G_E^p(t)/G_E^p(0)|}{t^2\sqrt{t - 4M_\pi^2}} dt$$

# UNPOLARIZED CROSS SECTION
# POLARIZATION OBSERVABLES

N.M.Rosenbluth,Phys.Rev.79,615

A.I.Akhiezer,M.P.Rekalo,Sov.Phys.Dokl.13,572




**Rosenbluth formula**

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \frac{1}{1-\tau} \left[G_E^2 - \frac{\tau}{\epsilon} G_M^2\right] \qquad \tau = \frac{q^2}{4M_N^2}$$

Mott pointlike cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} = \frac{4\alpha^2}{(-q^2)^2}\frac{E_2^3}{E_1}\cos^2(\theta_e/2)$$

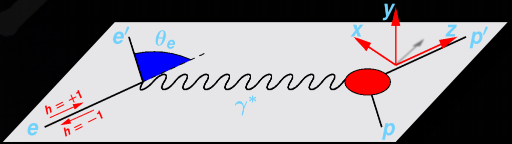
Degree of linear polarization of the virtual photon

$$\epsilon = \left[1 + 2\,(1-\tau)\tan^2(\theta_e/2)\right]^{-1}$$

In case of **polarized electrons** ($h = \pm 1$) on unpolarized nucleon target and measuring the polarization of the outgoing proton:

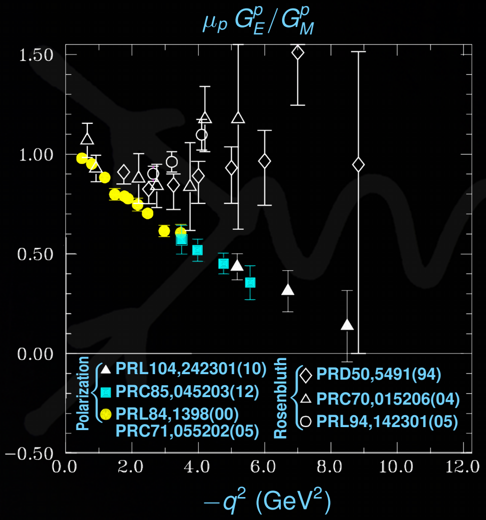
$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon}G_M^2}\, G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

$$P'_z = \frac{(E_e + E'_e)\sqrt{\tau(\tau-1)}}{M\left(G_E^2 - \frac{\tau}{\epsilon}G_M^2\right)}\, G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$



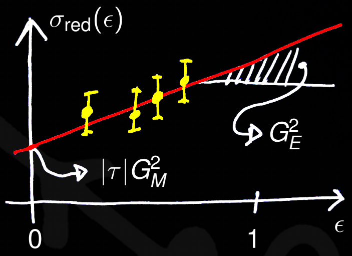

$$\frac{P'_x}{P'_z} = -\frac{2M\cot(\theta_e/2)}{E_e + E'_e}\,\frac{G_E}{G_M}$$

# $\mu_p G_E^p / G_M^p$: Rosenbluth and polarization techniques




- "Standard" dipole for the proton magnetic form factors $G_M^p$
- Linear deviation from the dipole for the electric proton form factor $G_E^p$
- **QCD scaling still not reached**
- **Zero crossing for $G_E^p$**
- Polarization data do not agree with old Rosenbluth data (◇)
- New Rosenbluth separation data from JLab **still do not agree** with polarization data

# The Rosenbluth method

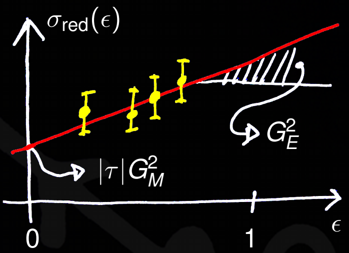



Reduced cross section $\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\rm Mott}}\epsilon(1-\tau)$

$$\sigma_{\rm red}(\epsilon;\theta,Q^2) = \epsilon G_E^2 + |\tau|\, G_M^2$$

- Function of $\epsilon$ at fixed $\theta$ and $Q^2$
- Slope $\rightarrow G_E^2$
- Intercept $\rightarrow G_M^2$

# The Rosenbluth method




Reduced cross section $\dfrac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{Mott}}}\epsilon(1-\tau)$

$$\sigma_{\text{red}}(\epsilon;\theta,Q^2) = \epsilon G_E^2 + |\tau|\, G_M^2$$

- Function of $\epsilon$ at fixed $\theta$ and $Q^2$
- Slope $\rightarrow G_E^2$
- Intercept $\rightarrow G_M^2$

$$\sigma_{red}(\varepsilon) = \underbrace{\epsilon G_E^2}_{<3\%} + \underbrace{|\tau| G_M^2}_{>97\%}$$

$$\delta \sigma_{red}(\varepsilon) \sim 1.5\,\%$$

For $Q^2 \geq 4$ GeV$^2$ the $G_E^2$ contribution to the reduced cross section can be of the same order of the error

$\Longrightarrow$ Large correlation

$\Longrightarrow$ Large errors

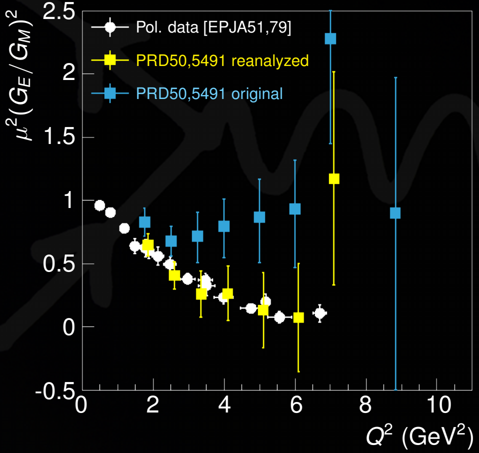
$\Longrightarrow$ Large rad. corr.

# THE ROSENBLUTH RE-ANALYSIS[1]

ARXIV:1604.02421

"New" fit function

$$\sigma_{red}(\epsilon) = G_M^2 \left(R^2 \epsilon + |\tau|\right)$$




- It reduces the effect of the corrections on the individual form factors
- The general normalization and systematic errors absorbed by $G_M^2$

$$R^2 = \frac{G_E^2}{G_M^2}$$

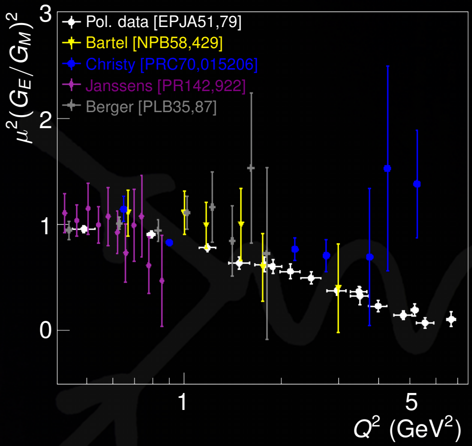
Reference data from PRD50,5491

Good agreement with pol. data

No other reaction mechanisms (two-photon exchange) are needed

# THE ROSENBLUTH RE-ANALYSIS$_2$

ARXIV:1604.02421




Other sets of data agree with polarization measurements

Those not in agreement, Qattan [PRL94, 142301] and Walker [RD49, 5671], show values of $R^2$ growing with $Q^2$

For these experiments radiative corrections and/or correlations are especially large

# THE ROSENBLUTH RE-ANALYSIS$_2$

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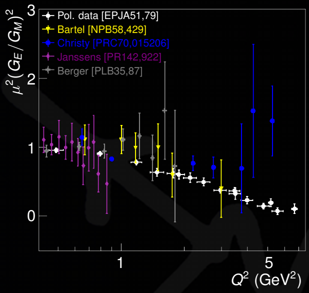
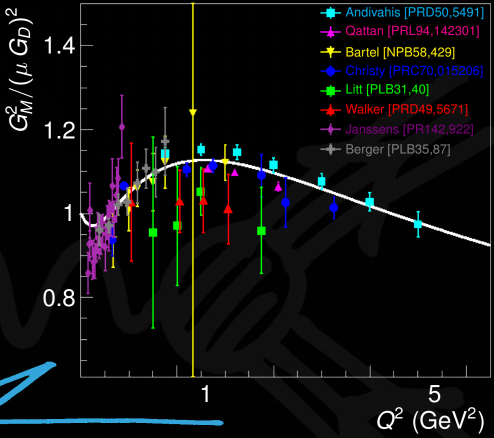
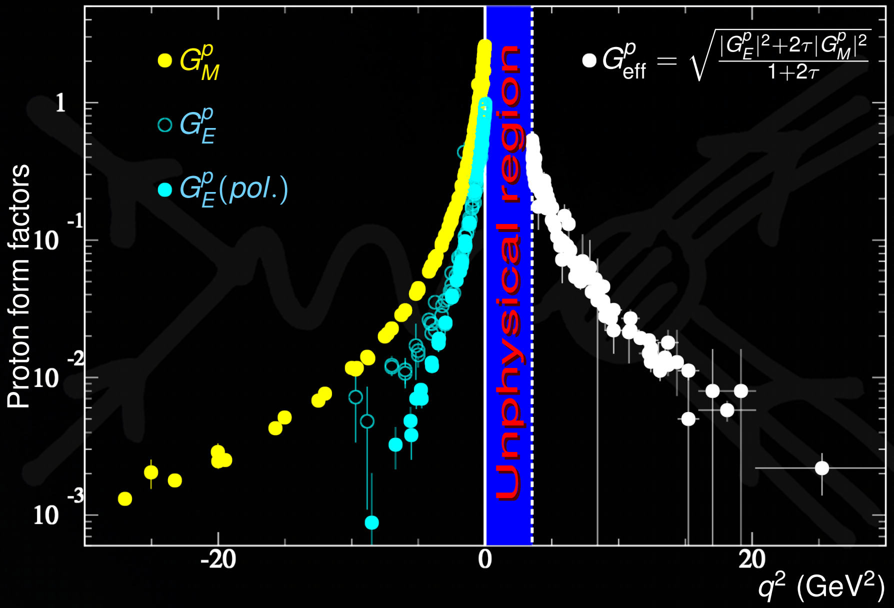


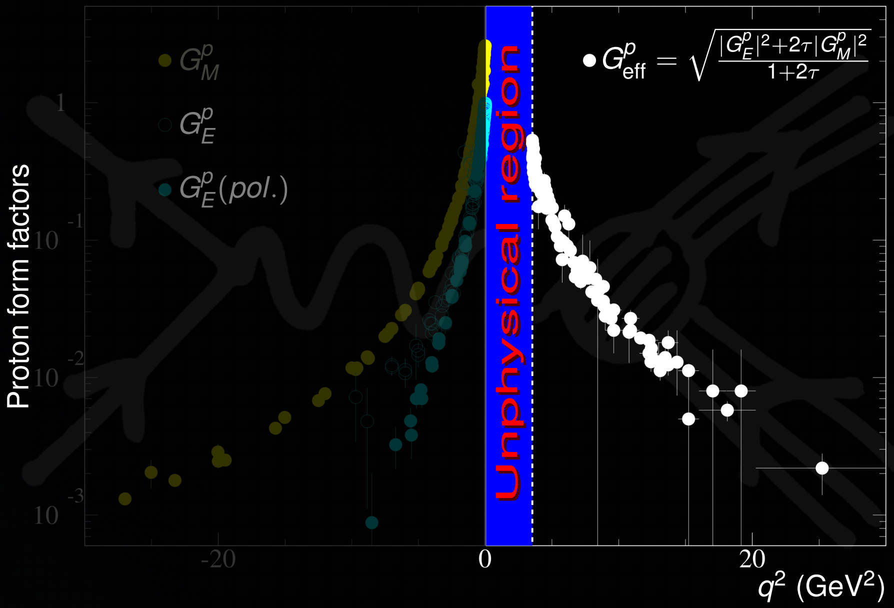




- The parameter $G_M^2$ is better determined than $R^2$ by the Rosenbluth fit
- Its values are well described by two component nucleon model based on VMD [PRC69, 068201]
- Values consistent with the model that represents, in fact, a global fit to the data

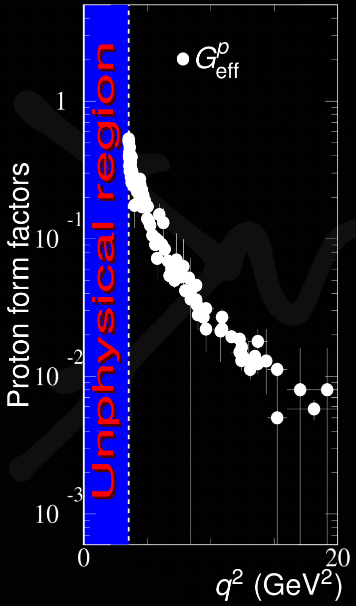
# THE TIME-LIKE REGION



# THE TIME-LIKE REGION



# The time-like region




Differential cross section $e^+e^- \to p\bar{p}$

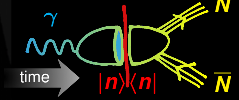
A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto [NC XXIV (1962) 170]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M^p|^2 + \frac{1}{\tau}\sin^2\theta|G_E^p|^2\right]$$

Optical theorem

$$\mathrm{Im}\langle \overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$$

$|n\rangle$ are on-shell intermediate states: $2\pi$, $3\pi$, $4\pi$, ...



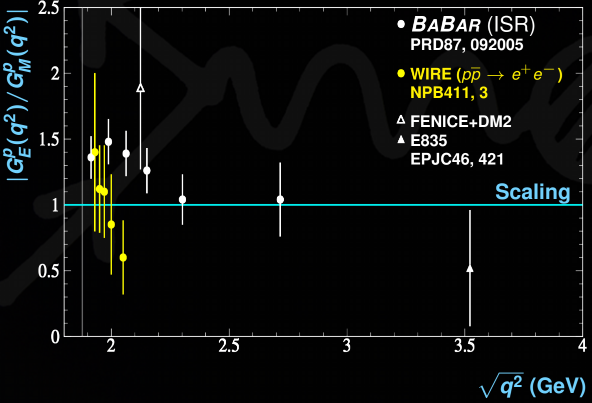

Form factors are complex for $q^2 > 4M_\pi^2$

- The cross section is an even function of $\cos\theta$
- It does not depend on the form factor phases
- At high $q^2$ the $|G_E^p|^2$ contribution is suppressed
- The unphysical region is not accessible through the annihilations $e^+e^- \leftrightarrow p\bar{p}$

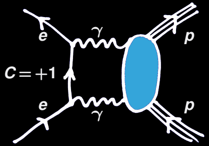
# TIME-LIKE $|G_E^p/G_M^p|$ MEASUREMENTS

See next talks by
Vladimir Druzhinin
Cristina Morales
Iris Zimmermann

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2}\sin^2\theta\left|\frac{G_E^p}{G_M^p}\right|^2\right]$$




## $\gamma\gamma$ exchange



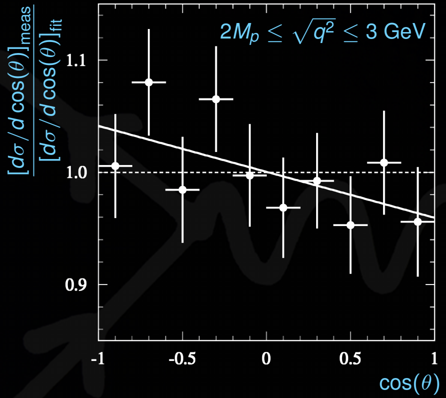

$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[E. Tomasi-Gustafsson, G.I. Gakh, NPA771,169(06)]

# $\gamma\gamma$ EXCHANGE FROM $e^+e^- \to p\bar{p}\gamma$ BABAR 2013 DATA

E. Tomasi-Gustafsson, E.A. Kuraev, S. Bakmaev, SP, Phys. Lett. B659 (2008) 197
*BABAR* Phys. Rev. D87 (2013) 092005




See next talk by Vladimir Druzhinin

Integrated over the $p\bar{p}$-CM energy from threshold up to 3 GeV

The MC-fit assumes **one-photon exchange**

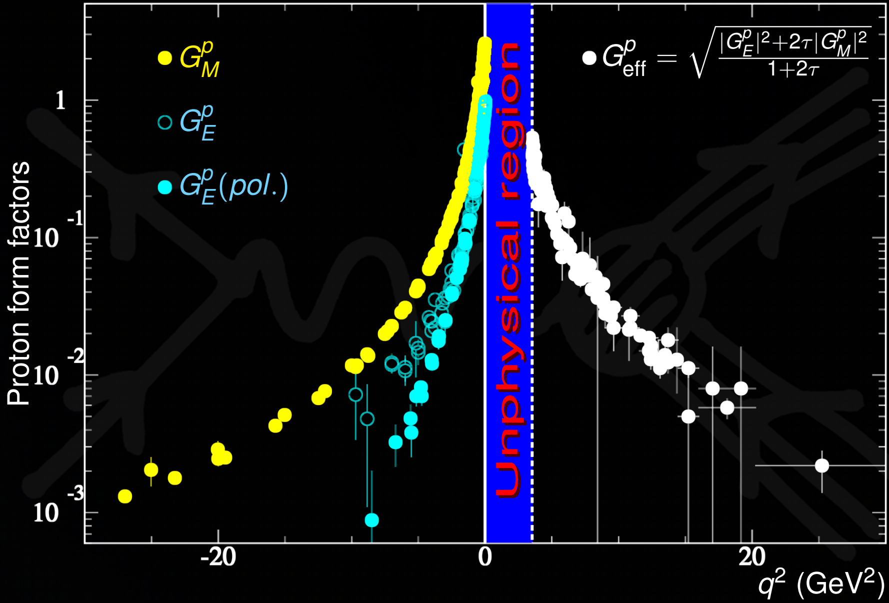
**Slope $= -0.041 \pm 0.026 \pm 0.005$**

**Integral asymmetry**

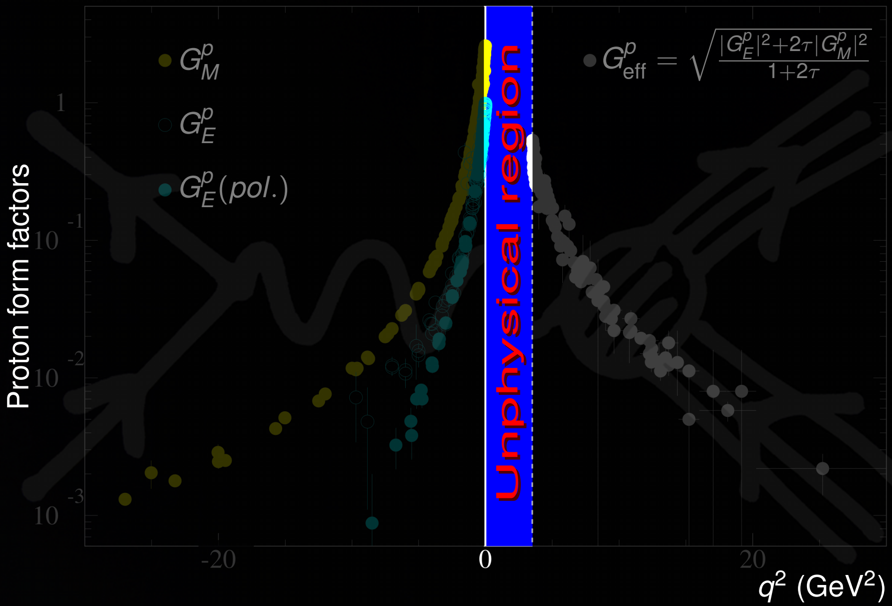
$$\langle \mathcal{A} \rangle_{\cos\theta_p} = \frac{\sigma(\cos\theta_p > 0) - \sigma(\cos\theta_p < 0)}{\sigma(\cos\theta_p > 0) + \sigma(\cos\theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos\theta_p \gtrless 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3$ GeV and $\cos\theta_p \gtrless 0$

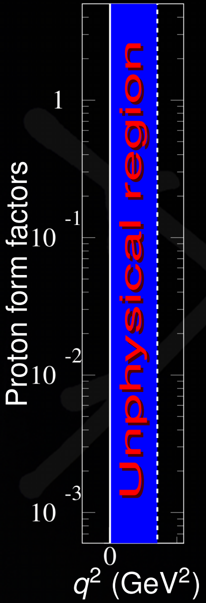
# THE UNPHYSICAL REGION



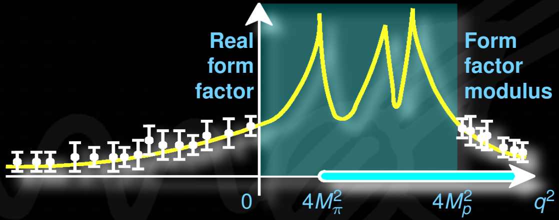
# THE UNPHYSICAL REGION



# THE UNPHYSICAL REGION




Unphysical region goes from $q^2 = 0$ up to the physical threshold $q^2 = 4M_p^2$




In that region, form factors

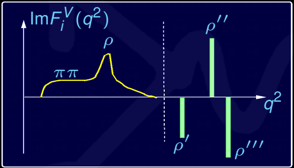
- are still well defined but not (directly) experimentally accessible
- are complex and, following VMD-based models, receive their main contribution from intermediate resonances

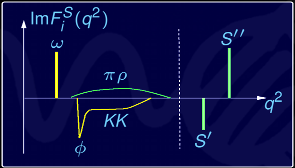
# HANDLING THE UNPHYSICAL REGION[1]

## **Model dependent disclosing** [Höler, Mergell, Meissner, Hammer]

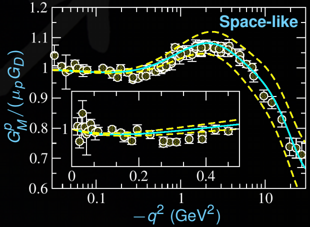
**Optical theorem** ................. $\text{Im}\langle \overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$

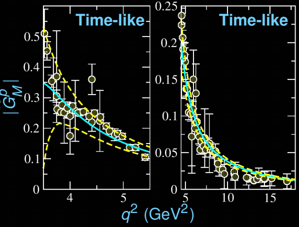
**Dispersion relations for the imaginary part** ....... $F(q^2_{\text{SL}}) = \dfrac{1}{\pi}\displaystyle\int_{4M^2_\pi}^{\infty} \dfrac{\text{Im}F(q^2_{\text{TL}})}{q^2_{\text{TL}} - q^2_{\text{SL}}}\, dq^2_{\text{TL}}$




- $2\pi$ and $2K$ continua are known
- The $\rho$ resonance with finite width
- Dirac delta poles for higher mass states




- Super convergence relations

$$\int_{4M^2_\pi}^{\infty} \text{Im}\, F_{1,2}(q^2)dq^2 = 0$$

$$\int_{4M^2_\pi}^{\infty} q^2\,\text{Im}\, F_2(q^2)dq^2 = 0$$

- Asymptotic behaviors from perturbative QCD

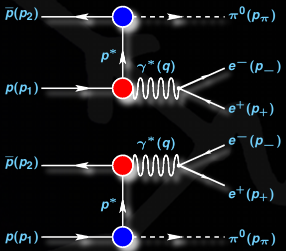
# HANDLING THE UNPHYSICAL REGION$_2$

## Accessing the unphysical region

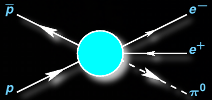
[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas]

The initial state $\pi$-production

$p\bar{p} \to \pi^0 e^+ e^-$



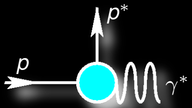

The process $p\bar{p} \to \pi^0 e^+ e^-$




Described in general by **six** amplitudes which depend on **three** kinematical variables

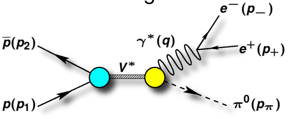
**Hadronic current** [PRC75 045205]

- $J_\mu = \phi_\pi(p_\pi)\,\bar{v}(p_2)\,O_\mu\,u(p_1)$
- $O_\mu = O_\mu[\Gamma_\mu(q)]$
- $\langle N(p')|\Gamma_\mu(q)|N(p)\rangle = \bar{u}(p')\left[F_1(q^2)\gamma_\mu + \frac{i\,\sigma_{\mu\nu}q^\nu}{4M_p^2}F_2(q^2)\right]u(p)$




Background:

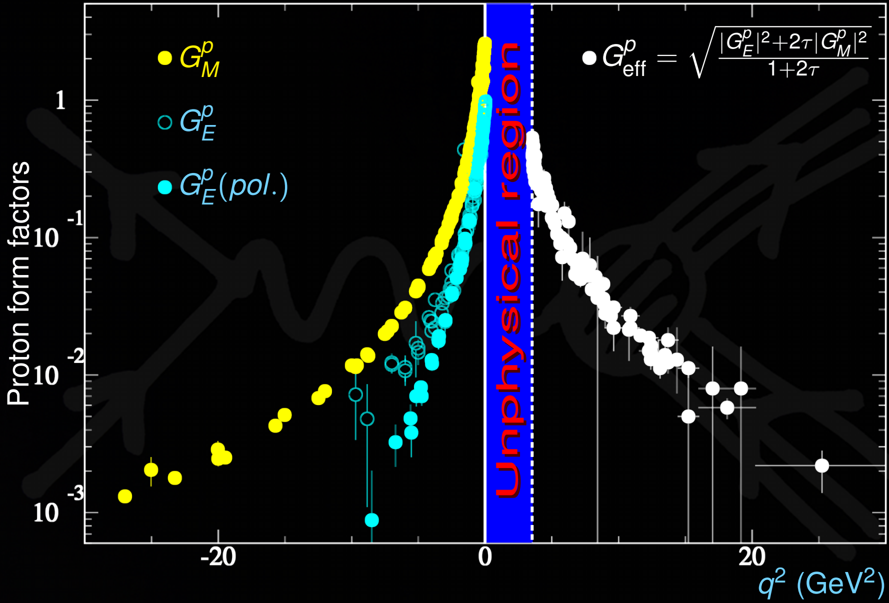
**Annihilation** diagram




**Polarization observables help in disentangle reaction mechanisms**

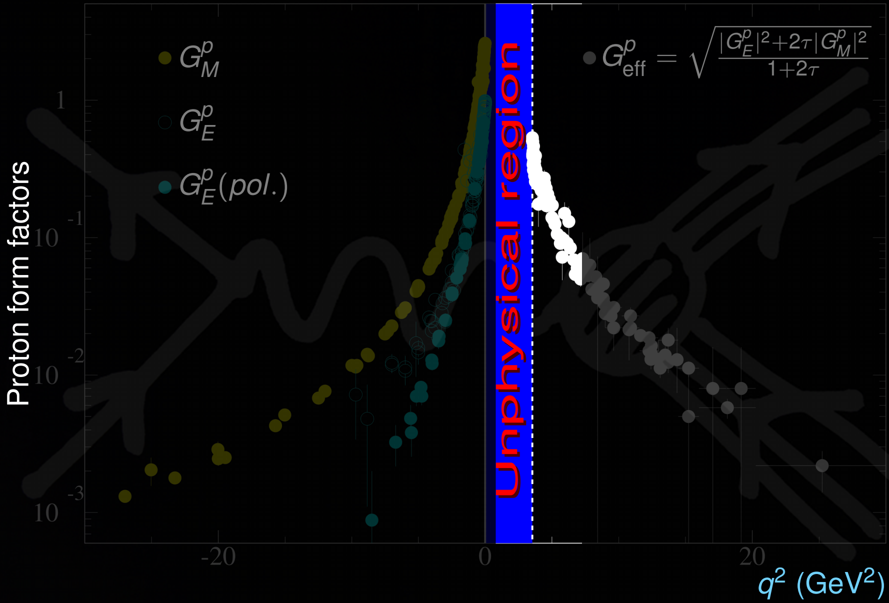
[E. A. Kuraev *et al*., J. Exp. Theor. Phys. 115 (2012) 93
G.I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh
PhysRevC86 (2012) 025204]

# THE THRESHOLD REGION

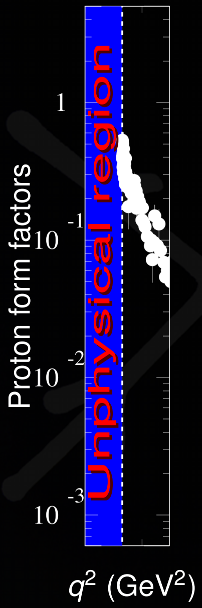



Probing transverse nucleon structure at high momentum transfer - ECT*

# THE THRESHOLD REGION



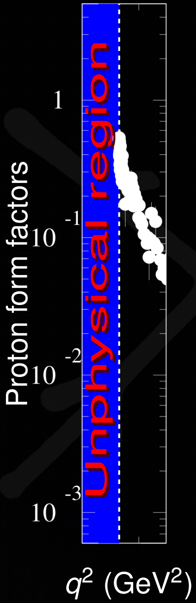
# The threshold region




Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$

# THE THRESHOLD REGION




## Annihilation cross section

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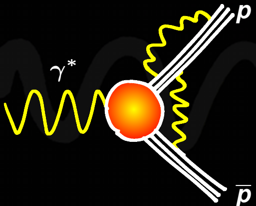



**Enhancement factor** $\mathcal{E} = \frac{\pi\alpha}{\beta}$

It is responsible for the one-photon exchange $p\bar{p}$ final state interaction, dominates at threshold and cancels the phase-space factor.

**Resummation factor** $\mathcal{R} = \frac{1}{1 - e^{-\frac{\pi\alpha}{\beta}}}$

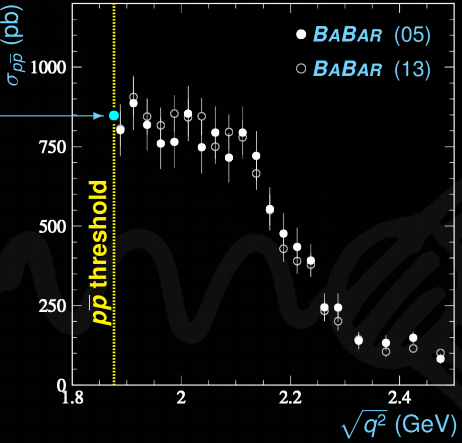
It is responsible for the multi-photon $p\bar{p}$ final state interaction, becomes ineffective few MeV above threshold and accounts also for gluon exchange.

$$\mathcal{C} = \mathcal{E} \times \mathcal{R} = \frac{\pi\alpha/\beta}{1 - \exp(-\pi\alpha/\beta)} \xrightarrow[\beta\to 0]{} \frac{\pi\alpha}{\beta} = \mathcal{E}$$

# STEP AND PLATEAU IN *BABAR* DATA

Eur. Phys. J. A39 (2009) 315

Expected cross section with $|G^p(4M_p^2)| = 1$




**At threshold**

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\,\frac{\beta_p}{\beta_p}\,|G^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\,|G^p(4M_p^2)|^2 \text{ nb}$$

$\Rightarrow$

$|G_S^p(4M_p^2)| \equiv 1$

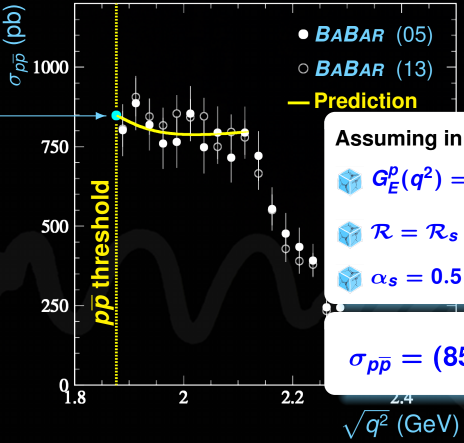
**as pointlike fermion pairs!**

# Step and plateau in BaBar data

Eur. Phys. J. A39 (2009) 315

Expected cross section with

$|G^p(4M_p^2)| = 1$




**Assuming in $q^2 \leq (2.1\text{GeV})^2$:**

- $G_E^p(q^2) = G_M^p(q^2) = 1$
- $\mathcal{R} = \mathcal{R}_s = \left(1 - e^{-\frac{\pi \alpha_S}{\beta}}\right)^{-1}$
- $\alpha_s = 0.5$

$$\sigma_{p\bar{p}} = (850\,\text{pb})\,\frac{1}{\tau}\,\mathcal{R}_s$$

**At threshold**

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\,\frac{\beta_p}{\beta_p}\,|G^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\,|G^p(4M_p^2)|^2\ \text{nb}$$

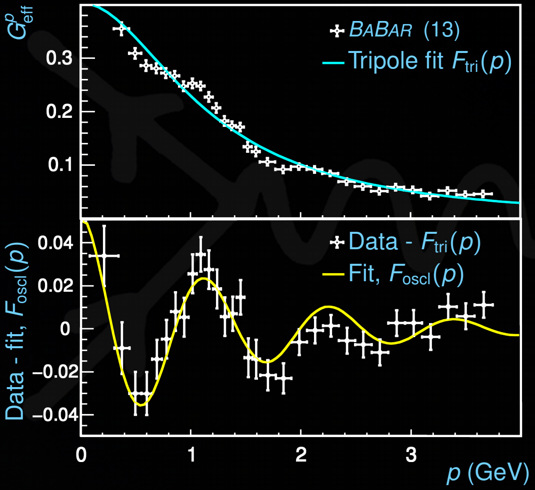
$\Longrightarrow$

$|G_S^p(4M_p^2)| \equiv 1$

**as pointlike fermion pairs!**

# Periodic interference near threshold

A. Bianconi, E. Tomasi-Gustafsson, Phys. Rev. Lett. 114, 232301




$$F_{\rm oscl}(p) = Ae^{-Bp}\cos(Cp + D)$$

| | |
|---|---|
| $A \ll 1$ | $B$ damp. par. |
| $C = r < 1$ fm | $D$ tresh. shift |

$p$ is the momentum of the proton in the anti-proton rest frame.

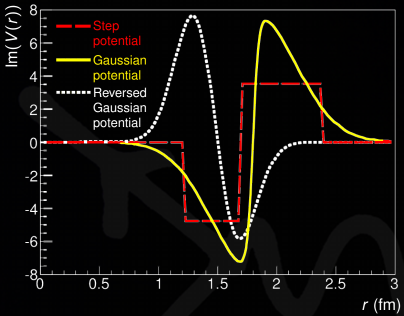
The periodical behavior suggests an interference due to a rescattering of proton and antiproton at low kinetic energy and separation $\sim$ 1 fm.

Proton and antiproton interact when they are almost phenomenological.

Unitarity implies a large imaginary part of form factors.

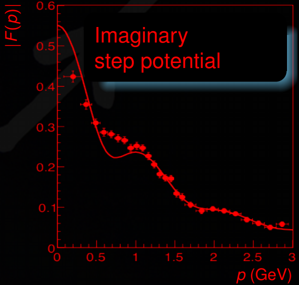
# Double hollow potentials

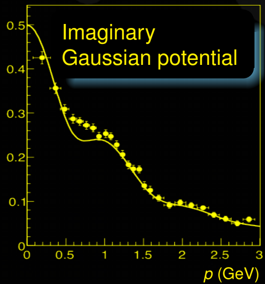
A. Bianconi, E. Tomasi-Gustafsson, PRC93, 035201

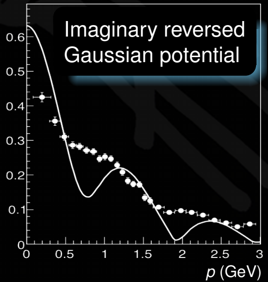



## Double-layer optical potentials are phenomenologically required

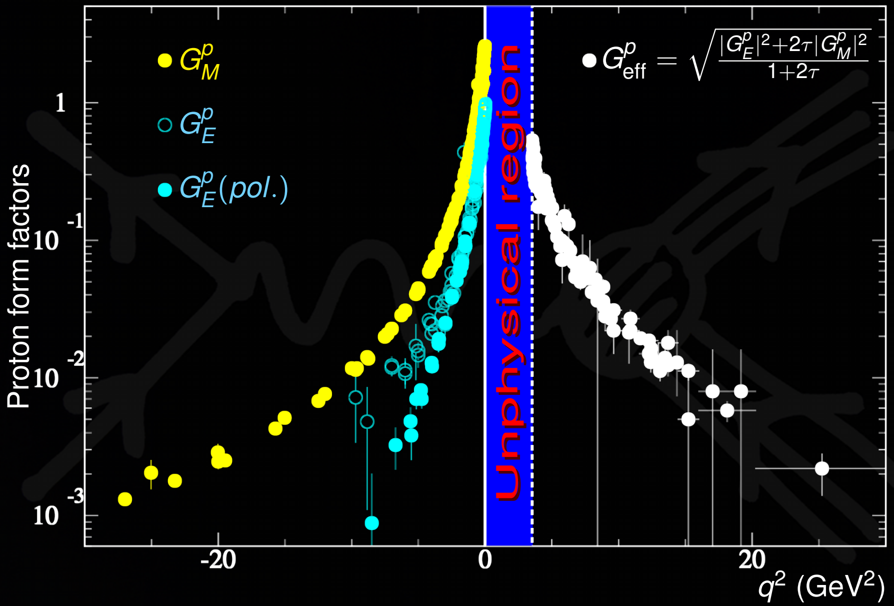
- Purely absorptive at $r > 1.7$ fm
- Flux-generating at $r < 1.7$ fm
- Quick ($< 2$ fm) transition absorption to flux-generating to get the period
- Maximum at $p = 0$ $\Rightarrow$ threshold enhancement
- Reversed potentials reproduce the oscillation amplitude not the period



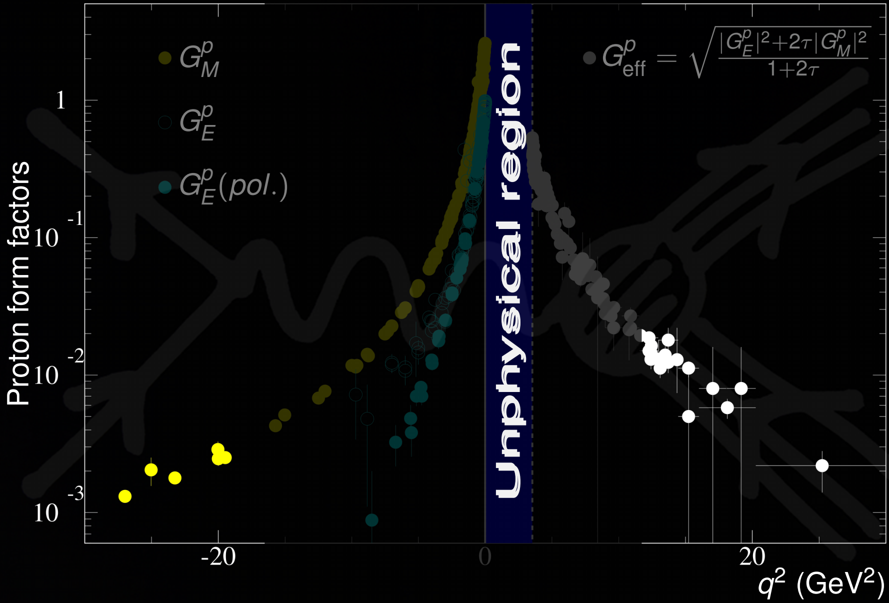



# THE ASYMPTOTIC REGIONS$_1$

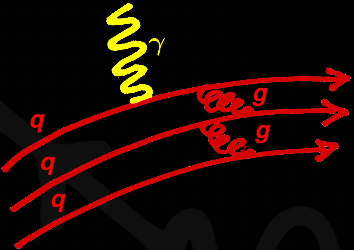


# THE ASYMPTOTIC REGIONS$_1$

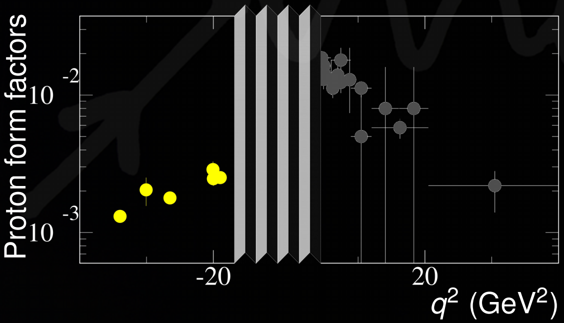


# THE ASYMPTOTIC REGIONS$_1$

## Space-like dimensional scaling




- pQCD: as $q^2 \rightarrow -\infty$, asymptotic behaviors of $F_1$ and $F_2$, and $G_E$ and $G_M$ must follow counting rules
- Valence quarks exchange gluons to distribute the momentum transfer $q$




- Dirac and Pauli form factors

$$F_i(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-1-i}$$

- Sachs form factors

$$G_{E,M}(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$$

$$\frac{G_E(q^2)}{G_M(q^2)} \underset{q^2 \to -\infty}{\sim} \text{cost.}$$

# THE ASYMPTOTIC REGIONS[1]

## Time-like asymptotic behavior

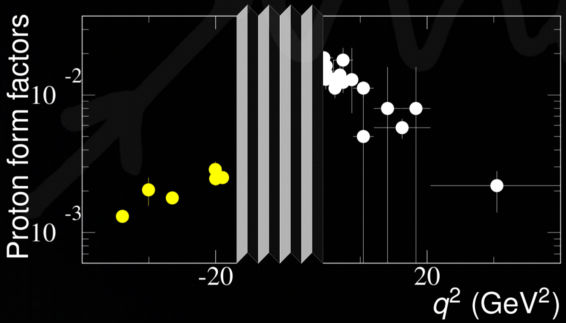
**Phragmèn Lindelöf theorem:**

If a function $f(z) \to a$ as $z \to \infty$ along a straight line, and $f(z) \to b$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \to a$ uniformly in this angle.

$$\underbrace{\lim_{q^2 \to -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2 \to +\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

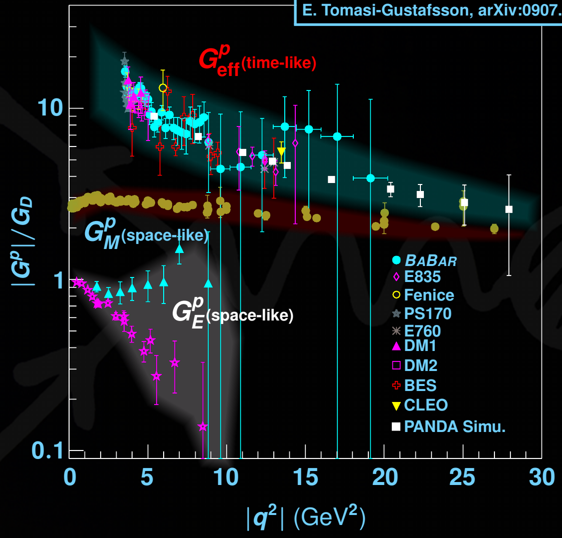
$$G_{E,M} \underset{q^2 \to +\infty}{\sim} (q^2)^{-2}$$

**real**



# THE ASYMPTOTIC REGIONS$_2$

**E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291**
**E. Tomasi-Gustafsson, arXiv:0907.4442**




**pQCD**

$$G^p_{\rm eff}(q^2) \underset{q^2\to\infty}{\sim} G^p_M(q^2)$$

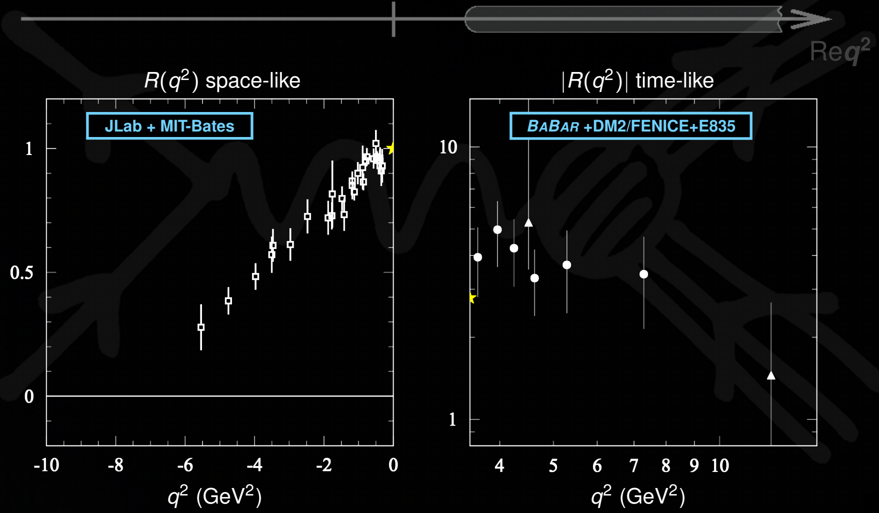
**Phragmèn Lindelöf**

$$\lim_{q^2\to\infty} \frac{G^p_{\rm eff}(q^2)}{G^p_M(-q^2)} = 1$$

# ANALYTIC $R = \mu G_E/G_M$

Eur. Phys. J. A32 (2007) 421

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$



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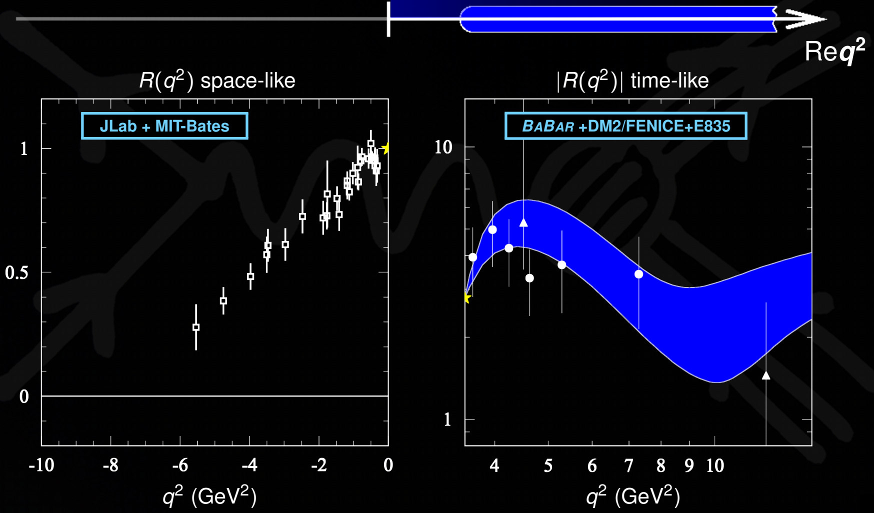
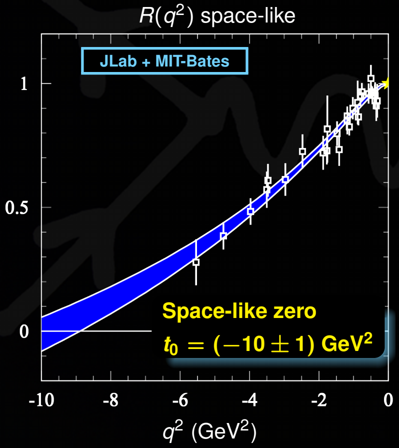


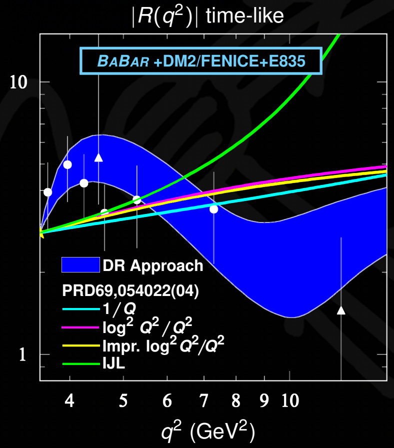

Probing transverse nucleon structure at high momentum transfer - ECT*

# Analytic $R = \mu G_E / G_M$

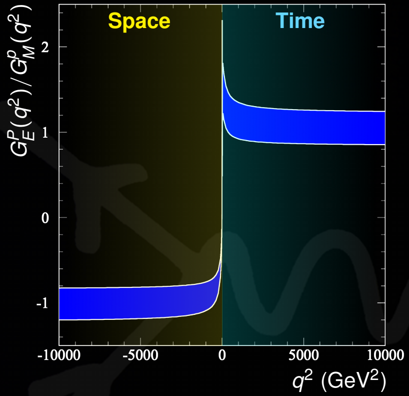
Eur. Phys. J. A32 (2007) 421

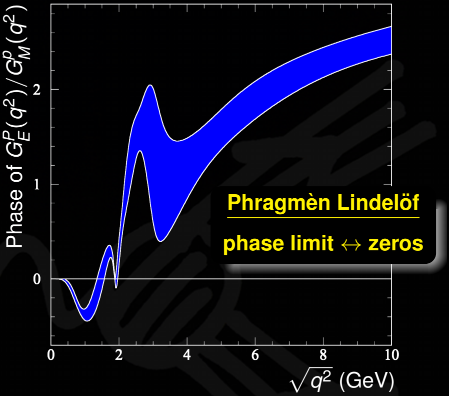
$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)} ds$$

# ASYMPTOTIC $G_E/G_M$ AND PHASE




**pQCD prediction**

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} \underset{|q^2|\to\infty}{\longrightarrow} -1$$

**Phase from DR**

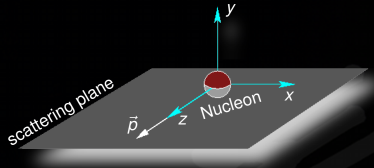
$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{th}}}{\pi} \,\mathrm{Pr}\int_{s_{th}}^{\infty} \frac{\ln|R(s)|\,ds}{\sqrt{s - s_{th}}\,(s - q^2)}$$

# Polarization formulae in the time-like region

The ratio $R(q^2)$ is complex for $q^2 \geq 4M_\pi^2$

$$R(q^2) = \mu_p \frac{G_E(q^2)}{G_M(q^2)} = |R(q^2)| e^{i\rho(q^2)}$$

The polarization depends on the phase $\rho$




[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

$$\mathcal{P}_y = -\frac{\sin(2\theta)|R|\sin(\rho)}{D\sqrt{\tau}} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \equiv \mathcal{A}_y$$

Does not depend on $P_e$

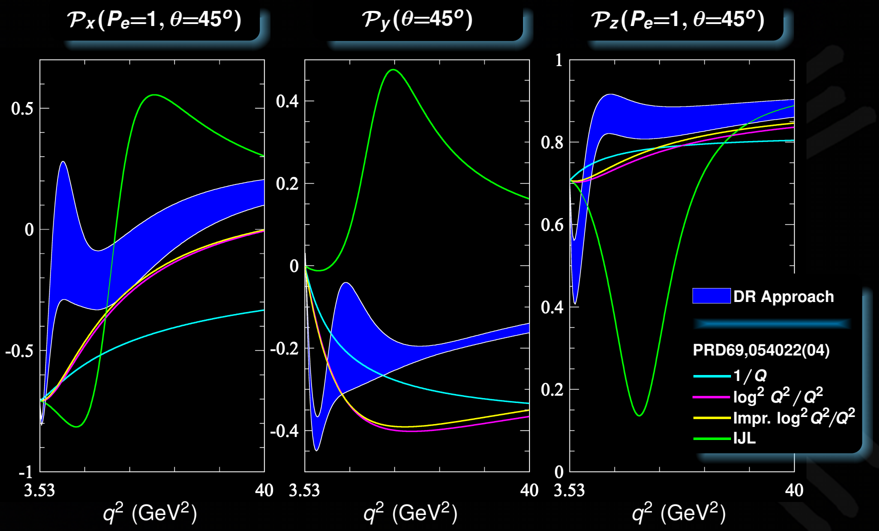
$$\mathcal{P}_x = -P_e \frac{2\sin(2\theta)|R|\cos(\rho)}{D\sqrt{\tau}}$$

$$\mathcal{P}_z = P_e \frac{2\cos(\theta)}{D}$$

Does not depend on $\rho$

$$D = \frac{1 + \cos^2\theta + \frac{1}{\tau}|R|^2 \sin^2\theta}{\mu}, \quad \tau = \frac{q^2}{4M^2}, \quad P_e = \text{electron polarization}$$

# Single Polarization



# Conclusions Phys. Rev. **550-551** (2015) 1

**Global models in space and time-like regions** for proton and neutron, electric and magnetic form factors must be encouraged.

They can help in understanding:

- the threshold behavior
- the proton radius
- the presence of zeros
- the asymptotic behavior
- the unphysical region
- ...

**To measure:**

- zero of $G_E^p$ in space-like region
- moduli of $G_E$ and $G_M$ in time-like region
- complex structure of form factors (polarization)
- unphysical time-like form factors ($p\bar{p} \to \pi^0 e^+ e^-$)
- ...

# EXPERIMENTS: NOW AND FUTURE

See next talks by
Vladimir Druzhinin
Cristina Morales
Iris Zimmermann

**Space-like region**

**A1 Mainz**

- $G_E^n$ at $-q^2 = 1.5$ GeV$^2$ (Pol. $^3$He)
- $G_E^p$ and $G_M^p$ for $-q^2 \leq 2.0$ GeV$^2$

**Jefferson Lab**

- [Hall A] $G_E^p / G_M^p$ up to 14 GeV$^2$
- [Hall A] $G_M^n$ (ratio) up to 18 GeV$^2$
- [Hall A] $G_E^n / G_M^n$ up to 10.2 GeV$^2$
- [Hall B] $G_M^n$ (deuterium) up to 14 GeV$^2$
- [Hall C] $G_E^n$ up to 7 GeV$^2$

**Time-like region**

**SND, CMD3 at VEPP-2000**
$e^+e^-$ collider
$|G_{\text{eff}}^p|$, $|G_{\text{eff}}^n|$ (scan)
$q^2 \leq (4\text{ GeV})^2$

**BESIII at BEPCII**
$e^+e^-$ collider
$|G_E^p|$, $|G_M^p|$, $|G_{\text{eff}}^n|$ (scan and ISR)
$q^2 \leq (3.5\text{ GeV})^2$

**$\bar{\text{p}}$anda at FAIR**
$p\bar{p}$ collider
$|G_E^p|$, $|G_M^p|$, $G_E^p / G_M^p$ phase ($\bar{p}$ polarization?)
$(2.4\text{ GeV})^2 \leq q^2 \leq (3.7\text{ GeV})^2$

**Belle II ? at SuperKEKB**
$e^+e^-$ collider
$|G_E^p|$, $|G_M^p|$, (ISR)
$q^2 \leq (4.5\text{ GeV})^2$