Overview of Nucleon Form Factors on a Lattice

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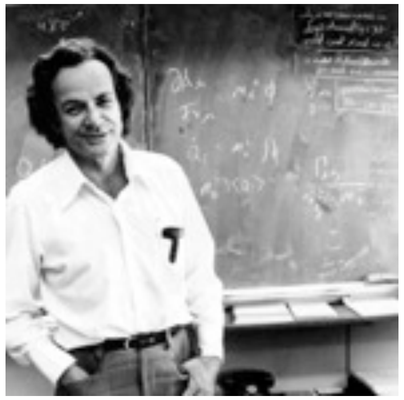
ECT* *Workshop*
"Probing transverse nucleon structure at high momentum transfer"
Trento, April 18-22, 2016

Outline

How well can we simulate nucleon structure on a lattice?

- Introduction
  lattice methodology and systematic errors
- Nucleon Electromagnetic Form Factors
  Nucleon form factors, radii, magnetic moment
- Nucleon Axial Form Factors
  Nucleon axial charge, axial radius, induced pseudoscalar form factor
- Transversely Polarized Quark Form Factors & Densities
- Summary

Quantum Chromodynamics on a Lattice (LQCD)



R. Feynman
Path integrals in
Quantum Mechanics

Lattice Field Theory $\Leftrightarrow$ Numerical evaluation of the Path Integral

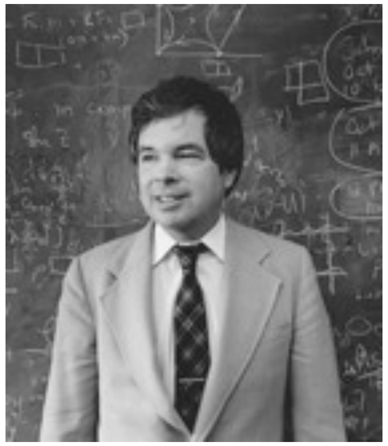
$$\langle q_x \bar{q}_y \ldots \rangle = \int \mathcal{D}\big(Glue\big) \int \mathcal{D}\big(Quarks\big)\; e^{-S_{Glue} - \bar{q}\left(\not{D}+m\right)q} \; \big[q_x \bar{q}_y \ldots\big]$$

Grassmann integration

$(\not{D}+m)\cdot q = 0$
quark motion in
gluon background

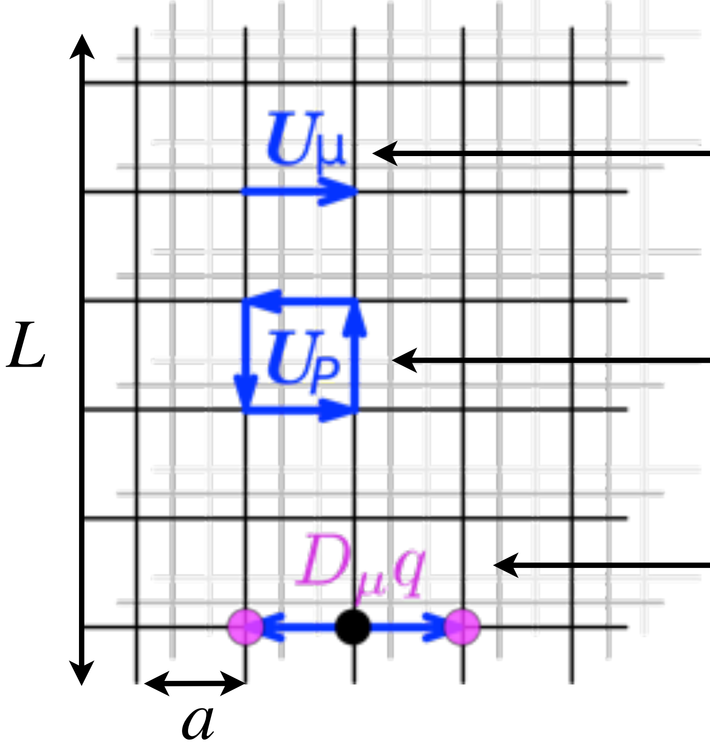
$$= \int \underbrace{\mathcal{D}\big(Glue\big)\; e^{-S_{Glue}}\; \mathrm{Det}\big(\not{D}+m\big)}_{\text{Markov Chain Monte Carlo sampling}} \; \big[\big(\not{D}+m\big)^{-1}_{x,y} \ldots\big]$$

Extremely demanding
numerical problem



K. Wilson
Renormalization
Continuum limit

Euclidean 4D lattice



$U_\mu \approx e^{ig\,aA_\mu}$

$U_P \approx e^{ig\,a^2 F_{\mu\nu}}$
$(\text{"curl"}\,A_\mu)$

quark covariant
derivative

IBM BlueGene/P
Argonne National Laboratory

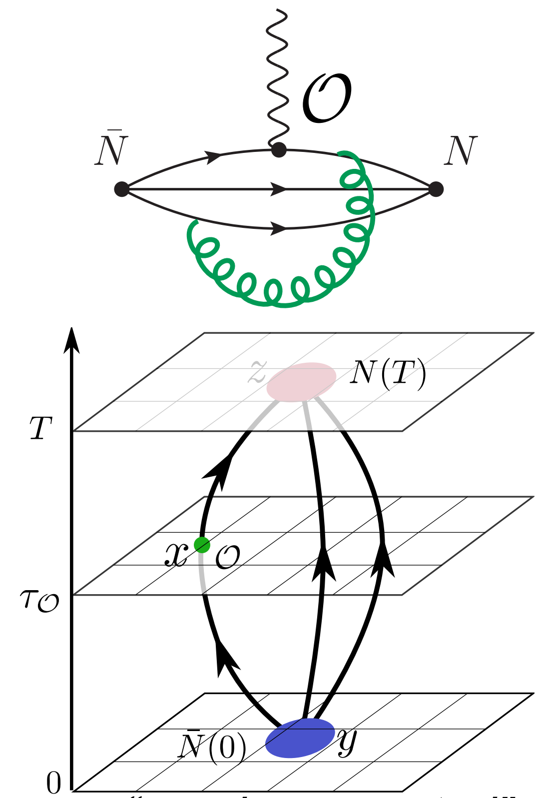
Nucleon Correlators and Matrix Elements

- Hadron correlators with quark(-bilinear) operators

observable, e.g. $\bar{q}\gamma_\mu q$

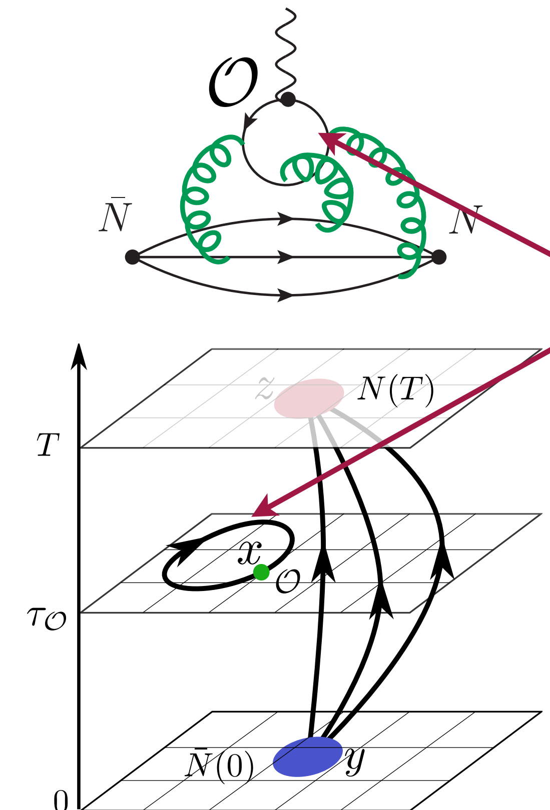
$$C_{3\mathrm{pt}}^{\mathcal{O}}(T) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$

sink source



"quark-connected"

+



"quark-disconnected"

hard numerical problem

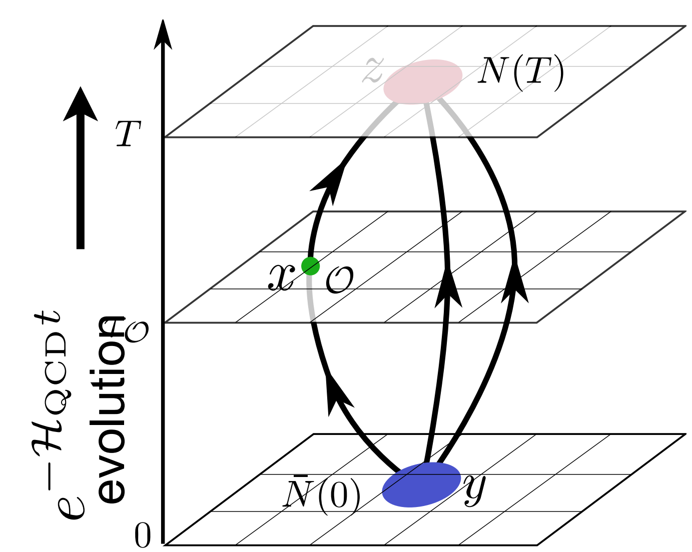
$$\sum_x \not{D}^{-1}(x,x)$$

$$F_{1,2}^u = (F_{1,2}^u)_{\mathrm{conn}} + (F_{1,2}^{u/d})_{\mathrm{disc}}$$
$$F_{1,2}^d = (F_{1,2}^d)_{\mathrm{conn}} + (F_{1,2}^{u/d})_{\mathrm{disc}}$$
$$F_{1,2}^s = \qquad\qquad + (F_{1,2}^s)_{\mathrm{disc}}$$

- Euclidean propagation selects the ground state

$$\langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle = \sum_{n,m}\sqrt{Z_n}e^{-E_n(T-\tau)}\langle n|\mathcal{O}|m\rangle e^{-E_m\tau}\sqrt{Z_m^*}$$

$$\overset{T\to\infty}{\approx} Z_0 e^{-E_0 T}\left[\langle 0|\mathcal{O}|0\rangle + O(\underbrace{e^{-\Delta E_{10}T}, e^{-\Delta E_{10}\tau}, e^{-\Delta E_{10}(T-\tau)}}_{\text{excited states}})\right]$$



Systematic & Stochastic Error in Lattice M.E.

- Matrix elements : C_{3pt}/C_{2pt} ratio or multi-exp. fits

$$R_{\mathcal{O}}(T,\tau;P,P') = \frac{\langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle}{\langle N(T)\bar{N}(0)\rangle} \xrightarrow[T,\tau,(T-\tau)\to\infty]{} \langle P'|\mathcal{O}|P\rangle$$

- Stochastic noise grows rapidly with *T*, especially with light pions [Lepage'89]:

$$\begin{array}{lll} \text{Signal} & \langle N(T)\bar{N}(0)\rangle & \sim e^{-M_N T} \\ \text{Noise} & \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 & \sim e^{-3m_\pi T} \\ \text{Signal/Noise} & & \sim e^{-(M_N - \frac{3}{2}m_\pi)T} \end{array}$$

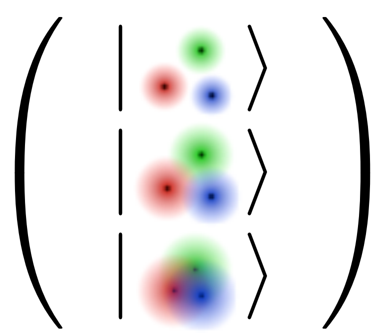
- Treating excited states:

$$\bar{N}_{\text{lattice}}|vacuum\rangle = |\text{nucleon}\rangle + |X\rangle$$

  - Multi-exponential fits (typically 2 states in practice)
  - Variational methods

$$\begin{pmatrix} \bar{N}|vac\rangle \\ e^{-1a_t\cdot\mathcal{H}}\bar{N}|vac\rangle \\ e^{-2a_t\cdot\mathcal{H}}\bar{N}|vac\rangle \\ \dots \end{pmatrix}$$

time-evolved states ("GPoF")



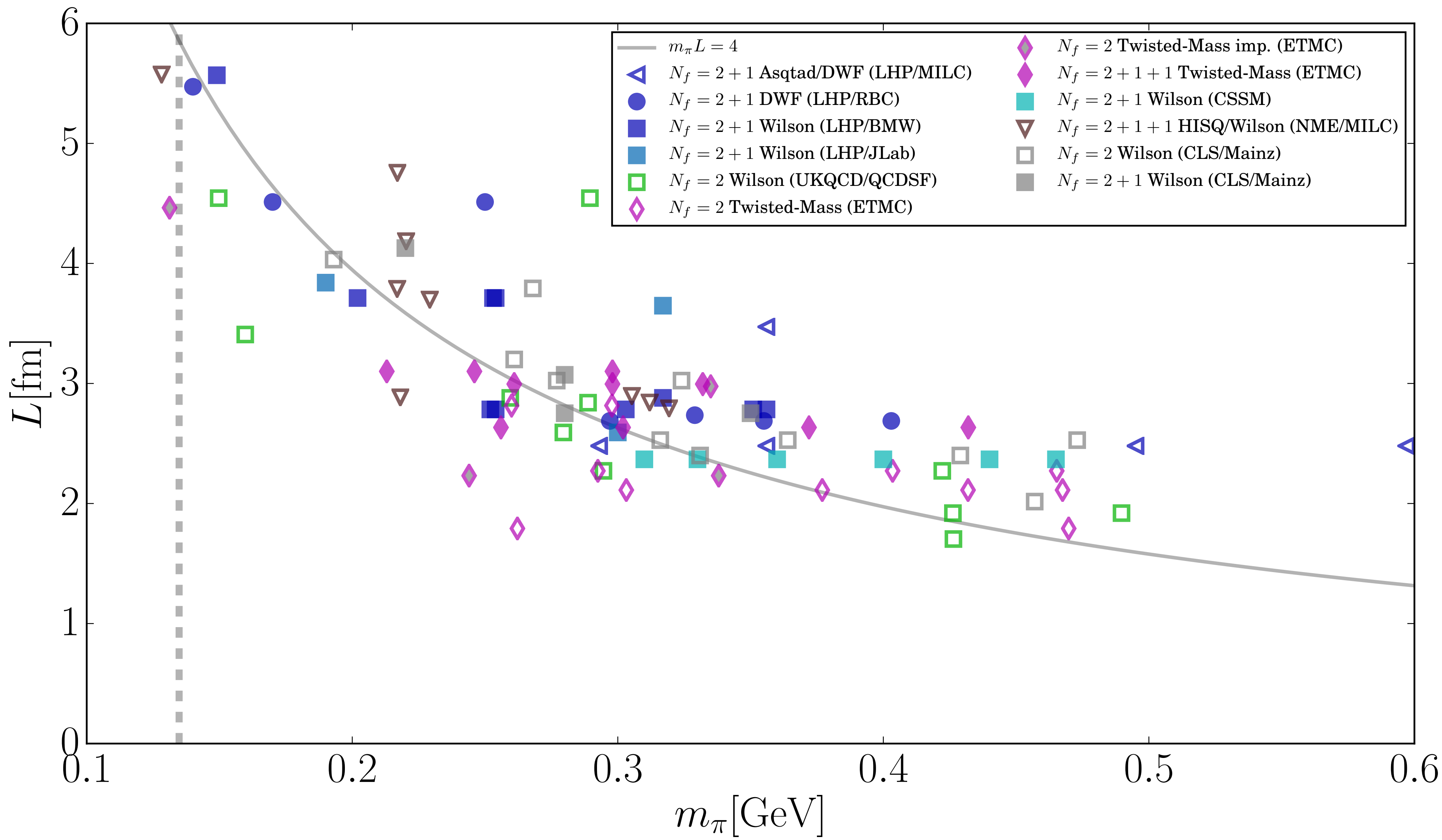
quark "smearing"

"distillation" : a large basis of states classified by H(3) : (JLab hadron spectrum calculations)

Nucleon Structure Calculations

For final answer, take

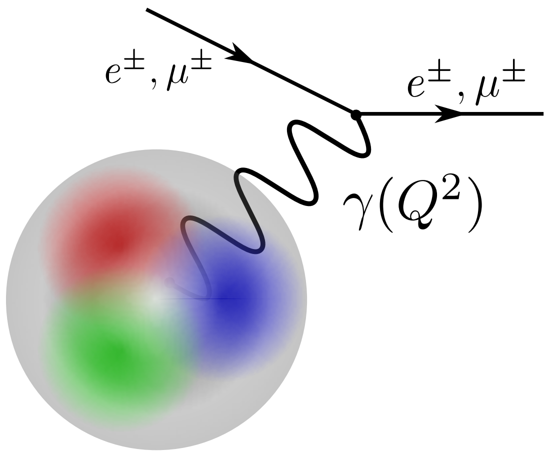
- physical pion/kaon masses
- continuum limit $a \to 0$
- large-volume limit $V \to \infty$; in practice, $L > 4m_\pi^{-1}$

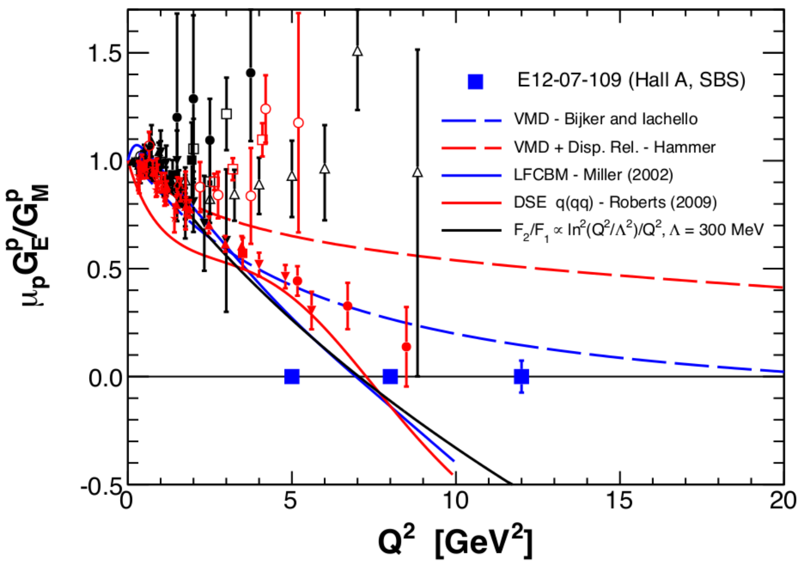


Nucleon Electromagnetic Form Factors

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\left[F_1(Q^2)\,\gamma^\mu + F_2\,(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]U_P$$

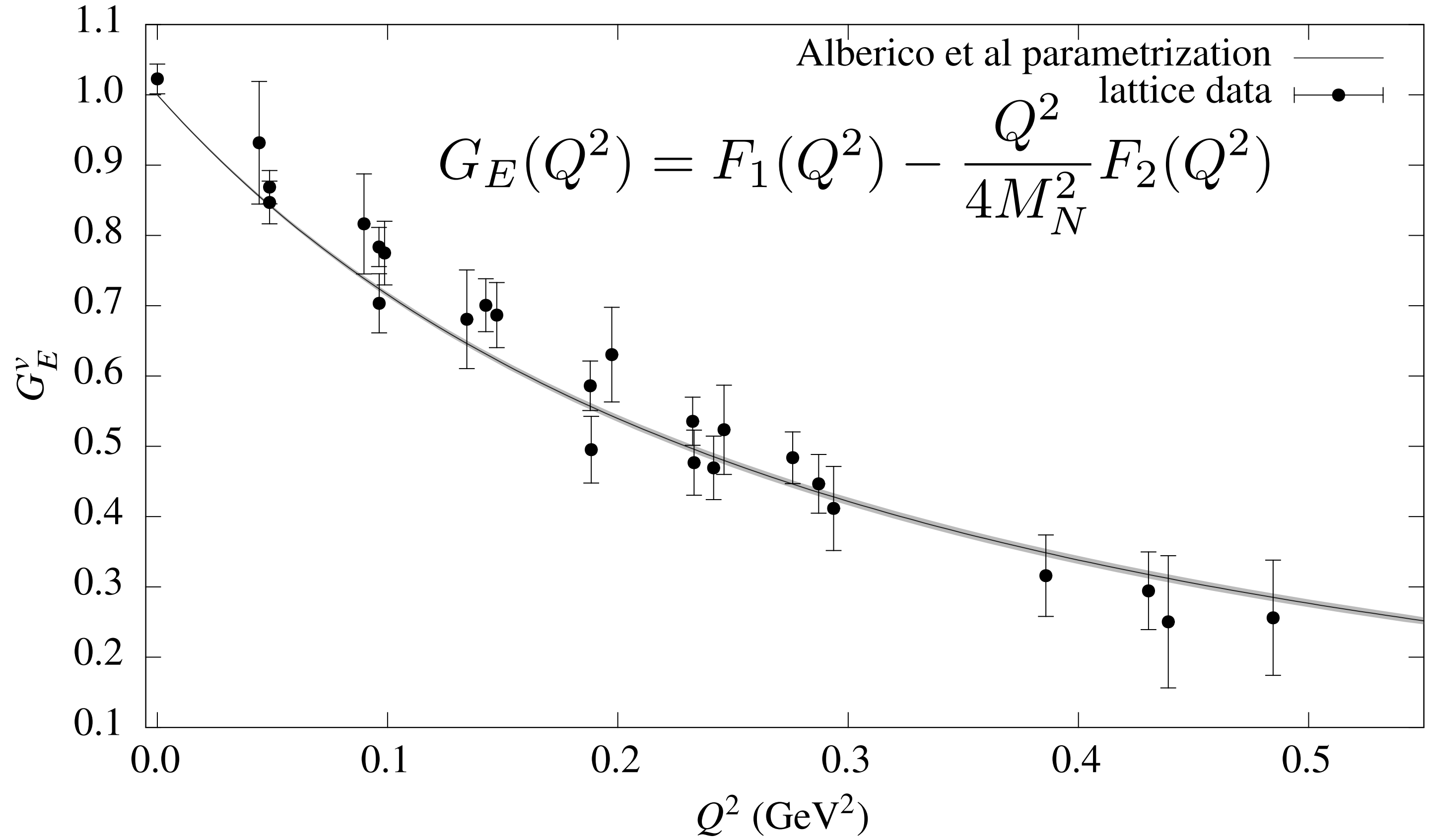
- ✦ JLab@12GeV : explore form factors at $Q^2 >= 10$ GeV^2
  - (F_1 / F_2) scaling at $Q^2 \to \infty$
  - (G_E/G_M) dependence up to Q^2=18 GeV^2
  - *u-*, *d*-flavor contributions to form factors
- ✦ Proton radius puzzle: 7σ difference
  - JLab pRAD experiment
  - MUSE@PSI : $e^\pm$ / $\mu^\pm$-scattering off the proton





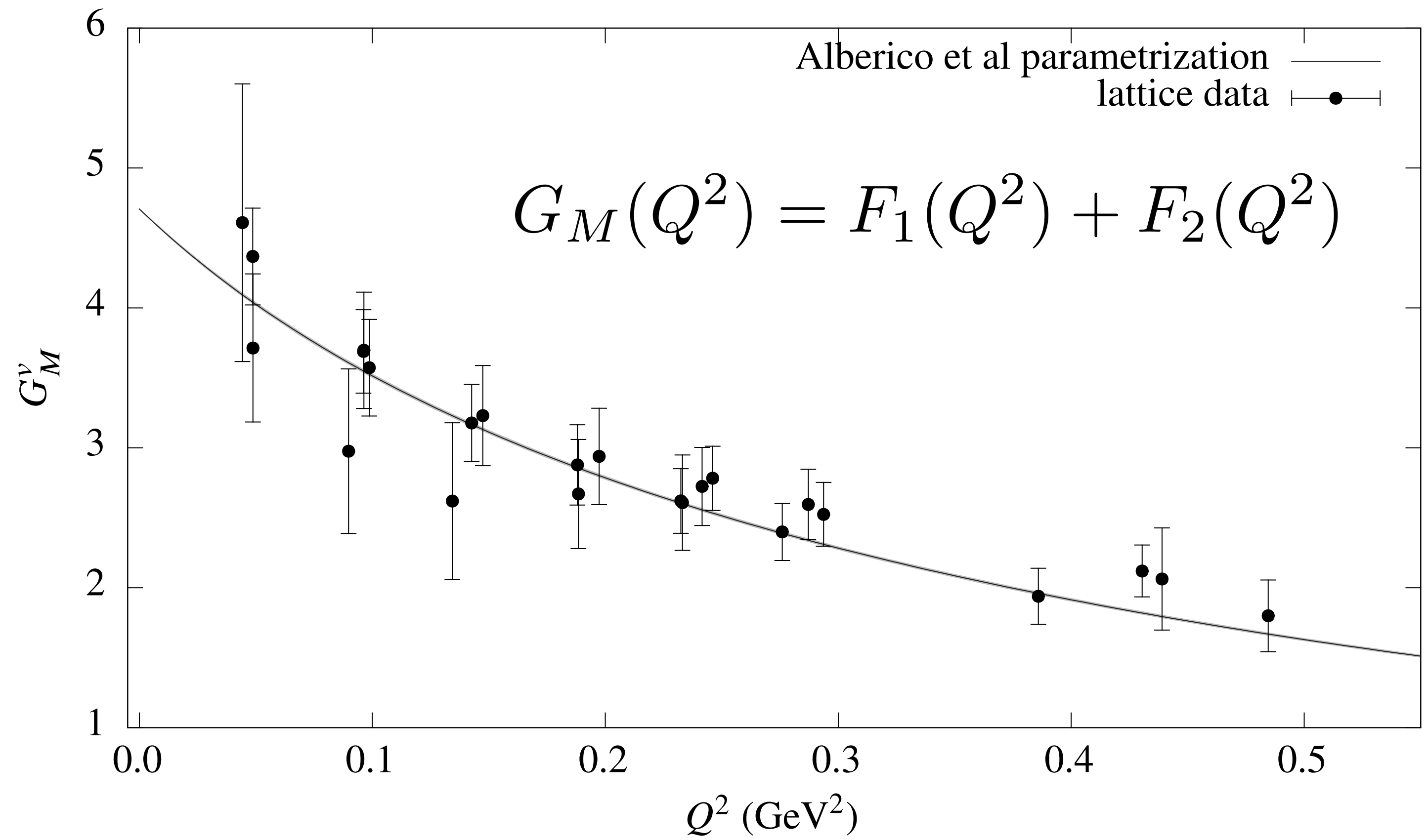
[Research Mgmt. Plan for SBS(JLab Hall A)]

Isovector (p-n) Sachs Form Factors vs Pheno.fits



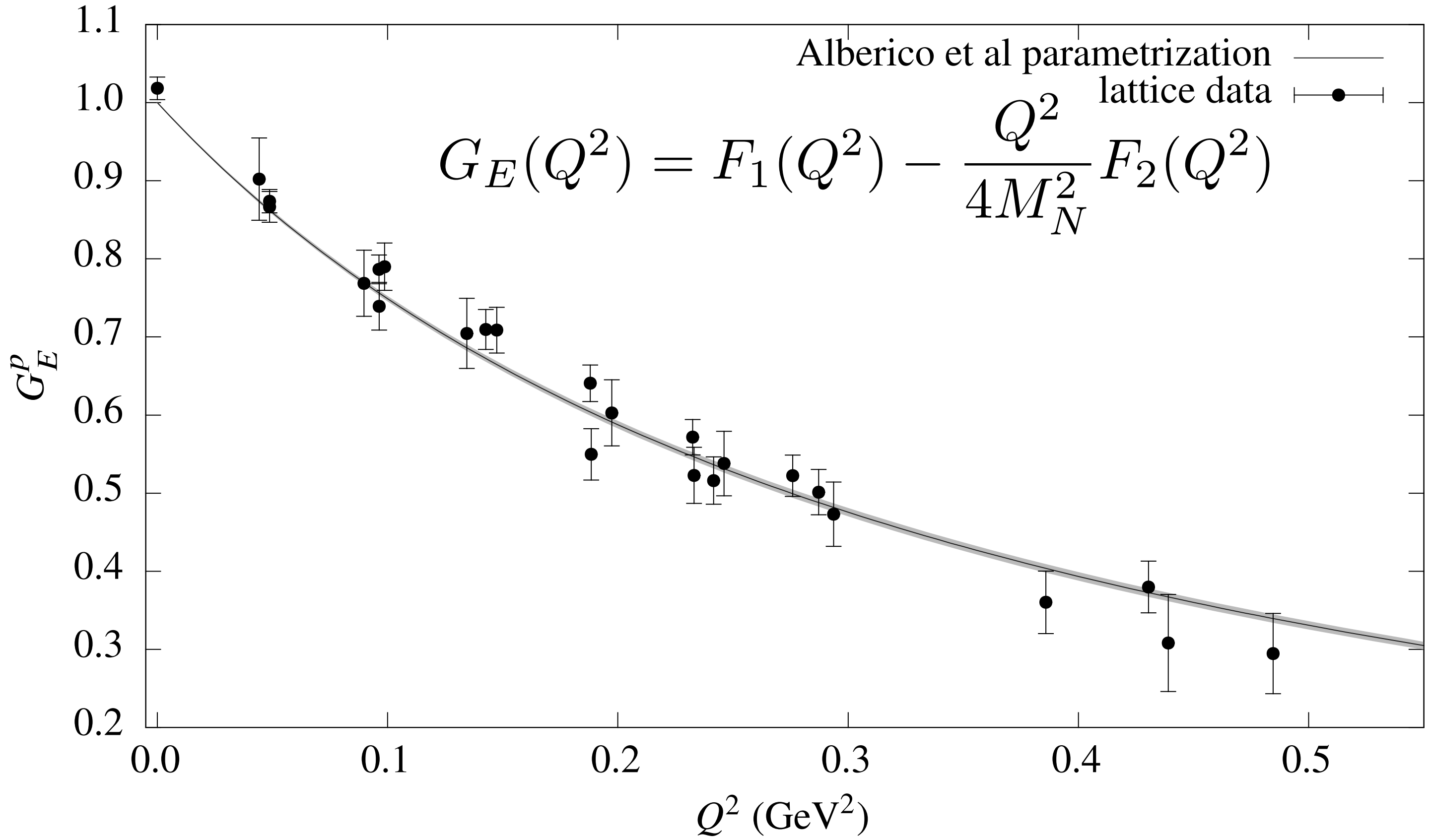
m_π=149 MeV data vs Phenomenology

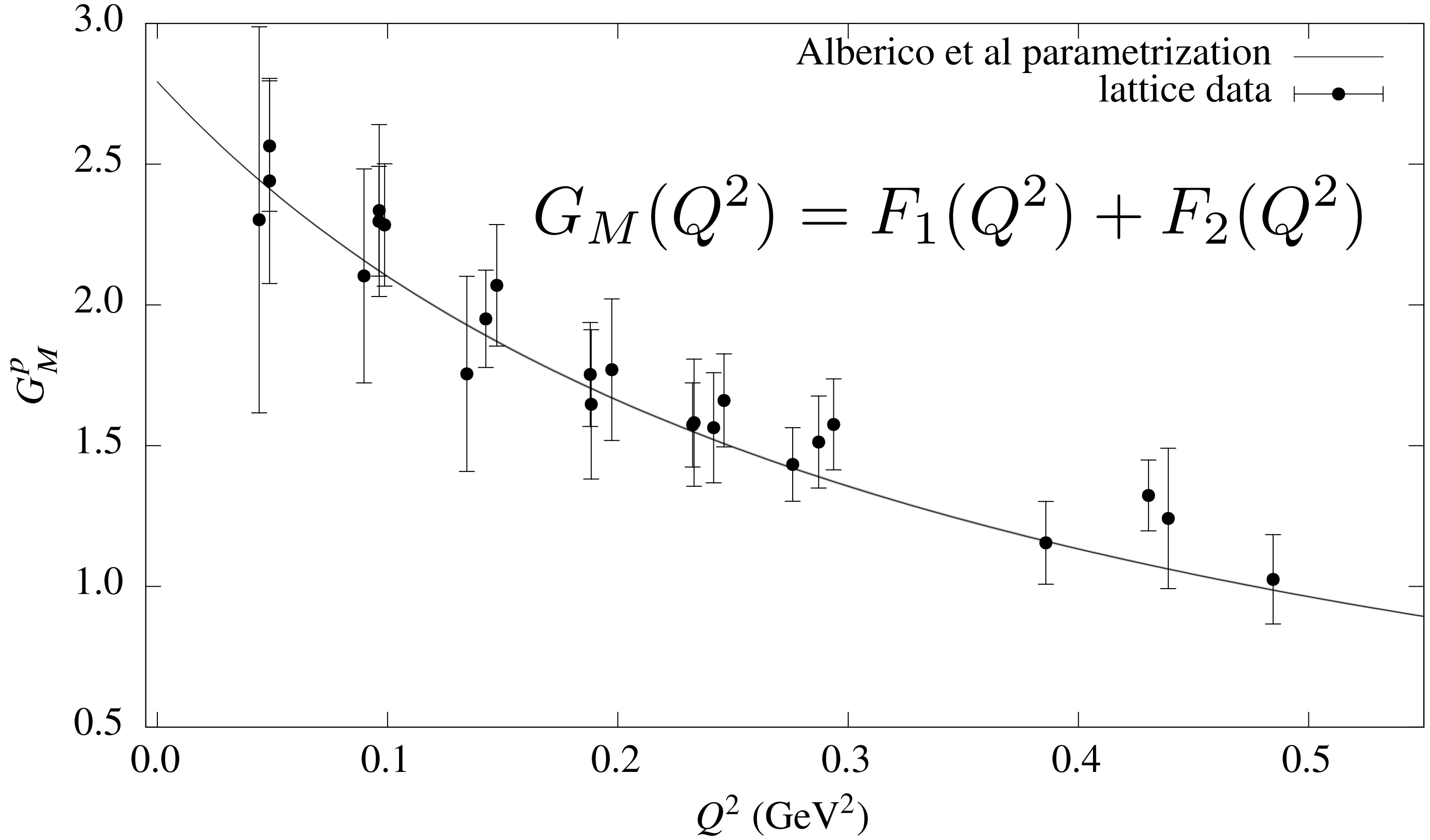
[J.R.Green et al (LHPc), PRD90:054507(2014); 1404.4059]



Pheno. Parameterization:
[W.M.Alberico et al, PRC79:065204(2009)])

Proton (conn. only) Sachs Form Factors vs Pheno.





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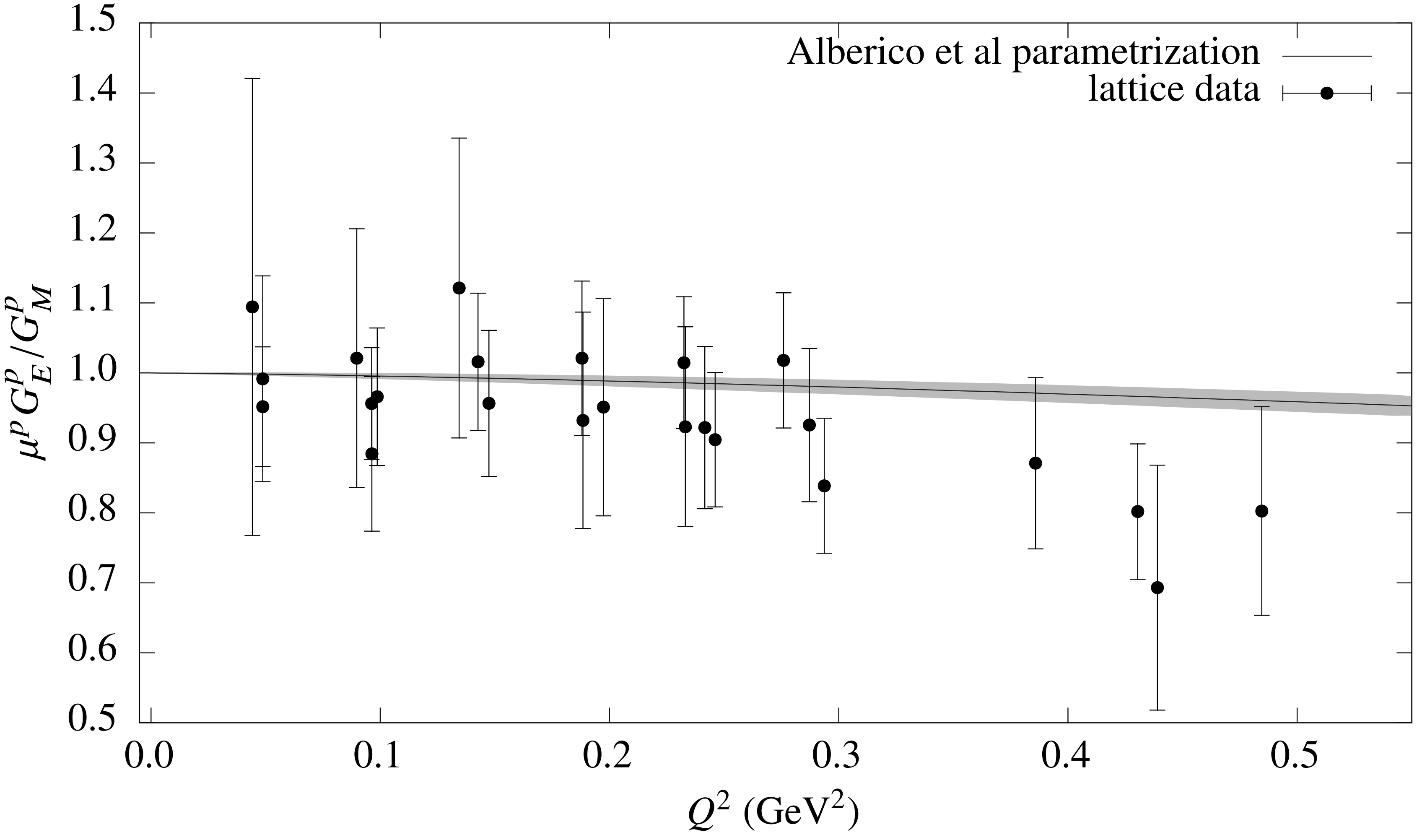
No disconnected part!
(apparently negligible)

Proton Sachs Form Factors : GEp/GMp Ratio



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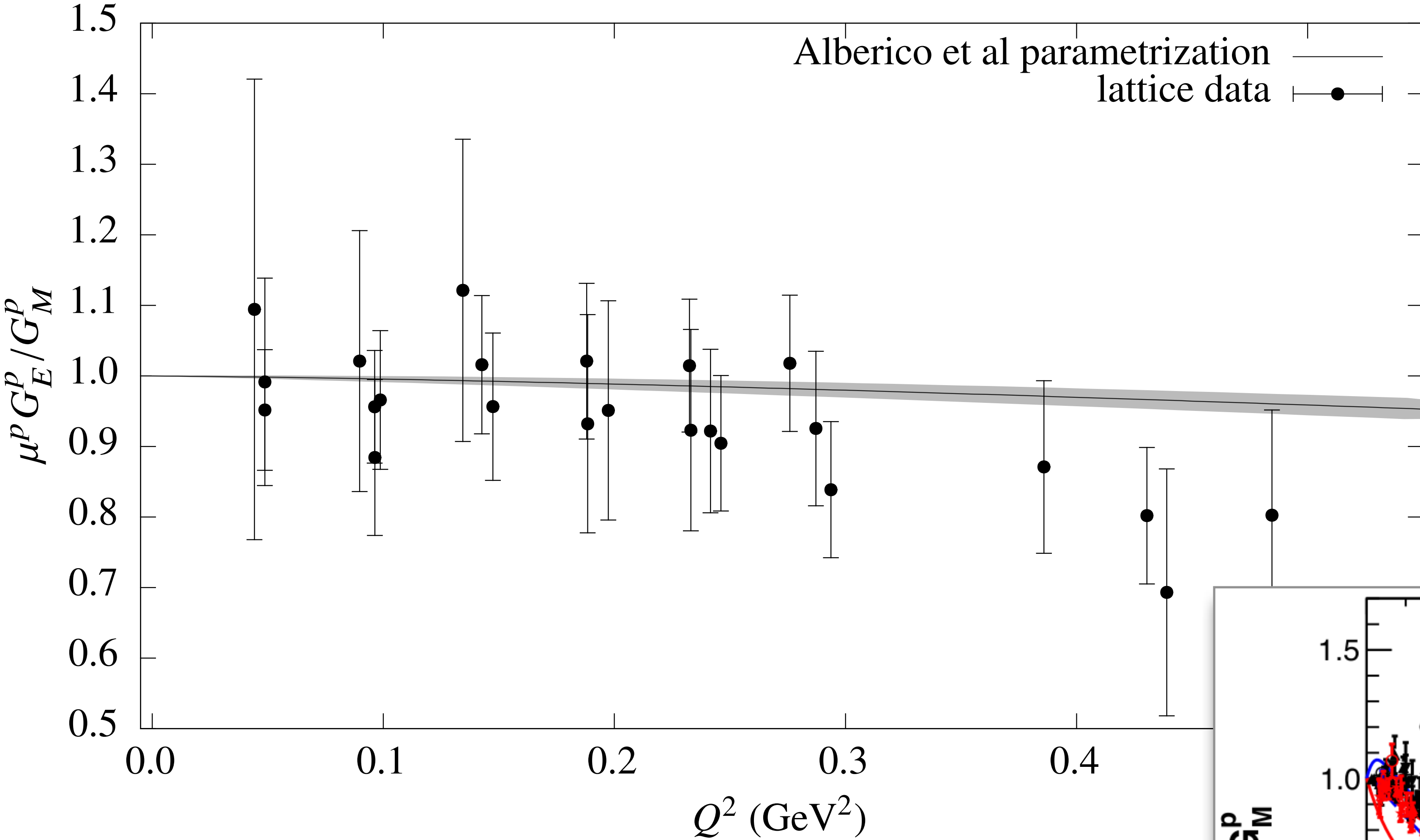
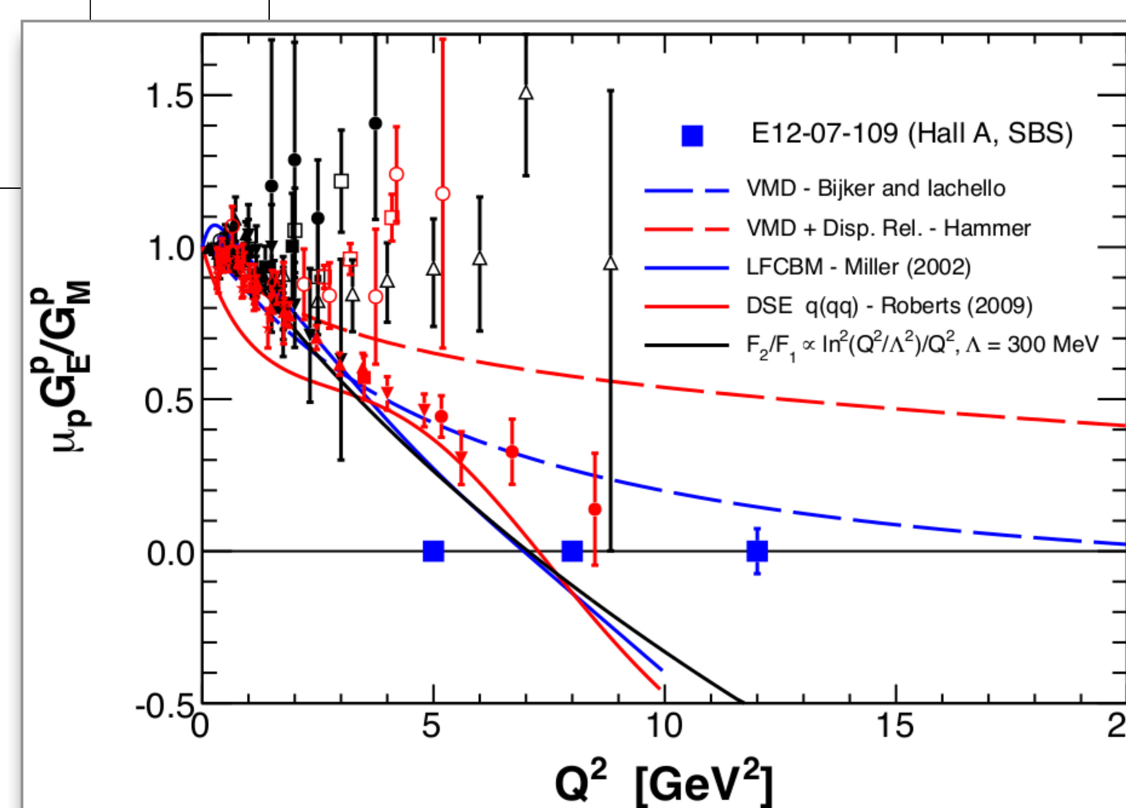


Current plans include direct lattice calculations with Q^2 up to ≈10 GeV^2

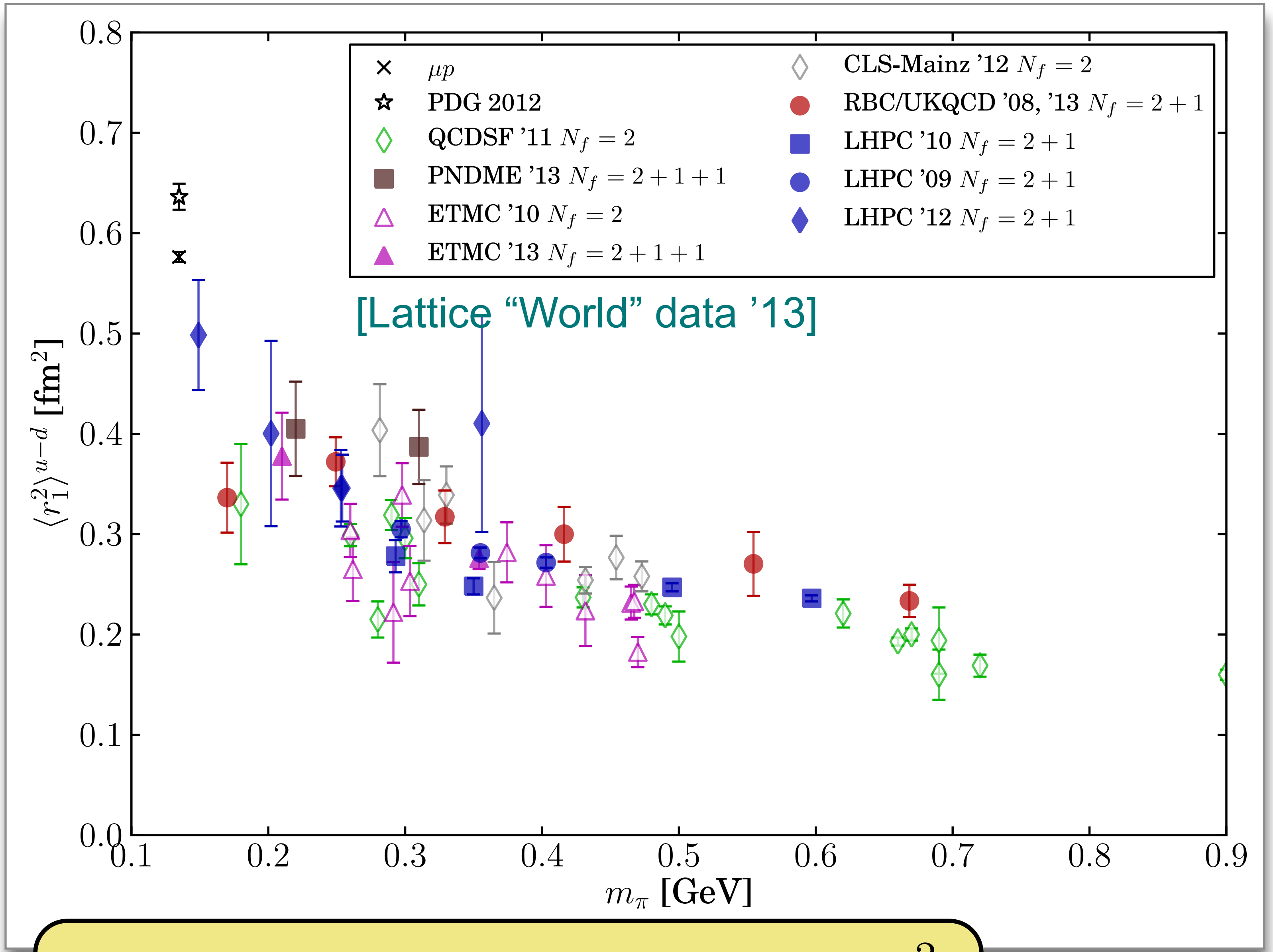
Require $P_z \gtrsim 1.5$ GeV



Dirac Radius vs. m_π and Proton Size Puzzle

$$F_1^{u-d}(Q^2) \approx F(0)\left[1 - \frac{1}{6}Q^2\langle r_1^2\rangle^{u-d} + \mathcal{O}(Q^4)\right]$$

(usually extracted from dipole fits in Q^2<0.5 GeV^2)



ChPT predicts divergence $\sim \log m_\pi^2$

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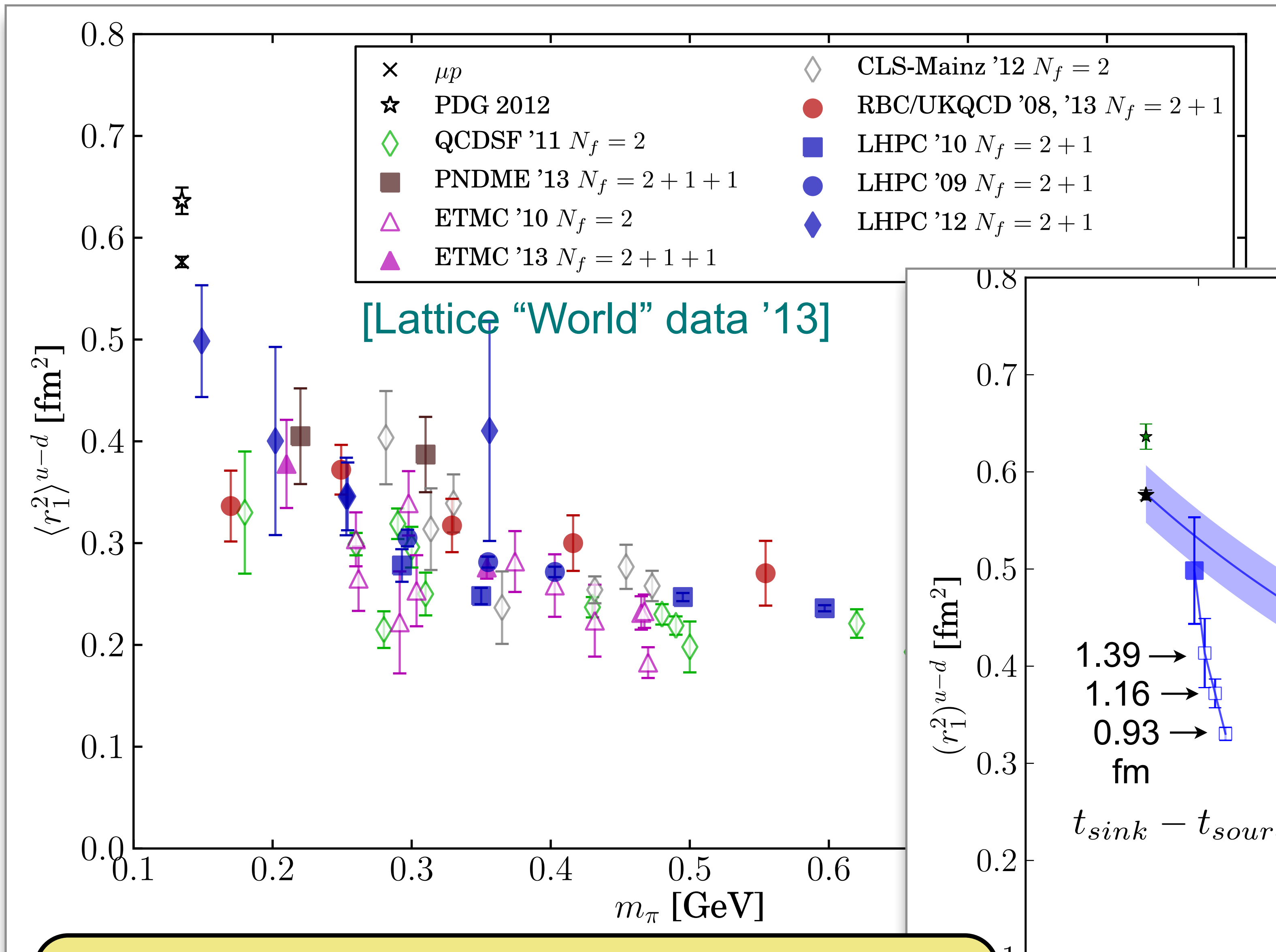
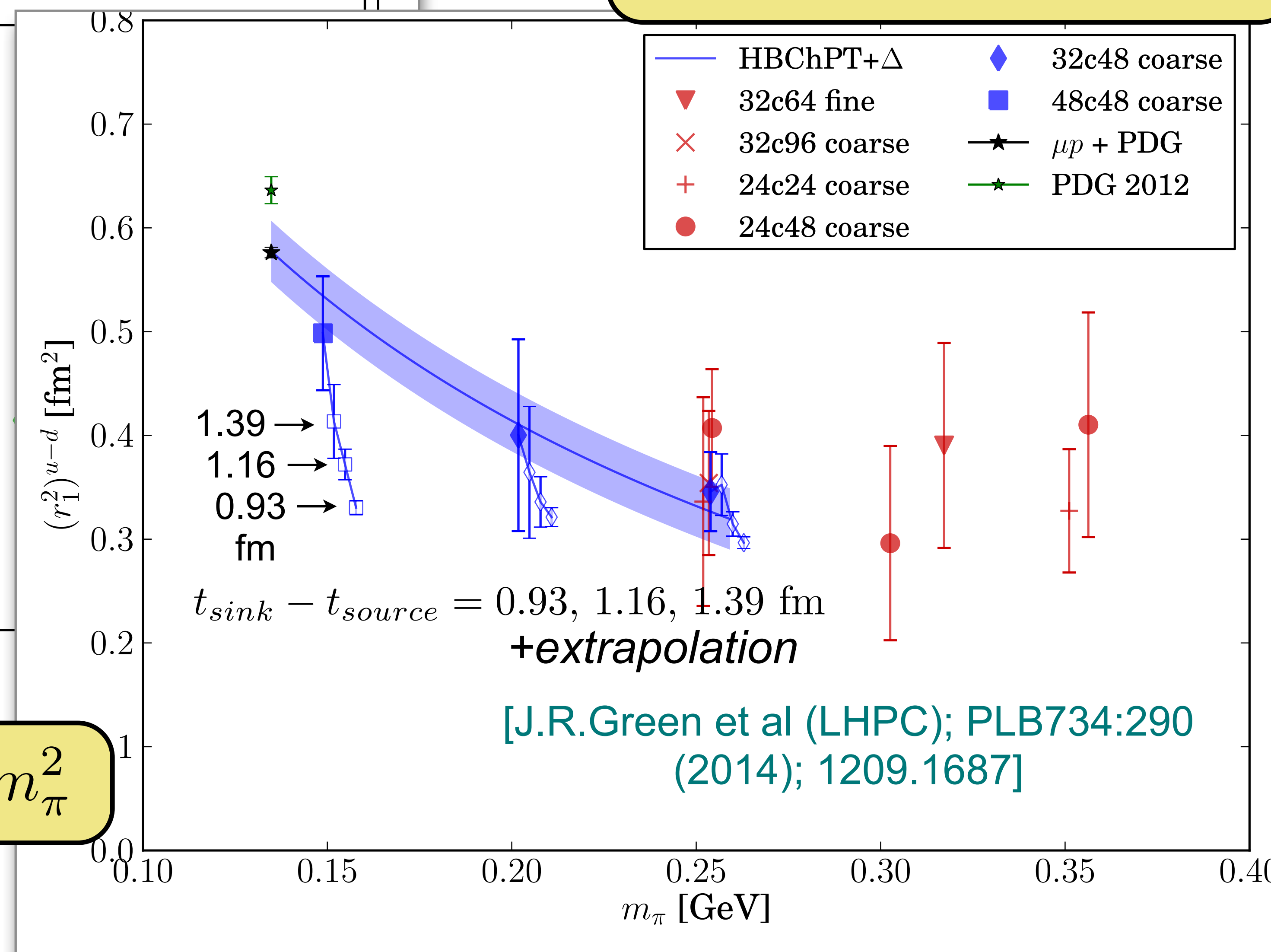


ChPT predicts divergence $\sim \log m_\pi^2$

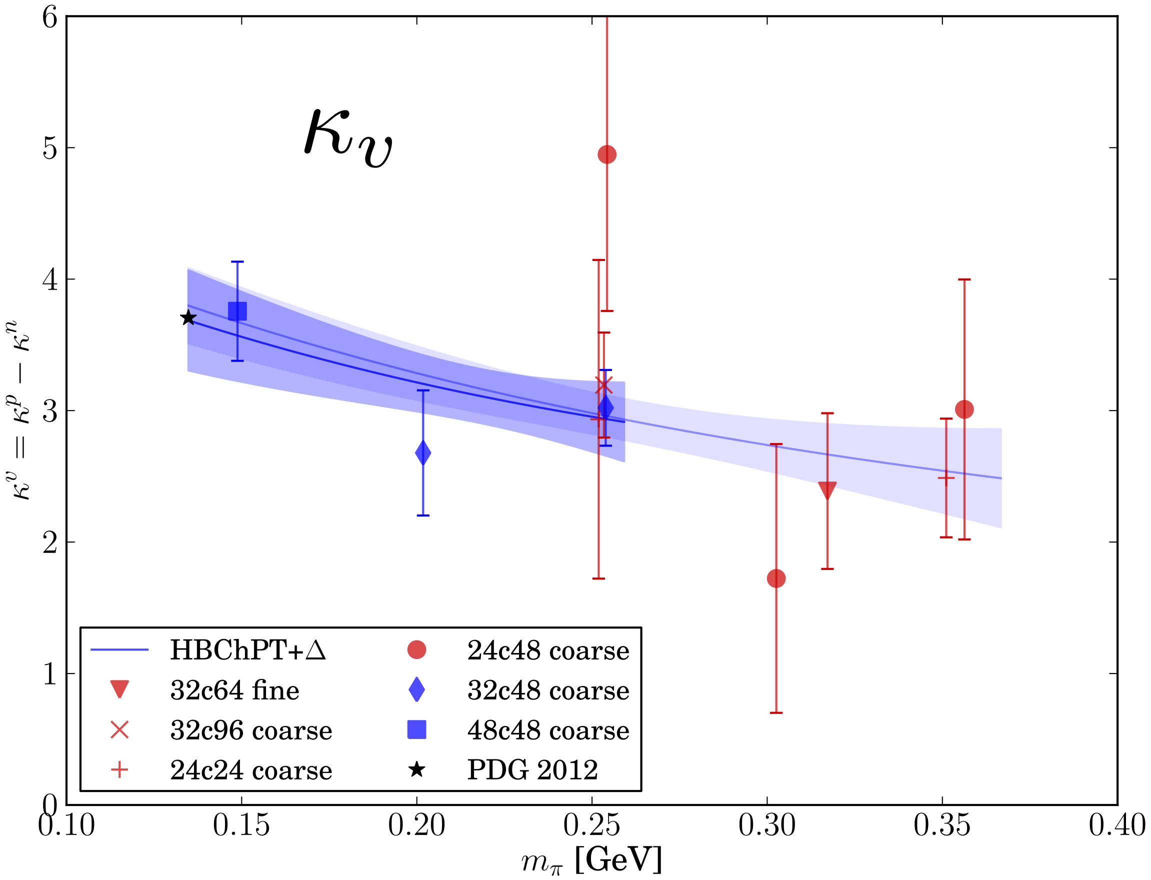
Excited states problem: Worse for m_π<200 MeV?

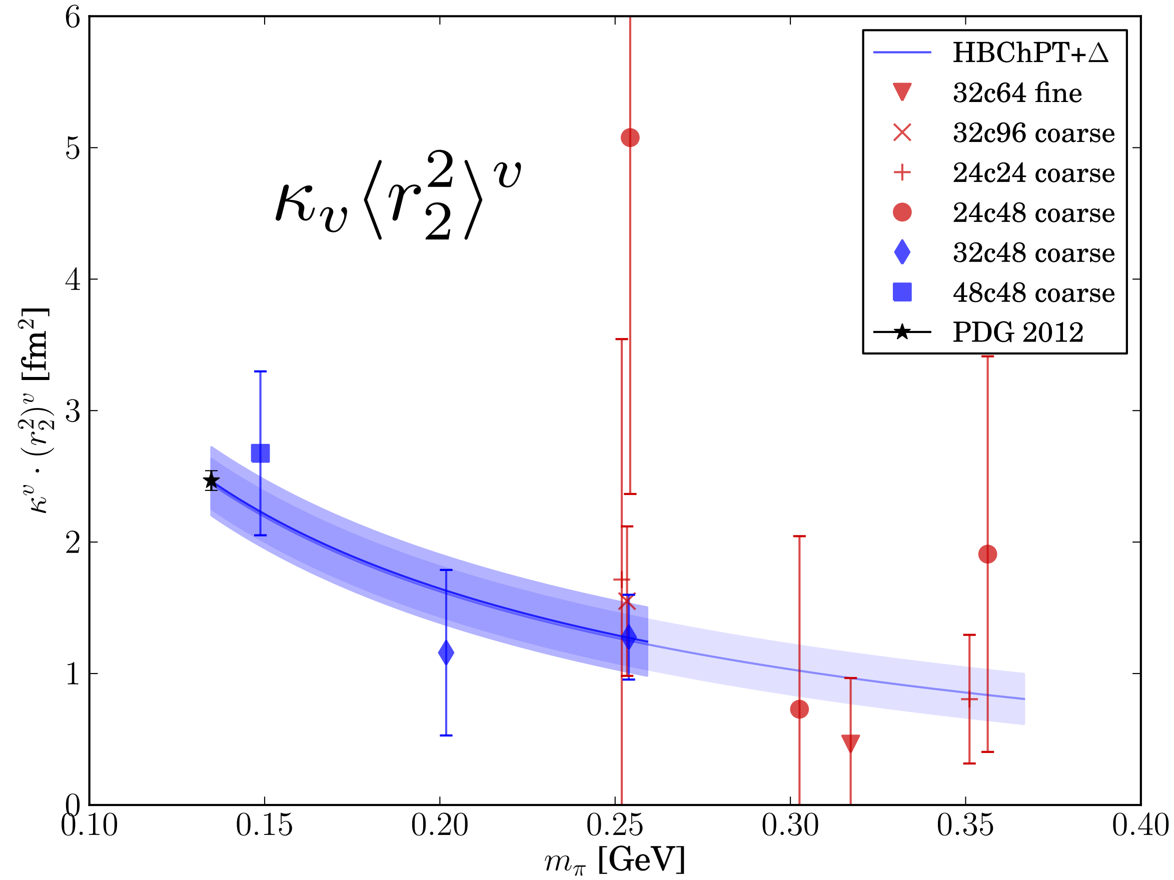


[J.R.Green et al (LHPC); PLB734:290 (2014); 1209.1687]

Isovector Magnetic Moment vs. m_π

$$F_2^{u-d}(Q^2) \approx \kappa_v \left[1 - \frac{1}{6} Q^2 \langle r_2^2 \rangle^v + \mathcal{O}(Q^4)\right]$$





m_π=149 MeV Nf=2+1 clover-imp.Wilson

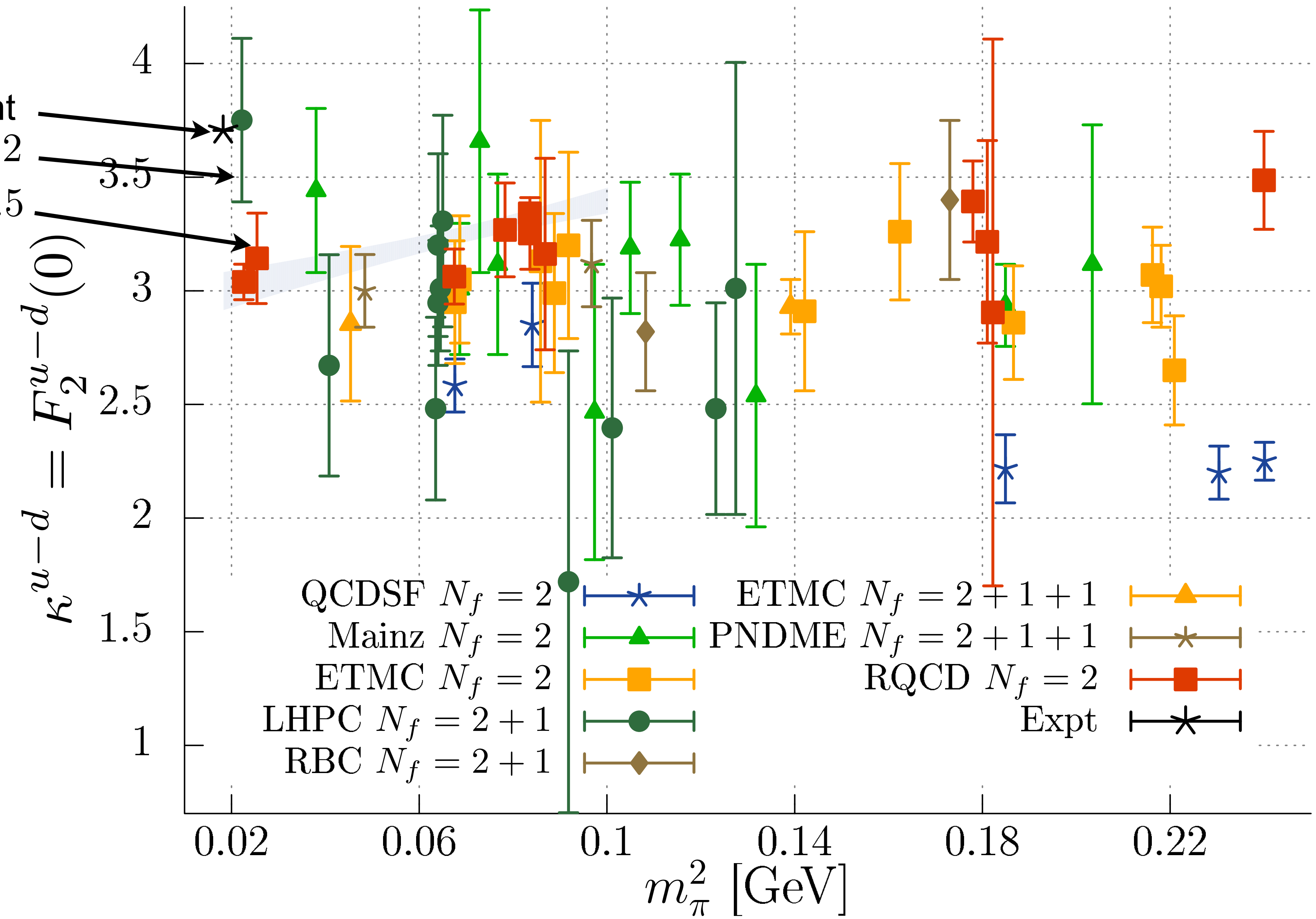
[J.R.Green et al (LHPC); PLB734:290(2014); 1209.1687]

Larger L_s, smaller $Q^2_{\min}$ are desirable

Nucleon Magnetic Moment

"World" Summary [G.Bali et al (RQCD), PRD91:054501(2015)]

experiment
$m_\pi L = 4.2$
$m_\pi L = 3.5$



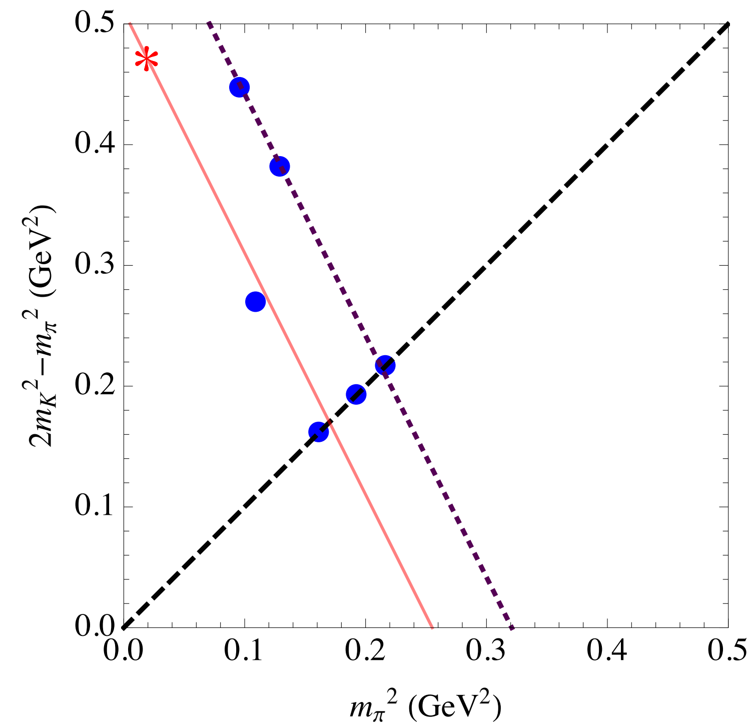
- finite-volume effects?
  FVE $\sim e^{-m_\pi L}$

ChPT-extrapolated Form Factors

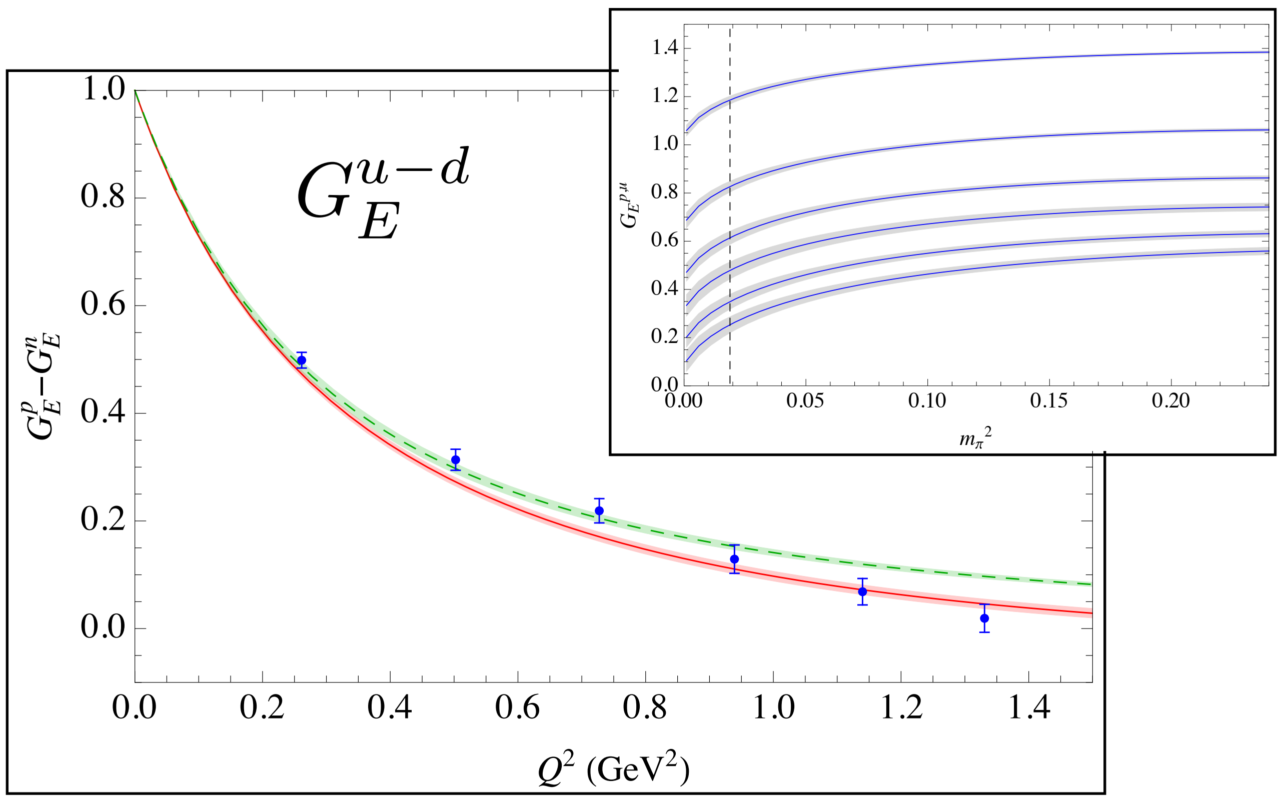
[P.Shanahan et al(CSSM), PRD89:074511; PRD90:034502]

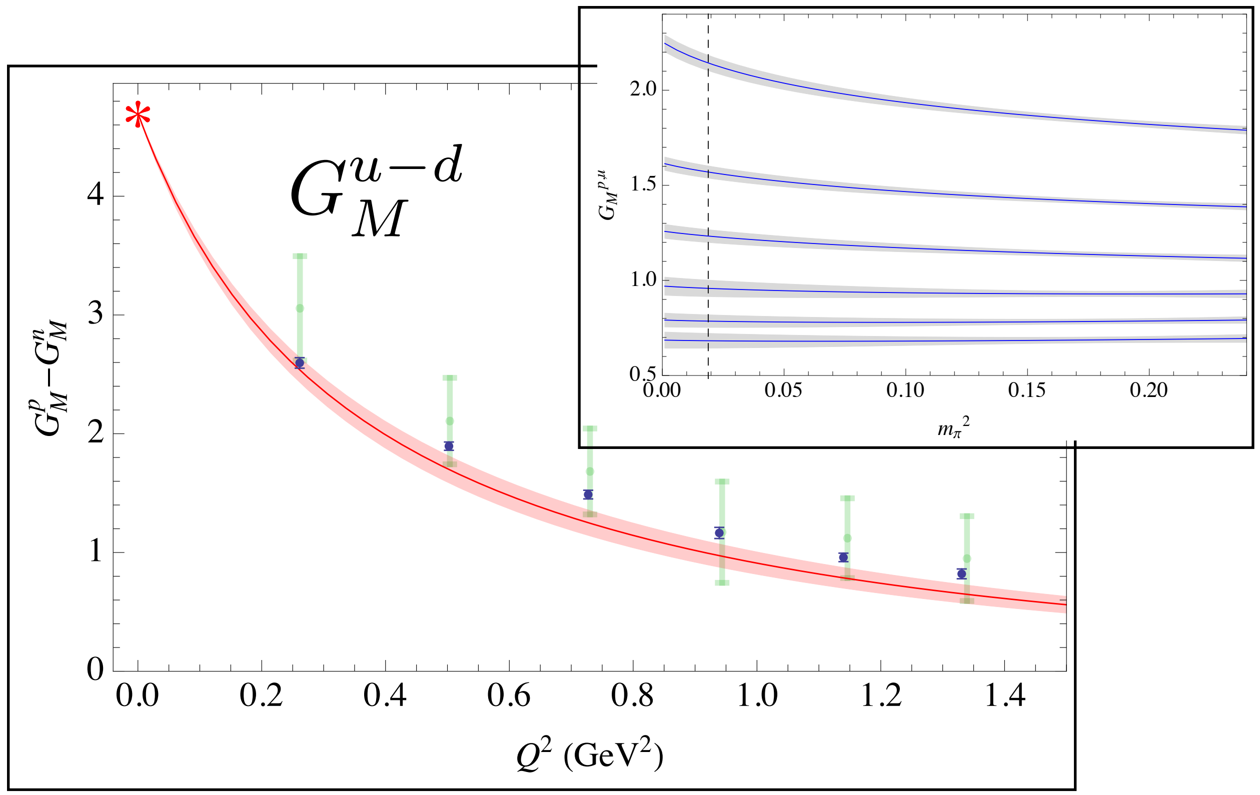
Heavy-baryon PQChPT for Octet baryon Form Factors

$$
\begin{cases}
G_E^{B,q}(Q^2) = G_E^{B,q}(Q^2=0) + Q^2\alpha^{Bq} + Q^2 \sum_{q'} \overline{\alpha}^{Bq(q')}\mathcal{B}m_q \\
\quad + \frac{1}{16\pi^3 f^2}\sum_{\phi}\Big(\frac{1}{2}\beta_O^{Bq(\phi)}\mathcal{I}_O(m_\phi,Q^2) - \beta_D^{Bq(\phi)}\mathcal{I}_D(m_\phi,Q^2) \\
\qquad + \beta_T^{Bq(\phi)}\mathcal{I}_T(m_\phi,Q^2)\Big) \\
G_M^{B,q}(Q^2) = \alpha^{Bq} + \sum_{q'}\overline{\alpha}^{Bq(q')}\mathcal{B}m_{q'} \\
\quad + \frac{m_N}{16\pi^3 f^2}\sum_{\phi}\Big(\beta_O^{Bq(\phi)}\mathcal{I}_O(m_\phi,Q^2) + \beta_D^{Bq(\phi)}\mathcal{I}_D(m_\phi,Q^2)\Big)
\end{cases}
$$



Independent ChPT fits at each fixed Q^2





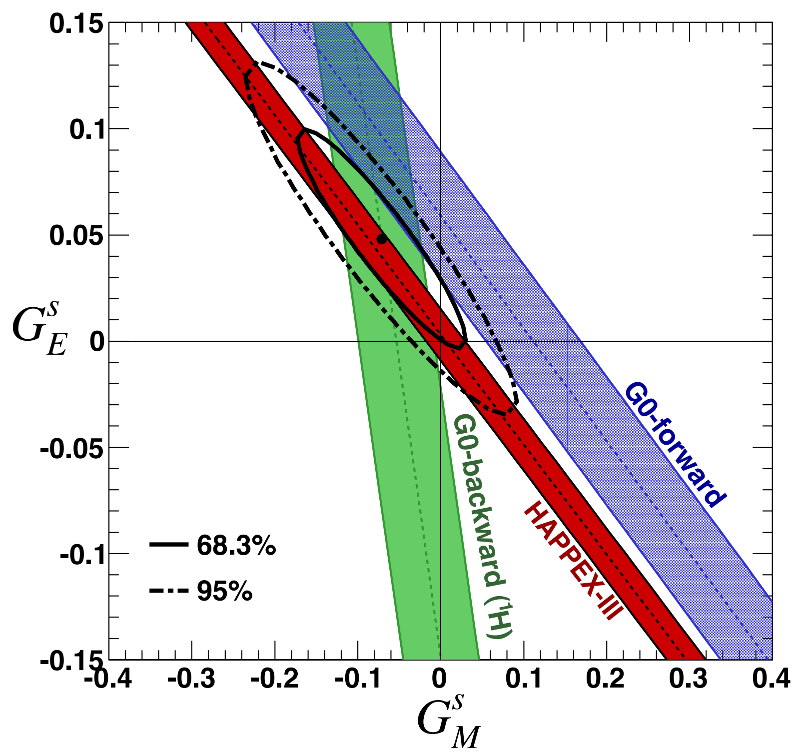
Strangeness in EM form factors

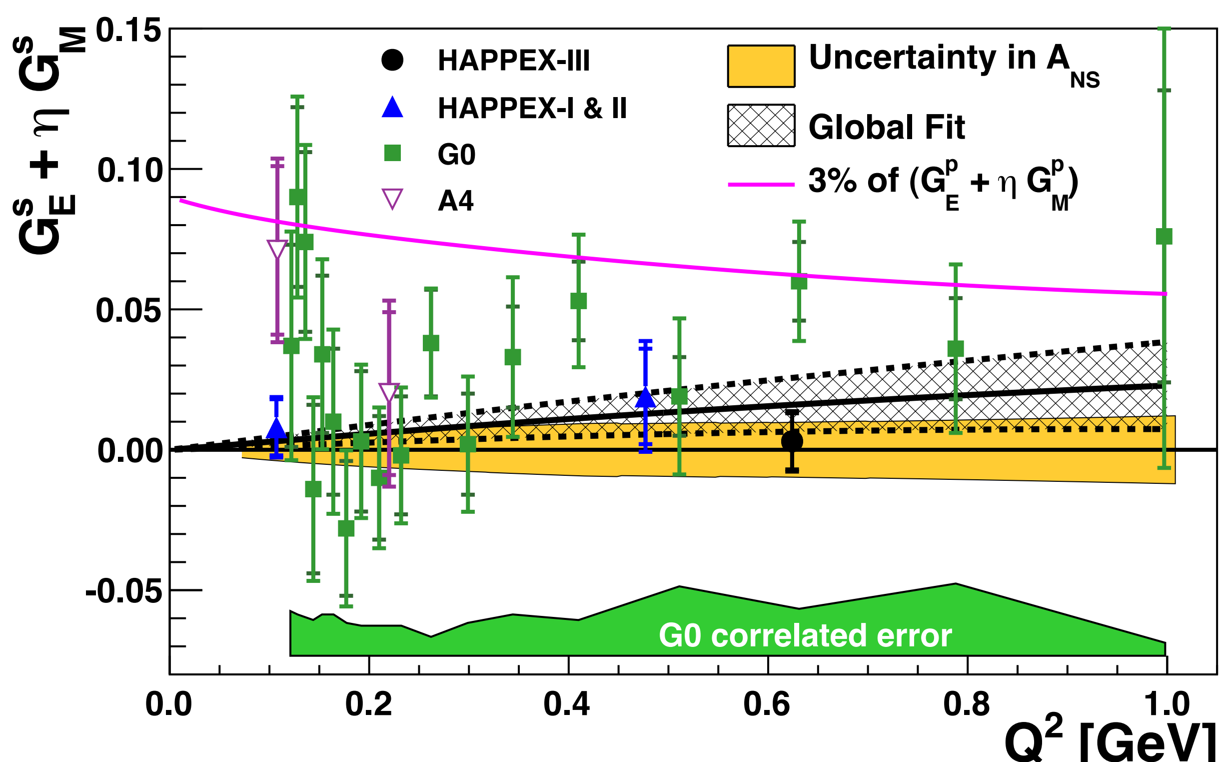
Strange quark form factors

$$G^p_{E,M} = \frac{2}{3} G^u_{E,M} - \frac{1}{3} G^d_{E,M} - \frac{1}{3} G^s_{E,M}$$

$G^s_{E,M}$ are measured e.g. in *e–p* elastic scattering asymmetry (SAMPLE, HAPPEX, G0, A4)

$$A_{LR} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = A_{non-strange} + \eta_E G^s_E + \eta_M G^s_M$$

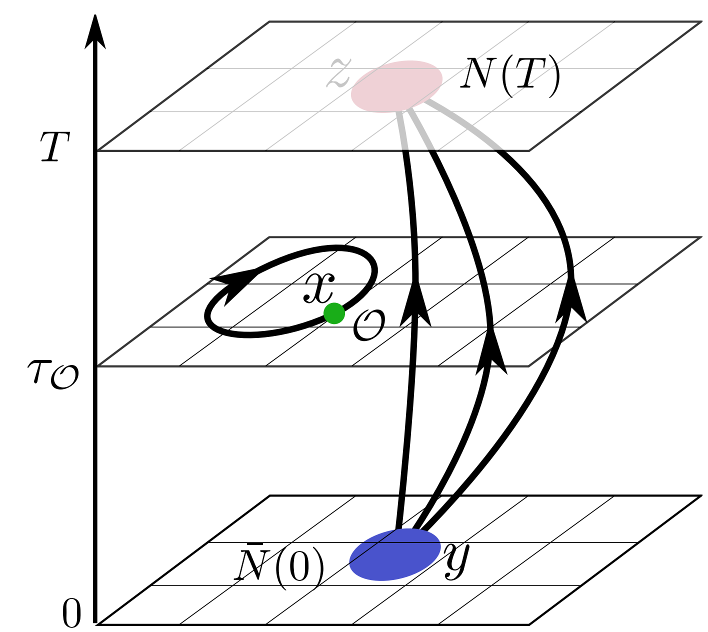




[HAPPEX collab., PRL108:102001(2012)]

Lattice Evaluation of Disconnected Contractions

- Stochastic evaluation: $\begin{cases} \xi(x) = \text{ random } Z_2\text{-vector} \\ E\left[\xi^\dagger(x)\xi(y)\right] = \delta_{x,y} \end{cases}$



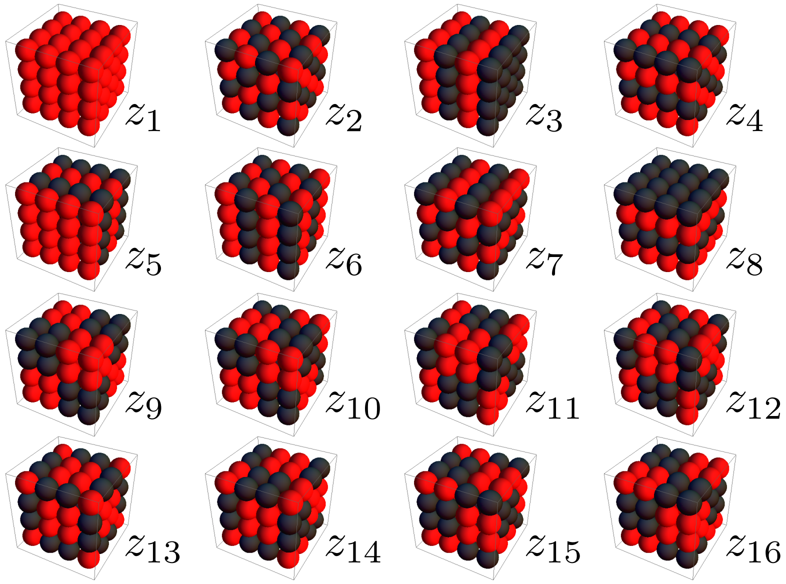
$$\sum_x e^{iqx} \not{D}^{-1}(x,x) \approx \frac{1}{N_{MC}} \sum_i^{N_{MC}} \xi^\dagger_{(i)} \left(e^{iqx} \not{D}^{-1} \xi_{(i)}\right)$$

$$\mathrm{Var}\Big(\sum_x \not{D}^{-1}(x,x)\Big) \sim \frac{1}{N_{MC}}$$ (contributions from $\not{D}^{-1}(x\neq y)$)

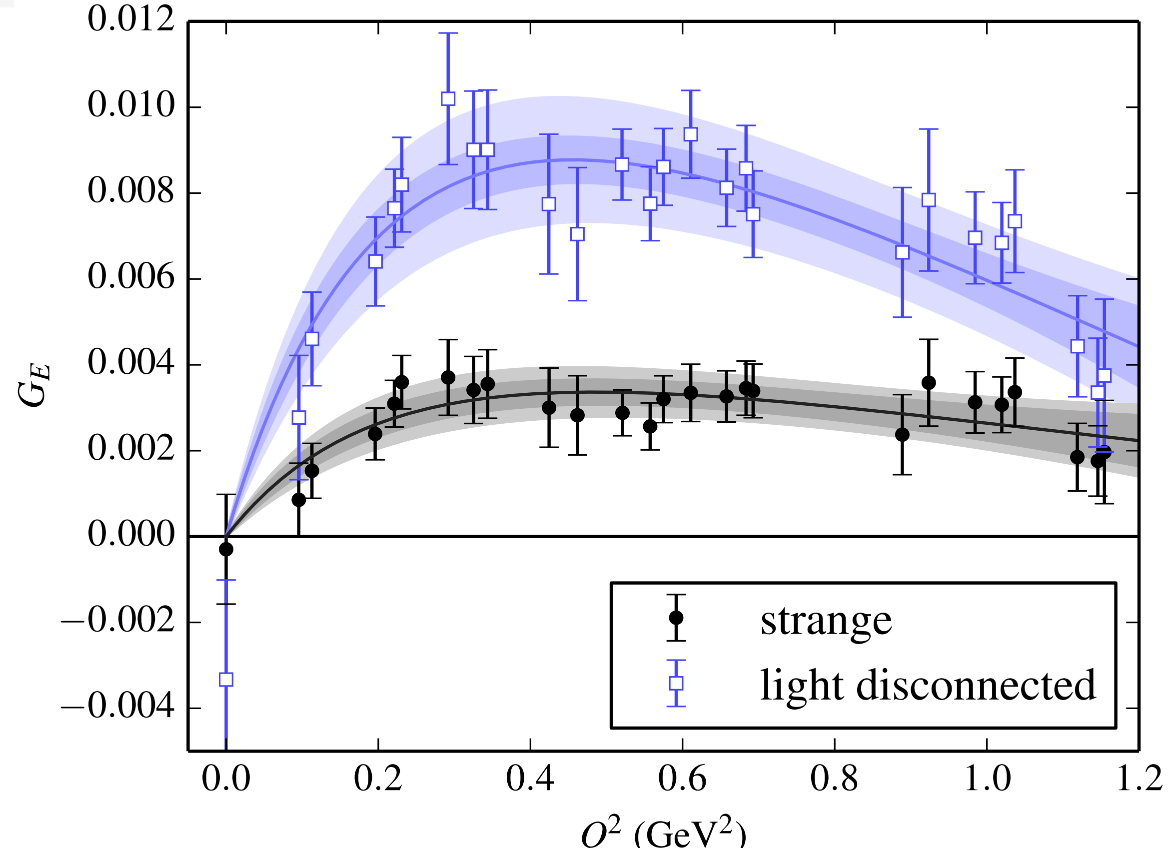
- Exploit $\not{D}^{-1}(x,y)$ FALLOFF to reduce $\sum_{x\neq y} |\not{D}^{-1}(x,y)|^2$

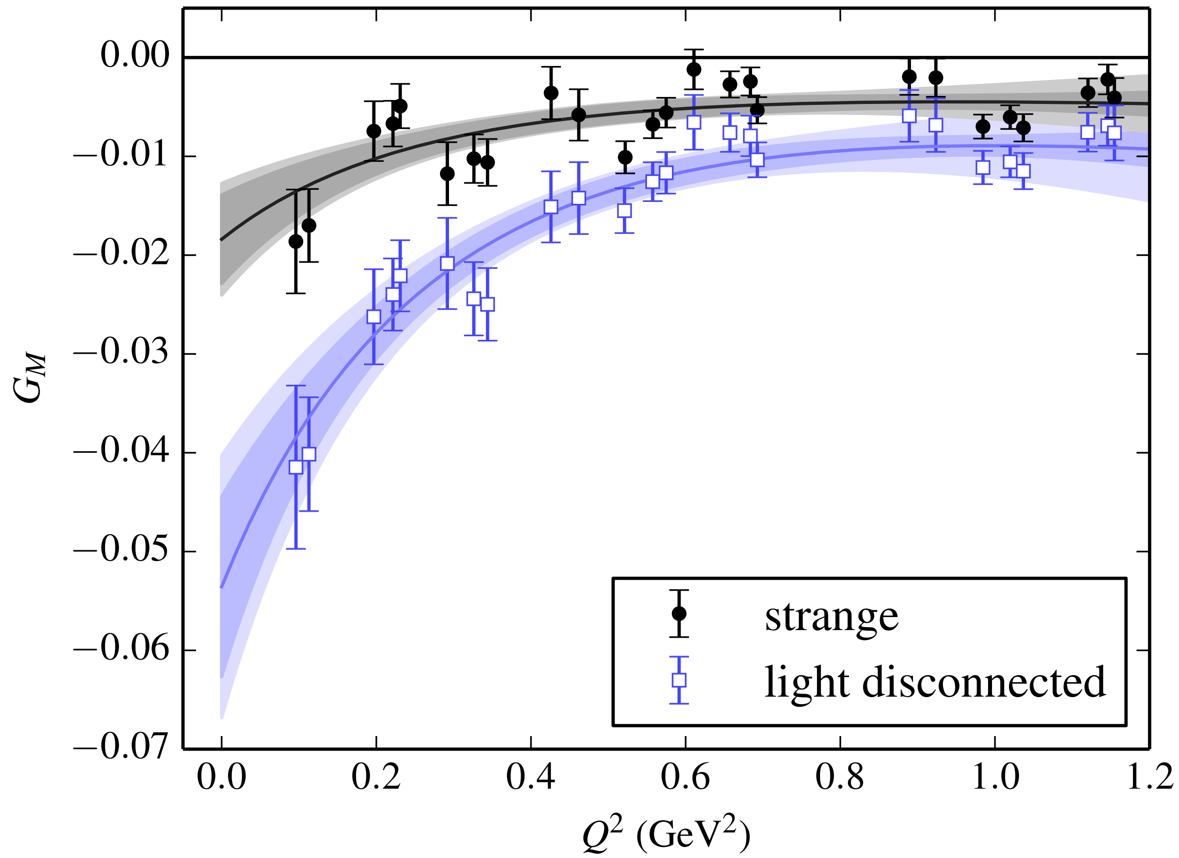
Hierarchical probing method [K.Orginos, A.Stathopoulos, '13] :
In sum over N=2^{nd+1} 3D(4D) Hadamard vectors,
near-(x,y) terms cancel:

$$\frac{1}{N}\sum_i z_i(x) z_i(y)^\dagger = \begin{cases} 0, & 1 \le |x-y| \le 2^k, \\ 1, & x = y \text{ or } 2^k < |x-y| \end{cases}$$



Disconnected Contractions for Nucleon FF's





[J. Green, S. Meinel, et al (LHPc); PRD92:031501]

N_f=2+1 dynamical fermions

m_π=319 MeV

(USQCD/JLab clover Wilson)

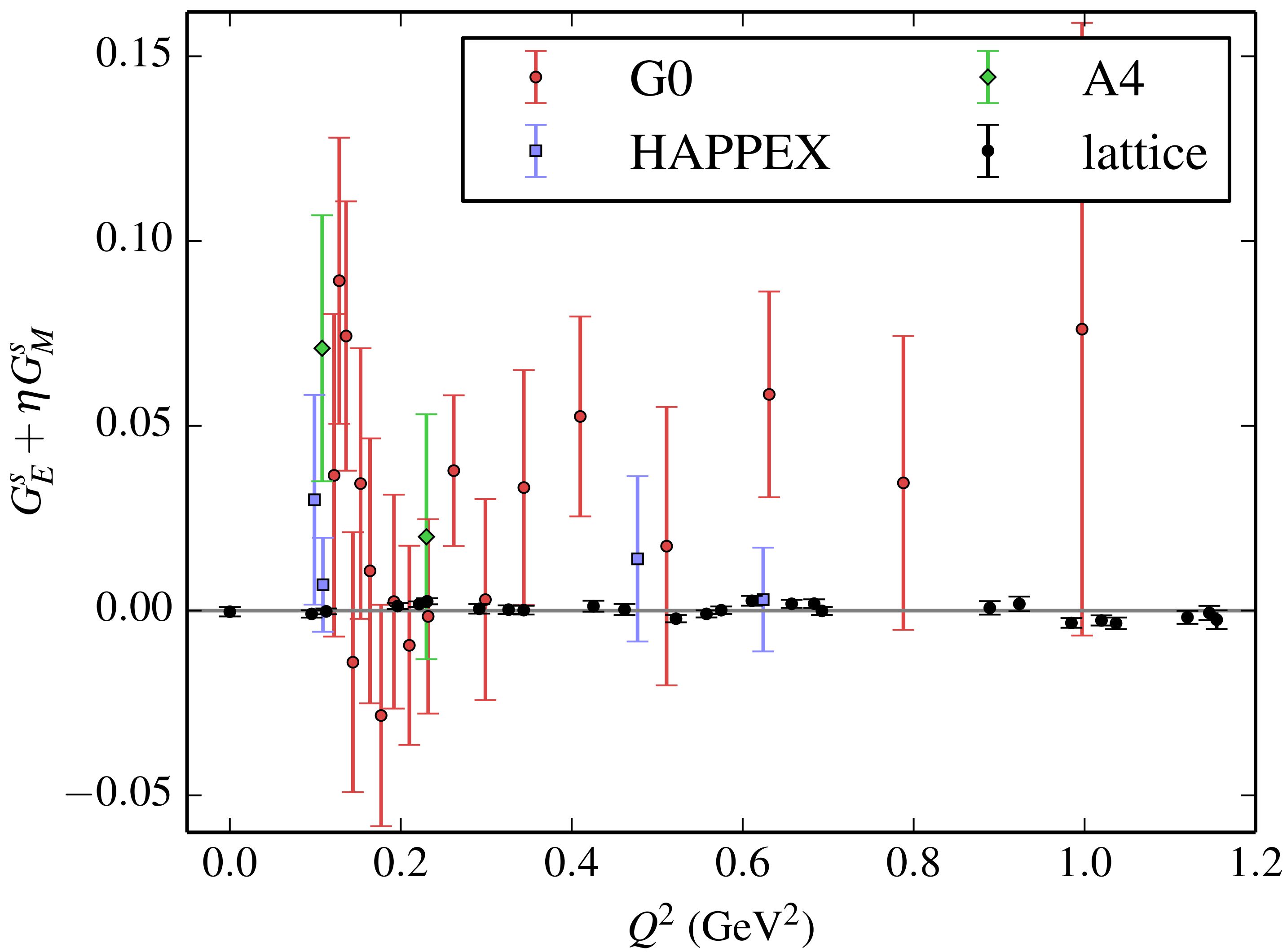
$$|(G_E^{u/d})_{\rm disc}| \lesssim 0.010 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

$$|(G_E^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

$$|(G_M^{u/d})_{\rm disc}| \lesssim 0.015 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

$$|(G_M^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

Strange Form Factors from PV: Exp't vs. Lattice



HAPPEX, G0, A4 data
[PRL108:102001(2012)]
vs.
Lattice QCD (m_π = 317 MeV)
[J. Green, S. Meinel; PRD92:031501]

Lattice data plotted with

$$\eta = \frac{Q^2}{0.94\mathrm{Ge}V^2}$$

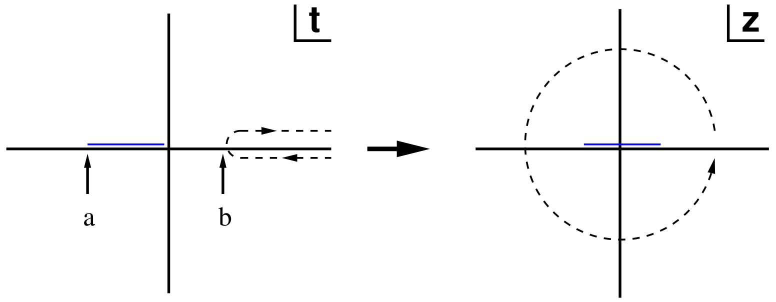
$(G_E^s + \eta G_M^s) \sim$ Strange part in the elastic *e–p* scattering asymmetry (forward angles)

Radii and Magnetic Moment of Strange Quark

z-expansion fit $\quad G(Q^2) = \sum_k^{k_\text{max}} a_k\, z^k, \quad z = \frac{\sqrt{t_\text{cut}+Q^2}-\sqrt{t_\text{cut}}}{\sqrt{t_\text{cut}+Q^2}+\sqrt{t_\text{cut}}}$

(analytic for $|z| < 1$)

[R.Hill, G.Paz, PRD82:113005 (2010)]



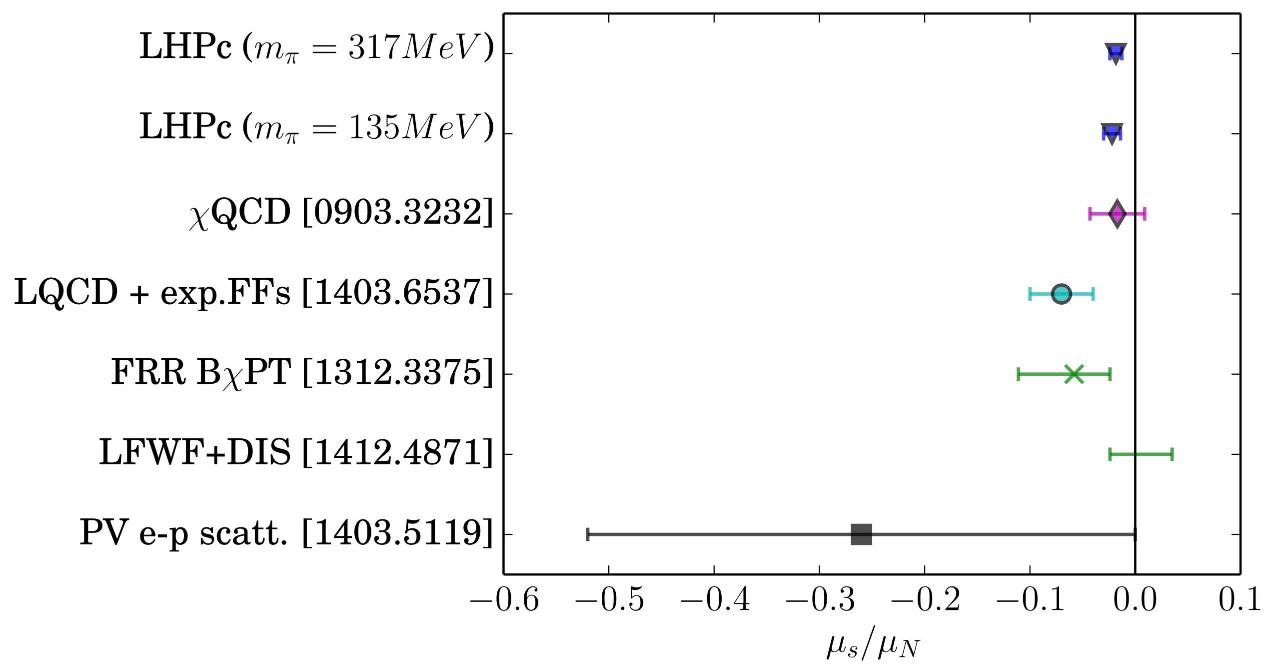
$$t = -Q^2\,, \quad t_\text{cut} = 4m_\pi^2$$

$$\mu = G_M(0) \sim a_0^M\,, \quad (r_{E,M})^2 = -6\frac{d\,G_{E,M}(Q^2)}{d\,Q^2}\Big|_{Q^2=0} \sim a^{E,M\,1}\,,$$

PQChPT-inspired linear extrapolation in
$(m_\text{loop})^2 \sim (m_{light} + m_{disconn})$ to phys.point

$$(r_E^2)^s = -0.00535(89)(56)(113)(20)\ \text{fm}^2$$
$$(r_M^2)^s = -0.0147(61)(28)(34)(5)\ \text{fm}^2$$
$$\mu^s = -0.0184(45)(12)(32)(1)\ \mu_N^\text{lat}$$

Errors = (statistic)(fit)(exc.state)(discr.)

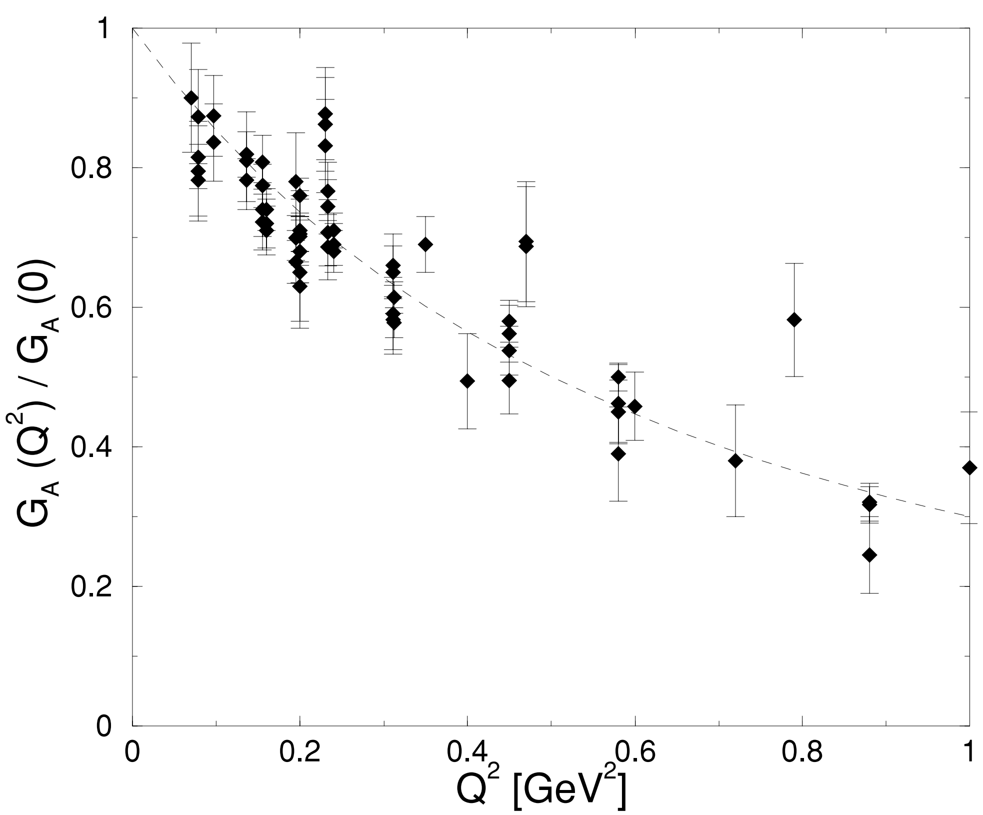


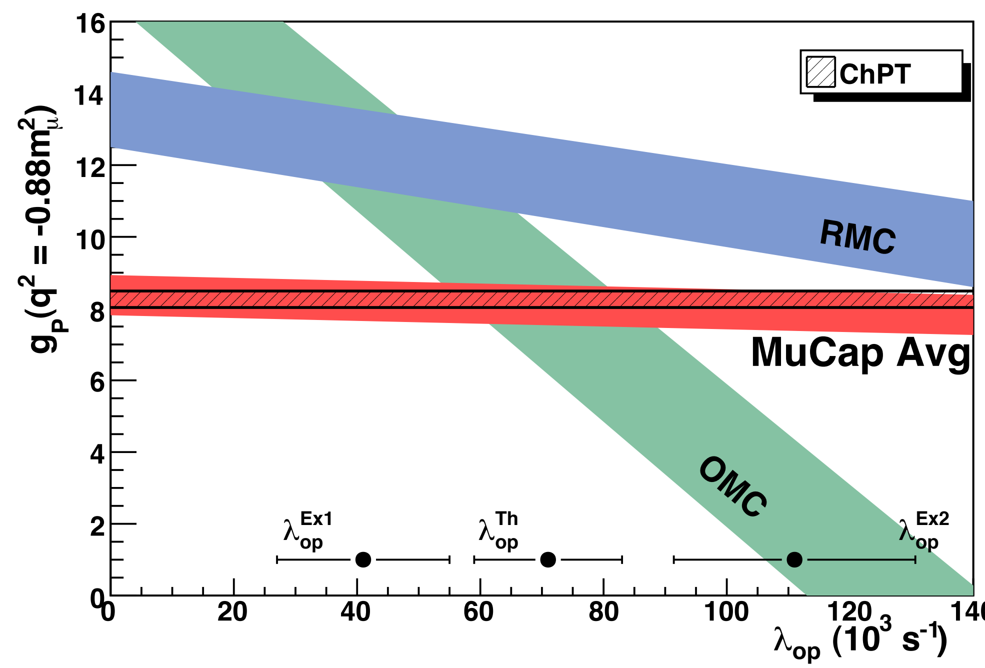
[J. Green, S. Meinel, et al (LHPc); PRD92:031501]

Axial-Vector Current Form Factors

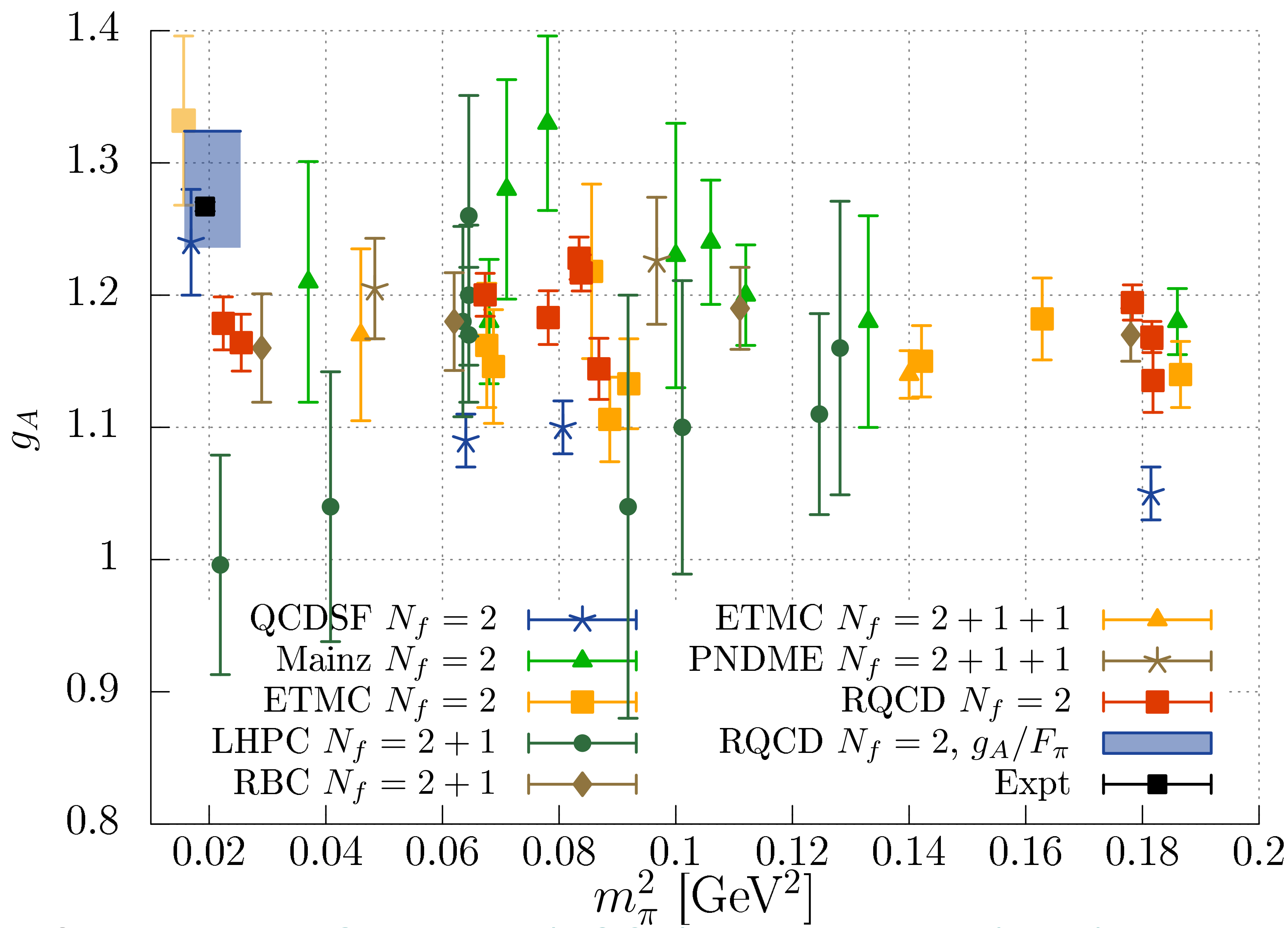
$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\,\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$

- Axial form factor $G_A(Q^2)$
  - Interaction with neutrinos: MiniBooNE
- Induced pseudoscalar form factor $G_P(Q^2)$
  - Charged pion electroproduction
  - Muon capture (MuCAP@UW): $g_P \sim G_P(Q^2 = 0.88\, m_\mu^2)$
- Strange axial form factor $G_A^s(Q^2)$: studied at MiniBooNE





Axial Charge in Lattice QCD



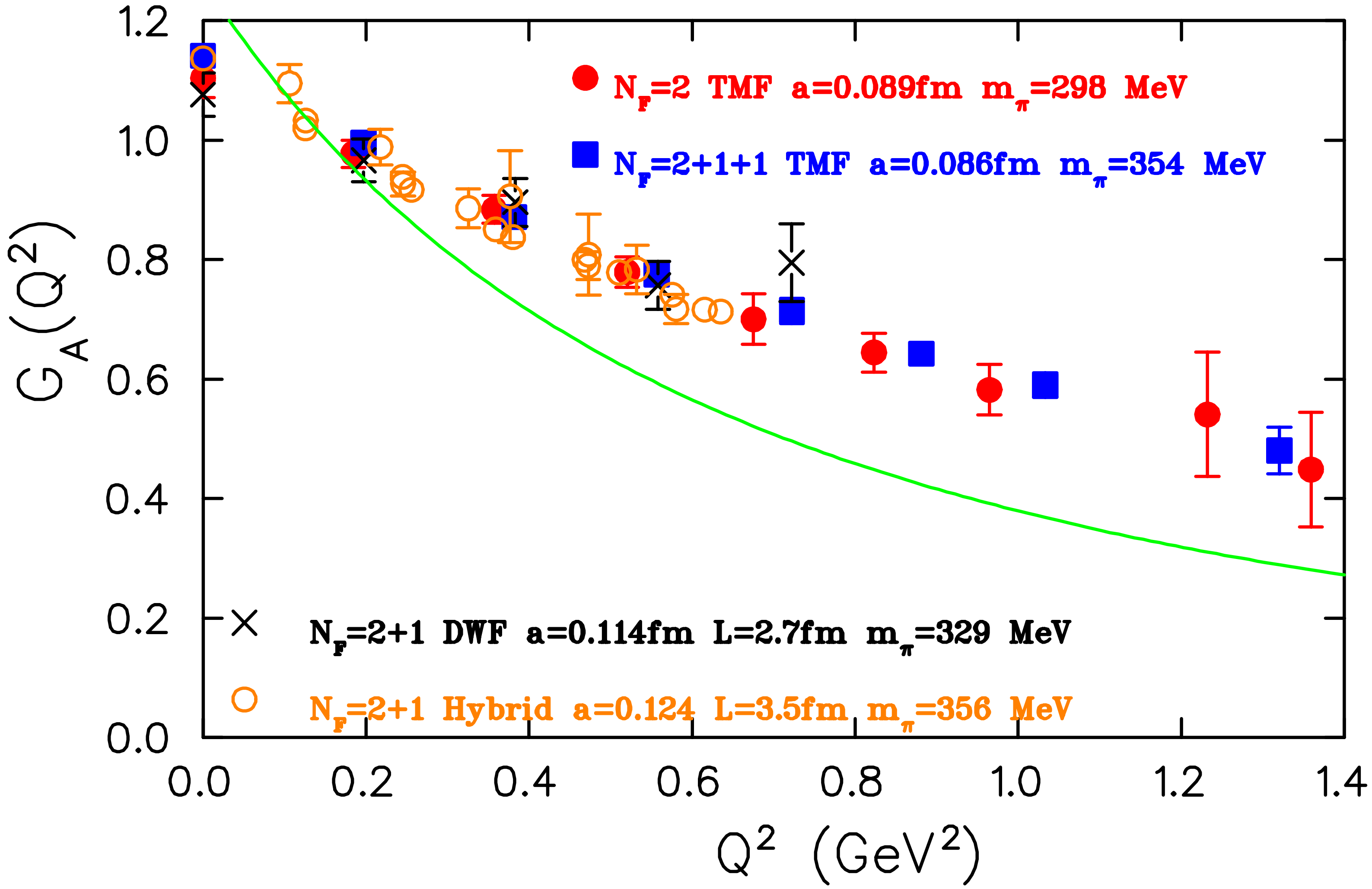
Summary plot [G.Bali et al (RQCD) PRD91:054501(2015); 1412.7336]

- g_A may be especially subject to effects of exc.states *and* finite volume
- evidence for cancellation of systematic effects in $(g_A/f\pi)$

Nucleon Axial Form Factor

$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle = \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$

[C.Alexandrou (ETMC), 1303.5979]



Nucleon Axial Form Factor

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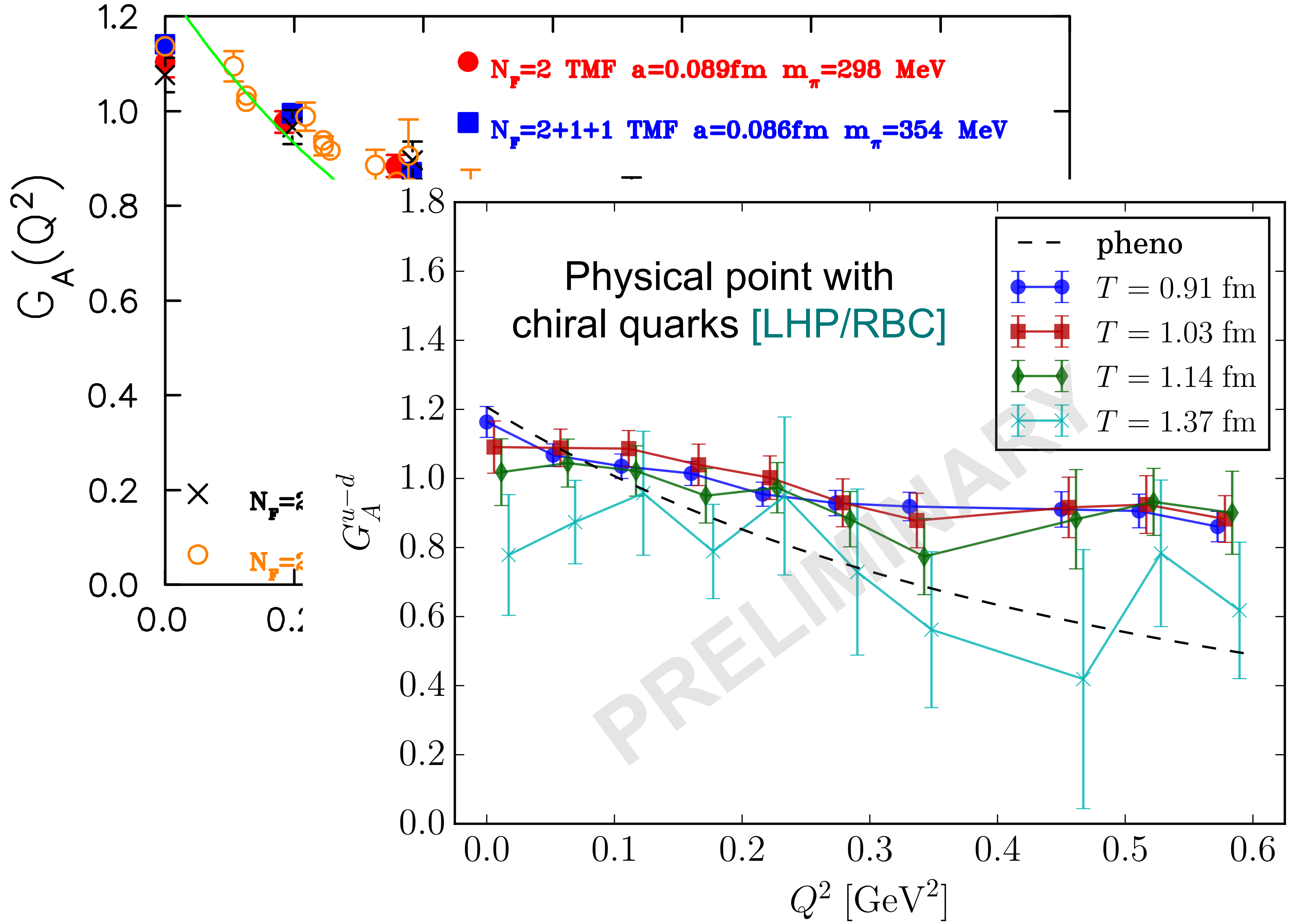
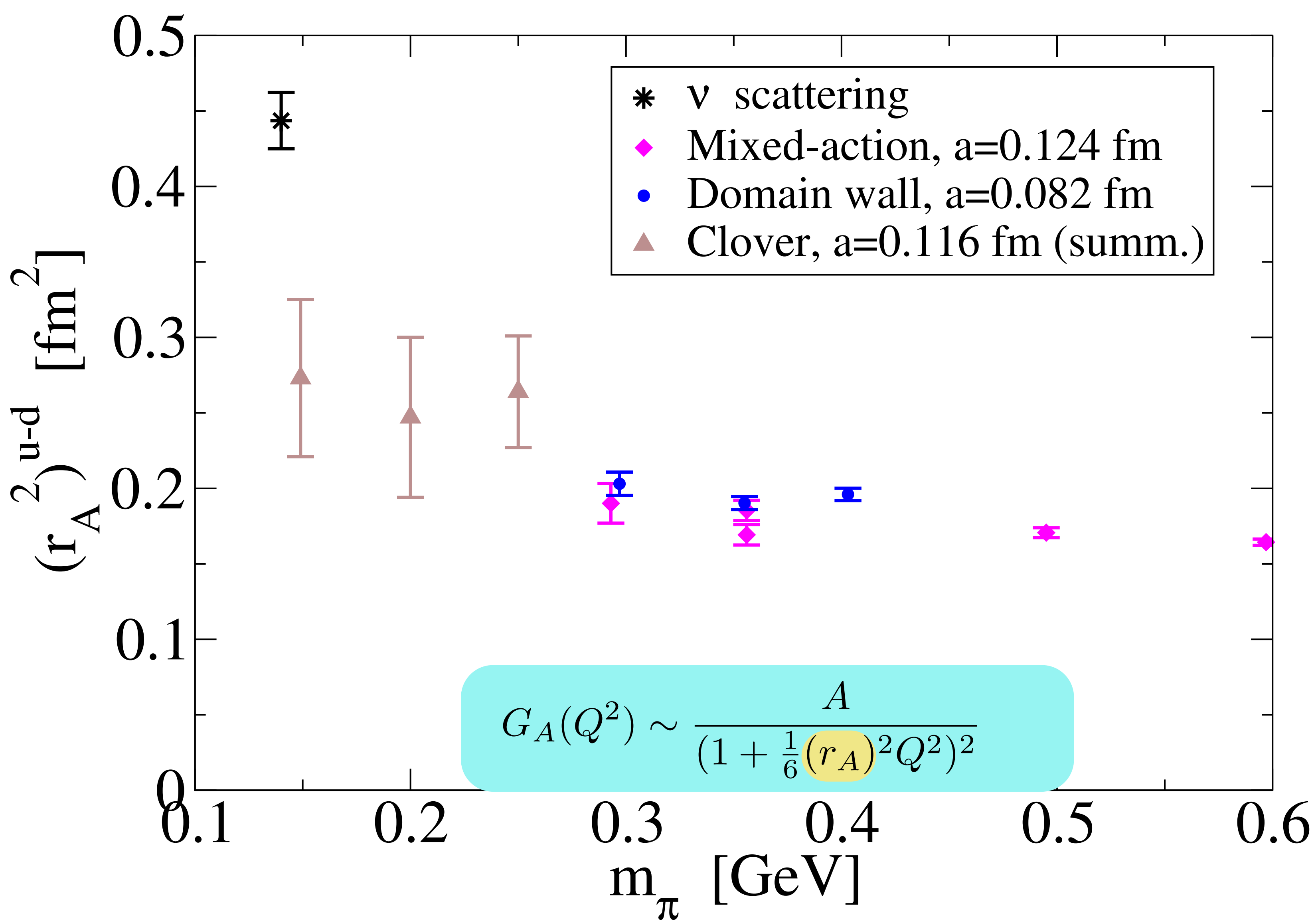


Nucleon Axial Radius

- 5% discrepancy in exp. values of r_A (from $G_A(Q^2)$ dipole fits)

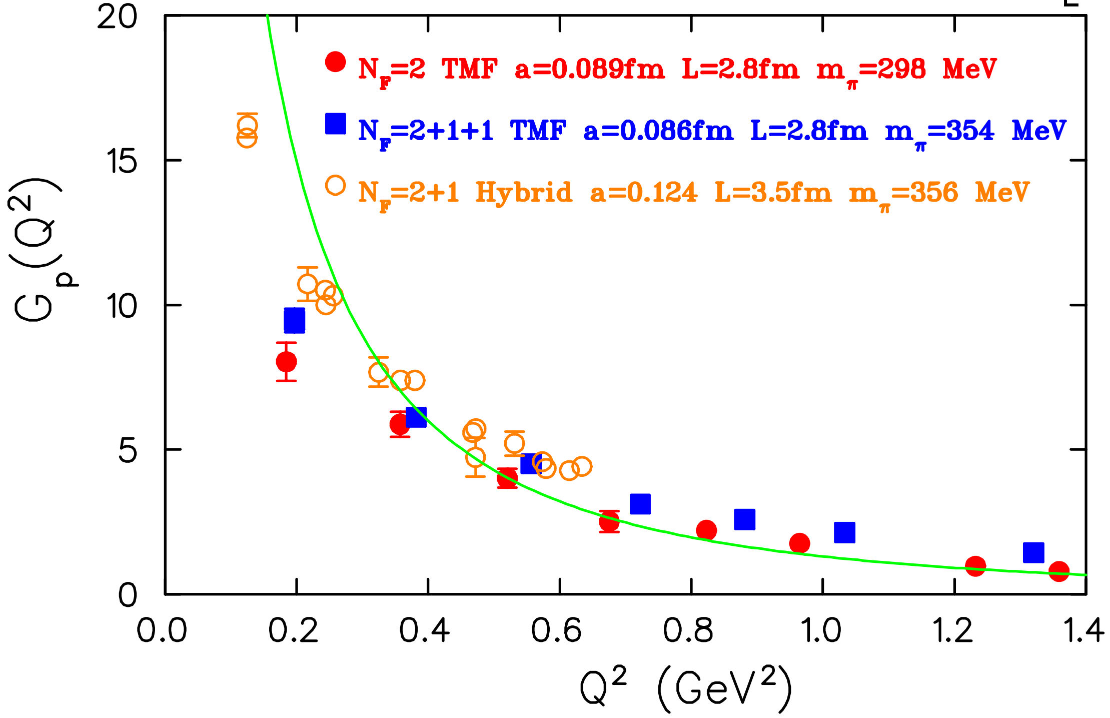
$$\sqrt{\langle r_A^2\rangle_{\nu-\mathrm{scatt.}}} = (0.666 \pm 0.014)\ \mathrm{fm}$$
$$\sqrt{\langle r_A^2\rangle_{el-prod}} = (0.639 \pm 0.010)\ \mathrm{fm}$$



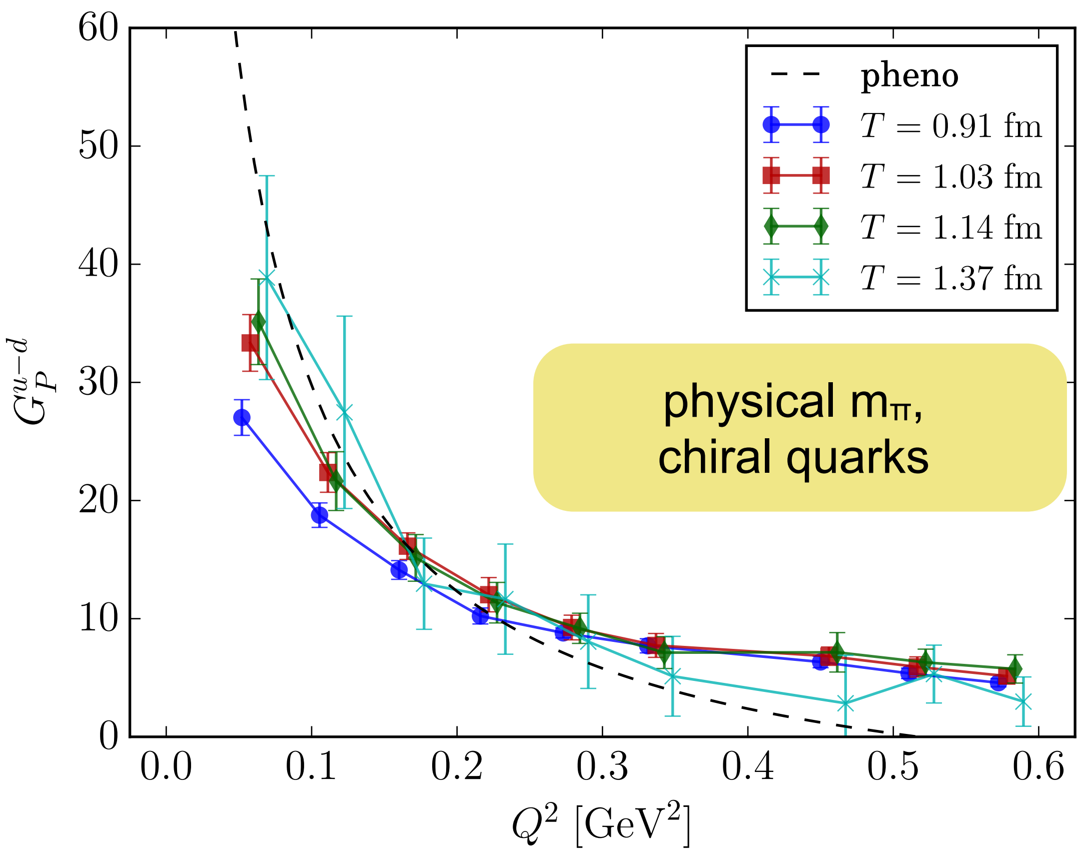
- Weak dependence on m_π and disagreement at m_π^{phys} : same problem as g_A ?
- Study required for volume dependence and exc.states.

Nucleon Pseudoscalar Form Factor $G_P(Q^2)$

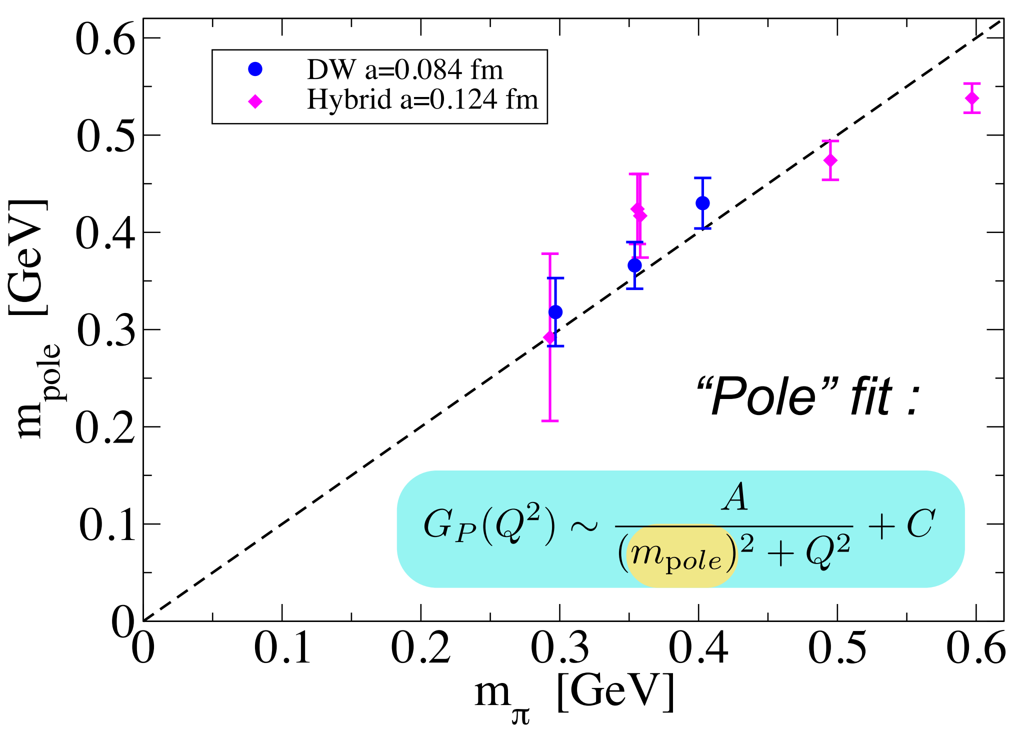
$$\langle P+q|\, \bar{q}\gamma^\mu\gamma^5 q\,|P\rangle = \bar{U}_{P+q}\left[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\right]U_P$$



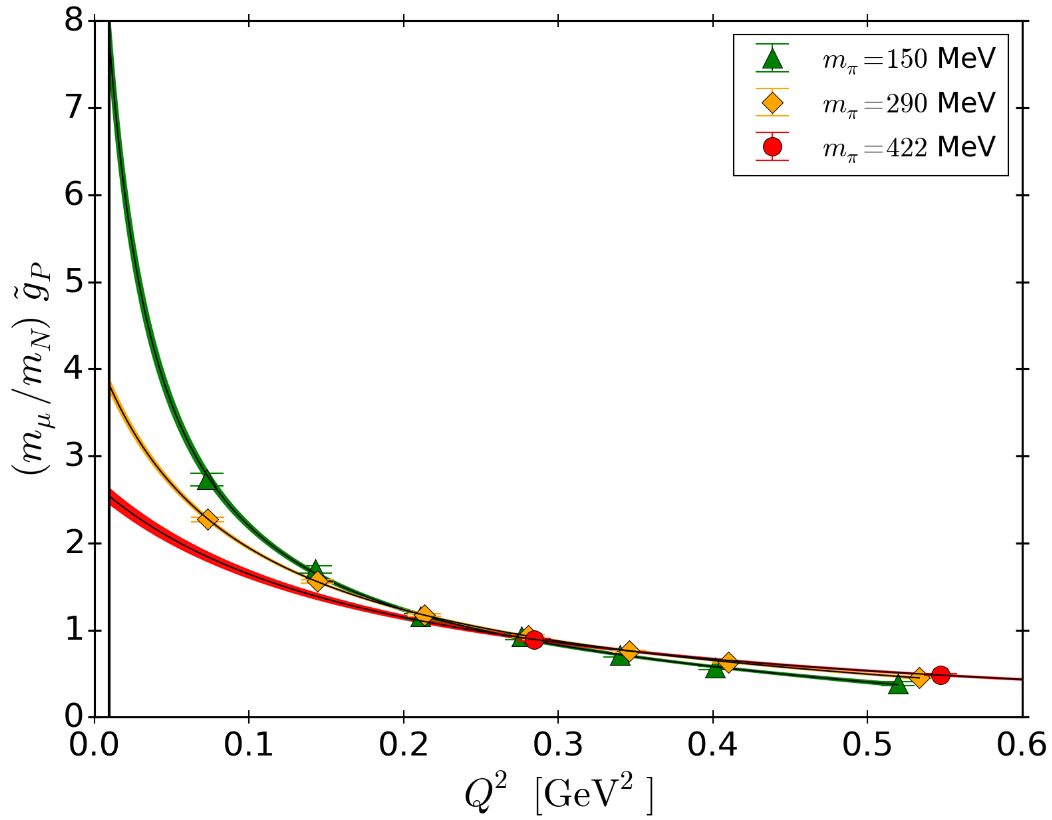
- G_P at the physical point : large excited states contrib.

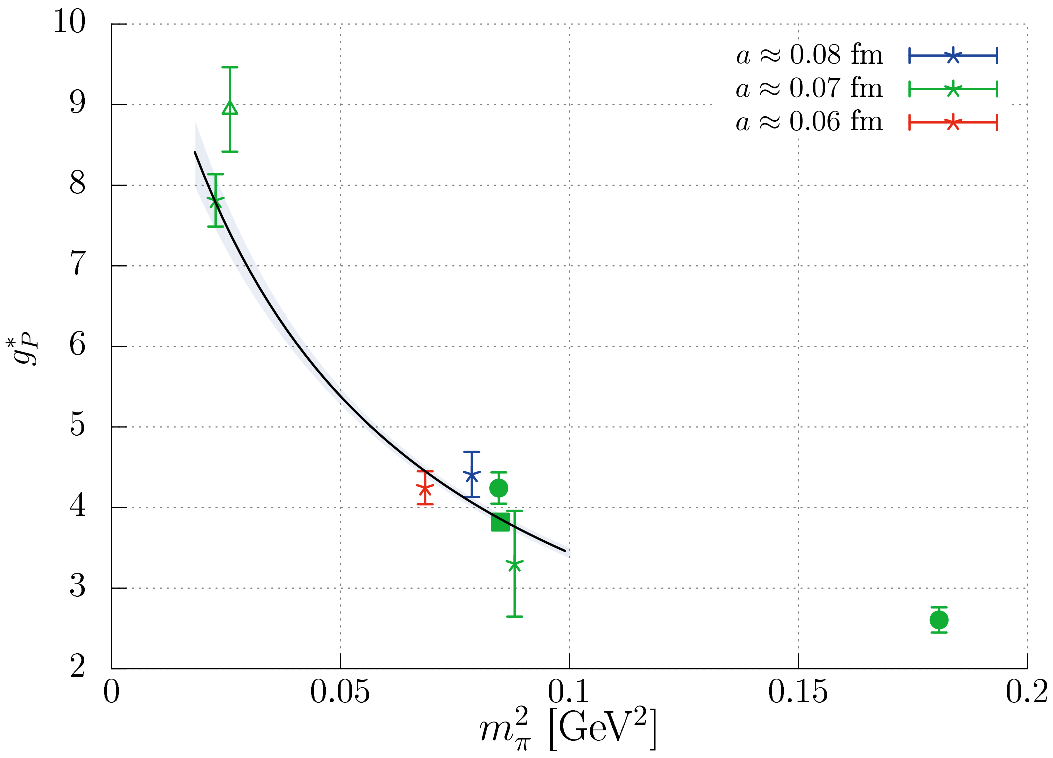


- Is G_P dominated by the pion pole ?



G_p Form Factor and μ-capture





Muon-capture coupling $g_P^* = \frac{m_\mu}{m_N} g_P(0.88 m_\mu^2)$

N_f=2 calculation with Wilson-Clover fermions [G.Bali et al (RQCD), PRD91:054501]

pion-pole extrapolation to extract g_P^*

$$\frac{m_\mu}{m_N} g_P(Q^2) = \frac{b_1}{Q^2 + m_\pi^2} + b_2 + b_3 Q^2$$

Fit & exptrapolation to phys.point

$$g_P^*(m_\pi^2) = \frac{a_1}{a_2 + m_\pi^2} \longrightarrow 8.40(40)$$

Agrees with MuCap result [PRL 110:012504]

$$g_P^* = 8.06(55)$$

Tensor GPDs and Transverse Plane Density

Transversity PDF operator

$\Big\downarrow$ OPE

Off-forward matrix elements of twist-2 operators

Mellin moments of tensor GPDs $H_T, E_T, \tilde{H}_T, \tilde{E}_T$

E.g. for n=1

$$\langle P', S'|\bar{q}\sigma^{\mu\nu}\gamma_5 q|P, S\rangle = \bar{U}_{P',S'}\Big[\sigma^{\mu\nu}\gamma_5\Big(A_{T10}(t) - \frac{t}{2m^2}\tilde{A}_{T10}(t)\Big) + \frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\overline{B}_{T10}(t) - \frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\Delta_\alpha\gamma_5}{2m^2}\tilde{A}_{T10}(t)\Big]u_{P,S}$$

Only n=1,2 moments due to operator mixing in O(4) → H(4)

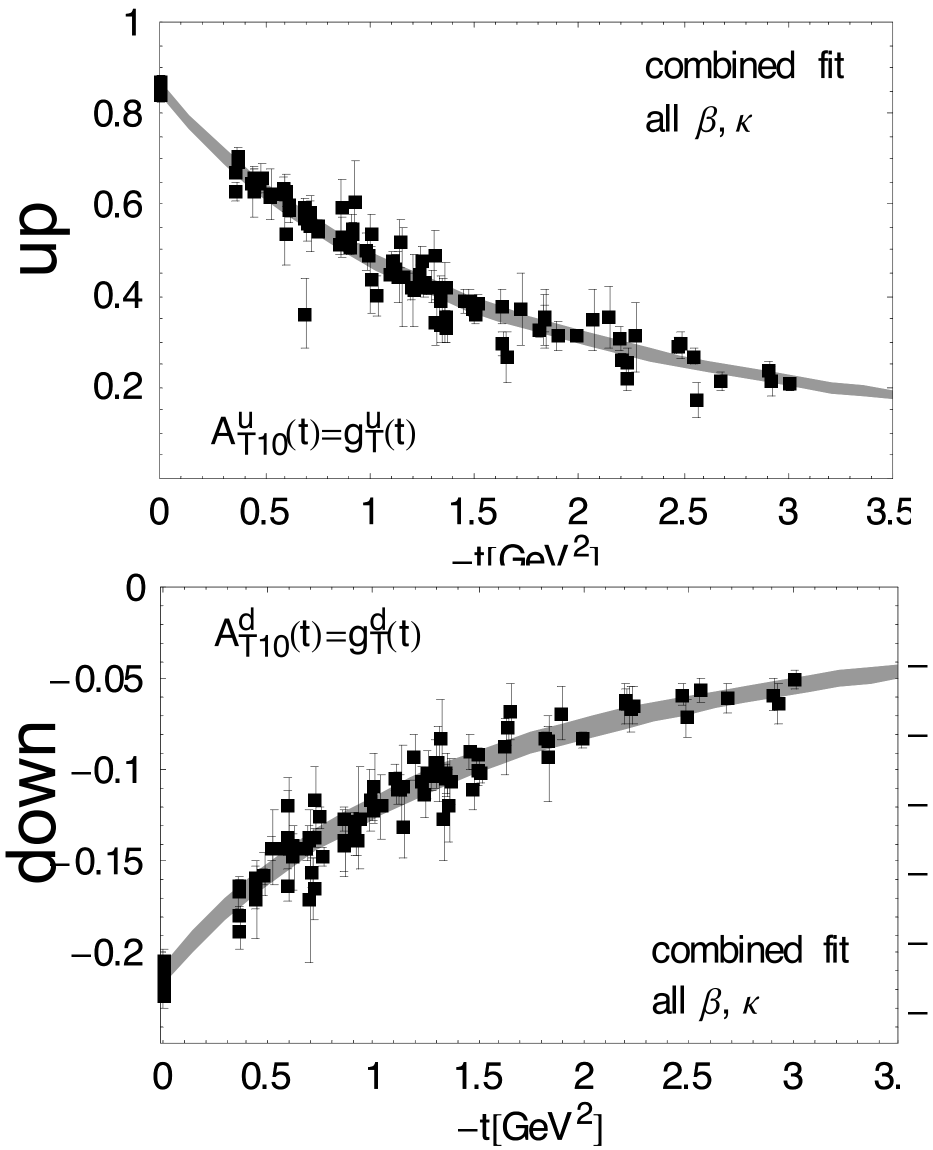
Zero-skewness $\xi = 0 \iff P'^+ = P^+$
Density of T-polarized quarks in the transverse plane

Moments of Tensor GPDs (GFFs)

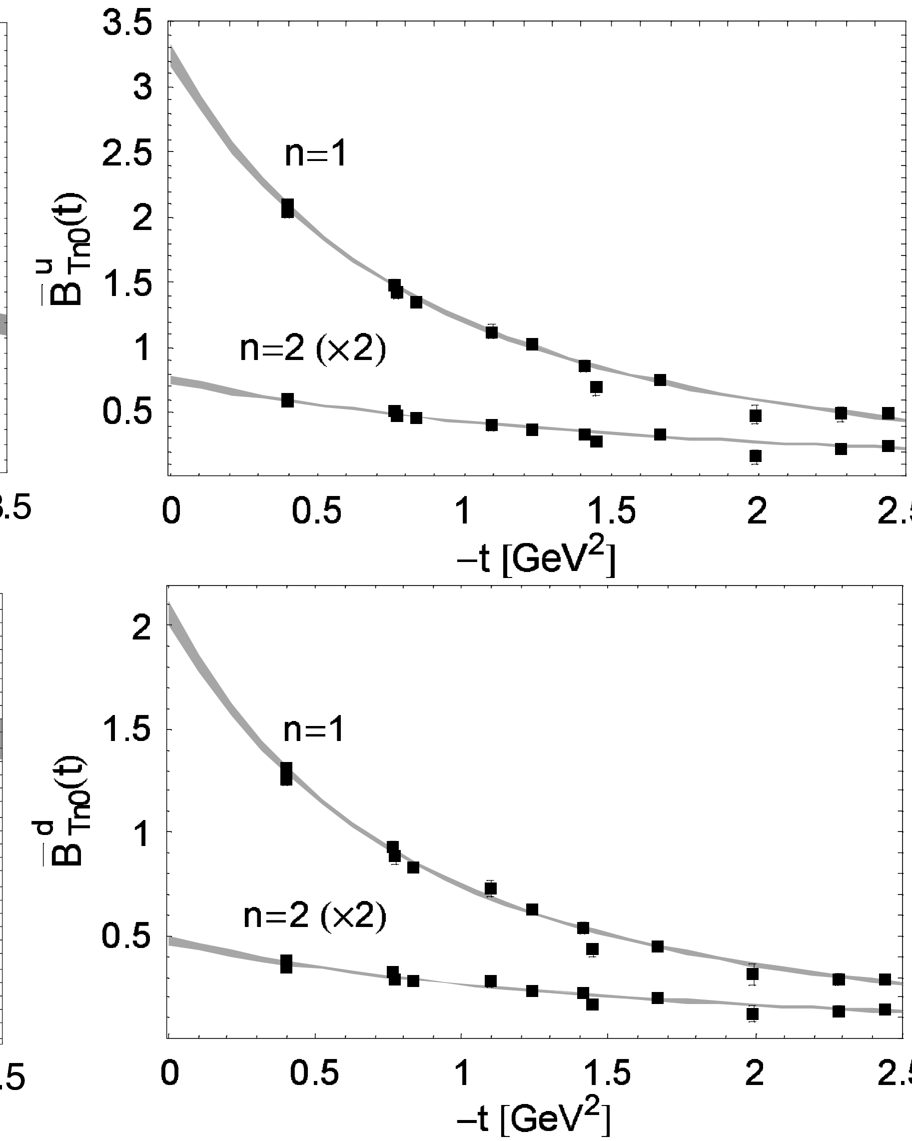
$$\langle P', S' | \bar{q}\sigma^{\mu\nu}\gamma_5 q | P, S\rangle = \bar{U}_{P',S'}\Big[\sigma^{\mu\nu}\gamma_5\Big(\boxed{A_{T10}(t)} - \frac{t}{2m^2}\boxed{\tilde{A}_{T10}(t)}\Big) + \frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\boxed{\overline{B}_{T10}(t)} - \frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\Delta_\alpha\gamma_5}{2m^2}\boxed{\tilde{A}_{T10}(t)}\Big] u_{P,S}$$

$A_{T10}(0) = \langle 1\rangle_{\delta q} = g_T$

$A_{T20}(0) = \langle x\rangle_{\delta q}$



[Gockeler et al, PoS Lattice 2005:55]



[M.Gockeler et al (QCDSF and UKQCD collab.) PRL 98:222001 (2007)]

Transverse Spin Densities

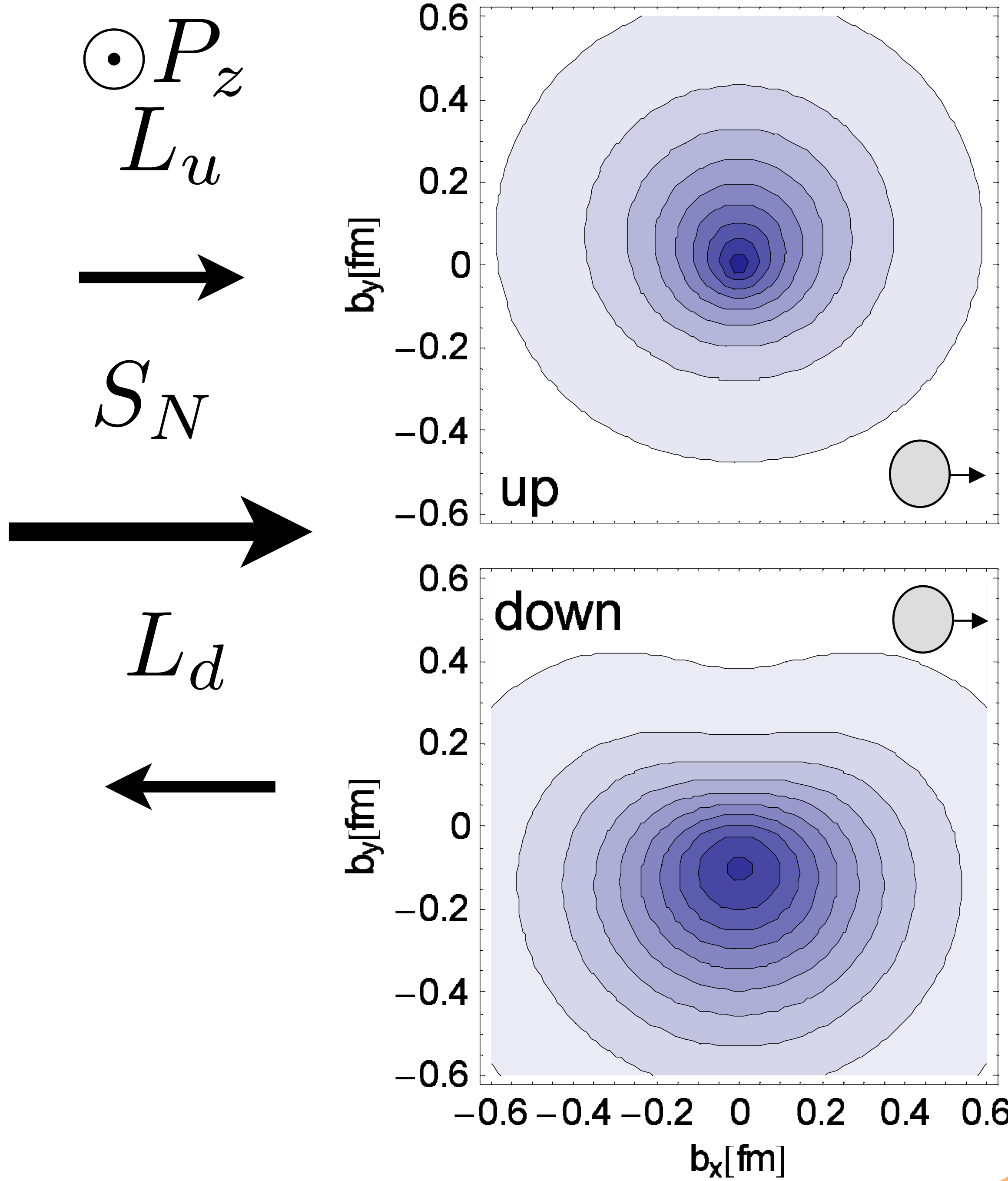
$$\rho(b_\perp^2) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \vec{\Delta}_\perp} \rho(\Delta_\perp^2 = -t)$$

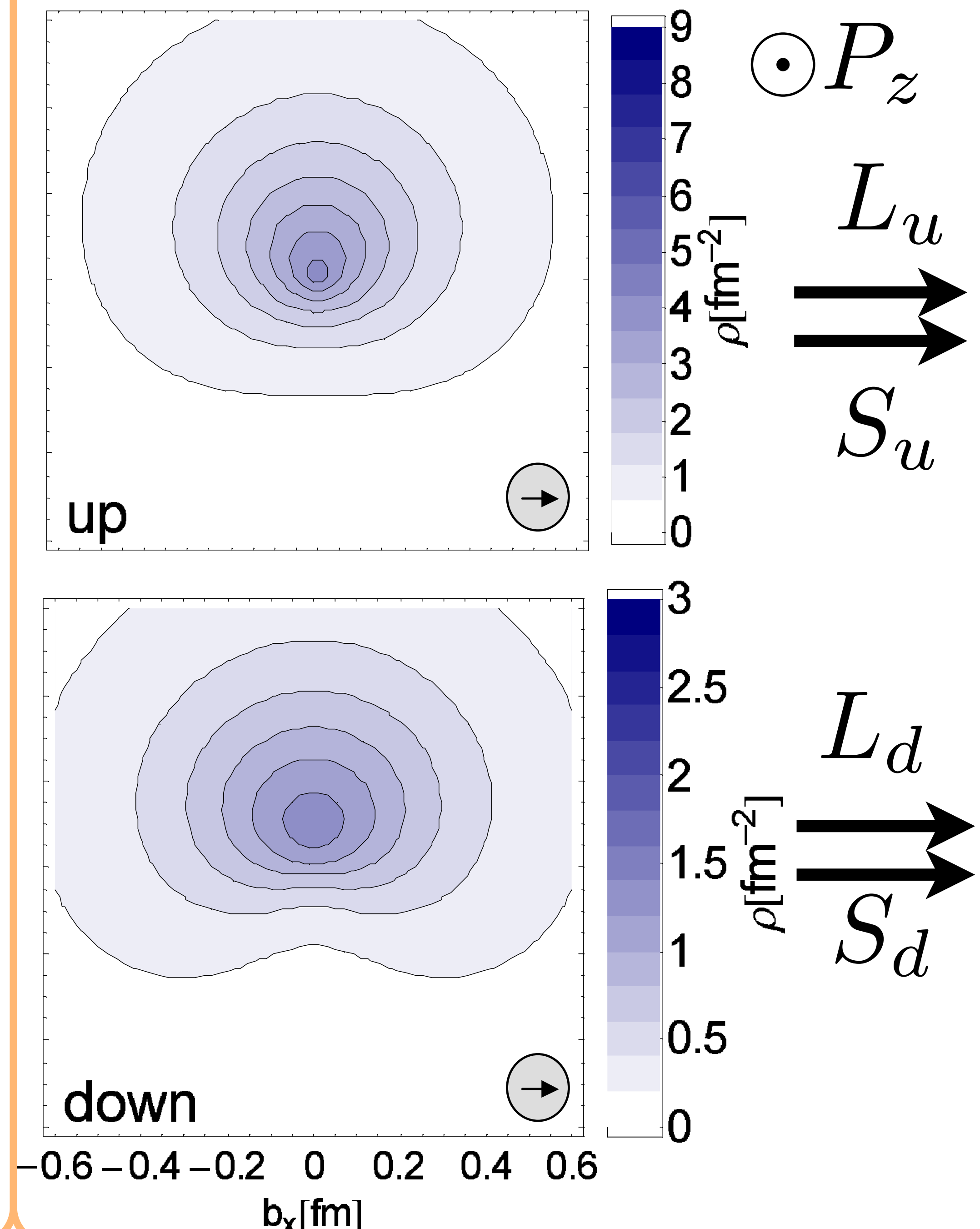
T-polarized nucleon

$$\rho(\vec{b}_\perp)|_{\text{N-pol}} = \frac{1}{2}\left[A_{10}(b_\perp^2) + \frac{(\vec{b}_\perp \times \vec{S}_\perp^N)_z}{m}\frac{\partial}{\partial b_\perp^2} B_{10}(b_\perp^2)\right]$$

T-polarzed quarks

$$\rho(\vec{b}_\perp)|_{\text{q-pol}} = \frac{1}{2}\left[A_{10}(b_\perp^2) + \frac{(\vec{b}_\perp \times \vec{s}_\perp^q)_z}{m}\frac{\partial}{\partial b_\perp^2} \overline{B}_{10}(b_\perp^2)\right]$$





[M.Gockeler et al (QCDSF and UKQCD collab.) PRL 98:222001 (2007)]

Quark Spin & OAM in Longitudinal Picture

Belinfante–Rosenfeld energy-momentum tensor in QCD:

$$T^q_{\mu\nu} = \bar{q}\,\gamma_{\{\mu} \overset{\leftrightarrow}{D}_{\nu\}}\, q \qquad \text{Quarks}$$

$$T^{\text{glue}}_{\mu\nu} = G^a_{\mu\lambda} G^a_{\nu\lambda} - \frac{1}{4}\delta_{\mu\nu}(G_{\mu\nu})^2 \qquad \text{Gluons}$$

Nucleon form factors of the Energy-Momentum tensor <=> n=2 Mellin Moments of GPDs

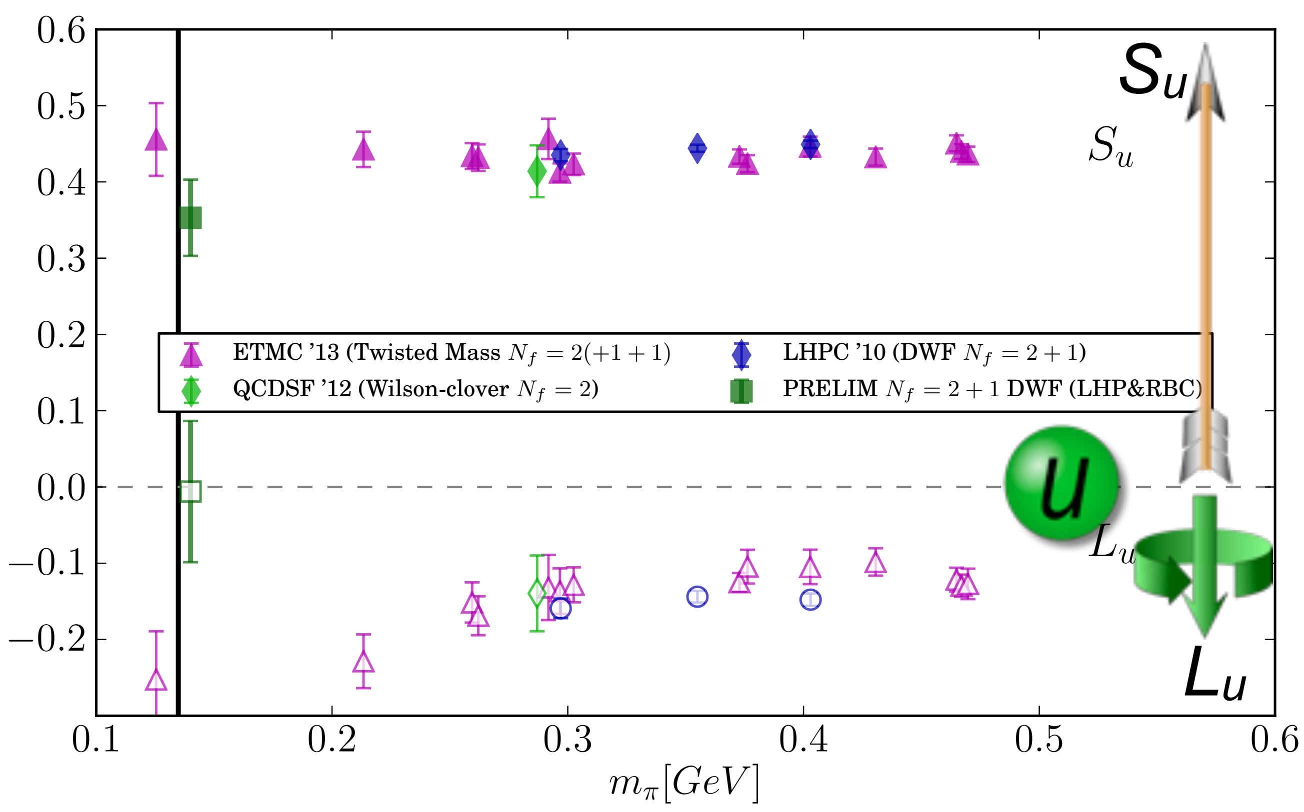
$$\langle N(p+q)|\, T^{q,glue}_{\mu\nu}\, |N(p)\rangle \to \Big\{A_{20},\, B_{20},\, C_{20}\Big\}(Q^2)$$

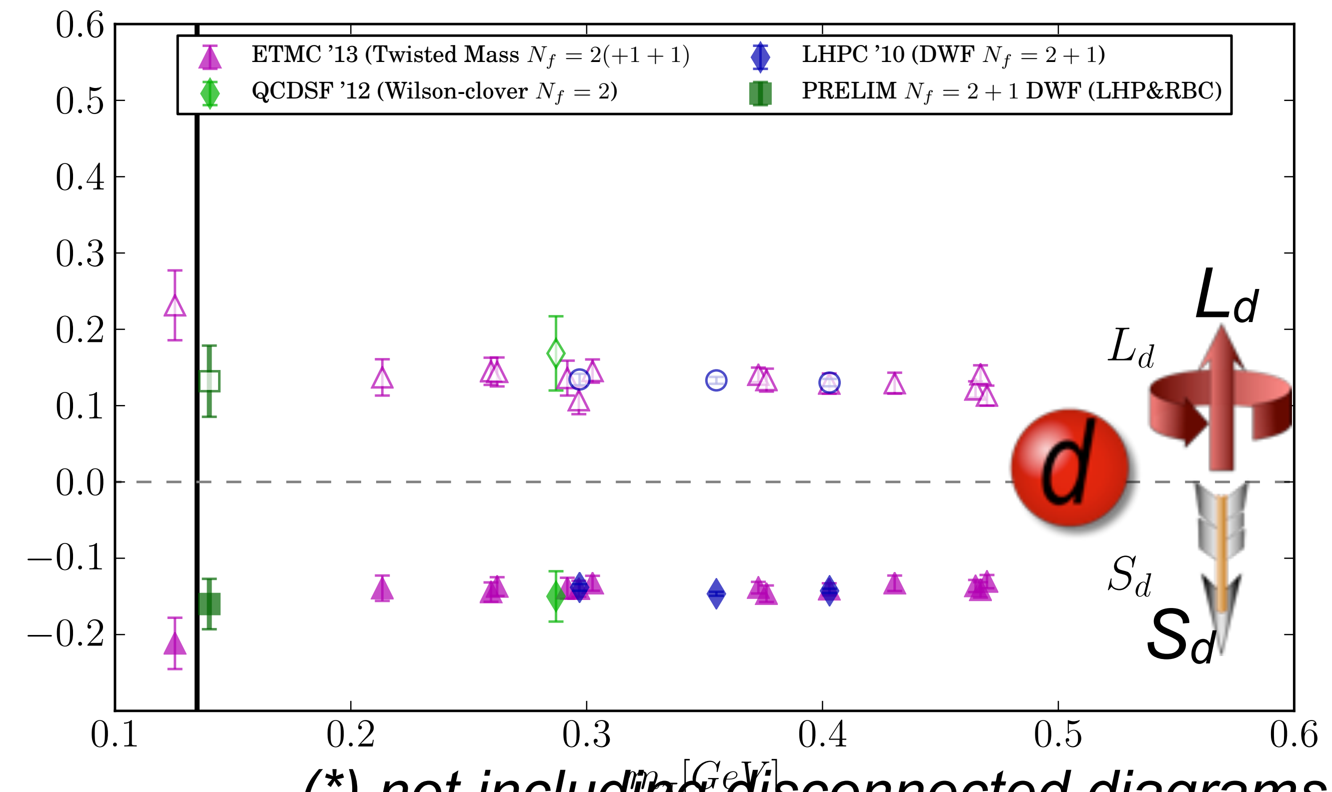
$$A_{20}(Q^2) = \int dx\, x\, H(x,0,Q^2)$$

$$B_{20}(Q^2) = \int dx\, x\, E(x,0,Q^2)$$

Quark & Gluon Angular Momentum [X.Ji, PRL 78:610 (1997)]

$$J_{q,glue} = \frac{1}{2}\Big[A^{q,glue}_{20}(0) + B^{q,glue}_{20}(0)\Big]$$





(*) not including disconnected diagrams

Summary

- Realistic calculations of nucleon structure on a lattice
  multiple lattice groups pursue calculations with physical light quarks

- Nucleon electromagnetic form factors agree with experiment
  lattice QCD results may be important for the "proton radius puzzle"

- Nucleon axial charge and radius : persistent disagreement
  axial charge : 10-15% ; axial radius : x(1/2)

- Lattice calculations provide direct access to transverse quark density distributions and contributions to the proton spin (to be examined by EIC)